Competition among the Big and the Small with Different Product Substitution*

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Abstract

This paper employs a linear monopolistic competition model to revisit the impacts of the large firm's entry in the context of a mixed market structure where large and small firms coexist. In our model, the large firm determines both the range of product varieties and the quantity of each variety while the small firm produces only one variety of output and freely enters the market. We argue that the different substitutablitities between the products of large firms and those of small firms play a critical role in determining the impacts exerted by the entry of the large firm. Specifically, if the products of large firms and those of small firms have the same substitutability, the entry of the large firm has no impact on the incumbent large firms' variety choice, output of each variety or prices. If the products of large firms and those of small firms have different substitutablitities, however, the entry of the large firm may cause a rise or a fall of the incumbent large firms' output, price and profit, depending on the comparison of the substitutability within large firms and small firms and the substitutability across these two types of firms. In particular, the entry of the large firm may reduce the profits of incumbent large firms and social welfare. Our welfare analysis implies that it may be reasonable for the government to conduct the regulation against the entry of large firms in local markets.

Keywords: big firms, small firms, product substitution, entry behavior, market impacts

JEL classification: D21, D43, L11, L13

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1 Introduction

Many industries consist of a small number of large firms (usually the firms with brand fame) and a large number of small firms (usually the firms without fame), such as bookstores, retailing and information technology industries. The large firms are usually influential in the market, able to affect the prices of the products and bargain with local government, while the small firms' impacts are negligible. This invites us to ask whether the standard imperfect competition theory still works to describe the market where the large and small firms coexist. Unfortunately, neither oligopoly nor monopolistic competition can completely capture such a market structure. Thus, it is worth investigating firms' behavior and social welfare in this market structure. Moreover, some governments also enforce industry laws to implicitly or explicitly restrict the entry of large firms into local markets. However, it is worth examining whether the barriers to the large firms' entry set by the government has a sound theoretical ground.

Shimomura and Thisse (2012) provides a pioneering study on the above two issues. Based on a CES utility framework, they combine large firms with a positive measure and small firms with zero measure so that the price elasticity of demand differs between large and small firms. Shimomura and Thisse (2012) build a bridge connecting oligopoly and monopolistic competition. With the presence of income effects, their theoretical model implies that in this mixed market structure, the entry of large firms has no impact on small firms' output, increases the incumbent large firms' profit, and reduces the market price level facing consumers. Furthermore, they find that the entry of large firms is welfare-improving, theoretically casting doubt on governments' restrictions on the large firm's entry. How robust are Shimomura and Thisse (2012)'s results?

The present paper revisits the impacts of large firms' entry by using a linear monopolistic competition model where the income effect is eliminated. Our model is characterized by the following three features: i) each large firm is multiproduct, producing a non-negligible range of product varieties [1]; ii) each small firm produces only one variety with a negligible amount, but can freely enter or exit the market, and more importantly, iii) the substitutability of products remains to be the same within large firms and small firms, but may be different across these two types of firms[2]. We consider a two-stage game. In stage 1, each large firm determines the range of product varieties, and small firms decide whether to enter the market. In stage 2, all the firms produce and compete with each other. We find that the different substitutablitities of products between large and small firms play a critical role in determining the impacts exerted by the entry of the large firm. When the substitutability of products is the same across large and small firms, the entry of large firms has no impact

[1] Parenti (2012) also distinguishes the big firm from small firm by assuming that the big firm is featured with mutliproduct production and the small one is featured with single-product production.

[2] Our analytical framework is similar to Singh and Vives (1984), Ottaviano et al, (2002) and Ottaviano and Thisse (2011), but is distinct from them in the above-mentioned three respects.

on the incumbent large firms' variety choice, output of each variety, price, or profit. These findings are different from the main results of Shimomura and Thisse (2012), because we do not take income effect into account. When the products of large firms and those of small firms have different substitutablitities, however, the entry of the large firm may cause a rise or a fall of the incumbent large firms' output, price and profit, depending on the comparison of the substitutability within large firms and small firms and the substitutability across these two types of firms. If the substitutability across these two types of firms is larger than the substitutability within large firms and small firms, the entry of the large firm will squeeze more small firms, which outweighs the competition effect among large firms and hence provides the room for the incumbent large firms to expand their production. Otherwise, the squeezing effect is not strong enough to compensate for the competition effect among large firms, and consequently the large firms have to reduce their output. Unlike Shimomura and Thisse (2012), who show that income effect amplifies the market expansion of large firms due to the exit of small firms when a new large firm enters the market, we find that the different substitutabilities across large and small firms play a critical role in determining whether entry is beneficial or harmful to large firms. In both cases, the entry of large firm decreases the size of small firms (shrinks the competitive fringe), which is commonly achieved by this paper and Shimomura and Thisse (2012). For the welfare anlysis, the entry of large firms will reduce the consumer surplus, but raise both producer surplus and social welfare if the substituility of products is the same across large and small firms. When the substituility of products is different across large and small firms, however, the entry of a large firm may reduce social welfare. Hence, our results may provide a theoretical foundation for the government's regulation policy against the large firms' entry behavior.

