The Vicious Cycle of Sovereign Debt and Interest Rates in the Euro Area

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ABSTRACT

In this paper we present an alternative via on modeling the fair value of Sovereign bonds based on the expected evolution of debt ratios. To simulate government debt ratios, we estimate a Vector Autoregressive (VAR) model for the nominal economic growth rate, the primary balance, and the parameters of the sovereign yield curve for the main economies of the euro area. This model contains forward-looking expectations of the interactions between the yield curve and fiscal variables relevant to forecast the debt ratio. From these simulations, we obtain a probability of unsustainable path of the debt ratio for each country and period. Finally, we link these probabilities with the observed yields by a Kalman filter to obtain a price of risk and a measure of a fair yield.

Key words: Sovereign Bonds, Fair Yields, Debt sustainability

JEL codes: G12; G17; H63

Introduction

The purpose of this research is to revisit the concept of sovereign bonds "Fair Value" from a *fundamental* point of view. This fundamental value should be the result of adding to the risk-free rate a credit risk premia. The main problem of such a model is that the credit risk premia is unobservable, as well as time variant. So, the yield of a sovereign bond with a given maturity could be decomposed in three components, all of them unobservable (with the exception of the risk-free rate that could be identified with the OIS rate) and time variant:

$$y_{it} = \alpha_t + \beta_t PD_{it} + \eta_{it} \qquad (1)$$

In this model (1), the sovereign bond **yield** (y_{it}) for a country *i* and a quarter *t* **depends** on the **probability of default of the sovereign** (PD_{it}). β_t is the price of risk that market participants demand for the perceived risk of their investment, that is generally considered to be time-variant and related to investors risk-aversion. α_t is the risk-free rate, i.e. the expected yield conditional on a null probability of default, that is also time-variant. In principle, we can consider the OIS for the same maturity as a good proxy for the risk-free rate. Finally, there is an error term (η_{it}) that will be greater as the bond yield depart from its fundamental value. Therefore, (η_{it}) could be considered as the outcome of market malfunctioning. This variable could be either positive or negative: positive, if investors ask for a yield well above the fundamental value (e.g.: due to an excessive penalty on the sovereign bond); or negative, if yield is below fundamental (e.g.: if the bond is benefiting from a flight-to-safety episode). Thus, we can consider that a "fair" yield would be $E[y_{it}|PD_{it}, \alpha_t, \beta_t]$.

Sustainability of the Government Debt

The probability of default of a sovereign (PD_{it}) used in model (1) is not directly observable, thus we need to find a way to estimate it. We can safely assume that the probability of default of a sovereign will mainly depend on the **perceived sustainability of the Government Debt** over GDP (D_t). Investors would consider to invest in a sovereign only if they are convinced that Government Debt is sustainable in the sense of not being a Ponzi-scheme (new bonds are issued to pay interests and amortize older bonds). By contrast, in a non-sustainable debt dynamic, the Government can only service its debt via new debt which would escalate out of control and the Government would ultimately default.

So, we have to forecast the evolution of the debt ratio in order to assess if the dynamic behaviour of the ratio is sustainable or not. The evolution of the debt ratio is a well-known identity,

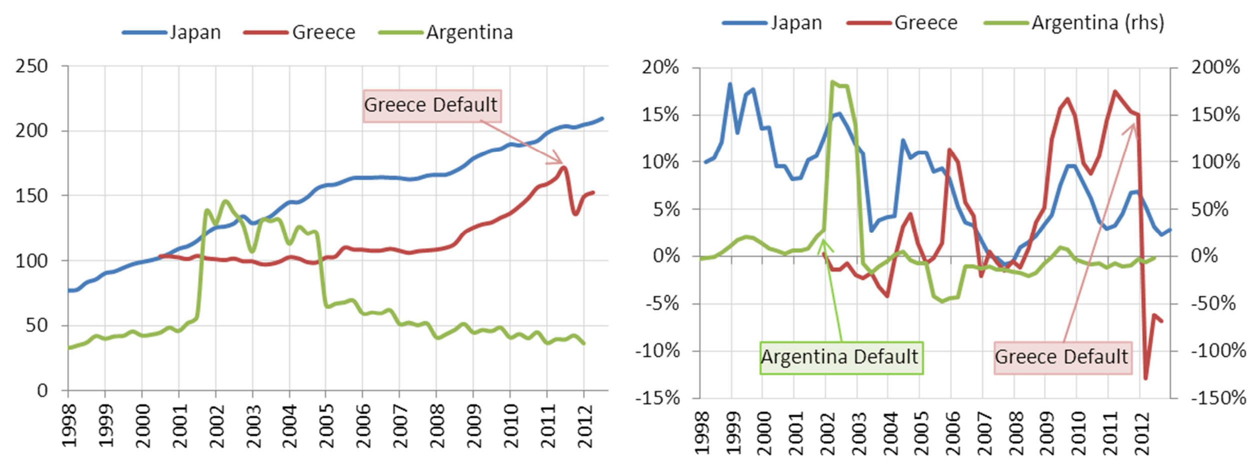
$$D_t = \frac{1+i_t}{1+g_t} D_{t-1} - pb_t + rs_t \qquad (2)$$

where D_t is the debt to GDP ratio, i_t is the interest rates paid for the outstanding debt, g_t is the GDP growth in nominal terms, pb_t is the public primary balance (public surplus pre interest payments), whereas rs_t are other shocks to the debt not coming from a primary deficit. These shocks could be, for instance, the product of injections into the financial sector in the form of asset purchases, movements of exchange rates affecting debt issued in other currencies, or privatization of public companies (in the part not accounted as a gain).

High debt levels produce large interest payments accelerating the process of debt growth. Nevertheless, high level of debt is not a sufficient condition for unsustainability. As can be seen in equation 2, increases in debt payments can be compensated by increases in the economic growth rate and/or the primary balance. It could also be compensated by a lower interest rate. In fact, a level of debt that can be sustainable for a country with high growth potential and solid fiscal position could be unsustainable for other with weak economic growth or with fiscal problems.

Governments can finance large amounts of debt if investors consider that their fundamentals (economic growth and primary deficit) are strong enough to potentially decelerate any debt expansion, while investors might penalize other countries with lower level of debts if they consider that the debt is not going to stabilise in the foreseeable future. Japan is a paradigmatic example of a country with a high debt over GDP that had no problem in financing its debt. By contrast, other defaulting countries like Argentina (2002) or Greece (2012) had no higher debt than Japan in the same moment (Figure 1).

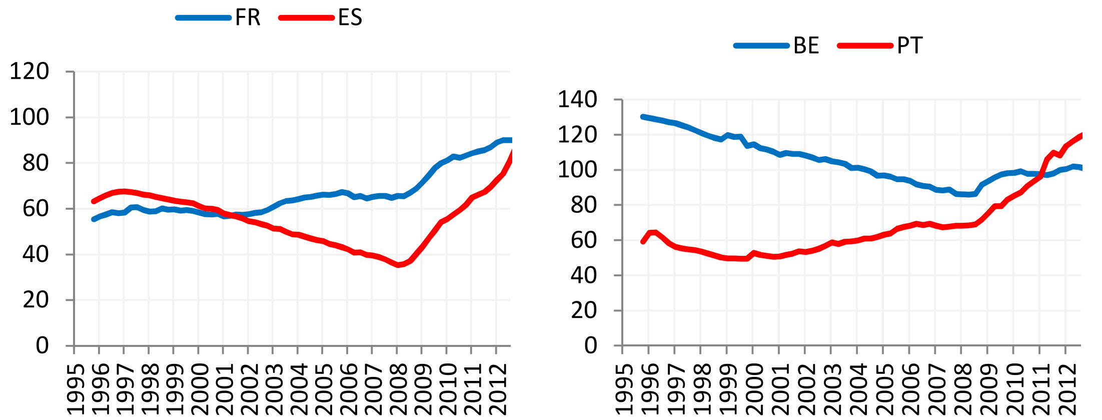
Figure 1: Debt over GDP for Argentina, Greece and Japan.
Level of debt in the left chart and annual growth rate on the right.



Sources: ECB SDW for Greece, INDEC and MECON for Argentina, Cabinet Office and Ministry of Finance for Japan

Something similar happened in the European Sovereign crisis, where those countries most affected by the crisis have not been necessarily those with the higher level of debt, but those where debt was growing at a faster path (Figure 2).

Figure 2: Level of debt over GDP for selected euro area countries



Evolution of the Government Debt over GDP ratio for France and Spain (left) and Belgium and Portugal (right). Source: ECB SDW

So, if the debt level is not the relevant feature for a default we have to look at the dynamics. A straightforward way to look at the sustainability of the government debt is analysing the growth rate of debt. From equation 1, and considering that rs_t is the result of one-off shocks to the level of debt, and can be discarded in the long run, the evolution of debt could be described by equation 3,

$$E[D_t] = \left[\frac{1+i_t}{1+g_t} - \frac{pb_t}{D_{t-1}}\right] D_{t-1} \tag{3}$$

If the number in brackets is expected to be consistently above 1, we could consider that the debt dynamics is unsustainable.This is the case if the interest rate satisfies the following inequality:

$$i \quad > (1+g^*)\left(1+\frac{pb^*}{D_{t-1}}\right)-1 \tag{4}$$

So, we can consider sustainable interest rates as a function of the actual level of debt, economic growth rate and fiscal primary balance. Alternative estimates for sustainable interest rates are shown in Table 1 for different assumptions about the values of g^* and pb^*.

Table 1: Computation of sustainable interest payments

	DE	FR	NL	ES	IT	BE	AT	FI
i_{2012Q4}	3.04	2.85	2.69	3.53	4.40	3.44	3.47	2.52
Maximum Potential								
$g*$	1.46	1.44	2.21	2.31	1.47	1.57	1.79	2.18
$pb*$	1.14	0.45	1.41	1.08	1.36	1.71	0.91	2.45
$i*$	11.51	7.77	17.13	14.98	10.22	13.10	12.20	27.04
Average Potential								
$g*$	0.54	0.76	0.85	1.13	0.61	0.87	0.85	0.89
$pb*$	0.20	-0.25	0.21	-0.19	0.47	0.78	0.18	1.05
$i*$	3.15	1.91	4.64	3.55	3.92	6.59	4.36	11.42
Average Potential in economic expansions								
$g*$	0.68	0.87	1.06	1.52	0.86	0.96	0.98	1.17
$pb*$	0.16	-0.21	0.29	0.20	0.50	0.82	0.18	1.11
$i*$	3.54	2.50	5.94	7.13	5.05	7.13	4.89	12.93

*g** is the quarterly nominal GDP growth rate, *pb** is the quarterly fiscal primary balance over GDP, and *i** is the level of interest rates that produce no growth in the level of debt for a given level of D_t, *g** and *pb**. As a term of comparison, we show in the first row the actual interest payments in 2012q4. *Maximum Potential* refers to the maximum level of both economic growth rate and primary balance in the period 2001-2012. *Average Potential* refers to the average of the same variables. Finally, *Average Potential in economic expansions* refers the mean values over those periods where the economic growth rate was positive. In the three cases, the debt level considered is the one prevailing at the end of 2012q4.

