Lifestyle Habits and International Transmission of Business Cycles

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Abstract

Our model reconciles with the data by closing the gap between cross-country correlations of consumption and output ('quantity anomaly'). The model predicts positive international correlations in investment and employment of the magnitude observed in the data ('international comovement puzzle'). The key ingredient is the preference ordering that implies internal habit formation over the composite of consumption and leisure. These lifestyle habits (i) reduce wealth effect on labor supply, which helps explain comovement of employment and output; (ii) reduce Edgeworth substitutability between consumption and leisure, which helps explain comovement of consumption; and (iii) discourage changes in consumption growth, which helps explain comovement of investment.

Keywords : Quantity anomaly; International comovement puzzle; Lifestyle habits; Wealth effects

JEL Codes: E32; F41; G15

1 Introduction

The predictions of two-country equilibrium business cycle models conflict with the data in several respects. The models tend to predict (i) negative cross-country correlations of investment and employment, and (ii) high international consumption correlations in excess of output correlations. Postwar data from the industrialized economies suggest the opposite. These well-known discrepancies have received considerable attention in the literature.[1]

This paper shows that a departure from the assumption that preferences are time-separable goes a long way toward accounting for the comovement puzzles. Often, deviation from time-separability

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[1]Baxter (1995) and Crucini (2008) offer reviews of the international business cycle puzzles.

is achieved by introducing habit formation or recursive preference structure.[2] Habits are typically defined over an aggregate consumption good as in Boldrin et al. (2001), individual varieties of goods as in Ravn et al. (2006), or leisure as in Lettau and Uhlig (2000). Instead, we consider a preference ordering that implies internal habit formation over a composite of consumption and leisure. This captures the notion that consumers get accustomed to a certain lifestyle and dislike changes in their standard of living. As shown by Jaccard (2014), such lifestyle habits help to account for asset pricing and business cycle facts in a standard Real Business Cycle (RBC) model.

We show that preferences featuring lifestyle habits possess several properties that are useful for modeling business cycles. First, they are consistent with balanced growth. Second, degree of habit formation governs the wedge between wealth elasticity and Frisch elasticity of labor supply. For a given level of Frisch elasticity, increasing habit intensity and reducing its persistence scales down the wealth effect to a desirable level. Third, lifestyle habits reduce Edgeworth substitutability between consumption and leisure. Finally, under lifestyle habits, consumers dislike not only changes in the composite of consumption and leisure but also changes in its growth rate.

Augmented with lifestyle habits, a two-country, two-good business cycle model driven by productivity shocks reconciles with the data along several important dimensions. First, the model predicts cross-country consumption and output correlations of the same magnitude. Both correlations are within one standard deviation of their empirical counterparts reported by Ambler et al. (2004). In fact, cross-correlation of consumption matches exactly. Most international business cycle models tend to predict excessively high consumption correlations and excessively low output correlations ('quantity anomaly'). Second, the model delivers positive cross-country correlations of employment and investment. Two-country business cycle models tend to predict the opposite ('international comovement puzzle'). Getting investment comovement right does not come at a cost of counterfactual implications for the trade balance.[3]The model is successful at predicting the countercyclical net export. Finally, the model is consistent with the balanced growth and within-country business cycle properties.

Our work builds upon and contributes to a large and growing literature that explores the international dimension of business cycles.[4] As in Heathcote and Perri (2002), we consider a two-country, two-good environment of Backus, Kehoe, and Kydland (1994) (henceforth, BKK) with exogenous incomplete markets. Within this environment, we explore the role of consumer preferences in accounting for international comovement of quantity aggregates. Our work is related to other studies that investigate the role of nonstandard preferences within two-country business cycle models. For instance, recent work by Colacito and Croce (2011) and Gourio et al. (2013) investigate how Epstein and Zin (1989) preferences interact with different sources of stochastic disturbances. Devereux et al. (1992) and Raffo (2008) examine the role of preferences introduced by Greenwood et al. (1988) that eliminate the wealth effect on the labor supply (GHH preferences). Engel and Wang (2011) and Dmitriev and Roberts (2012) consider versions of GHH preferences featuring nonseparability over time due to the presence of durables or habits. Dmitriev and Krznar (2012) discuss the role of linear consumption habits.

The rest of the paper proceeds as follows. The next section describes the model economy as

[2]See Backus et al. (2008) and references therein.

[3]Models that are successful at accounting for positive international comovement of investment often fail to account for countercyclical trade balance and vice versa. For instance, Kehoe and Perri (2002) and Dmitriev and Roberts (2012) get investment comovement right at the cost of predicting procyclical net exports. On the other hand, Raffo (2008) obtains countercyclical net export only to end up with negative investment correlations.

[4]Prominent examples include Backus et al. (1992), Baxter and Crucini (1995), Stockman and Tesar (1995), Kollmann (1996), Heathcote and Perri (2002), and Kehoe and Perri (2002). Some of the recent contributions include Mandelman et al. (2011), Olivero (2010), Rabanal and Tuesta (2010), Thoenissen (2011), Gourio et al. (2013), Karabarbounis (2014) and Gao et al. (2014).

well as properties of lifestyle habits. Section 3 discusses our parameterization of the model. Section 4 presents our quantitative results and explains how each feature of the model contributes to reproducing observed features of the data. Section 5 concludes the paper.

2 The Model Economy

The model economy mimics an environment of Heathcote and Perri (2002) with incomplete markets. The world consists of two countries, home and foreign. Each country is populated by a continuum of identical infinitely lived individuals. The same parameters describe technology and preferences in both countries.

Labor is immobile across countries. In each period t, the world economy experiences an event s_t drawn from the countable set of events, S. Let $s^t = (s_0, s_1, ..., s_t) \in S^t$ be the history of events from time 0 to time t. The probability at time 0 of any given history s^t is denoted by $\pi(s^t)$.

Our specification of preferences and technology ensures that the model is consistent with balanced growth. Labor augmenting technological progress, Γ_t, grows at a constant rate of

$$g = \Gamma_{t+1}/\Gamma_t > 1.$$

Following King et al. (1988, 2002), we rescale all trending variables by dividing them with Γ_t. This implies an adjustment to the discount factor and the equations of motion for the state variables. Throughout the paper, lowercase letters denote detrended variables.

2.1 Lifestyle Habit-Forming Consumers

Consumers have their preferences defined over stochastic sequences of consumption, lifestyle habits, and leisure

$$U = \sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} \pi\left(s^t\right) u(C_j\left(s^t\right), H_j\left(s^{t-1}\right), L_j\left(s^t\right)), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor. Household consumption in country $j \in \{1,2\}$ after realization of history s^t is denoted by $C_j(s^t)$. The time endowment in each period is normalized to 1 and is divided between leisure, $L_j(s^t)$, and hours worked, $N_j(s^t)$:

$$L_j\left(s^t\right) + N_j\left(s^t\right) = 1.$$

The way we model habit formation has four distinct features. First, consumers form habits over the composite of consumption and leisure. As shown by Jaccard (2014), this way of modeling habits helps a stochastic growth model explain simultaneously asset pricing and business cycle facts in a closed-economy setting.

Second, habits are persistent. The stock of habits depends on the entire past history rather than the previous period only. We define the stock of habits, $H_j(s^t)$, with which the agent enters period $t+1$ as a convex combination of her current stock of habits, $H_j(s^{t-1})$, and her current standard of living, $C_j(s^t)\Phi(L_j(s^t))$:

$$H_j(s^t) = \lambda H_j(s^{t-1}) + (1-\lambda) C_j\left(s^t\right) \Phi\left(L_j(s^t)\right). \tag{2}$$

Under this specification, the habit stock depreciates at a constant rate as in Ferson and Constantinides

(1991). The parameter $\lambda \in (0,1]$ determines the degree of habit persistence. The lower the λ, the more weight the agents place on recent lifestyle history relative to the past. That is, as λ declines, so does the persistence of habits. When $\lambda = 0$, the next period's habit stock is simply the current standard of living.

Third, habits are internal. The agent cares about her current standard of living relative to the lifestyle she is accustomed to. Consumers internalize the effect their current behavior has on their future felicity.

Finally, habits are additive. The agent smooths the quasi-difference between the composite of her consumption and leisure, $C_j\left(s^t\right)\Phi\left(L_j(s^t)\right)$, and her current stock of habits, $H_j\left(s^{t-1}\right)$. This helps to ensure that the consumer's objective function remains concave.

The instantaneous utility function defined over the habit-adjusted lifestyle, $C_j\left(s^t\right)\Phi\left(L_j(s^t)\right) - \chi H_j(s^{t-1})$, belongs to the CRRA family

$$u(C_j\left(s^t\right), H_j\left(s^{t-1}\right), L_j\left(s^t\right)) = \frac{\left[C_j\left(s^t\right)\Phi\left(L_j(s^t)\right) - \chi H_j(s^{t-1})\right]^{1-\sigma}}{1-\sigma},$$

where $\sigma > 0$ is the curvature parameter, and $\Phi(L) \equiv \psi + L^{\nu}$. The parameter $\chi \in [0,1]$ denotes the intensity of habit formation and introduces time nonseparability of preferences. Coefficients ψ and ν jointly control the steady-state level of employment and the Frisch elasticity of labor supply.

This specification of preferences nests several well-known special cases. First, when $\chi = 0$, it reduces to the time-separable specification of King et al. (1988), which satisfies restrictions necessary for balanced growth (henceforth, KPR preferences). Second, if labor supply becomes inelastic, the preference structure will feature persistent habit formation in consumption as in Ferson and Constantinides (1991). If, in addition, $\lambda = 0$, the preference ordering reduces to nonpersistent consumption habits popularized by Constantinides (1990).

2.2 Some Properties of Lifestyle Habit Formation

Preferences featuring lifestyle habits possess several properties useful for modeling international business cycles. First, lifestyle habits are consistent with balanced growth. Second, they allow changing the wedge between wealth elasticity and Frisch elasticity of labor supply. For example, one can reduce the magnitude of the wealth effects on labor supply for a given level of Frisch elasticity. Finally, they modify the degree of Edgeworth complementarity between consumption and leisure.

