When Does Better Product Quality Imply More Advertising?

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Abstract

Existing research debates if products of better quality are more heavily advertised. Results seem contradictory. This article solves this seeming contradiction by answering the question when does better quality lead to more advertising. It models the advertising-quality relationship in an optimal control setting. On the supply-side, a firm carries out advertising and product innovation policies. Product innovation drives product quality. On the demand-side, heterogeneous consumers are sensitive to product price, product quality, and advertising expenditure. This article proposes a rule for the advertising-quality relationship generating both positive and negative relationships: Advertising increases with quality if the demand effects (quality and advertising effects on demand) outweigh the supply effect (quality effect on cost); alternatively, advertising decreases with quality if the demand effects are lower than the supply effect. Consequently, despite consumer awareness of quality, to maximize profit the firm may advertise a product of lower quality more.

Keywords: Dynamic advertising, product quality, advertising-quality relationship, product innovation, optimal control
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1 Introduction

Is a product of better quality more heavily advertised? To this classical question, the two main views on advertising, namely the informative view (Ozga, 1960; Stigler, 1961; Telser, 1964) and the persuasive view (Marshall, 1890; Chamberlin, 1933; Kaldor, 1950), provide elements supporting contradictory answers. First, based on the informative view, Nelson (1974) replies yes: When the consumer can verify the objective characteristics of a product, misleading advertising is unlikely. In this situation, a firm may advertise a product of better quality more, constituting a positive advertising-quality relationship. Second, based on the persuasive view, Comanor and Wilson (1979) respond no: If advertising can increase preferences for products of the same objective characteristics, advertising may achieve subjective product differentiation. In this case, a firm may compensate lower quality with higher advertising, creating a negative advertising-quality relationship. Empirical studies summarized by Bagwell (2007) present conflicting evidence in support of these opposing viewpoints. Yet, how can both views be correct, and how can the controversy be resolved? Presumably by recognizing that the problem is not unidimensional but multidimensional. Under some conditions, one view may apply, whereas under other conditions, the alternative view may be appropriate. Following this contingency approach, Tellis and Fornell (1988) propose a conjecture yielding both positive and negative relationships. But theoretical studies fail to prove any common explanation of such opposing relationships (see the surveys of Feichtinger et al. 1994; Bagwell 2007; Huang et al. 2012). This paper fills the gap by formally deriving both positive and negative advertising-quality relationships from demand- and supply-sides effects.

This paper studies the impact of product quality on advertising expense. As such, it analyzes the conditions under which better quality leads to more or less advertising, that is, when the advertising-quality relationship is positive or negative. The analysis, based on optimal control, links supply and demand to firm organization and consumers preferences: A monopoly simultaneously conducts dynamic advertising and product innovation policies; product innovation raises product quality; production cost is based on product quality; demand of heterogeneous consumers augments with advertising expense and product quality. This work thus builds on literature on advertising-quality relationship and dynamic advertising.

A stream of literature analyzes the advertising-quality relationship. Most papers consider a one-dimensional issue in the sense that the relationship is either positive or negative. In the informative view, the relationship is positive. A large part of research in the informative view focuses on three effects identified by Nelson (1974), which are signaling-efficiency, repeat-business, and match-products-to-buyers effects. According to the signaling-efficiency effect, more efficient firms, characterized by lower production cost, have greater incentives to develop demand by providing better quality and more advertising (Kihlstrom and Riordan, 1984; Kirmani, 1997; Hertzendorf and Overgaard, 2001; Fluet and Garella, 2002; Linnemer, 2002, 2012; Horstmann and Moorthy, 2003). Following the repeat-business effect, a firm advertises a better product more as this product generates additional future purchases (Schmalensee, 1972, 1978; Milgrom and Roberts, 1986; Hertzendorf, 1993; Horstmann and MacDonald, 1994; Moraga-González, 2000; Zhao, 2000; Orzach et al., 2002). The match-products-to-buyers effect states that a better quality product is more heavily advertised so that it matches consumers who most value its quality (Grossman and Shapiro, 1984; Bagwell and Ramey, 1993; Meurer and Stahl II, 1994; Johnson and Myatt, 2006; Anderson and Renault, 2006). Conversely, in the persuasive view, the relationship is negative. Indeed, Dorfman and Steiner (1954)'s condition suggests a negative relationship as high advertising may be used to increase consumer preferences for low quality goods (Comanor and Wilson, 1979). If quality is endogenous, lower quality firms may use more efficient advertising technologies (Colombo and Lambertini, 2003). Following a parametric approach, Doganoglu and Klapper (2006) note the importance of in advertising intensity. More recently, Chioveanu (2008) shows that persuasive advertising softens competition and drives higher price dispersion. A large part of the theoretical literature considers either informative or persuasive advertising. Extensive empirical research has therefore been conducted to arbitrate between both views (Thomas et al., 1998; Moorthy and Zhao, 2000; Ackerberg, 2001, 2003; Tsui, 2012). The main empirical finding implies little or no systematic relationship between advertising and quality. Such mixed support reflects the contingency of the relationship linked to demand and supply circumstances.