A first alternative for calculating an upper bound to the sustainable interest rate is to consider the best possible situation, where both variables are in the maximum of the registered data for each country over the period 2001-2012. This produce interest rates that are, by far, in excess to the ones government are actually paying. For instance, in the case of Spain, while the current average interest rate is 3.5%, the sustainable rate obtained with this assumption is equal to almost 15%.

A second alternative is to consider for potential economic growth rates and primary balance the average valuesover the period 2001-2012. Results on this second option are also presented in Table 1. Under this assumption, actual interest payments would be above the sustainable rate for both France and Italy, and close to it in the cases of Spain and Germany. This would imply that countries affected by the crisis are in the same position as others that have been considered safe havens.

If we consider the second option as too pessimistic, the third option considers the average of economic growth rates and primary balance only when GDP growth is positive. This also produces some outputs difficult to reconcile with market developments (i.e. France is the only one with “unsustainable debt”; Germany is closer to unsustainable rates than either Spain or Italy).

We can argue that these alternatives are backward looking and not consider, as market participants do, the future evolution of these economic variables and the potential to enhance them via reforms. In order to consider a forward looking perspective, we need to specify a model that allow us to replace both g^* and pb^* by their expectations.

In fact, macroeconomic variables, like GDP growth, interest rates or the primary balance, tend to be highly correlated. Changes in the economic growth trigger automatic stabilisers in government budgets (reduction of tax income, increase in unemployment benefits). Government tend to respond to rising indebtedness by stepping up fiscal consolidation (Bohn, 1998; Ostry, et al., 2010).Growth is impeded by high levels of debt (Reinhart and Rogoff, 2010; Kumar and Woo, 2010; Checherita-Westphal and Rother, 2010). Finally, high government debt ratios contribute to rising sovereign long term interest rates (Codogno, et al., 2007; Ardagna, et al., 2004; Attinasi, et al., 2009; Von Hagen, et al., 2011). Therefore, we need to specify a model where all variables involved in equation (2) can affect all the other ones. The simplest way to do so is to use a VAR model.

Note that the interest rate obtained from the VAR model at a given point in time refers to the interest rate to be paid on debt issued at that point in time. The average interest rate on debt, i.e. the one entering equation (3) is a (weighted) average of previous interest rates. By replacing interest payments with market interest rates we are able to include in the model market expectations on the future evolution of both economic growth rates and primary balance that are closely followed by market participants.

Macro-finance model

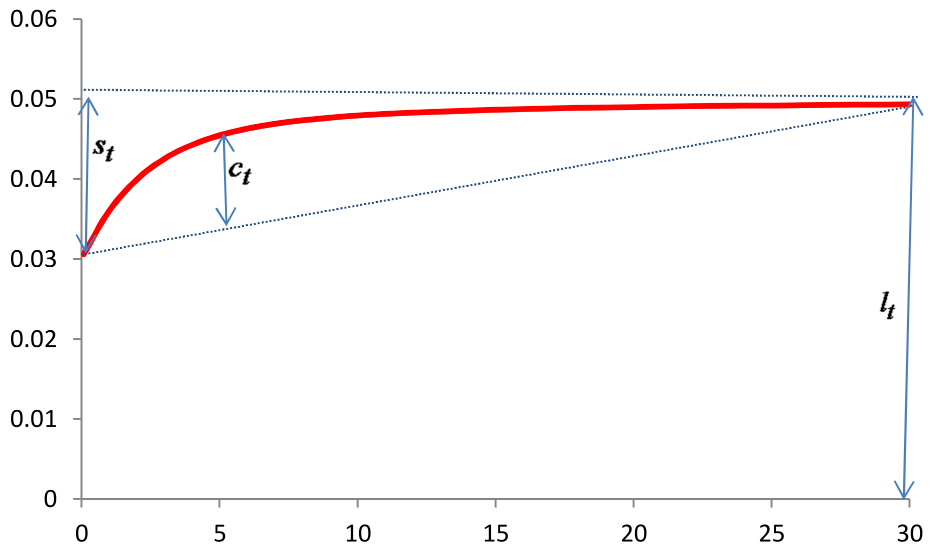
The model we are going to use for describing the evolution of the variables discussed in previous section is a simple VAR model,

$$X_{it} = \mu + \Phi X_{it-1} + \varepsilon_{it} \quad (5)$$

Where $X_{it} = (l_{it},\ s_{it},\ c_{it}, pb_{it}, g_{it})'$ and $\varepsilon_{it} \sim N(0, \Sigma)$. l_{it}, s_{it} and c_{it} correspond to the level, slope and curvature parameters in a Nelson-Siegel yield curve model (NSS, Nelson and Siegel, 1989), as in equation (6), estimated from sovereign bond market prices. Litterman and Scheinkman (1991) found in a component analysis of the term structure of US bonds that 3 factors (that they also identified with the level, slope and curvature) are enough to describe 98% of the differences across maturities. In a NSS specification, these parameters allow determining the shape of the sovereign yield curve (Figure 3): l_{it} is the long term rate (maturity→∞); $l_{it} + s_{it}$ is the short term rate (maturity→0); while c_{it} is multiplied by zero at both extremes of the curve, thus representing the shape for the middle of the curve.

$$y_{t,t+k} = l_t + s_t \frac{1 - e^{-k/\tau}}{k/\tau} + c_t \left(\frac{1 - e^{-k/\tau}}{k/\tau} - e^{-k/\tau} \right) \quad (6)$$

Figure 3: Interpretation of level (l_{it}), slope (s_{it}) and curvature (c_{it}) in a NSS model



Diebold and Li (2006) used these same parameters of the yield curve to successfully forecast the evolution of interest rates in a VAR model. Gimeno and Marqués (2009, 2012) used the same parameters as well as inflation showing the forecasting potential that they have over inflation. In our model, by combining interest rates parameters, as well as primary deficit and nominal economic growth rate, we are able to capture the information that financial markets (NSS parameters) have about macroeconomic variables, as well as how markets react to changes in these macro variables. This implies that the model will produce expectations on macro variables that are more forward-looking than just using the interest rate paid on outstanding debt.

Interest rates in the fiscal policy literature usually differ from the ones considered in the analysis of financial markets. In the later (as in this model), the relevant magnitude is the yield curve of the secondary markets. This contains information on investors' expectations, but from a fiscal point of view would be relevant only in the hypothetical case of the full substitution of the stock of debt. Fiscal literature is generally more interested in the amount that a government is paying in a given moment, although the stock of debt has interests agreed in the moment of issuance (maybe many years ago) that have a weaker link with macro variables (and usually backward-looking). In the next section, we will try to reconcile both magnitudes.

One of the caveats in this case for the VAR model is the scarcity of data. Relevant macro variables (pb_{it} and g_{it}) have a quarterly frequency and we have only national yield curves from 2004. Therefore we only have around 30 quarters to estimate the model. In order to avoid this problem we consider that the parameters of the model should be the same for all countries, estimating a single model for the eight countries considered[1]. Results can be shown in Table 2.

We have also estimated an affine model, where equation 5 is jointly estimated with an arbitrage free term structure of interest rates (see annex 1). Results of this alternative model are also displayed in Table 2. As can be seen, estimated parameters are quite in line with the ones of the unrestricted VAR model[2].Since the purpose of the specified model is to recover the dynamical evolution of macro-finance variables, we have opted for the simpler VAR model.

Results are in line with what we would expect. Increases in nominal economic growth or primary balance reduce the long term level of interest rates. An increase in the later, also produce a negative effect in both economic growth and primary balance. Similarly, a reduction in the short term of interest rates produces a clear decline in economic growth. Impulse-Response Functions are showed in Annex 2.

Table 2: VAR and Affine estimates of equation 5

	l_t		s_t		c_t		g_t		pb_t	
	VAR	Affine	VAR	Affine	VAR	Affine	VAR	Affine	VAR	Affine
Constant	0.402	0.379	0.642	0.646	0.940	0.991	1.277	1.171	0.109	0.090
l_{t-1}	0.991	0.994	-0.488	-0.489	-0.253	-0.258	-0.310	-0.294	-0.073	-0.065
s_{t-1}	0.082	0.083	0.640	0.640	-0.008	-0.010	-0.113	-0.119	-0.006	0.000
c_{t-1}	0.003	0.002	-0.076	-0.081	0.999	1.005	-0.006	0.000	-0.001	0.000
g_{t-1}	-0.105	-0.105	0.574	0.575	0.043	0.044	0.759	0.790	0.266	0.265
pb_{t-1}	-0.056	-0.058	-0.018	-0.019	-0.177	-0.176	-0.164	-0.161	0.763	0.758

Estimates of $X_{it} = \mu + \Phi X_{it-1} + \varepsilon_{it}$. Parameter$l_{it}$ represents the level of the yield curve (NSS) of country i and quarter t, $s_{it}$is the slope and c_{it} is the

