Interest rate pass-through in the Euro Area: a time-varying cointegration approach

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Abstract

This paper study the mechanism of transmission between the money and the retail credit markets stated in terms of the long-run relationship between the harmonized interest rates for different credit categories and for a subset of countries of the EMU. This mechanism, known as the interest rate pass-through (IRPT) phenomenon, has been analyzed in many empirical studies using a variety of econometric techniques, for different samples of countries and periods of time, and the general conclusion is that the pass-through seems to be incomplete in the long-run. Except for a few recent works, the analysis is performed on the basis on a time-invariant long-run relationship which may not be appropriate in this case and could condition this result. To evaluate the robustness of these findings we extend the analysis through a non-linear model for the long-run relationship between the money and the retail markets that incorporates in a very flexible form, and with minimum requirements on tuning parameters, the nonlinearity in the form of time-varying parameters. To that end we follow the approach initiated in Bierens (1997) and also propose some new tools to test for the existence of a stable time-varying cointegration relationship. The results obtained seems to support the former evidence of an incomplete pass-through.

Keywords and phrases: retail interest rates, monetary policy, cointegration analysis, structural instability, time-varying cointegration

JEL classification: E52, F36, C22

1. Introduction

Monetary transmission is a key issue when analyzing monetary policy decisions. In this sense, the transmission of monetary policy relies on how policy rate changes, measured as changes in money market interest rates, are transferred to the bank system via changes in the retail rates for each possible credit category in the economy, which is called the interest rate pass-through (IRPT) effect. This mechanism is important for achieving the aims of monetary policy, such as achieving price stability and influencing the path of the real economy through influencing aggregate demand at least to some extent. This phenomenon is closely related to the analysis of the stability properties of monetary policy rules in terms of giving rise to a unique and stable equilibrium if the implied response of the nominal interest rates to inflation changes is sufficiently strong (Taylor principle). An incomplete IRPT could violate the Taylor principle and monetary policy would fail to be stabilizing in the sense that retail interest rates do not respond sufficiently to ensure that real rates are stabilizing. This appears to be particularly important for the Euro Area, usually taken as an example of a bank-based financial system, for which the empirical evidence seems to indicate a limited IRPT (retail interest rates responding less than one-to-one to policy rates).[1]

This paper contributes to the empirical analysis of measuring the magnitude of the adjustment in the framework of analysis of a non-linear model for long-run relationship allowing for a time-varying relationship between the money market and the retail interest rates for a set of EMU countries selected by the criterion of having the longest available series.

Thus, in section 2 we contribute to the analysis of the specification of a general time-varying cointegrating model, both in the form of a time-varying cointegrating regression model or, alternatively, as a reduced rank time-varying error-correction model (ECM), and discuss some of their main features. Section 3 introduce the empirical analysis based on evaluating the existence of a time-varying cointegration relationship between the selected interest rate series, adopting the methodology introduced in Bierens (1997) that propose to model the parameters as smooth functions of time through a weighted average of Chebyshev time polynomials. This methodology has been used before in Bierens and Martins (2010) and in Neto (2012, 2014), but we propose some new tools to empirically assess the stability of the non-linear relationship allowing for consistent estimates of the instantaneous and time-varying magnitudes of the IRPT.

Some theoretical developments are presented in Appendixes A to C, while the main empirical results are presented in Appendixes D to F.

2. Econometric analysis

We consider the case where the nonstationary observed (k+1)-dimensional time series $\mathbf{z}_t = (y_t, \mathbf{x}'_{k,t})'$ t = 1, ..., n, where $\mathbf{x}_{k,t} = (x_{1,t}, ..., x_{k,t})'$ $k \geq 1$, is generated as $\mathbf{z}_t = \mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t$, where $\boldsymbol{\varepsilon}_t = (\varepsilon_{0,t}, \boldsymbol{\varepsilon}'_{k,t})'$ is a zero-mean (k+1)-dimensional weakly dependent and stationary error sequence, but it is assumed that it can also be embedded into the following general Error-Correction Model (ECM) representation

$$\tilde{\boldsymbol{\Phi}}(L)[(1-L)\mathbf{I}_{k+1} - (\alpha-1)\boldsymbol{\lambda}\boldsymbol{\kappa}'_t L]\mathbf{z}_t = \mathbf{e}_t \qquad (2.1)$$

[1] See, e.g., Kwapil and Scharler (2010) and the references cited on earlier empirical studies for EMU countries and different periods of time.

where $\tilde{\mathbf{\Phi}}(L) = \mathbf{\Phi}(L)\mathbf{M}_{k+1,t}$, with $\mathbf{\Phi}(L) = \mathbf{I}_{k+1} - \sum_{j=1}^{m} \mathbf{\Phi}_j L^j$ a stationary matrix polynomial of finite order m in the lag operator L (i.e. $|\mathbf{\Phi}(z)| = 0$ has roots outside the unit circle), $\boldsymbol{\lambda} = (\lambda_0, \boldsymbol{\lambda}_k')'$, $\boldsymbol{\kappa}_t = (1, -\boldsymbol{\beta}_{k,t}')'$ and the k+1 square matrix $\mathbf{M}_{k+1,t}$ can be either the identity matrix or, more generally, be defined as a time-varying rotation matrix of the form

$$\mathbf{M}_{k+1,t} = \begin{pmatrix} 1 & -\boldsymbol{\beta}_{k,t}' \\ \mathbf{0}_k & \mathbf{I}_{k,k} \end{pmatrix} \qquad (2.2)$$

thus preserving the equivalence of the roots of each lag polynomial, with $\boldsymbol{\beta}_{k,t}$ the k-dimensional single time-varying cointegrating vector. For the last term in (2.1), $\mathbf{e}_t = (e_{0,t}, \mathbf{e}_{k,t}')'$, it is assumed to be a zero-mean iid sequence with finite covariance matrix $\mathbf{\Sigma}_e = E[\mathbf{e}_t\mathbf{e}_t'] > 0$, and $E[\|\mathbf{e}_t\|^{2+\delta}] < \infty$ for some $\delta > 0$. This is a modified version of the ECM representation used in Elliott et.al. (2005) to derive a family of optimal testing procedures for cointegration in the case of a known and time-invariant cointegrating vector, $\boldsymbol{\beta}_{k,t} = \boldsymbol{\beta}_{k,0}$ $t = 1, ..., n$. Taking (2.2), equation (2.1) can also be rewritten as

$$\begin{pmatrix} \boldsymbol{\kappa}_t'\Delta\mathbf{z}_t \\ \Delta\mathbf{x}_{k,t} \end{pmatrix} = (\alpha - 1)\begin{pmatrix} \boldsymbol{\kappa}_t'\boldsymbol{\lambda} \\ \boldsymbol{\lambda}_k \end{pmatrix}\boldsymbol{\kappa}_t'\mathbf{z}_{t-1} + \boldsymbol{\xi}_t \qquad (2.3)$$

with $\boldsymbol{\xi}_t = (\upsilon_t, \boldsymbol{\varepsilon}_{k,t}')' = \mathbf{C}(L)\mathbf{e}_t$, $\mathbf{C}(L) = \mathbf{\Phi}(L)^{-1} = \sum_{j=0}^{\infty} \mathbf{C}_j L^j$, and $u_t = \boldsymbol{\kappa}_t'\mathbf{z}_t = y_t - \boldsymbol{\beta}_{k,t}'\mathbf{x}_{k,t}$ the cointegrating error term. Under the summability condition $\sum_{j=1}^{\infty} jTr(\mathbf{C}_j'\mathbf{C}_j) < \infty$ and the properties of the error sequence $\mathbf{e}_t$, the process $\boldsymbol{\xi}_t$ satisfy a multivariate invariance principle such as

$$n^{-1/2}\sum_{t=1}^{[nr]}\boldsymbol{\xi}_t = n^{-1/2}\sum_{t=1}^{[nr]}\begin{pmatrix} \upsilon_t \\ \boldsymbol{\varepsilon}_{k,t} \end{pmatrix} \Rightarrow \mathbf{B}_{\xi}(r) = \begin{pmatrix} B_{\upsilon}(r) \\ \mathbf{B}_k(r) \end{pmatrix} = \mathbf{\Omega}_{\xi}^{1/2}\mathbf{W}_{\xi}(r)$$

with $\mathbf{\Omega}_{\xi} = \mathbf{C}(1)\mathbf{\Sigma}_e\mathbf{C}(1)'$ the long-run covariance matrix of $\boldsymbol{\xi}_t$, and $\mathbf{W}_{\xi}(r) = (W_{\upsilon}(r), \mathbf{W}_k'(r))'$ a k+1-variate standard Brownian process. Given that $\boldsymbol{\kappa}_t'\Delta\mathbf{z}_t = \Delta u_t - \Delta\boldsymbol{\kappa}_t'\mathbf{z}_{t-1}$, and $\boldsymbol{\kappa}_t'\mathbf{z}_{t-1} = u_{t-1} + \Delta\boldsymbol{\kappa}_t'\mathbf{z}_{t-1}$ with $\Delta\boldsymbol{\kappa}_t = (0, \Delta\boldsymbol{\beta}_{k,t}')'$, then the first component of the vector in (2.3) allows to represent u_t as a time-varying AR(1) process $u_t = \rho_t(u_{t-1} + \Delta\boldsymbol{\kappa}_t'\mathbf{z}_{t-1}) + \upsilon_t$, where the time-varying autoregressive coefficient is given by $\rho_t = 1 + (\alpha - 1)\boldsymbol{\kappa}_t'\boldsymbol{\lambda} = \alpha + (1-\alpha)(1 - \boldsymbol{\kappa}_t'\boldsymbol{\lambda})$, that becomes fixed in the case of a time-invariant cointegrating vector, i.e. $\boldsymbol{\kappa}_t = \boldsymbol{\kappa} = (1, -\boldsymbol{\beta}_k')'$, or, alternatively, under the normalization restriction $\boldsymbol{\kappa}_t'\boldsymbol{\lambda} = 1$, in which case $\rho_t = \alpha$. Under this last condition we obtain the following static time-varying cointegrating regression model (a generalization of the so-called Phillip's triangular model) given by

$$y_t = \boldsymbol{\beta}_{k,t}'\mathbf{x}_{k,t} + u_t \qquad (2.4)$$

with a cointegrating error term of the form

$$u_t = \alpha u_{t-1} + \upsilon_t + \alpha\Delta\boldsymbol{\beta}_{k,t}'\mathbf{x}_{k,t-1} \qquad (2.5)$$

and

$$\mathbf{x}_{k,t} = \mathbf{x}_{k,t-1} + (\alpha - 1)\boldsymbol{\lambda}_k\boldsymbol{\kappa}_t'\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_{k,t} \qquad (2.6)$$

Under time-invariance of the cointegrating relation, equation (2.5), together with the assumption on the stationarity of υ_t, allows to differentiate between the existence of a cointegration relationship among y_t and $\mathbf{x}_{k,t}$ when $|\alpha| < 1$, and the absence of such a stable long-run relationship (no cointegration) when $\alpha = 1$. The extra term appearing in the right-hand side of equation (2.5) is nonstationary in general, due to the inclusion of the integrated regressors $\mathbf{x}_{k,t}$, except in the case of a time-invariant cointegrating vector with $\boldsymbol{\beta}_{k,t} = \boldsymbol{\beta}_k$, or trivially when α = 0 so that $u_t = \upsilon_t$ and all the serial correlation in the regression error term is through the dynamics in υ_t, although its behavior, properties and influence will depend on the particular mechanism that determines the changes in $\boldsymbol{\beta}_{k,t}$. As examples, we consider three very different mechanisms: (a) the case of a single discrete change possibly affecting to all the coefficients in $\boldsymbol{\beta}_{k,t}$ at a given break point as

$$\boldsymbol{\beta}_{k,t} = \boldsymbol{\beta}_{k,0} + \boldsymbol{\lambda}_k H_t(\tau_0) \tag{2.7}$$

where $H_t(\tau_0) = I(t > [n\tau_0])$ with $\tau_0 \in (0,1)$ the standard step function, (b) a martingale process as in Hansen (1992) given by

$$\boldsymbol{\beta}_{k,t} = \boldsymbol{\beta}_{k,t-1} + \boldsymbol{\upsilon}_{k,t} \tag{2.8}$$

with $\boldsymbol{\upsilon}_{k,t}$ a zero-mean error iid sequence with finite covariance matrix $E[\boldsymbol{\upsilon}_{k,t}\boldsymbol{\upsilon}'_{k,t}] = \boldsymbol{\Sigma}_{\boldsymbol{\upsilon}_k}$ and $\boldsymbol{\beta}_{k,0}$ a k-vector of fixed values, and (c) a time-varying cointegrating vector via Chebishev time polynomials proposed by Bierens and Martins (2010) extending the results in Bierens (1997), which is given by

$$\boldsymbol{\beta}_{k,t} = \sum_{j=0}^{m} \mathbf{b}_{kj,n} G_{j,n}(t) \tag{2.9}$$

where $G_{0,n}(t) = 1$ and $G_{j,n}(t) = \sqrt{2}\cos(j\pi(t-0.5)/n)$, j = 1, 2, …, $m \le n-1$ (see Appendix A for more details). First, under the stationarity condition on the regression error term given by |α| < 1 the scaled partial sum of u_t can be written as

$$n^{-1/2}\sum_{t=1}^{[nr]} u_t = \frac{1}{1-\alpha}\left\{ n^{-1/2}\sum_{t=1}^{[nr]} \upsilon_t + \alpha n^{-1/2}(u_0 - u_{[nr]}) + \alpha n^{-1/2}\sum_{t=1}^{[nr]} \Delta\boldsymbol{\beta}'_{k,t}\mathbf{x}_{k,t-1} \right\} \tag{2.10}$$

