An Economic Geography Model with Natural Resources*

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Draft

Abstract

In this paper we present a dynamic model of trade between two regions which are endowed with natural resouces (used for consumption and/or production). It is an economic geography model because it is based on the interaction of economies of scale with transportation costs as in Krugman (1991). With this model, we show the existence of a "resource effect". When the migration of workers is not allowed, a shock of natural resources, brings down wages and reduces growth rates in the transitional dynamics. With migration, economies must grow at the same rate, but the one with greater endowment of natural resources, have a lower levels of industrialization and nominal wages.

JEL classification: F12, F43, O41, Q01

1 Introduction

The model is a variant in the monopolistic competition framework initially proposed by Dixit and Stiglitz (1977). It is a dynamic version of the two-region model by Krugman (1991) which incorporates differences between the natural endowments of two regions with the aim of studying the causes of a resource curse. It can also be seen as an open-economy version of the growth model by Grossman and Helpman (1991) which incorporates natural resources as consumption good. The main elements of the new economic geography like transportation costs and economies of scale are also present.

The resource curse refers the paradox that countries and regions with an abundance of natural resources tend to have less economic growth and worse development outcomes than countries with fewer natural resources (Sachs and

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Warner, 1997, 2001). This paper analyzes the curse of natural resources using the approach of the New Economic Geography, which was initiated by P. Krugman in the early 1990´s. The Economic Geography studies the causes of the uneven geographical distribution of economic activities and its evolution through time.

Although there is abundant literature, theoretical and empirical1, on the resource curse, the issue has not been addressed from the spatial perspective, as we do in this paper. In that context, imperfect market competition and migratory flows (of labor and firms) can affect the economic growth rate, and the distribution of the industrial activity.

We study the effects of an unexpected increase in the natural resource sector in one of the countries. This is what happened in Venezuela with the Mene Grande discoveries in 1914; in Nigeria with the discoveries of oil in the Niger Delta (1956) and during the seventies in Northway, when the magnitude of its oilfields was discovered; among others. In all these examples, the resource curse hypothesis was satisfied. The countries experimented a decrease in the growth rates.

In this paper we present the first results of our research. Although it is a work in progress, the model without migration, predicts that a positive shock in the resource endowment in one of the countries, will rise the wages in this country in the short-run; and lower their growth rates in the transition, back to the sustainable balance growth path, in the long-run. When migration flows are allow, both economies must grow at the same rate, along a balance growth path, to ensure the equality of real wages. However, the distribution of the industry presents a negative relation with the natural resource endowments of the regions, as a consecuence of what we call: the *resource effect*. The dynamics of the latter model is still under study, so it is not included in this draft.

2 A dynamic two-region model

We consider a model of two regions. In this model there are assumed to be two kinds of goods: manufactures, which are tradable goods produced by an increasing-returns sector that can be located in either region, and natural resource, which is considered as an endowment that each region possesses. These endowments have the characteristic of being non-tradable across regions.

The number of varieties of industrial goods n_1 and n_2 for each region can be enlarged devoting efforts to the R&D sectors. Trade is not free, their exists transportation cost. The production of industrial goods needs labor and natural resources. Moreover, there are fixed costs that guarantee the existence of economies of scale.

All individuals in this economy are assumed to share the intertemporal utility function of the form

$$U = \int_0^\infty \ln \left[C_M^{\mu} C_A^{1-\mu} \right] e^{-\rho t} dt, \tag{1}$$

where C_A is consumption of the natural (agricultural) good and C_M is consumption of manufactures aggregate which is defined by

$$C_M = \left(\int_0^{n_1} c_{1i}^{\frac{\sigma-1}{\sigma}} di + \int_0^{n_2} c_{2i}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

being n_1 and n_2 the large number of potential products from each region at a given instant of time and $\sigma > 1$ is the elasticity of substitution among the products.

Individuals hold also assets in the form of ownership claims on innovative firms. We denote the net assets per person in region j by $a_j(t)$. Individuals are competitive in that each takes as given the interest rate, $r_j(t)$, the price of the natural resource $p_{A_j}(t)$, and the wage rate, $w_j(t)$, paid per unit of labor services. Each individual supplies inelastically one unit of labor services per unit of time. The total income received by an individual per unit of time is, therefore, the sum of labor income, w_j, and assets income, $r_j \cdot a_j$. Individuals use the income that they do not consume to accumulate more assets:

$$\dot{a}_j = w_j + r_j a_j - p_{A_j} C_{A_j} - \sum_{i=1}^{n_1} p_{1i} c_{1i} - \sum_{i=1}^{n_2} \frac{p_{2i}}{\tau} c_{2i} \tag{3}$$

where p_{ji} is the price of the good i produced in region j and $\tau < 1$ is the constant transportation cost. Transportation costs for manufactured goods will be assumed to take Samuelson´s "iceberg" form, in which transport costs are incurred in the good transported. And finally, L_j is the total labor force in region j.

The individual´s optimization problem is to decide on variables C_{A_j}, c_{1i} and c_{2i} to maximize U in equation (1) subject to the budget constrain in equation (3).

The usual Euler equation arises, for both countries, that is

$$\frac{\dot{E}_1}{E_1} = r_1 - \rho, \qquad \frac{\dot{E}_2}{E_2} = r_2 - \rho \tag{4}$$

where E_j denotes expenditures in consumption (manufactures and natural resource) in region j. And the houshold demand for primary goods is:

$$C_{A_j} = \frac{(1-\mu) E_j}{p_{A_j}} \tag{5}$$

We can now turn to the behavior of firms. The production of an individual manufactured good i involves a fixed cost and a constant marginal cost, giving rise to economies of scale:

$$x_i = \frac{\left(l_{x_i} - \phi_j\right)^{\alpha} A_i^{1-\alpha}}{\beta} \tag{6}$$

where l_{x_i} is labor used in producing i and x_i is the good´s output. The fixed cost ϕ_j depends on the region j and it is defined as $\phi_j = \phi / n_j$, that is, the higher the number of firms in a region n_j the lower the fixed cost. The production function (6) is similar to the one proposed by Krugman (1980, 1991).