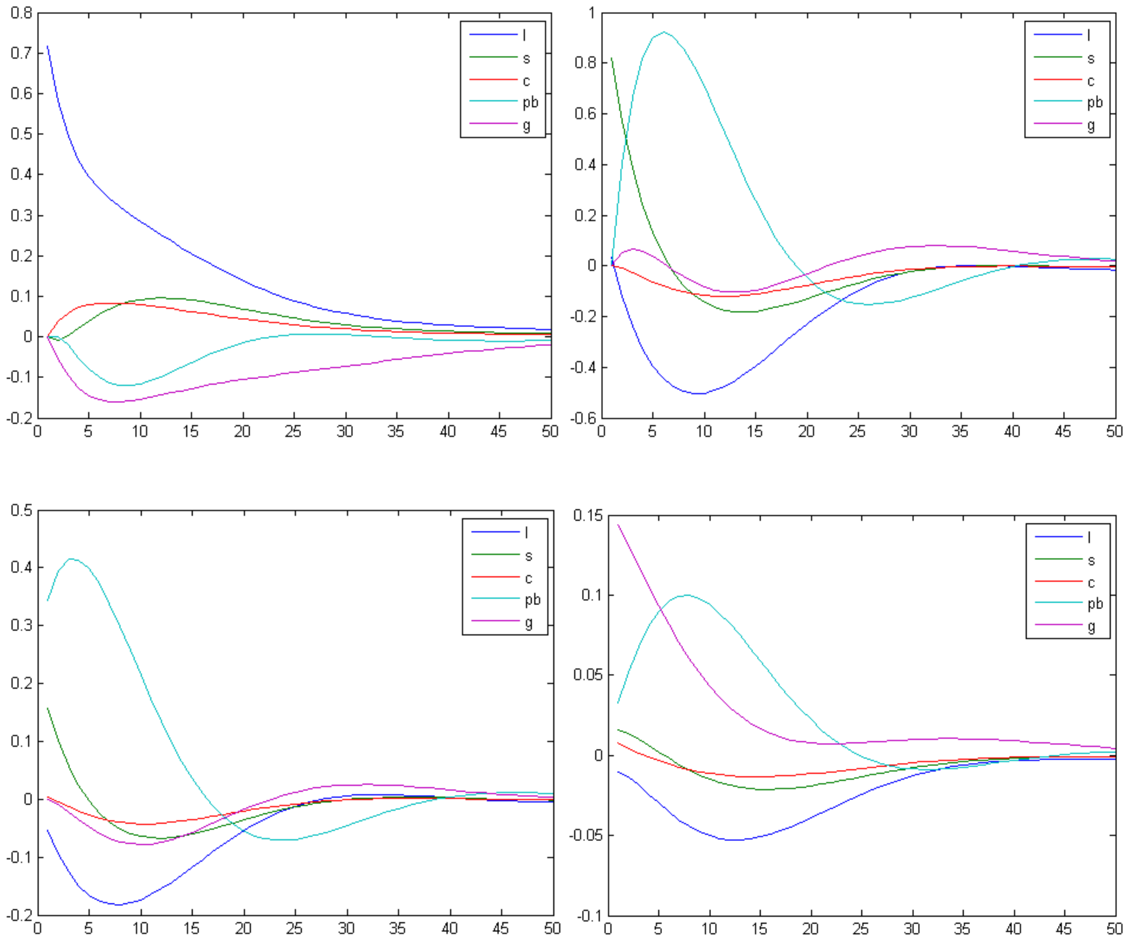
[1] Austria, Belgium, Finland, France, Germany, Italy, Netherlands and Spain.

[2]Joslin et al. (2012) showed that in an affine model the no arbitrage conditions will have a very limited effect on the VAR equation.

curvature. *g* is the nominal economic growth rate and *pb* is the fiscal primary balance. Observations correspond to quarterly data of Austria, Belgium, Finland, France, Germany, Italy, Netherlands and Spain from 2004Q1 to 2012Q4. Source: ECB SDW. VAR has been estimated by least squares. Affine model has been estimated jointly with an arbitrage-free term structure via maximum likelihood (Annex 1).

Results of this VAR model are quite in line with we would expect. As can be seen by the Impulse Response Functions of Figure 4, an increase in the level of the yield curve (l_{it}), which implies a general increase in market rates, produce a decline both in the economic growth and the primary balance. An increase in the slope of the yield curve (s_{it}), while the long term level of yields (l_{it}) remains unchanged represents a decline in the short end of the yield curve, anticipates a decline of the long end of the curve, and an advance in the primary balance. An improvement in the primary balance (pb_{it}), reduce market interest rates, and a slight decline in economic growth rate. Finally, an increasein the nominal economic growth rates (g_{it}) increases the primary balance and decreases market interest rates.

Figure 4: Impulse-Response Functions of the VAR model



Impulse Response Functions from the VAR are obtained by a Cholesky decomposition of the variance covariance matrix of the residuals. In the figures, the shocks are identified with an increase in the level (*l*) in the upper left panel; an increase in the slope (*s*), or a decrease in the short rate since the level is not contemporaneously changed, in the upper right panel; an increase in the primary balance (*pb*) in the lower left panel, and an increase in the nominal economic growth rate. Units in the x-axis represents quarters.

Monte Carlo Simulation

With the estimated VARmodel, we can produce Monte Carlo simulations for the variables involved in the debt ratio equation (2) (rs_t can be considered as the residual of (2), andsimulated as a noise variable), with the exception of interest rates, which differ from equation (2) to (5). NSS Parameters of equation (5) provide information on the secondary market price of sovereign debt dependent on the maturity, but in model (2) the interest rate used is a single value of the outstanding debt. Prevailing market interest rates only affect to new issuances of bonds, corresponding to the increase in the outstanding debt ($\frac{\Delta D_{it}}{D_{it}}$) and the rolling over of maturing debt (m).So, the interest rates on the stock of debt(i_{it}) can be computed as a weighted average of previous interest rates (i_{it-1}) and market interest rates (i_{it}^{M}).

$$i_{it} = \left\{\frac{\Delta D_{it}}{D_{it}} + m\right\} i_{it}^{M} + \left\{1 - \left(\frac{\Delta D_{it}}{D_{it}} + m\right)\right\} i_{it-1} \quad (7)$$

In equation (7), market interest rates is a single value, but model (5) gives us the whole yield curve ($l_{it},\ s_{it},\ c_{it}$) that we will have to transform into i_{it}^{M}. Nevertheless, this is straight forward since spot interest rates are a linear combination of NSS parameters (see equation 6) and the market interest rates is a weighted average of those spot interest rates (it is a bond portfolio). So, we could establish the following relationship:

$$i_{it}^{M} = \alpha_1 l_{it-k} + \alpha_2 s_{it-k} + \alpha_3 c_{it-k} \quad (8)$$

In order to simplify the model we have assumed that parameters in (7) and (8) do not change either with time or country. Therefore, we will consider that Treasuries have a similar issuance strategy that does not change in time. Estimates by MSE for both m and α_j are showed in (9) and (10).This simplification seems to be not too relevant, since the standard error of the joint model is lower than 2bp from the observed interest rate.

$$i_{it} = \left\{\frac{\Delta D_{it}}{D_{it}} + 0.0503\right\} i_{it}^{M} + \left\{1 - \left(\frac{\Delta D_{it}}{D_{it}} + 0.0503\right)\right\} i_{it-1} \quad (9)$$
$$i_{it}^{M} = 0.0146 l_{it-k} + 0.0087 s_{it-k} + 0.0035 c_{it-k} \quad (10)$$

Then, from model (5), it is possible to produce of future evolution of debt interest rates, primary deficit and economic growth rates[3] to obtain simulated

[3] In order to speed-up computation, we have used i_{it-1} in equation 2, instead of contemporaneous interest rates. Nevertheless, given the persistence in this variable, this lag has a minor effect on the final outcome.

paths of the debt ratio according to model (2)[4]. Results of these simulations are presented in annex 3. As can be seen in those simulations, although the VAR of equation (5) is obviously linear, these variables enter into equation(2) that is non-linear, producing debt paths that could be explosive.

Estimation of the Probability of Default

The simulated Debt paths previously obtained allow us to approximatemarket implied default probabilities as the proportion of those paths that can be considered explosive. We need an automatic procedure to classify paths as explosive or not. This classification is obviously the most delicate issue when calculating default probabilities. Although sometimes this is resolved by fixing an arbitrary threshold, this is not free of criticism, as we have shown above (Sustainability of Debt section). In this research project, we propose to estimate a quadratic tendency in order to obtain such a classification.

If $\widehat{D}_{t+k}^{(j)}$is the j-simulation of Debt k quarters ahead, then for each simulated path (j)we can estimate regression (13),

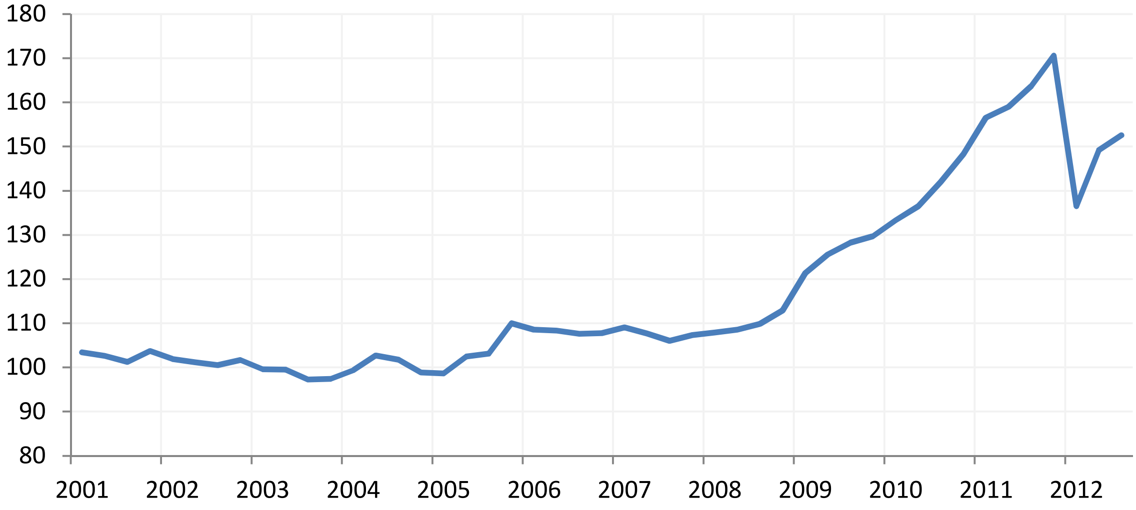
$$\widehat{D}_{t+k}^{(j)} = \gamma_0^{(j)} + \gamma_1^{(j)} k + \gamma_2^{(j)} k^2 + \omega \quad (11)$$

We will consider that a debt path is explosive (unsustainable) if both $\gamma_1^{(j)}$ and $\gamma_2^{(j)}$ are greater than zero. By doing so, we discard as explosive those cases where we have just an increase in the level of debt (something that happened for all countries in the sample in different moments), or cases that have decreasing debts although slowing down. The only cases where both parameters will be positive are those where the debt ratio is growing in an accelerated manner.