where $n^{-1/2}\sum_{j=1}^{t} \Delta\boldsymbol{\beta}'_{k,j}\mathbf{x}_{k,j-1}$ is $O_p(1)$ in the cases (a) and (c), while that it is $O_p(n^{1/2})$ in the case (b), and hence diverging with the sample size. Specifically we obtain

$$\begin{aligned} n^{-1/2}\sum_{t=1}^{[nr]} \Delta\boldsymbol{\beta}'_{k,t}\mathbf{x}_{k,t-1} &= \boldsymbol{\lambda}'_k \left\{ n^{-1/2}\mathbf{x}_{k,[nr]} - n^{-1/2}\left(\sum_{t=1}^{[nr]} \boldsymbol{\varepsilon}_{k,t} - \sum_{t=1}^{[n\tau_0]} \boldsymbol{\varepsilon}_{k,t} \right) \right\} I(r > \tau_0) \\ &= \boldsymbol{\lambda}'_k n^{-1/2} \left\{ \mathbf{x}_{k,0} + \sum_{t=1}^{[n\tau_0]} \boldsymbol{\varepsilon}_{k,t} \right\} I(r > \tau_0) \\ &= \boldsymbol{\lambda}'_k n^{-1/2}\mathbf{x}_{k,[n\tau_0]} I(r > \tau_0), \end{aligned} \tag{2.11}$$

$$n^{-1/2}\sum_{t=1}^{[nr]} \Delta\boldsymbol{\beta}'_{k,t}\mathbf{x}_{k,t-1} = n^{-1/2}\sum_{t=1}^{[nr]} \mathbf{x}'_{k,t-1}\boldsymbol{\upsilon}_{k,t} = O_p(n^{1/2}) \tag{2.12}$$

with

$$n^{-1}\sum_{t=1}^{[nr]} \mathbf{x}'_{k,t-1}\boldsymbol{\upsilon}_{k,t} \Rightarrow \int_0^r \mathbf{B}'_k(s)\,d\mathbf{B}_{\upsilon_k}(s) + r\sum_{h=1}^{\infty} E[\boldsymbol{\varepsilon}'_{k,t-h}\boldsymbol{\upsilon}_{k,t}]$$

and

$$\begin{aligned} n^{-1/2}\sum_{t=1}^{[nr]} \Delta\boldsymbol{\beta}'_{k,t}\mathbf{x}_{k,t-1} = &-\pi\sum_{j=1}^{m} j\mathbf{b}'_{kj,n} \left\{ n^{-3/2}\sum_{t=1}^{[nr]} H_{j,n}(t)\mathbf{x}_{k,t} - n^{-1}H_{j,n}([nr])n^{-1/2}\mathbf{x}_{k,[nr]} \right\} \\ &+ O(n^{-1})\sum_{j=1}^{m} \mathbf{b}'_{kj,n} \left\{ n^{-3/2}\sum_{t=1}^{[nr]} \mathbf{x}_{k,t-1} \right\} \end{aligned} \tag{2.13}$$

in each case, where $H_{j,n}(r)=\sqrt{2}\sin(j\pi(t-05)/n)$ in (2.13) for j = 1, …, m. On the other hand, under no stationarity of the regression error term in (2.4) with α = 1, we get the representation

$$n^{-1/2}u_t = n^{-1/2}u_0 + n^{-1/2}\sum_{j=1}^{t}\upsilon_j + n^{-1/2}\sum_{j=1}^{t}\Delta\boldsymbol{\beta}'_{k,j}\mathbf{x}_{k,j-1} \qquad (2.14)$$

where $\boldsymbol{\varepsilon}_{k,t}=\Delta\mathbf{x}_{k,t}$, with $n^{-1/2}u_{[nr]}=n^{-1/2}\sum_{t=1}^{[nr]}\upsilon_t+O_p(n^{-1/2})\Rightarrow B_\upsilon(r)$, under time invariance of the regression coefficients and the usual assumption of the initial value $u_0=O_p(1)$, where the last term $n^{-1/2}\sum_{j=1}^{t}\Delta\boldsymbol{\beta}'_{k,j}\mathbf{x}_{k,j-1}$ is given as in (2.11)-(2.13). Thus, assuming the validity of the time-varying ECM representation in (2.1), the cointegration assumption implies the extra condition α = 0, while that under no cointegration the disequilibrium error term u_t contains an additional term incorporating the changes in the values of $\boldsymbol{\beta}_{k,t}$, whenever it has a clear definition.

Also, as can be seen from (2.6), the assumption that $\mathbf{x}_{k,t}$ are not mutually cointegrated and have roots that are known a priori to be equal to one (i.e. $\mathbf{x}_{k,t}$ are $k \geq 1$ integrated but not-cointegrated regressors), corresponds to the restriction $\boldsymbol{\lambda}_k=\mathbf{0}_k$ (which implies $\lambda_0=1$), and hence (2.6) must be replaced by the usual representation as a k-dimensional integrated process[2]

$$\mathbf{x}_{k,t}=\mathbf{x}_{k,t-1}+\boldsymbol{\varepsilon}_{k,t} \qquad (2.6')$$

which also results in the case of no cointegration, i.e. when α = 1 in (2.5), irrespective of the value of $\boldsymbol{\lambda}_k$. A second form of the model is the ECM representation (2.1) with $\tilde{\boldsymbol{\Phi}}(L)$ replaced by $\boldsymbol{\Phi}(L)$, i.e. $\boldsymbol{\Phi}(L)\Delta\mathbf{z}_t=(\alpha-1)\boldsymbol{\Phi}(L)\boldsymbol{\lambda}\boldsymbol{\kappa}'_t\mathbf{z}_{t-1}+\mathbf{e}_t$, that can be written as a time-varying reduced rank ECM of the form

$$\Delta\mathbf{z}_t=(\alpha-1)\boldsymbol{\Phi}(1)\boldsymbol{\lambda}\boldsymbol{\kappa}'_t\mathbf{z}_{t-1}+\boldsymbol{\Lambda}_t(L)\Delta\mathbf{z}_{t-1}+\mathbf{e}_t \qquad (2.15)$$

by making use of the BN decomposition $\boldsymbol{\Phi}(L)=\boldsymbol{\Phi}(1)+(1-L)\boldsymbol{\Phi}^*(L)$ where $\boldsymbol{\Phi}^*(L)=\sum_{j=1}^{m}\boldsymbol{\Phi}_j^*L^{j-1}$ and $\boldsymbol{\Phi}_j^*=\sum_{i=j}^{m}\boldsymbol{\Phi}_i$, with the lag polynomial $\boldsymbol{\Lambda}_t(L)=\sum_{j=0}^{m-1}\boldsymbol{\Lambda}_{j,t}L^j$ and time-varying coefficients $\boldsymbol{\Lambda}_{j,t}=\boldsymbol{\Phi}_{j+1}^*(\alpha-1)\boldsymbol{\lambda}\boldsymbol{\kappa}'_t+\boldsymbol{\Phi}_{j+1}$, unless $\boldsymbol{\kappa}_t=\boldsymbol{\kappa}$ or, alternatively, α = 1 irrespective of the behaviour of $\boldsymbol{\kappa}_t$.

Under the restrictions considered we have

$$(\alpha-1)\boldsymbol{\Phi}(1)\boldsymbol{\lambda}=(\alpha-1)\begin{pmatrix}\phi_{00}(1)\\ \boldsymbol{\phi}_{k0}(1)\end{pmatrix}$$

where we have partitioned $\boldsymbol{\Phi}(L)$ after the first row and column, so that $\mathbf{x}_{k,t}$ is weakly

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[2] By recursive substitutions, equation (3.6) can also be written as $\mathbf{x}_{k,t}=\mathbf{x}_{k,0}+\sum_{j=1}^{t}\boldsymbol{\varepsilon}_{k,j}+(\alpha-1)\boldsymbol{\lambda}_k\sum_{j=1}^{t}\boldsymbol{\kappa}'_j\mathbf{z}_{j-1}$, where the last term is decomposed as $\sum_{j=1}^{t}\boldsymbol{\kappa}'_j\mathbf{z}_{j-1}=\sum_{j=1}^{t}u_{j-1}+\sum_{j=1}^{t}\Delta\boldsymbol{\kappa}'_j\mathbf{z}_{j-1}$, with $u_t=\boldsymbol{\kappa}'_t\mathbf{z}_t$. With a fixed cointegrating vector, $\boldsymbol{\kappa}_t=\boldsymbol{\kappa}$, $\sum_{j=1}^{t}\Delta\boldsymbol{\kappa}'_j\mathbf{z}_{j-1}=0$, while that with a time-varying cointegrating vector we have the representation $\sum_{j=1}^{t}\Delta\boldsymbol{\kappa}'_j\mathbf{z}_{j-1}=-\sum_{j=1}^{t}\boldsymbol{\beta}'_{k,j}\boldsymbol{\varepsilon}_{k,j}-(\boldsymbol{\beta}'_{k,0}\mathbf{x}_{k,0}-\boldsymbol{\beta}'_{k,t}\mathbf{x}_{k,t})$. In the fixed parameter case and under cointegration we get $n^{-1/2}\mathbf{x}_{k,t}\Rightarrow\mathbf{B}_k(r)+(\alpha-1)\boldsymbol{\lambda}_kB_u(r)$, with t = [nr], $r \in (n^{-1},1]$, and $B_u(r)=(1-\alpha)^{-1}B_\upsilon(r)$ as $n\to\infty$, while that under no cointegration the weak limit is $n^{-1/2}\mathbf{x}_{k,t}\Rightarrow\mathbf{B}_k(r)$, implying a different behavior in each situation that is unlikely in any real analysis.

exogeneous for $\boldsymbol{\beta}_{k,t}$ if and only if $\boldsymbol{\Phi}(1)$ is block upper triangular (i.e. $\boldsymbol{\phi}_{k0}(1) = \mathbf{0}_k$),[3] in which case only the first row of equation (2.15) includes the $\boldsymbol{\kappa}_t'\mathbf{z}_{t-1} = u_{t-1} + \Delta\boldsymbol{\kappa}_t'\mathbf{z}_{t-1}$, which equals the usual error correction term $u_{t-1} = \boldsymbol{\kappa}_{t-1}'\mathbf{z}_{t-1}$ only under constancy of the cointegrating vector. Note that (2.15) is the ad hoc specification of a time-varying ECM proposed by Bierens and Martins (2009, 2010), except for the fact that the finite order lag polynomial $\boldsymbol{\Lambda}_t(L)$ is assumed to be time-invariant. If instead of (2.15), and given the decomposition $\boldsymbol{\kappa}_t'\mathbf{z}_{t-1} = u_{t-1} + \Delta\boldsymbol{\kappa}_t'\mathbf{z}_{t-1}$ with $u_t = \boldsymbol{\kappa}_t'\mathbf{z}_t$, we consider the alternative ECM representation including the lagged valued of the error correction term resulting from (2.4) as

$$\Delta\mathbf{z}_t = (\alpha - 1)\boldsymbol{\Phi}(1)\boldsymbol{\lambda}\boldsymbol{\kappa}_{t-1}'\mathbf{z}_{t-1} + \boldsymbol{\Lambda}_t(L)\Delta\mathbf{z}_{t-1} + \mathbf{e}_{\alpha,t} \tag{2.16}$$

where the error term in (2.16) is given by $\mathbf{e}_{\alpha,t} = \mathbf{e}_t + (\alpha - 1)\boldsymbol{\Phi}(1)\boldsymbol{\lambda}\Delta\boldsymbol{\beta}_{k,t}'\mathbf{x}_{k,t-1}$, which behaves as a nonstationary sequence for $|\alpha| < 1$ under time-varying cointegration. Bierens and Martins (2009, 2010) propose a likelihood ratio test for time-invariant cointegration from (2.15), with a fixed lag polynomial $\boldsymbol{\Lambda}_t(L)$, against the alternative of a smoothly varying cointegrating relationship over time. Instead of relying on the use of the ECM in (2.15), the rest of the paper rests on the analysis of the time-varying cointegrating regression model in (2.4) for a particular choice of a smooth mechanism driving the coefficients in $\boldsymbol{\beta}_{k,t} = (\beta_{1,t},...,\beta_{k,t})'$, t = 1, ..., n. Also, if we consider the inclusion of some deterministic time trends in the generating mechanism of the observations of $\mathbf{z}_t = (y_t, \mathbf{x}_{k,t}')'$, such as $\mathbf{z}_t = \mathbf{d}_t + \boldsymbol{\eta}_t$ where $\mathbf{d}_t = (d_{0,t}, \mathbf{d}_{k,t}')'$, $\boldsymbol{\eta}_t = (\eta_{0,t}, \boldsymbol{\eta}_{k,t}')'$ and $\boldsymbol{\eta}_t = \boldsymbol{\eta}_{t-1} + \boldsymbol{\varepsilon}_t$, then we can obtain an augmented version of (2.4) given by

$$y_t = \alpha_t + \boldsymbol{\beta}_{k,t}'\mathbf{x}_{k,t} + u_t \tag{2.17}$$

with a possibly time-varying deterministic trend function.

3. Empirical analysis

In this section we focus on the analysis of the time-varying cointegrating relationship among the retail interest rates for different maturities and two definitions of credit variables, namely credits for house purchase and loans for consumption, to evaluate the magnitude of the long-run IRPT for a subset of countries in the Euro Area for which the longest and complete series is available.