We assume that there are a large number of manufacturing firms, each producing a single product. Then, given the definition of manufacturing aggregate (2) and the assumption of iceberg transportation costs, the elasticity of demand facing any individual firm is σ (see Krugman 1980, 1991). The profit-maximizing pricing behavior of a representative firm in region 1 is therefore to set a price equal to

$$p_1 = \frac{\sigma}{\sigma - 1}\beta \left(\frac{w_1}{\alpha}\right)^{\alpha} \left(\frac{p_{A_1}}{1-\alpha}\right)^{1-\alpha} \tag{7}$$

and the resource demand equal to:

$$A_1 = \frac{1-\alpha}{\alpha}\frac{w_1}{p_{A_1}}\left(l_{x_i} - \phi_j\right) \tag{8}$$

where w_1 is the wage rate of workers in region 1. Since firms are identical and they face the same prices, labor force $l_{x_i} = l$ for all i.

$$x_1 = \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \left(\frac{w_1}{p_{A_1}}\right)^{1-\alpha} \frac{l_{x_i} - \phi_j}{\beta}$$

Similar equations applies in region 2. Comparing the prices of representative products, we have

$$\frac{p_1}{p_2} = \left(\frac{w_1}{w_2}\right)^{\alpha} \left(\frac{p_{A_1}}{p_{A_2}}\right)^{1-\alpha} \tag{9}$$

At a point in time, the technology exists to produce n_1(in country 1) and n_2 (in country 2) varieties of consumption goods. An expansion of these numbers requires a technological advance in the sense of an invention that permits the production of the new kind of consumption goods. Following Romer/Crossman-Helpman/Aghion-Howit, the change in n_j ($j = 1, 2$) will be equal to the number of people attempting to discover new ideas, l_{n_j}, multiplied by the rate at which R&D generates new ideas, $\bar{\eta}_j$:

$$\dot{n}_j = \bar{\eta}_j l_{n_j} \tag{10}$$

where $\bar{\eta}_j = \eta n_j$ in its simplest form. Any individual is allowed to enter the R&D sector and prospect for new designs, so that labor must receive the same compensation in its two uses:

$$w_j = \bar{\eta}_j V_j \tag{11}$$

where V_j is the net value of future expected profits linked to the discovery in country j, whose expression is

$$V_j(t) = \int_t^{\infty} \pi_j(s) e^{-\bar{r}_j(t,s)(s-t)} ds \text{ with } \bar{r}_j(t,s) \equiv \frac{1}{(s-t)} \int_t^s r_j(v) dv \tag{12}$$

$$\pi_j(s) = p_j x_j - w_j l_{x_j} - p_{A_j} A_j \tag{13}$$

Function $\pi_j(s)$ represents instantaneous profits of monopolist.

Differentiating (12) it is obtained that

$$r_j(t) = \frac{\pi_j}{V_j} + \frac{\dot{V}_j}{V_j}, \tag{14}$$

that is, the rate of returns r_j equals the rate of return to investing in R&D. The R&D rate of return equals the profit rate, π_j/V_j, plus the rate of capital gain or loss derived from the change in the value of the research firm, $\dot{V}_j/V_j$.

Combining equations (7, 6, 11 and 13), it yields that

$$\pi_j(t) = \frac{\eta}{\alpha(\sigma-1)} V_j(t) \left\{ L_{E_j} - \phi \left[1 + \alpha \left(\sigma - 1 \right) \right] \right\} \tag{15}$$

where $L_{E_j} = \sum_{i=1}^{n_j} l_{x_i}$.

While in the natural resource sector, a tipical primary firm will maximize their benefits, choosing the amount of labor to employ in the extraction of the resource, subject to the extraction function:

$$\begin{aligned} & \max \pi_{\Omega_j} = p_{A_j} \Omega_j - w_j l_{\Omega_j} \\ s.t \quad : \quad & \Omega_1 = \epsilon S_j l_{\Omega_j} \end{aligned} \tag{16}$$

The production (or extraction) of the resource is a direct function of: the aviable stock of the natural resource in the region, S_1, the labor employ in the extraction, l_{Ω_1}, and ϵ, that is a productivity parameter. The first order condition of this maximization problem is:

$$p_{A_j} \frac{\Omega_j}{l_{\Omega_j}} = w_j \tag{17}$$

Notice that this condition also implies that the profits of these firms will be equal to zero.

3 Short-Run Equilibrium

Let c_{11} be the consumption in region 1 of a representative region 1 product, and c_{12} be the consumption in region 1 of a representative region 2 product. From the first order optimality conditions derived from the optimization of (1) subject to (3) it is obtained that

$$\frac{c_{11}}{c_{12}} = \left(\frac{p_1 \tau}{p_2} \right)^{-\sigma}$$

If z_{11} is the ratio of region 1 spending on local manufactures to that on manufactures from the other region, and z_{12} is the ratio of region 2 expenditure on region 1 products to spending on local products, we have:

$$z_{11} \equiv \frac{n_1 c_1 p_1}{n_2 c_2 p_2/\tau} \tag{18}$$

$$z_{12} \equiv \frac{n_1 c_1 p_1/\tau}{n_2 c_2 p_2} \tag{19}$$

Lets seize this opportunity to define the following variables as well:

$$\begin{aligned} w &\equiv \frac{w_1}{w_2} \;\; ; \;\; p_A \equiv \frac{p_{A_1}}{p_{A_2}} \;\; ; \;\; n = \frac{n_1}{n_2} \;\; ; \;\; T = \tau^{\sigma-1} \\ \varepsilon_j &\equiv \frac{E_j}{w_j} \;\; ; \;\; S \equiv \frac{S_1}{S_2} \;\; ; \;\; X \equiv n w^{1-\sigma} S^{(1-\alpha)(\sigma-1)} \end{aligned} \tag{20}$$

The expenditure in manufactured goods produced in one region, have to pay all the factor involved in their production, plus the profits:

$$\pi_1 n_1 + p_{A_1} A_1 n_1 + w_1 L_{E_1} = \mu \left\{ \frac{z_{11}}{1+z_{11}} E_1 + q \frac{z_{12}}{1+z_{12}} E_2 \right\} \tag{21}$$

$$\pi_2 n_2 + p_{A_2} A_2 n_2 + w_2 L_{E_2} = \mu \left\{ \frac{1}{1+z_{11}} \frac{E_1}{q} + \frac{1}{1+z_{12}} E_2 \right\} \tag{22}$$

The terms on the right, of expression (21) are the region 1 and 2, expenditures, devoted to industrial goods produced in region 1. While, on the left are the payments to productive factors, plus the benefit of the industrial firms. Where q are the terms of trade; equal to the ratio of wages ($q = w_1/w_2$). Expression (22), states the same equality for region 2. Making use of (8 and 13) the previous equations can be expressed as follows:

$$\frac{\pi_1 n_1}{w_1} + \phi = \frac{\mu}{\sigma} \left\{ \frac{z_{11}}{1+z_{11}} \varepsilon_1 + \frac{z_{12}}{1+z_{12}} \varepsilon_2 \right\} \tag{23}$$

$$\frac{\pi_2 n_2}{w_2} + \phi = \frac{\mu}{\sigma} \left\{ \frac{1}{1+z_{11}} \varepsilon_1 + \frac{1}{1+z_{12}} \varepsilon_2 \right\} \tag{24}$$

Now that we already have an expression to determine the beneftis, equation (13) gives the amount of work in the industrial sector:

$$L_{E_j} = \alpha (\sigma - 1) \left(\frac{\pi_j n_j}{w_j} + \phi \right) + \phi \tag{25}$$