Another stream of research focuses on dynamic advertising. Nerlove and Arrow (1962) first investigate dynamic advertising. In this context, Piga (1998, 2000) studies dynamic advertising together with product differentiation and sticky prices. Erickson (2009) and Grosset et al. (2011) analyze the goodwill impact, and Jørgensen et al. (2009) study the effect of an entertainment event on the advertising policy. The competition between national and store brands that affects advertising is analyzed by Karray and Martín-Herrán (2009). Gupta and Di Benedetto (2007) consider the threat of competitive entry. Dynamic sponsored search advertising is examined by Yao and Mela (2011); Zhang and Feng (2011); Ye, Aydin, and Hu (Ye et al.). New product diffusion has been extensively studied by Krishnan and Jain (2006); Sethi et al. (2008); Krishnamoorthy et al. (2010); Swami and Dutta (2010); Fruchter and Van den Bulte (2011); Chutani and Sethi (2012); Helmes et al. (2013); Yenipazarli (Yenipazarli). An element of this literature is considering advertising implications over time with parametric models, as assessed in the surveys by Huang et al. (2012); Jørgensen and Zaccour (2014) (an exception using a structural model is for example Dockner and Jørgensen, 1988).

Prior literature considers the advertising-quality relationship as a one-dimensional issue. Yet, Tellis and Fornell (1988) do study this relationship as a two-dimensional matter. They conjecture that advertising increases with quality if quality is produced at a lower cost and if consumers respond cautiously to advertising. As they acknowledge, however, consumers may be misled by unknown quality and they provide no formal guarantee of their results. The main contribution of the present manuscript that builds on a multi-dimensional approach is detailed below.

This paper provides a rule for the advertising-quality relationship. The model at the base of the rule builds on the properties of the general functions of demand, cost, and innovation, yielding structural results as opposed to parametric results. The underlying model is thus loosely constrained. As such, it accounts for both informative and persuasive views of advertising. The rule of advertising-quality relationship identifies four effects, three on the demand-side (the direct advertising and quality effects and the indirect advertising effect) and one on the supply-side (the quality effect on production cost). Because it formally considers the impact of quality on advertising as a four-dimensional problem, this work provides better understanding of the controversy on this impact. The direct advertising and cost effects are in line with Tellis and Fornell (1988), whereas the quality and indirect advertising effects are new insights deriving from this article. More generally, the rule of the advertising-quality relationship shows that better quality increases advertising (dominance of the informative view) if the demand effects overcome the supply effect. Alternatively, greater quality decreases advertising (preeminence of the persuasive view) if the demand effects are lower than the supply effect. Although the model here is not an attempt to formalize Tellis and Fornell (1988)'s conjecture, it is nevertheless in the spirit of their works. Of interest, it proves their conjecture with no need to assume that consumers may be mislead about product quality.

This article proposes the first formal model with known quality, which integrates the opposing informative and persuasive views on advertising. By considering jointly these classic views, it reconciles their different predictions on the advertising-quality relationship in a contingency perspective. This contingency perspective yields a rule indicating the demand and supply conditions for which the relationship is positive (efficient market in the informative view) or negative (perverse market in the persuasive view). As a result, the rule that I propose founds an original articulation of the demand- and supply-side, shedding new light on the advertising-quality relationship.

2 Model Formulation

2.1 Model Development

Monopoly behavior is modeled in an optimal control setting. The planning horizon T is finite and the time $t \in [0, T]$ is continuous.

2.1.1 Quality

The firm chooses the level of innovation (or product innovation) $u(t) \in \mathbb{R}^+$ that improves quality (or product quality) $q(t) \in \mathbb{R}^+$. Thus, innovation $u(t)$ is a decision (or control) variable and quality $q(t)$ is a state variable. The quality dynamics evolve according to

$$\dot{q}(t) = K(u(t), q(t)), \tag{1}$$

where $K : \mathbb{R}^{2+} \to \mathbb{R}^+$ is twice continuously differentiable. The notations $\dot{z}$ and z_x state for the time derivative of z and the first order derivative of z with respect to x; the notations z_{xx} and z_{xy} denote the second order derivative of z with respect to x and the cross derivative of z with respect to x and y.

To simplify presentation, I shall omit the arguments from the functions where there is no confusion. Innovation u increases quality q with diminishing returns:

$$K_u > 0,\ K_{uu} < 0. \tag{2}$$

The model allows for autonomous quality dynamics. The case $K_q \geqslant 0$ captures autonomous improvement of quality, and any quality improvement is cumulative. The case $K_q < 0$ stands for autonomous deterioration.

2.1.2 Cost

The unitary production cost function $C : \mathbb{R}^+ \to \mathbb{R}^+$ is twice continuously differentiable and increases with quality q. Therefore the cost is $C = C(q(t))$ with

$$C_q \geqslant 0. \tag{3}$$

The effect of quality on cost is C_q. The independence of cost to quality $C_q = 0$ and the increase of cost with quality $C_q > 0$ describe, for example, the software and hardware industries (Shy, 2001).

2.1.3 Demand

The advertising expense $a(t) \in \mathbb{R}^+$ is a firm decision variable. The demand function $D : \mathbb{R}^{2+} \to \mathbb{R}^+$ is twice continuously differentiable. The demand of heterogeneous consumers D depends jointly on advertising a and quality q, that is $D = D(a(t), q(t))$.

Demand rises with advertising with diminishing returns. Empirical validation of the diminishing returns of advertising is synthesized in Bagwell (2007). Demand increases with product quality and the advertising effect is higher for better product quality.

$$D_a > 0,\ D_{aa} < 0,\ D_q \geqslant 0,\ D_{aq} \geqslant 0. \tag{4}$$

The direct effects of advertising and quality on demand are D_a and D_q. The indirect effect of advertising on demand D_{aq} indicates an increasing return phenomenon ($D_{aq} \geqslant 0$).

This general demand function places little restriction on the way advertising affects demand. Indeed, this demand function is compatible with the persuasive and informative views (Bagwell, 2007). According to the persuasive view, advertising changes consumer preferences (Marshall, 1890; Chamberlin, 1933; Kaldor, 1950); following the informative view, advertising provides product information (Ozga, 1960; Stigler, 1961; Telser, 1964). In each view, more advertising implies greater demand.