[4]In a Monte Carlo simulation, shocks are obtained by drawing values of $\varepsilon_{it}$from a normal multivariate distribution. Therefore, all the debt paths obtained have the same likelihood of happening, although the Debt density is far from uniform.

As a way to illustrate this issue we can analyse the Greek case (Figure 5). Although it is not a direct comparison[5], we can get an idea of the method for identifying explosive debt ratios since, in this case, the Debt started to grow rather quickly from the beginning of the economic crisis reaching a peak in February 2012 where the private sector involvement in the debt restructuring deal allowed a strong reduction in the ratio of debt.

Figure 5: Greek Government evolution

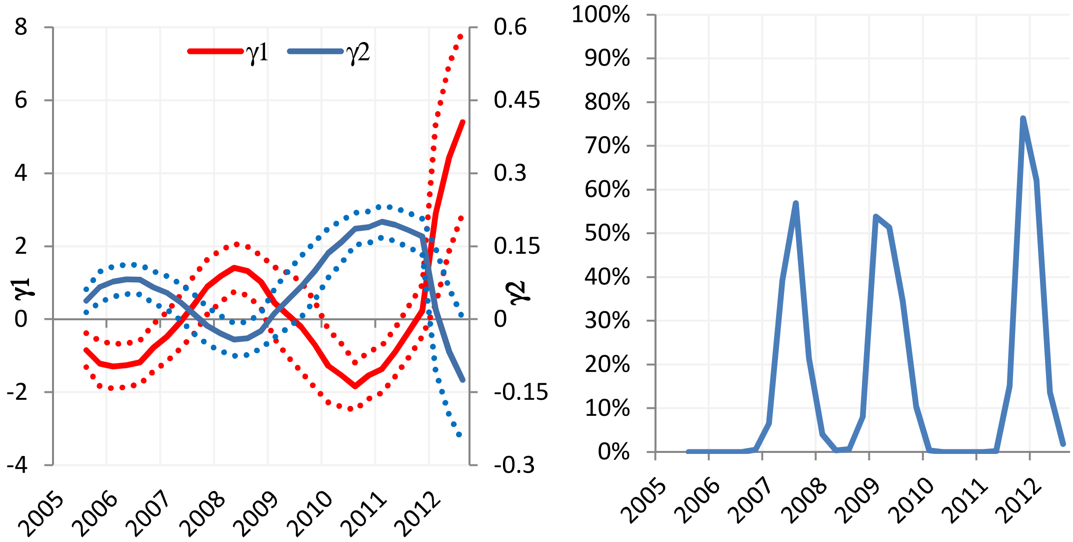


Source ECB SDW.

If we estimate model 11 for a rolling window of 20 quarters, we get the estimations of Figure 6. As can be seen, for most of the sample period, either of the parameters was positive, but rarely both at the same time. In these rare quarters (2007Q3, 2009 Q1 and Q2), they are close to zero. This is a signal of the high correlation between both regressors.

[5] In the model we use future “expected” evolution of debt, but in this exercise, we use past observed debt. To properly compare the model we would have need to obtain projections of the debt paths from 2011Q4 and compute the proportion of them with explosive paths.

Figure 6: Estimated γ_1 and γ_2 for Greek debt (left) and the probability for both parameters to be positive (right).



For that reason, in order to avoid false positive identifications of explosive paths, we need to test that both coefficients are not individually positive, but take into account the correlation between them. In order to do so, from the estimated values of $\gamma_i^{(j)}$ that would be normally distributed ($\hat{\Gamma} \sim N(\Gamma, \Sigma_\Gamma)$), we will consider being explosive those cases where:

$$1 - \Phi_1\left(\hat{\gamma}_1^{(j)}\right) - \Phi_2\left(\hat{\gamma}_2^{(j)}\right) + \Phi_{1,2}\left(\hat{\gamma}_1^{(j)}, \hat{\gamma}_2^{(j)}\right) > 0.95 \qquad (14)$$

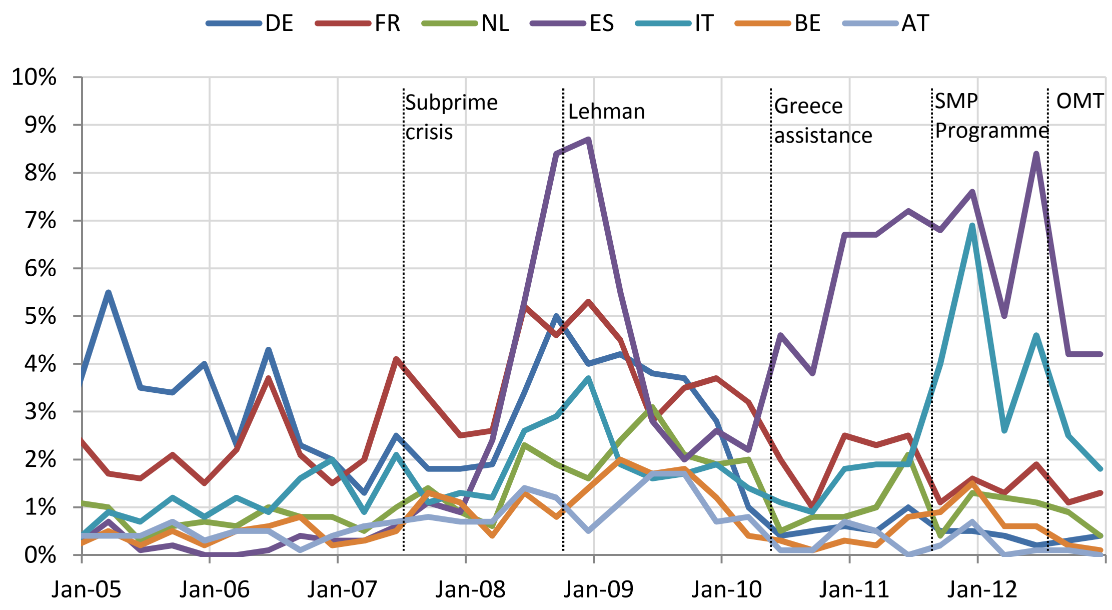
Where Φ_1 and $\Phi_1$represent the univariate cumulative normal distribution ($N(0, \sigma_i)$) and $\Phi_{1,2}$the multivariate normal distribution ($N(0, \Sigma_\Gamma)$). Therefore, the probability of unsustainable debt (PD_{it}) for a country *i* at a given moment *t* will be the proportion of simulated paths where (14) inequality is true.

For the Greek example, when considering the probability of being both positives, we observe that probabilities are in the range of 50-60% in those periods. We obtain a maximum (close to 80%) in 2011Q4 just before the Greek debt restructuring[6]. Applying the same criteria to the forward-looking simulations obtained in previous sections, we get the proportion of unsustainable debt paths of Figure 7. As can be seen, there was a big

[6] Although this value is below the 95% threshold we have chosen, we may assumed that if there would have not been a debt reduction in 2012Q1, this threshold would have been reached in this or the following quarter.

deterioration of the fiscal position after Lehman Brothers Bankruptcy, which it was reduced at the end of 2009, although it return with the sovereign debt crisis, first to Spain and later to Italy.

Figure 7: Probabilities of unsustainable debt paths



Fair value yields

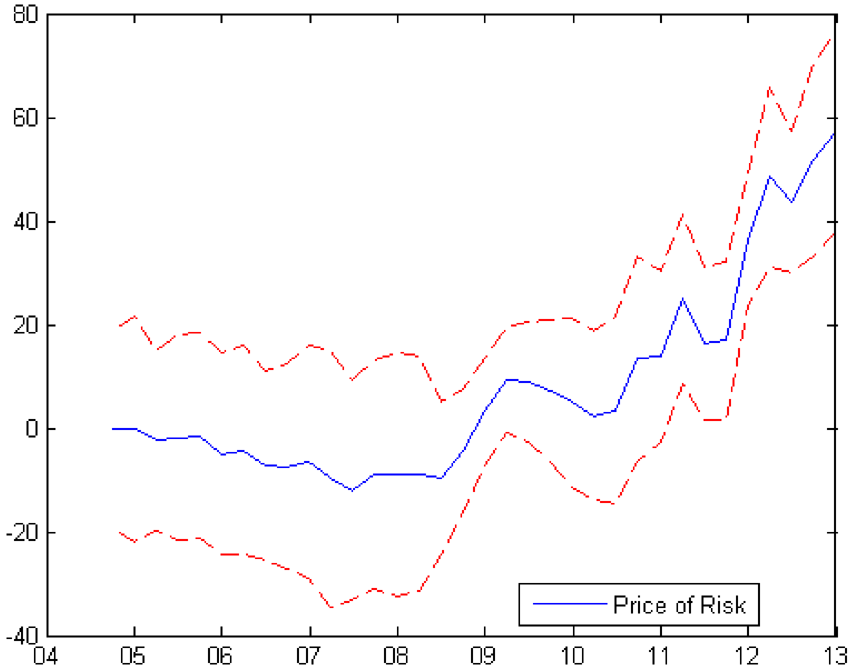
Now that we have computed for each quarter t and country i the probability of default, we are able to estimate model 1 via a Kalman filter, assuming that the risk free rate (α_t) is equal to the OIS to the same term (and therefore, observable) and that the price of risk (β_t) follows a random walk. We consider that the price of risk is common for all countries, so we can estimate a single model for all of them. A single price of risk is consistent with Ross (1976), assuming that all euro area bond market share a similar pool of investors (investors are free to invest in any of those countries).

$$y_{it} - OIS_t = \beta_t PD_{it} + \eta_{it} \qquad \eta_{it} \sim N(0, \sigma_\eta) \qquad (15)$$

$$\beta_t = \beta_{t-1} + \nu_t \qquad \nu_t \sim N(0, \sigma_\nu) \qquad (16)$$

We have considered the 2 years market par yields for y_{it}, although the same model can be applied to any other maturity. The model can be estimated with a Kalman filter. Results from this estimation are showed in figure 8:

Figure 8: The price of risk for sovereign 2 year par yields.