2.2.1 Lifestyle Habits and Balanced Growth

Constant growth in per capita income characterizes the behavior of most industrialized countries. Lifestyle habits are consistent with balanced growth.[5] Jaccard (2014) has established this result in a special case where the growth rate is strictly positive, and habit intensity χ is fixed at 1. Generalizing the specification of preferences to allow for variation in habit intensity does not change this result, nor does the introduction of a multigood environment. To see this, consider a household's optimality conditions for consumption,

$$\Lambda_{Cj}\left(s^t\right) = \left(\left[c_j\left(s^t\right)\Phi\left(L_j(s^t)\right) - \chi h_j(s^{t-1})\right]^{-\sigma} + \theta_j\left(s^t\right)(1-\lambda)\right)\Phi\left(L_j(s^t)\right), \quad (3)$$

[5]Popular GHH preferences used in Raffo (2008) and their time-nonseparable version used in Dmitriev and Roberts (2012) are inconsistent with balanced growth.

and leisure,

$$\Lambda_{Lj} = \Lambda_{Cj}\left(s^t\right) w_j\left(s^t\right), \tag{4}$$

where Λ_{Cj} is the marginal utility of consumption, Λ_{Lj} is the marginal utility of leisure,

$$\Lambda_{Lj} = \left(\left[c_j\left(s^t\right)\Phi\left(L_j(s^t)\right) - \chi h_j(s^{t-1})\right]^{-\sigma} + \theta_j\left(s^t\right)(1-\lambda)\right) c_j\left(s^t\right)\Phi'\left(L_j(s^t)\right), \tag{5}$$

and $w_j\left(s^t\right)$ is the real wage in country j measured in terms of its own consumption good. Notice that the marginal utility of consumption and leisure are forward-looking. Both contain a negative expectational term,

$$\theta_j\left(s^t\right) = \tilde{\beta}\sum_{\tau=t+1}^{\infty}\left(\tilde{\beta}\lambda\right)^{\tau-1}\sum_{s^\tau}\pi\left(s^\tau \mid s^t\right)\left[(-\chi)\left[c_j\left(s^\tau\right)\Phi\left(L_j(s^\tau)\right) - \chi h_j(s^{\tau-1})\right]^{-\sigma}\right],$$

which measures an expected decrease in lifetime utility from raising the future standard of living by increasing today's consumption or leisure. However, a static labor supply equation (6)

$$c_j\left(s^t\right)\frac{\Phi'\left(L_j(s^t)\right)}{\Phi\left(L_j(s^t)\right)} = w_j\left(s^t\right), \tag{6}$$

obtained by dividing equation (5) by equation (3), is exactly as one would obtain in the model without habits. Hence, the restrictions imposed on the utility function make our model consistent with balanced growth (see King et al., 1988). Along a nonstochastic balanced growth path (BGP), leisure must remain constant, whereas real wage grows at a constant rate. The labor supply condition (6) shows that the choice of leisure depends on the ratio of consumption and wages. While both consumption and wages grow at a rate g, leisure remains constant along the BGP.

2.2.2 Lifestyle Habits and Wealth Effects on Labor Supply

Lifestyle habits modify the wedge between wealth elasticity and Frisch elasticity of labor supply. This preference ordering can substantially decrease wealth effects on labor supply for a given level of Frisch elasticity, ϵ_{frisch}. Both habit intensity χ and habit persistence λ affect wealth effects by altering the elasticity of the marginal utility of consumption with respect to consumption, η_{CC}. These properties of the preferences can be summarized more formally as follows:

(i) Given ϵ_{frisch}, higher elasticity of marginal utility of consumption with respect to consumption, η_{CC}, implies lower wealth effect on labor supply;

(ii) Because η_{CC} is decreasing in habit intensity χ, higher χ implies lower wealth effect on labor supply;

(iii) Because η_{CC} is decreasing in λ, a lower persistence implies lower wealth effect on labor supply;

(iv) As χ tends to 1, wealth effect on labor supply tends to 0;

(v) As habits become nonpersistent ($\lambda \longrightarrow 1$), η_{CC} tends to $\frac{\sigma}{(1-\chi)}$, and therefore wealth elasticity of labor supply tends to $\frac{1-\chi}{\sigma}\epsilon_{frisch}$.

To see this, consider a log-linearized version of the labor supply equation (4),

$$\frac{\Lambda_{LC}C}{\Lambda_L}\widehat{c}_t + \frac{\Lambda_{LL}L}{\Lambda_L}\widehat{l}_t = \widehat{w}_t + \frac{\Lambda_{CC}C}{\Lambda_C}\widehat{c}_t + \frac{\Lambda_{CL}L}{\Lambda_C}\widehat{l}_t, \tag{7}$$

where $\widehat{l}_t$ denotes log-deviation of leisure from its deterministic steady state L, Λ_L is the steady-state value of the marginal utility of leisure, and Λ_{LL} denotes the derivative of the marginal utility of leisure with respect to leisure evaluated at the steady state. Analogous notation applies to the rest of the variables. Because $L(s^t) + N(s^t) = 1$, and therefore $\widehat{l}_t = -\frac{N}{L}\widehat{n}_t$, we can rewrite the log-linearized labor supply equation as

$$\left(\frac{\Lambda_{CL}N}{\Lambda_C} - \frac{\Lambda_{LL}N}{\Lambda_L}\right)\widehat{n}_t = \widehat{w}_t - \left(\frac{\Lambda_{LC}C}{\Lambda_L} - \frac{\Lambda_{CC}C}{\Lambda_C}\right)\widehat{c}_t.$$

The expression for the log-deviation of the marginal utility of consumption

$$\widehat{\lambda}_{ct} = \frac{\Lambda_{CC}C}{\Lambda_C}\widehat{c}_t + \frac{\Lambda_{CL}L}{\Lambda_C}\widehat{l}_t,$$

can be solved for consumption

$$\widehat{c}_t = \frac{\Lambda_C}{\Lambda_{CC}C}\left(\widehat{\lambda}_{ct} - \frac{\Lambda_{CL}L}{\Lambda_C}\widehat{l}_t\right),$$

and substituted into equation (7) to yield

$$-\frac{\Lambda_{LL}L}{\Lambda_L}\left(1 - \frac{\Lambda_{LC}}{\Lambda_{LL}}\frac{\Lambda_{CL}}{\Lambda_{CC}}\right)\frac{N}{L}\widehat{n}_t = \widehat{w}_t + \left(1 - \frac{\Lambda_{LC}}{\Lambda_L}\frac{\Lambda_C}{\Lambda_{CC}}\right)\widehat{\lambda}_{ct}. \tag{8}$$

From equation (8), we can express (inverse) Frisch elasticity of labor supply, $1/\epsilon_{frisch}$, as

$$\left.\frac{\partial\widehat{w}_t}{\partial\widehat{n}_t}\right|_{\lambda_c} = -\frac{\Lambda_{LL}L}{\Lambda_L}\left(1 - \frac{\Lambda_{LC}}{\Lambda_{LL}}\frac{\Lambda_{CL}}{\Lambda_{CC}}\right)\frac{N}{L}, \tag{9}$$

and the wealth elasticity of labor supply, ϵ_{wealth}, as

$$\left.\frac{\partial\widehat{n}_t}{\partial\widehat{\lambda}_{ct}}\right|_{w} = \left(1 - \frac{\Lambda_{LC}}{\Lambda_L}\frac{\Lambda_C}{\Lambda_{CC}}\right)\epsilon_{frisch}. \tag{10}$$

Let η_{CC} denote the elasticity of marginal utility of consumption with respect to consumption at the steady state:

$$\eta_{CC} = -\frac{\Lambda_{CC}C}{\Lambda_C}. \tag{11}$$

As shown in Appendix B, we can express the elasticity marginal utility of leisure with respect to leisure as

$$\frac{\Lambda_{LL}L}{\Lambda_L} = \frac{\Phi''(L)\,L}{\Phi'(L)} - \eta_{CC}\frac{\Phi'(L)\,L}{\Phi(L)}, \tag{12}$$

and the elasticity marginal utility of consumption with respect to leisure as

$$\frac{\Lambda_{CL}L}{\Lambda_C} = (1-\eta_{CC})\frac{\Phi'(L)\,L}{\Phi(L)}. \tag{13}$$

Substituting equations (11), (12), and (13) into equations (9) and (10) and taking into account that $\frac{\Lambda_L L}{\Lambda_C C} = \frac{\Phi'(L)L}{\Phi(L)}$ yields the expression for (inverse) Frisch elasticity as

$$1/\epsilon_{frisch} = \left(-\frac{\Phi''(L)\,L}{\Phi'(L)} + \left(2 - \frac{1}{\eta_{CC}}\right)\frac{\Phi'(L)\,L}{\Phi(L)}\right)\frac{N}{L}, \tag{14}$$

and the wealth elasticity of labor supply as

$$\epsilon_{wealth} = \frac{1}{\eta_{CC}} \epsilon_{frisch}. \tag{15}$$

The elasticity of marginal utility of consumption w.r.t. consumption, η_{CC}, can be expressed as a function of the discount factor β, utility curvature σ, habit intensity χ, and habit persistence λ (see Appendix B):

$$\eta_{CC} = \sigma \frac{\left(1 + \chi^2 \beta \frac{(1-\lambda)^2}{1-\beta\lambda^2}\right)}{(1-\chi)\left(1 - \beta\chi \frac{(1-\lambda)}{(1-\beta\lambda)}\right)}. \tag{16}$$

Given an expression for η_{CC}, it is straightforward to show that $\frac{\partial \eta_{CC}}{\partial \chi} > 0$ and $\frac{\partial \eta_{CC}}{\partial \lambda} < 0$. Therefore, equation (15) implies how wealth elasticity responds to changes in habit parameters while keeping Frisch elasticity constant:

$$\left.\frac{\partial \epsilon_{wealth}}{\partial \chi}\right|_{\epsilon_{frisch}} = -\frac{\epsilon_{frisch}}{(\eta_{CC})^2} \frac{\partial \eta_{CC}}{\partial \chi} < 0,$$

$$\left.\frac{\partial \epsilon_{wealth}}{\partial \lambda}\right|_{\epsilon_{frisch}} = -\frac{\epsilon_{frisch}}{(\eta_{CC})^2} \frac{\partial \eta_{CC}}{\partial \lambda} > 0.$$

To summarize, the higher importance consumers place on their stock of habits, the lower is their wealth elasticity of labor supply. The wealth effect on labor supply is lower when consumers' memory is more short-lived.

2.2.3 Lifestyle Habits and Edgeworth Complementarity of Consumptions and Leisure

The introduction of lifestyle habits affects Edgeworth substitutability between consumption and leisure. The reason lies behind adjacent complementarity in the agent's standard of living. A higher standard of living today increases the marginal utility of consumption and leisure in the near future. After a positive productivity shock, consumers do their best to avoid simultaneously increasing their consumption and leisure. Hence, lifestyle habits reduce Edgeworth substitutability between consumption and leisure.

