

Modeling Housing Prices using a Present Value State Space Model

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Abstract

This paper introduces a State Space approach to explain the dynamics of rent growth, expected returns and Price-Rent ratio in housing markets. According to the present value model, movements in price to rent ratio should be matched by movements in expected returns and expected rent growth. The state space framework assume that both variables follow an autoregressive process of order one. The model is applied to the US and UK housing market, which yields series of the latent variables given the behaviour of the Price-Rent ratio. Resampling techniques and bootstrapped likelihood ratios show that expected returns tend to be highly persistent compared to rent growth. The filtered expected returns is considered in a simple predictability of excess returns model with high statistical predictability evidenced for the UK. Overall, it is found that the present value model tends to have strong statistical predictability in the UK housing markets.

1 Introduction

Over the past decades, housing prices in many countries have witnessed interesting trends such as sharp increases (formation of bubbles) followed by collapse in the aftermath (crash). Moreover, they tend to be very volatile and respond to bank rate cuts and hikes. In both the US and UK, house prices showed massive increases before the financial crisis and collapsed afterwards. It is interesting to note that in both countries, the price to rent ratio has a similar trend. For instance before the financial turmoil, the price to rent ratio showed an increasing trend which implies that the rate of growth in house prices are higher than the rate of growth in rent. A microfounded model which can explain such behaviour is the present value model in standard asset pricing. According to the present value model, the price of an asset is equal to the conditional expectations of discounted future payoffs. Houses are both a consumption and an investment good, and therefore they should be priced in the same way. The price of houses is equal to the conditional expectations of future price sales and rent paid to homeowners.

In this paper, we propose a methodology based on the dynamic form of the present value to estimate the *expected* returns and rent growth rate in a time varying world. The empirical methodology which is used in the literature to model house price-rent ratio makes use of vector autoregression (VAR) where both short-run and long run dynamics can be decomposed, and also to test for the presence of bubbles. Interesting papers in this literature include Engsted (2006), Engsted and Nielsen (2012), Engsted et al (2012), Kivedal (2013), Liu et. al (2014). The main starting point of these models is to assume that the dynamic present value structure holds, and estimate coefficients on price-rent growth, rent growth and returns. These models build on Campbell and Shiller (1988) who used a similar structure to model stock market price bubbles. The VAR models tend to assume that the estimated parameters are fixed over the sample of estimation, and are very reliable towards modelling long run relationships.

In this paper, an alternative model is considered to take into account the time variation of expected returns and expected rent growth rate. The proposed model is a dynamic state space present value, which follows closely from Kojien and Van Binsbergen (2010) in the context of stock prices. The empirical advantage which the model has is that it allows parameters in the discount rate and rent growth process to respond to new information in the market. Such variation ensures that the discount rate is time-varying in the short-run, which may be overlooked using the standard VAR model.

In the state space framework, there are observable (price to rent ratio, and rent growth) and unobservable variables (expected returns and expected rent growth) at time t . The unobserved variables are assumed to have a dynamic structure, which is assumed to be a simple autoregressive process. The unobserved variables are knotted to the measured or observables through the dynamic present value identity. The dynamic present value model (Campbell and Shiller, 1988) stipulates that the price to rent ratio is a loglinear function of expected returns (discount rate) and the rent growth. The model is then optimized through a Kalman Filter which yields a series of long run parameters, which are useful for deriving the series of expected returns and expected dividend growth.

The model helps to explain what moves house prices and above all, provide a predictor variable for housing market returns. What moves house prices? Both fundamentals and speculative channels have been put forward. As pointed out by Cochrane(2011), house prices increases can result from speculative behaviour but also low risk premiums, and it is hard to distinguish between both. However the market discount rate will reflect both channels. The expected returns (discount rate) is made up of the risk-free rate and the risk premium ¹. Lower risk premiums imply a lower expected returns (discount rates), which according to the present value would lead to increases in house prices. The current framework simply assumes that any speculative activity will be reflected in the risk premiums as in Shiller (2014).

The paper is divided as follows. Section 2 illustrates the present value model

¹Alternative present values formula have been considered in the presence of bubbles, where discount rates and bubbles are considered separately.

and the derivation of the state space model. Section 3 discusses the construction of the data set and reports the results from the state space model. Three further tests are discussed namely: simple autoregressive regressions, bootstrapped likelihood ratio tests and the predictability of expected returns. Section 4 concludes.

2 The Present Value Model

The dynamic present value model can be traced back to Campbell and Shiller (1988) who showed that the price dividend ratio can be log-linearized in into expected dividend growth and expected discount rate. In this framework, unexpected stock returns are driven by shocks to expected cash flows and shocks to discount rates. Applications of the present value model to the housing market counterpart are many. However the application of the dynamic framework - explaining movements of the price rent ratio according to changes in rent growth and expected returns is fairly recent.

