Wicksell versus Taylor: A Quest for Determinacy and the (Ir)relevance of the Taylor Principle *

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Abstract

In a new-Keynesian model we compare the determinacy regions of price-level targeting rules (called Wicksellian rules) and Taylor rules. We conclude that Wicksellian rules do not require the Taylor principle to be satisfied to induce determinacy. Moreover, the areas of determinacy are generally larger under Wicksellian rules. Our results have two implications. First, we show that in a univariate setting estimating simple Taylor rules, when the true rule is Wicksellian, may lead to erroneously conclude that the equilibrium is indeterminate even if the true data generating process is such that indeterminacy is absent. Second, if the policy estimation is performed using system based methods, indeterminacy is ruled out. However, the policy misspecification will led to conclude that the central bank is less averse to inflation movements.

Keywords: Wicksellian rules, Taylor rules, determinacy.

JEL codes: E31, E52, E58, C62.

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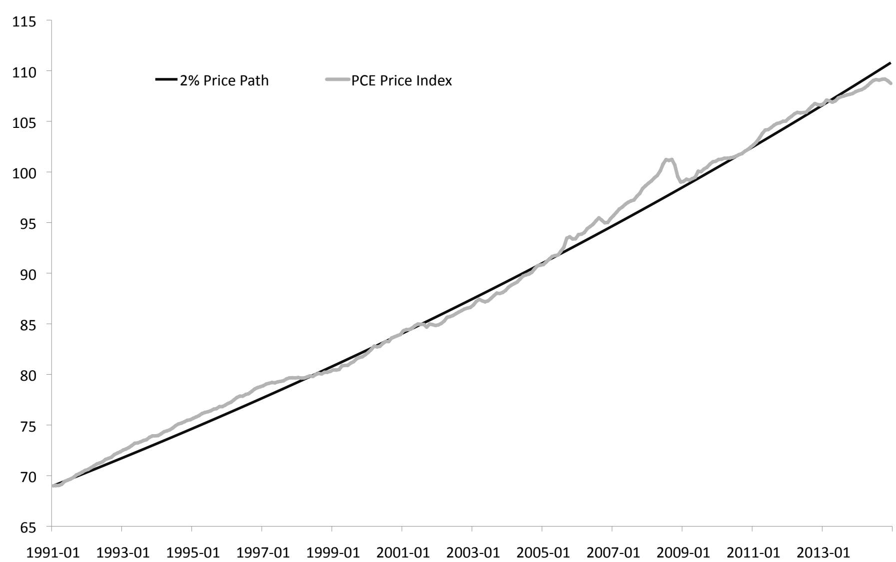
1 Introduction

Since Taylor (1993) many studies document that the Taylor rule, in which the policy rate reacts to deviations of the inflation rate and a measure of the output gap, describes fairly well the monetary policy in the USA and in other economies [1]. In recent years Wicksellian rules, in which the policy rate reacts to deviations of the *price level* instead of inflation have regained attention among academics and policymakers. Several elements have contributed to this fact. Gorodnichenko and Shapiro (2007) show that the FED, in the Volcker-Greenspan era, was behaving consistently with having the price level in its objectives. They also concluded that price-level targeting (PLT) is superior to inflation targeting (IT) in a wide range of situations. Further evidence is provided by Bullard (2012) who noticed that prices in the USA have fluctuated, between 1995 to 2012, around a 2% price path. Expanding the sample from 1991 to 2014 reinforces this finding (Figure 1).

Figure 1: Personal Consumption Expenditure Price Index and 2% Price Path: 1991-2014



Source: author's calculation based on US. Bureau of Economic Analysis. Last observation December 2014.

[1] See, among others, Clarida et al. (2000) for evidence in the USA, Clarida et al. (1998) and Lubik and Schorfheide (2007) for estimation of Taylor rules in advanced economies and Aizenman et al. (2011) for evidence in emerging countries.

In a recent contribution, Giannoni (2014) concludes, in the context of a New Keynesian model, that simple Wicksellian rules perform better in terms of welfare and are more robust to alternative shock processes than Taylor rules. Evans (2012), on the other hand, argues that PLT would be a helpful complement of FED's strategies in the aftermath of the 2008 global financial crisis.

In the case of Canada Kamenik et al. (2013) and Ruge-Murcia (2014) provide similar evidence to the one reported in Figure 1. In particular, they show that, since the mid 1990s, in Canada consumer prices, inflation expectations and the policy rate are determined in a way that is consistent with an element of price-level-targeting.

Despite the renewed interest in PLT there are only few studies investigating the properties of Wickesllian rules in the context of New Keynesian models. Furthermore, the existing studies concentrate only on simple rules that react to contemporaneous values of the price level and the output gap [2]. As noted by Bullard and Mitra (2002) contemporaneous data rules place unrealistic informational demands on the monetary authority: precise information on the current level of prices and the output gap are usually not available to policymakers. In the case of Taylor rules the properties of alternative specifications, that consider more realistic informational sets for the central bank, have already been derived in closed economies (Bullard and Mitra (2002)) and small open economies (Llosa and Tuesta (2008)).

In the case of Wicksellian rules, however, the properties of non-contemporenous specifications, both in terms of determinacy and learnability, have not been derived. In this context, the objective of this paper is twofold. First, in a standard New Keynesian model we derive the areas of determinacy of forward-looking, backward-looking and hybrid Wicksellian rules (rules that react to the expected level of prices and contemporaneous output gap). Second, we assess the implication of estimating Taylor rules in a context in which the central bank is setting its policy based on Wicksellian rules. The purpose of this exercise is to determine

[2] For instance, Kerr and King (1996) and Woodford (2003) who conclude, in different setups, that if the monetary authority adjusts the interest rate in response to deviations of the price level from a target path, then there is a unique equilibrium under a wide range of parameter choices: all that is required is that the authority raise the nominal rate when the price level is above the target path and lower it when the price level is below the target path. By contrast, if the monetary authority responds to deviations of the inflation rate from a target path, then a much more aggressive pattern is needed: the monetary authority must make the nominal rate rise by more than one-for-one with the inflation rate (the so called Taylor principle). A similar result emerges in the case in which central banks, besides reacting to prices, also respond to contemporaneous output (Giannoni (2014))

the extent to which misleading conclusions about the nature of the central bank are drawn if the econometrician fails to recognize that prices are trend stationary (i.e the monetary authority is implementing PLT).

The main findings of this paper are as follows. First, Wicksellian rules do not require the Taylor principle, understood as an increase of the policy rate by more than one-for-one with inflation, to be satisfied in order to generate determinacy. This is true regardless of whether the rule reacts to lagged, contemporaneous, or expected inflation and output. Second, as in the case of non-contemporaneous Taylor rules, we show that Wicksellian rules can induce indeterminacy if the response to prices is overly aggressive. We find, however, that in all cases the maximum response to prices is twice as large as the maximum response to inflation in the Taylor rule. Third, we demonstrate that the estimated response to inflation, in a simple Taylor rule that only reacts to inflation, is a downward-biased estimator of the true response to prices if the monetary authority sets its policy based on a simple PLT rule. The bias is such that the estimated inflation response is half the size of the response to prices in the PLT. As a consequence, the econometrician may conclude, erroneously, that the system is not determined if he fails to recognize that the central bank follows a PLT rule. Misleading conclusions regarding the nature of the central bank emerge also in the case of rules that incorporate a reaction to the output gap, as well as rules that are system based estimated.

The paper is organized as follows. In Section 2 we derive the determinacy regions of alternative Wicksellian rules and compare them with the areas of analogous Taylor rules. We do so in a standard New Keynesian model. Section 3 assess the implications, in terms of the dynamic followed by prices and inflation, of implementing either a PLT rule or a Taylor rule. This section provides evidence suggesting that prices, in the USA, are trend stationary, so it is not possible to reject the hypothesis that the FED follows a PLT strategy. In Section 4 we show that estimating Taylor rules may lead to misleading conclusions regarding the nature of the central bank, if the central bank follows a PLT rule. This results is derived analytically in the case of simp rules and through stochastic simulations in the case of rules that, besides reacting to inflation or prices, introduce policy inertia and reaction to the output gap. Section 5 concludes.

2 Determinacy regions in a New Keynesian model

The scope of this section is to determine the parameter regions for which a (linearized) standard monetary policy model has a unique solution, under a particular specification of the monetary policy rule followed by the central bank. We focus our attention in areas where a unique solution exists because areas where there are infinite solutions are areas in which some undesirable outcomes may arise, such as sunspots and equilibria in which fluctuations in inflation and the output gap are driven by self-fulfilling expectations. To this end, we study determinacy properties of different specifications of monetary policy rules in the standard workhorse New Keynesian model analyzed by, among many, Clarida et al. (1999), Woodford (2003) and more recently Giannoni (2014).

The model economy can be represented by two equations, one intertemporal IS equation that is obtained from the intertemporal optimality condition of households, and a forward-looking New Keynesian Phillips curve that summarizes the optimal pricing decision of monopolistically competitive producers that cannot adjust prices every period.

The IS equation is

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - r_t^e), \tag{1}$$

where $x_t = (y_t - y_t^N)$ is the output gap defined as the difference between the level of output, y_t, and its flexible price level, y_t^N, π_t is inflation in period t, i_t is the nominal interest rate, r_t^e is the *natural rate of interest*, or the real interest rate of the flexible price economy, and σ is the inverse of the intertemporal elasticity of substitution. Each variable is represented as a percentage deviation from its steady state value.

The New-Keynesian Phillips curve (NKPC) reads

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t, \tag{2}$$

where κ is related to the frequency of price-adjustment by producers, β is the discount factor and u_t is a cost push shock that introduces a trade-off in the policy maker's decision problem[3].

[3]The shock u_t represents, for example, changes is distortionary taxes or exogenous variations in the degree of market power of firms. More generally, this shock may be interpreted as the difference between the efficient level of output, y_t^E, and its natural level, y_t^N. The inclusion of such shocks does not alter the results on determinacy of different monetary policy rules.

To close the model, we need to specify a monetary policy rule to provide a specification for the nominal interest rate. In what follows we consider simple rules, that is, monetary policy rules that depend on variables observed by the central bank and that, consequently, should be feasible to adopt. We compare two different types of rules: Taylor rules and Wicksellian rules. Taylor rules have been widely analyzed in both the theoretical and empirical literature of monetary policy. Under this type of rules, the nominal interest rate reacts to deviations of the output gap and the inflation rate from specified target values. Wicksellian rules, on the other hand, have received much less attention in the literature of simple rules[4]. A Wicksellian rule implies that the nominal interest rate reacts to deviations of the output gap and the *price level* from target values.

In the next sections, we study the determinacy properties of different specifications for Taylor and Wicksellian rules. In particular, we follow Bullard and Mitra (2002) and consider interest rate equations that react to forward expectations and contemporaneous data. We incorporate in the analysis an additional rule, which we call a *hybrid* rule, that reacts to expected inflation and contemporaneous data. This rule has not received too much attention in the theoretical literature despite the fact that it has been used extensively in empirical research[5]. In the Appendix we study the empirically less relevant case of rules with lagged data.

We analytically characterize the regions of determinacy of different rules. In all cases we compare the determinacy areas of Taylor and Wicksellian rules. We follow the calibration by Woodford (1999) and Bullard and Mitra (2002) and set $\beta = 0.99$, $\sigma = 0.157$ and $\kappa = 0.024$.

2.1 Contemporaneous rules

Contemporaneous rules are are widely used both for theoretical and empirical purposes. From the seminal contribution of Taylor (1993), papers such as Adolfson (2007), Lubik and Schorfheide (2007) and Aizenman et al. (2011) have estimated this type of rules, and many papers and some books, of which Gali (2008) and Gali and Monacelli (2005) are notable examples, have assessed the behavior of such rules in theoretical frameworks.