To quantify the relation between lifestyle habits and Edgeworth complementarity, consider what Frisch (1959) refers to as utility acceleration.[6] Notice that utility acceleration defined by Frisch (1959) as

$$\iota = \frac{\partial \Lambda_C}{\partial L} \frac{L}{\Lambda_C},$$

is given by equation (13). As shown in Section 3, the term $\frac{\Phi'(L)L}{\Phi(L)} > 0$ is determined by the factor share α, the steady-state investment-to-GDP ratio, and the average hours worked N. Therefore, habit parameters affect utility acceleration entirely through their effect on the elasticity of marginal utility of consumption w.r.t. consumption, η_{CC}. First, consider the case of time-separable KPR preferences, which we obtain by setting χ to 0. As follows from equations (16) and (13), in this case, $\eta_{CC} = \sigma$, and the utility acceleration is simply

$$\iota_{KPR} = (1-\sigma) \frac{\Phi'(L) L}{\Phi(L)}.$$

[6] Bilbiie (2011) explored the role of utility acceleration on comovement between consumption and leisure following a government spending shock. Furlanetto and Seneca (2014) extended the comovement analysis to the case of investment-specific technology shocks. Their discussions are limited to time-separable preferences.

Utility acceleration ι_{KPR} is negative as long as the curvature parameter σ is above 1, which is most common in the business cycle literature. Therefore, in our time-separable case, consumption and leisure are Edgeworth complements. Introducing habit formation further increases the degree of complementarity between consumption and leisure. Utility acceleration under lifestyle habits ι_{LH} is decreasing in habit intensity χ and increasing in persistence parameter λ:

$$\frac{\partial \iota_{LH}}{\partial \chi} = -\frac{\Phi'(L)\,L}{\Phi(L)}\frac{\partial \eta_{CC}}{\partial \chi} < 0,$$

$$\frac{\partial \iota_{LH}}{\partial \lambda} = -\frac{\Phi'(L)\,L}{\Phi(L)}\frac{\partial \eta_{CC}}{\partial \lambda} > 0.$$

In other words, the higher importance consumers place on their customary lifestyle relative to the current standard of living, the more complementary are their consumption and leisure in the Edgeworth sense. In addition, the degree of complementarity between consumption and leisure is higher when consumers' memory is more short-lived.

2.3 Producers

There are two types of firms in this environment: intermediate good firms and final good firms.

2.3.1 Intermediate Good Producers

Intermediate good firms in each country specialize in the production of a single country-specific good. Country 1 produces good a, whereas country 2 produces good b. Households in country j own the domestic capital stock, $K_j(s^t)$, and rent it to domestic intermediate good firms. Installed capital is immobile across countries. It evolves according to the law of motion

$$K_j(s^t) = (1-\delta)K_j(s^{t-1}) + \phi\left(\frac{I_j(s^t)}{K_j(s^{t-1})}\right)K_j(s^{t-1}), \text{ for } j \in \{1,2\}, \tag{17}$$

where δ is the depreciation rate, and $I_j(s^t)$ is gross investment. Capital accumulation is subject to convex adjustment costs described in Hayashi (1982). The adjustment cost function ϕ satisfies $\phi(\cdot) > 0$, $\phi'(\cdot) > 0$, and $\phi''(\cdot) < 0$. Let i/k denote the investment-to-capital ratio along deterministic BGP, and then the restrictions $\phi(i/k) = i/k$ and $\phi'(i/k) = 1$ will ensure that incorporation of the adjustment cost does not affect the deterministic BGP of the model.

Households supply labor to the intermediate firms at the wage $W_j(s^t)$ and capital at the rental price $R_j(s^t)$. The production function for the intermediate goods is Cobb-Douglas with labor-augmenting productivity growth:

$$Y_j(s^t) = z_j(s^t)\,K_j^{\alpha}(s^{t-1})\left(\Gamma_t N_j(s^t)\right)^{1-\alpha}, \text{ for } j = 1, 2. \tag{18}$$

Production is subject to country-specific exogenous random shock, $z_j(s^t)$, to total factor productivity (TFP). The TFP shocks follow a stationary vector autoregressive process in logs:

$$\begin{bmatrix} \log(z_1(s^t)) \\ \log(z_2(s^t)) \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{12} & \rho_{11} \end{bmatrix}\begin{bmatrix} \log(z_1(s^{t-1})) \\ \log(z_2(s^{t-1})) \end{bmatrix} + \begin{bmatrix} \varepsilon_1(s^t) \\ \varepsilon_2(s^t) \end{bmatrix}.$$

Diagonal elements of the transition matrix, ρ_{11}, determine the degree of persistence in productivity within each country. Off-diagonal elements, ρ_{12}, determine the speed with which productivity innovations spill over national borders. The innovations to the productivity process are serially uncorrelated bivariate normal random variables with zero mean and the contemporaneous covariance matrix

$$E\left[\varepsilon_t \varepsilon_t'\right] = \sigma_\varepsilon^2 \cdot \begin{bmatrix} 1 & \rho_\varepsilon \\ \rho_\varepsilon & 1 \end{bmatrix}.$$

2.3.2 Final Good Producers

The final good firms in country 1 use as inputs the quantities $A_1(s^t)$ and $B_1(s^t)$ of the two intermediate goods. They produce an amount $G(A_1(s^t), B_1(s^t))$ of the final good, using a constant returns-to-scale technology

$$G(a,b) = \left(\omega a^{1-\eta} + (1-\omega) b^{1-\eta}\right)^{\frac{1}{1-\eta}},$$

where $1/\eta$ is the elasticity of substitution between intermediate inputs. The constant $\omega \in (1/2, 1)$ reflects the extent of home bias. In the same way, the firms in country 2 produce an amount $G(B_2(s^t), A_2(s^t))$ of the final good using $A_2(s^t)$ and $B_2(s^t)$ of the intermediate inputs.

2.4 Markets

Markets are incomplete. Agents have access to one-period riskless bonds traded internationally. The claims are denominated in terms of the intermediate good a. Let $Q(s^t)$ be the price of a claim sold after realization of s^t, which delivers a unit of good a at time $t+1$ irrespective of the realized state of nature. Let $B_1^f(s^t)$ be the quantity of such claims held by the residents of country 1 after history s^t. Then, the consumers in country 1 face the budget constraint

$$\begin{aligned} & C_1(s^t) + I_1(s^t) + q_1^a(s^t) Q(s^t) B_1^f(s^t) \\ = \; & q_1^a(s^t)\left[R_1(s^t) K_{1t}(s^{t-1}) + W_1(s^t) \cdot N_1(s^t)\right] + q_1^a(s^t)\left[B_1^f(s^{t-1}) - \Psi\left(B_1^f(s^t)\right)\right], \end{aligned} \quad (19)$$

where $q_1^a(s^t)$ denotes the price of good a in terms of the final good of country 1. As in Mandelman et al. (2011), to ensure stationarity of detrended variables, we introduce an adjustment cost for holding bonds of

$$\Psi\left(B_{1t}^f(s^t)\right) = \frac{\varsigma}{2}\Gamma_t \left(\frac{B_{1t}^f(s^t)}{\Gamma_t}\right)^2,$$

where ς is an arbitrary small positive number. The agents in the foreign country face a similar constraint, except for adjustment cost.

2.5 Equilibrium

The equilibrium consists of the state-contingent sequences of factor prices $R_j(s^t), W_j(s^t)$; intermediate good prices $q_j^a(s^t), q_j^b(s^t)$; bond prices $Q(s^t)$; and allocations $A_j(s^t)$, $B_j(s^t)$, $C_j(s^t)$, $I_j(s^t)$, $N_j(s^t)$, $K_j(s^t)$, $B_j^f(s^t)$ that satisfy the following conditions. Given the prices:

(i) Consumers in country j choose state-contingent sequences $\{C_j(s^t)\}_{t=0}^{\infty}$, $\{N_j(s^t)\}_{t=0}^{\infty}$, $\{I_j(s^t)\}_{t=0}^{\infty}$ and bond holdings $\{B_j(s^t)\}_{t=0}^{\infty}$ for all $s^t \in S^t$ to maximize expected utility in equation (1) subject to the flow budget constraint in equation (19), the laws of motion in equations (17) and (2), and the initial conditions.

(ii) Intermediate firms in country j choose $N_j(s^t)$ and $K_j(s^{t-1})$ to maximize profits

$$Y_j(s^t) - R_j(s^t)K_j(s^{t-1}) - W_j(s^t)N_j(s^t),$$

subject to the technological constraint in equation (18) and the nonnegativity constraints $N_j(s^t) \geq 0$ and $K_j(s^{t-1}) \geq 0$.

(iii) The final good producers in country j choose $A_j(s^t)$ and $B_j(s^t)$ to maximize profits $\Pi_j(s^t)$ given by

$$\Pi_j\left(s^t\right) = \begin{cases} G\left(A_j\left(s^t\right), B_j\left(s^t\right)\right) - q_j^a(s^t)A_j(s^t) - q_j^b(s^t)B_j(s^t), & \text{for} \quad j = 1; \\ G\left(B_j\left(s^t\right), A_j\left(s^t\right)\right) - q_j^a(s^t)A_j(s^t) - q_j^b(s^t)B_j(s^t), & \text{for} \quad j = 2. \end{cases}$$

The prices ensure that for all $t \geq 0$ and for all $s^t \in S^t$:

(iv) Intermediate good markets clear

$$\begin{aligned} A_1\left(s^t\right) + A_2\left(s^t\right) &= Y_1(s^t), \\ B_1\left(s^t\right) + B_2\left(s^t\right) &= Y_2(s^t). \end{aligned}$$

v) Final good markets clear

$$\begin{aligned} C_1\left(s^t\right) + I_1\left(s^t\right) &= G\left(A_1\left(s^t\right), B_1\left(s^t\right)\right), \\ C_2\left(s^t\right) + I_2\left(s^t\right) &= G\left(B_2\left(s^t\right), A_2\left(s^t\right)\right). \end{aligned}$$

vi) Asset markets clear

$$B_1^f(s^t) + B_2^f(s^t) = 0, \text{ for all } s_t \in S^t.$$

2.6 Other Variables of Interest

We measure GDP in country 1 as

$$GDP_1(s^t) = q_1^a(s^t)Y_1(s^t).$$

The real exchange rate for country 1 is defined as

$$RER\left(s^t\right) = q_1^a(s^t)/q_2^a(s^t),$$

and the terms of trade are defined as

$$TOT\left(s^t\right) = q_1^b(s^t)/q_1^a(s^t).$$

We define the trade balance as the net exports for country 1 as a fraction of GDP for country 1, both measured in current prices

$$NX\left(s^t\right) = \frac{q_1^a(s^t)A_2(s^t) - q_1^b(s^t)B_1(s^t)}{GDP_1(s^t)}.$$

3 Model Calibration, Estimation, and Solution

3.1 Calibration

To solve the benchmark model numerically, we use the parameter values reported in Table 1. The first group includes parameters whose values are common to the international business cycle literature. The capital income share α and the utility curvature σ take the values found in BKK and Kehoe and Perri (2002). We rely on BKK's estimates of the parameters governing the stochastic process for productivity. Similar to BKK, we set the elasticity of substitution between intermediate traded goods $1/\eta$ to 1.5. The bond adjustment cost parameter ς is small enough to have virtually no effect on the cyclical properties of the model. It is set to one basis point.