3 Methodology

In this section, we illustrate the log-linearized present value and follow up with an application of the state space model in the present value. Denoting X_t and P_t as the rent and price at time t , the price the log return on selling a house (r_{t+1}), rent growth (ΔX_{t+1}) and the Price to Rent ratio (PX_t) can be defined as follows:

$$r_{t+1} = \log\left(\frac{P_{t+1} + X_{t+1}}{P_t}\right), \quad (1)$$

$$\Delta x_{t+1} = \log\left(\frac{X_{t+1}}{X_t}\right). \quad (2)$$

$$PX_t = \frac{P_t}{X_t} \quad (3)$$

The log linearized returns equation can be written as follows (See the appendix for details) :

$$r_{t+1} = \kappa + \rho px_{t+1} + \Delta x_{t+1} - px_t, \quad (4)$$

where

$$px_t = E[\log(PX_t)]. \quad (5)$$

The linearization parameter κ is an arbitrary constant defined as:

$$\kappa = \log(1 + \exp(\overline{px})) - \rho\overline{px},$$

where

$$\rho = \frac{\exp(\overline{px})}{1 + \exp(\overline{px})}.$$

A major assumption is the type of process of expected returns and rent growth. The intuitive idea concerning the functional form of the process is that it should be able to illustrate the dynamics of the variables. However, finding a model that resembles the true data generating process is problematic and requires significant data mining. The mean-adjusted conditional expected capital gains returns and rent growth rate are modelled as an autoregressive process of order 1 (AR(1)) process and are shown in equations 6 and 7 respectively :

$$\mu_{t+1} - \delta_0 = \delta_1(\mu_t - \delta_0) + \varepsilon_{t+1}^\mu, \quad (6)$$

$$g_{t+1} - \gamma_0 = \gamma_1(g_t - \gamma_0) + \varepsilon_{t+1}^g, \quad (7)$$

where $\mu_t = E_t(r_{t+1})$ and $g_t = E_t(\Delta x_{t+1})$.

Equation 6 and 7 refer to the mean deviation of the expected returns and expected rent growth rate where δ_0 and γ_0 represents the unconditional mean of the expected returns and rent growth, respectively. δ_1 and γ_1 represents the autoregressive parameters. ε_{t+1}^μ and ε_{t+1}^g represents the shocks to the expected returns and the payoff growth rate processes. The realized growth rate is defined as the expected rent growth rate plus the unobserved rent shock ε_{t+1}^g , where by :

$$\Delta x_{t+1} = g_t + \varepsilon_{t+1}^g. \quad (8)$$

ε_{t+1}^g and g_t are assumed to be orthogonal to each other. $E(\varepsilon_{t+1}^g, g_t) = 0$.

To study the dynamics of the price-rent ratio, (4) may be written with px_t as the subject of the formula:

$$px_t = \kappa + \rho px_{t+1} + \Delta x_{t+1} - r_{t+1}.$$

By replacing lagged iterated values of px_{t+1} in the equation, the process may be written as :

$$\begin{aligned} px_t &= \sum_{i=0}^{\infty} \rho^i \kappa + \rho^\infty px_\infty + \sum_{i=1}^{\infty} \rho^{i-1} (\Delta x_{t+i} - r_{t+i}), \\ px_t &= \frac{\kappa}{1 - \rho} + \rho^\infty px_\infty + \sum_{i=1}^{\infty} \rho^{i-1} (\Delta x_{t+i} - r_{t+i}). \end{aligned} \quad (9)$$

Equation 9 relates the dynamics of the price to payoff ratio to the expectations of future expected returns and expected future payoff growth. Since $\rho < 1$, the infinite price to payoff ratio is negligible. Because $\frac{\kappa}{1-\rho}$ does not depend on time, it will not affect the movement of the price to payoff ratio.

3.1 State Space Model

The construction of the state space model is similar to Kojien and Van Binsbergen (2010). The basic state space model² is made up of a transition equation and a measurement equation. The transition equation models the dynamics of a nonmeasurable or nonobserved variable. The transition variable is a variable which cannot be easily measured in ex ante. The measurement equation defines the relationship between the non-measured variable and the observed variable. In the context of the present value, the non-measurable variables are expected returns and expected rent growth. The observed variables are the price-rent ratio and realized rent growth. Starting with the transition equation, which relates to the non-measurable terms, an autoregressive process of order one is assumed for the demeaned expected dividend growth (6) and conditional expected returns (7).

$$\widehat{g}_{t+1} = \gamma_1 \widehat{g}_t + \varepsilon_{t+1}^g, \quad (10)$$

$$\widehat{\mu}_{t+1} = \delta_1 \widehat{\mu}_t + \varepsilon_{t+1}^\mu. \quad (11)$$

where \widehat{g}_{t+1} and $\widehat{\mu}_{t+1}$ are demeaned expectations of growth and returns. The two measurement equations are given by 12 and 13 :

$$\Delta x_{t+1} = \gamma_0 + \widehat{g}_t + \varepsilon_{t+1}^x, \quad (12)$$

$$px_t = A - B_1 \widehat{\mu}_t + B_2 \widehat{g}_t. \quad (13)$$

(12) states that the realized rent growth is equal to the its expected counterpart plus a stochastic error term ε_{t+1}^x . This is another assumption of our model. (13) is the dynamic discount model where the price to rent ratio is equal a fixed term A and varies with the expected returns and expected rent growth, with parameters B_1 and B_2 . The terms A_1, B_1 and B_2 are defined as follows:

²A more indepth exposure to such models are available in Durbin and Koopman (2012).

$$\begin{aligned}
A &= \frac{\kappa}{1-\rho} + \frac{\gamma_0 - \delta_0}{1-\rho} \\
B_1 &= \frac{1}{1-\rho\delta_1} \\
B_2 &= \frac{1}{1-\rho\gamma_1}
\end{aligned}$$

Equation 11 can be rearranged into 13 such that there are only two measurement equations and only one state equation.

$$\widehat{g}_{t+1} = \gamma_1 \widehat{g}_t + \varepsilon_{t+1}^g, \quad (14)$$

$$\Delta x_{t+1} = \gamma_0 + \widehat{g}_t + \varepsilon_{t+1}^x, \quad (15)$$

$$px_{t+1} = (1 - \delta_1)A - B_2(\gamma_1 - \delta_1)\widehat{g}_t + \delta_1 px_t - B_1 \varepsilon_{t+1}^\mu + B_2 \varepsilon_{t+1}^g. \quad (16)$$

Equation 14 defines the transition equation. Equation 15 and 16 relate to the measurement equation. These equations can be rearranged into a state space form, as illustrated in the next section. Since all equations are linear, we can implement the Kalman Filter and obtain the maximum likelihood to maximise the following vector of parameters:

$$\Theta = (\gamma_0, \delta_0, \gamma_1, \delta_1, \sigma_g, \sigma_\mu, \sigma_x, \rho_{g\mu}, \rho_{gx}, \rho_{\mu x})$$

3.2 Kalman Filter

In this section, we describe the Kalman filter procedure. From the paper, there are two measurement equation and one transition equation. Equations 14, 15 and 16 can be written in this form:

$$G_t = FG_{t-1} + R\varepsilon_t,$$

$$Y_t = M_0 + M_1 Y_{t-1} + M_2 X_t,$$

$$\text{where } Y_t = \begin{bmatrix} \Delta x_t \\ px_t \end{bmatrix}.$$

The variables of the transition equation are X_t and ε_{t+1}^x and consists of the following elements:

$$G_t = \begin{bmatrix} \widehat{g}_{t-1} \\ \varepsilon_t^x \\ \varepsilon_t^g \\ \varepsilon_t^\mu \end{bmatrix}, \quad F = \begin{bmatrix} \gamma_1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \varepsilon_{t+1}^x = \begin{bmatrix} \varepsilon_{t+1}^x \\ \varepsilon_{t+1}^g \\ \varepsilon_{t+1}^\mu \end{bmatrix}.$$

The parameters of the measurement equation include parameters of the net present value model to be estimated. These are defined as :

$$M_0 = \begin{bmatrix} \gamma_0 \\ (1 - \delta_1) * A \end{bmatrix} \quad M_1 = \begin{bmatrix} 0 & 0 \\ 0 & \delta_1 \end{bmatrix} \quad M_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ B_2(\gamma_1 - \delta_1) & 0 & B_2 & -B_1 \end{bmatrix}$$

The variance covariance matrix from the state space model is given by :

$$\Sigma = \text{var} \begin{bmatrix} \varepsilon_{t+1}^g \\ \varepsilon_{t+1}^\mu \\ \varepsilon_{t+1}^d \end{bmatrix} = \begin{bmatrix} \sigma_g^2 & \sigma_{g\mu} & \sigma_{gd} \\ \sigma_{g\mu} & \sigma_\mu^2 & \sigma_{x\mu} \\ \sigma_{gd} & \sigma_{x\mu} & \sigma_x^2 \end{bmatrix}.$$

The Kalman Filter procedure is given by the following equations :

$$\begin{aligned} G_{0|0} &= E[G_0], \\ P_{0|0} &= E[G_0 G_0'], \\ G_{t|t-1} &= G G_{t-1|t-1}, \\ P_{t|t-1} &= F P_{t-1|t-1} F' + R \Sigma R', \\ \eta_t &= Y_t - M_0 - M_1 Y_{t-1} - M_2 G_{t|t-1}, \\ S_t &= M_2 P_{t|t-1} M_2', \\ K_t &= P_{t|t-1} M_2' S_t^{-1}, \\ X_{t|t} &= X_{t|t-1} + K_t \eta_t, \\ P_{t|t} &= (I - K_t M_2) P_{t|t-1}. \end{aligned}$$