The present paper is related to the studies on the issues concerning the coexistence of large and small firms. There are several ways to model the market structure where the large and small firms coexist. The first way is the widely used dominant firm model. The dominant firm is modeled as the leader and the price maker while small firms are modeled as the followers who face increasing marginal cost and behave like price takers. Representative works include Chen (2003) and Growrisankaran and Holmes (2004). The second way is to use the traditional Stackelberg model to deal with such issues, as represented by Etro (2004, 2006) etc. In their models, the firm is large in the sense that it is both the leader and the first mover. The small firms are followers but their production behavior can influence the market price. The small firms and large firms can share the same marginal costs and they conduct a Stackelberg game. Besides, some scholars assume that the differences of large firms and small firms can be also from the perspective of consumers, namely the high-end firms and low-end firms (Ishibashi and Matsushima 2009). Another way to differentiate large and small firms is that large firms enjoy the cost advantage over small firms (Matsumura and Matsushima, 2010). Unlike the dominant firm model, we do not assume that small firms have increasing marginal cost or are price takers. In addition, our model can be treated as a modified version of stackelberg leader where the large firms in some sense also behave like stackelberg leaders and determine the range of varieties first, but produce with small firms in the later stage. More importantly, the present paper studies different issues compared with the above-mentioned literature. Here it is worth noting that in order to guarantee the coexistence of the large and small firms, we need to assume that large firms enjoy cost advantages over small firms.

The rest parts are organized as follows. We construct the model in Section 2. Results are given in Section 3. Section 4 provides the robustness check of the established results and policy implications.

2 The model

2.1 Preference and demand

Consider a closed economy consisting of two sectors. Sector 1 produces the differentiated products with a mixed market structure, where large and small firms coexist. Small firms freely enter or exit the market, while the number of large firms is exogeneously given. Firms in sector 2 are perfectly competitive and produce the homogenous product under constant return to scale.

On the demand side, the large and small firms differ in three respects. First, each large firm has a positive mesure, whereas each small firm has a zero measure. Thus, each large firm imposes a direct effect on the market, while each small firm is negligible. This assumption is the same as Shimomura and Thisse (2012). Second, each large firm produces a multiple range of varieties, and strategically determines both the range of product varieties and the quantity of each variety, while each small firm only produces one variety of product. Third, the varieties are equally substitutable within the group of large firms and that of small firms, but may have different substitutabilities across these two types of firms.

The utility of the representative consumer U is described by:

$$\begin{aligned} U \quad = \quad & \alpha[\int_0^N q_S(i)di + \sum_{m=1}^M \int_{\omega\in\Omega_m} q_L^m(\omega)d\omega] \qquad (1) \\ & -\frac{\beta}{2}\sum_{m=1}^M \int_{\omega\in\Omega_m} [q_L^m(\omega)]^2 d\omega - \frac{\beta}{2}\int_0^N [q_S(i)]^2 di \\ & -\frac{\gamma_1}{2}[\int_0^N q_S(i)di]^2 - \frac{\gamma_2}{2}[\sum_{m=1}^M \int_{\omega\in\Omega_m} q_L^m(\omega)d\omega]^2 \\ & -\gamma_3[\int_0^N q_S(i)di][\sum_{m=1}^M \int_{\omega\in\Omega_m} q_L^m(\omega)d\omega] + q_0 \end{aligned}$$

where $q_S(i)$ is the quantity of small firm i with $i \in [0, N]$. N is the size of small firms, describing the competitive fringe. Ω_m and $q_L^m(\omega)$ are the set of varieties produced by the large firm m and the quantity for variety $\omega \in \Omega_m$. M is the total number of the incumbent large firms, with $M \geq 2$. For simplicity, M and $|\Omega_m|$ can be treated as continuous variables in our later analysis. q_0 is the output of sector 2. $\alpha > 0$ shows the market size of sector 1. $\beta > 0$ represents the consumer's preference for diversified outputs of small firms and those of large firms. γ_i $(i = 1, 2, 3)$ expresses the substitutability between varieties: a higher γ_i implies a closer substitute for that variety. Specifically, γ_1 expresses the substitutability among the varieties produced by small firms, γ_2 expresses the substitutability among the varieties produced by large firms, and γ_3 expresses the substitutability between the varieties produced by large firms and those produced by small firms. The varieties of the small and large firms are equally substitutable when $\gamma_1 = \gamma_2 = \gamma_3$ and have different substituabilities otherwise. q_0 is the output of sector 2. We treat sector 2's output as the numeraire.

The representative consumer's budget constraint is:

$$\int_0^N p_S(i)q_S(i)di + \sum_{m=1}^{M} \int_{\omega\in\Omega_m} p_L^m(\omega)q_L^m(\omega)d\omega + q_0 = I$$

where $p_S(i)$ and $p_L^m(\omega)$ are the prices of small firm i and large firm m's variety ω. I is the representative consumer's income, which is exogeneously given. The demand functions facing small firms and large firms are implied by the maximization of the consumer's utility subject to the budget constraint:

$$p_S(i) = \alpha - \beta q_S(i) - \gamma_1 Q_S - \gamma_3 Q_L \tag{2}$$

$$p_L^m(\omega) = \alpha - \beta q_L^m(\omega) - \gamma_3 Q_S - \gamma_2 Q_L \tag{3}$$

where $Q_S = \int_0^N q_S(i)di$ and $Q_L = \sum_{m=1}^{M} \int_{\omega\in\Omega_m} q_L^m(\omega)d\omega$.

Hence, small firms face demands with the same constant marginal price of quantity. whereas the demands facing large firms vary according to the large firms' range of varieties.

2.2 Firms' behavior

For both large and small firms, we assume the marginal cost of producing each variety is zero. The fixed cost of producing each variety is the same within large firms as well as within small firms. But the fixed cost of each variety for large firms can be different from that of small firms.

2.2.1 Small firms

The profit function of the small firms is:

$$\Pi_S(i) = p_S(i)q_S(i) - F_S$$

where $\Pi_S(i)$ is the profit of the small firm i, and F_S is the fixed cost of the small firm.