As argued in Belke et.al. (2013), when analyzing aggregated micro data from many banks, each of these institutions might face different information and transaction costs, a smooth transition pattern seems to be a plausible mechanism. These authors use a smooth transition regression to incorporate different patterns of nonlinearity in the adjustment and short-run dynamics for the relationship between the Euro OverNight Index Average (EONIA), as a global indicator of the money market rate in the Euro Area, and credit categories with various maturities. Marotta (2009) considers the possibility of allowing for multiple unknown structural breaks in the cointegrating relationship based on unharmonized retail rates for several EMU countries, and found

[3] From this, the restriction $\boldsymbol{\lambda}_k = \mathbf{0}_k$ implies weak exogeneity of the integrated regressors in the particular cases where $\boldsymbol{\Phi}(L) = \mathbf{I}_{k+1}$ or, more generally, $\boldsymbol{\Phi}(L) = \mathrm{diag}(\phi_{00}(L), \boldsymbol{\Phi}_{kk}(L))$.

different estimates of the equilibrium pass-through indicating a slow adjustment to the monetary regimes. From these results and the evidence presented in ECB (2009), indicating no evidence for a structural change in the IRPT mechanism during the recent period of the financial crisis, instead of relying of these particular choices for explaining the possible variability of the magnitude of the long-run relationship between the retail and the market interest rates we consider the more flexible and general approach based on the assumption of time-varying parameters in the cointegrating regression model modelled as a weighted average of Chebyshev time polynomials with deterministic weights, following the proposal by Bierens (1997). For a formal description of this approach and some important results arising from fitting a time-varying cointegrating regression model via Chebyshev time polynomials see Appendix A. Next we describe the data used in the empirical analysis and the structure of the econometric study.

3.1. The data and some initial basic results

Following Belke et.al. (2013), for the harmonized retail rates data we use the harmonized interest rate series from the Monetary Financial Institutions (MFI) interest rate statistics of the European Central Bank (ECB) for the seven countries and periods appearing in Table 1, representing the longest series for which complete data are available. All data refer to loans for households and non-profit institutions and are monthly averages and exclusively new business. For the credit categories we consider credits for house purchase and loans for consumption with short, medium and long maturities (up to 1 year, over 1 and up to 5 years, and over 5 years, respectively).

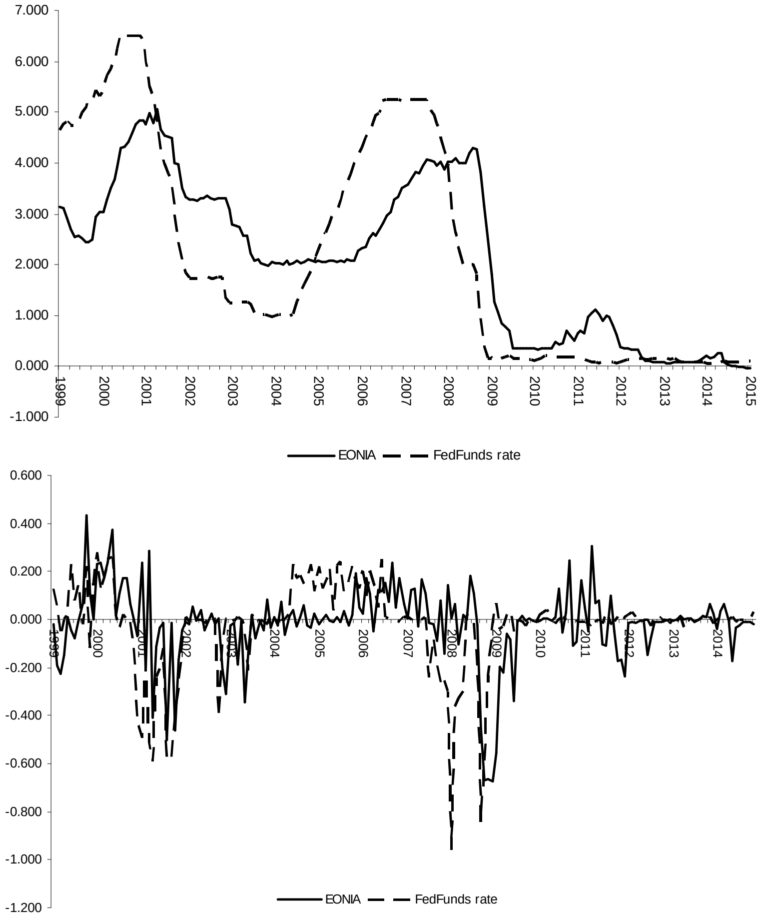
Table 1. Monthly retail rates by country

	Credits for house purchase		Loans for consumption	
Country	Period	*n*	Period	*n*
Austria	01.2003-12.2014	144	01.2000-12.2014	180
Belgium	10.2006-12.2014	99	10.2006-12.2014	99
Finland	01.2003-12.2014	144	01.2000-12.2014	180
France	″	″	″	″
Germany	″	″	″	″
Italy	″	″	01.2003-12.2014	144
Spain	″	″	01.2000-12.2014	180

As the money market rate for all the countries we consider the EONIA,[4] because it seems to better reflect the stance of the monetary policy. Figure 1 shows monthly series of the EONIA and the three-month Effective Federal Funds Rates (EFFR) as a proxy for the policy rate in the US economy, which can be described as a market-based system as opposed to the bank-based system for the Euro Area, for the period january 1999 to december 2014. The time path of both series closely resembles, displaying an apparent nonstationary behavior but with a certain time delay in the response of the EONIA rates to changes in the EFFR. Cross contemporaneous correlation between both series in first differences is 0.358, while cross autocorrelations are 0.41, 0.402 and 0.332 for lags 1-3 of the EFFR series.

[4] EONIA is the effective overnight reference rate for the euro computed as a weighted average of all overnight unsecured lending transactions in the interbank market undertaken in the EMU and European Free Trade Association (EFTA) countries.

Figure 1. Monthly EONIA and Effective Federal Funds Rates (01.1999-12.2014) in levels and in first differences



Next, figures 2-8 shows the time pattern of the retail interest rates for each of the seven countries for both types of credit categories and the three maturities considered in the analysis.

Figure 2. *Retail rates of credits for house purchase (left) and loans for consumption (right): Austria*

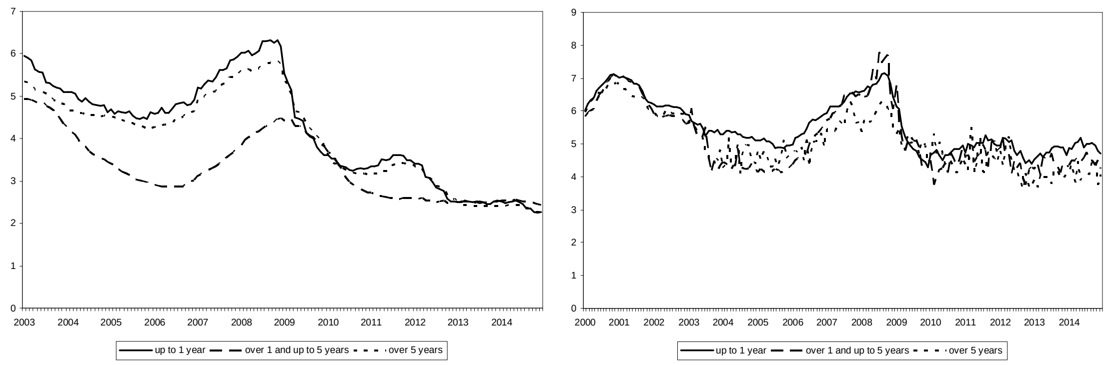


Figure 3. *Retail rates of credits for house purchase (left) and loans for consumption (right): Belgium*

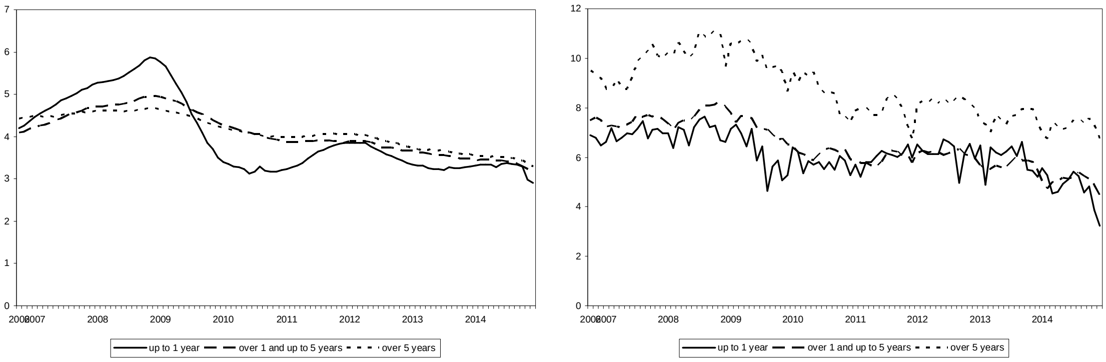


Figure 4. *Retail rates of credits for house purchase (left) and loans for consumption (right): Finland*

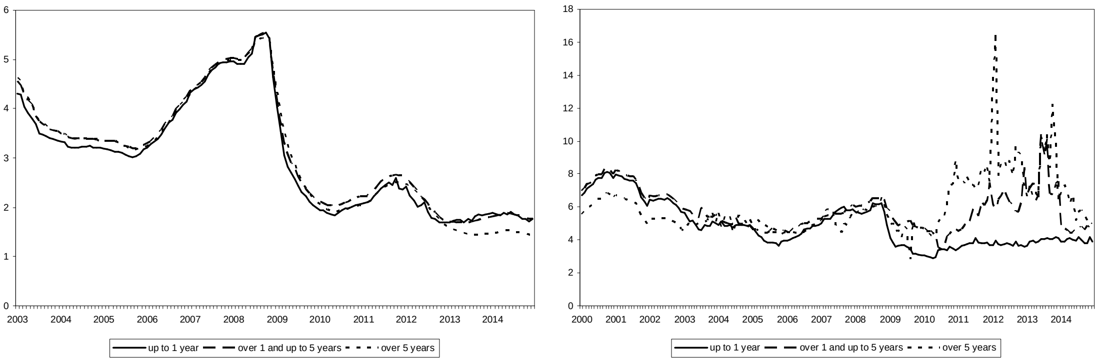


Figure 5. *Retail rates of credits for house purchase (left) and loans for consumption (right): France*

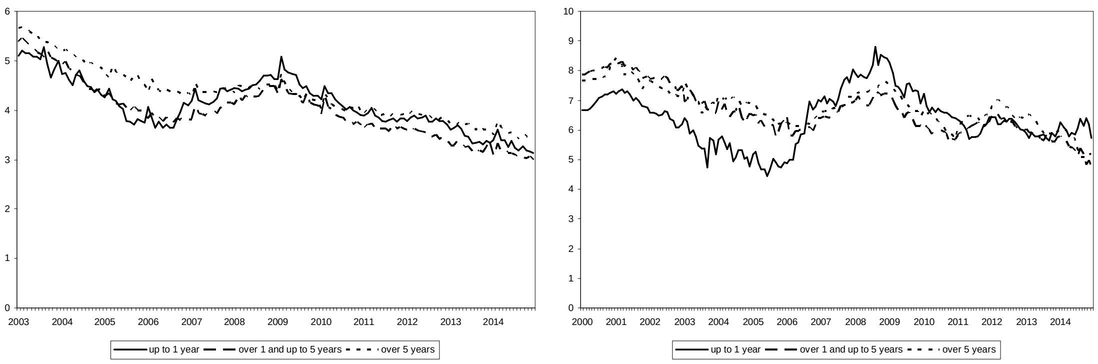


Figure 6. *Retail rates of credits for house purchase (left) and loans for consumption (right): Germany*

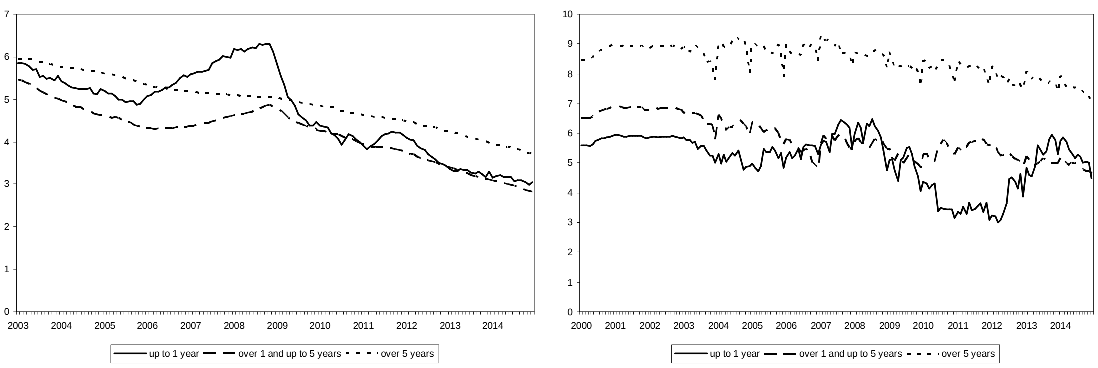


Figure 7. *Retail rates of credits for house purchase (left) and loans for consumption (right): Italy*