A contemporaneous Taylor rule is represented by

[4]Some exceptions are Giannoni (2014) and Dittmar and Gavin (2005).

[5]See Clarida et al. (1998), Clarida (2001), Engel and West (2006) and Adolfson et al. (2011) among others.

$$i_t = \varphi_\pi \pi_t + \varphi_x x_t \tag{3}$$

where φ_π and φ_x represent the reaction of the nominal interest rate to deviations of the output gap and the inflation rate from target values[6] [7].

As shown by Bullard and Mitra (2002) the necessary and sufficient condition for a rational expectations equilibrium to be unique (determinate) is that

$$\varphi_\pi + \frac{1-\beta}{\kappa}\varphi_x > 1. \tag{4}$$

Notice that, from the NKPC (2), a one percent permanent change in inflation implies that the output gap increases by $(1-\beta)/\kappa$, under the assumption that $u_t = 0$. Then, condition (4) states that the increase in the nominal interest rate given a permanent one percent increase in inflation should be higher than one percent. In other words, the nominal interest rate should react by more than the increase in permanent inflation. This guarantees that the *real interest rate* increases as well, therefore dampening aggregate demand through the IS equation (1) and, consequently, reducing the output gap and inflation.

The contemporaneous Wicksellian rule is

$$i_t = \psi_p p_t + \psi_x x_t, \tag{5}$$

where ψ_p and ψ_x represent the magnitude of the reaction of the nominal interest rate to deviations of the output gap and the price level [8] [9].

As shown by Giannoni (2014) under contemporaneous Wicksellian rules such, as (5), all possible values of $\psi_p > 0$ and $\psi_x > 0$ yield a rational expectations equilibrium that is unique

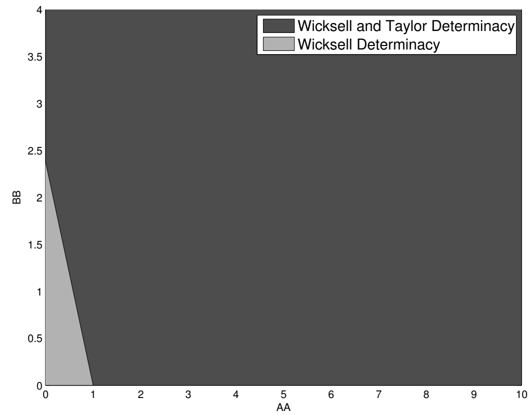
[6] These coefficients can be determined so as to minimize the loss function of the central bank. In this case, we say that the simple rule is optimal, in the sense that it is the best possible rule given the restriction imposed by the specific functional form of the rule we are assuming. This "optimal simple rule", however, will in general not implement the first best allocation.

[7] For simplicity, we are assuming that the target values for inflation and the output gap are zero. Considering any other constant target would not change the results on determinacy.

[8] Notice that, as we are working with variables expressed in deviations from the steady state, the variable p_t can be interpreted as the deviation of the price level from a predetermined target.

[9] We are implicitly assuming that the steady state of this economy is an undistorted one, i.e., inflation and the output gap are zero. If we considered a model with a different steady state, we would need to define the Wicksellian rule in terms of the deviation of the price level with respect to some deterministic trend for the price level. See Giannoni (2014) for an example.

Figure 2: Determinacy areas of contemporaneous rules



Note: The darker area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are determinate. The lighter area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is indeterminate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate.

(determinate). As a result, under contemporaneous PLT rules, the policy rate does not need to increase by more than one-for-one with inflation to ensure determinacy.

Notice that a comparison between the areas of determinacy in the space of coefficients $\{\varphi_x, \varphi_\pi\}$ and $\{\psi_x, \psi_p\}$ is not immediate, because φ_π and ψ_p are coefficients accompanying different variables. They are, however, comparable in the following sense: consider shocks that cause inflation and the output gap to react on impact in the same magnitude under both types of rules, then $\varphi_\pi = \psi_p$ implies the same reaction of the nominal interest rate *on impact* to such shocks. Then, a larger determinacy area for combinations of the coefficients of the Wicksellian rule translates into a larger set of actions that the monetary authority can take following a shock that causes a given initial reaction of inflation and the output gap, compared to the case in which the monetary authority follows a Taylor rule (Figure 2). In Section **??** and 4 we further discuss about both, the dynamics under each rule and the practical implications of estimating a Taylor rule if prices are stationary (i.e the central bank follows a PLT rule).

2.2 Forward-looking rules

In line with Clarida et al. (2000), Woodford (2000), Bullard and Mitra (2002) and Svensson and Woodford (2004) a purely forward-looking Taylor rule can be expressed as:

$$i_t = \varphi_\pi E_t \pi_{t+1} + \varphi_x E_t x_{t+1}, \tag{6}$$

where φ_π and φ_x represent the reaction of the nominal interest rate to deviations of expected inflation and output from target or long-run values. As noted by Bernanke and Woodford (1997), the use of this type of rules overcomes the problems associated with the long lag between changes in policy and changes int in inflation. In particular, the forecast of future inflation, unlike contemporaneous inflation, can be affected by changes in the policy rate. Bullard and Mitra (2002) show that, in rules such as (6), the necessary and sufficient conditions for a rational expectations equilibrium to be unique (determinate) are [10]

$$\kappa(\varphi_\pi - 1) + (1+\beta)\varphi_x < 2\sigma(1+\beta), \tag{8}$$

and

$$\varphi_\pi + \frac{1-\beta}{\kappa}\varphi_x > 1. \tag{9}$$

Notice that this rule requires the *Taylor principle*, equation (9), to be satisfied: the equilibrium, under rule (6), will be unique if φ_π and φ_x are large enough to guarantee that the real rate eventually rises in the face of an increase in inflation. Now, satisfying the Taylor principle is a necessary, but not suffcient, conditions for determinacy. As shown by Bernanke and Woodford (1997), Bullard and Mitra (2002) and Levine et al. (2007) this rule can induce indeterminacy for aggressive responses to inflation and output, if condition (8) is not satisfied.

Now, a forward-looking Wicksellian rule can be specified as follows

$$i_t = \psi_p E_t p_{t+1} + \psi_x E_t x_{t+1}, \tag{10}$$

where ψ_p and ψ_x represent the policy response to deviations of expectations of the price level and to expectations of the output gap, respectively.

The system composed by equations (1), (2) and (10) can be written as

$$y_t = BE_t y_{t+1} + Cz_t,$$

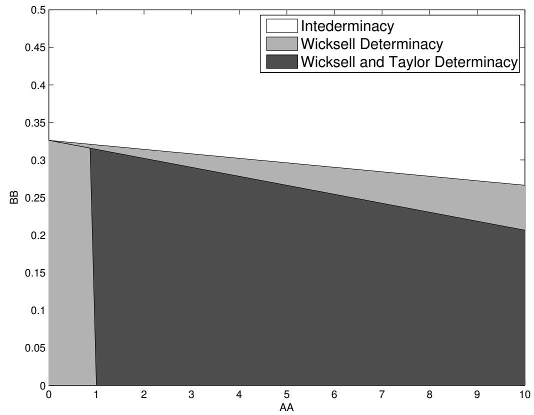
where $y_t = [x_t; \pi_t; p_{t-1}]$, $z_t = [r_t^e; u_t]$ and

[10] Bullard and Mitra (2002) list as an additional condition

$$\varphi_x < \sigma(1+\beta^{-1}). \tag{7}$$

It can be easily shown that (8) and (9) imply (7), so we do not include it as a necessary and sufficient condition.

Figure 3: Determinacy areas of forward-looking rules



Note: The darker area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are determinate. The lighter shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is indeterminate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate. The non-shaded area corresponds to corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which both systems are indeterminate.

$$B = \begin{pmatrix} \frac{\sigma\beta+\kappa(1-\psi_p)}{(\sigma-\psi_x)\beta} & -\frac{1-\psi_p(1+\beta)}{(\sigma-\psi_x)\beta} & \frac{\psi_p}{\sigma-\psi_x} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ 0 & 1 & 1 \end{pmatrix}^{-1} \tag{11}$$

This system has two endogenous non-predetermined variables, x_t and π_t, and one endogenous predetermined variable, p_{t-1}. For the equilibrium to be determinate, that is, for the system to have a unique solution, it is required that the the matrix B has two eigenvalues inside the unitary circle and one outside it. The following proposition characterizes the necessary and sufficient conditions for determinacy when rule (10) is implemented.

Proposition 1. *Under forward-looking Wicksellian rules such as (10), in which $\psi_x \geq 0$, the necessary and sufficient conditions for a rational expectations equilibrium to be unique (determinate) is that*

$$\psi_p > 0 \tag{12}$$

$$\kappa(\psi_p - 2) + 2(1+\beta)\psi_x < 4\sigma(1+\beta). \tag{13}$$

Proof. See Appendix A.2 . □

A natural corollary of the previous proposition is that the Taylor principle, understood as an increase of the nominal rate by more than on-for-one with inflation, is not a condition for determinacy. In particular, in the limiting case in which $\psi_x = 0$, determinacy is achieved by an arbitrarily small and positive ψ_p coefficient (condition (12)). Also, as in the case of forward-looking Taylor rules, overly aggressive responses to prices and to the output gap in the PLT rule induce indeterminacy (condition (13)). However, the region for which forward-looking Wicksellian rules are able to induce determinacy is larger than under forward-looking Taylor rules. To see this, we use conditions (8), (9) and (13) to derive the maximum policy response to inflation and prices for a given reaction to the output gap:

$$\varphi_\pi < \frac{2\sigma(1+\beta) - (1+\beta)\varphi_x}{\kappa} + 1 \tag{14}$$

$$\psi_p < 2\left(\frac{2\sigma(1+\beta) - (1+\beta)\psi_x}{\kappa} + 1\right) \tag{15}$$

It is clear, from (14) and (15), that for a common policy reaction to the output gap, $\varphi_x = \psi_x$, the maximum response to prices in the Wicksellian rule is twice as large as the maximum response to inflation in the Taylor rule. As shown in Figure 3, in the case of Wicksellian rules there is a larger area from which the monetary authority can choose its response to inflation and output gap, given a shock.

2.3 Hybrid rules

In many empirical exercises[11] Taylor rules are estimated using a specification in which the policy instrument reacts to expected inflation and contemporaneous output (and some other external variables in the case of open economies). This type of rules can be expressed as:

$$i_t = \varphi_\pi E_t \pi_{t+1} + \varphi_x x_t, \tag{16}$$

As before, the rationale for this specification is that expected inflation, rather than the contemporaneous level of it, could be influenced by monetary policy. This type of rule is usually referred in the literature as a forward-looking Taylor rule. In order to distinguish this specification from the pure forward-looking one considered in the previous subsection, we call it a *hybrid* Taylor rule.

[11]See Clarida et al. (1998), Clarida (2001) and Engel and West (2006) among others.

As shown by Caputo and Herrera (2013) under hybrid Taylor rules such as (16) the necessary and sufficient conditions for a rational expectations equilibrium to be unique (determinate) are that

$$\kappa(\varphi_\pi - 1) - (1+\beta)\varphi_x < 2\sigma(1+\beta), \tag{17}$$

and

$$\varphi_\pi + \frac{1-\beta}{\kappa}\varphi_x > 1. \tag{18}$$

Notice that in the case of the hybrid Taylor rule, determinacy also requires the Taylor principle to be satisfied (conditions (9) and (18) are identical). However, comparing (8) and (17), it is easy to see that the previous conditions imply a larger area of determinacy for the hybrid rule than for the forward-looking one, but a smaller one than for the contemporaneous rule. This is to be expected, as this rule uses a combination of contemporaneous data and expectations of future variables.