The second group includes parameters calibrated to match long-run averages in the U.S. data, as described in Cooley and Prescott (1995). We interpret one period as a quarter and set the gross trend growth rate g to 1.005. This reflects the fact that U.S. real GDP per capita grew, on average, at 2.004 percent per year over the period from 1960 to 2011. Given the growth rate, the quarterly depreciation rate is set at 0.02 to ensure that the steady-state investment-to-GDP ratio is 0.25, and the capital-to-GDP ratio is 10. Growth-adjusted discount factor β^* follows from the consumer's Euler equation in the steady state. Given the values for g, α, δ, and the capital-to-GDP ratio, we compute the discount factor as

$$\beta^* = \frac{g}{\alpha\,(gdp/k) + 1 - \delta}.$$

The degree of home bias ω is chosen to match the observed long-run import share of GDP, im. The average value of the import share for the Unites States, Japan, and the EU-15 for the period from 1995 to 2010 is 0.15. Because we consider a symmetric equilibrium, we compute the home bias parameter as

$$\omega = \frac{(1/im - 1)^{\eta}}{1 + (1/im - 1)^{\eta}}.$$

Utility function parameters ψ and ν are chosen to match the following two features of labor supply: (i) consumers spend on average one-third of their time endowment working; (ii) the Frisch elasticity of labor supply is 1.5. As shown in equation (14), Frisch elasticity depends on the elasticity of marginal utility of consumption with respect to consumption, η_{CC}. The latter varies with habit intensity χ and habit persistence λ according to equation (16). For a given choice of the habit parameters and, therefore, η_{CC}, we set ν to keep Frisch elasticity constant across parameterizations. Because $\Phi\,(L) = \psi + L^{\nu}$, it follows that

$$-\frac{\Phi''\,(L)\,L}{\Phi'\,(L)} = 1 - \nu,$$

which can be substituted into equation (14) to yield

$$\nu = 1 - 1/\epsilon_{frisch}\frac{L}{N} + \left(2 - \frac{1}{\eta_{CC}}\right)\frac{\Phi'\,(L)\,L}{\Phi\,(L)}.$$

Notice that the term $\frac{\Phi'(L)L}{\Phi(L)}$ is determined by the factor share α, the steady-state investment-to-GDP ratio, and average hours worked N. Labor supply equation (6) in the steady state can be written as

$$\frac{\Phi'\,(L)\,L}{\Phi\,(L)} = \frac{wL}{c}.$$

Table 1: Baseline Parametrization

Notation	Definition	Value
Intermediate good production		
g	gross trend growth rate	1.005
ξ	elasticity of investment w.r.t. Tobin's q	12.01
α	capital income share	0.36
σ	utility curvature	2
δ	depreciation rate	0.02
Final good production		
$1/\eta$	elasticity of substitution between domestic and foreign inputs	1.5
ω	home bias parameter	0.7607
Preferences		
β^*	adjusted discount factor	0.9892
σ	utility curvature	2
υ	preference parameter that controls Frisch elasticity of labor supply	3.08
ψ	preference parameter that controls steady-state hours worked	0.2308
λ	lifestyle habit persistence	0.00
χ	lifestyle habit intensity	1.00
Stochastic process for productivity		
ρ_{11}	persistence of the TFP shocks	0.906
ρ_{12}	TFP spillover parameter	0.088
σ_ε	standard deviation of the innovations to the TFP process	0.00852
ρ_ε	contemporaneous correlation of the innovations	0.25

Note: The two countries share the same parameterization. One time period is a quarter.

In the steady state, $c + i = gdp$ and $wN = (1-\alpha)\, gdp$, which implies that

$$\frac{\Phi'(L)\, L}{\Phi(L)} = \frac{(1-\alpha)}{1 - i/gdp}\frac{L}{N}.$$

To calculate ψ, recall that $\Phi(L) = \psi + L^\nu$, from which it follows that

$$\frac{\Phi'(L)\, L}{\Phi(L)} = \frac{\nu L^\nu}{\psi + L^\nu}.$$

Given the value of $\frac{\Phi'(L)L}{\Phi(L)}$ that corresponds to $N = 1/3$, and the value of ν that keeps Frisch elasticity at the target level, we have

$$\psi = \left(\nu\left(\frac{\Phi'(L)\, L}{\Phi(L)}\right)^{-1} - 1\right) L^\nu.$$

Capital adjustment cost and habit parameters constitute the last group. Their values are chosen to match the second moments of the data. We start with capital adjustment cost $\phi(x)$, whose functional form follows from Boldrin et al. (2001):

$$\phi(x) = \frac{\Omega_1}{1 - 1/\xi}(x)^{1-1/\xi} + \Omega_2.$$

Parameters Ω_1, Ω_2 must satisfy $\phi'(i/k) = 1$ and $\phi'(i/k) = i/k$ to ensure that the introduction of adjustment costs does not affect the BGP of the model. Therefore, it follows that

$$\Omega_1 = (i/k)^{-1/\xi},$$

and

$$\Omega_2 = \frac{i/k}{1-\xi}.$$

Because the elasticity parameter ξ controls cyclicality of trade balance, its baseline value set to equate the correlation of net exports and GDP predicted by the model to that in the data.

3.2 Estimation

In the absence of any econometric estimates for lifestyle habit parameters, we rely on the simulated method of moments (SMM). Having the 'quantity anomaly' in mind, we choose habit intensity χ and persistence λ to maximize the model's ability to account for international comovement of consumption and output. In particular, let $\mathbf{m}$ denote a (2×1) vector of cross-country correlations of GDP and consumption

$$\mathbf{m} = \left(\begin{array}{cc} Corr(\widetilde{c}_1, \widetilde{c}_2) & Corr(\widetilde{gdp}_1, \widetilde{gdp}_2) \end{array} \right)',$$

where tildes indicate that we deal with cyclical components of the variables obtained by removing a trend with an HP filter. Empirical correlation $\mathbf{m}_{data}$ for the Unites States and an aggregate of 15 European countries are reported in the Data row of Table 2. We chose the lifestyle habit parameters by minimizing a distance $L(\chi, \lambda)$

$$(\chi, \lambda) = \arg \min_{(\chi,\lambda)\in\Theta} L(\chi, \lambda),$$

between empirical moments $\mathbf{m}_{data}$ and their simulated counterparts $\mathbf{m}_{sim}(\chi, \lambda)$, obtained after solving the model for given parameter pair χ and λ :

$$L(\chi, \lambda) = (\mathbf{m}_{data} - \mathbf{m}_{sim}(\chi, \lambda))' \mathbf{W} (\mathbf{m}_{data} - \mathbf{m}_{sim}(\chi, \lambda)).$$

In the equation above, $\mathbf{W}$ denotes a positive semi-definite weighting matrix, and $\Theta = [0,1] \times [0,1]$ is an admissible range for χ and λ. In practice, we discretize Θ into an equally spaced grid and perform maximization over it. Surprisingly, our estimates reported in Table 1 are very close to those chosen by Jaccard (2014) to match observed equity premium and the risk-free rate in a closed-economy RBC model.

3.3 Solution Method

The model is solved using the first order perturbation method implemented in DYNARE. Derivations of the equilibrium conditions reported in Appendix A are provided in an online supplement.[7]

[7]The supplement is available at the author's website (`http://www.admitriev.net`).

4 The Results

4.1 The Quantity Anomaly and the International Comovement Puzzle

We start by reviewing the puzzles. The first row of Table 2 displays correlations between cyclical components of the quantity aggregates of the United States and an aggregate of 15 European countries.

Predictions of the model with time-separable KPR preferences are reported in the second row. The model fails along two dimensions. First, it predicts negative cross-country correlations in investment and hours worked when these are positive in the data (international comovement puzzle). Second, in the data, international correlations of output and consumption are of similar magnitude, with the former slightly exceeding the latter. Relative to the data, the model predicts a too high consumption correlation and too low output correlation. The fact that consumption correlation is far in excess of output correlation is often referred to as the quantity anomaly.

Table 2: International comovements

	Cross-Country Correlations			
	Consumption	GDP	Investment	Employment
Data (U.S.-Europe)	0.47	0.48	0.49	0.65
	(0.04)	(0.03)	(0.04)	(0.02)
Standard Preferences (KPR)	0.71	0.26	-0.34	-0.57
Baseline Model	0.47	0.46	0.32	0.46

Note: International statistics in the Data row follow from Ambler et al. (2004) who also report GMM standard errors given in parentheses. All correlations are calculated from U.S. data and aggregated data for 15 European countries covering 1960:1-2000:4. The model's statistics are computed from a single simulation of 100,000 periods. All the statistics are based on logged and HP-filtered data with a smoothing parameter of 1600.

The introduction of lifestyle habits helps our model address both puzzles. It resolves what Baxter (1995) calls the international comovement puzzle. It matches the data by predicting positive cross-country correlations of investment and hours worked. The model moves in the right direction in accounting for the quantity anomaly by predicting a realistic international correlation of consumptions (0.47 vs. 0.47 in the data) and outputs (0.46 vs. 0.48 in the data). Whereas the standard model counterfactually predicts that consumptions are far more correlated than outputs, our benchmark eliminates the gap between consumption and output correlations. In fact, our model matches consumption correlation exactly. However, it still fails to reverse the order of the correlations.