The likelihood function which is maximised over the set of parameters Θ is given by :

$$L = -\sum_{t=1}^T \log(\det(S_t)) - \sum_{t=1}^T \eta_t' S_t^{-1} \eta_t.$$

4 Data and Results

The model is applied to the US and UK housing markets using quarterly data. Prior to the estimation of the state space model, explanation of the construction of the three indices is required. For the US, two indices are used - the Case-Shiller files and the FHFA (Federal Housing Finance Agency). In the case of the UK, the price-rent ratio is not available. We use the price-rent index from the OECD to infer rent based on ratios of housing prices (available from Nationwide) and Rent (From the Valuation Office Agency). Before we can use the crude figures, the data is deflated according to the consumer price index

(excluding shelter), both time series are available from the FRED database. The time span for the series are from 1970Q2 -2013Q4 and 1960Q2-2014Q1 for the UK and the US market respectively.

The results from the optimization is illustrated in table 1.

Sample	Case- Shiller (US)		FHFA (US)		UK	
	Θ	std errors.	Θ	std errors.	Θ	std errors.
γ_0	0.024	0.001	0.003	0.001	0.003	0.004
δ_0	0.014	0.001	0.021	0.038	0.015	0.004
γ_1	0.580	0.152	0.666	0.196	0.446	0.059
δ_1	0.981	0.002	0.999	0.001	0.951	0.020
σ_g	0.004	0.001	0.003	0.001	0.028	0.003
σ_x	0.001	0.001	0.001	0.001	0.004	0.002
σ_μ	0.003	0.001	0.004	0.001	0.026	0.002
$\rho_{g\mu}$	0.492	0.078	0.721	0.088	0.252	0.032
$\rho_{\mu x}$	-0.083	0.182	-0.231	0.098	0.121	0.051
Likelihood	-1370.8		-1457		-542.42	

Table 1: Estimation of Parameters of Present Value Model. The different parameters making up the present value are reported in the rows: namely the intercept term and the autoregressive parameters of expected returns and expected rent growth. The standard deviation and correlation of realized rent growth, expected returns and expected rent growth are reported next. The likelihood ratio from the three indices are also reported. The standard errors from each estimate is also reported. The standard errors are computed from the Hessian Matrix in the optimizer.

From table 1, the rate of mean rent growth between the two US indices differ across the sample (0.024 and 0.003). Average expected returns tend to be different across both samples. It is interesting to note that the expected returns from house sales are roughly similar for both markets (0.014%–0.021%). It shows that the discount factor in the long run is roughly similar in both countries. It equals to the quarterly returns on houses in both countries. In both cases, the unconditional expected returns is higher than the unconditional rent growth. The dynamics of the growth rate is very interesting for they are very persistent in the US. The autoregressive term lies between 0.58 –0.67 in the US. The persistence in rent growth is high, and less than market dividends as evidenced by Kojien and Van Binbergen (2010). This could be due to wealth effects as has been demonstrated in this literature. Sustained cyclical trends pressure rent growth which hence assumes this persistence. A similar finding is found in the UK, where the persistence is 0.45. This figure reflects the interest rate transmission mechanism from the bank rate to the whole of the economy, as has been common in the UK system. However, in all three cases, the standard errors

are very high, and higher than the other parameters in general. This suggests that there may be a lot of volatility surrounding periods of turmoil, which in this literature may be thought as either different regimes or breakpoints³.

A very interesting point worth mentioning is the relatively high persistence in discount rates in both US and UK. Near unit root behaviour is noticed, especially in the case of US. Such findings are common in valuation ratios such as the price dividend and price to rent ratio. However, this is the first time that these findings are witnessed in the discount factor. Regarding the time series properties of both series, it is very important to remark that expected returns are more persistent than rent growth. Habit formation arguments can be used to supplement this idea. An alternative reason which is very important to mention in this framework is the possible specification of the model. The model is currently considering a simple autoregressive process of order one. More complicated structures can also be thought of such ARFIMA models, which may account for long memory in the line of the aggregation phenomenon (Granger 1980), learning theory (Chevillon and Mavroeidis 2013) and unbalanced regression models (Golinski et. al. 2015).

Shocks to the rent market appear to be very small as noted by σ_d hovering around 0. The standard error for the expected returns tend to be higher in the case of the UK housing market compared to other markets. Worthy of notice is that the correlation between expected rent growth and expected returns is higher in the US than in the UK - it would mean that there is a relatively similar pattern in buying of houses and rent. Such a pattern is weaker for the UK market. Based on the state space model, it may be inferred that expected return on sale of houses tend to be higher than the growth in rent. The pattern between observed rent growth and expected returns are relatively weak in most cases. In the Case-Shiller index, it is negative and close to zero. In the case of the UK, it is positive and fairly high. Time series plots of expected returns and expected rent growth are shown in Figures 1 and 2 respectively:

The expected returns plot (Figure 1) shows cyclical trends which can be matched with business cycles. Interestingly in the US, prior to 2000, the cycles do not appear to be highly volatile - expected returns is very persistent though. It is very noticeable that the expected returns from the UK market tend to be very volatile. Expected returns were very high at the start of the sample until 1974, where there was a massive fall in house prices, coinciding with the 74 oil shock. The expected returns is still positive but then becomes negative in 1981 which again coincides with another economic shock. Expected Returns go through a boom and bust cycle of approximately 5 years. Prior to the financial

³Structural breaks represent a very interesting challenge in this literature. However in a time-varying framework such as the Kalman Filter, breaks have little incidence as parameters in the transition equation are switching over time.