In the equilibrium, the supply of natural resource, equation (17) must equal the sum of individual consumption (5) and industrial use (8) in each country:

$$l_{\Omega_j} = (1-\mu)\varepsilon_j + \frac{1-\alpha}{\alpha} \left(L_{E_j} - \phi_j \right) \tag{26}$$

Turning now to the R&D sector, the total investment of a region must be equal to the total income minus the expenditure, in that region. Considering $(A_j = n_j V_j)$, and using expresions (10, 11 and 14), the labor engaged in research can be obtain:

$$l_{n_j} = L_j + \frac{\pi_j n_j}{w_j} - \varepsilon_j \tag{27}$$

We also assume full employment, so that total labor force must be exhausted by labor used in production, in R&D sector and in the resource sector:

$$L_j = \sum_{i=1}^{n_j} l_{x_i} + l_{n_j} + l_{\Omega_j} = L_{E_j} + l_{n_j} + l_{\Omega_j} \tag{28}$$

Then, the markets equilibria can be resume by the system (23 to 28). Nevertheless, we can do some manimpulations of this equations in order to simplify the system. First, replacing all the labor forces, into the full employment condition:

$$\frac{\pi_j n_j}{w_j} + \phi = \frac{\mu}{\sigma}\varepsilon_j \tag{29}$$

Second, with this equality and some of the transform variables (20), the system can be rewrite as:

$$\frac{\varepsilon_1}{\varepsilon_2} = X\frac{T+X}{1+XT} \tag{30}$$

$$L_{E_j} = \frac{\mu\alpha(\sigma-1)}{\sigma}\varepsilon_j + \phi \tag{31}$$

$$l_{\Omega_j} = \frac{\sigma - \mu\left[1+\alpha(\sigma-1)\right]}{\sigma}\varepsilon_j \tag{32}$$

$$l_{n_j} = L_j - \frac{\sigma-\mu}{\sigma}\varepsilon_j - \phi \tag{33}$$

We turn now to study the consecuences of differents shocks, over the wages, in the short-run. The following porpositions study the effect of a suddenly increase of the natural resource, the variety of industrial goods, and the expenditures, in one of the regions:

Proposition 1 *In a context without migration, when a shock in the natural resources take place, the distribution of the labor forces between the different sectors won't change.*

Proof. This comes straightforward from the previous system. ■

This happen becouse, from the begining, we are assuming that wages, in each region, are equal across sectors. Another interesting fact is that as corollary of Proposition (1), this model doesn't admit total concentration of the labor force in one region, otherwise l_{n_j} would be negative. Each region must have at least a minimum of population to produce industrial goods and extract natural resources. However, the labor force engaged in research activities can be equal to zero.

Proposition 2 *Starting from a position of symmetry between the two countries ($L_1 = L_2$ and S_1=S_2, so $w_1 = w_2$), when the natural resource is devoted only to final consumption ($\alpha = 1$), a shock of natural resources that takes place in one of the two regions, such that $S > 1$, will not modify the equilibrium wage rate, and hence $w_1 = w_2$.*

Proof. See the appendix. ■

If a suddenly increase of S_1 happens, since it is not a tradable good, the excess of natural resource supply in region 1 will generate a decrease in the price of this resource until the population of the same country be willing to consume all of it, so their incomes will not be affected by this shock.

But if ($0 < \alpha < 1$), changes in the price of the resource will affect the price of industrial goods, and wages will change due to the *resource effect.* To isolate this effect, we hold constant the expenditures and the ratio of variety of goods, allowing the ratio of resource stocks, to change.

Proposition 3 *When the natural resource is devoted to final consumption as well as to industrial production, ($0 < \alpha < 1$), an increase in the resource endowment of one of the regions, will increase the wage of this same region.*

Proof. See the appendix. ■

For example, assuming that we start form a symmetric position between regions, if the ratio of resource endowments increases, due an external shock, since the resource is not tradable, the ratio of prices (p_A) will fall, and initially, the agregate expenditure will not be affected (as in the case $\alpha = 1$). However, the industrial firms in region 1 will benefit from a cheaper input. This will allow them to reduce the price of industrial goods, increasing sales at expense of firms in region 2, causing an increase of the exports and a reduction of the imports of region 1. The trade balance of region 2 moves in the opposite direction. This increased demand for industrial goods in region 1, generates upward pressure over the price-ratio and wage-ratio, that will increase until the pressure disappears, ie when the trade balance again be in equilibrium.

Proposition 4 *When region 1 agregate expenditure is larger than region 2 agregate expenditure, the "expenditure effect" will reduce the ratio of wages (w).*

Proof. See the appdendix. ■

What happen is that, at the initial wages, an increase of the agregate expenditure (in region 1 for example), will rise the imports of region 1, wile export reduces. This causes an excess of supply of industrial goods produced in region 1, and an excess of demand of the goods produced in region 2, generating a downward pressure on the price ratio (p_1/p_2). Then, wages (w) and prices must fall, in order to reduce imports and to increase exports, on industrial goods from the region 1, untill the equilibrium is restored.

The expenditure effect works in the same direction as Krugman's *competition effect* but, the economic intuition behind these two, is different. The competition effect ocurs becouse a larger number of industrial firms have to compete for the same market.

The *home-market effect*, takes into account the creation of new varieties of industrial goods, assuming that the regions are symmetrical in their expenditures and resource stocks.

Proposition 5 *When a region produces a greater variety of industrial goods, the wage of that region will tend to rise, due to the "home-market effect".*

Proof. See the appdendix. ■

Because consumers prefer variety to quantity, the proportion of expenditure, form region 1 and 2, dedicated to industrial goods of region 1, will increase if $n_1 > n_2$. At the initial prices, this will causes an excess demand of goods produced in region 1. Then, prices and wages must increases too.

Before turning to analyse the long-run, we have one more question: What happen with transportation costs? The following proposition try to shed some light on this issue.

Proposition 6 *a) If expenditure of region 1 is greater than the expenditure of region 2 ($\varepsilon_1/\varepsilon_2 > 1$), an increase of τ will reduce the ratio of nominal wages. b) When region 2 has the biggest expenditure ($\varepsilon_1/\varepsilon_2 < 1$), an increase of τ will rise the wage ratio. c) And, if both regions are equal ($\varepsilon_1/\varepsilon_2 = 1$), an increase of τ won"t change the ratio of nominal wages.*

Proof. See the appendix. ■

The effect of a reduction of τ depends on the size of the economies. When the transport cost reduces, the locals will transfer expenditure from local to foreing goods, if we are in case a), where the region 1 is larger, in terms of total expenditure, the households in this region will find that import industrial goods from region 2, is now cheaper, so imports will grow. At the same time, industrial goods of region 1 will become more accessible for residents of region 2, so exports of region 1 will grow too. But becouse the agregate expenditure of region 1 is greater, imports grow more than exports, resulting in a trade imbalance, at the initial prices. The only way to correct the excess supply, is reducing prices, ie, reducing the ratio of wages. The opposite happens when the economies are in case b); the exports grow more than the imports, becouse the foreing market is "bigger", in terms of expenditures, generating an excess of demand, pressing prices and wages upward. When both economies have the same size, case c), the reduce in the transportation costs, causes displacement of local spending by foreign spending in equal measure. Exports and imports increase by the same amount. In conclusion, the largest economy, will suffer in terms of nominal wages due to lower transport costs, while the small economy will be benefit.