A hybrid Wicksellian rule can be defined as:

$$i_t = \psi_p E_t p_{t+1} + \psi_x x_t, \tag{19}$$

As before, the system composed by equations (1), (2) and (19) can be cast in the form:

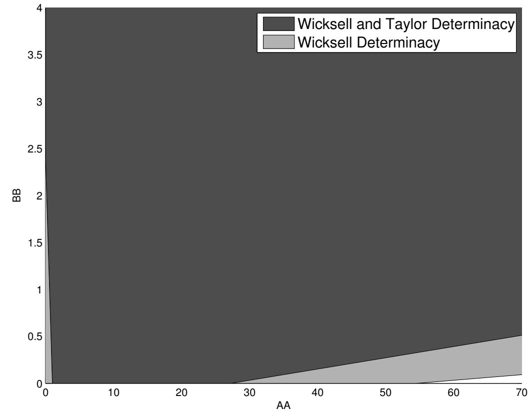
$$y_t = B E_t y_{t+1} + C z_t,$$

where $y_t = [x_t; \pi_t; p_{t-1}]$, $z_t = [r_t^e; u_t]$. In this particular case, the $\tilde{B}$ matrix takes de form:

$$B = \begin{pmatrix} \frac{\sigma\beta+\kappa(1-\psi_p)+\beta\psi_x}{\sigma\beta} & -\frac{1-\psi_p(1+\beta)}{\sigma\beta} & \frac{\psi_p}{\sigma} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ 0 & 1 & 1 \end{pmatrix}^{-1}. \tag{20}$$

This system has two endogenous non-predetermined variables, x_t and π_t, and one endogenous predetermined variable, p_{t-1}. Thus, as in the case of the pure forward-looking Wickesellian rule analyzed previously, for the equilibrium to be determinate, that is, for the system to have a unique solution, it is required that the the matrix B has two eigenvalues inside the unitary circle and one outside it.

Figure 4: Determinacy areas of hybrid rules



Note: The darker area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are determinate. The lighter shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is indeterminate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate.

The following proposition characterizes the necessary and sufficient conditions for determinacy when the rule (19) is implemented.

Proposition 2. *Under hybrid Wicksellian rules such as (19) the necessary and sufficient condition for a rational expectations equilibrium to be unique (determinate) is that*

$$\psi_p > 0, \tag{21}$$

and

$$\kappa(\psi_p - 2) - 2(1+\beta)\psi_x < 4\sigma(1+\beta). \tag{22}$$

Proof. See Appendix A.3 . □

As in the case of forward-looking Wicksellian rules, under hybrid rules the policy rate does not need to increase one-for-one with inflation (condition (21)). Also, an overly aggressive response to prices may generate indeterminacy, although in this case there is no limit for the values that ψ_x and φ_x can take (condition (22) and condition (17)).

Based on the determinacy conditions for each rule, the upper limit of ψ_p and φ_π can be expressed as:

$$\varphi_\pi < \frac{2\sigma(1+\beta) + (1+\beta)\varphi_x}{\kappa} + 1, \tag{23}$$

$$\psi_p < 2\left(\frac{2\sigma(1+\beta)+(1+\beta)\psi_x}{\kappa}+1\right). \tag{24}$$

Based on the previous conditions, again the maximum response to prices, ψ_p, is twice as large as the maximum response to inflation in the Taylor rule, φ_π.

Now, when compared to forwad-looking rules, the determinacy areas under hybrid rules is much larger. In particular, Figure 4 suggests that the kind of overreaction that would be conducive to indeterminacy would require rather extreme values of the price and inflation coefficients in the policy rules. In this context, a hybrid specification is able to expand the determinacy area of inflation-forecast and price-forecast rules. As a consequence, this type of rules can solve the problems related to the poor stabilization properties of inflation and price forecast-based rules. Levine et al. (2007) show that an alternative way of expanding the determinacy areas of forward-looking Taylor rules is to specify rules that depend on a discounted sum of current and future rates of inflation, or rules that are expressed in first differences.

2.4 Backward-Looking Rules

Under backward-looking rules, the policy maker adjusts the policy rate in response to deviations of the lagged values of the variables of interest with respect to their target values. Taylor rules that react to lagged inflation and output gap, in general, are not considered in the theoretical literature on monetary policy, with the exception of Bullard and Mitra (2002). These rules, however, have been used in studies that attempt to estimate the actual rules used by central banks, such as Rotemberg and Woodford (1999) and Giannoni and Woodford (2004).

A backward-looking Taylor rule can be expressed as follows:

$$i_t = \varphi_\pi \pi_{t-1} + \varphi_x x_{t-1}, \tag{25}$$

The following propositions, derived in Appendix A.4, characterize the necessary and sufficient conditions for the existence of a unique equilibrium [12]

[12]The necessary and sufficient conditions for determinacy are not derived by Bullard and Mitra (2002), who only present the set of sufficient conditions. Figure 1 in Bullard and Mitra (2002), however, shows the necessary and sufficient conditions for determinacy.

Proposition 3. *Under backward-looking Taylor rules such as (25) the necessary and sufficient conditions for a rational expectations to be unique (determinate) are either that*

$$\kappa(\varphi_\pi - 1) + \varphi_x(1+\beta) > 2\sigma(\beta+1), \tag{26}$$

$$\varphi_\pi + \frac{1-\beta}{\kappa}\varphi_x < 1. \tag{27}$$

or

$$\kappa(\varphi_\pi - 1) + \varphi_x(1+\beta) < 2\sigma(\beta+1), \tag{28}$$

$$\varphi_\pi + \frac{1-\beta}{\kappa}\varphi_x > 1. \tag{29}$$

As is clear, if conditions (26) and (27) hold, the Taylor principle does not need to be satisfied to induce determinacy. On the contrary, if conditions (28) and (29) hold, the Taylor principle is relevant and the area of determinacy is identical to the determinacy area in forward-looking rules (conditions (28) and (29) are identical to conditions (8) and (9)).

A backward-looking Wicksellian rule reads:

$$i_t = \psi_p p_{t-1} + \psi_x x_{t-1}, \tag{30}$$

Proposition 4. *Under backward-looking Wicksellian rules such as (30) the necessary and sufficient condition for a rational expectations equilibrium to be unique (determinate) is that*

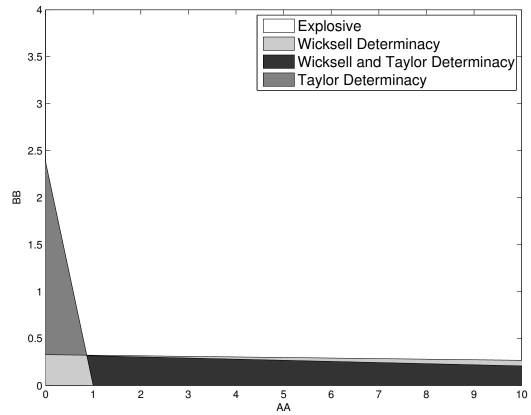
$$\psi_p > 0 \tag{31}$$

$$\kappa(\psi_p - 2) + 2(1+\beta)\psi_x < 4\sigma(1+\beta). \tag{32}$$

In the case of backward-looking Wicksellian rules, the conditions for determinacy, equations (31) and (32)), are also identical to the conditions for forward-looking rules (equations (12) and (13))).

We conclude that, as in the case of forward-looking rules, the determinacy area is considerable smaller than in the case of contemporaneous and hybrid rules (see Figure 5). In the

Figure 5: Determinacy areas of backward-looking rules



Note: The darker area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are determinate. The semi-dark shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is determinate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is explosive. The lighter shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is indeterminate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate. Finally, the non-shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are explosive.

case of backward-looking rules an overly aggressive response to prices (inflation) and output induce explosive solutions. Finally, as in the case of forward-looking and hybrid rules, the maximum response to prices in the Wicksellian rule is twice as large as the maximum response to inflation in the backward-looking Taylor rule.

3 Price Level Targeting and Inflation Targeting: Dynamic Implications

Taylor and Wicksellian rules not only differ in terms of the determinacy areas, as shown in the previous section, they imply very different path for prices. We will show that these two elements may determine that, in practice, it may be difficult for the econometrician to accurately identify the nature of the central bank. The aim of this section is to show how the path for prices evolves under each type of rules. The next section assesses the importance of this element to infer the nature of the central bank.

Under PLT an increase in the price level, beyond the price-path, has to be offset in subsequent periods. In theory, IT does not impose such a restriction so temporary shocks that impinge on the price level may be accommodated. As a consequence, under any of the Taylor rules specifications discussed in the previous section, the price level is a non-stationary variable, whereas in the case of Wicksellian rules, the price level is a stationary variable.

To understand the different path that prices may follow under alternative monetary regimes, it is useful to analyze the solution under each regime. The New Keynesian model with a Taylor rule is such that the dynamics of the endogenous variables, $y_t = [x_t; \pi_t]$, is a function of the exogenous, stationary shocks r_t^e and u_t

$$y_t = ar_t^e + bu_t$$

Under this solution inflation and the output gap are stationary, but the price level has a unit root. Hence exogenous shock generate a permanent movement in the price level.

Now, in the case of the model with a Wicksellian rule, there is an additional endogenous predetermined variable, p_{t-1}, so the system has a solution of the form

$$y_t = ar_t^e + bu_t + cp_{t-1}$$

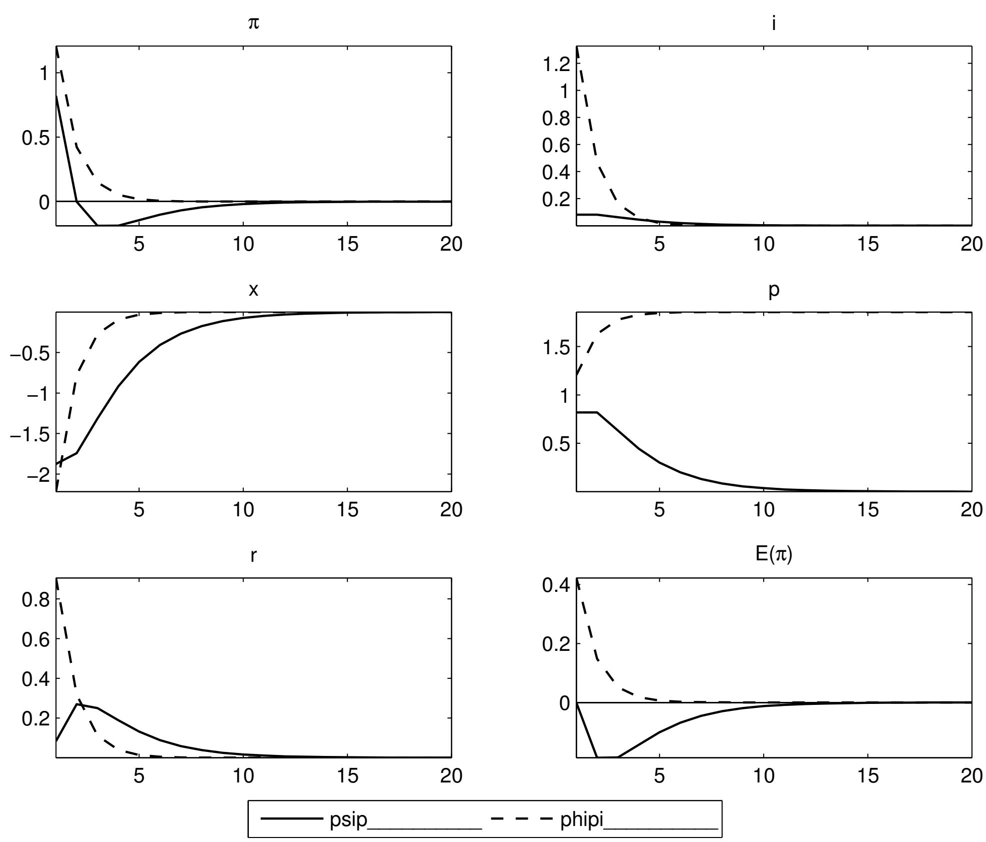
where $y_t = [x_t; \pi_t; p_t]$. In the case of the Wicksellian rule, the element of vector c that corresponds to the price level, c_p, is smaller than one, that is

$$p_t = a_p r_t^e + b_p u_t + c_p p_{t-1} \tag{33}$$

Then, the price level is stationary and uniquely determined by this equation, given that the initial condition for the price level and the law of motion for prices determines a non-explosive path.