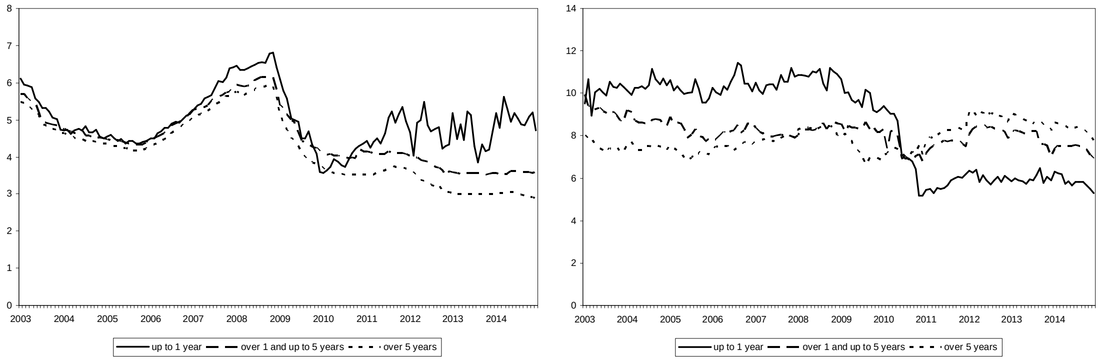
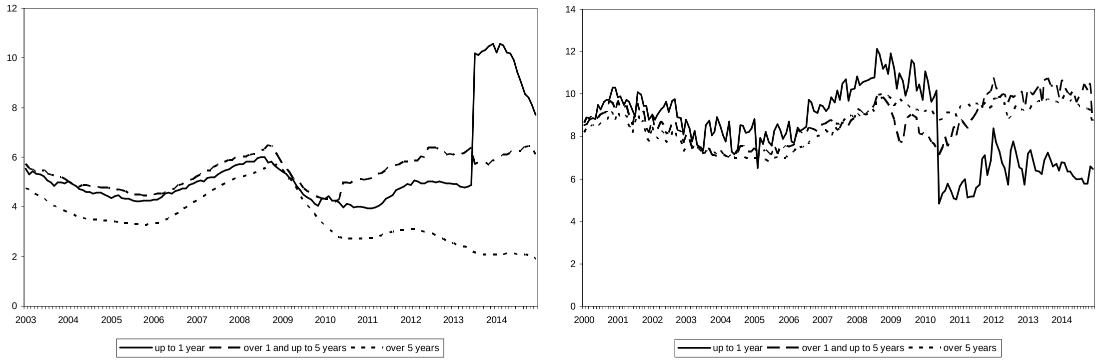


Figure 8. *Retail rates of credits for house purchase (left) and loans for consumption (right): Spain*



Simple visual inspection of all these series reveals a very different behavior of the retail credit markets in each country in the sample, with a certain homogeneity among maturities for each economy and credit category. Particularly interesting in the case of Spain, where the series of interest rates for house purchase and short-term maturity has experienced a wide growth from 2013, while the short-term rate of loans for consumption displays a sharp fall at the end of 2010, and has remained since then between 6 and 8% which is the highest value for the seven countries. These different behaviours seems to anticipate the differences encountered in practice in the estimation of the magnitude of the short and long-run IRPT measures, but can also serve as a justification for the use of a flexible modelling such as the one considered in this paper. The main tool proposed to the analysis of the mechanism of transmission of the monetary policy from the money to the retail markets is a simple regression model of the form

$$y_t = \alpha + \beta x_t + u_t \tag{3.1}$$

that falls into the class of cointegrating regression models given the no stationary behavior of the series involved, with the dependent variable, y_t, given by the retail rate for different maturities of the credit for house purchase and loans for consumption as credit categories, and explanatory variable, x_t, given by the EONIA interest rate. The results of the analysis of integration and stationarity for all the series are not presented here,[5] but strongly indicates that the variables are no stationary, thus supporting the analysis of the regression model as a way of representing the cointegrating relationship among the retail interest rates and the money market rate. Table E.1 in Appendix E presents the results of a variety of testing procedures for cointegration, roughly

[5] The analysis was performed based on the usual ADF and PP test statistics for the null of integration against the alternative of stationarity, and the KPSS test statistic for the hypothesis in reverse order. In all the cases, the stationarity hypothesis is rejected at the 5% level of signification.

indicating the no existence of a stable long-run relationship in all the cases when based on the time-invariant regression model (3.1). One possible explanation for these results could be attributed to the existence of a time-varying stable relationship omitted in (3.1), as is apparent from the results of Hansen's (1992) tests for parameter instability in cointegration regressions with integrated regressors.[6] Appendix A contains the theoretical analysis of the consistency of this testing procedure against the alternative of a time-varying cointegrating regression where the pattern of changes in the parameters is modelled via Chebyshev time polynomials. This is the approach taken in the rest of this section.

3.2. Time-varying cointegrating regression analysis

With the aim to explore the capability of the approach proposed by Bierens (1997), and extended by Bierens and Martins (2009, 2010) and Neto (2012, 2014) to the cointegration analysis, we propose the following generalization of equation (3.1) as

$$y_t = \alpha_t(m) + \beta_t(m) x_t + u_t \tag{3.2}$$

where the time-varying intercept and slope are defined as

$$\alpha_t(m) = \sum_{j=0}^{m} a_{j,n} G_{j,n}(t) \tag{3.3}$$

and

$$\beta_t(m) = \sum_{j=0}^{m} b_{j,n} G_{j,n}(t) \tag{3.4}$$

respectively, with $G_{0,n}(t) = 1$, $G_{j,n}(t) = \sqrt{2}\cos(j\pi(t-0.5)/n)$, j = 1, 2, ..., m, $m \leq n-1$. This general specification allows to obtain three alternative models given by

Model 1. No intercept and TV slope, $a_{j,n} = 0$, j = 0, 1, ..., m

Model 2. Fixed intercept and TV slope, $a_{j,n} = 0$, j = 1, ..., m

and

Model 3. TV intercept and slope.

This model does not allow to capture in general structural changes in the cointegrating relationship since the functions $\alpha_t(m)$ and $\beta_t(m)$ are assumed to be smooth and slow time-varying deterministic functions of time. However, there exists the possibility of easily combine the proposed formulation with a structural break, unless the magnitude shifts be small enough to be subsumed by the time-varying structure of the model parameters, as shown in the analysis of Appendix B. The analysis performed in this section based on (3.2)-(3.3) requires the OLS estimation of the coefficients $a_{j,n}, b_{j,n}$ j = 0, 1, ..., m for a particular choice of $1 \leq m < n$, and the computation of the test statistics

$$\hat{K}_n(m) = \frac{1}{n^2 \hat{\omega}_{u,n}^2(q_n)} \sum_{t=1}^{n} \left(\sum_{j=1}^{t} \hat{u}_j(m) \right)^2 \tag{3.5}$$

and

$$C\hat{S}_n(m) = \frac{1}{\hat{\omega}_{u,n}(q_n)\sqrt{n}} \max_{t=1,\dots,n} \left| \sum_{j=1}^{t} \hat{u}_j(m) \right| \tag{3.6}$$

---

[6] Quintos and Phillips (1993) also propose a number of related procedures to test the null hypothesis of time-invariant cointegration against specific directions of departures from the null, including the possibility to test the stability of a subset of coefficients. For more results related to testing for partial parameter instability in cointegrating regressions see also Kuo (1998) and Hsu (2008).

where $\hat{\omega}^2_{u,n}(q_n) = \hat{\sigma}^2_{u,n} + 2\hat{\lambda}_{u,n}(q_n)$ is a kernel-type estimator of the long-run variance of u_t, with $\hat{\sigma}^2_{u,n} = n^{-1}\sum_{t=1}^{n}\hat{u}_t^2(m)$ and $\hat{\lambda}_{u,n}(q_n) = \sum_{h=1}^{n-1} w(h/q_n) n^{-1} \sum_{t=h+1}^{n} \hat{u}_{t-h}(m)\hat{u}_t(m)$ for some weighting function $w(\cdot)$ and bandwidth $q_n = o(n^{1/2})$, based on the autocovariances of the residuals

$$\hat{u}_t(m) = u_t - \sum_{j=0}^{m}((\hat{a}_{j,n} - a_{j,n}),(\hat{b}_{j,n} - b_{j,n}))G_{j,n}(t)\begin{pmatrix}1\\ x_t\end{pmatrix}$$

The statistic (3.5) is the so-called KPSS test statistic for testing the null hypothesis of stationarity for the regression error term u_t, and hence cointegration, while that $\hat{CS}_n(m)$ in (3.6) is the Xiao and Phillips (2002) test statistic adapted to the residuals from (3.2) as has been considered by Neto (2014). In the case of endogeneous regressors, the OLS version of these test statistics cannot be used in practical applications given that their limiting null distributions depend on some nuisance parameters, and hence must be computed on the basis of residuals from some asymptotically efficient estimation such as the FM-OLS method (see Neto (2014) and Appendix B). Table D.1 in Appendix D contains the finite-sample upper critical values for Models 1-3 with one integrated regressor and *m* = 1, …, 5, thus generalizing the results in Neto (2014). The limiting null distribution of these test statistics is model-dependent in the sense that the critical values differ for each model and value of *m*. To avoid this dependence on the model specification and dimension when testing for time-varying cointegration, we also propose the use of the test statistics proposed by McCabe et.al. (2006) (MLH) described in Appendix C, which also have the advantages of relying only on the OLS estimation of the time-varying cointegrating regression (3.2), even under endogeneity of the integrated regressors.

The results of these testing procedures presented in Table E.2 and E.3 in Appendix E are mixed, both for each country and for the different maturities of the two types of credit categories analyzed in terms of the stability of the long-run relationship between the retail and the market rates, with different conclusions depending on the order of approximation of the time-evolving parameters given by *m*. However, when based on the results of the MLH tests, the overall conclusion is that of stationary time-varying cointegration for almost all the cases, particularly when focus on the results of the statistic labelled MLH2 (see equation (C.6) in Appendix C) and for moderate values of *m* ranging from 1-4.

Finally, based on these results, Appendix F shows the estimated values of the long-run IRPT for the series of each country and for Models 2 and 3 with values of *m* ranging from 1 to 10. From these estimates we cannot conclude a clear evidence on the degree of adjustment of the retail and money markets for the series and models considered, although there is some indication that the pass-through is incomplete.

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Appendix A. *Hansen's tests for parameter instability under time-varying cointegrating regression via Chebyshev time polynomials*

Let us assume that the time-varying cointegrating vector, $\boldsymbol{\beta}_{k,t} = (\beta_{1,t},\dots,\beta_{k,t})'$, in the specification of the time-varying (TV) cointegrating regression model, $y_t = \boldsymbol{\beta}'_{k,t}\mathbf{x}_{k,t} + u_t$, with $\Delta\mathbf{x}_{k,t} = \boldsymbol{\varepsilon}_{k,t}$, is given by

$$\boldsymbol{\beta}_{k,t} = \sum_{j=0}^{n-1} \boldsymbol{\alpha}_{kj,n} G_{j,n}(t) \tag{A.1}$$

where $\boldsymbol{\alpha}_{kj,n} = n^{-1}\sum_{t=1}^{n} \boldsymbol{\beta}_{k,t} G_{j,n}(t)$ by the orthogonality property of Chebyshev polynomials $G_{j,n}(t)$, i.e., $n^{-1}\sum_{t=1}^{n} G_{j,n}(t)G_{i,n}(t) = I(i=j)$, with

$$\begin{aligned} G_{j,n}(t) &= 1 && j=0 \\ &= \sqrt{2}\cos\left(\frac{j\pi(t-0.5)}{n}\right) && j=1,2,\dots,n-1 \end{aligned} \tag{A.2}$$

which implies that $n^{-1}\sum_{t=1}^{n} G_{j,n}(t) = 0$ for any j = 1, 2, ... Also, given that $\boldsymbol{\beta}_{k,t}$ can be written as $\boldsymbol{\beta}_{k,t} = \boldsymbol{\beta}_{k,t}(m) + \mathbf{b}_{k,t}(m)$, with

$$\boldsymbol{\beta}_{k,t}(m) = \sum_{j=0}^{m} \boldsymbol{\alpha}_{kj,n} G_{j,n}(t) \tag{A.3}$$

for some fixed natural number $m < n-1$, and the fact that for the remaining term $\mathbf{b}_{k,t}(m) = \sum_{j=m+1}^{n-1} \boldsymbol{\alpha}_{kj,n} G_{j,n}(t)$ we have that $\lim_{m,n\to\infty} n^{-1}\sum_{t=1}^{n} \mathbf{b}'_{k,t}(m)\mathbf{b}_{k,t}(m) = 0$ and $\lim_{n\to\infty} n^{-1}\sum_{t=1}^{n} \mathbf{b}'_{k,t}(m)\mathbf{b}_{k,t}(m) \le (m+1)^{-2q}$ for $q \ge 2$ and $m \ge 1$ (see Lemma 1 in Bierens and Martins (2010)), then our TV cointegrating regression model is given by

$$y_t = \boldsymbol{\beta}'_{k,t}(m)\mathbf{x}_{k,t} + u_t = \sum_{j=0}^{m} \boldsymbol{\alpha}'_{kj,n}\mathbf{x}_{k,t}G_{j,n}(t) + u_t = (\boldsymbol{\alpha}'_{k0,n}, \mathbf{A}'_{km,n})\begin{pmatrix}\mathbf{x}_{k,t}\\ \mathbf{X}_{km,t}\end{pmatrix} + u_t \tag{A.4}$$