Now that we have clarify the behaivor of the economies in the short-run, we can begin to study their evolution over time. However, we need more information about the dynamic of the resource stocks. Regarding this, we can assume that its dynamics are driven by a logistic function:

$$\dot{S}_j = g_j S_j \left(1 - \frac{S_j}{CC_j}\right) - \Omega_j \tag{34}$$

We are considering renewable resources, wichs reproductions depends on the rate (g_j), the stocks of resources at each moment of time $(S_{j(t)})$, and the carring capacity (CC_j). The growth rate of the ratio of stocks (S) is:

$$\gamma_S \equiv g_1 \left(1 - \frac{S_1}{CC_1}\right) - g_2 \left(1 - \frac{S_2}{CC_2}\right) - \epsilon \left(l_{\Omega_1} - l_{\Omega_2}\right) \tag{35}$$

To this point, we can thus summarize the evolution of the economies by a system of four differential equations in ε_1, ε_2 , S_1 and S_2. Substituing equation (14) in the euler equations (4); taking into account the R&D production function (10), the free entry condition (11), and equation (27), we have:

$$\dot{\varepsilon}_1 = \varepsilon_1 \left[\eta \left(\varepsilon_1 - L_1\right) - \rho\right] \tag{36}$$

$$\dot{\varepsilon}_2 = \varepsilon_2 \left[\eta \left(\varepsilon_2 - L_2\right) - \rho\right] \tag{37}$$

$$\dot{S}_1 = S_1 \left\{ g_1 \left(1 - \frac{S_1}{CC_1}\right) - \epsilon \frac{\sigma - \mu \left[1 + \alpha \left(\sigma - 1\right)\right]}{\sigma} \varepsilon_1 \right\} \tag{38}$$

$$\dot{S}_2 = S_2 \left\{ g_2 \left(1 - \frac{S_2}{CC_2}\right) - \epsilon \frac{\sigma - \mu \left[1 + \alpha \left(\sigma - 1\right)\right]}{\sigma} \varepsilon_2 \right\} \tag{39}$$

Let γ_y be the growth rate of the variable y. Using the definition of the variable X and equation (30), we arrive to the growth rate of the wages:

$$\gamma_w = \left(1 - \alpha\right) \gamma_S + \frac{\gamma_n}{\sigma - 1} - \left[\frac{1 + XT}{1 + X^2 \left(\varepsilon_2 / \varepsilon_1\right)}\right] \frac{\gamma_{\varepsilon_1} - \gamma_{\varepsilon_2}}{\sigma - 1} \tag{40}$$

4 Long-Run Equilibrium without Migration

In this section of the paper, we assume that there is no migration between the regions. In a perfect-foresight growth equilibrium, each agent behaves optimally (taken as given the time paths of variables out of their control), and the growth rates of all variables are constant (ε_1, ε_2 , S_1 and S_2). A quick inspection of equation (30), reveals that $\dot{X} = 0$ when both economies are in their respective balance growth path (BGP). From now on, let γ_j^* $(j = 1, 2)$ be the growth rate of the economy, in the BGP; and asterisc in the superscript of a variable, indicates the equilibrium value. The solution of system (36 to 39) is:

$$\varepsilon_1^* = L_1 + \frac{\rho}{\eta} \tag{41}$$

$$\varepsilon_2^* = L_2 + \frac{\rho}{\eta} \tag{42}$$

$$S_1^* = \left\{ g_1 - \epsilon \frac{\sigma - \mu\left[1 + \alpha\left(\sigma - 1\right)\right]}{\sigma} \left(L_1 + \frac{\rho}{\eta} \right) \right\} \frac{CC_1}{g_1} \tag{43}$$

$$S_2^* = \left\{ g_2 - \epsilon \frac{\sigma - \mu\left[1 + \alpha\left(\sigma - 1\right)\right]}{\sigma} \left(L_2 + \frac{\rho}{\eta} \right) \right\} \frac{CC_2}{g_2} \tag{44}$$

The distribution of the labor force (L_1/L_2) is given, because of the no-migration assumption. And the resources stocks (43 and 44) ensure a sustainable path for both economies. By using the BGP values of the variables, and the system (30-33) we arrive to the following proposition:

Proposition 7 *Given a population distribution (L_1/L_2), and some initial conditions for the variety of goods and the resource stocks, a unique positive solution for the system (30 to 33 and 41 to 44) exists, along the balance growth path:*

$$X^* = \frac{\left(\varepsilon_1^*/\varepsilon_2^* - 1\right) T + \sqrt[2]{\left(\varepsilon_1^*/\varepsilon_2^* - 1\right)^2 T^2 + 4\left(\varepsilon_1^*/\varepsilon_2^*\right)}}{2}$$

$$L_{E_j}^* = \frac{\mu\alpha\left(\sigma - 1\right)}{\sigma}\left(L_j + \rho/\eta\right) + \phi \tag{45}$$

$$l_{\Omega_j}^* = \frac{\sigma - \mu\left[1 + \alpha\left(\sigma - 1\right)\right]}{\sigma}\left(L_j + \rho/\eta\right) \tag{46}$$

$$l_{n_j}^* = \frac{\mu}{\sigma}\left(L_j + \rho/\eta\right) - \left(\phi + \rho/\eta\right) \tag{47}$$

Proof. See in the appendix. ■

In the BGP, the economies growth rates are: $\gamma_j^* = \gamma_{w_j}$. We can only find the differential growth rates between the two economies ($\gamma^* = \gamma_1^* - \gamma_2^*$), just what equation (40) shows, but with the first and third terms equal to cero:

$$\gamma^* = \frac{\gamma_n}{\sigma - 1} \tag{48}$$

$$\gamma^* = \frac{1}{(\sigma - 1)} \frac{\eta\mu}{\sigma}\left(L_1 - L_2\right) \tag{49}$$

Proposition 8 *In a BGP, the ratio of aggregate consumption expenditure (E_1/E_2), grow at the rate γ^* given by (48).*

Proof. This comes straightfoward from equation (40) and the definitions of ε_1^* and ε_2^*. Since $r_j^* > \gamma_j^*$, household utility is always finite, and the relevant transversality condition is satisfied. ■

A few things can be said of distribution of the labor force between sectors and the growth rate, through equations (45, 46, 47 and 48). When the economies of scale increases (a reduce of σ), the industrial sector is more efficent and the labor force, released by this sector, can be devoted to research activities; although both regios will increase their work in research, it increases more in the most populous country, acelereting it's growth. If what rises is the proportion of the expenditure in industrial goods (μ), the demand for this goods will be greater, and more labor will be devoted to production and research of new varieties of goods; the primary sector will decline, and the largest region will benefit again, in terms of growth rates. If (η) is bigger, the rate of succes of the research is greater, other things equal, the R&D sector is more productive, attracting labor force from the others sectors, promoting the growth.