4.2 Intuition

4.2.1 Consumption Correlation

To understand why our model delivers a realistic consumption correlation, first consider an extreme case. Suppose that utility is separable in consumption and leisure. In this case, utility acceleration is 0, which makes consumption and leisure neither Edgeworth complements nor substitutes. If markets were complete, the risk-sharing condition would imply equality of all agents' marginal utilities for every time and state. Hence, consumption would be perfectly correlated across counties. Departing from separability in consumption and leisure somewhat reduces consumption correlation. For instance, BKK's model which relies on a special case of KPR preferences - Cobb-Douglas preferences

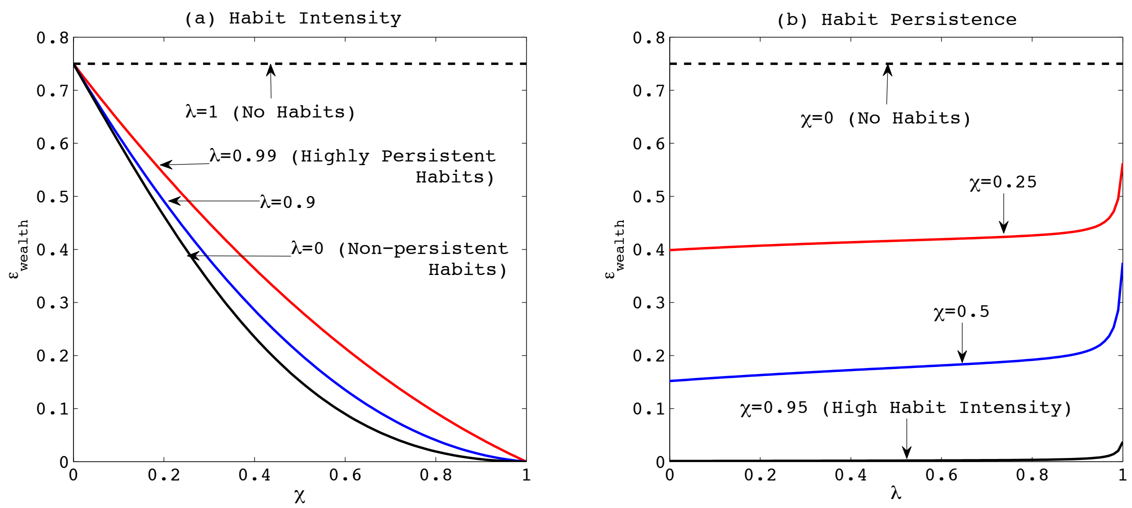
- predicts a cross-country consumption correlation of 0.72 rather than 1. Under KPR preferences, utility acceleration is negative which means that consumption and leisure are complements in the Edgeworth sense. Reducing Edgeworth substitutability between consumption and leisure lowers the consumption correlation. Our discussion in Section 2.2.3 indicates that increasing intensity of lifestyle habit formation further reduces utility acceleration. Additional reduction in Edgeworth substitutability between consumption and leisure is partially responsible for equating consumption correlation with its empirical counterpart (0.47 vs. 0.47 in the data).

Lifestyle habits also increase the role of market incompleteness in explaining international consumption correlations. Under incomplete markets, marginal utilities are no longer equated for all times and states. Instead, stochastic discount factors are equated but only in expected terms. This promising avenue for reducing consumption correlation has delivered disappointing quantitative results so far. The reason is that stochastic discount factors turned out not to be volatile enough to make this financial friction matter. Introducing lifestyle habits changes this result. By increasing the elasticity of marginal utility of consumption with respect to to consumption, lifestyle habits dramatically increase volatility of stochastic discount factors for a given level of consumption volatility. As a consequence, the financial friction becomes more effective in breaking the link between domestic and foreign consumption.

4.2.2 Employment Correlation

The introduction of lifestyle habits allows reducing the wealth effect on labor supply for a given level of Frisch elasticity. How important is this result quantitatively? Figure 1 shows how habit intensity and habit memory affect wealth elasticity of labor supply, given our choice of β, g, and ϵ_{frisch}.

Figure 1: How wealth effects on labor supply are governed by habit parameters

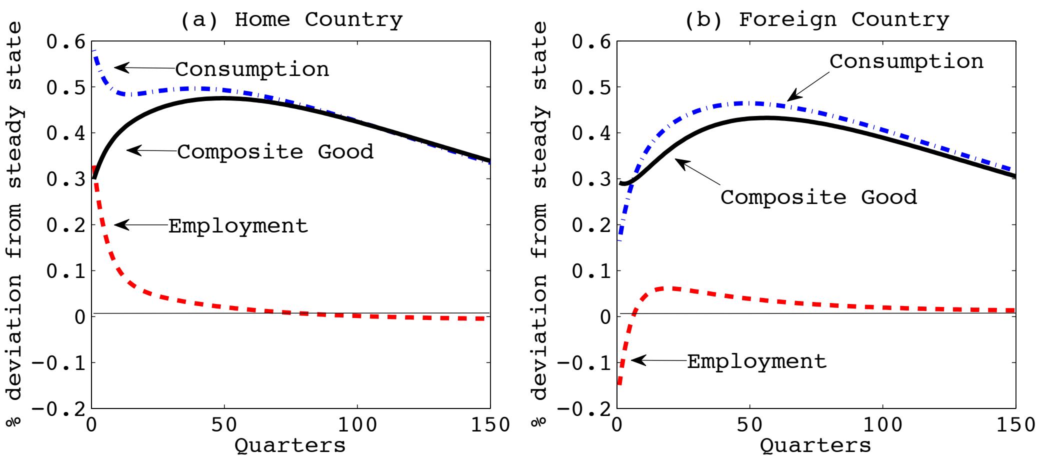


Note: Both panels display how the wealth elasticity of labor supply varies for a given level of Frisch elasticity. Wealth elasticity is decreasing in habit intensity and increasing in habit persistence.

Increasing habit intensity makes the wealth effect on labor supply arbitrarily small irrespective of habit persistence. The converse is not true. Reducing habit persistence lowers the wealth elasticity to a level determined by intensity of the habit formation. Habit intensity, rather than its persistence, is the dimension more quantitatively important for the magnitude of the wealth effect. For a given level of intensity χ, changing depreciation of habits from 100 percent ($\lambda = 0$) to 1 percent ($\lambda = 0.99$) reduces wealth elasticity by less than 15 percent of its initial value.

Now, consider the effect of a one-standard-deviation increase in TFP at home (Figures 2-5). The home country becomes more productive in manufacturing good A. Domestic real wage in terms of good A, $w_1(s^t)$, goes up. Domestically produced goods are now relatively more abundant, but they are only imperfect substitutes for imported goods. The price of good A relative to good B falls, which corresponds to a jump in the terms of trade, $TOT(s^t)$ (see Figure 3). Despite the deterioration in the terms of trade, domestic wages in terms of the domestic final good, $q_1^a(s^t)w_1(s^t)$, go up. While the substitution effect compels the consumer to increase the hours worked, the wealth effect makes her demand more leisure. The relative magnitude of the two forces determines the response of employment to the shock.

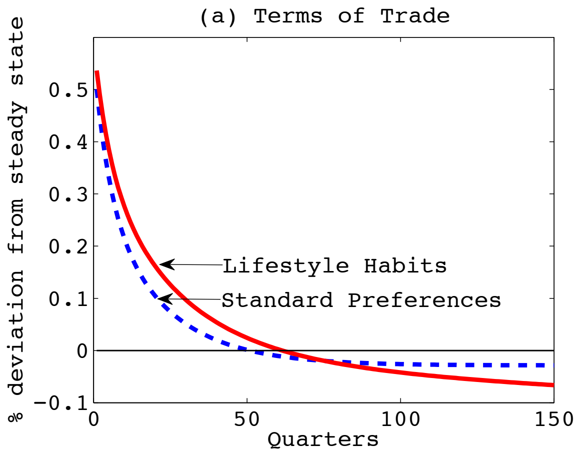
Figure 2: Responses of consumption and employment to a TFP shock at home: KPR preferences



Note: The figure reports responses of consumption $C_j(s^t)$, employment $N_j(s^t)$, and a composite of consumption and leisure $C_j(s^t)\Phi(L_j(s^t))$ to a one-standard-deviation TFP shock at home. All variables are expressed in terms of percentage deviation from their respective steady-state values.

Under standard preferences, wealth effects are quite substantial (dashed line in Figure 1). At home, the substitution effect dominates, and domestic employment rises. Abroad, the terms-of-trade effect increases the wages measured in terms of foreign final goods, $q_2^b(s^t)w_2(s^t)$. The substitution effect compels the agents to increase their hours worked. However, the wealth effect dominates, and employment abroad falls, causing negative cross-country comovement of hours worked (Figure 2). This mechanism is known as "make hay where the sun shines."

Figure 3: Responses of the terms of trade to a TFP shock: Lifestyle habits versus KPR preferences



Note: The figure reports responses of the terms of trade, $TOT\left(s^t\right) = q_1^b(s^t)/q_1^a(s^t)$, to a one-standard-deviation TFP shock at home. The variable is expressed in terms of percentage deviation from its steady-state values.

Lifestyle habits improve the model's predictions for comovement of hours through two channels. First, they affect the magnitude of the substitution effect by increasing the volatility of the terms of trade. A more pronounced fall in the price of domestic goods further discourages domestic workers from supplying more labor. On the other hand, a more pronounced hike in the price of imported goods compels foreign workers to increase their labor supply. Second, lifestyle habits reduce the wealth elasticity of labor supply. This prevents workers in both countries from demanding more leisure after a productivity improvement. In the home country, the wealth effect and terms-of-trade effect move employment in opposing directions. As shown in Figures 2 and 4, the response of employment in the immediate aftermath of the shock almost doubles. Abroad, the wealth effect and terms-of-trade effect move employment in the same direction. The two forces make foreign employment rise after a positive productivity shock at home.

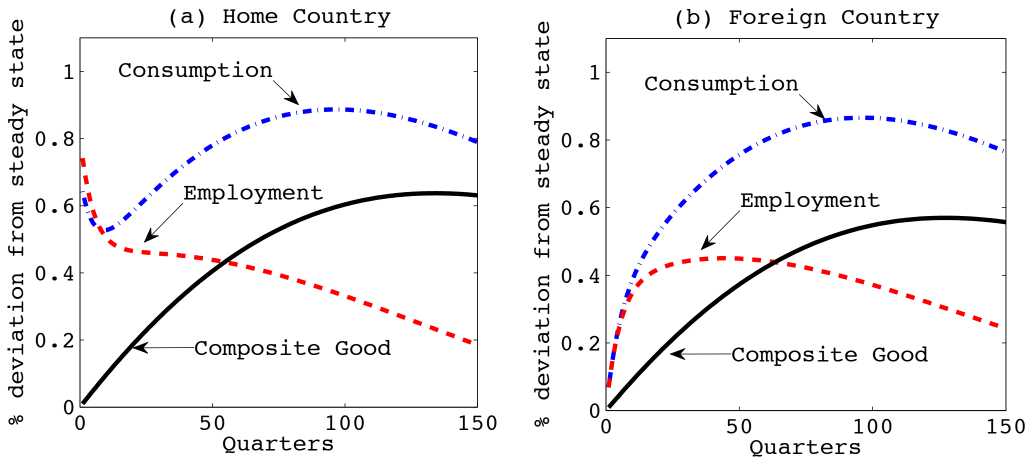
4.2.3 Output Correlation

Getting comovement of employment right directly translates into a plausible cross-country correlation of GDP. Under standard preferences, positive comovement of productivity and negative comovement of intermediate good prices are partially offset by a counterfactual negative correlation of hours worked (-0.57 vs. 0.65 in the data). As a result, the model predicts output correlations that are too low relative to the data (0.26 vs. 0.48 in the data). When the employment correlation approaches its empirical counterpart (0.48 vs. 0.65 in the data), so does the cross-country correlation of GDP (0.46 vs. 0.48 in the data).

4.2.4 Investment Correlation

To understand international comovement of investment, consider how the dynamics of world savings and world investment differs across models. Notice that under standard preferences, households dislike changes in the composite of consumption of leisure, $c_j\left(s^t\right)\Phi\left(L_j(s^t)\right)$. Under lifestyle habits, they also dislike changes in the growth rate of the composite good, which makes the optimal response of the composite good to a productivity shock hump-shaped. In contrast, the optimal response of the composite good under standard preferences is a jump (solid lines in Figure 2 vs. Figure 4).