with $\mathbf{A}_{km,n} = (\boldsymbol{\alpha}'_{k1,n},...,\boldsymbol{\alpha}'_{km,n})'$, $\mathbf{X}_{km,t} = (\mathbf{x}'_{k1,t},...,\mathbf{x}'_{km,t})'$, and $\mathbf{x}_{kj,t} = \mathbf{x}_{k,t}G_{j,n}(t)$, j = 1, ..., m. The necessary tools required for the asymptotic analysis of the estimation results arising from this specification are provided by Bierens (1997) (see Lemmas A.1-A.5) and Bierens and Martins (2009, 2010). Thus, under the assumption that the regression error term u_t is given by $u_t = \alpha u_{t-1} + \upsilon_t$, with $|\alpha| \leq 1$ and υ_t a zero-mean weakly stationary error sequence with finite variance, and defining the partial sum process of u_t as $U_n(r) = 0$ when $r \in [0, n^{-1}]$, and $U_n(r) = \sum_{t=1}^{[nr]} u_t$ for $r \in [n^{-1}, 1]$, then we get

$$n^{-1/2}\sum_{t=1}^{[nr]} F(t/n)u_t \Rightarrow \int_0^r F(s)dB_u(s) = F(1)B_u(1) - \int_0^r f(s)B_u(s)ds \tag{A.5}$$

and

$$n^{-3/2}\sum_{t=1}^{[nr]} F(t/n)U_n(t/n) = n^{-1}\sum_{t=1}^{[nr]} F(t/n)(n^{-1/2}U_n(t/n)) \Rightarrow \int_0^r F(s)B_u(s)ds \tag{A.6}$$

where $B_u(r)$ is a Brownian motion process characterizing the weak limit of $n^{-1/2}U_n(r)$, for any differentiable real function on [0,1], $F(\cdot)$, with derivative $f(\cdot)$. Also, given that $\mathbf{x}_{k,n[nr]} = n^{-1/2}\mathbf{x}_{k,0} + \mathbf{B}_{k,n}(r)$, with $\mathbf{B}_{k,n}(r) = n^{-1/2}\sum_{j=1}^{[nr]} \boldsymbol{\varepsilon}_{k,j}$ for $r \in [n^{-1}, 1]$, and $\mathbf{B}_{k,n}(r) = \mathbf{0}_k$ for $r \in [0, n^{-1}]$, then we get that $n^{-1}\sum_{t=1}^{n} \mathbf{x}_{kj,nt}\mathbf{x}'_{k,nt}$ and $n^{-1}\sum_{t=1}^{n} \mathbf{x}_{kj,nt}\mathbf{x}'_{kj,nt}$ are both $O_p(1)$. Taking now $\boldsymbol{\beta}_{k,0} = \boldsymbol{\alpha}_{k0,n}$, the TV cointegrating regression can be rewritten as

$$y_t = \boldsymbol{\beta}'_{k,0}\mathbf{x}_{k,t} + u_t + \sum_{j=1}^{n} \boldsymbol{\alpha}'_{kj,n}\mathbf{x}_{k,t}G_{j,n}(t) = \boldsymbol{\beta}'_{k,0}\mathbf{x}_{k,t} + v_t \tag{A.7}$$

where $v_t = u_t + \mathbf{X}'_{km,t}\mathbf{A}_{km,n}$, where the OLS estimation error of $\boldsymbol{\beta}_{k,0}$ is given by

$$\begin{aligned}\hat{\boldsymbol{\beta}}_{k0,n} - \boldsymbol{\beta}_{k0} &= n^{-(1/2+\kappa)}\left(n^{-1}\sum_{t=1}^{n}\mathbf{x}_{k,nt}\mathbf{x}'_{k,nt}\right)^{-1} n^{-(1-\kappa)}\sum_{t=1}^{n}\mathbf{x}_{k,nt}v_t \\ &= n^{-(1/2+\kappa)}\left(n^{-1}\sum_{t=1}^{n}\mathbf{x}_{k,nt}\mathbf{x}'_{k,nt}\right)^{-1} n^{-(1-\kappa)}\sum_{t=1}^{n}\mathbf{x}_{k,nt}(u_t + \sqrt{n}\mathbf{X}'_{km,nt}\mathbf{A}_{km,n})\end{aligned} \tag{A.8}$$

with $\mathbf{x}_{k,nt} = n^{-1/2}\mathbf{x}_{k,t}$, $\mathbf{X}_{km,nt} = n^{-1/2}\mathbf{X}_{km,t} = (\mathbf{x}'_{k1,nt},...,\mathbf{x}'_{km,nt})'$, and $\mathbf{x}_{kj,nt} = \mathbf{x}_{k,nt}G_{j,n}(t)$, j = 1, ..., m. Given that $\Delta\mathbf{x}_{kj,nt} = G_{j,n}(t)\boldsymbol{\varepsilon}_{k,t} + (G_{j,n}(t) - G_{j,n}(t-1))\mathbf{x}_{k,n(t-1)}$, it comes that the variance of $\Delta\mathbf{x}_{kj,nt}$ is not constant since it depends on t, but as $G_{j,n}(t+h) - G_{j,n}(t) = -(hj\pi/n)H_{j,n}(t) + O(n^{-2})$ with $H_{j,n}(t) = \sqrt{2}\sin(j\pi(t-0.5)/n)$ for each j = 1, 2, ... the second term becomes asymptotically negligible and hence $Var[\Delta\mathbf{x}_{kj,nt}] \to G^2_{j,n}(t)E[\boldsymbol{\varepsilon}_{k,t}\boldsymbol{\varepsilon}'_{k,t}]$ as $n\to\infty$, that depends on t only through the Chebyshev polynomial. The scaled OLS estimation error is then given by

$$n^{1/2+\kappa}(\hat{\boldsymbol{\beta}}_{k0,n} - \boldsymbol{\beta}_{k0}) = \mathbf{Q}^{-1}_{kk,n}\left\{n^{-(1-\kappa)}\sum_{t=1}^{n}\mathbf{x}_{k,nt}u_t + n^{1/2+\kappa}n^{-1}\sum_{t=1}^{n}\mathbf{x}_{k,nt}\mathbf{X}'_{km,nt}\mathbf{A}_{km,n}\right\} \tag{A.9}$$

where $\mathbf{Q}_{kk,n} = n^{-1}\sum_{t=1}^{n}\mathbf{x}_{k,nt}\mathbf{x}'_{k,nt}$, with the index κ taking the values κ = 1/2 under cointegration, that is when the error term u_t is stationary, and κ = −1/2 under no cointegration, so that the second term between brackets will dominates the behavior of $n(\hat{\boldsymbol{\beta}}_{k0,n} - \boldsymbol{\beta}_{k0})$ under cointegration when $\mathbf{A}_{km,n} \neq \mathbf{0}_{km}$. From this result, the t-th OLS residual is given by

$$\hat{v}_t = v_t - n^{-\kappa}\mathbf{x}'_{k,nt}[n^{1/2+\kappa}(\hat{\boldsymbol{\beta}}_{k0,n} - \boldsymbol{\beta}_{k0})] = \tilde{u}_t + n^{1/2}\mathbf{d}'_{km,nt}\mathbf{A}_{km,n} \tag{A.10}$$

where the two terms composing these residuals are

$$\tilde{u}_t = u_t - n^{-\kappa}\mathbf{x}'_{k,nt}\mathbf{Q}^{-1}_{kk,n}n^{-(1-\kappa)}\sum_{j=1}^{n}\mathbf{x}_{k,nj}u_j \tag{A.11}$$

and

$$\mathbf{d}_{km,nt} = \mathbf{X}_{km,nt} - n^{-1}\sum_{j=1}^{n}\mathbf{X}_{km,nj}\mathbf{x}'_{k,nj}\mathbf{Q}^{-1}_{kk,n}\mathbf{x}_{k,nt} \tag{A.12}$$

Hansen (1992) proposed a set of statistics to test for parameter instability in regression models with integrated regressors that are based on different measures of excessive variability of the partial sums of the estimated scores from the model fitting. First, we consider the case of the OLS estimated scores, $\hat{\mathbf{s}}_{k,t} = \mathbf{x}_{k,t}\hat{v}_t$, that can be decomposed as

$$\hat{\mathbf{s}}_{k,t} = n^{1/2}\tilde{\mathbf{s}}_{k,nt} + n\mathbf{x}_{k,nt}\mathbf{d}'_{km,nt}\mathbf{A}_{km,n} \tag{A.13}$$

where $\tilde{\mathbf{s}}_{k,nt} = \mathbf{x}_{k,nt}\tilde{u}_t = O_p(1)$ under cointegration, so that $\hat{\mathbf{S}}_{k,t} = \sum_{j=1}^{t}\hat{\mathbf{s}}_{k,j}$ can be written as

$$\begin{aligned}\hat{\mathbf{S}}_{k,t} &= n\left\{\left(n^{-1/2}\sum_{j=1}^{t}\tilde{\mathbf{s}}_{k,nj}\right) + n\left(n^{-1}\sum_{j=1}^{t}\mathbf{x}_{k,nj}\mathbf{d}'_{km,nj}\right)\mathbf{A}_{km,n}\right\} \\ &= n(\tilde{\mathbf{V}}_{k,nt} + n\mathbf{V}_{k(km),nt}\mathbf{A}_{km,n})\end{aligned} \tag{A.14}$$

which implies that $\hat{\mathbf{S}}_{k,t} = n\tilde{\mathbf{V}}_{k,nt} = O_p(n)$ when $\mathbf{A}_{km,n} = \mathbf{0}_{km}$, where $n^{-1}\hat{\mathbf{S}}_{k,t}$ has a well defined weak limit. Under the general assumption that the model parameters $\boldsymbol{\beta}_{k,t}$ follow a martingale process $\boldsymbol{\beta}_{k,t} = \boldsymbol{\beta}_{k,t-1} + \boldsymbol{\eta}_{k,t}$, with $E[\boldsymbol{\eta}_{k,t}\boldsymbol{\eta}'_{k,t}] = \delta^2\mathbf{G}_t$ and $\mathbf{G}_t^{-1} = \omega^2_{v.k}\mathbf{M}_{kk,n}$, where $\mathbf{M}_{kk,n} = \sum_{t=1}^{n}\mathbf{x}_{k,t}\mathbf{x}'_{k,t}$ and $\omega^2_{v.k} = \omega^2_v - \boldsymbol{\omega}'_{kv}\boldsymbol{\Omega}^{-1}_{kk}\boldsymbol{\omega}_{kv}$ is the conditional long-run variance of v_t given the sequence of error terms driving the integrated regressors, $\boldsymbol{\varepsilon}_{k,t}$, then the OLS version of the LM-type test statistic is given by

$$\hat{L}_n = \frac{1}{n\hat{\omega}^2_{v,n}}\sum_{t=1}^{n}\hat{\mathbf{S}}'_{k,t}\mathbf{M}^{-1}_{kk,n}\hat{\mathbf{S}}_{k,t} = \frac{1}{\hat{\omega}^2_{v,n}}n^{-1}\sum_{t=1}^{n}(n^{-1}\hat{\mathbf{S}}'_{k,t})\mathbf{Q}^{-1}_{kk,n}(n^{-1}\hat{\mathbf{S}}_{k,t}) \tag{A.15}$$

with $\hat{\omega}^2_{v,n}$ a consistent estimator of ω^2_v under parameter stability and strict exogeneity of the integrated regressors, i.e. $\delta^2 = 0$ and $\boldsymbol{\omega}_{kv} = \mathbf{0}_k$ respectively, usually given by a kernel-type estimator based on the sequence of sample serial covariances of the OLS residuals such as $\hat{\omega}^2_{v,n} = n^{-1}\sum_{t=1}^{n}\hat{v}_t^2 + 2\sum_{h=1}^{n-1}w(h/q_n)n^{-1}\sum_{t=h+1}^{n}\hat{v}_t\hat{v}_{t-h}$, with $w(\cdot)$ the kernel (weighting) function and bandwidth $q_n = o(n^{1/2})$. Given that the residual covariance of order h can be written as

$$\begin{aligned}n^{-1}\sum_{t=h+1}^{n}\hat{v}_t\hat{v}_{t-h} &= n^{-1}\sum_{t=h+1}^{n}\tilde{u}_t\tilde{u}_{t-h} \\ &\quad + \mathbf{A}'_{km}n^{-1/2}\sum_{t=h+1}^{n}(\mathbf{d}_{km,nt}\tilde{u}_{t-h} + \mathbf{d}_{km,n(t-h)}\tilde{u}_t) \\ &\quad + \mathbf{A}'_{km}\sum_{t=h+1}^{n}\mathbf{d}_{km,nt}\mathbf{d}'_{km,n(t-h)}\mathbf{A}_{km}\end{aligned} \tag{A.16}$$

then $\hat{\omega}^2_{v,n}$ can be decomposed as

$$\hat{\omega}^2_{v,n} = \hat{\omega}^2_{u,n} + 2\mathbf{A}'_{km}\left(n^{-1/2}\sum_{t=1}^{n}\mathbf{d}_{km,nt}\tilde{u}_t + \sum_{h=1}^{n-1} w(h/q_n) n^{-1/2}\sum_{t=h+1}^{n}(\mathbf{d}_{km,nt}\tilde{u}_{t-h} + \mathbf{d}_{km,n(t-h)}\tilde{u}_t)\right)$$
$$+ n\mathbf{A}'_{km}\left(n^{-1}\sum_{t=1}^{n}\mathbf{d}_{km,nt}\mathbf{d}'_{km,nt} + 2\sum_{h=1}^{n-1} w(h/q_n) n^{-1}\sum_{t=h+1}^{n}\mathbf{d}_{km,nt}\mathbf{d}'_{km,n(t-h)}\right)\mathbf{A}_{km} \tag{A.17}$$