2.1.4 Price

The entire demand is satisfied and there is no inventory; demand equals sales and production. The price (or unit product price) P is given by the inverse demand function $P : \mathbb{R}^+ \to \mathbb{R}^+$ that is twice continuously differentiable. Price P depends of the demand D, and $P = P(D(a(t), q(t)))$. The price decreases with the demand

$$P_D \leqslant 0. \tag{5}$$

The case $P_D < 0$ refers to the monopolistic case, where the firm has market power, and the price reduces if the quantity sold increases. The case $P_D = 0$ represents a first approximation of the competitive case, in which the firm has no market power, and the price is given by the market (Schmalensee, 1978).

2.2 Model Analysis

Table 1 defines the notations used in the model analysis.

Table 1: Notation

T	= fixed terminal time of the planning horizon,
r	= interest rate,
$a(t)$	= advertising expense at time t (decision variable),
$u(t)$	= product innovation at time t (decision variable),
$q(t)$	= product quality at time t (state variable),
$\dot{q}(t)$	$= dq(t)/dt = K(u, q) =$ quality dynamics at time t,
$\lambda(t)$	= current-value adjoint variable at time t,
$C(q)$	= unit production cost,
$D(a, q)$	= demand,
$P(D)$	= unit price,
$\pi(t)$	= current profit at time t,
$H(a, u, q, \lambda)$	= current-value Hamiltonian.

The current profit $\pi(t)$, with values in $\mathbb{R}$, is

$$\pi(t) = [P(D(a(t), q(t))) - C(q(t))]D(a(t), q(t)) - a(t) - u(t).$$

The firm maximizes the intertemporal profit (or total present value of profit) by simultaneously finding the optimal trajectories of advertising and innovation over the planning horizon. The firm accounts for the quality dynamics and the discount rate $r \in \mathbb{R}$. Formally, the objective function of the firm is

$$\max_{a(t)\geqslant 0, u(t)\geqslant 0} \int_0^T e^{-rt}\pi(t)dt,$$
$$\text{subject to } \dot{q}(t) = K(u(t), q(t)).$$

The intertemporal profit maximization problem is solved with the necessary and sufficient optimality conditions of Pontryagin's maximum principle. On this basis, the shadow price (or current-value adjoint variable) $\lambda(t)$ represents the marginal value of quality on the intertemporal profit at t, and the current-value Hamiltonian H is

$$H(a, u, q, \lambda) = [P(D(a, q)) - C(q)]D(a, q) - a - u + \lambda K(u, q).$$

The current-value Hamiltonian H sums the current profit $(P - C)D - a - u$ and the future profit λK. As such, H measures the intertemporal profit.

The maximum principle implies the dynamic of the shadow price λ:

$$\begin{aligned} &\dot{\lambda} = r\lambda - H_q, \text{ with } \lambda(T) = 0, \\ \implies &\dot{\lambda} = r\lambda - [(P_D D_q - C_q)D + (P - C)D_q + \lambda K_q], \text{ with } \lambda(T) = 0. \end{aligned} \tag{6}$$

Assuming the existence of interior solutions for advertising and innovation, the monopolist maximizes the intertemporal profit H if and only if a and u satisfy the necessary first-order conditions:

$$H_a = 0 \implies P - C - \frac{1}{D_a} + P_D D = 0, \tag{7a}$$

$$H_u = 0 \implies K_u - \frac{1}{\lambda} = 0. \tag{7b}$$

Let $a_M(u)$ be the advertising rate that satisfies (7a). This advertising level maximizes the intertemporal profit for any level of innovation. In a similar vein, let $u_M(a)$ denotes the innovation rate that satisfies (7b), and it maximizes the intertemporal profit for any level of advertising. The intertemporal profit is maximal when the firm jointly selects the advertising and innovation pair such that $(a_M, u_M) = (a_M(u_M), u_M(a_M))$.

Following (7a) the markup $P - C$ is strictly positive and the firm never sells at loss because $D_a > 0$ from (4) and $P_D \leqslant 0$ from (5). With regard to (7b), innovation $u_M(a)$ increases with the shadow price of quality λ. With higher λ, $u_M(a)$ rises, and the impact of additional innovation on quality K_u falls since the diminishing returns of innovation $K_{uu} < 0$ from (2).

For the maximization of the intertemporal profit H, I further assume the three following second-order conditions (concavity of H with respect to a and u):

$$H_{aa} < 0 \implies -D_a(2P_D + P_{DD}D) - \frac{D_{aa}}{(D_a)^2} > 0, \tag{8a}$$

$$H_{uu} < 0 \implies \lambda K_{uu} < 0, \tag{8b}$$

$$H_{aa}H_{uu} - H_{au} > 0. \tag{8c}$$

The detailed proof of the implication of Condition (8a) is in Appendix A.1.

The implication of Condition (8b) together with (2) is

$$\lambda(t) > 0, \, \forall \, t \, \in [0, T), \tag{9}$$

which means that better quality always augments the intertemporal profit.

There is no additional constraint with Condition (8c) which is verified because $H_{aa} < 0$, $H_{uu} < 0$ and $H_{au} = 0$.

2.2.1 Value of $\lambda(t)$

Note $\eta_q \equiv \dfrac{\partial D}{\partial q}\dfrac{q}{D}$ the quality elasticity of demand and $\eta_a \equiv \dfrac{\partial D}{\partial a}\dfrac{a}{D}$ the advertising elasticity of demand and assume K_q constant. The intertemporal value at time t of a marginal increase in quality q is given by the integration of (6) that yields[1]

$$\lambda(t) = \int_t^T e^{-(r-K_q)(s-t)} \left(\frac{\eta_q}{\eta_a}\frac{a}{q} - C_q D\right) ds. \tag{10}$$

The shadow price of quality λ is the net result of the effects of advertising adjustment to quality $\dfrac{\eta_q}{\eta_a}\dfrac{a}{q}$ and total cost $C_q D$.