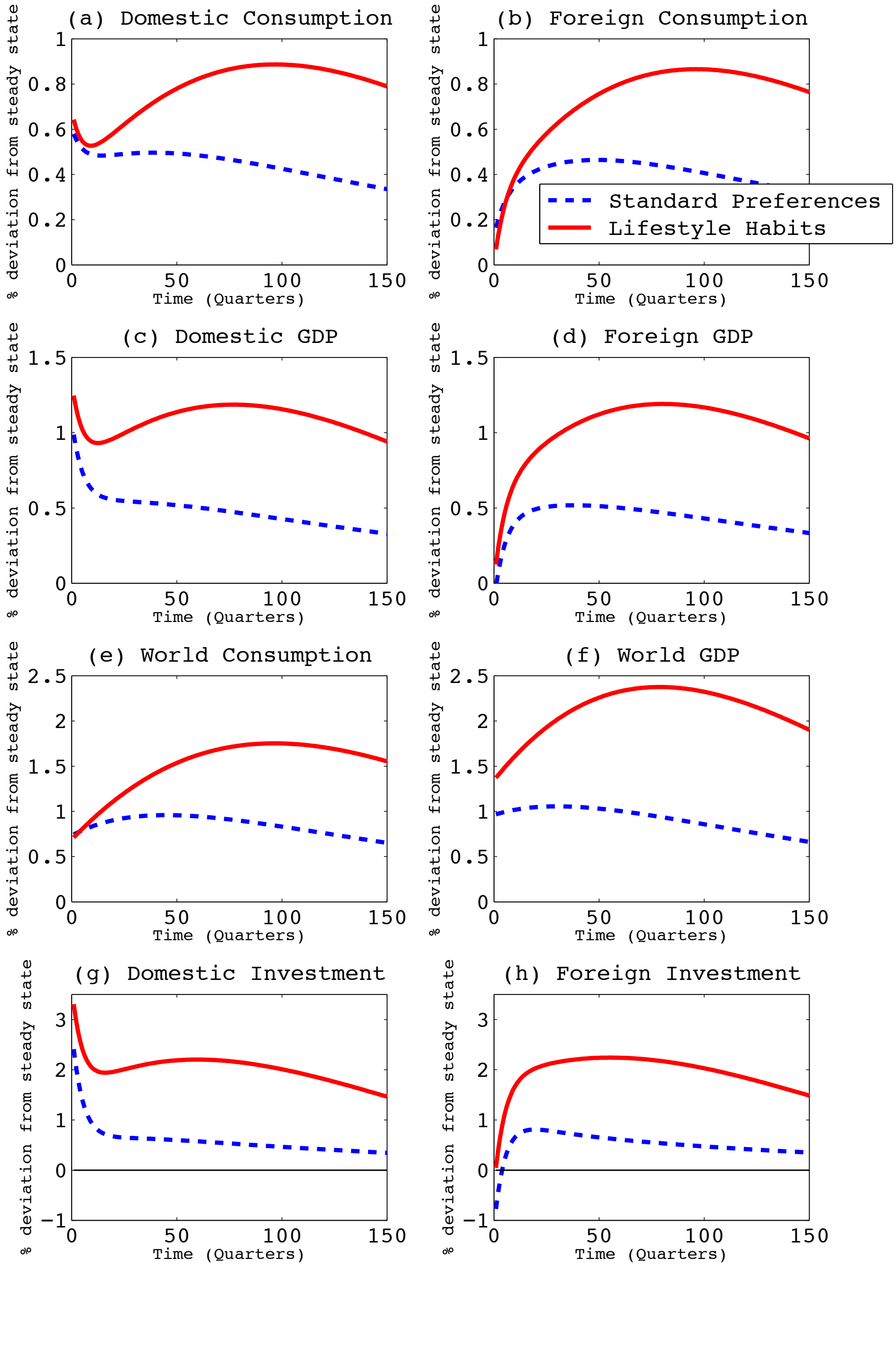
Figure 4: Responses of consumption and employment to a TFP shock at home: lifestyle habits



Note: The figure reports responses of consumption $C_j\left(s^t\right)$, employment $N_j\left(s^t\right)$, and the composite of consumption and leisure $C_j\left(s^t\right)\Phi\left(L_j(s^t)\right)$ to a one-standard-deviation TFP shock at home. All variables are expressed in terms of percentage deviation from their respective steady-state values.

The desire to smooth the growth rate of the composite good under lifestyle habits translates into a hump-shaped response of consumption to the productivity shock (top row of Figure 5). On impact, consumption in both countries does not jump as much as it does under standard preferences. What happens to worldwide savings after the shock? A lower wealth effect on the labor supply increases the response of GDP to higher productivity in both countries (row 2 of Figure 5). The positive response of world GDP is exacerbated, whereas the positive response of world consumption is dampened. Hence, the world experiences a massive increase in savings in the immediate aftermath of the shock. Most of these extra savings are invested in the relatively more productive domestic economy. However, imperfect substitutability between domestic and foreign goods coupled with adjustment cost to capital formation ensures that foreign investment increases as well.

Figure 5: Impulse responses: lifestyle habits versus KPR preferences



Note: The figure reports responses of consumption, GDP and investment to a one-standard-deviation TFP shock at home. Dashed line corresponds to KPR preferences, whereas solid line corresponds to lifestyle habits.

4.3 Other Business Cycle Regularities

Explaining international comovements does not come at a cost of deteriorating the domestic business cycle statistics reported in Tables 3 and 4. Two observations are in order. First, the model reconciles with data in predicting countercyclical trade balance. Second, introduction of lifestyle habits only marginally affects the volatility of the terms of trade and the real exchange rate. Still, the models predictions fall short of their empirical counterpart.

Table 3: Business cycles: volatilities

	$\sigma(GDP)$	$\frac{\sigma(C)}{\sigma(GDP)}$	$\frac{\sigma(I)}{\sigma(GDP)}$	$\frac{\sigma(N)}{\sigma(GDP)}$	$\frac{\sigma(NX)}{\sigma(GDP)}$	$\sigma(TOT)$	$\sigma(RER)$
Data (U.S.)	1.27	0.81	2.77	0.75	0.48	2.15	3.66
Standard Preferences (KPR)	1.25	0.66	2.35	0.33	0.09	0.77	0.54
Baseline Model	1.63	0.52	2.58	0.60	0.11	0.82	0.58

Note: Statistics in the Data row are calculated from U.S. quarterly time series covering 1973:1-2007:4. The model's statistics are computed from a single simulation of 100,000 periods. All the statistics are based on logged (except for the net exports) and HP-filtered data with a smoothing parameter of 1600.

Table 4: Business cycles: correlations with GDP

	$corr(C, GDP)$	$corr(I, GDP)$	$corr(N, GDP)$	$corr(NX, GDP)$
Data (U.S.)	0.84	0.93	0.84	-0.43
Standard Preferences	0.96	0.95	0.90	-0.52
Baseline Model	0.99	0.99	0.99	-0.43

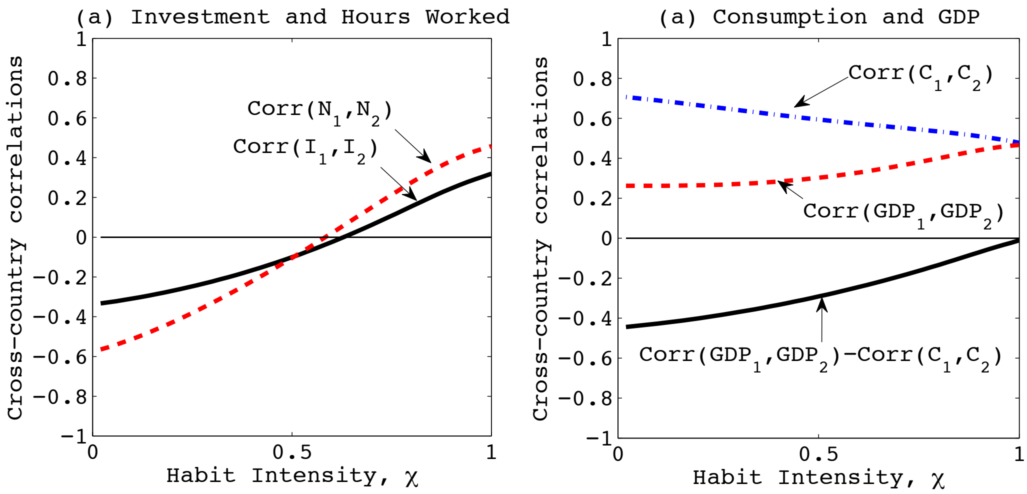
Note: Statistics in the Data row are calculated from U.S. quarterly time series covering 1973:1-2007:4. The model's statistics are computed from a single simulation of 100,000 periods. All the statistics are based on logged (except for the net exports) and HP-filtered data with a smoothing parameter of 1600.

The model's prediction that trade net exports are countercyclical matters for two reasons. From the empirical standpoint, countercyclical trade balance is a very robust and well documented fact (see e.g. Raffo, 2008). From the modeling perspective, cyclical behavior of net exports is closely related to the model's prediction for cross-country correlation of investment. In the canonical international RBC model, investment rushes to the most productive location ('make hay where the sun shines') causing countercyclical net exports at the cost of negative cross-country investment correlation ('international comovement puzzle'). A similar scenario occurs in more elaborate international business cycle models (see e.g. Raffo, 2008). On the other hand, models that get investment comovement right often fail to account for the countercyclical net export by either imposing financial autarky (e.g Heathcote and Perri (2002), Yakhin (2007), and Gao et al. (2014)) or by predicting procyclical trade balance (e.g. Kehoe and Perri (2002), Dmitriev and Krznar (2012), and Dmitriev and Roberts (2012)).

4.4 Sensitivity Experiments

This section discusses how robust our model's predictions are to variations in the intensity and persistence of lifestyle habits. The results are reported in Figures 6 and 7.