Thus, when the productive structure favors the industrial sector (low value of σ and high value of μ) and innovation (high value of η), the region with the largest populations can gain a greater advantage, in terms of growth.

5 Stability and Transition without Migration

In this section, we analyze the stability of the unique BGP found in the previous section. That is, although the economies might eventually grow indefinitely, could this sustained growth path be reached if the economies are not initially in equilibrium? The engienvalues asociated with our differential system, equations (36 to 39), are:

$$e_1 = \eta L_1 + \rho > 0 \tag{50}$$

$$e_2 = \eta L_2 + \rho > 0 \tag{51}$$

$$e_3 = -\left\{ g_1 - \epsilon \frac{\sigma - \mu \left[1 + \alpha\left(\sigma - 1\right)\right]}{\sigma} \left(L_1 + \frac{\rho}{\eta} \right) \right\} < 0 \tag{52}$$

$$e_2 = -\left\{ g_2 - \epsilon \frac{\sigma - \mu \left[1 + \alpha\left(\sigma - 1\right)\right]}{\sigma} \left(L_2 + \frac{\rho}{\eta} \right) \right\} < 0 \tag{53}$$

Where the model has two control variables (S_1 and S_2), therefore, the BGP can be caracterize as a saddle-point equilibrium.

5.1 Growth and Natural Resources

We have now all the elements to study the consecuences of a resource shock. It is evident from equation (48), that a shock in the endowment of natural resources, does not affect growth in the balanced growth path. However, it does have effects on wage levels, and the transition to the long-run equilibrium.

Because we are interested in the *resource curse*; assume that economies are in their respective balanced growth path, and that at some point in time, a positive shock take place, such that region 1 have, a resource stock greater than the equilibrium level ($S_1 > S_1^*$). This shock can be the result of a reduction in the regeneration rate (g_1) or the carring capacity (CC_1); or an increase in the productivity of extracction, but then, we should differentiate between the productivity of regions (ϵ_j). It is important to see that S_1 does not increase, is S_1^* what decreases.

In the short-run, the resource effect take place. The primary good is cheaper due to the increase in the supply, and the industrial firms take advantage of this situation. The exports of the region grow, and the excess of demand for industrial goods produced in region 1, start pushing prices upward; finally, the ratio of wages rise until the excess demand vanishes. But, the interpretation is quite different however. Instead of thinking in the resource effect as an increase in the ratio of wages from the path in which it was; what happens is that the path moves downwards. Both ways of seeing the effects are qualitatively similar, but the second, will tell us, if the ratio of wages is higher or lower than before the shock.

The effects over the transition to the balance growth path, however, are more complex, and relys on: the parameter values and the populations sizes. Lets call γ^T to differential growth rates in transición:

$$\gamma^T = (1-\alpha)\,\gamma_S + \frac{\gamma_n}{\sigma - 1} \tag{54}$$

Lets begin with the more interesting case, when ($L_1 > L_2$). In this case, we have two alternative scenarios, depending on the size of the resource shock ($\triangle S_{1t} \equiv S_{1t} - S_1^*$). When the shock is lower than certain threshold ($\triangle S^{th}$), we can observe a jump of the wage-ratio due to the resource effect, follow by a transition towards the BGP, where: $\gamma^* > \gamma^T > 0$. In the second scenario, the shock is bigger, such that $\triangle S_{1t} > \triangle S^{th}$, so the jump of the wage ratio is greater too, and the transition have two faces: 1) $\gamma^T < 0 < \gamma^* and;$ 2) $\gamma^* > \gamma^T > 0$, as in the first scenario. Where the threshold is:

$$\triangle S^{th} = \frac{\eta\mu}{\sigma}\frac{(L_1 - L_2)}{(\sigma - 1)}\frac{CC_1/g_1}{(1-\alpha)} \tag{55}$$

When the initial shock is larger than the threshold, the wage ratio is much larger than it's long-run value, due to the resource effect; so that the economy begins to experience negative growth rates, in a first face. Together with the wages, the initial shock start to vanishes, until it becomes lower than the threshold. At this point, the economy, again begin to grow, albeit at a lower rate than in the BGP. Finally, the transition growth rate converges to the long-run rate, which is equal to the growth rate before the shock (γ^* don't change at all). However, the ratio of wages in each moment of time (t) is now lower than before the shock.

Now we can turn to the simple cases. What happen when ($L_1 < L_2$)? We start with a negative differential growth rate. When the shock take place, as before, the resource effect move the wages path downward. Then, the growth rates in the transition to long-run equilibrium, will be even more negative than in the BGP. And, finally, if both economies were growing at the same rate (ie. $L_1 = L_2$), a natural resource shock, generates a negative growth rate during the transition to the BGP. Again, the ratio of wages, in the long-run, will be lower than before the shock.

As in the case of γ^*, certain regularities are observed with respect to pro-industry and pro-primary economies. When the values of the parameters are such that favor the primary activity (high values of σ and ϵ_1; and lower values of μ and α), the growth rate during the transition to the BGP will be lower than if the production structure favors the industrial sector (high values of μ, α and η; and lower values of σ and ϵ_1). This happens because the parameters affect both, the size of the shock, and the threshold, in opposite directions, accentuating adverse effects in the case of pro-primary economies. Furthermore, low values of α accentuates the resource effect, which decreases the ratio of wages in the long run.