The effect of advertising adjustment to quality increase $\dfrac{\eta_q}{\eta_a}\dfrac{a}{q}$ measures the optimal advertising adjustment to stimulate demand after an increase in quality. This adjustment depends on the demand sensitivity to quality η_q, the demand sensitivity to advertising η_a, and the advertising expenditure by level of quality $\dfrac{a}{q}$. The advertising-quality effect has a positive impact on λ, that is $\dfrac{\eta_q}{\eta_a}\dfrac{a}{q} > 0$, because any increase in quality fosters the demand, and thus the future profits.

The term $C_q D$ simply represents the rate of increase of total production cost as product quality q increases, demand D remaining constant. The total cost effect $C_q D$ represents the increase of total cost as quality q increases, demand D remaining constant. The total cost effect has a negative impact on λ since $C_q \geqslant 0$ and $D \geqslant 0$. Better quality augments cost, and diminishes future profits. If the marginal impact of quality on cost is null $C_q = 0$, then the total cost effect disappears and only the advertising-adjustment effect remains. But if $C_q > 0$, the shadow price λ reduces, and hence innovation according to innovation rule (7*b*).

Conditions (9) and (10) imposes

$$\frac{\eta_q}{\eta_a}\frac{a}{q} > C_q D,\ \forall t \in [0, T). \tag{11}$$

A positive shadow price of quality λ requires that the advertising-quality effect $\dfrac{\eta_q}{\eta_a}\dfrac{a}{q}$ dominates the total cost effect $C_q D$. This condition is natural because quality rises (innovation u remaining constant), the firm gains more from higher demand than it looses from higher cost: the net result of better quality on profit is positive. As a result, the firm invests in innovation and develops quality such that increase in demand after adjusting advertising is higher than the total cost of quality. This condition makes sense because if quality increase would deteriorate profit, then the firm would not innovate to promote quality.

Result (11) changes from Dorfman and Steiner (1954)'s approach that would yield $\dfrac{\eta_q}{\eta_a}\dfrac{a}{q} = C_q D$. The difference originates from the nature of quality q, a

[1] The proof lies in Appendix A.2.

decision variable in their approach and a state variable in my approach. In their approach, the comparison of the first-order conditions on advertising and quality imposes the equality $\frac{\eta_q}{\eta_a}\frac{a}{q} = C_q D$. In my approach, $\lambda > 0$ from (9) only requires the inequality $\frac{\eta_q}{\eta_a}\frac{a}{q} > C_q D$, and my model provides more flexibility for the level of c_q, and thus of q. For instance, I am able to analyze the case $c_q = 0$ that is not possible under Dorfman and Steiner's approach (because $\frac{\eta_q}{\eta_a}\frac{a}{q} > 0$ and $D > 0$ involve $C_q > 0$). Quality q originating from innovation u differs therefore from quality q chosen, and yields a richer view.

Equations (7*b*) and (10) indicate that both the demand-side, with consumer preferences for advertising and quality, and the supply-side, with the firm capability for cost and quality, determine the product innovation policy u over time. The model takes thus into account two main views on innovation. Innovation is driven by the consumer in the market pull view and by the firm in the technology push view. Both views taken together explain most innovation features (Teng and Thompson, 1996; Adner and Levinthal, 2001; Chenavaz, 2012).

2.2.2 Variations of $u(t)$

The transversality condition $\lambda(T) = 0$ in (6) and the condition on the shadow price of quality $\lambda(t) > 0$ for all $t \in [0, T)$ in (9) imply that there is $t_1 \in [0, T)$ such that $\dot{\lambda}(t) > 0$ for all $t \in [0, t_1)$ and $\dot{\lambda}(t) < 0$ for all $t \in [t_1, T)$. The shadow price λ augments before time t_1 and declines after. In addition, because $K_u = \frac{1}{\lambda}$ from (7*b*), then $\dot{K}_u = -\frac{\dot{\lambda}}{\lambda^2}$. So, $\operatorname{sgn} \dot{K}_u = -\operatorname{sgn} \dot{\lambda}$ and for all $t \in [t_1, T)$, there is $\dot{K}_u > 0$. Recalling that $\dot{K}_u = K_{uu}\dot{u}$ and $K_{uu} < 0$ in (2), I derive $\operatorname{sgn} \dot{u} = \operatorname{sgn} \dot{\lambda}$. Therefore innovation rises before time t_1 and falls after. Formally:

$$\exists\, t_1 \in [0, T) \mid \dot{u}(t) > 0,\ \forall t \in [0, t_1),\ \dot{u}(t) < 0,\ \forall t \in [t_1, T). \tag{12}$$

In the first part of the product life cycle, from $t = 0$ to t_1, innovation rises ($\dot{u} > 0$). In the second part of the product life cycle, from t_1 to T, innovation falls ($\dot{u} < 0$). In the case $t_1 = 0$, innovation always falls. Because of innovation rule (7*b*) though, the firm always invests in innovation, even at a decreasing rate.

Result (12) links the sensitivity of the consumer to advertising and quality and to the possibility of the firm in quality and cost. In line with Teng and Thompson (1996); Adner and Levinthal (2001); Chenavaz (2012), the main innovation is achieved at the beginning of the product life cycle. At the beginning, innovation stabilizes the product and develops new features, which interest the consumer. With product maturity, innovation becomes less essential and falls.