Figure 1: Expected Returns: The graph illustrates the expected returns on houses over time. The red, blue and green line relate to the UK, US-Case-Shiller and US-FHFA.

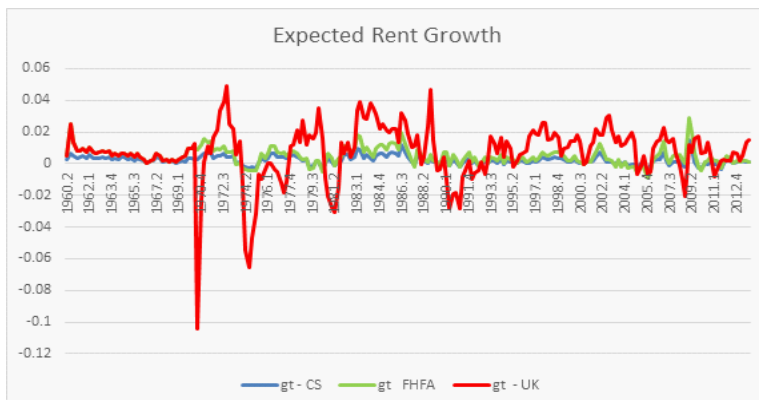


Figure 2: Rent Growth: The figure illustrates the expected rent growth in UK, and US. The graph illustrates the expected rent growth over time. The red, blue and green line relate to the UK, US-Case-Shiller and US-FHFA.

crash, the expected returns on the housing market was extremely high, especially in the UK. In the immediate periods before the crash, expected returns tend to fall - The Case-Shiller Index tends to show a steeper fall. All this evidence tend to suggest that the housing crash could also be explained by the lower expected returns in the market prior to the crash. The timings of the dip in expected returns tend to differ, as the UK market tends to fall after the housing market collapsed in the US.

The movement of the expected rent growth are very different across the UK and the US. Figure 2 tends to show that there is a higher volatility in expected rent growth in the UK than in the US. There are indeed variations which coincides with the business cycles but it is to be noted than in generally the magnitude of the increases are lower. After a period of instability back in the early 1980's, the rent growth tends to settle for some regular patterns of ups and downs in the market. Both the Case-Shiller and the FHFA tends to show dampened fluctuations, with much less volatility than in the UK case. Statistical properties from the time series variables are given in 2.

Properties of Filtered Returns			
	CS	FHFA	UK
Mean	0.015	0.014	0.015
Std. Deviation	0.004	0.002	0.015
Skewness	-1.532	-0.891	1.058
Kurtosis	5.475	3.742	3.812
ADF	-2.227	2.120	-2.872
Properties of Filtered Rent Growth			
Mean	0.002	0.002	0.003
Std. Deviation	0.003	0.003	0.018
Skewness	0.721	1.394	-2.049
Kurtosis	5.231	8.822	12.50
ADF	-4.619	-4.267	-4.652

Table 2: Summary Statistics of Expected Returns. The table shows moments and tests of normality of expected returns from both ratios in periods 1927-2012 and 1947-2012.

The summary statistics offer interesting insights on the distribution and stationarity of the series. It can be seen that the expected returns in the US tends to be left-skewed which can be easily associated with housing crashes. On the other hand housing crashes have not been yet been witnessed on a large scale in the UK. It is also interesting to note that the expected returns in the US tend to harbour a unit root, as shown by the ADF test. On the other hand, tin the UK, the unit root hypothesis is only rejected at the 10 %. On the

other hand, rent growth is stationary all across the different markets. It can easily be seen that there is an distribution for the UK which is left skewed and has fat tails. Tests of normality (not reported) tend to reject normality in all variables. It is worth noting that the actual observed data (House Prices and Rent growth) between the Case Shiller and FHFA tend to differ only after 2000, which consists of 32 % of the sample. However the correlation in rent growth and expected returns are respectively 0.87 and 0.98, which implies that the state space offers a nice and parsimonious relationship which can be adapted for the long run equilibrium. No evidence of correlation is found between the UK and US markets in terms of rent growth. However, there is a relatively strong positive correlation in the case of expected returns (0.52 with the Case-Shiller and 0.58 with the FHFA).