6 Migratory Flows

Until this point, we had analyze a model where people work in the same region where they were "born". In this section, we want to change this assumption, allowing migratory flows between regions. We need to model this flows. First, we have to keep in mind that workers are interested not in nominal wages, but in real wages. Following Krugman (1991), the real wages of workers in each region are:

$$\omega_1 = w_1 P_1^{-\mu} p_{A_1}^{-(1-\mu)} \tag{56}$$

$$\omega_2 = w_2 P_2^{-\mu} p_{A_2}^{-(1-\mu)} \tag{57}$$

with P_j the price index of manufactured goods for consumers in region j, given by

$$P_1 = \left[n_1 p_1^{1-\sigma} + n_2 \left(\frac{p_2}{\tau} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{58}$$

in region 1 and, that for consumers residing in region 2 is

$$P_2 = \left[n_1 \left(\frac{p_1}{\tau} \right)^{1-\sigma} + n_2 p_2^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{59}$$

Price indexes P_j are weighted sums of the manufactures' prices given in (7). Lets normalize the total population to one ($L_1 + L_2 = 1$), where the dynamic of the population is driven by:

$$\dot{L}_1 = L_1 (1 - L_1) \left(\frac{\omega_1}{\omega_2} - 1 \right)$$

On the long-run, the labor force will move between the two regions until both real wages will be equal. Note that when L_1 approaches the unit, $\dot{L}_1$ slows down, until it becomes zero. The same happens when L_1 approaches zero. From equations (9, 17 and 30), and after some manipulation:

$$\dot{L}_1 = L_1 (1 - L_1) \left[\left(\frac{S_1}{S_2} \right)^{-\frac{(1-\mu\alpha)(\sigma-1)}{\mu}} \frac{X^2}{\frac{\varepsilon_1}{\varepsilon_2} \frac{n_1}{n_2}} - 1 \right] \tag{60}$$

Where X is a function of (ε_1 and ε_2) through equation (30). Using equations (10, 33 and 35) we found our last differential equation:

$$\dot{n} = \left(\frac{n_1}{n_2}\right)\eta\left[(2L_1 - 1) - \frac{(\sigma - \mu)}{\sigma}(\varepsilon_1 - \varepsilon_2)\right] \tag{61}$$

Now, the dynamics of our economies can be summarize by the system (36, 37, 38, 39, 60 and 61), in the variables: ε_1, ε_2, S_1, S_2, L_1 and n.

7 Long-Run Equilibrium with Migration

In this extended model, the Balance Growth Path is more restrictive, impling that: $\dot{\varepsilon}_1 = \dot{\varepsilon}_2 = \dot{S}_1 = \dot{S}_2 = \dot{n} = \dot{L}_1 = 0$. For now, lets focus only on the interior solutions, therefore two conditions must hold: $\omega_1 = \omega_2$ and $0 < L_1 < 1$.

Before beginning the analysis of the model itself, the following propositions shows the existence of an interior solution, and characterize it.

Proposition 9 *Along the BGP there exist unique solution for L_1^* and L_2^* for which real wages are equal in both economies.*

$$\varepsilon_1^* = \frac{1}{2} + \frac{\rho}{\eta} \tag{62}$$

$$\varepsilon_2^* = \frac{1}{2} + \frac{\rho}{\eta} \tag{63}$$

$$S_1^* = \left\{g_1 - \epsilon\frac{\sigma - \mu\left[1 + \alpha(\sigma - 1)\right]}{\sigma}\left(\frac{1}{2} + \frac{\rho}{\eta}\right)\right\}\frac{CC_1}{g_1} \tag{64}$$

$$S_2^* = \left\{g_2 - \epsilon\frac{\sigma - \mu\left[1 + \alpha(\sigma - 1)\right]}{\sigma}\left(\frac{1}{2} + \frac{\rho}{\eta}\right)\right\}\frac{CC_2}{g_2} \tag{65}$$

$$n^* = \left(\frac{S_1^*}{S_2^*}\right)^{-\frac{(1-\mu\alpha)(\sigma-1)}{\mu}} \tag{66}$$

$$L_1^* = \frac{1}{2} \tag{67}$$

Proof. See in the appendix. ∎

Observe that we have the "standar" result of the model without migration, for the variables ε, ε_2, S_1 and S_2. As Proposition (8) states, the agregate consumption expenditure in both regions, hence the economies, grows according to (48). However, the size of both economies L_1 and L_2 are not given, they are determined by expression (60). In the sustainable BGP, the population is equally distributed between regions.

Proposition 10 *In the sustainable BGP, the industrial price indexes are equal ($P_1 = P_2$).*

Proof. See in the appendix. ∎

The regions having the same sizes imply that their expeditures, expressed in a common currency, are equal too ($\varepsilon_1 w_1 = \varepsilon_2 w_2 q$). In this case, to maintain the trade balance in equilibrium, equation (77), the proportion of expenditure devoted to imports, must be equal to the share devoted for exports. In other words, the relative weight of the set of goods of region 1, against the set of goods in region 2, must be equal to one ($X = 1$). Ultimately, this means that both baskets of goods have the same weight. Because transportation cost are equal in both regions, and now, for the equilibrium of the trade balance, the baskets of industrial goods weight the same, the industrial price indexes are equal. However, the composition of the basket of goods (n, w and S), may vary among regions, depending on what are the initial conditions and the value of the parameters.

Proposition 11 *In the BGP, both economies grow at the same rate ($\gamma^* = 0$).*

Proof. It is obtained by substituting L_1^* into equation (48). ■

This comes as a corollary of the condition that ensure a sustainable balance growth path, and an interior solution. In terms of growth rates, this condition can be write as:

$$\gamma_{w_1} - \gamma_{w_2} = \mu \left(\gamma_{P_1} - \gamma_{P_2}\right) + (1-\mu)\left(\gamma_{p_{A_1}} - \gamma_{p_{A_2}}\right)$$

As a consecuence of Proposition (10) and equation (30), the industrial price indexes are constant in the long-run equilibrium. According with the previous equation, the ratio of wages can only grow if the price ratio of the resorces grow too. But, since we are in a sustainable equilibrium, the stock of natural resorces are constant. To maintain the real wages equal (ie. to be in the interior solution), nominal wages must remain constant over time.

7.1 Effects of Differences in the Natural Resources

Although, in the model with migration, economies grow at the same rate, we can analyze what happens to the level of industrialization, and nominal wages, when a region has more natural resources than the other.

Proposition 12 *In a BGP, the economy with greater stock of natural resources will have lower wages and less variety of industrial goods, according to:*

$$\frac{w_1}{w_2} = \left(\frac{S_1}{S_2}\right)^{-\frac{(1-\mu)}{\mu}} \tag{68}$$

$$\frac{n_1}{n_2} = \left(\frac{S_1}{S_2}\right)^{-\frac{(1-\mu\alpha)(\sigma-1)}{\mu}} \tag{69}$$

Proof. See in the appendix. ■

Supose that region 1 have a gretar natural resource stock than region 2. The supply of primary goods is greater in the region with more natural resource stock, making its price lower. As $P_1 = P_2$, according to Proposition (10), the nominal wage should be higher in the region with the lowest natural endowment to maintain the balance of real wages, and to avoid migratory flows. However, with other things equal, the highest nominal wage create a trade imbalance between regions. Spending on goods in region 1 are now greater than the income of the region. Or, exports of region 1, exceed imports. To prevent this imbalance, region 2 must have a greater variety of industrial goods, to attract the excess demand, through the channel of the home-market effect.