To illustrate the path followed by all variables we simulate the dynamic response of the economy, characterized by (1) and (2), in the face of a cost push shock, under two alternative instrument rules. The first one is a simple Wicksellian rule in which the policy rate reacts only to contemporaneous prices, whereas the second rule is a simple Taylor rule which only reacts to contemporaneous inflation. We impose a small policy response to the price level, $\psi_p = 0.1$, which ensures determinacy but implies, on impact, that the Taylor principle is not satisfied. In fact, in this case the nominal interest rate moves less than one to one with inflation. On the other hand, we set the policy response to inflation to $\varphi_\pi = 1.1$, so determinacy conditions (and the Taylor principle) are satisfied.

Figure 6: Responses to a 1% Cost Push Shock (in percentage deviation from steady state)



The response of the economy to a cost push shock is presented in Figure 6. In the case of the simple Taylor rule, as expected, the interest rate increases, on impact, more than inflation (contemporaneous and expected). As a consequence, the real ex-ante interest rate increases, pushing down the output gap. The decline in the output gap reduces marginal costs, inducing a decline in inflation over time. The price level, on the other hand, increases and does not return to its initial level. As consequence under IT prices have a unit root: transitory shocks have a permanent effect.

In the case of the Wicksellian rule the transmission mechanism is quite different. The cost push shock generates, on impact, an increase in the price level and in inflation of the same magnitude. Given the fact that $\psi_p = 0.1$, it follows that the increase in the nominal rate is well below the increase in inflation (see top row of Figure 6). Hence, on impact, the Taylor principle is not satisfied. In subsequent periods, the policy rate remains above its steady state level as long as the price level does not adjust to its initial value. To induce this adjustment, inflation should decline to negative values. This, in turn, requires a contraction in the output gap, which is both more severe and persistent than in the case of IT. This contraction is only possible if the real rate increases. Under PLT prices eventually converge to its initial level, so prices are stationary.

Under PLT, the increase in the real rate is driven, mostly, by expected future deflation. As a consequence, determinacy is achieved despite the fact that the Taylor principle is not satisfied on impact [13].

3.1 Is Price Level Trend Stationary? Evidence for the USA

In theory, PLT and IT predict that inflation and the output gap are stationary variables. Both regimes, however, have very different testeable implications for prices. As noted by Ruge-Murcia (2014), PLT generates two alternative predictions for the price level. First, since price-level deviations from the targeted path must be offset in future periods, the price level should follow a stationary process around a deterministic trend and, second, the deviations from this trend should follow a stationary process around zero. In contrast, under IT shocks have a permanent effect on prices, so the price level should have a unit root. In

[13] After the initial period, the policy rate is slightly above its steady state level, whereas inflation is systematically below its long-run value. As a consequence, there is a negative relationship between the interest rate and inflation after the initial period, so the Taylor principle is not satisfied in subsequent periods.

addition, the deviation of the price index from the trend implied by the inflation target should have a unit root as well.

The evidence regarding the properties of the price level in IT countries is mixed. On the one hand, Ruge-Murcia (2014) concludes that in Canada the price level is trend stationary and that deviations from this trend follows a stationary process around zero. Similar results for Canada are reported by Kamenik et al. (2013). On the other hand, Ruge-Murcia (2014) shows that in Australia, New Zealand, Sweden and the United Kingdom the price level deviates from the price-path implied from by the inflation target [14]. As a consequence, the evidence suggest that for Canada the predictions of IT are rejected in favor of those of PLT.

Despite the fact that prices in the USA seem to be trend stationary (Figure 1 and discussion in Bullard (2012)), there are, to our knowledge, no formal tests as to whether prices evolve as predicted by PLT. In this context, we test the two main predictions regarding PLT: i) Whether prices are trend stationary in the USA and, if so, ii) if deviations from this trend follow and stationary process around zero. We use monthly data from1959.01 to 2014.12 for the personal consumption expenditures (PCE) price index. This is the price measure officials chose for their inflation target since 2012 [15]. In addition, we perform the stationary test on the GDP deflator series during the same sample period [16]. This price measure has been used in empirical exercises that attempt to characterize the monetary policy in the USA [17].

Table 1 reports the KPSS stationary test for the PCE price index and the GDP deflator. For the pre-Volcker era, 1959.01 to 1979.08, the null hypothesis of trend stationarity is rejected for both price indices. In the period after the appointment of Volcker as FED Chairman, the post-Volcker era, the price dynamics changed importantly. In particular, from 1979.09 to 2014.12, the hypothesis of trend stationarity cannot be rejected. In this period the slope of the trend, which could be interpreted as the long-run price/inflation target, is 2.46% in the case of the PCE price index and 2.40% in the case of the GDP deflator. Furthermore, the hypothesis that price deviations from this trend follows a stationary process

[14] A caveat in Ruge-Murcia (2014) is that the price-path implied by the inflation target is constructed assuming that the price target at the beginning of the period is equal to the actual price level. As a consequence, results may change if one takes an agnostic view as to whether prices, at the beginning of the IT period, differ from the price-target path.

[15] On January 25th 2012 FED Governor B.Bernake announced an inflation target of 2% based on the PCE price index.

[16] This series is available on a quarterly basis from 1959.Q1 to 2014.Q4.

[17] See Clarida et al. (2000) and Gorodnichenko and Shapiro (2007).

around zero cannot be rejected. Now, in the period after the appointment of Greenspan, the post-Greenpan era, results are qualitativey similar, although in this case the long-run price/inflation target implicit in the trend is very close to 2%, which as discussed previously, is the explicit target of the FED since 2012 [18]

Table 1: KPSS Trend Stationary Tests for the Price Level

		1959.01-2014.12 Full Sample	1959.01-1979.08 Pre-Volcker	1979.09-2014.12 Post-Volcker	1987.09-2014.12 Post-Greenspan
PCE	LM-Stat.	0.401	1.042	0.147***	0.141**
	$\Delta\overline{p}$ %	-	-	2.46%	2.07%
	Zero Trend Deviation (p-value)			0.978	0.995
GDP	LM-Stat.	0.447	0.814	0.152***	0.201***
	$\Delta\overline{p}$ %	-	-	2.40%	2.09%
	Zero Trend Deviation (p value)			0.998	0.994

LM asymptotic critical values: 1% level (0.216), 5% level (0.146), 10% level (0.119)
*** No rejection of null at 1%, **No rejection of null at 5%, *No rejection of null at 10%

4 Wicksell or Taylor? Practical Implications and Misleading Conclusion

In previous sections we have shown that the Taylor principle, understood as an increase in the nominal rate by more than one-to-one with inflation, is irrelevant when monetary policy is conducted following a Wicksellian rule. Also, we have shown that the price level dynamics differ importantly across monetary regimes. These two elements may determine that, in practice, it would be difficult for the econometrician to accurately identify the nature of the central bank. In this section we show, in three different contexts, how misleading conclusions may emerge if the econometrician estimates a Taylor rule when the true underlying policy governing monetary policy is of the Wicksellian type.

4.1 Simple Wicksellian Rules and Systematic Bias in Taylor Rules

To illustrate the problems that emerge when estimating Taylor rules in practice, we assess the implication of estimating a simple, non-inertial, Taylor rule in a context in which the

[18]Our results are robust to the use of alternative unit root test, the ADF and the Phillips-Perron test as well as to considering the periods from 1980 to 2014 or 1990 to 2014.

true data generating process is derived from a simple Wicksellian rule. In particular, we show that estimating a Taylor rule of the form:

$$i_t = \varphi_\pi \pi_t + \epsilon_t \tag{34}$$

may lead to conclude, erroneously, that the system is not determined if the central bank sets its policy according to:

$$i_t = \psi_p p_t \tag{35}$$

The disturbance term in (34), ϵ_t, is the lagged interest rate, i_{t-1}. As a consequence, the OLS estimation of φ_π is going to be biased and inconsistent. In particular, it can be proved that in the case of (34) the OLS estimator of φ_π is a downward biased estimator of ψ_p.

Proposition 5. *If the central bank follows a policy rule like (35), then the OLS estimator of φ_π in (34) is a downward biased estimator of ψ_p. The bias is such that $\hat{\varphi}_\pi$ tends, asymptotically, to half the value of ψ_p. This result is independent of the number and structure (persistence, variance and covariance) of the stochastic shocks in the economy. It is also independent from the structure of the model, as long as prices follow a stationary process like (33)*

To be more specific, the OLS estimator of φ_π is such that:

$$\begin{aligned} E_t\left(\hat{\varphi}_\pi\right) &= \frac{1}{2}\psi_p + E_t\left(\frac{\sum_{t=0}^{T}(k_t + z_t)}{\sum_{t=0}^{T}\pi_t^2}\right) \\ &= \frac{1}{2}\psi_p \end{aligned} \tag{36}$$

where the second term in (36) tends to zero for large samples (i.e. large values of T).

Proof. See Appendix A.5 . □

A natural corollary is that there is a range of values for ψ_p that induce determinacy in the system, but generate OLS estimates of φ_π that do not satisfy the Taylor principle. In particular:

Corollary 1. *For values of ψ_p between (0 and 2] the Wicksellian rule in (35) generates a determinate system. However, estimating a Taylor rule like (34) by OLS will lead to the conclusion that the system is indeterminate.*

We have shown that the OLS estimator of φ_π converges, asymptotically, to $\frac{1}{2}\psi_p$. In order to assess how the bias changes for different sample sizes, we simulate the economy characterized by the output gap in equation (1), the New Keynesian Phillips curve in (2), and the simple Wicksellian rule in (35). The stochastic simulation is performed with two different sample sizes, $T = 100$ and $T = 1000$ [19]. In each case, we estimate the Taylor rule in (35) using OLS. As expected, for $T = 1000$, the OLS estimator of φ_π is $\frac{1}{2}\psi_p$. The estimation is such that the null hypothesis that $\hat{\varphi}_\pi = \psi_p$ is rejected (see Table 2). For a smaller sample size, $T = 100$ which is equivalent to 25 years of data, $\hat{\varphi}_\pi$ is still a downward biased estimator of ψ_p and its value is very close to $\frac{1}{2}\psi_p$. Again, in this case is not possible to reject the null that $\hat{\varphi}_\pi = \psi_p$ (see Table 3).