Plugging $p_S(i)$ of equation (2) into the above profit function, $\Pi_S(i)$ can be rewritten as:

$$\Pi_S(i) = \alpha q_S(i) - \beta[q_S(i)]^2 - [\gamma_1 Q_S + \gamma_3 Q_L]q_S(i) - F_S \tag{4}$$

The free entry and exit of small firms implies that each small firm earn zero profit:

$$\Pi_S(i) = \alpha q_S(i) - \beta[q_S(i)]^2 - [\gamma_1 Q_S + \gamma_3 Q_L]q_S(i) - F_S = 0 \tag{5}$$

2.2.2 Big firms

The profit function of the large firm is:

$$\Pi_L^m(\Omega_m) = \int_{\omega\in\Omega_m} (p_L^m(\omega)q_L^m(\omega) - F_L)d\omega - G$$

where $\Pi_L^m(\Omega_m)$ is the profit of large firm m, F_L is the fixed cost for the large firm to produce each variety, and G is the entry cost of the large firm. Without loss of generality, we assume $G = 0$.[3]

Substituting $p_L^m(\omega)$ of equation (3) into the above profit function, $\Pi_L^m(\Omega_m)$ can be rewritten as:

$$\begin{aligned}\Pi_L^m(\Omega_m) \quad = \quad & (\alpha - \gamma_3 Q_S - \gamma_2)[\sum_{k\neq m}\int_{\omega\in\Omega_k} q_L^k(\omega)d\omega\ ][\int_{\omega\in\Omega_m} q_L^m(\omega)d\omega] \\ & -\beta\int_{\omega\in\Omega_m}[q_L^m(\omega)]^2 d\omega - \gamma_2[\int_{\omega\in\Omega_m} q_L^m(\omega)d\omega]^2 - F_L\,|\Omega_m|\end{aligned} \tag{6}$$

2.3 Welfare

The social welfare comprises consumer surplus and producer surplus. Consumer surplus is represented by:

$$CS = U - I$$

Hence, the change of consumer surplus moves in the same direction as that of consumer's utility.

Since small firms earn zero profit, producer surplus is given by the sum of all large firms' profits:

[3]Here we do not consider the free entry and exit of large firms. Therefore, we can assume $G = 0$ without loss of generality.

$$PS = \sum_{m=1}^{M} \Pi_L^m$$

Then, social welfare SW is the sum of consumer surplus and producer surplus:

$$SW = U - I + \sum_{m=1}^{M} \Pi_L^m \tag{7}$$

We consider the following two-stage game. In stage 1, each large firm determines the range of product varieties, and small firms decide whether to enter the market. In stage 2, all the firms produce and compete with each other in quantity. Here we only investigate the symmetric equilibrium depicted by the variables endowed with a superscript $*$.We use backwards induction to solve the established model.

3 Results

We consider two cases. In the first case, the substitutability is the same among the large and small firms; and in the second case, the substitutability differs across the large and small firms. First, we need to identify the conditions for the coexistence of large and small firms: (i) the measure of small firms N^* should be positive; (ii) large firms should earn positive profits with a positive range of varieties $|\Omega^*|$; (iii) the market is stable. Then we investigate the impacts of the entry of a large firm.

3.1 Case I: $\gamma_1 = \gamma_2 = \gamma_3$

In this case, the substitutability is the same among the large and small firms. We denote $\gamma_1 = \gamma_2 = \gamma_3 \equiv \gamma$.

Stage 2

In this stage, small firms compete with large firms with respect to quantity.

Small Firms

A small firm only accounts for the impact of the market's total production, but its own impact on the market is negligible. Hence the small firm maximizes its profit given by equation (4) with respect to its output $q_S(i)$, yielding the optimal quantity produced by each small firm, for a given total output of large firms Q_L and mass of small firms N:

$$q_S^*(Q_L, N) = \frac{\alpha - \gamma Q_L}{2\beta + \gamma N} \tag{8}$$

Using equation (2), the equilibrium price of the small firm is:

$$p_S^*(Q_L, N) = \beta \frac{\alpha - \gamma Q_L}{2\beta + \gamma N} \tag{9}$$

Accordingly, the equilibrium quantity and price of the small firm decrease with the mass of small firms and the total output of large firms.

Large Firms

Unlike small firms, large firms impose non-negligible effects on the market. Large firm m maximizes its profit given by equation (6) with respect to its output $q_m(\omega)$, yielding the optimal quantity of each variety, for a given total output of small firms Q_S, its own product range $|\Omega_m|$, and the product range of other large firms $(M-1)\,|\Omega|\,q_L$, where $|\Omega|$ and q_L are the product range and output of each variety for other large firms:

$$q_L^{m*}(Q_S, |\Omega|, q_L, |\Omega_m|) = \frac{\alpha - \gamma Q_S - \gamma(M-1)\,|\Omega|\,q_L}{2(\beta + |\Omega_m|)} \tag{10}$$

Everything else being equal, an increase in firm m's product range (larger $|\Omega_m|$) results in a reduction in the quantity of each variety, implying cannibalization.

Stage 1

In this stage, small firms enter the market, and the large firms determine the optimal range of varieties.

Small Firms

Entry and exit are free for small firms. Using equation (5) after plugging in (8) and (9), the equilibrium mass of small firms with a given total output of large firms Q_L is:

$$N^*(Q_L) = \frac{1}{\gamma}[\sqrt{\frac{\beta}{F_S}}(\alpha - \gamma Q_L) - 2\beta] \tag{11}$$

The equilibrium mass of small firms decreases with the total output of large firms.