with $\hat{\omega}^2_{u,n} = n^{-1}\sum_{t=1}^{n}\tilde{u}_t^2 + 2\sum_{h=1}^{n-1} w(h/q_n) n^{-1}\sum_{t=h+1}^{n}\tilde{u}_t\tilde{u}_{t-h}$ a consistent estimator of the long-run variance of u_t under cointegration given that $n^{-1}\sum_{t=h+1}^{n}\tilde{u}_t\tilde{u}_{t-h} \to^p E[u_t u_{t-h}]$. The second term on the right hand side is also $O_p(1)$ under the assumption of cointegration, while that for the last term on the right hand side between brackets we have

$$q_n n^{-1}\sum_{t=1}^{n}\mathbf{d}_{km,nt}\mathbf{d}'_{km,nt}\left\{\frac{1}{q_n}\left(1 + 2\sum_{h=1}^{n-1} w(h/q_n)\right)\right\} + o_p(1) = O_p(q_n) \tag{A.18}$$

so that $\hat{\omega}^2_{v,n} = O_p(nq_n)$ under time-varying cointegration of the type considered. Otherwise, under time-invariant cointegration (i.e. when $\mathbf{A}_{km,n} = \mathbf{0}_{km}$), the test statistic is given by

$$\hat{L}_n = \frac{1}{\hat{\omega}^2_{u,n}} n^{-1}\sum_{t=1}^{n}\tilde{\mathbf{V}}'_{k,nt}\mathbf{Q}^{-1}_{kk,n}\tilde{\mathbf{V}}_{k,nt} \tag{A.19}$$

where $\tilde{\mathbf{V}}_{k,n[nr]} = n^{-1/2}\sum_{t=1}^{[nr]}\tilde{\mathbf{s}}_{k,nt} \Rightarrow \mathbf{V}_k(r) = \int_0^r \mathbf{B}_k(s)dV_{u,k}(s) + (r\mathbf{I}_{k,k} - \mathbf{Q}_{kk}(r)\mathbf{Q}^{-1}_{kk}(1))\mathbf{\Delta}_{ku}$ as $n\to\infty$, with $V_{u,k}(r)$ the weak limit of $n^{-1/2}\sum_{t=1}^{[nr]}\tilde{u}_t$ under stationarity, given by $V_{u,k}(r) = B_u(r) - \mathbf{v}'_k(r)\mathbf{Q}^{-1}_{kk}(1)\int_0^1 \mathbf{B}_k(s)dB_u(s)$, with $\mathbf{v}_k(r) = \int_0^r \mathbf{B}_k(s)ds$, $\mathbf{Q}_{kk}(r) = \int_0^r \mathbf{B}_k(s)\mathbf{B}'_k(s)ds$, for $0 < r \leq 1$, and $\mathbf{\Delta}_{ku} = \sum_{h=0}^{\infty} E[\boldsymbol{\varepsilon}_{k,t-h}u_t]$ the one-sided long-run covariance between past values of $\boldsymbol{\varepsilon}_{k,t}$ and u_t. Also, taking into account that $\hat{\omega}^2_{u,n} \to^p \omega^2_u = \sum_{h=-\infty}^{\infty} E[u_{t-h}u_t]$, then $\hat{L}_n(q_n) \Rightarrow \omega_u^{-2}\int_0^1 \mathbf{V}'_k(r)\mathbf{Q}^{-1}_{kk}(1)\mathbf{V}_k(r)$. This limit distribution cannot be used in practice in the general case due to the presence of the measure of weak endogeneity of the regressors through $\mathbf{\Delta}_{ku}$ and the fact that $E[\mathbf{B}_k(s)V_{u,k}(s)] \neq \mathbf{0}_k$ under endogeneity of the integrated regressors, which implies that the component $\int_0^r \mathbf{B}_k(s)dV_{u,k}(s)$ has not a mixed Gaussian distribution. However, despite this result, given that under time-varying cointegration we have that the numerator of the test statistic can be written as

$$n^{-1}\sum_{t=1}^{n}(n^{-1}\hat{\mathbf{S}}'_{k,t})\mathbf{Q}^{-1}_{kk,n}(n^{-1}\hat{\mathbf{S}}_{k,t}) = n^{-1}\sum_{t=1}^{n}\tilde{\mathbf{V}}'_{k,nt}\mathbf{Q}^{-1}_{kk,n}\tilde{\mathbf{V}}_{k,nt}$$
$$+ 2\sum_{t=1}^{n}\tilde{\mathbf{V}}'_{k,nt}\mathbf{Q}^{-1}_{kk,n}\mathbf{V}_{k(km),nt}\mathbf{A}_{km,n} \tag{A.20}$$
$$+ n\mathbf{A}'_{km,n}\sum_{t=1}^{n}\mathbf{V}'_{k(km),nt}\mathbf{Q}^{-1}_{kk,n}\mathbf{V}_{k(km),nt}\mathbf{A}_{km,n} = O_p(n^2)$$

so that it is dominated for the last term, we get that $\hat{L}_n(q_n) = O_p(n/q_n)$ and hence it will diverge at the given rate in the case of a smooth time-varying cointegration relation as described by the representation based on Chebyshev time polynomials. In order to circumvent the problems associated with the use of the OLS version of the test statistic under the null of time-invariant cointegration with endogeneous regressors, Hansen (1992) propose a modified version based on an asymptotically efficient estimator such as the Fully-Modified OLS (FM-OLS) estimator by Phillips and Hansen (1990). From the computation of the element, $\hat{\boldsymbol{\gamma}}_{kv,n} = \hat{\boldsymbol{\Omega}}_{kk,n}^{-1}\hat{\boldsymbol{\omega}}_{kv,n}$ with $\hat{\boldsymbol{\Omega}}_{kk,n} = \hat{\boldsymbol{\Delta}}_{kk,n} + \hat{\boldsymbol{\Lambda}}'_{kk,n}$ and $\hat{\boldsymbol{\omega}}_{kv,n} = \hat{\boldsymbol{\Delta}}_{kv,n} + \hat{\boldsymbol{\Lambda}}'_{vk,n}$ consistent kernel-type estimators of the long-run covariance matrix of $\Delta\mathbf{x}_{k,t} = \boldsymbol{\varepsilon}_{k,t}$ and long-run covariance vector of $\boldsymbol{\varepsilon}_{k,t}$ and v_t, respectively, with components $\hat{\boldsymbol{\Delta}}_{kk,n} = n^{-1}\sum_{t=1}^{n}\Delta\mathbf{x}_{k,t}\Delta\mathbf{x}'_{k,t} + \hat{\boldsymbol{\Lambda}}_{kk,n}$, $\hat{\boldsymbol{\Lambda}}_{kk,n} = \sum_{h=1}^{n-1} w(h/q_n)n^{-1}\sum_{t=h+1}^{n}\Delta\mathbf{x}_{k,t-h}\Delta\mathbf{x}'_{k,t}$, $\hat{\boldsymbol{\Delta}}_{kv,n} = \sum_{h=0}^{n-1} w(h/q_n)n^{-1}\sum_{t=h+1}^{n}\Delta\mathbf{x}_{k,t-h}\hat{v}_t$, and $\hat{\boldsymbol{\Lambda}}'_{vk,n} = \sum_{h=1}^{n-1} w(h/q_n)n^{-1}\sum_{t=h+1}^{n}\Delta\mathbf{x}_{k,t}\hat{v}_{t-h}$, the FM-OLS estimator of $\boldsymbol{\beta}_k$ is $\hat{\boldsymbol{\beta}}_{k0,n}^{+} = (\sum_{t=1}^{n}\mathbf{x}_{k,t}\mathbf{x}'_{k,t})^{-1}(\sum_{t=1}^{n}\mathbf{x}_{k,t}y_t^{+} - n\hat{\boldsymbol{\Delta}}_{kv,n}^{+})$, where $y_t^{+} = y_t - \hat{\boldsymbol{\gamma}}'_{kv,n}\Delta\mathbf{x}_{k,t}$ and $\hat{\boldsymbol{\Delta}}_{kv,n}^{+} = \hat{\boldsymbol{\Delta}}_{kv,n} - \hat{\boldsymbol{\Delta}}_{kk,n}\hat{\boldsymbol{\gamma}}_{kv,n}$. In the case of parameter instability of the type considered, then we have that $y_t^{+} = \mathbf{x}'_{k,t}\boldsymbol{\beta}_{k0} + v_t^{+}$ with

$$v_t^{+} = v_t - \hat{\boldsymbol{\gamma}}'_{kv,n}\Delta\mathbf{x}_{k,t} = u_t^{+} + \sqrt{n}(\mathbf{X}'_{km,nt} - n^{-1/2}\Delta\mathbf{x}'_{k,t}\hat{\boldsymbol{\Omega}}_{kk,n}^{-1}\hat{\boldsymbol{\Omega}}_{k(km),n})\mathbf{A}_{km}$$

where $u_t^{+} = u_t - \Delta\mathbf{x}'_{k,t}\hat{\boldsymbol{\gamma}}_{ku,n}$, $\hat{\boldsymbol{\omega}}_{kv,n} = \hat{\boldsymbol{\omega}}_{ku,n} + \hat{\boldsymbol{\Omega}}_{k(km),n}\mathbf{A}_{km}$ and $\hat{\boldsymbol{\gamma}}_{kv,n} = \hat{\boldsymbol{\gamma}}_{ku,n} + \hat{\boldsymbol{\Omega}}_{kk,n}^{-1}\hat{\boldsymbol{\Omega}}_{k(km),n}\mathbf{A}_{km}$, with $\hat{\boldsymbol{\Omega}}_{k(km),n} = \hat{\boldsymbol{\Delta}}_{k(km),n} + \sum_{h=1}^{n-1} w(h/q_n)n^{-1/2}\sum_{t=h+1}^{n}\Delta\mathbf{x}_{k,t}\mathbf{d}'_{km,n(t-h)}$, and $\hat{\boldsymbol{\Delta}}_{k(km),n} = \sum_{h=0}^{n-1} w(h/q_n)n^{-1/2}\sum_{t=h+1}^{n}\Delta\mathbf{x}_{k,t-h}\mathbf{d}'_{km,nt}$. Also, taking into account that $\hat{\boldsymbol{\Delta}}_{kv,n}^{+}$ is decomposed as $\hat{\boldsymbol{\Delta}}_{kv,n}^{+} = \hat{\boldsymbol{\Delta}}_{ku,n}^{+} + \hat{\boldsymbol{\Delta}}_{k(km),n}^{+}\mathbf{A}_{km}$, with $\hat{\boldsymbol{\Delta}}_{ku,n}^{+} = \hat{\boldsymbol{\Delta}}_{ku,n} - \hat{\boldsymbol{\Delta}}_{kk,n}\hat{\boldsymbol{\gamma}}_{ku,n}$, and $\hat{\boldsymbol{\Delta}}_{k(km),n}^{+} = \hat{\boldsymbol{\Delta}}_{k(km),n} - \hat{\boldsymbol{\Delta}}_{kk,n}\hat{\boldsymbol{\Omega}}_{kk,n}^{-1}\hat{\boldsymbol{\Omega}}_{k(km),n}$, then the scaled FM-OLS estimation error is given by

$$n^{1/2+\kappa}(\hat{\boldsymbol{\beta}}_{k0,n}^{+} - \boldsymbol{\beta}_{k0}) = \left(n^{-1}\sum_{t=1}^{n}\mathbf{x}_{k,nt}\mathbf{x}'_{k,nt}\right)^{-1}\left(n^{-(1-\kappa)}\sum_{t=1}^{n}\mathbf{x}_{k,nt}v_t^{+} - n^{-(1/2-\kappa)}\hat{\boldsymbol{\Delta}}_{kv,n}^{+}\right) \quad \text{(A.21)}$$

where the last term between parenthesis can be expressed as

$$\begin{aligned} n^{-(1-\kappa)}\sum_{t=1}^{n}\mathbf{x}_{k,nt}v_t^{+} - n^{-(1/2-\kappa)}\hat{\boldsymbol{\Delta}}_{kv,n}^{+} &= n^{-(1-\kappa)}\sum_{t=1}^{n}\mathbf{x}_{k,nt}u_t^{+} - n^{-(1/2-\kappa)}\hat{\boldsymbol{\Delta}}_{ku,n}^{+} \\ &+ n^{1/2+\kappa}\Bigg\{n^{-1}\sum_{t=1}^{n}\mathbf{x}_{k,nt}\mathbf{X}'_{km,nt} \\ &-n^{-1}\left\{\left(n^{-1/2}\sum_{t=1}^{n}\mathbf{x}_{k,nt}\Delta\mathbf{x}'_{k,t}\right)\hat{\boldsymbol{\Omega}}_{kk,n}^{-1}\hat{\boldsymbol{\Omega}}_{k(km),n} - \hat{\boldsymbol{\Delta}}_{k(km),n}^{+}\right\}\Bigg\}\mathbf{A}_{km} \end{aligned} \quad \text{(A.22)}$$

which implies consistent estimation under cointegration under parameter stability, i.e. $\mathbf{A}_{km} = \mathbf{0}_{km}$. If we define the FM-OLS estimated scores as $\hat{\mathbf{s}}_{k,t}^{+} = \mathbf{x}_{k,t}\hat{v}_t^{+} - \hat{\boldsymbol{\Delta}}_{kv,n}^{+}$, so that $\sum_{t=1}^{n}\hat{\mathbf{s}}_{k,t}^{+} = \mathbf{0}_k$, the FM-OLS version of the test statistic $\hat{L}_n^{+} = (n\hat{\omega}_{v.k,n}^{2})^{-1}\sum_{t=1}^{n}\hat{\mathbf{S}}'^{+}_{k,t}\mathbf{M}_{kk,n}^{-1}\hat{\mathbf{S}}_{k,t}^{+}$, with $\hat{\omega}_{v.k,n}^{2} = \hat{\omega}_{v,n}^{2} - \hat{\boldsymbol{\omega}}'_{ku,n}\hat{\boldsymbol{\Omega}}_{kk,n}^{-1}\hat{\boldsymbol{\omega}}_{ku,n}$ and $\hat{\mathbf{S}}_{k,t}^{+} = \sum_{j=1}^{t}\hat{\mathbf{s}}_{k,t}^{+}$, will provide similar results to what obtained when using the OLS estimates and residuals. Also, similar consistency results are obtained for the sup-F and mean-F tests based on

$$\hat{F}_{nt} = \frac{1}{n\hat{\omega}_{v,n}^{2}}\hat{\mathbf{S}}'_{k,t}\mathbf{V}_{kk,n}^{-1}\hat{\mathbf{S}}_{k,t} \quad t = 1,\ldots,n$$

with $\mathbf{V}_{kk,n} = \mathbf{M}_{kk,t} - \mathbf{M}_{kk,t}\mathbf{M}_{kk,n}^{-1}\mathbf{M}_{kk,t}$, where the sup-F test is given by

$$S\hat{F}_n(\tau_0,\tau_1) = \max_{t=[n\tau_0],\ldots,[n\tau_1]}\hat{F}_{nt},\ 0 < \tau_0 < \tau_1 < 1$$

and allows to test for a single structural change at an unknown break point, while that the mean-F test, which is defined as

$$M\hat{F}_n(\tau_0,\tau_1)=\frac{1}{[n\tau_1]-[n\tau_0]+1}\sum_{t=[n\tau_0]}^{[n\tau_1]}\hat{F}_{nt}$$

is also designed to test against a martingale mechanism guiding the variability of the regression coefficients.