Proposition 13 *An increase of τ won't cause any changes.*

Proof. See in the appendix. ■

Becouse both regions are equaly populated, the relative weight of the set of goods from both regions are equal ($X = 1$), so the industrial price indexes are equal too, as we know from Proposition (10). In this context, a reduction of the transportation costs, benefits the regions equaly, mantaning $P_1 = P_2$ and $\omega_1 = \omega_2$. While imports and exports increases in the same amount in the two regions. As a result, nor the wages-ratio, nor varieties-ratio, changes.

8 Conclusions

We have studied a model of trade between two countries. Both regions are endowed with a renewable natural resource which is used as input and it is also a final consumption good . The natural resource is not tradable. The model has the main elements of the new economic geography like transportation costs and economies of scale.

In the model without migration, a positive shock in the resource sector, causes lower growth rates, or even negative, during the transition to the long-run equilibrium, and a lower level of wages, due to the *resource effect*. While, in the model that allow migratory flows, both economies grow at the same rate, but, there is a negative relationship between natural endowment and industrialization. This comes as a result of the interacction of the resource, expenditure and home-market effects, toguether with the real wage equalization.

The next step in our research is to identify the transitional dynamics of this last model, in order to clarify the interacctions, as we have done for the model without migration.

9 References

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10 Appendix

Proof of proposition 2: Solving the system (30 to 33) we arrive to:

$$(\varepsilon_1/\varepsilon_2 - 1)\,T + (\varepsilon_1/\varepsilon_2)\,X^{-1} - X = 0 \tag{70}$$

Making use of the definition of X, the derivative of the last expression with respect to the rate of wages is:

$$\frac{(\sigma-1)}{w_1/w_2}\left[(\varepsilon_1^*/\varepsilon_2^*)\,X^{-1} + X\right] > 0 \tag{71}$$

In the situation of symmetry, we have that $L_1 = L_2$ and $S_1 = S_2$, this implies that the labor devoted to research is the same in both regions ($l_{n_1} = l_{n_2}$, $L_{E_1} = L_{E_2}$ and $l_{\Omega_1} = l_{\Omega_2}$); so the variety of industrial goods will also be equal ($n_1 = n_2$). Is easy to see that $w_1 = w_2$ is a solution in this case, and considering (71), this is a unique solution.

If now we consider an increase in S_1, nothing will change in the last equations; and this is because nor prices of industrial goods nor expenditures are affected by this change in the endowments. The case of the prices need not demonstration, this is because the primary good take no place in the production of industrial goods. We turn to see what happen with the expenditure in the region affected by the boom. Making use of the definition of ε_1 and equations (17, 28, 45 and 47) we can express the total agregate expenditure of region one as:

$$E_1 = \frac{\mu\left[1+\alpha\left(\sigma-1\right)\right]}{\sigma}\left(L_1 + \frac{\rho}{\eta}\right)w_1 + \Omega_1 p_{A_1}$$

Now, holding constant the wages, we derivate this expression with respect to S_1.

$$\frac{\partial E_1}{\partial \Omega_1} = \frac{\partial p_{A_1}}{\partial S_1}\Omega_1 + p_{A_1}\frac{\Omega_1}{S_1} \tag{72}$$

$$\frac{\partial E_1}{\partial \Omega_1} = -\frac{p_{A_1}}{S_1}\Omega_1 + \frac{p_{A_1}}{S_1}\Omega_1 = 0 \tag{73}$$

Turning now to equation (70) with this result, the wages must remain equal. Because the resource is non-tradable, their owners must consume it all, so the price must fall until they are willing to do it, this means that their incomes will not change.

Proof of proposition 3: Note that expressions (70 and 71) are valid in the case of $\alpha < 1$; then, using the same reasoning as in Proposition (2), we can arrive to the same result for the wages ($w = 1$). But, what happen when S increases? Using the definition of X we have that:

$$\frac{\partial\left(w_1/w_2\right)}{\partial S} = (1-\alpha)\,\frac{w_1/w_2}{S_1/S_2} > 0$$

The ratio of wages must increase.

Why this happen? The boom has not effect over the expenditures of the regions, as before. However, it will affect the price-ratio of industrial goods, because the primary good is used for industrial production. Since the price of the resource reduces due to the increase in the supply, all firms in region 1 can benefit from this lower primary price. To see this, lets take the derivative of the price-ratio holding constant the ratio of wages (for now).

$$\frac{\partial\left(p_1/p_2\right)}{\partial S_1} = (1-\alpha)\,\frac{\left(p_1/p_2\right)}{p_{A_1}}\frac{\partial p_{A_1}}{\partial S_1} < 0$$

This reduction in the price-ratio will lead to an increase in the demand of the industrial goods of region 1 and a decrease of the demand of goods from region 2. Again, we can see this (holding constant the ratio of wages) looking at the following derivatives:

$$\frac{\partial z_{11}}{\partial\left(p_1/p_2\right)} < 0 \quad \text{and} \quad \frac{\partial z_{12}}{\partial\left(p_1/p_2\right)} < 0 \tag{74}$$

$$\frac{\partial\left(\frac{z_{11}}{1+z_{11}}\right)}{\partial z_{11}} > 0 \quad \text{and} \quad \frac{\partial\left(\frac{z_{12}}{1+z_{12}}\right)}{\partial z_{12}} > 0 \tag{75}$$

$$\frac{\partial\left(\frac{1}{1+z_{11}}\right)}{\partial z_{11}} < 0 \quad \text{and} \quad \frac{\partial\left(\frac{1}{1+z_{12}}\right)}{\partial z_{12}} < 0 \tag{76}$$

Now is time to see if the wages change. We can rewrite equation (30) as function of z_{11} and z_{12}, then replace the balance growth path values for ε_1 and ε_2, and find out that: w must increase, until the equilibrium between imports and exports, is restored.

$$\overbrace{\frac{\varepsilon_1/\varepsilon_2}{1+z_{11}}}^{\text{Imports of region 1}} = \overbrace{\frac{z_{12}}{1+z_{12}}}^{\text{Exports of region 1}} \tag{77}$$

Proof of proposition 4: To see the expenditure effect, we must assume that the regions are symmetrical in every aspect but their expenditures. This is equivalent to assuming that $n = 1$ and $S = 1$ (and constant). Consider the case in which the region 1 is the most populated. Due to our assumptions, the only difference is that: $\varepsilon_1 > \varepsilon_2$. From the definition of X and equation (30):

$$\frac{\partial (w_1/w_2)}{\partial (\varepsilon_1/\varepsilon_2)} = -\frac{1}{(\sigma-1)} \frac{w_1/w_2}{X} \frac{\partial X}{\partial (\varepsilon_1/\varepsilon_2)} < 0$$

$$\text{Where} \quad : \quad \frac{\partial X}{\partial \left(\frac{\varepsilon_1}{\varepsilon_2}\right)} = \frac{T + X^{-1}}{X^{-2}\frac{\varepsilon_1}{\varepsilon_2} + 1} > 0$$