Substituting (11) into (8), the optimal quantity of each small firm is:

$$q_S^* = \sqrt{\frac{F_S}{\beta}}$$

Plugging q_S^* into (9) yields the equilibrium price of small firms:

$$p_S^* = \sqrt{\beta F_S}$$

Owing to free entry and exit, the quantity produced by the small firm is independent of the behavior of large firms.

Large Firms

The product range of firm m, $|\Omega_m^*|$, that maximizes (6) after substituting (10) satisfies:

$$2(\beta + \gamma\,|\Omega_m^*|) = \sqrt{\frac{\beta}{F_L}}[\alpha - \gamma Q_S - \gamma(M-1)\,|\Omega|\,q_L] \tag{12}$$

Equating q_L^{m*} with q_L and $|\Omega_m^*|$ with $|\Omega|$, we obtain the symmetric Nash equilibrium from equations (10) and (12). The equilibrium output per variety for the large firm is:

$$q_L^{m*} = \sqrt{\frac{F_L}{\beta}}$$

The equilibrium output of each variety of the large firm is determined only by the fixed cost of large firms and the demand parameters, but independent of its range of varieties or other firms' behavior.

The equilibrium product range $|\Omega_m^*|$ with a given aggregate output of small firms Q_S is:

$$|\Omega_m^*|\,(Q_S) = \frac{\sqrt{\beta/F_L}(\alpha - \gamma Q_S) - 2\beta}{\gamma(M+1)} \tag{13}$$

In the symmetric Nash equilibrium, the total output of big firms can be expressed by $Q_L^* = M\,|\Omega_m^*|\,q_L^{m*}$, and $Q_S^* = N^* q_S^*$. Plugging these two expressions into (11) and (13), the mass of small firms and the product range of each big firm are:

$$\begin{aligned} N^* &= \frac{1}{\gamma}\sqrt{\frac{\beta}{F_S}}[\alpha - 2\sqrt{\beta F_S} - 2M\sqrt{\beta}(\sqrt{F_S} - \sqrt{F_L})] \\ |\Omega^*| &= \frac{2\beta}{\gamma}(\sqrt{\frac{F_S}{F_L}} - 1) \end{aligned}$$

Substituting N^*, $|\Omega^*|$, q_S^* and q_L^{m*} into equation (3), the price of large firms in equilibrium is:

$$p_L^* = 2\sqrt{\beta F_S} - \sqrt{\beta F_L}$$

Substituting the equilibrium range of varieties $|\Omega^*|$, the output of each variety q_L^{m*} and the equilbirum price of large firms p_L^* into equation (6), we obtain the equilibrium profit of each large firm:

$$\Pi_L^* = \frac{4\beta}{\gamma}(\sqrt{F_S} - \sqrt{F_L})^2$$

In this paper, we focus on the market with the coexistence of large and small firms. To ensure the market is mixed in equilibrium, all the endogeneous variables should be positive. The following lemma establishes the conditions:

Lemma 1 *To ensure positive results for the mixed market equilibrium, the following two conditions should hold:*
(1-i) $F_S - F_L > 0$;
(1-ii) $\alpha > 2(M+1)\sqrt{\beta F_S} - 2M\sqrt{\beta F_L}$.

Condition (1-i) indicates that the large firm enjoys a cost advantage. It is a natural assumption since large firms have good reputations and more sophisticated distributive channels, etc. Condition (1-ii) argues that the market size should be large enough.

Now we investigate the impacts of the entry of a large firm. Proposition 1 establishes the results:

Proposition 1 *The entry of a large firm exerts the following effects:*

(i) it has no impacts on the output and price level of each small firm;

(ii) it has no impacts on the output of each variety, the range of varieties and price level of each large firm;

(iii) it decreases the size of small firms, i.e., shrink the competitive fringe.

Proof. *From the above results,* $\frac{dq_S^*}{dM} = 0$, $\frac{dp_S^*}{dM} = 0$, $\frac{dq_L^*}{dM} = 0$, $\frac{dp_L^*}{dM} = 0$, $\frac{d|\Omega^*|}{dM} = 0$, $\frac{d\Pi_L^*}{dM} = 0$ *and* $\frac{dN^*}{dM} < 0$. ■

Different from the results of the Cournot oligopoly model, the price, quantity, product variety and profit of large firms remains to be the same after the entry of a large firm. That is because the shrinkage of competitive fringe provides large firms with expanded market space that completely buffers the negative competition effects generated by the entry of a large firm. Precisely, the intuition can be explained by the following expressions:

$$\alpha - \underbrace{(\gamma Q_S + \gamma Q_L)}_{\text{Market Impact on Small Firm}} - \underbrace{2\beta q_S^*}_{\text{Self Impact of Small Firm}} = 0 \tag{14}$$

$$\alpha - \underbrace{(\gamma Q_S + \gamma Q_L)}_{\text{Market Impact on Big Firm}} - \underbrace{2\beta q_L^*}_{\text{Self Impact of Big Firm}} - \underbrace{\gamma |\Omega| q_L^*}_{\text{Internalization}} = 0 \tag{15}$$

Expressions (14) and (15) determine the equilibrium size of small firms N^* and the equilibrium range of varieties for each large firm $|\Omega^*|$ when the products of large firms and those of small firms have the same substitutability, after plugging in the equilibrium output of small firms as well as the the equilibrium output of each variety of large firms and applying the symmetry rule. Equation (14) consists of only the market impact on the small firm and the self impact of the small firm because each small firm does not account for its production's impact on the market. Equation (15) consists of the market impact on the large firm, the self impact and the internalization effect of the large firm. With free entry and exit of small firms, the output of each firm is constant, as shown by equation (15). Then equation (14) implies that the market impact is also constant. We can also find that the market impacts on both small and large firms are the same here. In addtion, the large firm's output of each variety is constant according to equation (19), so the self impact of the large firm is also constant. Therefore, the internalization effect is also constant, which implies that the large firm's range of varieties and price are constant.