Table 2: Contemporaneous Taylor Rules Estimates (OLS w/ sample size T=1000)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$\psi_p = 0.1$	$\psi_p = 0.5$	$\psi_p = 1$	$\psi_p = 1.1$	$\psi_p = 1.5$	$\psi_p = 2$	$\psi_p = 3$	$\psi_p = 5$	$\psi_p = 10$
$\hat{\varphi}_\pi$	0.050***	0.250***	0.500***	0.550***	0.750***	1.000***	1.500***	2.501***	5.002***
	(0.006)	(0.025)	(0.046)	(0.050)	(0.066)	(0.084)	(0.119)	(0.184)	(0.333)
Observations	1000	1000	1000	1000	1000	1000	1000	1000	1000
R-squared	0.068	0.089	0.104	0.106	0.114	0.123	0.136	0.154	0.183

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.001$

Table 3: Contemporaneous Taylor Rules Estimates (OLS w/ sample size T=100)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$\psi_p = 0.1$	$\psi_p = 0.5$	$\psi_p = 1$	$\psi_p = 1.1$	$\psi_p = 1.5$	$\psi_p = 2$	$\psi_p = 3$	$\psi_p = 5$	$\psi_p = 10$
$\hat{\varphi}_\pi$	0.052***	0.260***	0.520***	0.572***	0.779***	1.038***	1.554***	2.585***	5.156***
	(0.014)	(0.064)	(0.120)	(0.130)	(0.172)	(0.221)	(0.314)	(0.489)	(0.889)
Observations	100	100	100	100	100	100	100	100	100
R-squared	0.107	0.132	0.150	0.153	0.163	0.173	0.188	0.211	0.244

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.001$

[19] We allow for demand and supply shocks, i.e. disturbances to the IS and New Keynesian Phillip curve. We consider i.i.d shocks to r_t^e and u_t, with mean zero and variances σ_{r^e}=1 and σ_u=0.05.

Alternatives to OLS, for instance a two-stage instrumental variables approach, may overcome the bias in (34). However, even if the bias is eliminated, the problem regarding the identification of the nature of the central bank persist. In particular, if an alternative procedure succesfully removes the bias in (34), values of ψ_p in the interval (0,1] will generate estimates of φ_π in the same interval, thus violating the Taylor principle. Not surprisingly, as long as the lagged interest rate is omitted from the simple Taylor rule in (34), it would not be possible for the econometrician to properly infer the nature of the central bank.

4.2 GMM Estimates of Taylor Rules and Indeterminacy

In many empirical applications, an inertial Taylor rule that reacts to expected inflation and to the contemporaneous output gap is specified. In particular, Clarida et al. (1998) and Clarida et al. (2000) among others, estimate for the USA and other developed countries, a partial adjustment hybrid Taylor rule of the form:

$$i_t = \rho i_{t-1} + \varphi_\pi E_t \pi_{t+1} + \varphi_x x_t + \varepsilon_t \tag{37}$$

The above rule is a misspecified version of the following hybrid Wicksellian rule:

$$i_t = \psi_p E_t p_{t+1} + \psi_x x_t, \tag{38}$$

As a consequence, if the true underlying policy governing monetary policy is the hybrid Wicksellian rule in (38), estimating a rule like (37) may lead to wrong inferences regarding the nature of the central bank. To assess the extend of this problem, we simulate the economy under the hybrid Wicksellian rule [20], and then estimate the Taylor rule in ((37)). We consider alternative positive values for the ψ_p coefficient and set ψ_x=2.0 [21].

In this particular case it can be shown that the error term in (37), ε_t, contains an endogenous variable (lagged value of the output gap) that is correlated to expected inflation, the output gap and the lagged interest rate. This may generate biased and inconsistent estimates, independently of the sample size T. To correct the bias generated by the correlation between the error term and the explanatory variables, we estimate the Taylor rule in (37)

[20] As before, we generate sequence of stochastic demand and supply i.i.d shocks with mean zero and variances σ_{r^e}=1 and σ_u=0.05.

[21] The results that follow are robust to alternative values of the ψ_x coefficient.

using an instrumental variable approach as in Clarida et al. (1998) and Clarida et al. (2000). In particular, we remove the unobserved expected inflation deviation by rewriting the policy rule (37) in terms of realized variables as follows:

$$i_t = \rho i_{t-1} + \varphi_\pi \pi_{t+1} + \varphi_x x_t + \nu_t \tag{39}$$

where the error term, ν_t, is a combination of the forecast errors of inflation and the disturbance ε_t. We define a vector of variables $\mathbf{u}_t$ within each central bank's information set, at the time each one chooses the interest rate, that is orthogonal to ν_t. Hence, $E[\nu_t|\mathbf{u}_t] = 0$. In order to estimate the parameters of interest, we use the generalized method of moments (GMM), instrumenting expected inflation [22]. The set of instruments $\mathbf{u}_t$ we use includes lagged values of inflation. We check the validity of instruments with the Hansen J overidentification test [23].

The results from estimating equation (37) are presented in Table (4). Despite the fact that the policy response to expected inflation is positive and statistically different from zero, it is always below one. The policy response to output, on the other hand, is positive for small values of ψ_p, but declines and is even negative as ψ_p increases. The persistence coefficient is, in general, not statistically different from zero. We conclude that estimated coefficients fall within the indeterminacy area, for this Taylor rule. In particular, for every value of ψ_p, the generalized Taylor principle for hybrid rules (condition (18)) is not satisfied by the estimated coefficients in Table (4). Now, based on the Hansen J test, we can reject, in almost all cases, the null hypothesis that our instrument set is appropriate.

The previous results illustrate the extent to which misleading conclusions regarding the nature of the central bank emerge if: i) the central bank follows a PLT strategy and ii) the econometrician estimates a misspecified Taylor rules . In particular, one may conclude that the Taylor principle is violated even in cases in which the system is determinate (results in Table 2 and Table 4).

[22]Our results are robust to instrumenting also the output ago, in the event this variable is not observed by the econometrician.

[23]We also perform the Kleibergen-Paap underidentification test, and we can reject, in all cases , the null that the equation is underidentified, so we do not report this test.

Table 4: Hybrid Taylor Rule Estimates (2-Step GMM-IV)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$\psi_p = 0.1$	$\psi_p = 0.5$	$\psi_p = 1$	$\psi_p = 1.1$	$\psi_p = 1.5$	$\psi_p = 2$	$\psi_p = 3$	$\psi_p = 5$	$\psi_p = 10$
$\hat{\rho}$	0.092	0.125*	0.132*	0.133*	0.133	0.131	0.124	0.106	0.051
	(0.063)	(0.074)	(0.078)	(0.079)	(0.081)	(0.083)	(0.086)	(0.087)	(0.065)
$\hat{\varphi}_\pi$	0.544***	0.603***	0.596***	0.593***	0.583***	0.572***	0.551***	0.514***	0.376*
	(0.057)	(0.061)	(0.061)	(0.061)	(0.059)	(0.056)	(0.051)	(0.060)	(0.122)
$\hat{\varphi}_x$	0.370***	0.095***	0.024	0.016	-0.010	-0.033**	-0.064**	-0.101**	-0.151***
	(0.077)	(0.029)	(0.017)	(0.016)	(0.013)	(0.011)	(0.008)	(0.006)	(0.003)
Observations	998	998	998	998	998	998	998	998	998
J-test(p-value)	0.935	0.589	0.451	0.431	0.366	0.304	0.214	0.107	0.006

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.001$

4.3 System Based Estimates: Misleading Conclusions Persists

We have shown that estimating the Taylor rule, in a univariate setting, generates downward biased estimates in the case of simple contemporaneous rules estimated with OLS. When a hybrid Taylor rule is estimated by GMM, as is standard in the empirical literature, the bias persist and, furthermore the estimated coefficients do not satisfy the Taylor principle, and thus fail to generate determinacy. To test whether our results are robust to alternative estimation procedures, we used a system based approach to estimate (37). As before, the true underlying policy governing monetary policy is the hybrid Wicksellian rule in (38), whereas the output gap and inflation are determined according to (1) and (2). We set the coefficients in the Wicksellian rule to ψ_p=1.1 and ψ_x=0.5 and simulate the model with the same sequence of stochastic demand and supply shocks used in the previous exercises (sample size T=1000). The coefficients in the IS and NKPC equations are, as before, those used by Giannoni (2014).

As noted by Lubik and Schorfheide (2007) a system based approach optimally adjusts the estimation of the policy rule coefficients for the endogeneity of the right-hand-side variables. Moreover, it is possible to exploit cross equation restrictions that link agents's decision rules to the policy parameters. We use Bayesian techniques to estimate the Taylor rule coefficients in (37). Bayesian estimation of the structural parameters is common in academic and policy circles, for instance in Smets and Wouters (2003), Ireland (2004), Canova and Gambetti (2009), Rabanal and Rubio-Ramirez (2005), and more recently in Lubik and Schorfheide (2007) and Kamenik et al. (2013).

We perform two exercises. First, we estimate the policy rule coefficients ρ, φ_π and φ_x in (37) given the parameters in the rest of the equations of the system, which includes actual volatility of shock, and the simulated data. This exercise shows the impact of misspecification on the Taylor rule coefficients. In the second exercise, we estimate the same Taylor rule coefficients, as well as the volatilities of the demand and supply shocks. In this case, the impact of misspecification goes beyond the policy rule equation and can, eventually, be transmitted to the stochastic processes in the system. The purpose of this last exercise is to assess the extend to which the impact of misspecification, on the policy rule coefficients, can be reduced (or exacerbated) once we let other coefficients of the system to adjust.

We assign prior distributions to Taylor rule coefficients and to the standard deviation of demand and supply shocks. Then, we used Bayesian techniques to compute the posterior distribution of relevant coefficients. It should be noticed that this system based approach requires the system to be determinate. As a consequence, the effective prior distribution is truncated at the boundary of the determinacy region. Hence, by construction, the prior and posterior distribution of the policy rule coefficients can not lie in the indeterminacy region. In short, the Taylor principle always holds.

We choose the density of the priors following Lubik and Schorfheide (2007). The interest rate smoothing coefficient, ρ, follows a beta prior distribution with mean of 0.5 with and a standard deviation of 0.25. The priors for φ_π and φ_x follow an gamma distribution and are centered at 1.1 and 0.5, respectevely. The coefficients σ_{r^e} and σ_u follow an inverse gamma distribution and are centered at their true values, 1 and 0.05 (Table 5). Under the prior mean, the determinacy conditions, equation (18), holds. In particular, in the long-run $\varphi_\pi + \frac{1-\beta}{\kappa}\varphi_x = 2.305$ which is well above 1, as required by the Taylor principle [24].

The Bayesian estimates of the Taylor rule coefficients, given the rest of parameters of the model at their true vales, can be found in Table 6. In addition to 90% posterior probability intervals we report the posterior mean. The degree of persistence increases substantially, from the 0.5 value of the prior mean to 0.722. The estimated response to inflation declines importantly from 1.1 to 0.223, whereas the response to the output gap increases just marginally from 0.5 to 0.546. For all coefficients the 90% interval is very narrow, suggesting, perhaps erroneously that the data is quite informative in order to identify the policy rule coefficients.

[24] With policy persistence the Taylor principle is such that: $\frac{\varphi_\pi + \frac{1-\beta}{\kappa}\varphi_x}{\rho} > 1$

The log data density declines to -1687 from 885, when only the Wicksellian coefficients are estimated [25]. The estimated Taylor rule imply a weaker response to inflation, both in terms of its own prior mean, as well as relative to the estimated response to the output gap. In terms of determinacy, our results show that in the long-run $\varphi_\pi + \frac{1-\beta}{\kappa}\varphi_x = 1.008$, so the Taylor principle in (18) holds just marginally. Based on these results, it would tempting to conclude the central bank is not reacting aggresivly to stabilize infaltion. In practice, however, the PLT policy followed by the central bank stabilizes both, inflation and the price level, so these conclusions may be incorrect

In order to asses the impact of policy misspecification in the rest of the equations that characterize the economy, we estimate the policy coefficients as well as the volatility of structural shocks. As before, policy misspecification tends to affect the estimation of the Taylor rule coefficients. In particular, the posterior mean of the the policy response to inflation falls to 0.04 , whereas the estimated response to the output gap increases to 0.718. The degree of policy persistence increases further to nearly 0.9 (Table 6, last two columns).