Appendix B. *OLS estimation of a time-varying cointegrating regression model via Chebyshev polynomials under a structural break in the cointegrating vector*

Let us assume that the time-varying cointegrating regression model is specified as

$$y_t=\boldsymbol{\beta}'_{k,t}(m)\mathbf{x}_{k,t}+u_t=(\boldsymbol{\alpha}'_{k0},\mathbf{A}'_{km})\begin{pmatrix}\mathbf{x}_{k,t}\\ \mathbf{X}_{km,t}\end{pmatrix}+v_t \tag{B.1}$$

where $\boldsymbol{\beta}_{k,t}=\boldsymbol{\beta}_{k,t}(m)$ is as in (A.3), so that the OLS estimation error of $(\boldsymbol{\alpha}'_{k0},\mathbf{A}'_{km})'$ is

$$\begin{pmatrix}\hat{\boldsymbol{\alpha}}_{k0,n}-\boldsymbol{\alpha}_{k0}\\ \hat{\mathbf{A}}_{km,n}-\mathbf{A}_{km}\end{pmatrix}=n^{-(1/2+\kappa)}\left(n^{-1}\sum_{t=1}^{n}\mathbf{X}_{nt}(m)\mathbf{X}'_{nt}(m)\right)^{-1}n^{-(1-\kappa)}\sum_{t=1}^{n}\mathbf{X}_{nt}(m)v_t \tag{B.2}$$

with $\mathbf{X}_{nt}(m)=(\mathbf{x}'_{k,nt},\mathbf{X}'_{km,nt})'$. However, the true mechanism driving the time-varying cointegrating vector is given by a permanent and abrupt change at the break point of the sample $\gamma_0=[n\tau_0]$ such as $\boldsymbol{\beta}_{k,t}=\boldsymbol{\alpha}_{k0}+\boldsymbol{\lambda}_{k0}H_t(\gamma_0)$, with $H_t(\gamma_0)=I(t>\gamma_0)$ and break fraction $\tau_0\in(0,1)$, where $\boldsymbol{\lambda}_{k0}=(\lambda_{10},\ldots,\lambda_{k0})'$ is the k-vector containing the shift magnitudes. Under this assumption, the correct specification of the cointegrating regression is as follows

$$y_t=\boldsymbol{\alpha}'_{k0}\mathbf{x}_{k,t}+\boldsymbol{\lambda}'_{k0}\mathbf{x}_{k,t}H_t(\gamma_0)+u_t \tag{B.3}$$

which implies that the regression error term v_t in (B.1) and (B.2) is given by

$$v_t=u_t+\boldsymbol{\lambda}'_{k0}\mathbf{x}_{k,t}H_t(\gamma_0)-\mathbf{A}'_{km}\mathbf{X}_{km,t}, \tag{B.4}$$

so that the last term in the right-hand side of (B.2) can be decomposed as

$$\begin{aligned}n^{-(1-\kappa)}\sum_{t=1}^{n}\mathbf{X}_{nt}(m)v_t&=n^{-(1-\kappa)}\sum_{t=1}^{n}\mathbf{X}_{nt}(m)u_t\\&\quad+n^{1/2+\kappa}n^{-1}\sum_{t=1}^{n}\mathbf{X}_{nt}(m)(\mathbf{x}'_{k,nt}H_t(\gamma_0)\boldsymbol{\lambda}_{k0}-\mathbf{X}'_{km,nt}\mathbf{A}_{km})\end{aligned} \tag{B.5}$$

where

$$\begin{aligned}n^{-1}\sum_{t=1}^{n}\mathbf{X}_{nt}(m)\mathbf{x}'_{k,nt}H_t(\gamma_0)&=n^{-1}\sum_{t=[n\tau_0]+1}^{n}\mathbf{X}_{nt}(m)\mathbf{x}'_{k,nt}\\&=n^{-1}\sum_{t=[n\tau_0]+1}^{n}\mathbf{X}_{nt}(m)\mathbf{X}'_{nt}(m)\begin{pmatrix}\mathbf{I}_{k,k}\\ \mathbf{0}_{km,k}\end{pmatrix}=(\mathbf{Q}_n(m)-\mathbf{Q}_{[n\tau_0]}(m))\begin{pmatrix}\mathbf{I}_{k,k}\\ \mathbf{0}_{km,k}\end{pmatrix}\end{aligned} \tag{B.6}$$

and

$$n^{-1}\sum_{t=1}^{n}\mathbf{X}_{nt}(m)\mathbf{X}'_{km,nt}=n^{-1}\sum_{t=1}^{n}\mathbf{X}_{nt}(m)\mathbf{X}'_{nt}(m)\begin{pmatrix}\mathbf{0}_{k,km}\\ \mathbf{I}_{km,km}\end{pmatrix}=\mathbf{Q}_n(m)\begin{pmatrix}\mathbf{0}_{k,km}\\ \mathbf{I}_{km,km}\end{pmatrix} \tag{B.7}$$

where $\mathbf{Q}_{[n\tau_0]}(m)=n^{-1}\sum_{t=1}^{[n\tau_0]}\mathbf{X}_{nt}(m)\mathbf{X}'_{nt}(m)$ and $\mathbf{Q}_n(m)$ is $\mathbf{Q}_{[n\tau_0]}(m)$ with $\tau_0=1$. Thus, equation (B.2) can be rewritten as

$$\begin{aligned}n^{1/2+\kappa}\begin{pmatrix}\hat{\boldsymbol{\alpha}}_{k0,n}-\boldsymbol{\alpha}_{k0}\\ \hat{\mathbf{A}}_{km,n}-\mathbf{A}_{km}\end{pmatrix}&=n^{1/2+\kappa}\begin{pmatrix}\boldsymbol{\lambda}_{k0}\\ -\mathbf{A}_{km}\end{pmatrix}+\mathbf{Q}_n^{-1}(m)n^{-(1-\kappa)}\sum_{t=1}^{n}\mathbf{X}_{nt}(m)u_t\\&\quad-n^{1/2+\kappa}\mathbf{Q}_n^{-1}(m)\mathbf{Q}_{[n\tau_0]}(m)\begin{pmatrix}\boldsymbol{\lambda}_{k0}\\ \mathbf{0}_{km,k}\end{pmatrix}\end{aligned}$$

which gives

$$n^{1/2+\kappa}\begin{pmatrix}\hat{\boldsymbol{\alpha}}_{k0,n}-(\boldsymbol{\alpha}_{k0}+\boldsymbol{\lambda}_{k0})\\ \hat{\mathbf{A}}_{km,n}\end{pmatrix}=\mathbf{Q}_n^{-1}(m)\left\{n^{-(1-\kappa)}\sum_{t=1}^{n}\mathbf{X}_{nt}(m)u_t-\mathbf{Q}_{[n\tau_0]}(m)\begin{pmatrix}n^{1/2+\kappa}\boldsymbol{\lambda}_{k0}\\ \mathbf{0}_{km,k}\end{pmatrix}\right\} \tag{B.8}$$

Under the assumption of cointegration, with the index κ taking the value κ = 1/2, then u_t is given by $u_t=(1-\alpha L)^{-1}\upsilon_t=\mathbf{c}_0'(L)\mathbf{e}_t=u_{0,t}-\Delta\tilde{e}_t$, with $u_{0,t}=\mathbf{c}_0'(1)\mathbf{e}_t$,and $\tilde{e}_t=\tilde{\mathbf{c}}_0'(L)\mathbf{e}_t$ where the lag polynomial $\mathbf{c}_0(L)$ is $\mathbf{c}_0(L)=(1-\alpha L)^{-1}\mathbf{d}_0(L)=\sum_{j=0}^{\infty}\mathbf{c}_{0,j}L^j$, with $\tilde{\mathbf{c}}_0(L)=\sum_{j=0}^{\infty}\tilde{\mathbf{c}}_{0,j}L^j$, and $\tilde{\mathbf{c}}_{0,j}=\sum_{i=j+1}^{\infty}\mathbf{c}_{0,i}$. Then, to obtain the limit for each component of the *km*-vector

$$n^{-1/2}\sum_{t=1}^{n}\mathbf{X}_{nt}(m)u_t=\begin{pmatrix}n^{-1/2}\sum_{t=1}^{n}\mathbf{x}_{k0,nt}u_t\\ n^{-1/2}\sum_{t=1}^{n}\mathbf{x}_{k1,nt}u_t\\ \vdots\\ n^{-1/2}\sum_{t=1}^{n}\mathbf{x}_{km,nt}u_t\end{pmatrix}$$

we have that

$$\begin{aligned}n^{-1/2}\sum_{t=1}^{n}\mathbf{x}_{kj,nt}u_t&=n^{-1/2}\sum_{t=1}^{n}G_{j,n}(t)\mathbf{x}_{k,nt}u_t\\&=n^{-1/2}\sum_{t=1}^{n}G_{j,n}(t)\mathbf{x}_{k,nt}u_{0,t}+n^{-1}\sum_{t=1}^{n}G_{j,n}(t+1)\boldsymbol{\varepsilon}_{k,t+1}\tilde{e}_t\\&\quad+n^{-1}\sum_{t=1}^{n}(G_{j,n}(t+1)-G_{j,n}(t))\mathbf{x}_{k,t}\tilde{e}_t\\&\quad+n^{-1/2}(G_{j,n}(1))\mathbf{x}_{k,n1}\tilde{e}_0-G_{j,n}(n+1))\mathbf{x}_{k,n(n+1)}\tilde{e}_n)\end{aligned} \tag{B.9}$$

for each *j* = 0, 1, …, *m*, with $n^{-1/2}\sum_{t=1}^{n}G_{j,n}(t)\mathbf{x}_{k,nt}u_{0,t}\Rightarrow\int_0^1\mathbf{B}_{kj}(r)dB_u(r)$, where $\mathbf{B}_{kj}(r)=G_j(r)\mathbf{B}_k(r)$, $G_0(r)=1$ and $G_j(r)=\sqrt{2}\cos(j\pi r)$ *j* = 1, …, *m*, while that for the second term in the right-hand side of (B.9) we have

$$\begin{aligned}n^{-1}\sum_{t=1}^{n}G_{j,n}(t+1)\boldsymbol{\varepsilon}_{k,t+1}\tilde{e}_t&=E[\boldsymbol{\varepsilon}_{k,t+1}\tilde{e}_t]n^{-1}\sum_{t=1}^{n}G_{j,n}(t)\\&\quad+n^{-1/2}\left\{n^{-1/2}\sum_{t=1}^{n}G_{j,n}(t+1)(\boldsymbol{\varepsilon}_{k,t+1}\tilde{e}_t-E[\boldsymbol{\varepsilon}_{k,t+1}\tilde{e}_t])\right\}+O(n^{-1})\\&=E[\boldsymbol{\varepsilon}_{k,t+1}\tilde{e}_t]I(j=0)+O_p(n^{-1/2})\end{aligned} \tag{B.10}$$

with $E[\boldsymbol{\varepsilon}_{k,t+1}\tilde{e}_t]\rightarrow^p\boldsymbol{\Delta}_{ku}=\sum_{h=0}^{\infty}E[\boldsymbol{\varepsilon}_{k,t-h}u_t]$. Also, given that $\Delta G_{0,n}(t+1)=0$ and $\Delta G_{j,n}(t+1)=-(j\pi/n)H_{j,n}(t)+O(n^{-2})$, for *j* = 1, …, *m* we obtain