The differences between the above findings and those obtained by Shimomura and Thisse (2012) are as follows. In their paper, the entry of the large firm will increase the output and profit of the large firm, decrease the price faced by the large firm and reduce the industrial price index. The similarities of the two papers are that the entry of the large firm decreases the number of the small firms, but exert no impact on the small firm's output decision. Without income effect, we show that the entry of a large firm leaves the incumbents' behavior unchanged except for the shrinkage of competitive fringe.

Based on the above resluts, we can obtain:

$$
\begin{aligned}
CS^* &= \frac{1}{2\gamma}[(\alpha - 2\sqrt{\beta F_S})(\alpha - \sqrt{\beta F_S}) \\
&\quad -4\beta M(\sqrt{F_S} - \sqrt{F_L})^2] \\
PS^* &= \frac{4\beta}{\gamma} M(\sqrt{F_S} - \sqrt{F_L})^2 \\
SW^* &= \frac{1}{2\gamma}[(\alpha - 2\sqrt{\beta F_S})(\alpha - \sqrt{\beta F_S}) \\
&\quad +4\beta M(\sqrt{F_S} - \sqrt{F_L})^2]
\end{aligned}
$$

We use Proposition 2 to show our findings.

Proposition 2 *The entry of a large firm (i) decreases consumer's utility (consumer surplus), (ii) raises producer surplus and (iii) increases social welfare.*
Proof. *The above results directly indicate that* $\frac{dCS^*}{dM} = -\frac{\beta}{\gamma}(\sqrt{F_S} - \sqrt{F_L})^2 < 0$, $\frac{dPS^*}{dM} = \frac{4\beta}{\gamma}(\sqrt{F_S} - \sqrt{F_L})^2 > 0$ *and* $\frac{dSW^*}{dM} = \frac{3\beta}{\gamma}(\sqrt{F_S} - \sqrt{F_L})^2 > 0$. ■

As shown by the results, consumer surplus decreases with the entry of a large firm although total welfare rises. This means that the benefits from the entry of a large firm are more than fully enjoyed by the large firms. Therefore, if the government is more concerned about consumers' interest, the restrictions on large firms may be justified.

3.2 Case II: $\gamma_1 \neq \gamma_3$ or $\gamma_2 \neq \gamma_3$.

In Case I, we assume that the varieties of large and small firms are equally differentiated. However, it is reasonable to question whether the products of these firms have the same level of substitution. In this subsection, we examine the case when the substitutability differs across the large and small firms, that is, $\gamma_1 \neq \gamma_3$ or $\gamma_2 \neq \gamma_3$.

Stage 2

In this stage, small firms compete with large firms with respect to quantity.

Small Firms

A small firm only accounts for the impact of the market's total production, but its own impact on the market is negligible. Hence the small firm maximizes its profit given by equation (4) with respect to its output $q_S(i)$, yielding the optimal quantity produced by each small firm, for a given mass of small firms N and total output of large firms Q_L:

$$q_S^*(Q_L, N) = \frac{\alpha - \gamma_3 Q_L}{2\beta + \gamma_1 N} \tag{16}$$

Using equation (2), the equilibrium price is of the small firm is:

$$p_S^*(Q_L, N) = \beta\frac{\alpha - \gamma_3 Q_L}{2\beta + \gamma_1 N} \tag{17}$$

Accordingly, the equilibrium quantity price of the small firm decrease with the mass of small firms and the total output of large firms.

Large Firms

Unlike small firms, large firms impose non-negligible effects on the market. Large firm m maximizes its profit given by equation (6) with respect to its output $q_m(\omega)$, yielding the optimal quantity of each variety, for a given total output of small firms Q_S, its own product range $|\Omega_m|$, and the product range of other large firms $(M-1)|\Omega| q_L$, where $|\Omega|$ and q_L are the product range and output of each variety for other large firms:

$$q_L^{m*}(Q_S, |\Omega|, q_L, |\Omega_m|) = \frac{\alpha - \gamma_3 Q_S - \gamma_2(M-1)|\Omega| q_L}{2(\beta + \gamma_2|\Omega_m|)} \tag{18}$$

Everything else being equal, an increase in firm m's product range (larger $|\Omega_m|$) result in a reduction in the quantity of each variety, implying cannibalization.

Stage 1

In this stage, small firms enter the market, and the large firms determine the optimal range of varieties.

Small Firms

Entry and exit are free small firms. Using equation (5) after plugging in (16) and (17), the equilibrium mass of small firms with a given total output of large firms Q_L is:

$$N^*(Q_L) = \frac{1}{\gamma_1}[\sqrt{\frac{\beta}{F_S}}(\alpha - \gamma_3 Q_L) - 2\beta] \tag{19}$$

The equilibrium mass of small firms decreases with the total output of large firms.

Substituting (19) into (16), the optimal quantity of each small firm is:

$$q_S^* = \sqrt{\frac{F_S}{\beta}}$$

Owing to free entry and exit, the quantity produced by the small firm is independent of the behavior of large firms.