4.1 Resampling

The results from the optimised parameters illustrated in table 1 requires inserting initial values. These initial values are set based on economic intuition and guesswork and as such may undermine the validity of the optimised models. As a misspecification test, we consider a simple resampling strategy, similar to the bootstrap, where the vector of initial values are randomly generated. The boundaries for generation of initial values for the intercept terms (γ_0, δ_0) , slope coefficients (γ_1, δ_1) , standard errors $(\sigma_g, \sigma_x, \sigma_\mu)$ and correlation coefficients $(\rho_{\mu x}$ and $\rho_{\mu g})$ were $[-0.5, 1]$, $(-1, 1)$, $(0, 0.6]$ and $[-1, 1]$ respectively. The number of draws for the resampled data was 200. The empirical distributions for the US using the FHPA dataset are illustrated in 3.

It is interesting to note that the empirical distributions are not normally distribution which invalidates inference on standard t-distributions. Considerable skewness is noticed in the case of $\gamma_0, \delta_0, \gamma_1$ and δ_1 . It is interesting to note that the estimated values tend to be close to the peak of the distributions most of times. It is interesting to note that the empirical distribution for the δ_1 suggests that rent growth should be more persistent. The intercept term for the expected returns and the correlation between realized rent growth and expected rent growth tends to be off the simulated means.

4.2 Univariate Autoregressive Models

In this section, expected returns is modeled by a univariate specification (AR(1)) instead of present value framework. It should be clarified that the expected returns from the state space and its simple univariate model may be different. This is simply the state space controls for the dynamics of the expected returns and expected rent growth at the same time through the correlation coefficient $\rho_{g\mu}$, and to a lesser extent ρ_{gx} . If the correlation between expected dividend growth and expected returns is equal to zero, it can be easily shown that the state space model reduces to two strictly uncorrelated univariate AR(1) processes.

The objective of running an AR(1) process is to convince for the robustness of the model and to test whether there are other underlying problems besides the unit root, evidenced in the previous section. Other interesting statistics we report are tests on serial correlation on the error term, functional form and test for structural break. This is shown in table 3:

	$\mu_{t+1,CS}$	$\mu_{t+1,FHFA}$	$\mu_{t+1,UK}$
Intercept	0.012	0.011	0.01
AR(1)	0.99	0.99	0.96
R^2	0.98	0.98	0.95
Error Variance	0.001	0.001	0.003
Diagnostic Tests			
Serial Correlation (p-val)	0	0	0.06
Functional Form (p-val)	0.76	0.60	0.97
Andrews Break Test (p-val)	0	0.01	1

Table 3: Univariate Models of Expected Returns. An AR(1) is fitted for each series of expected returns. The table shows the estimate of the intercept and autoregressive coefficient. The goodness of fit is represented by the R-squared and the Error Variance. Diagnostic tests on Serial Correlation, Functional Form and Structural Stability are also reported.

Table (3) illustrates the results from running an autoregressive process of order one to the filtered series. It is interesting to note that the expected returns from the simple univariate model tends to replicate the same performance as the present value model. The autoregressive parameter was found to be statistically significant. With regards to the intercept, the standard errors did not reject the null only at 10 % level. Bootstrapped Sieve-AR residuals were also computed which showed that the intercept were statistically significant. The goodness of fit tends to be very good, though spuriousness from the unit root needs to be taken into consideration. In all three cases, the errors are serially correlated (test conducted with 4 lags). The Ramsey Reset tests rejects linearity of variables in all three variables. It is interesting to note that the Andrews test for structural breaks tend to reject the presence of structural breaks in both US indices. However, structural breaks are very much present in the UK.

4.3 Bootstrapped Likelihood Tests

In the following section, we test whether the filtered expected returns have short range dependence, which includes testing whether the autoregressive parameter is different from zero within the states space model. We also test for time variation in expected returns, which imply that the under the null hypothesis, the autoregressive parameter and the standard error of the transition equation shock are equal to zero. The problem with testing these hypotheses using the conventional Wald tests is that the alternative might fall under an open set,

which hence requires using the bootstrap to find the distribution under the null and the alternative. The tests involve computing the likelihood ratio under alternative (L_1) and the null (L_0). The likelihood ratio test is computed as follows:

$$LR = 2(L_1 - L_0)$$

The likelihood ratio is distributed as $\chi^2(k)$ where k represents the number of restrictions or constrained parameters. The results for the null hypothesis is given in table 4:

Likelihood Ratio			
	$\mu_{t+1,CS}$	$\mu_{t+1,FHFA}$	$\mu_{t+1,UK}$
Persistence Test:			
$H_0 : \delta_1 = 0$	524	481	82
Time Variation Test			
$H_0 : \delta_1 = \sigma_\mu = 0$	560	512	82
Persistence Test:	$g_{t+1,CS}$	$g_{t+1,FHFA}$	$g_{t+1,Uk}$
$H_0 : \gamma_1 = 0$	16	87	11
Time Variation Test			
$H_0 : \gamma_1 = \sigma_\gamma = 0$	86	61	46

Table 4: Tests of persistence and time variation. The table illustrates the bootstrapped Likelihood Ratio for the null hypothesis of no persistence in expected returns and no time variation.