2.2.3 Variations of $a(t)$

Equation (7a) provides the static advertising condition. The advertising condition must hold during the whole planning period, on which the firm has an optimal behavior. At the optimum, marginal revenue variations balance marginal cost variations. Such variations also generate variations in advertising and quality. The link between the dynamics of advertising and quality becomes explicit with the differentiation with respect to time of the static advertising condition[2]:

$$\dot{a}\left(-D_a(2P_D + P_{DD}D) - \frac{D_{aa}}{D_a^2}\right) = \dot{q}\left(D_q(2P_D + P_{DD}D) + \frac{D_{aq}}{D_a^2} - C_q\right), \tag{13}$$

which is called the *rule of dynamic advertising*.

The rule of dynamic advertising (13) originates from the properties of demand $D(a, q)$ and cost $C(q)$ functions. Moreover, it establishes structural and analytical (in opposition to parametric and numerical) links between the dynamics on advertising and quality. Because (13) is solely tied to the static advertising condition (7a), it depends neither on the static innovation condition (7b) nor on the quality dynamics (1). In other words, the rule of dynamic advertising is robust to any innovation process, say an exogenous or a stochastic process that would drive quality dynamics.

It would be convenient to express advertising a in terms of quality q. Because advertising and quality are decision and state variables, it is possible to apply the time elimination method (Mulligan and Sala-i Martin, 1991). Let the decision a be a once continuously differentiable function of the state q. In this case, $\frac{\dot{a}}{\dot{q}}$ simplifies to a_q which directly measures the impact of quality on advertising. Therefore (13) rewrites as

$$\begin{aligned} &\underbrace{a_q\left(-D_a(2P_D + P_{DD}D) - \frac{D_{aa}}{D_a^2}\right)}_{\text{Second-order conditions } (+)} \\ &= \underbrace{\underbrace{D_q(2P_D + P_{DD}D)}_{\text{Quality } (?)} \quad \underbrace{+\frac{D_{aq}}{D_a^2}}_{\text{Advertising } (+)}}_{\text{Demand effects}} \quad \underbrace{\underbrace{-\,C_q}_{\text{Cost } (-)}}_{\text{Supply effect}}, \end{aligned} \tag{14}$$

which is identified as the *rule of advertising-quality relationship*.

The rule of advertising quality relationship (14) measures the impact of quality on advertising a_q. Because of the second-order condition (8a), on the left-hand side of (14), the second factor $\left(-D_a(2P_D + P_{DD}D) - \frac{D_{aa}}{D_a^2}\right)$ is positive. On the right-hand side, the total impact of quality on advertising depends on the direct quality effect D_q, on the direct and indirect advertising effects D_a and D_{aq}, and on the cost effect C_q. Quality and advertising effects stand for the demand effects, whereas the cost effect measures the supply effect. I detail deeper these four effects hereafter.

[2]The detailed proof is in Appendix A.3.

- The direct quality effect D_q captures the impact of better quality on demand. This impact sums two potentially competing effects. First, higher quality increases demand, and thus lowers price ($P_D < 0$). This slope effect is negative. Second, if higher demand reduces the price at a decreasing level (if $P_{DD} > 0$), then the curvature effect is positive because the firm takes advantage of larger demand (if $P_{DD}D > 0$). As a result, it is undetermined whether the firm looses more from lower price P than it benefits from higher demand D; the sign of $(2P_D + P_{DD}D)$ is unknown, and the direct quality effect is ambiguous.

- The direct advertising effect D_a captures the raise in demand after an advertising increase. If advertising is more effective, the firm needs to advertise less to reach the same demand; the direct advertising effect is negative.

- The indirect advertising effect D_{aq} measures the higher increase of demand following the advertising of a better quality product, reflecting a synergy effect. Because it captures that synergy, the indirect advertising effect is positive.

- The cost effect c_q accounts for the impact of an increase in quality on the unit cost. The higher the cost of quality, the less money remains to advertise. Advertising falls with any increase in cost, and the cost effect is negative.

Theorem 1. *The relationship between advertising and quality is characterized by*

Cases	Conditions	Results
Case 1	$D_q(2P_D + P_{DD}D) + \frac{D_{aq}}{D_a^2} > C_q$	$a_q > 0$
Case 2	$D_q(2P_D + P_{DD}D) + \frac{D_{aq}}{D_a^2} = C_q$	$a_q = 0$
Case 3	$D_q(2P_D + P_{DD}D) + \frac{D_{aq}}{D_a^2} < C_q$	$a_q < 0$

Proof. Immediate with (14). □

With regard to Theorem 1, the impact of quality on advertising depends on the relative weigh of the demand effects (the quality and advertising effects) and the supply effect (the cost effect).

- Case 1: In this situation, demand effects outweigh the supply effect. Therefore, higher quality involves more advertising, and the consumer may infer better quality from higher advertising. Case 1 corresponds to the efficient market of the informative view, in which there is complementarity between advertising and quality.

- Case 2: Under this scenario, demand effects exactly balance the supply effect. Consequently, better quality does not impact advertising. If quality is unknown, the consumer cannot infer greater quality from more advertising. There is independence between advertising and quality.
- Case 3: In this case, demand effects are dominated by the supply effect. Thus, higher quality yields less advertising, and the consumer may not deduce better quality from larger advertising. Case 3 matches the perverse market of the persuasive view, where there is substitutability between advertising and quality.

From these three cases the model suggests the following implications: The firm substitutes advertising for quality in the marketing mix in Case 3 but not in Cases 1 and 2. Consumers use advertising as a guide of high quality in Case 1 but not in Cases 2 and 3.