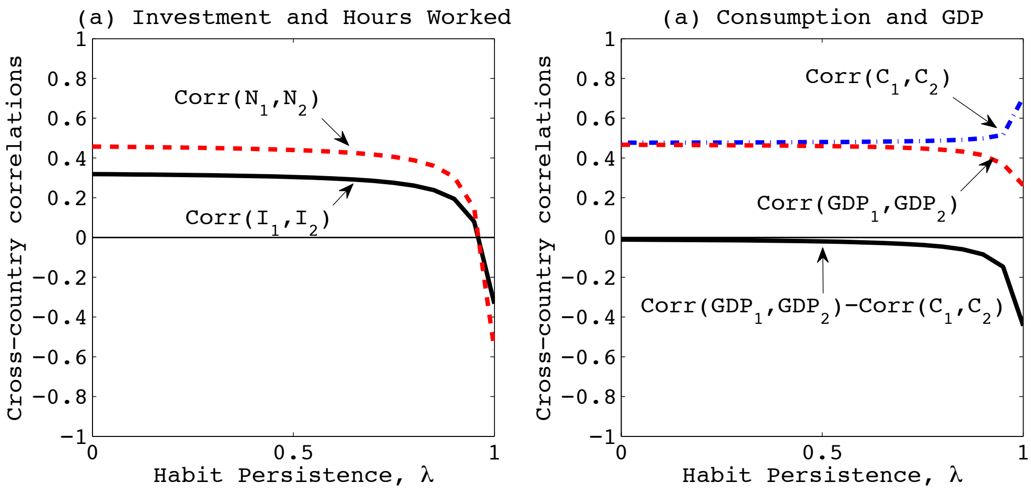
Figure 6: Habit intensity and cross-country correlations



Note: The model's predictions for cross-country correlations of cyclical components of consumption (C), employment (N), investment (I), and GDP are reported for different values of habit intensity. During the simulations the remaining model parameters were kept at the baseline values reported in Table 1.

Degree of Habit Intensity. International comovements depend on the persistence and intensity of lifestyle habit formation. Figure 6 shows that cross-country correlations of employment and investment become positive in our model for relatively modest habit intensity. Cross-correlations of consumptions and GDP get equalized for only for a very high degree of lifestyle habit formation.

Figure 7: Habit persistence and cross-country correlations



Note: Note: The model's predictions for cross-country correlations of cyclical components of consumption (C), employment (N), investment (I), and GDP are reported for different values of habit persistence. During the simulations the remaining model parameters were kept at the baseline values reported in Table 1.

Persistence of Memory. For a given level of habit intensity, variation in the persistence of habits has little effect on international comovements. The model's predictions barely change as long as the current lifestyle has sufficient importance (more that 10%) in updating the stock of habits. For extreme levels of persistence, the stock of habits barely changes over time, and the model behaves similar to the one with time-separable preferences.

Our sensitivity results seem to be related to the experiments reported in Figure1, where we show how wealth elasticity of labor supply depends on habit parameters. Both emphasize the prominent role of habit intensity and minor role of habit persistence on wealth effects and international comovements.

5 Conclusion

This paper shows that internal habit formation defined over a composite of consumption and leisure can bring a two-country business cycle model closer to the data. Our model reconciles with the data by closing the gap between cross-country correlations of consumption and output. It also predicts positive international correlations of investment and employment of the magnitude observed in the data. In other words, a rather parsimonious departure from a canonical two-country, two-good model goes a long way toward a resolution of two puzzles: the quantity anomaly and the international comovement puzzle.

A further contribution of this paper is that it formally describes several properties of lifestyle habit formation preferences that researchers might find appealing for modeling business cycles. For instance, by modifying lifestyle habit intensity, one can reduce the wealth elasticity of the labor supply to a desirable level while keeping Frisch elasticity constant. Unlike popular GHH preferences that entirely eliminate wealth on labor supply, lifestyle habits are consistent with positive wealth effects and balanced growth.

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A Optimality Conditions

This section summarizes the system of optimality conditions formulated in terms of stationary variables. In what follows, $c_j(s^t) \equiv C_j(s^t)/\Gamma_t$ and $k_j(s^{t-1}) \equiv K_j(s^{t-1})/\Gamma_t$ will be denoted as c_{jt} and k_{jt-1} respectively. Same notational convention applies to the rest of the variables. The equilibrium is characterized by the following set of optimality conditions:

i) marginal utilities of consumption

$$\Lambda_{jt} = (\psi + (1 - N_{jt})^{\nu}) \left[[c_{jt} (\psi + (1 - N_{jt})^{\nu}) - \chi h_{jt-1}]^{-\sigma} + \theta_{jt} (1 - \lambda) \right], \text{ for } j = 1, 2, \quad (20)$$

ii) labor supply equations

$$c_{1t} \frac{\nu(1 - N_{1t})^{\nu-1}}{\psi + (1 - N_{1t})^{\nu}} = q^a_{1t} w_{1t}, \quad (21)$$

$$c_{2t} \frac{\nu(1 - N_{2t})^{\nu-1}}{\psi + (1 - N_{2t})^{\nu}} = q^b_{2t} w_{2t}, \quad (22)$$

iii) demand for intermediate goods

$$a_{1t} = \omega^{1/\eta} \left(q^a_{1t}\right)^{-1/\eta} da_{1t}, \quad (23)$$

$$a_{2t} = (1 - \omega)^{1/\eta} \left(q^a_{1t}/RER_t\right)^{-1/\eta} da_{2t}, \quad (24)$$

$$b_{2t} = \omega^{1/\eta} \left(q^b_{2t}\right)^{-1/\eta} da_{2t}, \quad (25)$$

$$b_{1t} = (1 - \omega)^{1/\eta} \left(RER_t \cdot q^b_{2t}\right)^{-1/\eta} da_{1t}, \quad (26)$$

iv) production functions for the final goods

$$da_{1t} = \left(\omega a_{1t}^{1-\eta} + (1 - \omega)\, b_{1t}^{1-\eta}\right)^{\frac{1}{1-\eta}}, \quad (27)$$

$$da_{2t} = \left((1 - \omega)\, a_{2t}^{1-\eta} + \omega b_{2t}^{1-\eta}\right)^{\frac{1}{1-\eta}}, \quad (28)$$

v) market clearing for the final goods

$$c_{jt} + i_{jt} = da_{jt}, \text{ for } j = 1, 2, \quad (29)$$

vi) market clearing for intermediate goods

$$y_{1t} = a_{1t} + a_{2t} = z_{1t} k^{\alpha}_{1t-1} N^{1-\alpha}_{1t}, \quad (30)$$

$$y_{2t} = b_{1t} + b_{2t} = z_{2t} k^{\alpha}_{2t-1} N^{1-\alpha}_{2t}, \quad (31)$$

vii) factor prices

$$R_{jt} = \alpha z_{jt} k^{\alpha-1}_{jt-1} N^{1-\alpha}_{jt}, \text{ for } j = 1, 2, \quad (32)$$

$$w_{jt} = (1 - \alpha)\, z_{jt} k^{\alpha}_{1t-1} N^{-\alpha}_{jt}, \text{ for } j = 1, 2, \quad (33)$$

viii) risk sharing condition

$$E_t \left[\beta^* \frac{\Lambda_{2t+1}}{\Lambda_{2t}} \frac{q^a_{1t+1}}{q^a_{1t}} \frac{RER_t}{RER_{t+1}} \right] = E_t \left[\beta^* \frac{\Lambda_{1t+1}}{\Lambda_{1t}} \frac{q^a_{1t+1}}{q^a_{1t}} \right] - g\varsigma b^f_{1t}, \quad (34)$$

where $\beta^* = \beta(1 + g)^{1-\sigma}$.

ix) the Euler equations

$$gm_{1t} = \beta^* E_t \left[\Lambda_{1t+1} q^a_{1t+1} R_{1t+1} + m_{1t+1} \left(1 - \delta - \frac{\Omega_1}{1-\xi} \left(\frac{i_{1t}}{k_{1t-1}} \right)^{1-1/\xi} + \Omega_2 \right) \right], \tag{35}$$

$$gm_{2t} = \beta^* E_t \left[\Lambda_{2t+1} q^b_{2t+1} R_{2t+1} + m_{2t+1} \left(1 - \delta - \frac{\Omega_1}{1-\xi} \left(\frac{i_{2t}}{k_{2t-1}} \right)^{1-1/\xi} + \Omega_2 \right) \right], \tag{36}$$

where

$$m_{jt}\Omega_1 \left(\frac{i_{jt}}{k_{jt-1}} \right)^{-1/\xi} = \Lambda_{1t}, \text{ for } j = 1, 2. \tag{37}$$

x) the equations of motion for capital

$$gk_{jt} = (1-\delta)k_{jt-1} + \left(\frac{\Omega_1}{1-1/\xi} \left(\frac{i_{jt}}{k_{jt-1}} \right)^{1-1/\xi} + \Omega_2 \right) k_{jt-1}, \text{ for } j = 1, 2. \tag{38}$$

xi) intertemporal condition for habits

$$g\theta_{jt} = \beta^* E_t \left[\lambda\theta_{jt+1} - \chi \left[c_{jt+1} \left(\psi + (1 - N_{jt+1})^\nu \right) - \chi h_{jt} \right]^{-\sigma} \right], \text{ for } j = 1, 2, \tag{39}$$

xii) the equations of motion for habits

$$gh_{jt} = \lambda h_{jt-1} + (1-\lambda)\, c_{jt} \left(\psi + (1 - N_{jt})^\nu \right), \quad \text{for } j = 1, 2,$$

xiii) bond price

$$Q_t = E_t \left[\beta^* \frac{\Lambda_{1t+1}}{g\Lambda_{1t}} \frac{q^a_{1t+1}}{q^a_{1t}} \right] - \varsigma b^f_{1t},$$

xiv) equation of motion for bonds

$$Q_t b^f_{1t} = a_{2t} - \frac{q^b_{1t}}{q^a_{1t}} b_{1t} + \frac{1}{g} b^f_{1t-1} - \frac{\varsigma}{2} \left(b^f_{1t} \right)^2.$$

B Elasticity of marginal utility of consumption and leisure

This section contains derivations of the following expressions for the coefficients in the log-linearized labor supply equation (7) discussed in Section 2.2.2:

$$\eta_{CC} = -\frac{\Lambda_{CC} C}{\Lambda_C} = \sigma \frac{\left(1 + \chi^2 \beta \frac{(1-\lambda)^2}{1-\beta\lambda^2} \right)}{(1-\chi) \left(1 - \beta\chi \frac{(1-\lambda)}{(1-\beta\lambda)} \right)},$$

$$\frac{\Lambda_{LL} L}{\Lambda_L} = \frac{\Phi''(L)\, L}{\Phi'(L)} - \eta_{CC} \frac{\Phi'(L)\, L}{\Phi(L)},$$

$$\frac{\Lambda_{CL} L}{\Lambda_C} = (1 - \eta_{CC}) \frac{\Phi'(L)\, L}{\Phi(L)}.$$