Plugging q_S^* into (5) yields the equilibrium price of small firms:

$$p_S^* = \sqrt{\beta F_S}$$

Large Firms

The product range of large firm m, $|\Omega_m^*|$, that maximizes (6) after substituting (18) satisfies:

$$2(\beta + \gamma_2 |\Omega_m^*|) = \sqrt{\frac{\beta}{F_L}}[\alpha - \gamma_3 Q_S - \gamma_2(M-1)|\Omega| q_L] \tag{20}$$

Equating q_L^{m*} with q_L and $|\Omega_m^*|$ with $|\Omega|$, we obtain the symmetric Nash equilibrium from equations (18) and (20). The equilibrium output per variety for the large firm is:

$$q_L^{m*} = \sqrt{\frac{F_L}{\beta}}$$

The equilibrium output of each variety of the large firm is determined only by the fixed cost of large firms and the demand parameters, but independent of its range of varieties or other firms' behavior.

The equilibrium product range $|\Omega_m^*|$ with a given aggregate output of small firms Q_S is:

$$|\Omega_m^*|(Q_S) = \frac{\sqrt{\beta/F_L}(\alpha - \gamma_3 Q_S) - 2\beta}{\gamma_2(M+1)} \tag{21}$$

In the symmetric Nash equilibrium, the total output of big firms can be expressed by $Q_L^* = M|\Omega_m^*| q_L^{m*}$, and $Q_S^* = N^* q_S^*$. Plugging these two expressions into (19) and (21), the mass of small firms and the product range of each big firm are:

$$\begin{aligned}
N^* &= \sqrt{\frac{\beta}{F_S}} \frac{\alpha[\gamma_2(M+1) - \gamma_3 M] - 2\sqrt{\beta}[\gamma_2(M+1)\sqrt{F_S} - \gamma_3 M \sqrt{F_L}]}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M} \\
|\Omega^*| &= \sqrt{\frac{\beta}{F_L}} \frac{\alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F_L} - \gamma_3\sqrt{F_S})}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}
\end{aligned}$$

Substituting N^*, $|\Omega^*|$, q_S^* and q_L^{m*} into equation (3), the price of large firms in equilibrium is:

$$p_L^* = \sqrt{\beta F_L} + \frac{\gamma_2[\alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F_L} - \gamma_3\sqrt{F_S})]}{\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M}$$

Substituting the equilibrium range of varieties $|\Omega^*|$, the output of each variety q_L^{m*} and the equilbirum price of large firms p_L^* into equation (6), we obtain the equilibrium profit of each large firm:

$$\Pi_L^* = \frac{\gamma_2[\alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F_L} - \gamma_3\sqrt{F_S})]^2}{[\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M]^2}$$

Again, here we focus on the market with the coexistence of large and small firms. To ensure the market is mixed and stable in equilibrium, all the endogeneous variables should be positive. The following lemma establishes the conditions:

Lemma 2 *To ensure positive results for the mixed market equilibrium, the following three conditions should hold:*

(2-i) $\alpha[\gamma_2(M+1) - \gamma_3 M] > 2\sqrt{\beta}[\gamma_2(M+1)\sqrt{F_S} - \gamma_3 M\sqrt{F_L}]$*;*
(2-ii) $\alpha(\gamma_1 - \gamma_3) > 2\sqrt{\beta}(\gamma_1\sqrt{F_L} - \gamma_3\sqrt{F_S})$*;*
(2-iii) $\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$*.*

Conditions (2-i) and (2-ii) correspond to conditions (1-i) and (1-ii). Condition (2-iii) is the sufficient condition to guarantee the stability of the established model (see **Appendix A-1**). Here we also focus on the the market where both large and small firms exist.

Now we investigate the impacts of the entry of a large firm. Proposition 3 establishes the results:

Proposition 3 *The entry of a large firm will exert the following effects:*
(i) It has no impact on the output and price level of each small firm;
(ii) It has no impact on the output of each variety of each large firms;
(iii) It decreases the size of small firms, i.e., shrinks the competitive fringe;
(iv) The range of varieties, price, and profit of each large firm rise if $\gamma_1\gamma_2 < \gamma_3^2$*, and fall if* $\gamma_1\gamma_2 > \gamma_3^2$.

Proof. See **Appendix A-2**. ■

Here we obtain similar results as Shimomura and Thisse (2012), but the mechanism is different. In Shimomura the Thisse (2012), the increase in the outputs and profits of incumbent large firms arises from the income effect of higher profits of large firms. Without income effect, we show that the entry of a large firm may raise or reduce the outputs and profits of the incumbent large firms when the substitutability across the products of large firms and those of small firms are is different from the sustitutability within the groups of large and small firms. To illustrate the mechanism, we establish the following two expressions:

$$\underbrace{(\gamma_3 Q_S + \gamma_2 Q_L)}_{\text{Market Impact on Big Firm}} = \alpha - \underbrace{2\beta q_L^*}_{\text{Self Impact of Big Firm}} - \underbrace{\gamma_2 |\Omega| q_L^*}_{\text{Internalization}} \tag{22}$$

$$\underbrace{(\gamma_1 Q_S + \gamma_3 Q_L)}_{\text{Market Impact on Small Firm}} = \alpha - \underbrace{2\beta q_S^*}_{\text{Self Impact of Small Firm}} \tag{23}$$