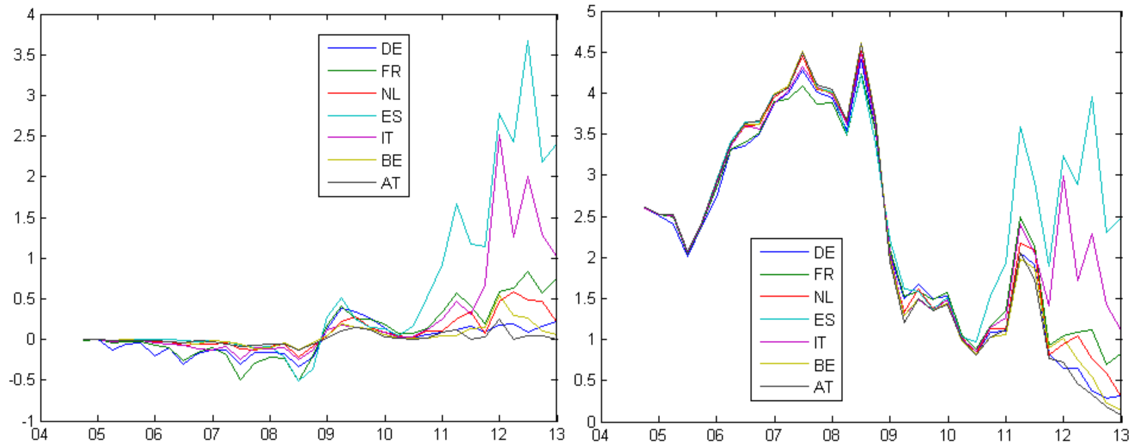


Kalman filter estimates of equations 15-16.

As can be seen, the price of risk was very close to zero before the crisis, but started to increase afterwards (especially with the sovereign debt crisis phase). This is consistent with asituation before the crisis, when investors did not take into account default probabilities, and considered all euro area sovereign debt to be equivalent. By contrast, after the crisis, investors started to look at each country's credit risk and discriminate sovereigns in accordance. The value of the parameter can be interpreted as the bp over the OIS for a 1% of probability of unsustainable debt.

Once we have estimated this time variant price of risk, we can recover the expected (fair) OIS spreads and par yields for each euro area country and quarter, as can be seen in Figure 9 ($y_{it}^{*} = OIS_t + \beta_t PD_{it}$).

Figure 9: 2 year OIS Spread (Left) and par yields (Right).



Comparison with alternative measures

Finally, as a way to compare the model with alternative procedures, Figure 10 and 11 represents the Fir Yield estimates for Italy and Spain respectively. At 2012 Q4 both fair yield estimates were around 1.1% for Italy and 2.4% for Spain (identified as Fundamental Fair Yields in figures 10 and 11), very close to observed yields.

Figure 10: Alternative Fair Yield estimates for Italy

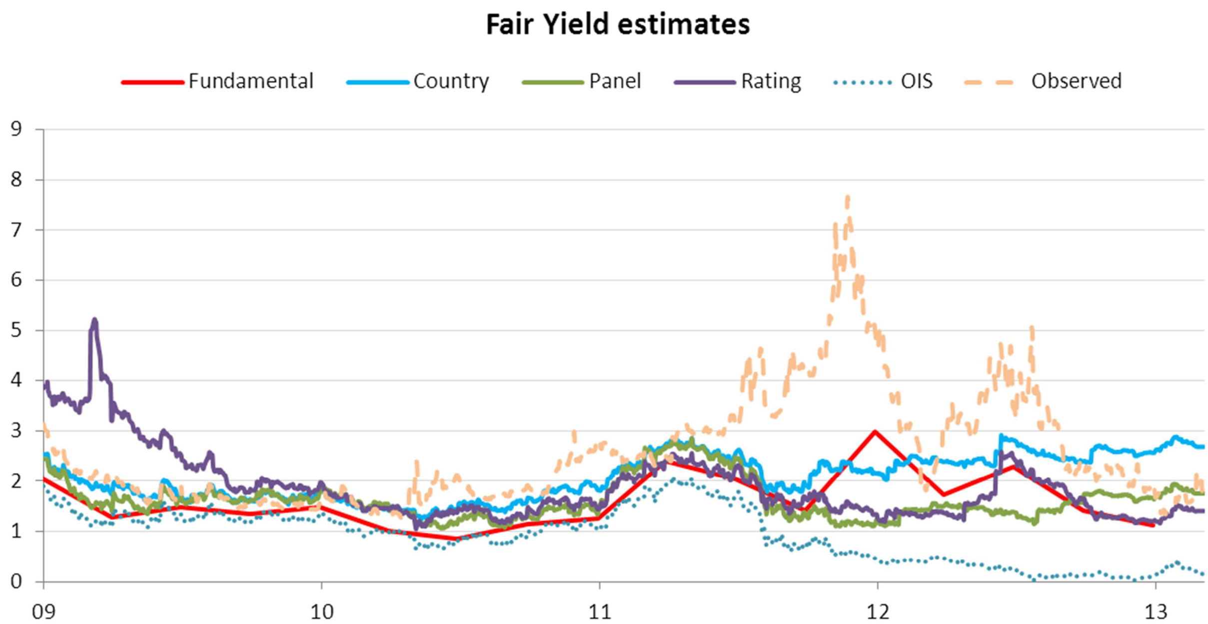
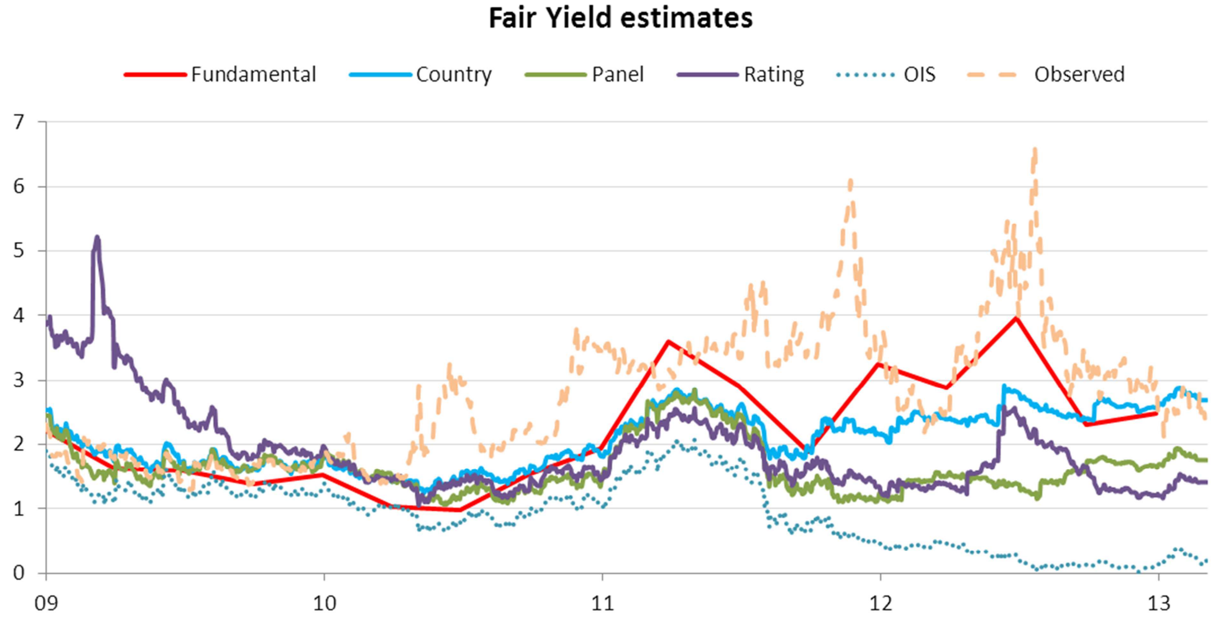


Figure 11: Alternative Fair Yield estimates for Spain



Inconsistency of the fair value estimates

The main drawback of the above framework is the fact that fair yields have been determined using prevailing market information, i.e., observed interest rates that might be *unfair*. The estimated probabilities of default depended on market interest rates, so we could say that these probabilities were also *unfair*. Therefore, one could argue that is not possible to consider y_{it}^{*} being a *fair value* if it is based on unfair market rates and unfair probabilities of default. This implies an endogeneity problem for the estimation of *fair values* should be computed from "fair" probabilities of default that can only be obtained from fair interest rates. A clear example of this endogeneity issue is the hypothetical event of an OMT that will trigger a swift in the yield curve, changing the probabilities of default, and therefore the value of the fair yields. If OMT targets were based on these fair value estimates, then the problem would compound.

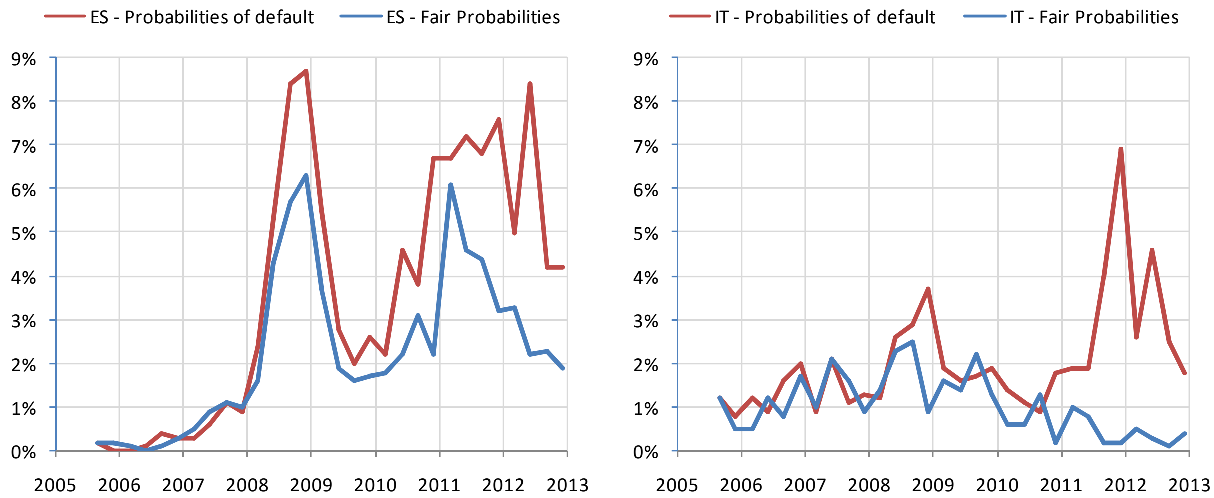
A way to solve this issue would be to use a recursive formula for obtaining fair yields. We start by getting fair estimates of the yield curve parameters $(l_{it}^{*}, s_{it}^{*}, c_{it}^{*})$. In order to do so, we estimate the Kalman filter (equations 15-16) for different maturities (1, 2,…, 10 years), and recover Nelson Siegel parameters (l_{it}^{*}, s_{it}^{*}, c_{it}^{*}) from these fair yields. For the Kalman filter, we use the same quantities of risk for all maturities (PD_{it}), and the maturity differences will arise from the OIS and the price of risk (β_t) that in both cases will be different for each maturity. Once we have these fair curve parameters, we have a "fair" macro-finance vector: $X_{it}^{*} = (l_{it}^{*},\ s_{it}^{*},\ c_{it}^{*}, pb_{it}, g_{it})'$. Although macroeconomic variables (primary balance and economic growth) are not contemporaneously changed, they will be affected in the simulations for subsequent periods.