The policy misspecification has also important consequences for the other equations in the system [26]. In particular, the estimated size of structural shocks (in absolute and relative terms) move away from its true value. The estimated demand shock volatility is twice as large as the true volatility, whereas the volatility of supply shocks declines both in absolute and relative terms, from 0.05 to 0.03. As before, the Taylor principle holds just marginally: in the long-run $\varphi_\pi + \frac{1-\beta}{\kappa}\varphi_x = 1.055$.

Table 5: Prior Distribution

Coefficient	Prior Mean	Density	Std. Dev
ρ	0.500	Beta	0.250
φ_π	1.100	Gamma	0.500
φ_x	0.500	Gamma	0.250
σ_{r^e}	1.000	Inv Gamma	4.000
σ_u	0.050	Inv Gamma	4.000

[25] To verify the estimation algorithms work well, we estimate the Wicksell coefficients ψ_p and ψ_x. Not surprisingly, the posterior mean is 1.1 for ψ_p and 0.51 for ψ_x with a very narrow 90% interval. In this case, the log data density is 885.

[26] In the context of DSGE models the impact that misspecification has on the proper estimation of structural coefficients is discussed in Canova and Gambetti (2009)

Table 6: Parameter Estimation Results: Taylor Rule Posteriors

Coefficient	Prior Mean	Taylor Rule		Taylor Rule/Shocks	
		Post. Mean	90% Interval	Post. Mean	90% Interval
ρ	0.500	0.722	[0.720, 0.724]	0.891	[0.886, 0.897]
φ_π	1.100	0.223	[0.223, 0.224]	0.040	[0.039, 0.041]
φ_x	0.500	0.546	[0.545, 0.548]	0.718	[0.714, 0.723]
σ_{r^e}	1.000	-	-	2.238	[2.162, 2.315]
σ_u	0.050	-	-	0.030	[0.029, 0.031]
Log Data Density		-1687		160	
Taylor Principle (condition 18)	2.305	1.008		1.055	

5 Conclusions and future research

In a standard New Keynesian model, we compare the areas of determinacy of two alternative instrument rules. The first one is a simple Taylor rule in which the policy rate reacts to movements in inflation and output. The second one is a Wicksellian rule in which the interest rate reacts to movements in the price level and output. Our main findings are as follows. First, the area of determinacy for the policy coefficients is, in general, larger for Wicksellian rules. Second, Wicksellian rules do not need to satisfy the Taylor principle in order to induce determinacy. In particular, such rules are able to generate determinacy even in cases in which the nominal rate reacts less than one to one with inflation. In those cases the main mechanism ensuring determinacy, or an increase in the real ex-ante interest rate in the face of supply shocks, is the expected future deflation which a Wicksellian rule generates. Finally, under Wicksellian rules the price level, as well as inflation, are uniquely determined given the initial conditions. As a consequence, this rule avoids nonlocal nominal paths or nominal explosions as defined by Cochrane (2011).

There are two practical implications related to the irrelevance of the Taylor principle for Wicksellian rules. The first is that estimating a Taylor rule when the true underlying rule is Wicksellian, may lead to conclude -erroneously- that the equilibrium is indeterminate even in the case in which the price level rules ensures determinacy. The second is that, when the central bank follows a Wicksellian rule, the frequency of occurrence of an active zero lower bound on the policy rate is greatly reduced.

Based on the findings of this paper, there are theoretical issues that could be addressed in future research. First, in open economies some forms of managed exchange rate rules may alleviate problems of indeterminacy. In other words, an augmented Taylor rule that, besides reacting to inflation and output, also moves in the face of changes in the exchange rate, increases the determinacy area of the policy coefficients (see Llosa and Tuesta (2008)). Second, the existence of trend inflation in a standard New Keynesian model modifies substantially the determinacy properties of simple Taylor rules (see Ascari and Ropele (2009)). In particular, when trend inflation is considered, neither the Taylor principle nor the generalized Taylor principle, which requires the nominal interest rate to be raised by more than the increase in inflation in the long run, is a sufficient condition for local determinacy of equilibrium. In this respect, it should be interesting to check the extent to which our main results for the Wicksellian rules hold in an open economy environment and in the presence of trend inflation.

From a practical perspective, we have shown that empirical results from Taylor rules estimates may lead to misleading conclusions about the nature of monetary policy. In this context, to see the extent to which, under the pre-Volcker period, the policy followed by the Fed was indeed stabilizing, we could estimate Wicksellian rules during that period in order to assess the extent to which determinacy conditions under such rules were satisfied. Finally, recent papers (Giannoni (2014)) have shown that Wicksellian rules have a better performance in terms of welfare than simple Taylor rules. This analysis does not incorporate the advantages of Wicksellian rules under the ZLB. In this respect, it would be useful to assess the extent to which the existence of a ZLB is an element that increases even further the welfare gains of price level targeting rules.

A Appendix: proofs

A.1 Necessary and sufficient conditions for determinacy

In order to prove Proposition **??**, Proposition 1 and Proposition 2, we need to determine if matrix $\tilde{B} = B^{-1}$ has exactly two eigenvalues outside the unitary circle. To this end, we follow Proposition C.2 in Woodford (2003) that lists a set of necessary and sufficient conditions for this to be the case. For ease of exposition, we reproduce this proposition here.

Let the characteristic polynomial of the 3x3 matrix $\tilde{B}$, be defined as

$$\mathcal{P}(\lambda) = \lambda^3 + \mathcal{A}_2\lambda^2 + \mathcal{A}_1\lambda + \mathcal{A}_0,$$

where λ are the eigenvalues of $\tilde{B}$. This equation has one root inside the unit circle and two roots outside if and only if:

- **Case I**

  $$\mathcal{P}(1) = 1 + \mathcal{A}_2 + \mathcal{A}_1 + \mathcal{A}_0 < 0, \tag{40}$$

  and

  $$\mathcal{P}(-1) = 1 + \mathcal{A}_2 - \mathcal{A}_1 + \mathcal{A}_0 > 0; \tag{41}$$

  or

- **Case II**

  $$\mathcal{P}(1) = 1 + \mathcal{A}_2 + \mathcal{A}_1 + \mathcal{A}_0 > 0, \tag{42}$$

  $$\mathcal{P}(-1) = 1 + \mathcal{A}_2 - \mathcal{A}_1 + \mathcal{A}_0 < 0, \tag{43}$$

  and

  $$\mathcal{A}_4 = \mathcal{A}_0^2 - \mathcal{A}_0\mathcal{A}_2 + \mathcal{A}_1 - 1 > 0, \tag{44}$$

  or

- **Case III**

  Conditions (42) and (43) hold in addition to

  $$\mathcal{A}_4 = \mathcal{A}_0^2 - \mathcal{A}_0\mathcal{A}_2 + \mathcal{A}_1 - 1 < 0 \tag{45}$$

  and

$$|\mathcal{A}_2| > 3. \tag{46}$$

A.2 Determinacy of forward-looking Wicksellian rules

In order to prove Proposition 1, we need to determine if matrix B defined in (11) has exactly two eigenvalues inside the unitary circle. To simplify the algebra, we define $\tilde{B} = B^{-1}$, so to achieve determinacy, $\tilde{B}$ needs to have two eigenvalues outside the unit circle.

The characteristic polynomial of matrix $\tilde{B}$ is

$$\mathcal{P}(\lambda) = \lambda^3 + \mathcal{A}_2\lambda^2 + \mathcal{A}_1\lambda + \mathcal{A}_0,$$

where λ are the eigenvalues of $\tilde{B}$ and

$$\mathcal{A}_2 = -\frac{(\sigma - \psi_x)(1+\beta) + \sigma\beta + \kappa(1-\psi_p)}{(\sigma - \psi_x)\beta},$$

$$\mathcal{A}_1 = \frac{\sigma(2+\beta) + \kappa - \psi_x}{(\sigma - \psi_x)\beta},$$

$$\mathcal{A}_0 = -\frac{\sigma}{(\sigma - \psi_x)\beta}.$$

As a result, we can compute $\mathcal{P}(1)$ and $\mathcal{P}(-1)$ to obtain

$$\mathcal{P}(1) = \frac{\kappa\psi_p}{(\sigma - \psi_x)\beta}, \tag{47}$$

$$\mathcal{P}(-1) = \frac{2(1+\beta)(\psi_x - 2\sigma) - \kappa(2-\psi_p)}{(\sigma - \psi_x)\beta}. \tag{48}$$

We restrict our attention to values of $\psi_x \geq 0$. In this case, the sign of $\mathcal{P}(1)$ and $\mathcal{P}(-1)$ depends on whether $(\sigma - \psi_x)$ is positive or negative. We consider each case separately.

A.2.1 Case I: $\sigma < \psi_x$

Notice that, in this case, $\mathcal{P}(1) < 0$ and $\mathcal{P}(-1) > 0$ if the following conditions are satisfied:

$$\psi_p > 0 \tag{49}$$

$$\kappa(\psi_p - 2) + 2(1+\beta)\psi_x < 4\sigma(1+\beta). \tag{50}$$

Hence, if $\sigma < \psi_x$ the system is determinate if condition (49) and (50) are satisfied.

A.2.2 Case II and III: $\sigma > \psi_x$

Notice that, in this case, $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) < 0$ hold if conditions (49) and (50) are satisfied. To have determinacy, however, it is requiered in addition that $\mathcal{A}_4 > 0$ (Case II) or $\mathcal{A}_4 < 0$ and $|\mathcal{A}_2| > 3$ (Case III).

Now, notice that:

$$\mathcal{A}_4 = \frac{\psi_x\beta(\sigma-\psi_x)(\beta-1)+(\sigma-\psi_x)\beta\kappa+\psi_x\sigma(1-\beta)+\sigma\kappa(\psi_p-1)}{(\sigma-\psi_x)^2\beta^2}. \tag{51}$$

For (51) to be positive, since the denominator is positive, the numerator should be positive as well. This is the case if:

$$\kappa(\psi_p-1) > -\psi_x(1-\beta)^2 - \frac{\psi_x^2(1-\beta)\beta}{\sigma} - \frac{(\sigma-\psi_x)\beta\kappa}{\sigma}. \tag{52}$$

If conditions (50) and (52) are satisfied, then the system is determinate (Case II).