So far, I studied the case where the firm has market power, that is $P_D < 0$. Now, I analyze the case where the firm has no market power, namely $P_D = 0$. There are two justifications for this situation. First, $P_D = 0$ represents a first approximation of the monopolistic case by conjecturing that the qualitative structure of the advertising policy holds. This conjecture is explicit in Dockner and Jørgensen (1988) and implicit in Erickson (2009). Second, in the competitive case, $P_D = 0$ relies on the "oligopolists' tendency to substitute non-price for price competition" (Schmalensee, 1978, p. 487). In each case, the assumption $P_D = 0$ is convenient, though restrictive, as it avoids the study of any pricing behavior over time.

Corollary 1. *If the price does not depend on the quantity sold $P_D = 0$, then the relationship between advertising and quality is characterized by*

Cases	Conditions	Results
Case 1	$\frac{D_{aq}}{D_a^2} > C_q$	$a_q > 0$
Case 2	$\frac{D_{aq}}{D_a^2} = C_q$	$a_q = 0$
Case 3	$\frac{D_{aq}}{D_a^2} < C_q$	$a_q < 0$

Proof. Substitute $P_D = 0$ and $P_{DD} = 0$ in (14). □

Corollary 1 also holds in the more general case of $D_q(2P_D + P_{DD}D) = 0$, that is, if the demand effect D_q is low enough to be approximated by 0 or if the slope effect and the curvature effect compensate each other, that is $2P_D + P_{DD}D = 0$.

According to Corollary 1, the impact of quality on advertising is positive if the advertising effects outweigh the cost effect (Case 1). If the advertising effects balance the cost effect, the impact of quality on advertising is null (Case 2). The impact of quality on advertising is negative if the advertising effects are below the cost effect (Case 3).

Remark 1. *The direct effect of quality on demand D_q does not impact the relationship between advertising and quality.*

Proof. Immediate with Corollary 1. □

If the firm has no market power ($P_D = 0$), the effect of quality on demand D_q does not influence the relationship between quality and advertising.

Remark 2. *If the cost is independent from quality $C_q = 0$, then quality has a positive impact on advertising.*

Proof. Immediate with Corollary 1 and $C_q = 0$. □

If the cost effect C_q vanishes, then Case 1 applies according to Corollary 1, and quality has a positive effect on advertising. This situation characterizes for instance digital goods. Indeed, for digital goods, the marginal cost is often assumed to be null or very low (Shy, 1995, 2001).

Remark 3. *If the demand function is additively separable $D(a, q) = h(a)+l(q)$, then quality has a negative impact on advertising.*

Proof. Consider $D(a, q) = h(a) + l(q)$ that imposes $D_{aq} = 0$. The proof is immediate with Corollary 1 and $D_{aq} = 0$. □

With an additive separable demand function, the indirect advertising effect D_{aq} vanishes. Case 3 from Corollary 1 applies, and quality has a negative effect on advertising. This case is worth noting since much research uses linear demand functions, which have the property of additive separability (Tirole, 1988; Shy, 1995).

3 Discussion

The results of this work are the basis for a discussion about its testable implications. Directly based on that rule, Theorem 1 states that advertising and quality are complements if the demand effects (or quality and advertising effects on demand) overcome the supply effect (or cost effect) and substitutes if the demand effects are dominated by the supply effect. When the price is constant, Corollary 1 reveals that the quality effect on demand does not work. In this situation, advertising and quality are complements if the advertising effects outweigh the cost effect and substitutes if the advertising effects are below the cost effect. Figure 1 represents the complement and substitute spaces for advertising and quality, as expressed by Corollary 1. The implications of Corollary 1 is clearer with the features of goods and industries on the supply- and demand-sides.

The supply-side is usefully characterized by low and high cost industries. The cost effect is low or null in digital industries, because the unit production cost (the cost of information copy or storage) is not larger for a software with more functionalities or for a more entertaining movie. But the cost effect is high for manufacturing industries like traditional or hardware industries. In effect,

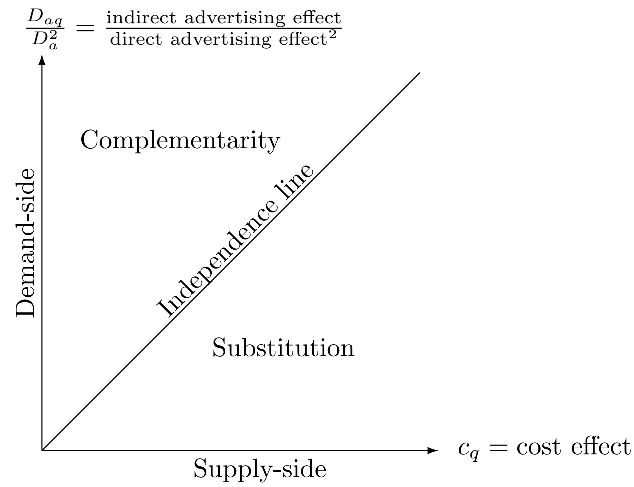

Figure 1: Complement and Substitute Spaces for Advertising and Quality with Respect to Cost and Advertising.

producing higher quality, such as a more powerful car or a smaller chip, is costly (Shy, 2001). Also, low cost may denote young or inefficient firms, whereas high cost may account for mature or efficient firms; cost signals efficiency (Bagwell, 2007). According to Corollary 1, advertising and quality are more likely to be complements for low cost and substitutes for high cost industries or firms.