Simulations starting with X_{it}^{*}, rather than observed X_{it}, allow getting new probabilities of default that are based on fair yields instead of observed yields. To do so, we will assume that the relationship between yield curves and macroeconomic variables are independent of the fairness of the yield curves. In that case, VAR equation (5) will still be valid regardless of the use of observed (X_{it}) or fair (X_{it}^{*}) macro-finance variables. Applying the above methodology for simulating future path of debt and analyzing what proportion of debt became explosive, we get "fair" probabilities of default/quantities of risk (PD_{it}^{*}).

We also assume that the price of risk (β_t) used in equation 15 is not affected by a replacement of PD_{it} by PD_{it}^{*} (If the relationship between the quantity of risk and the risk premia is linear, then the price of risk is not changed by a modification on the quantity). Therefore, we can consider that equation 15 is still valid. So, by replacing PD_{it}^{*} in equation 15, we can get a better estimate of the fair yields. We will repeat this process 20 times for each country and quarter to ensure that fair interest rates and probabilities estimates converge.

Results of this iterative process are showed in Figures 12 and 13, where previous estimates of the probabilities of default (figure 12) and the 2 year fair yields are plotted both for Spain (left hand side panels) and Italy (right hand side panels). As can be seen, probabilities of default are considerably reduced when market interest rates are replaced by fair interest rates. This outcome is consistent with the perception that the risk of default is considerably reduced (although it does not disappear in our model) if a country is able to finance itself at low interest rates. The impact is higher for 2011 and 2012 when the crisis is especially intense for both markets.

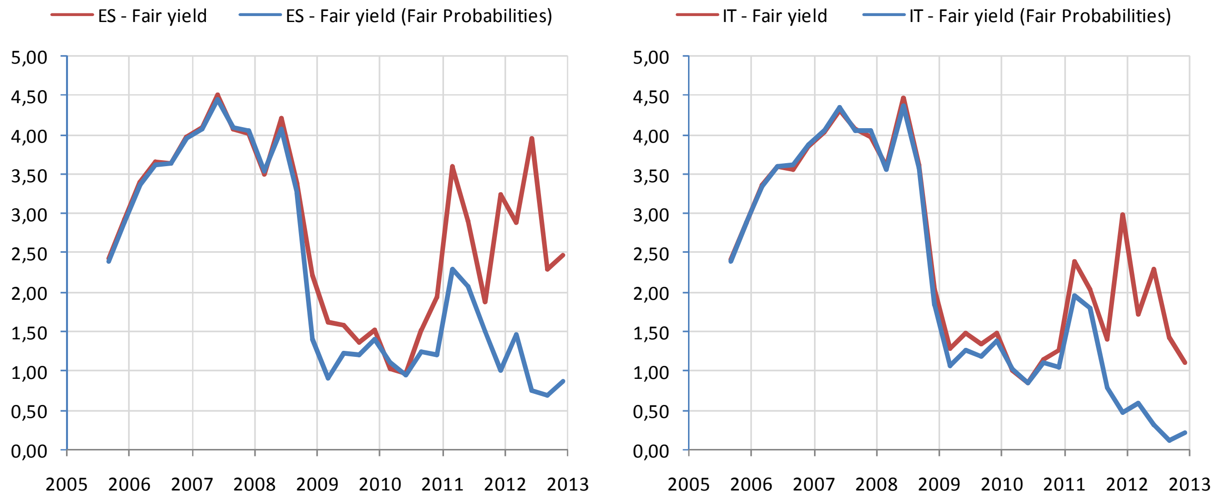
Figure 12: Estimated probabilities of default (unsustainable debt paths) for Spain and Italy



Proportion of simulated paths of Government Debt that growth explosively in the following five years. Red lines represent the simulations obtained from VAR models using market interest rates. Blue lines represent the same measures, but with simulations where market interest rates have been replaced by fair yields through a iterative process. Left chart represent estimates for Spain and right chart represent estimates for Italy.

Similarly, fair yield estimates are remarkably reduced once fair probabilities are used (Figure 13). This is clearer for Italy, where once market interest rates are replace by fair yields, the probabilities of default collapses, and the OIS-spreads are, therefore, minimized.

Figure 13: Estimated 2-year fair yields for Spain and Italy



2-year fair yields obtained from Kalman filter estimates (equation 15). Red lines represent the fair yields obtained from probabilities of default computed using market interest rates. Blue lines represent the same fair yields, but from default probabilities that are based on fair yields through a iterative process. Left chart represent estimates for Spain and right chart represent estimates for Italy.

Conclusions

We have presented a novel way to consider debt under stress. This allows us to get a measure of the quantity of credit risk in a sovereign, and recover from it the fair value of government bonds. Nevertheless, this is completely different from other authors like Reinhart and Rogoff (2010) or Reinhart at al. (2012) that consider that the threshold of 90% is the indicator of problems with the level of debt. To test the superiority of the proposed method, we will need to further compare our approach with alternative specifications. These could include the definition of thresholds either in the level or the growth rate of debt over GDP, and the proportion of simulations that are above those thresholds.

Extensions

Although macroeconomic variables present a quarterly frequency, yield curve parameters are obtained daily, so, in principle it is possible to obtain daily estimates of the fair yield. This would help to the Kalman Filter estimation of model 15-16 (time variant price of risk) and will allow us to having early values of the fair yields and not need to wait for end of quarter. The simplest way to do so is to add an additional step for those days that are not end of quarter. In this step we estimate an additional VAR model where quarterly market yield curve parameters are replace with the daily values *n* days ahead of the end of quarter, so we get simulations for the start of next quarter that can be an input for the MonteCarlo simulations for the quarterly model.

Countries includes in the model are limited by the availability of estimates of the yield curve provided by the statistics department. This excludes Ireland, Portugal and Greece. Nevertheless, if some of them regain access to financial markets they could, eventually, be eligible for the OMT. In this later case, it could be relevant to have also estimations of their fair values. In order to do so, we would need to recover data on individual bonds for these countries and estimate their yield curves. Nevertheless, this is a computer-intensive task.

Following Favero and Giavazzi (2007), VAR equation in model 5, can be extended adding and additional term to take into account the possible effects on X_{it} of the level of Debt:

$$X_{it} = \mu + \Phi X_{it-1} + \Gamma D_{it-1} + \varepsilon_{it} \qquad \varepsilon_{it} \sim N(0, \Sigma) \qquad (17)$$

This might help to better capture the reaction of the government to an increasing level of Debt.

Applications

The presented model also allows simulating the impact of adverse shocks into the probabilities of default and fair values. For instance, we can compare the standard simulations of debt with other simulation where we include the impact of a banking sector rescue, a higher than expected primary deficit, or a sluggish growth rate.

Other possible use for the model is looking for the interest rates level that produces unsustainable debt paths. To do so, we have to increase market interest rates for the same values of the macroeconomic variables until we reach a predetermined value for the probability of default (e.g.: 100%, 50%).

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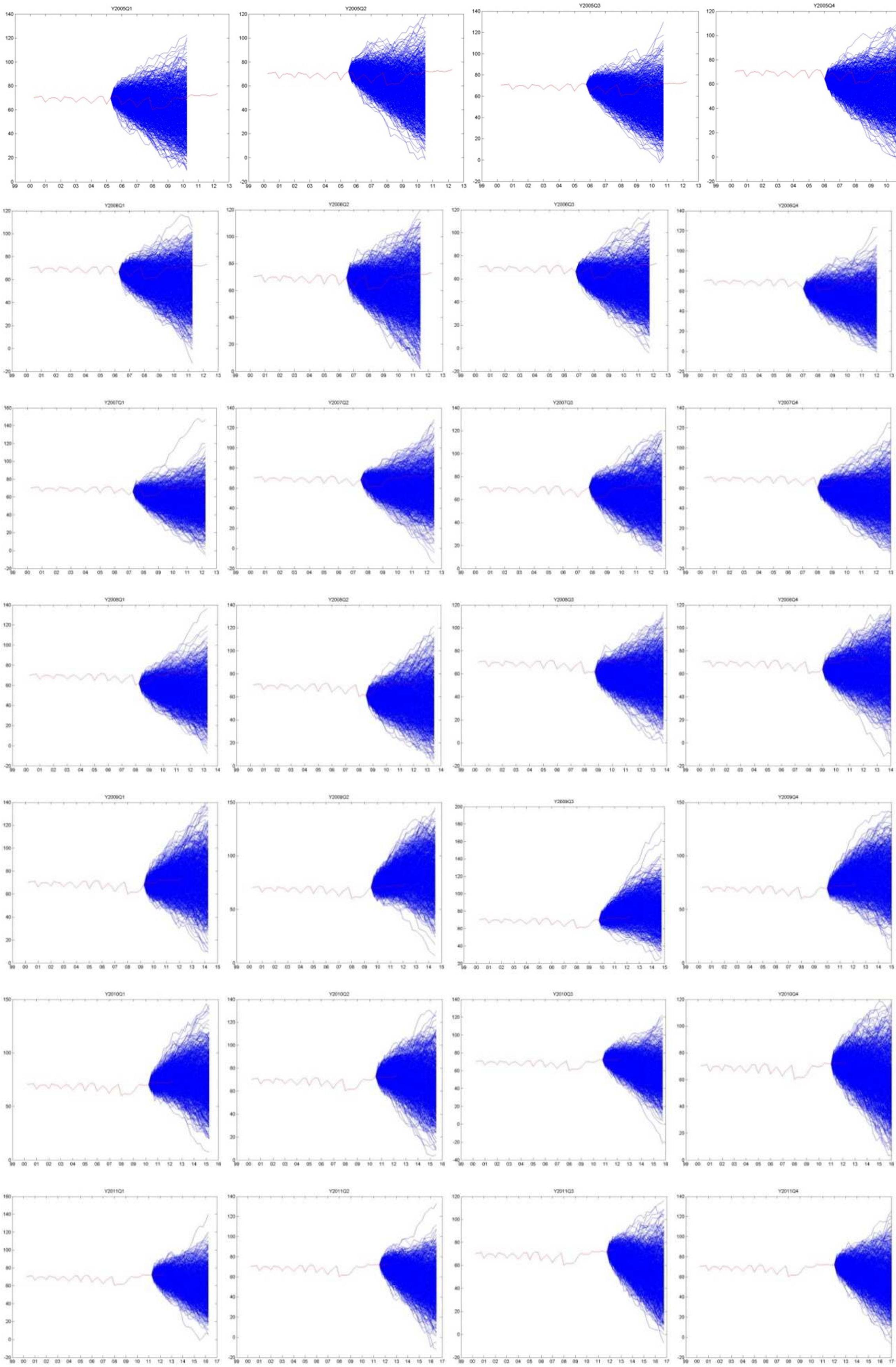
ANNEX 1. MONTECARLO SIMULATIONS:AUSTRIA