Proof of proposition 5: Using equation the definition of X, and holding constant the expenditures and the ratio of natural resources, we arrive to:

$$\frac{\partial (w_1/w_2)}{\partial (n_1/n_2)} = \frac{1}{(\sigma-1)} \frac{w_1/w_2}{n_1/n_2} > 0$$

Proof of proposition 6: Using expression (70) and the definition of X, we obtain:

$$\frac{\partial w}{\partial \tau} = \frac{w}{X} \frac{(1 - \varepsilon_1^*/\varepsilon_2^*)\, \tau^{\sigma-2}}{\varepsilon_1^*/\varepsilon_2^* X^{-2} + 1}$$

Proof of proposition 7: Given a population distribution ($L_1 > 0$ and $L_2 > 0$), the BGP values of ε_1 and ε_2 can be found from equations (36 and 36). With these, equations (31-33) can be solved. Then, equation (30) can be re-write as follows:

$$\overbrace{\frac{\varepsilon_1^*/\varepsilon_2^*}{X}}^{LHS} = \overbrace{\frac{X+T}{XT+1}}^{RHS}$$

$$\lim_{X \to 0} LHS = \infty \quad ; \quad \lim_{X \to 0} RHS = T$$

$$\lim_{X \to \infty} LHS = 0 \quad ; \quad \lim_{X \to \infty} RHS = T^{-1}$$

$$\frac{\partial LHS}{\partial X} = -\frac{\varepsilon_1^*/\varepsilon_2^*}{X^2} < 0 \quad ; \quad \frac{\partial RHS}{\partial X} = \frac{1 - T^2}{(XT+1)^2} > 0$$

Then, there must be a unique positive solution for this problem, when $\varepsilon_1^*/\varepsilon_2^* > 0$.

Proof of proposition 9: The balance growth values: ε_1^*, ε_2^*, S_1^* and S_2^* are the same as for the model without migration. Making $\dot{n} = 0$, and replacing equations (41 and 42) into (61) we obtain: $L_1 = 1/2$. Then, replacing this value into (41, 42, 43 and 44), the rest of the balance growth values are obtain. Solving equation (30) we found that $X = 1$. Finally, in order to obtain an interior solution, we have, from equation (60), that: $n^* = (S^*)^{-\frac{(1-\mu\alpha)(\sigma-1)}{\mu}}$.

Provided that certain restrictions on the parameters are fulfilled, will be $0 < L_1 < 1$, and therefore the solutions for X and Y are also positive.

Proof of proposition 10: The ratio of the between the price index of manufactured goods can be transform such that:

$$P \equiv \left(\frac{P_1}{P_2}\right)^{1-\sigma} = \frac{X+T}{XT+1} \tag{78}$$

$$\frac{\partial P}{\partial X} = \frac{1-T^2}{(XT+1)^2} > 0 \tag{79}$$

After a quick inspection, is easy to see that: when $X = 1$, $P = 1$. The unique solution for equation (30) when $\varepsilon_1^*/\varepsilon_2^* = 1$ (ie. populations are equal), is $X = P = 1$, this comes straightforward from substituing P into equation (30).

Proof of proposition 12: In the interior solutions, real wages must be equal, two way to write this equality are:

$$\frac{w_1}{w_2} = \left(\frac{S_1}{S_2}\right)^{-\frac{1-\mu}{\mu}} P^{\frac{1}{1-\sigma}} \tag{80}$$

$$\frac{X}{P} = \frac{n_1}{n_2}\left(\frac{S_1}{S_2}\right)^{\frac{(1-\mu\alpha)(\sigma-1)}{\mu}} \tag{81}$$

Substituing, in this two equations, the balance growth values ($X = P = 1$), expressions (68 and 69) can be easily obtain.

Proof of proposition 13: through equation (80) and the definition of the variable P (78), we can obtain the elasticity of the wage ratio with respect to the transportation costs:

$$\xi^m_{w,\tau} = -\frac{1}{\sigma-1}\xi^m_{P,\tau}$$

$$\xi^m_{P,\tau} = \frac{(\sigma-1)T}{P(XT+1)^2}\left[\frac{\left(\frac{\varepsilon_1}{\varepsilon_2}X^{-2} - X^2\right) + T^2\left(1-\frac{\varepsilon_1}{\varepsilon_2}\right)}{\frac{\varepsilon_1}{\varepsilon_2}X^{-2}+1}\right]$$

While, making use of the last two expressions and (70), toguether with the definition of X, the elasticity of the varieties-ratio is:

$$\xi^m_{n,\tau} = \xi_{X,\tau} - \xi^m_{P,\tau}$$

$$\xi^m_{n,\tau} = \frac{-T}{\left(\frac{\varepsilon_1}{\varepsilon_2}X^{-2}+1\right)}\left[\frac{(\sigma-1)\left(1-\frac{\varepsilon_1}{\varepsilon_2}\right)}{X} + \frac{\left(\frac{\varepsilon_1}{\varepsilon_2}X^{-2} - X^2\right) + T^2\left(1-\frac{\varepsilon_1}{\varepsilon_2}\right)}{P(XT+1)^2}\right]$$

On one hand, we know that when $\frac{\varepsilon_1}{\varepsilon_2} = 1$ the unique positive solution is: $X = P = 1$, and that P is an increasing function of X. On the other hand, from equation (70), we have:

$$\frac{\partial X}{\partial \left(\frac{\varepsilon_1}{\varepsilon_2}\right)} = \frac{T + X^{-1}}{X^{-2}\frac{\varepsilon_1}{\varepsilon_2} + 1} > 0$$

With this information and comparing the variables $(\varepsilon_1/\varepsilon_2)$, X and P, we can obtain this relations:

When: $\frac{\varepsilon_1}{\varepsilon_2} < 1 \rightarrow \frac{\varepsilon_1}{\varepsilon_2} < X < P < 1$ and $\frac{\varepsilon_1}{\varepsilon_2} > X^2$ then $\xi^m_{w,\tau} < 0$, $\xi^m_{n,\tau} < 0$

When: $\frac{\varepsilon_1}{\varepsilon_2} = 1 \rightarrow \frac{\varepsilon_1}{\varepsilon_2} = X = P = 1$ and $\frac{\varepsilon_1}{\varepsilon_2} = X^2$ then $\xi^m_{w,\tau} = 0$, $\xi^m_{n,\tau} = 0$

When: $\frac{\varepsilon_1}{\varepsilon_2} > 1 \rightarrow \frac{\varepsilon_1}{\varepsilon_2} > X > P > 1$ and $\frac{\varepsilon_1}{\varepsilon_2} < X^2$ then $\xi^m_{w,\tau} > 0$, $\xi^m_{n,\tau} > 0$