In the case of some quality uncertainty, the demand-side is better qualified by Nelson (1970)'s distinction between search and experience goods. The quality of the good is known before purchase for a search good and after purchase for an experience good. A labeled coffee, whose taste is known to the consumer before he drinks it, and a movie, for which the consumer knows if he likes it after he watches it, are examples of search and experience goods. According to Nelson (1974, p. 734), "the advertising for experience qualities is dominantly indirect information and the advertising for search qualities is dominantly direct information". Indeed, the direct advertising effect is lower for search goods for which only information about price and location is needed, but it is greater for experience goods that require more information to convince the consumer. Thus, and in line with Bagwell (2007)'s great synthesis, the indirect advertising effect is larger for experience goods than for search goods and the direct advertising effect is lower for experience goods than for search goods. Consequently, the ratio of advertising effects D_{aq}/D_a^2 is higher for experience goods than for search goods. With regard to Corollary 1, advertising and quality are more likely to be substitutes for search goods and complements for experience goods.

Summarizing the characterization of industries and goods, Figure 2 presents the following testable implications of Corollary 1: The relationship between advertising and quality is more likely to be: positive for an experience good of

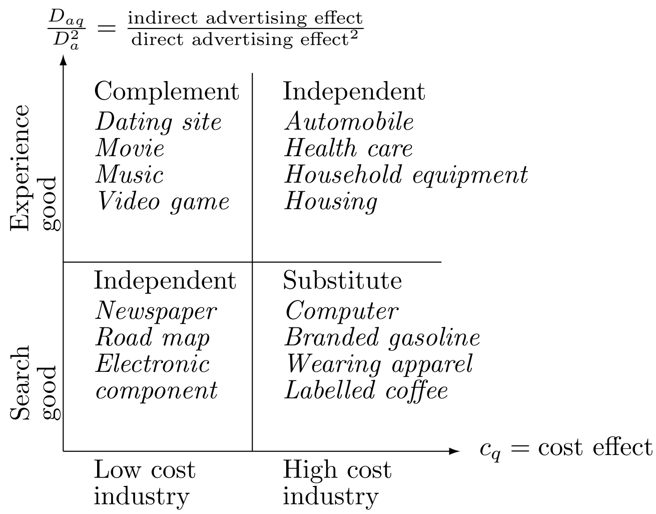

Figure 2: Complement and Substitute Spaces for Advertising and Quality with Respect to Industry and Good.

a low-cost industry, negative for a search good of a high-cost industry, mitigate for a search good of a low-cost industry, and mitigate for an experience good of a high-cost industry.

4 Conclusion

This paper analyzes the conditions under which better product quality involves more or less advertising. For this analysis, I develop an optimal control model with general non-linear functions for demand, cost, and innovation. On this basis, I derive a rule of advertising-quality relationship, based on structural and analytical results – as opposed to parameter and numerical results. According to this rule, quality has a positive effect on advertising if the demand effects (quality and advertising effects on demand) overcome the supply effect (quality effect on cost). On the contrary, quality has a negative effect on advertising if the demand effects fall behind the supply effect. This rule represents the first theoretical foundation to both positive and negative relationships between advertising and quality.

The rule of advertising-quality relationship provides formal guarantee to and expands prior results. More specifically, the rule proves the conjecture of Tellis and Fornell, according to which advertising and quality are substitutes if the direct advertising effects and the cost are high enough. More generally, the rule newly articulates the demand- (through research and experience goods) and supply-sides (via low and high cost industries). This articulation, providing a deeper understanding of the links between advertising and quality, paves the way to theoretical implications that require further research for empirical validation.

A Appendix

A.1 Proof of Equation (8a)

Recalling (7a), the first-order condition with respect to a is:

$$H_a = 0 \implies P - C - \frac{1}{D_a} + P_D D = 0,$$

which is a rearrangement of

$$H_a = 0 \implies P_D D_a D + (P - C)D_a - 1 = 0.$$

Assuming an interior solution, the Hamiltonian is concave in the decision variable a, and the second-order condition with respect to a writes

$$\begin{aligned}
&H_{aa} < 0 \\
&\implies P_{DD} D_a D_a D + D_{aa} P_D D + P_D D_a D_a + P_D D_a D_a + (P - C)D_{aa} < 0, \\
&\implies D_a^2 (2P_D P_{DD} D) + D_{aa}(P_D D + P - C) < 0.
\end{aligned}$$

Substitute in this result $P - C = \frac{1}{D_a} - P_D D$ from (7a) gives

$$D_a^2 (2P_D P_{DD} D) + D_{aa}(\frac{1}{D_a}) < 0.$$

Multiply by $-\frac{1}{D_a}$ yields

$$-D_a(2P_D + P_{DD} D) - \frac{D_{aa}}{(D_a)^2} > 0,$$

which completes the proof.

A.2 Proof of Equation (10)

Recall that the dynamic of λ writes in (6)

$$\dot{\lambda} = r\lambda - ((P_D D_q - C_q)D + (P - C)D_q + \lambda K_q), \text{ with } \lambda(T) = 0.$$

Substitute in this result $P - C = \frac{1}{D_a} - P_D D$ from (7a) and rearrange

$$\dot{\lambda} = (r - K_q)\lambda + C_q D - \frac{D_q}{D_a}, \text{ with } \lambda(T) = 0.$$

Recall $\eta_q \equiv \frac{\partial D}{\partial q}\frac{q}{D}$ and $\eta_a \equiv \frac{\partial D}{\partial a}\frac{a}{D}$ and substitute

$$\dot{\lambda} = (r - K_q)\lambda + C_q D - \frac{\eta_q}{\eta_a}\frac{a}{q};\ \lambda(T) = 0.$$

Consider the integrating factor $e^{-(r-K_q)t}$, such that

$$\frac{d\lambda(t)e^{-(r-K_q)t}}{dt} = e^{-(r-K_q)t}(\dot{\lambda} - (r - K_q)\lambda).$$