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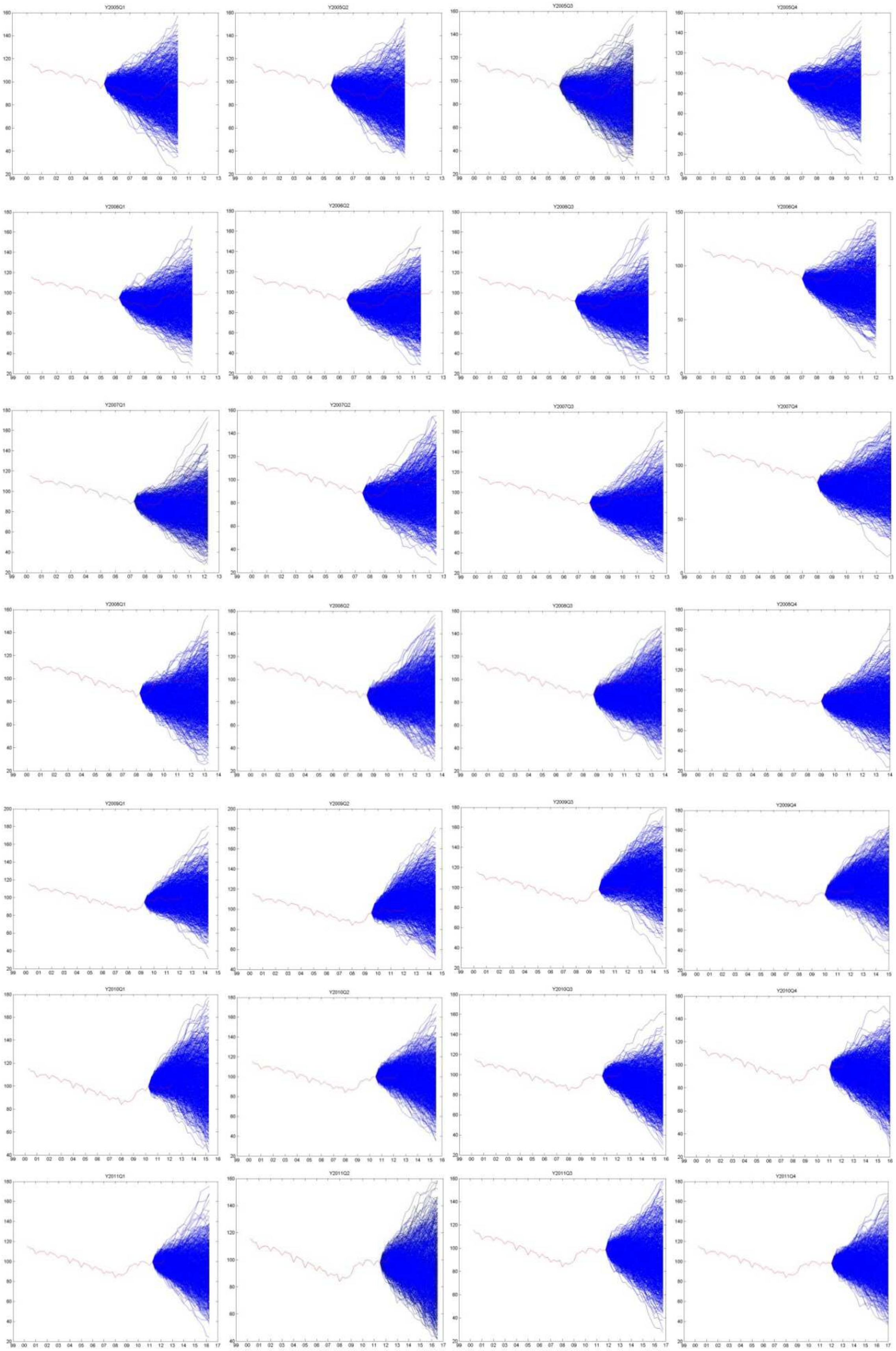
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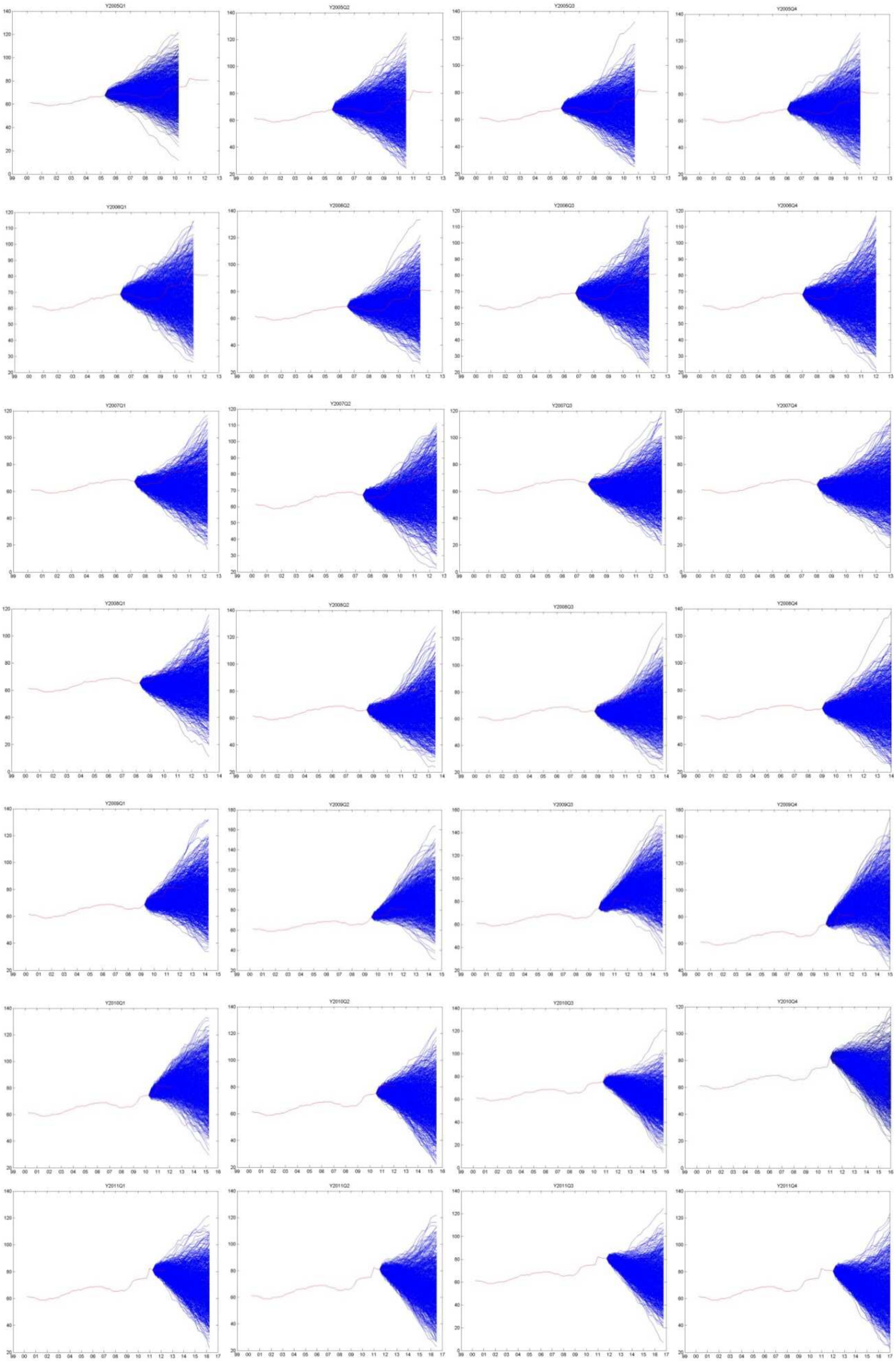
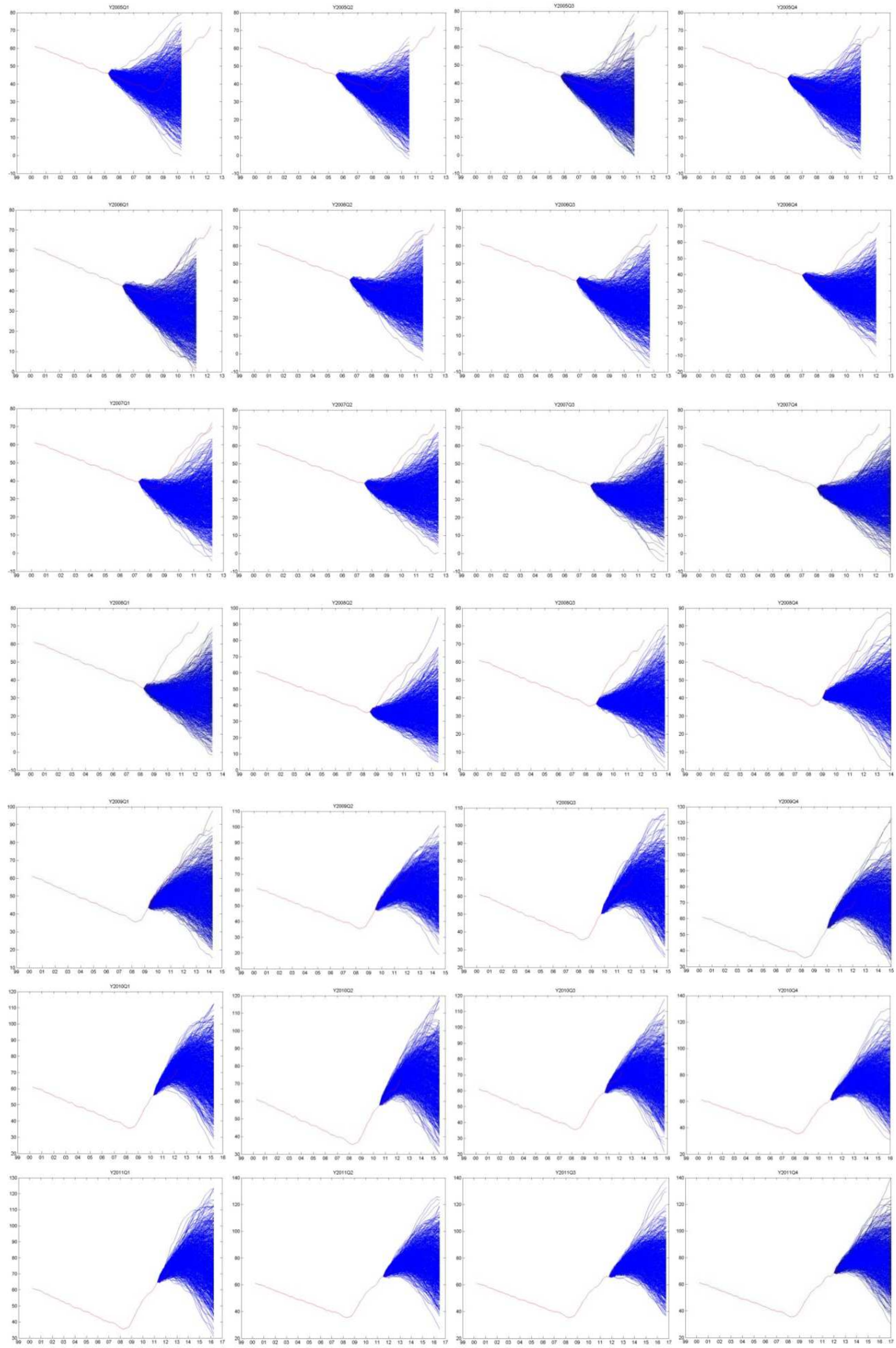
GERMANY