The results show that the null hypothesis of persistence and time variation are both rejected at the 5 % level from a Chi-square distribution. This finding has been explored in the asset pricing literature through various studies including Kojien and Van Binsebergen (2011), Rychkov (2012) and Campbell and Cochrane (1999). From simply comparing the likelihood ratio, it appears that there is much more persistence in the expected returns from the price dividend for the period 1927-2012. For the latter periods, it appears that the price rent tend to display more persistence. The same findings apply to the time variation of expected returns.

4.4 Predictability of Excess returns on Housing Market

This section investigates whether excess returns on the housing market can be predicted by lagged expected returns. The model is illustrated as follows:

$$r_{h,t+1} - r_{f,t+1} = \alpha + \beta\mu_t + v_{t+1} \tag{17}$$

Equation 17 illustrates the excess returns from the housing market (minus the riskless asset proxied here by 3 month Treasury Bill Rate) is being predicted

by the expected returns. The error terms is assumed to be well-behaved. It must be noted that equation is one of the simplest predictive ability regression. This may be improved by considering long range horizon regressions and also invoking out of sample predictability. Moreover VAR models can be considered to include feedback effects and consideration of other macroeconomic variables. However, it may be a different study in itself. The results are reported in

	Likelihood Ratio	
	$r_{h,t+1} - r_{f,t+1}(US)$	$r_{h,t+1} - r_{f,t+1}(UK)$
Intercept	-0.016	-0.047
Expected Returns	1.33	5.839
R-squared	0.019	0.40

Table 5: Tests of insample predictability. The shows the estimated parameters from regression excess returns on lagged expected returns..

The table shows the insample predictability from the two regression models. The table shows that lagged expected returns has predictive ability on the quarterly excess returns in the case of UK. The R-squared is 40 %, which is very strong in the predictability literature. However in the case of the US the predictability is 1.9 %, which is very low.

5 Conclusion

This paper sheds light on the dynamic modeling of expected returns, expected rent growth from the state space approach for the UK and US. For both countries, expected returns is found to be persistent and vary over time as proved by the bootstrapped likelihood ratio test. The expected returns tends to show characteristics of persistence similar to the price to rent ratio. The distributional properties show that expected returns are nonnormal and different markets tend to observe different skewness and tail behaviour. Univariate autoregression models were used as a further test to reinforce the results of high persistence in expected returns. It was found to be high in the autoregressive parameter, with the volatility parameters similar to the state space model. If there is any effect from the present value variables, it is only reinforcing the persistence in expected returns. The state space model is robust across both samples, and across the two different measures of price to rent ratio in the US. Another time series property uncovered in this paper is that expected returns tend to fail the standard stationarity test and nonstationarity is rejected at high levels. It is also worth mentioning that structural breaks were found in the case of the UK but not in the US.

At the outset, it is important to mention the main assumption regarding the dynamics of expected returns and expected rent growth. They both follow an AR(1) process. It is worth mentioning if this is not true based on the unknown

data generating process, sizable distortions may be observed in the parameters. However, this is a further topic of research in itself. A simple application of the filtered series of expected returns is provided. The lagged of expected returns is assessed as a potential predictor of quarterly excess returns. It was noted that although predictability was very strong in the case of the UK, the statistical evidence is too weak for the US. This study opens various avenues for further studies in the area of predictability in terms of assessing the performance of VAR models against state space models. Moreover, the expected returns series may be used as an indicator of housing market bubbles.

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6 Appendix

6.1 Dynamic Present Value

Equations 1 , 2 and 3 are shown again:

$$r_t = \log\left(\frac{P_{t+1} + X_{t+1}}{P_t}\right) \quad (18)$$

$$PX_t = \frac{P_t}{X_t} \quad (19)$$

$$\Delta x_{t+1} = \log\left(\frac{X_{t+1}}{X_t}\right) \quad (20)$$

The return process can be written as

$$\begin{aligned} r_t &= \log\left(\left(\frac{X_{t+1} + X_{t+1}}{P_t}\right) \cdot \frac{X_t}{X_t} \cdot \frac{X_{t+1}}{X_{t+1}}\right) \\ &= \log\left(\left(\frac{P_{t+1}X_t + X_{t+1}X_t}{P_tX_{t+1}}\right) \cdot \frac{X_{t+1}}{X_t}\right) \end{aligned} \quad (21)$$

$$\log\left(\left(\frac{X_t}{P_t} \cdot \frac{P_{t+1}}{X_{t+1}} + \frac{X_t}{P_t}\right) \cdot \frac{X_{t+1}}{X_t}\right) \quad (22)$$

$$\log\left(\left(\frac{P_{t+1}}{X_{t+1}} + 1\right) \frac{X_{t+1}}{X_t} \cdot \frac{X_t}{P_t}\right) \quad (23)$$

$$\log(1 + e^{px_{t+1}}) + \Delta x_{t+1} - px_t \quad (24)$$

Assuming the log linearization of Campbell and Shiller (1988) the returns can be written as

$$r_t \simeq \log((1 + e^{px_{t+1}})) + \frac{\exp(\overline{px})}{1 + \exp(\overline{px})} + \Delta x_{t+1} - px_t$$

$$r_t = \kappa + \rho px_{t+1} + \Delta x_{t+1} - px_t$$

where $\kappa = \log((1 + e^{px_{t+1}}))$ and $\rho = \frac{\exp(\overline{px})}{1 + \exp(\overline{px})}$