Expressions (22) and (23) determine the equilibrium size of small firms N^* and the equilibrium range of varieties for each large firm $|\Omega^*|$, after plugging in the equilibrium output of small firms as well as the the equilibrium output of each variety of large firms and applying the symmetry rule. Equation (23) consists of only the market impact on the small firm and the self impact of the small firm because each small firm does not account for its production's impact on the market.Equation (22) consists of the market impact on the large firm, the self impact and the internalization effect of the large firm. Similar to Case I, the self impacts of both large and small firms are constant, which implies that the weighted market impact on small firms is constant. When $\gamma_1\gamma_2 < \gamma_3^2$, the substitutability across the large and small firms is relatively larger than the substitutability within the small firms and within the large firms. Then the entry of a large firm will squeeze a larger market share of small firms, which provides the room for large firms to expand their production. In this case, the squeezing effect of the small firms outweighs the competition effect among the large firms. As shown by (22), in addition, the internalization effect becomes stronger, hence raising the price of the large firm. With more varieties and a higher price, the profit of the large firm also rises. If $\gamma_1\gamma_2 < \gamma_3^2$, on the other hand, the substitutability across the large and small firms is relatively smaller than the substitutability within the small firms and within the large firms. Then the squeezing effect of the small firms is weaker than the competition effect among the large firms. In this case, each large firm reduces its range of varieties and price, and its profit falls.

In addition, if $\gamma_1\gamma_2 = \gamma_3^2$, (22) can be rewritten as $\frac{\gamma_3}{\gamma_1}(\gamma_1 Q_S + \gamma_3 Q_L) = \alpha - 2\beta q_L^* - \gamma_2 |\Omega| q_L^*$. Since the LHS of (23) does not change with M, as shown earlier, the LHS of (22) does not change with M either. Accordingly, the product range of incumbent large firms does not change with the entry of a large firm.

Now let us consider how the entry of large firm influences the consumer surplus, producer surplus and social welfare. We use Proposition 4 to show the results.

Proposition 4 *The entry of a large firm generates the following impacts on social welfare:*

(i) decreases consumer's utility (consumer surplus) if $\frac{EM(\gamma_3^2-\gamma_1\gamma_2)}{D} + \frac{\sqrt{\beta}}{2}(\gamma_3\sqrt{F_S} - \gamma_1\sqrt{F_L}) > 0$*;*

(ii) Produer surplus rises if $1+\frac{2EM}{D}(\gamma_3^2-\gamma_1\gamma_2) > 0$, *and falls if* $1+\frac{2EM}{D}(\gamma_3^2-\gamma_1\gamma_2) < 0$*;*

(iii) raises social welfare if $1 + \frac{EM}{D}(\gamma_3^2 - \gamma_1\gamma_2) - \frac{\sqrt{\beta}}{2}(\gamma_3\sqrt{F_S} - \gamma_1\sqrt{F_L}) > 0$.

where $D = \gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M$, *and* $E = \alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F_L} - \gamma_3\sqrt{F_S})$.

Proof. See **Appendix A-3**. ■

Therefore, the entry of the large firm will only conditionally raise consumer surplus, producer surplus and social welfare.

4 Discussion

It is natural to question the robustness of Propositions 1, 2, 3 and 4. Here we ignore the detailed derivation processes and just report our results. We find that the entry of a large firm will qualitatively exert the same impacts achieved by Propositions 1, 2, 3 and 4 if we consider the following cases: (i) large firms and small firms face different market sizes ($\alpha \to \alpha_L, \alpha_S$), and different consumer's preferences for diversity ($\beta \to \beta_L, \beta_S$); (ii) large firms and small firms have the same or) different marginal costs, saying for the linear marginal cost case $c_L q_L$, $c_S q_S$ or the quadratic marginal cost case $\frac{c_L}{2}q_L^2$, $\frac{c_S}{2}q_S^2$; (iii) varieties of large firms are exogenously given.

Our results also have some policy implications. In Shimomura and Thisse (2012), they suggest that government should not restrict the large firm's entry because it raises the profits of large firms and reduces the market price level, leading to a welfare-improving result. In our model, the entry of the large firm may be harmful to consumer surplus and social welfare if we account for different levels of substitution across large and small firms, as suggested by Propositions

2 and 4. If the varieties of small and large firms are equally differentiated, consumer surplus is reduced by the entry of a big firm although social welfare is raised. If the subsitutability differs across the varieties produced by small firms and those produced by large firms, the entry of the big firm may reduce both consumer surplus and social welfare. In this sense, this paper provides a theoretical foundation for government's regulation policy of the large firms' entry behavior.

In reality, lots of industries are featured with the coexistence of large and small firms. More work can be done in the studies of such a mixed market structure. All in all, the present paper is just an attempt in this field.

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Appendices

Appendix A-1: Proof of Assumption II-3.

Given the equilibirum values of $q_S^* = \sqrt{\frac{F_S}{\beta}}$ and $q_L^* = \sqrt{\frac{F_L}{\beta}}$, the free entry condition of small firm and the profit maximization of large firm yield the following two expressions of dynamic adjustment process:

$$\dot{N}(N,\Omega) = d_1[\alpha q_S^* - \beta q_S^{*2} - (\gamma_1 N q_S^* + \gamma_3 M\Omega q_L^*)q_S^* - F_S],$$

$$\dot{\Omega}(N,\Omega) = d_2\{[\alpha - \beta q_L^* - \gamma_3 N q_S^* - \gamma_2(M+1)\Omega q_L^*]q_L^* - F_L\}.$$

Where $\dot{N} = \frac{dN}{dt}$, $\dot{\Omega} = \frac{d\Omega}{dt}$, $d_1 > 0$ and $d_2 > 0$. To ensure the local stability of the established model, the Jacobian matrix derived from the above two expressions is required to be negative definite:

$$J = \begin{pmatrix} \frac{\partial \dot{N}}{\partial N} & \frac{\partial \dot{N}}{\partial \Omega} \\ \frac{\partial \dot{\Omega}}{\partial N} & \frac{\partial \dot{\Omega}}{\partial N} \end{pmatrix} = \begin{pmatrix} -\gamma_1 q_S^{*2} & -\gamma_3 M q_S^* q \\ -\gamma_3 q_S^* q_L^* & -\gamma_2(M+1)q_L^{*2} \end{pmatrix}$$

$J^1 = -\gamma_1 q_S^* < 0$, and $J^2 = [\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M]\frac{q_S^* q_L^*}{4} > 0$. Hence$\gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M > 0$.