Now, if condition (52) is not satisfied, so that $\mathcal{A}_4 < 0$, we require that $|\mathcal{A}_2| > 3$ (Case III). Notice that

$$|\mathcal{A}_2| = \left|-\frac{(\sigma-\psi_x)(1+\beta)+\sigma\beta+\kappa(1-\psi_p)}{(\sigma-\psi_x)\beta}\right|. \tag{53}$$

Assume ψ_p is not very large, so $\mathcal{A}_2 < 0$. Then, condition $|\mathcal{A}_2| > 3$ requires that [27]

$$-\frac{(\sigma-\psi_x)(1+\beta)+\sigma\beta+\kappa(1-\psi_p)}{(\sigma-\psi_x)\beta} < -3.$$

or

$$\kappa(\psi_p-1) < \sigma(1-\beta)+\psi_x(2\beta-1). \tag{54}$$

We show next that if condition (52) is not satisfied, then condition (54) is automatically satisfied. To this end, we need to check if, when

$$\kappa(\psi_p-1) < -\psi_x(1-\beta)^2 - \frac{\psi_x^2(1-\beta)\beta}{\sigma} - \frac{(\sigma-\psi_x)\beta\kappa}{\sigma},$$

it is always the case that condition (54) holds. This is the case if the right hand side of equation (54) is larger than the right hand side of equation (52). We prove this by contradiction. Suppose

$$-\psi_x(1-\beta)^2 - \frac{\psi_x^2(1-\beta)\beta}{\sigma} - \frac{(\sigma-\psi_x)\beta\kappa}{\sigma} > \sigma(1-\beta)+\psi_x(2\beta-1).$$

Rearranging,

[27] If ψ_p is larger than one, condition (52) is always satisfied, so Case III becomes irrelevant.

$$-\left(\psi_x(1-\beta)^2+\frac{\psi_x^2(1-\beta)\beta}{\sigma}+\frac{(\sigma-\psi_x)\beta\kappa}{\sigma}+(\sigma-\psi_x)(1-\beta)+\psi_x\beta\right)>0,$$

which is clearly a contradiction because all terms inside the brackets in the left hand side of the previous expression are positive. We can conclude that all cases that satisfy $\mathcal{P}(1)>0$ and $\mathcal{P}(-1)<0$, either satisfy condition (52) or condition (54). This completes the proof.

A.3 Determinacy of hybrid Wicksellian rules

In order to prove Proposition 2, we need to determine if matrix B defined in (20) has exactly two eigenvalues inside the unitary circle. To simplify the algebra, we define $\tilde{B}=B^{-1}$, so to achieve determinacy, $\tilde{B}$ needs to have two eigenvalues outside the unit circle.

The characteristic polynomial of matrix $\tilde{B}$ is

$$\mathcal{P}(\lambda)=\lambda^3+\mathcal{A}_2\lambda^2+\mathcal{A}_1\lambda+\mathcal{A}_0,$$

where λ are the eigenvalues of $\tilde{B}$ and

$$\mathcal{A}_2=\frac{\kappa(\psi_p-1)-\psi_x\beta-\sigma}{\sigma\beta}-2,$$

$$\mathcal{A}_1=\frac{\beta(\sigma+\psi_x)+\kappa+2\sigma+\psi_x}{\sigma\beta},$$

$$\mathcal{A}_0=-\frac{\sigma+\psi_x}{\sigma\beta}.$$

As a result, we can compute $\mathcal{P}(1)$ and $\mathcal{P}(-1)$ to obtain

$$\mathcal{P}(1)=\frac{\kappa\psi_p}{\sigma\beta}>0, \tag{55}$$

$$\mathcal{P}(-1)=\frac{\kappa(\psi_p-2)-4\sigma(1+\beta)-2\psi_x(1+\beta)}{\sigma\beta}. \tag{56}$$

Since $\mathcal{P}(1)>0$ for all values of the parameters, as long as $\psi_p>0$, we can already discard Case I as a relevant case to establish areas of determinacy. Instead, we consider Case II and Case III.

A.3.1 Case II and III:

These conditions require that $\mathcal{P}(1)>0$ and $\mathcal{P}(-1)<0$. The first condition is automatically satisfied. To satisfy the second condition, given that the denominator is always positive, the numerator must be negative, that is

$$\kappa(\psi_p - 2) - 2(1+\beta)\psi_x < 4\sigma(1+\beta). \tag{57}$$

Case II requires the following additional condition to hold

$$\mathcal{A}_4 = \frac{\psi_x^2(1-\beta) + \kappa\sigma(\psi_p + \beta - 1) - \psi_x(\kappa(\psi_p - 1) + \sigma(1-\beta)^2)}{\sigma^2\beta^2} > 0. \tag{58}$$

For (58) to be positive, since the denominator is positive, the numerator should be positive as well. This is the case if

$$\psi_p > \frac{-\psi_x^2(1-\beta) + \kappa\sigma(1-\beta) + \psi_x\kappa - \psi_x\sigma(1-\beta)^2}{\kappa(\sigma + \psi_x)}. \tag{59}$$

If conditions (57) and (59) are satisfied, then the system is determinate (Case II). If condition (59) is not satisfied, so that $\mathcal{A}_4 < 0$, we require $|\mathcal{A}_2| > 3$ (Case III). Notice that

$$|\mathcal{A}_2| = \left| \frac{\kappa(\psi_p - 1) + \psi_x\beta - \sigma - 2\sigma\beta}{\sigma\beta} \right|.$$

Assume ψ_p is not very large, so $\mathcal{A}_2 < 0$. Then, condition $|\mathcal{A}_2| > 3$ requires that [28]

$$\frac{\kappa(\psi_p - 1) - \psi_x\beta - \sigma - 2\sigma\beta}{\sigma\beta} < -3,$$

or

$$\psi_p < \frac{\sigma(1-\beta) + \psi_x\beta}{\kappa} + 1. \tag{60}$$

We show next that if condition (59) is not satisfied, then condition (60) is automatically satisfied. To this end, we need to check if, when

$$\psi_p < \frac{-\psi_x^2(1-\beta) + \kappa\sigma(1-\beta) + \psi_x\kappa - \psi_x\sigma(1-\beta)^2}{\kappa(\sigma + \psi_x)},$$

it is always the case that condition (60) holds. This is the case if the right hand side of equation (60) is larger than the right hand side of equation (59). We prove this by contradiction. Suppose

$$\frac{-\psi_x^2(1-\beta) + \kappa\sigma(1-\beta) + \psi_x\kappa - \psi_x\sigma(1-\beta)^2}{\kappa(\sigma + \psi_x)} > \frac{\sigma(1-\beta) + \psi_x\beta}{\kappa} + 1.$$

Rearranging,

$$\sigma^2(1-\beta) + \sigma\psi_x\beta + \psi_x\sigma(1-\beta) + \psi_x^2 + \sigma\kappa\beta + \psi_x\sigma(1-\beta)^2 < 0,$$

[28] If ψ_p is larger than one, condition (59) is always satisfied, so Case III becomes irrelevant.

which is clearly a contradiction because all terms in the left hand side are positive. We can conclude that all cases that satisfy $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) < 0$, either satisfy condition (59) or condition (60). Then, the only relevant condition for determinacy is condition (57). This completes the proof.

A.4 Backward-looking rules

A backward-looking Taylor rule can be expressed as:

$$i_t = \varphi_\pi \pi_{t-1} + \varphi_x x_{t-1}, \tag{61}$$

The system formed by equations (1), (2) and (61) can be rewritten as

$$E_t y_{t+1} = \tilde{B} y_t + \tilde{C} z_t,$$

where $y_t = [x_t; \pi_t; i_t]$, $z_t = [r_t^e; u_t]$ and

$$\tilde{B} = \begin{pmatrix} \frac{\beta\sigma+\kappa}{\beta\sigma} & -\frac{1}{\beta\sigma} & \frac{1}{\sigma} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \varphi_x & \varphi_\pi & 0 \end{pmatrix}. \tag{62}$$

This system has two endogenous non-predetermined variables, x_t and π_t, and one endogenous predetermined variable, i_t. The following proposition characterizes the necessary and sufficient conditions for determinacy when the rule (61) is implemented [29].

Proposition 6. *Under backward-looking Taylor rules such as (61) the necessary and sufficient conditions for a rational expectations to be unique (determinate) are either that*

$$\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta) < 0 \quad \textit{and}$$
$$\kappa(\varphi_\pi - 1) + \varphi_x(1+\beta) > 2\sigma(\beta+1),$$

or

$$\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta) > 0 \quad \textit{and}$$
$$\kappa(\varphi_\pi - 1) + \varphi_x(1+\beta) < 2\sigma(\beta+1),$$

Proof. In order to prove the proposition, we need to determine if matrix $\tilde{B}$ defined in (62) has exactly two eigenvalues outside the unitary circle.

The characteristic polynomial of $\tilde{B}$ is given by

$$\mathcal{P}(\lambda) = \lambda^3 + \mathcal{A}_2\lambda^2 + \mathcal{A}_1\lambda + \mathcal{A}_0,$$

[29] Bullard and Mitra (2002) also study the determinacy areas of these type of rules, but only determine analytically sufficient conditions for determinacy, while we establish necessary and sufficient conditions.

where λ are the eigenvalues of $\tilde{B}$ and

$$\mathcal{A}_2 = -\frac{\beta\sigma + \kappa + \sigma}{\beta\sigma},$$
$$\mathcal{A}_1 = -\frac{\beta\varphi_x - \sigma}{\beta\sigma},$$
$$\mathcal{A}_0 = \frac{\varphi_\pi\kappa + \varphi_x}{\beta\sigma}.$$

We can compute $\mathcal{P}(1)$ and $\mathcal{P}(-1)$ to obtain

$$\mathcal{P}(1) = \frac{\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta)}{\beta\sigma},$$
$$\mathcal{P}(1) = \frac{-2\beta\sigma - \kappa(1-\varphi_\pi) - 2\sigma + \varphi_x(1+\beta)}{\beta\sigma}.$$

We consider cases I and II separately.

A.4.1 Case I:

Notice that, in this case, $\mathcal{P}(1) < 0$ and $\mathcal{P}(-1) > 0$ if the following conditions are satisfied:

$$\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta) < 0,$$
$$\kappa(\varphi_\pi - 1) + \varphi_x(1+\beta) > 2\sigma(1+\beta).$$

A.4.2 Case II:

Notice that, in this case, $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) < 0$ if the following conditions are satisfied:

$$\kappa(\varphi_\pi - 1) + \varphi_x(1-\beta) > 0,$$
$$\kappa(\varphi_\pi - 1) + \varphi_x(1+\beta) < 2\sigma(1+\beta).$$

Moreover, we need that

$$\mathcal{A}_4 = \frac{(\varphi_\pi\kappa + \varphi_x)(\varphi_\pi\kappa + \varphi_x + \beta\sigma + \kappa + 1) - \beta^2\sigma\varphi_x + \sigma^2\beta - \beta^2\sigma^2}{\beta^2\sigma^2} > 0.$$

For this expression to be positive, it is required that the numerator is positive. Rearranging the numerator, it can be expressed as

$$\varphi_\pi\kappa(\varphi_\pi\kappa + \varphi_x + \beta\sigma + \kappa + \sigma) + \varphi_x(\varphi_\pi\kappa + \varphi_x + \kappa + \sigma) + \varphi_x\beta\sigma(1-\beta) - \beta\sigma^2(\beta - 1),$$

which is clearly positive because $\beta < 1$ and, thus, all terms in the previous expression are positive. This completes the proof.

$\square$

A backward-looking Wicksellian rule reads:

$$i_t = \psi_p(p_{t-1} - \bar{p}) + \psi_x x_{t-1}, \tag{63}$$

As before, the system composed by equations (1), (2) and (63) can be cast in the form:

$$E_t y_{t+1} = \tilde{B} y_t + \tilde{C} z_t + \tilde{D} \bar{p},$$

where $y_t = [x_t; \pi_t; i_t; p_{t-1}]$, $z_t = [r_t^e; u_t]$. In this particular case, the $\tilde{B}$ matrix takes de form:

$$\tilde{B} = \begin{pmatrix} \frac{\beta\sigma+\kappa}{\beta\sigma} & -\frac{1}{\beta\sigma} & \frac{1}{\sigma} & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 & 0 \\ \psi_x & \psi_p & 0 & \psi_p \\ 0 & 1 & 0 & 1 \end{pmatrix}. \tag{64}$$

Notice that this system has two endogenous non-predetermined variables, x_t and π_t, and two endogenous predetermined variables, i_t and p_{t-1}. In this case, for the equilibrium to be determinate it is required that the matrix $\tilde{B}$ has two eigenvalues outside the unitary circle and two inside.