SPAIN



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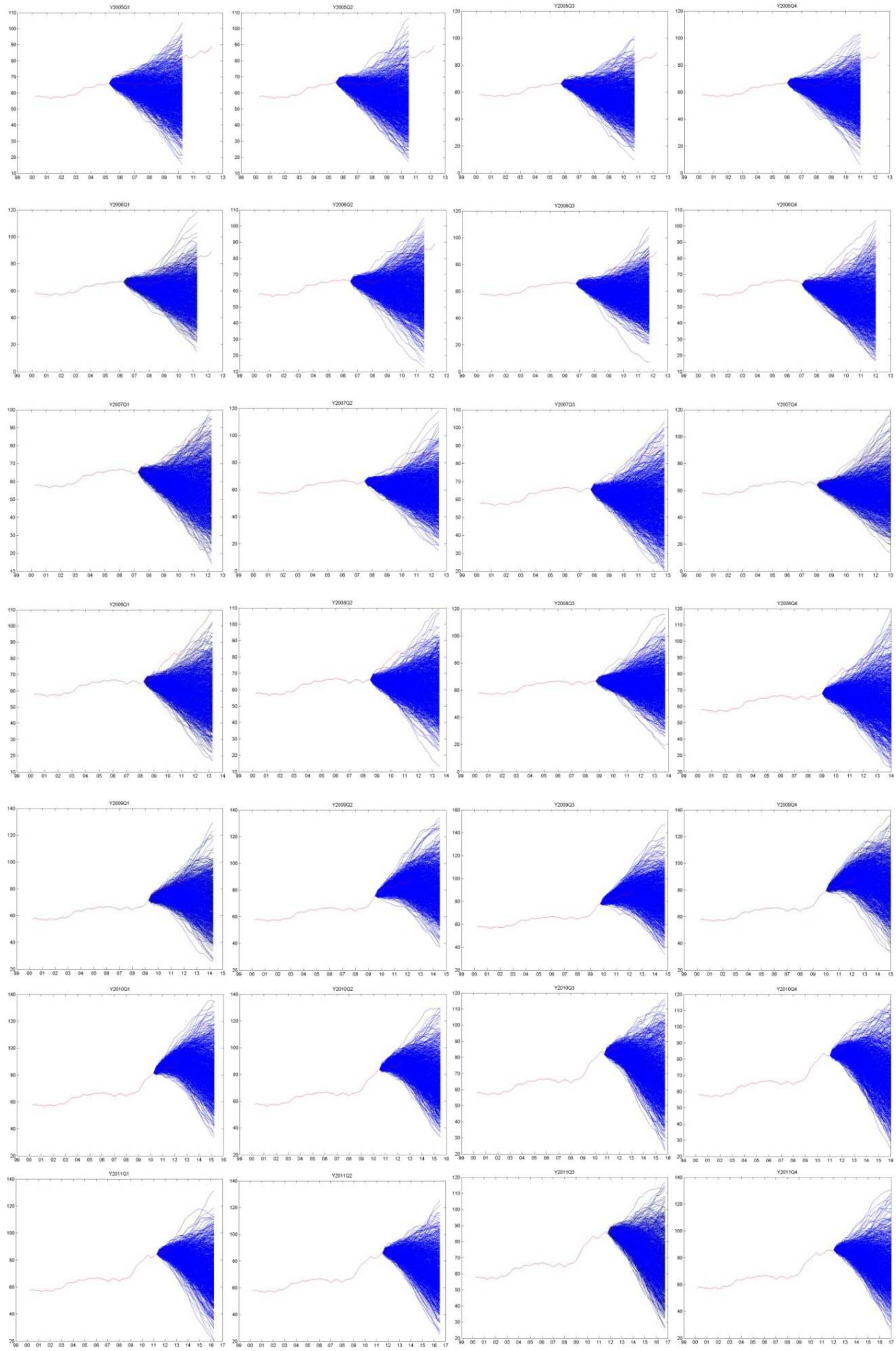
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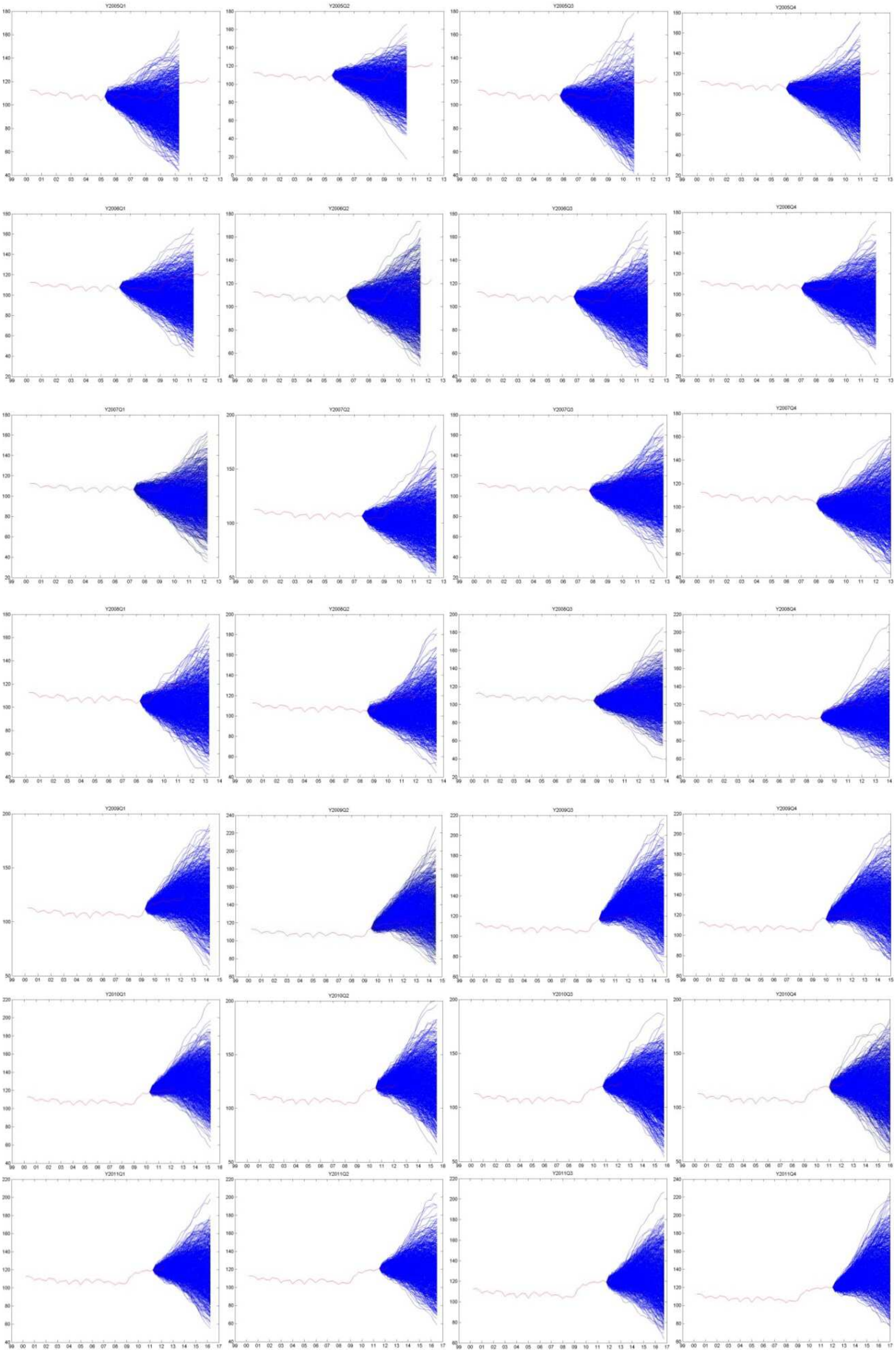
ITALY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NETHERLANDS