Appendix A-2: Proof of Proposition 3.

Let $D = \gamma_1\gamma_2 + (\gamma_1\gamma_2 - \gamma_3^2)M$, and $E = \alpha(\gamma_1 - \gamma_3) - 2\sqrt{\beta}(\gamma_1\sqrt{F_L} - \gamma_3\sqrt{F_S})$. By **Lemma 2**, $D > 0$ and $E > 0$. From the obtained results, we have:

$\frac{dq_S^*}{dM} = 0$, $\frac{dp_S^*}{dM} = 0$, $\frac{dq_L^*}{dM} = 0$, and $\frac{dN^*}{dM} = -\sqrt{\frac{\beta}{F_S}}\frac{\gamma_2\gamma_3 E}{D^2} < 0$. $\frac{dp_L^*}{dM} = \frac{\gamma_2(\gamma_3^2 - \gamma_1\gamma_2)E}{D^2}$, $\frac{d|\Omega^*|}{dM} = \sqrt{\frac{\beta}{F_L}}\frac{(\gamma_3^2 - \gamma_1\gamma_2)E}{D^2}$, and $\frac{d\Pi_L^*}{dM} = \frac{2\gamma_2(\gamma_3^2 - \gamma_1\gamma_2)E^2}{D^3}$, which are positive if $\gamma_1\gamma_2 < \gamma_3^2$ and negative if $\gamma_1\gamma_2 > \gamma_3^2$.

Appendix A-3: Proof of Proposition 4.

The consumer surplus, producer surplus and social welfare can be expressed as:

$$\begin{aligned} CS^* &= \alpha Q^* - \frac{\beta}{2}(N^* q_S^{*2} + M\,|\Omega^*|\, q_L^{*2}) - \frac{\gamma_1}{2}Q_S^{*2} \\ &\quad -\frac{\gamma_2}{2}Q_L^{*2} - \gamma_3 Q_S^* Q_L^* - p_S^* Q_S^* - p_L^* Q_L^* \\ PS^* &= \frac{\gamma_2 E^2}{D^2} \\ SW^* &= \alpha Q^* - \frac{\beta}{2}(N^* q_S^{*2} + M\,|\Omega^*|\, q_L^{*2}) - \frac{\gamma_1}{2}Q_S^{*2} \\ &\quad -\frac{\gamma_2}{2}Q_L^{*2} - \gamma_3 Q_S^* Q_L^* - p_S^* Q_S^* - p_L^* Q_L^* + \frac{\gamma_2 E^2}{D^2} \end{aligned}$$

The impact of a marginal increase of M on consumer surplus is:

$$\frac{dCS^*}{dM} = -\frac{\gamma_2 E}{D^2}[\frac{EM(\gamma_3^2 - \gamma_1\gamma_2)}{D} + \frac{\sqrt{\beta}}{2}(\gamma_3\sqrt{F_S} - \gamma_1\sqrt{F_L})]$$

$\frac{dCS^*}{dM}$ is positive if $\frac{EM(\gamma_3^2-\gamma_1\gamma_2)}{D}+\frac{\sqrt{\beta}}{2}(\gamma_3\sqrt{F_S}-\gamma_1\sqrt{F_L})<0$ and negative if $\frac{EM(\gamma_3^2-\gamma_1\gamma_2)}{D}+\frac{\sqrt{\beta}}{2}(\gamma_3\sqrt{F_S}-\gamma_1\sqrt{F_L})>0$. In particular, a sufficient condition for $\frac{dCS^*}{dM}<0$ is $\gamma_3^2>\gamma_1\gamma_2$ and $\gamma_3\sqrt{F_S}>\gamma_1\sqrt{F_L}$, and a sufficient condition for $\frac{dCS^*}{dM}>0$ is $\gamma_3^2<\gamma_1\gamma_2$ and $\gamma_3\sqrt{F_S}<\gamma_1\sqrt{F_L}$.

The impact of a marginal increase of M on producer surplus is:

$$\frac{dPS^*}{dM}=\frac{\gamma_2 E}{D^2}[1+\frac{2EM}{D}(\gamma_3^2-\gamma_1\gamma_2)]$$

A sufficient condition for $\frac{dPS^*}{dM}>0$ is $\gamma_3^2>\gamma_1\gamma_2$.

The impact of a marginal increase of M on social welfare is:

$$\frac{dSW^*}{dM}=\frac{\gamma_2 E}{D^2}[1+\frac{EM}{D}(\gamma_3^2-\gamma_1\gamma_2)-\frac{\sqrt{\beta}}{2}(\gamma_3\sqrt{F_S}-\gamma_1\sqrt{F_L})].$$

$\frac{dSW^*}{dM}$ is positive if $1+\frac{EM}{D}(\gamma_3^2-\gamma_1\gamma_2)-\frac{\sqrt{\beta}}{2}(\gamma_3\sqrt{F_S}-\gamma_1\sqrt{F_L})>0$ and negative if $1+\frac{EM}{D}(\gamma_3^2-\gamma_1\gamma_2)-\frac{\sqrt{\beta}}{2}(\gamma_3\sqrt{F_S}-\gamma_1\sqrt{F_L})<0$.