Given that matrix $\tilde{B}$ is a 4×4 matrix, we cannot characterize analytically the regions of determinacy, indeterminacy and explosive solutions [30]. Instead, we claim that the region of determinacy is the same as the one for forward-looking and hybrid Wicksellian rules.

Claim 1. *Under backward-looking Wicksellian rules such as (63) the necessary and sufficient condition for a rational expectations equilibrium to be unique (determinate) is that*

$$\kappa(\psi_p - 2) + 2(1+\beta)\psi_x < 4\sigma(1+\beta).$$

Notice that, if Claim 1 is correct, backward-looking Wicksellian rules do not need to satisfy the Taylor principle. In this case, however, we cannot ascertain that a Wicksellian rule leads to a larger area of determinacy than a Taylor rule, because there is an additional area of determinacy in the Taylor rule for which the Taylor principle is not satisfied.

We assess the validity of Claim 1 through a numerical example in the spirit of the ones conducted before. The darker area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are determinate. The semi-dark shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$

[30] To our knowledge, in the case of a 4×4 matrix there are no theorems that characterize regions where the eigenvalues are inside or outside the unitary circle, as is the case with 3×3 matrices (see Woodford (2003)).

of the Taylor rule for which the system is determinate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is explosive. The lighter shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule for which the system is indeterminate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate. Finally, the non-shaded area corresponds to combinations of parameters $\{\varphi_x, \varphi_\pi\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are explosive.

As it is clear from Figure 5, the region of determinacy of a Wicksellian backward-looking rule is identical to the region of determinacy under a forward-looking rule. Then, Claim 1 is confirmed (in this particular example). From the numerical example, it is not possible to conclude that one area of determinacy is larger than the other. It is still the case, however, that the Wicksellian rule does not need to satisfy the Taylor principle. The Taylor rule, on the contrary, has to satisfy this principle for some combinations of parameters $\{\varphi_x, \varphi_\pi\}$ in order to attain determinacy.

A.5 Systematic Bias in Simple Taylor Rules

Proof. The Wicksellian rule in (35) is equivalent to a simple Taylor rule of the form:

$$i_t = i_{t-1} + \psi_p \pi_t \tag{65}$$

As a consequence, the OLS estimator of φ_π in (34) is a biased estimator of ψ_p. The bias is determined by the correlation between the explanatory variable in (34), π_t, and the omitted variable, i_{t-1}. In particular,

$$E_t\left(\hat{\varphi}_\pi\right) = \psi_p + E_t\left(\frac{\sum_{t=0}^{T} \pi_t i_{t-1}}{\sum_{t=0}^{T} \pi_t^2}\right) \tag{66}$$

where $E_t = \left(\frac{\sum_{t=0}^{T} \pi_t i_{t-1}}{\sum_{t=0}^{T} \pi_t^2}\right)$ is the bias. Under the Wicksellian rule prices are stationary and determined according to equation (33), $p_t = a_p r_t^e + b_p u_t + c_p p_{t-1}$, where $0 < c_p < 1$. As a consequence, it can be shown that:

$$E_t\left(\pi_t^2\right) = \frac{2\left[(c_p-1)b_p E_t\left(p_{t-1}u_t\right) + (c_p-1)a_p E_t\left(p_{t-1}r_t^e\right) + b_p^2 E_t\left(u_t^2\right) + a_p^2 E_t\left(r_t^e\right)^2 + 2a_p b_p E_t\left(u_t r_t^e\right)\right]}{(1+c_p)} \tag{67}$$

Now, under the Wicksellian rule (35) it is possible to derive the following expression for the lagged interest rate, $i_{t-1} = \psi_p\left(\pi_t + \pi_{t-1} + ... + \pi_{t-\tau+1}\right)$, where $p_{t-\tau}$ converges to zero as τ increases. Based on the previous expression, it is possible to express $\pi_t i_{t-1}$ as:

$$
\begin{aligned}
E_t(\pi_t i_{t-1}) &= \psi_p \left[\pi_t\pi_{t-1} + \pi_t\pi_{t-2} + ... + \pi_t\pi_{t-1+\tau}\right] \\
&= \psi_p[(c_p - 1)(1 - c_p)E_t(p_t^2)(1 + c_p + c_p^2 + ... + c_p^{\tau-1}) + \\
&+ b_p E_t\left(p_{t-1}u_t\right) + a_p E_t\left(p_{t-1}r_t^e\right) + E_t(k_t) + E_t(z_t)] \\
&= -\psi_p \frac{1}{2} E_t\left(\pi_t^2\right) + E_t(k_t) + E_t(z_t)
\end{aligned}
\tag{68}
$$

where $k_t = -p_{t-\tau}\left[(c_p - 1)b_p \sum_{j=1}^{\tau-1} c_p^{j-1} u_{t-j} + b_p u_t\right]$ and $z_t = -p_{t-\tau}\left[(c_p - 1)a_p \sum_{j=1}^{\tau-1} c_p^{j-1} r_{t-j}^e + a_p r_t^e\right]$, and we have used the fact that $E_t(p_t^2) = E_t(p^2)$ for any t. Notice that both, k_t and z_t converge to zero as τ increases. Hence, based on (67) and (68), we conclude that:

$$
\begin{aligned}
E_t\left(\hat{\varphi}_\pi\right) &= \frac{1}{2}\psi_p + E_t\left(\frac{\sum_{t=0}^{T}(k_t + z_t)}{\sum_{t=0}^{T}\pi_t^2}\right) \\
&= \frac{1}{2}\psi_p
\end{aligned}
\tag{69}
$$

where, for a large τ, k_t and z_t converge to zero for any $t = 0$ to T.

□

References

Adolfson, M. (2007). Incomplete exchange rate pass-through and simple monetary policy rules. *Journal of International Money and Finance 26*, 468–494.

Adolfson, M., S. Laseen, J. Linde, and L. E. Svensson (2011). Optimal monetary policy in an operational medium-sized dsge model. *Journal of Money, Credit and Banking 43*, 1287–1331.

Aizenman, J., M. Hutchison, and I. Noy (2011). Inflation targeting and real exchange rates in emerging markets. *World Development 39*, 712–724.

Ascari, G. and T. Ropele (2009). Trend inflation, taylor principle, and indeterminacy. *Journal of Money, Credit and Banking 41*, 1557–1584.

Bernanke, B. S. and M. Woodford (1997, November). Inflation forecasts and monetary policy. *Journal of Money, Credit and Banking 29*(4), 653–84.

Bullard, J. (2012). Price level targeting: the Fed has it about right. *Presentation at Economic Club of Memphis, Memphis, Tennessee.*.

Bullard, J. and K. Mitra (2002). Learning about monetary policy rules. *Journal of Monetary Economics 49*, 1105–1129.

Canova, F. and L. Gambetti (2009, February). Structural changes in the US economy: Is there a role for monetary policy? *Journal of Economic Dynamics and Control 33*(2), 477–490.

Caputo, R. and L. O. Herrera (2013, July). Efficient CPI-Based Taylor Rules in Small Open Economies. Working Papers Central Bank of Chile 694, Central Bank of Chile.

Clarida, R. (2001, October). The empirics of monetary policy rules in open economies. *International Journal of Finance & Economics 6*(4), 315–23.

Clarida, R., J. Gali, and M. Gertler (1998, June). Monetary policy rules in practice: some international evidence. *European Economic Review 42*(6), 1033–1067.

Clarida, R., J. Gali, and M. Gertler (1999). The science of monetary policy - a new keynesian perspective. *Journal of Economic Literature XXXVII*, 1661–1707.

Clarida, R., J. Gali, and M. Gertler (2000, February). Monetary policy rules and macroeconomic stability: evidence and some theory. *The Quarterly Journal of Economics 115*(1), 147–180.

Cochrane, J. (2011). Determinacy and identification with taylor rules. *Journal of Political Economy 119*, 565–615.

Dittmar, R. and W. Gavin (2005). Inflation-targeting, price-path targeting and indeterminacy. *Economics Letters 88*, 336–342.

Engel, C. and K. West (2006, August). Taylor rules and the deutschmark: dollar real exchange rate. *Journal of Money, Credit and Banking 38*(5), 1175–1194.

Evans, C. (2012). Monetary policy in a low-inflation environment: developing a state-contingent price-level target. *Journal of Money, Credit and Banking 44*, 147–155.

Gali, J. (2008). *Monetary Policy, Inflation, and the Business Cycle.* Princeton University Press.

Gali, J. and T. Monacelli (2005). Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies 72*, 707–734.

Giannoni, M. and M. Woodford (2004). Optimal inflation-targeting rules. In *The Inflation-Targeting Debate*, NBER Chapters, pp. 93–172. National Bureau of Economic Research.

Giannoni, M. P. (2014). Optimal interest-rate rules and inflation stabilization versus price-level stabilization. *Journal of Economic Dynamics and Control 41*(C), 110–129.

Gorodnichenko, Y. and M. Shapiro (2007, May). Monetary policy when potential output is uncertain: understanding the growth gamble of the 1990s. *Journal of Monetary Economics 54*(4), 1132–1162.

Ireland, P. N. (2004, November). Technology Shocks in the New Keynesian Model. *The Review of Economics and Statistics 86*(4), 923–936.

Kamenik, O., H. Kiem, V. Klyuev, and D. Laxton (2013, 02). Why Is Canada's Price Level So Predictable? *Journal of Money, Credit and Banking 45*(1), 71–85.

Kerr, W. and R. G. King (1996). Limits on interest rate rules in the IS model. *Economic Quarterly* (Spr), 47–75.

Levine, P., P. McAdam, and J. Pearlman (2007, December). Inflation-Forecast-Based Rules and Indeterminacy: A Puzzle and a Resolution. *International Journal of Central Banking 3*(4), 77–110.

Llosa, L. and V. Tuesta (2008). Determinacy and learnability of monetary policy rules in small open economies. *Journal of Money, Credit and Banking 40*, 1033–1063.

Lubik, T. and F. Schorfheide (2007). Do central banks respond to exchange rate movements? a structural investigation. *Journal of Monetary Economics 54*, 1069–1087.

Rabanal, P. and J. F. Rubio-Ramirez (2005, September). Comparing New Keynesian models of the business cycle: A Bayesian approach. *Journal of Monetary Economics 52*(6), 1151–1166.

Rotemberg, J. and M. Woodford (1999). Interest rate rules in an estimated sticky price model. In *Monetary Policy Rules*, NBER Chapters, pp. 57–126. National Bureau of Economic Research, Inc.

Ruge-Murcia, F. (2014, June). Do Inflation-Targeting Central Banks Implicitly Target the Price Level? *International Journal of Central Banking 10*(2), 301–326.

Smets, F. and R. Wouters (2003, 09). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association 1*(5), 1123–1175.

Svensson, L. and M. Woodford (2004). Implementing optimal policy through inflation-forecast targeting. In *The Inflation-Targeting Debate*, NBER Chapters. National Bureau of Economic Research, Inc.

Taylor, J. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy 39*, 195–214.

Woodford, M. (1999, Supplemen). Optimal monetary policy inertia. *Manchester School 67*(0), 1–35.

Woodford, M. (2000, May). Pitfalls of forward-looking monetary policy. *American Economic Review 90*(2), 100–104.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press.