Hence,

$$px_t = \kappa + \rho px_{t+1} + \Delta x_{t+1} - r_{t+1}$$

6.2 Resampled Distributions

The resampled distribution for the US data are as follows:

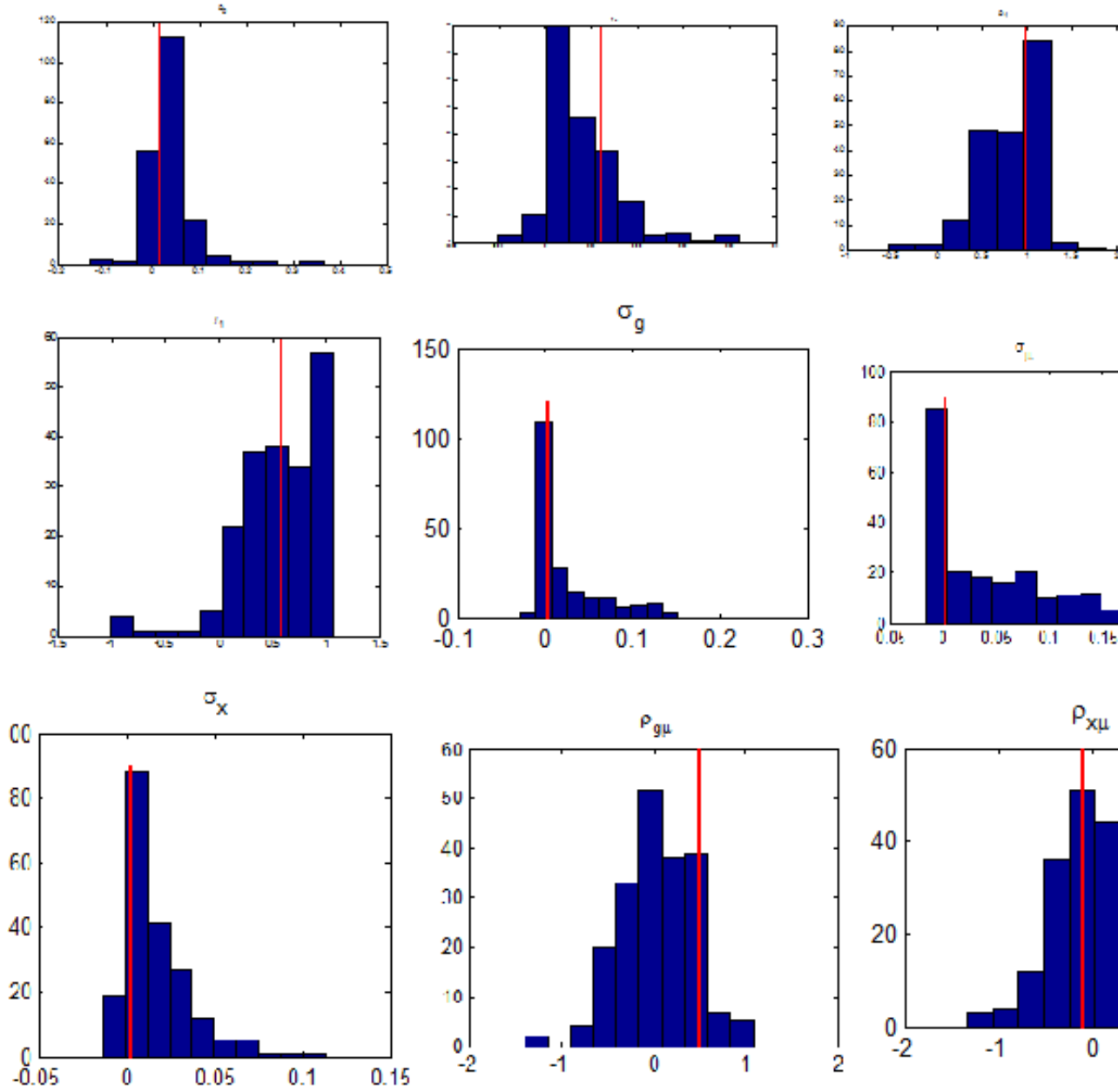


Figure 3: Empirical Distribution of optimal parameters for US-FHPA dataset