First, we note that Frisch demand functions $C_t = C(W_t, \Lambda_{C,t})$ and $L_t = L(W_t, \Lambda_{C,t})$ are implicitly defined by the first order conditions

$$\Lambda_{C,t} = \left[\left(C_t \Phi(L_t) - \chi H_t \right)^{-\sigma} + \theta_t (1-\lambda) \right] \Phi(L_t), \tag{40}$$

$$\Lambda_{C,t} W_t = \Lambda_{L,t},$$

where

$$\Lambda_{L,t} = \left[\left(C_t \Phi(L_t) - \chi H_t \right)^{-\sigma} + \theta_t (1-\lambda) \right] C_t \Phi'(L_t), \tag{41}$$

$$H_t = \lambda H_{t-1} + (1-\lambda) C_{t-1} \Phi(L_{t-1}), \tag{42}$$

and

$$\theta_t = \beta E_t \left[\lambda \theta_{t+1} - \chi \left(C_{t+1} \Phi(L_{t+1}) - \chi H_{t+1} \right)^{-\sigma} \right]. \tag{43}$$

Notice that from equation (42) we can derive the following expression for the stock of habits by simply using recursive substitution

$$H_t = (1-\lambda) \sum_{\tau=1}^{t} \lambda^{\tau-1} C_{t-\tau} \Phi(L_{t-\tau}). \tag{44}$$

Furthermore, by applying recursive substitution and the law of iterated projections to equation (43), we obtain the following expression for the multiplier θ_t :

$$\theta_t = -\chi \beta E_t \left[\sum_{\tau=1}^{\infty} (\lambda\beta)^{\tau-1} \left(C_{t+\tau} \Phi(L_{t+\tau}) - \chi H_{t+\tau} \right)^{-\sigma} \right]. \tag{45}$$

Now we differentiate (40) and (41) with respect to C_t and L_t. Denoting $\Lambda_{CC,t}$ the derivative of $\Lambda_{C,t}$ w.r.t. C_t and applying the same notational convention to the rest of the second order partial derivatives of the marginal utility, we obtain

$$\Lambda_{CC,t} = \left[-\sigma \Phi(L_t) \left(C_t \Phi(L_t) - \chi H_t \right)^{-(1+\sigma)} + (1-\lambda) \frac{\partial \theta_t}{\partial C_t} \right] \Phi(L_t),$$

$$\begin{aligned} \Lambda_{LL,t} &= \left[\left(C_t \Phi(L_t) - \chi H_t \right)^{-\sigma} + (1-\lambda) \theta_t \right] C_t \Phi''(L_t) \\ &+ \left[-\sigma \left(C_t \Phi(L_t) - \chi H_t \right)^{-\sigma-1} C_t \Phi'(L_t) + (1-\lambda) \frac{\partial \theta_t}{\partial L_t} \right] C_t \Phi'(L_t), \end{aligned}$$

$$\begin{aligned} \Lambda_{CL,t} &= \left[\left(C_t \Phi(L_t) - \chi H_t \right)^{-\sigma} + (1-\lambda) \theta_t \right] \Phi'(L_t) \\ &+ \left[-\sigma \left(C_t \Phi(L_t) - \chi H_t \right)^{-\sigma-1} C_t \Phi'(L_t) + (1-\lambda) \frac{\partial \theta_t}{\partial L_t} \right] \Phi(L_t). \end{aligned}$$

To simplify the expressions above we use (45) and express the partial derivatives of the multiplier θ_t as

$$\frac{\partial \theta_t}{\partial C_t} = \sigma \chi \beta E_t \left[\sum_{\tau=1}^{\infty} (\lambda\beta)^{\tau-1} \left(C_{t+\tau} \Phi(L_{t+\tau}) - \chi H_{t+\tau} \right)^{-(\sigma+1)} \left(-\chi \frac{\partial H_{t+\tau}}{\partial C_t} \right) \right], \tag{46}$$

and

$$\frac{\partial \theta_t}{\partial L_t} = \sigma\chi\beta E_t \left[\sum_{\tau=1}^{\infty} (\lambda\beta)^{\tau-1} \left(C_{t+\tau}\Phi\left(L_{t+\tau}\right) - \chi H_{t+\tau}\right)^{-(\sigma+1)} \left(-\chi\frac{\partial H_{t+\tau}}{\partial L_t}\right)\right]. \tag{47}$$

Equation (44) implies that

$$\frac{\partial H_{t+\tau}}{\partial C_t} = \lambda^{\tau-1}(1-\lambda)\Phi\left(L_t\right), \text{ for } \tau = 1, 2, ...,$$

and

$$\frac{\partial H_{t+\tau}}{\partial L_t} = \lambda^{\tau-1}(1-\lambda)C_t\Phi'\left(L_t\right), \text{ for } \tau = 1, 2, ...,$$

which allows us to rewrite the partial derivatives of the multiplier θ_t as

$$\frac{\partial \theta_t}{\partial C_t} = -(1-\lambda)\sigma\chi^2\beta E_t \left[\sum_{\tau=1}^{\infty} \left(\lambda^2\beta\right)^{\tau-1} \left(C_{t+\tau}\Phi\left(L_{t+\tau}\right) - \chi H_{t+\tau}\right)^{-(\sigma+1)}\right] \Phi\left(L_t\right), \tag{48}$$

and

$$\frac{\partial \theta_t}{\partial L_t} = -(1-\lambda)\sigma\chi^2\beta E_t \left[\sum_{\tau=1}^{\infty} \left(\lambda^2\beta\right)^{\tau-1} \left(C_{t+\tau}\Phi\left(L_{t+\tau}\right) - \chi H_{t+\tau}\right)^{-(\sigma+1)}\right] C_t\Phi'\left(L_t\right). \tag{49}$$

Let's denote steady-state values of the variables by eliminating their time subscripts. We obtain the following expressions for the steady-state values of marginal utility of consumption and leisure and their partial derivatives:

$$\Lambda_C = \left[\left(C\Phi\left(L\right) - \chi H\right)^{-\sigma} + (1-\lambda)\,\theta\right] \Phi\left(L\right), \tag{50}$$

$$\Lambda_L = \left[\left(C\Phi\left(L\right) - \chi H\right)^{-\sigma} + (1-\lambda)\,\theta\right] C\Phi'\left(L\right), \tag{51}$$

$$\Lambda_{CC} = \left[-\sigma\Phi\left(L\right)\left(C\Phi\left(L\right) - \chi H\right)^{-(1+\sigma)} + (1-\lambda)\theta_C\right] \Phi\left(L\right), \tag{52}$$

$$\Lambda_{LL} = \left[\left(C\Phi\left(L\right) - \chi H\right)^{-\sigma} + (1-\lambda)\,\theta\right] C\Phi''\left(L\right) + \left[-\sigma\left(C\Phi\left(L\right) - \chi H\right)^{-\sigma-1} C\Phi'\left(L\right) + (1-\lambda)\,\theta_L\right] C\Phi'\left(L\right), \tag{53}$$

$$\Lambda_{CL} = \left[\left(C\Phi\left(L\right) - \chi H\right)^{-\sigma} + (1-\lambda)\,\theta\right] \Phi'\left(L\right) + \left[-\sigma\left(C\Phi\left(L\right) - \chi H\right)^{-\sigma-1} C\Phi'\left(L\right) + (1-\lambda)\,\theta_L\right] \Phi\left(L\right), \tag{54}$$

where θ_C and θ_L are the values of the partial derivatives in equations (48) and (49) in the steady state:

$$\begin{aligned}\theta_C &= -(1-\lambda)\sigma\chi^2\beta \left[\sum_{\tau=1}^{\infty} \left(\lambda^2\beta\right)^{\tau-1}\right] \Phi\left(L\right)\left(C\Phi\left(L\right) - \chi H\right)^{-(\sigma+1)} \\ &= -\sigma\frac{(1-\lambda)\chi^2\beta}{1-\beta\lambda^2}\Phi\left(L\right)\left(C\Phi\left(L\right) - \chi H\right)^{-(\sigma+1)},\end{aligned} \tag{55}$$

$$\begin{aligned}\theta_L &= -(1-\lambda)\sigma\chi^2\beta \left[\sum_{\tau=1}^{\infty} \left(\lambda^2\beta\right)^{\tau-1}\right] C\Phi'\left(L\right)\left(C\Phi\left(L\right) - \chi H\right)^{-(\sigma+1)} \\ &= \frac{C\Phi'\left(L\right)}{\Phi\left(L\right)}\theta_C.\end{aligned} \tag{56}$$

Finally using equations (50)-(54), equations (55)-(56), the steady-state value for the habit stock

$$H = C\Phi\left(L\right),$$

and the steady-state value for the multiplier

$$\theta = -\frac{\beta\chi}{(1-\beta\lambda)}\left(C\Phi\left(L\right) - \chi H\right)^{-\sigma},$$

we obtain the coefficients of the log-linearized labor supply equation (7) as follows:

$$\begin{aligned}
\frac{\Lambda_{CC}C}{\Lambda_C} &= \frac{\left[-\sigma\Phi\left(L\right)\left(C\Phi\left(L\right) - \chi H\right)^{-(1+\sigma)} + (1-\lambda)\theta_C\right]C\Phi\left(L\right)}{\left[\left(C\Phi\left(L\right) - \chi H\right)^{-\sigma} + (1-\lambda)\,\theta\right]\Phi\left(L\right)} \\
&= -\sigma\frac{\left(1+\chi^2\beta\frac{(1-\lambda)^2}{1-\beta\lambda^2}\right)}{(1-\chi)\left(1-\beta\chi\frac{(1-\lambda)}{(1-\beta\lambda)}\right)},
\end{aligned}$$

$$\begin{aligned}
\frac{\Lambda_{LL}L}{\Lambda_L} &= \frac{\Phi''\left(L\right)L}{\Phi'\left(L\right)} + \frac{-\sigma\left(C\Phi\left(L\right) - \chi H\right)^{-\sigma-1}C\Phi'\left(L\right)L + (1-\lambda)\,\theta_L L}{\left[\left(C\Phi\left(L\right) - \chi H\right)^{-\sigma} + (1-\lambda)\,\theta\right]} \\
&= \frac{\Phi''\left(L\right)L}{\Phi'\left(L\right)} + \frac{\Lambda_{CC}C}{\Lambda_C}\frac{\Phi'\left(L\right)L}{\Phi\left(L\right)},
\end{aligned}$$

$$\begin{aligned}
\frac{\Lambda_{CL}L}{\Lambda_C} &= \frac{\left[\left(C\Phi\left(L\right) - \chi H\right)^{-\sigma} + (1-\lambda)\,\theta\right]\Phi'\left(L\right)L + \left[-\sigma\left(C\Phi\left(L\right) - \chi H\right)^{-\sigma-1}C\Phi'\left(L\right) + (1-\lambda)\,\theta_L\right]\Phi\left(L\right)L}{\left[\left(C\Phi\left(L\right) - \chi H\right)^{-\sigma} + (1-\lambda)\,\theta\right]\Phi\left(L\right)} \\
&= \frac{\Phi'\left(L\right)L}{\Phi\left(L\right)} + \frac{\Lambda_{CC}C}{\Lambda_C}\frac{\Phi'\left(L\right)L}{\Phi\left(L\right)}.
\end{aligned}$$