Since $\dot{\lambda} - (r - K_q)\lambda = C_q D - \frac{\eta_q}{\eta_a}\frac{a}{q}$, then

$$\frac{d\lambda(t)e^{-(r-K_q)t}}{dt} = e^{-(r-K_q)t}\left(C_q D - \frac{\eta_q}{\eta_a}\frac{a}{q}\right),$$

and thus

$$d\lambda(t)e^{-(r-K_q)t} = e^{-(r-K_q)t}\left(C_q D - \frac{\eta_q}{\eta_a}\frac{a}{q}\right)dt.$$

Consequently,

$$\int_t^T d\lambda(s)e^{-(r-K_q)s} = \int_t^T e^{-(r-K_q)s}\left(C_q D - \frac{\eta_q}{\eta_a}\frac{a}{q}\right)ds,$$

and

$$\lambda(T)e^{-(r-K_q)T} - \lambda(t)e^{-(r-K_q)t} = \int_t^T e^{-(r-K_q)s}\left(C_q D - \frac{\eta_q}{\eta_a}\frac{a}{q}\right)ds.$$

The substitution of the transversality condition $\lambda(T) = 0$ yields

$$\lambda(t) = \int_t^T e^{-(r-K_q)(s-t)}\left(\frac{\eta_q}{\eta_a}\frac{a}{q} - C_q D\right)ds,$$

which completes the proof.

A.3 Proof of Equation (13)

The first-order condition with respect to a (7a) that writes $P - C - \frac{1}{D_a} + P_D D = 0$ is a rearrangement of the immediate condition

$$H_a = 0 \implies P_D D_a D + (P - C)D_a - 1 = 0.$$

Derivate the last condition with respect to t:

$$\begin{aligned} &P_{DD}(D_a\dot{a} + D_q\dot{q})D_a D + P_D(D_{aa}\dot{a} + D_{aq}\dot{q})D + P_D D_a(D_a\dot{a} + D_q\dot{q}) \\ &\quad + P_D(D_a\dot{a} + D_q\dot{q})D_a - C_q\dot{q}D_a + (P - C)(D_{aa}\dot{a} + D_{aq}\dot{q}) = 0. \end{aligned}$$

A rearrangement yields

$$-\dot{a}(D_a^2(2P_D + P_{DD}D) + D_{aa}(P_D D + P - C))$$
$$= \dot{q}(D_q(P_{DD}D_a D + 2P_D D_a) + D_{aq}(P_D + P - C) - C_q D_a)$$

Substitute $P - C = \frac{1}{D_a} - P_D D$ from (7a) and divide by D_a:

$$\dot{a}\left(-D_a(2P_D + P_{DD}D) - \frac{D_{aa}}{D_a^2}\right) = \dot{q}\left(D_q(2P_D + P_{DD}D) + \frac{D_{aq}}{D_a^2} - C_q\right),$$

which completes the proof.

B Supplementary Online Material

B.1 Comparaison with Tellis and Fornell (1988)

As the spirit of the present article is linked to Tellis and Fornell (1988)'s conjecture, I compare here deeper both articles. Table 2 contrasts the present article with Tellis and Fornell (1988). Both articles study the advertising-quality relationship. The main difference is that Tellis and Fornell (1988) formulate a conjecture with empirical support, whereas I develop a formal theoretical framework which proves their conjecture as a special case. They conjecture that the advertising-quality relationship may be negative if (1) quality is produced at high cost and (2) consumers respond strongly to advertising. Such a negative relationship is explained by assuming unknown product quality and heterogeneous production cost of competing firms. In other words, consumers may be misled about quality and firms are heterogeneous. They write (Tellis and Fornell, 1988, p. 66)

> "Because of the uncertainty about quality, consumers will be responsive to advertising (...) If quality costs substantially more to produce, the low quality producers, with lower costs, will advertise heavily enough to attract a larger share of consumers (...) Consequently, lower quality would lead to higher levels of advertising."

In this work, I relax the assumptions of unknown quality (the consumer cannot be mistaken) and firm heterogeneity (I study a monopolist, for which the cost is implicitly homogeneous). I show that even with known quality and homogeneous cost, Fornell and Tellis' conjecture holds. Indeed, a negative advertising-quality relationship may arise if direct advertising and cost effects are sufficiently large (Case 3 of Theorem 1 in the monopoly scenario and Case 3 of Corollary 1 in the competitive scenario). Further, they acknowledge the limitations of their modeling of quality (Tellis and Fornell, 1988, p. 68):

> "The major assumption in our theoretical and empirical model is that quality is exogenous and fixed for each business. The assumption of exogenous quality may appear intuitively unreasonable."

The assumption of exogenous and fixed quality is also relaxed in my work, which considers endogenous and variable quality depending on innovation. At least, they discuss two effects, namely the direct advertising effect and the cost effect, while I highlight two additional effects that are the indirect advertising and the direct quality effects. In a nutshell, Tellis and Fornell (1988) made a conjecture about the possible positive and negative articulation of advertising and quality. In this paper, I prove this conjecture with fewer assumptions and more effects at work.

Table 2: Comparison of this Work with Tellis and Fornell (1988)

Tellis and Fornell (1988)	This work
Conjecture	Proof
Unknown quality	Known quality
Exogenous and fixed quality	Endogenous and variable quality
Heterogeneous cost	Homogeneous cost
Two effects:	Four effects:
1 Direct advertising effect	1 Direct advertising effect
2 Cost effect	2 Cost effect
	3 Indirect advertising effect
	4 Direct quality effect

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