$$\begin{aligned}n^{-1}\sum_{t=1}^{n}(G_{j,n}(t+1)-G_{j,n}(t))\mathbf{x}_{k,t}\tilde{e}_t&=-(j\pi/n)n^{-1/2}\sum_{t=1}^{n}H_{j,n}(t)\mathbf{x}_{k,nt}\tilde{e}_t\\&\quad+O(n^{-2})n^{-1/2}\sum_{t=1}^{n}\mathbf{x}_{k,nt}\tilde{e}_t=O_p(n^{-1})\end{aligned} \tag{B.11}$$

so that it is asymptotically negligible. Taking together these results we obtain

$$n^{-1/2}\sum_{t=1}^{n}\mathbf{X}_{nt}(m)u_t\Rightarrow\int_0^1\mathbf{m}^{(m)}(r)dB_u(r)+\boldsymbol{\Delta}_{ku}^{(m)} \tag{B.12}$$

where $\mathbf{m}^{(m)}(r)=(\mathbf{B}_k'(r),\mathbf{B}_k'^{(m)}(r))'$, with $\mathbf{B}_k^{(m)}(r)=(G_1(r)\mathbf{B}_k'(r),...,G_m(r)\mathbf{B}_k'(r))'$ and $\boldsymbol{\Delta}_{ku}^{(m)}=(\boldsymbol{\Delta}_{ku}',\mathbf{0}_k',...,\mathbf{0}_k')'$. Finally, from (B.8), under the assumption of a structural change of decreasing magnitude with the sample size at the rate $\boldsymbol{\lambda}_{k0} = O(n^{-1})$ when κ = 1/2 (that is under cointegration), then we have

$$n\begin{pmatrix} \hat{\boldsymbol{\alpha}}_{k0,n} - (\boldsymbol{\alpha}_{k0} + \boldsymbol{\lambda}_{k0}) \\ \hat{\mathbf{A}}_{km,n} \end{pmatrix} = \mathbf{Q}_n^{-1}(m)\left\{ n^{-(1-\kappa)} \sum_{t=1}^{n} \mathbf{X}_{nt}(m)u_t - \mathbf{Q}_{[n\tau_0]}(m)\begin{pmatrix} \boldsymbol{\lambda}_{k0} \\ \mathbf{0}_{km,k} \end{pmatrix} \right\}$$
$$\Rightarrow \mathbf{Q}^{-1}(1,m)\left\{ \int_0^1 \mathbf{m}^{(m)}(r)dB_u(r) + \boldsymbol{\Delta}_{ku}^{(m)} - \mathbf{Q}(\tau_0,m)\begin{pmatrix} \boldsymbol{\lambda}_{k0} \\ \mathbf{0}_{km,k} \end{pmatrix} \right\} \quad \text{(B.13)}$$

with $\mathbf{Q}(r,m) = \int_0^r \mathbf{m}^{(m)}(s)\mathbf{m}'^{(m)}(s)ds$, so that the limiting distribution under cointegration of the parameters in the time-varying regression model will contain as nuisance parameters those measuring the magnitude of the shifts $\boldsymbol{\lambda}_{k0}$. If we introduce the stronger condition $\boldsymbol{\lambda}_{k0} = O(n^{-(1+\delta)})$, for some δ > 0, then

$$n\begin{pmatrix} \hat{\boldsymbol{\alpha}}_{k0,n} - (\boldsymbol{\alpha}_{k0} + \boldsymbol{\lambda}_{k0}) \\ \hat{\mathbf{A}}_{km,n} \end{pmatrix} = \mathbf{Q}_n^{-1}(m) n^{-(1-\kappa)} \sum_{t=1}^{n} \mathbf{X}_{nt}(m)u_t + O_p(n^{-\delta}) \quad \text{(B.14)}$$

and hence the abrupt change can be captured by the smooth function of time characterizing the specification of the estimated regression in (B.1). Observe that the same result is obtained for the estimator of $\boldsymbol{\beta}_{k0}$ in the time-invariant specification of the cointegrating regression $y_t = \boldsymbol{\beta}'_{k0}\boldsymbol{x}_{k,t} + v_t$, although (B.14) has the advantage of providing superconsistent estimates of the time changing parameters under cointegration.

Appendix C. *McCabe et.al. (MLH) (2006) tests for stochastic cointegration based on OLS residuals*

Let us assume that the k+1-dimensional vector $\boldsymbol{\zeta}_t = (u_t, \boldsymbol{\varepsilon}'_{k,t})'$ follows a linear process such as $\boldsymbol{\zeta}_t = \mathbf{C}(L)\mathbf{e}_t = \sum_{j=0}^{\infty} \mathbf{C}_j \mathbf{e}_{t-j}$, with $\sum_{j=0}^{\infty} j\|\mathbf{C}_j\|^2 < \infty$,and $\mathbf{e}_t = (e_{0,t}, \mathbf{e}'_{k,t})' \sim iid(\mathbf{0}, \boldsymbol{\Sigma}_e)$. If we now define the augmented k+2-dimensional vector $\boldsymbol{\xi}_t = (u_t, \upsilon_t, \boldsymbol{\varepsilon}'_{k,t})'$, with $\upsilon_t = u_t^2 - \sigma_u^2$, under the above assumption we get

$$n^{-1/2} \sum_{t=1}^{[nr]} \boldsymbol{\xi}_t \Rightarrow \mathbf{B}_{\boldsymbol{\xi}}(r) = \begin{pmatrix} B_u(r) \\ B_\upsilon(r) \\ \mathbf{B}_k(r) \end{pmatrix} = \boldsymbol{\Omega}_{\boldsymbol{\xi}}^{1/2} \mathbf{W}_{\boldsymbol{\xi}}(r) \quad \text{(C.1)}$$

where $\mathbf{B}_{\boldsymbol{\xi}}(r)$ is a k+2-vector Brownian process with covariance matrix $\boldsymbol{\Omega}_{\boldsymbol{\xi}}$ given by

$$\boldsymbol{\Omega}_{\boldsymbol{\xi}} = \begin{pmatrix} \omega_u^2 & \omega_{u\upsilon} & \boldsymbol{\omega}_{uk} \\ & \omega_\upsilon^2 & \boldsymbol{\omega}_{\upsilon k} \\ & & \boldsymbol{\Omega}_{kk} \end{pmatrix} \quad \text{(C.2)}$$

and $\mathbf{W}_{\boldsymbol{\xi}}(r) = (W_u(r), W_\upsilon(r), \mathbf{W}'_k(r))'$ a k+2-vector standard Brownian process. Taking the upper Cholesky decomposition of $\boldsymbol{\Omega}_{\boldsymbol{\xi}}$,

$$\boldsymbol{\Omega}_{\boldsymbol{\xi}}^{1/2} = \begin{pmatrix} \omega_{u.k}\sqrt{1 - \left(\dfrac{\omega_{u\upsilon} - \boldsymbol{\omega}_{uk}\boldsymbol{\Omega}_{kk}^{-1}\boldsymbol{\omega}_{k\upsilon}}{\omega_{u.k}\omega_{\upsilon.k}} \right)^2} & \omega_{u.k}\left(\dfrac{\omega_{u\upsilon} - \boldsymbol{\omega}_{uk}\boldsymbol{\Omega}_{kk}^{-1}\boldsymbol{\omega}_{k\upsilon}}{\omega_{u.k}\omega_{\upsilon.k}} \right) & \boldsymbol{\omega}_{uk}\boldsymbol{\Omega}_{kk}^{-1/2} \\ 0 & \omega_{\upsilon.k} & \boldsymbol{\omega}_{\upsilon k}\boldsymbol{\Omega}_{kk}^{-1/2} \\ \mathbf{0}_k & \mathbf{0}_k & \boldsymbol{\Omega}_{kk}^{1/2} \end{pmatrix} \quad \text{(C.3)}$$

with $\omega_{u.k}^2$ and $\omega_{\upsilon.k}^2$ the long-run variances of u_t and υ_t conditional on $\boldsymbol{\varepsilon}_{k,t}$, given by $\omega_{u.k}^2 = \omega_u^2 - \boldsymbol{\omega}_{uk}\boldsymbol{\Omega}_{kk}^{-1}\boldsymbol{\omega}'_{uk} = \omega_u^2(1-\rho_{uk}^2)$ and $\omega_{\upsilon.k}^2 = \omega_\upsilon^2 - \boldsymbol{\omega}_{\upsilon k}\boldsymbol{\Omega}_{kk}^{-1}\boldsymbol{\omega}'_{\upsilon k} = \omega_\upsilon^2(1-\rho_{\upsilon k}^2)$, with $\rho_{uk}^2 = \omega_u^{-2}\boldsymbol{\omega}_{uk}\boldsymbol{\Omega}_{kk}^{-1}\boldsymbol{\omega}'_{uk}$ and $\rho_{\upsilon k}^2 = \omega_\upsilon^{-2}\boldsymbol{\omega}_{\upsilon k}\boldsymbol{\Omega}_{kk}^{-1}\boldsymbol{\omega}'_{\upsilon k}$ the squared long-run correlation coefficients between these error terms, then the limiting process in (C.1) can be expressed as

$$\begin{pmatrix} B_u(r) \\ B_\upsilon(r) \\ \mathbf{B}_k(r) \end{pmatrix} = \begin{pmatrix} \omega_{u.k}W_u(r)\sqrt{1-\rho_{u\upsilon.k}^2} + \omega_{u.k}W_\upsilon(r)\rho_{u\upsilon.k} + \boldsymbol{\omega}_{uk}\boldsymbol{\Omega}_{kk}^{-1}\mathbf{B}_k(r) \\ \omega_{\upsilon.k}W_\upsilon(r) + \boldsymbol{\omega}_{\upsilon k}\boldsymbol{\Omega}_{kk}^{-1}\mathbf{B}_k(r) \\ \boldsymbol{\Omega}_{kk}^{1/2}\mathbf{W}_k(r) \end{pmatrix} \tag{C.4}$$

where $\rho_{u\upsilon.k} = (1/\omega_{u.k}\omega_{\upsilon.k})(\omega_{u\upsilon} - \boldsymbol{\omega}_{uk}\boldsymbol{\Omega}_{kk}^{-1}\boldsymbol{\omega}_{k\upsilon})$.

C.1. The case of stationary cointegration under time invariance of the cointegrating vector

Taking $\mathbf{A}_{km,n} = \mathbf{0}_{km}$ in (A.10), so that the correctly specified cointegrating regression has fixed coefficients, the sequence of OLS residuals is given by $\hat{v}_t = \tilde{u}_t = u_t - n^{-\kappa}\mathbf{x}'_{k,nt}\hat{\boldsymbol{\Theta}}_{k0,n}$, where $\hat{\boldsymbol{\Theta}}_{k0,n} = n^{1/2+\kappa}(\hat{\boldsymbol{\beta}}_{k0,n} - \boldsymbol{\beta}_{k0}) = \mathbf{Q}_{kk,n}^{-1}n^{-(1-\kappa)}\sum_{j=1}^n \mathbf{x}_{k,nj}u_j$. To test the null hypothesis of stationary cointegration against no cointegration, McCabe et.al. (2006) have proposed the test statistics

$$\hat{H}_{v,n}(q_n) = \frac{1}{\hat{\omega}_{v,n}(q_n)} n^{-1/2} \sum_{t=s_n+1}^n \hat{v}_t\hat{v}_{t-s_n} \tag{C.5}$$

and

$$\hat{H}_{\upsilon,n}(q_n) = \frac{\sqrt{12}}{\hat{\omega}_{\upsilon,n}(q_n)} n^{-3/2} \sum_{t=1}^n t(\hat{v}_t^2 - \hat{\sigma}_{v,n}^2) \tag{C.6}$$

with $\hat{\omega}_{v,n}^2(q_n)$ and $\hat{\omega}_{\upsilon,n}^2(q_n)$ estimates of ω_u^2 and ω_υ^2, such as the kernel-based heteroskedasticity and autocorrelation consistent (HAC) estimators given by $\hat{\omega}_{v,n}^2(q_n) = \sum_{h=-(n-1)}^{n-1} w(h/q_n)n^{-1}\sum_{t=s_n+|h|+1}^n \hat{a}_t\hat{a}_{t-h}$, where $\hat{a}_t = \hat{v}_t\hat{v}_{t-s_n}$ t = 1, 2, …, s_n, and $\hat{\omega}_{\upsilon,n}^2(q_n) = \sum_{h=-(n-1)}^{n-1} w(h/q_n)n^{-1}\sum_{t=|h|+1}^n \hat{b}_t\hat{b}_{t-h}$, with $\hat{b}_t = \hat{v}_t^2 - \hat{\sigma}_{v,n}^2$. Given that the numerator of $\hat{H}_{v,n}(q_n)$ in (C.5) can be decomposed as

$$n^{-1/2}\sum_{t=s_n+1}^n \hat{v}_t\hat{v}_{t-s_n} = n^{-1/2}\sum_{t=s_n+1}^n u_t u_{t-s_n} + n^{-\kappa}\hat{\boldsymbol{\Theta}}'_{k0,n} n^{-1/2}\sum_{t=s_n+1}^n (\mathbf{x}_{k,n(t-s_n)}u_t + \mathbf{x}_{k,nt}u_{t-s_n})$$
$$+ n^{1/2-2\kappa}\hat{\boldsymbol{\Theta}}'_{k0,n} n^{-1}\sum_{t=s_n+1}^n \mathbf{x}_{k,nt}\mathbf{x}'_{k,n(t-s_n)}\hat{\boldsymbol{\Theta}}_{k0,n}$$

under the cointegration assumption, with $\kappa = 1/2$, we get

$$n^{-1/2}\sum_{t=s_n+1}^n \hat{v}_t\hat{v}_{t-s_n} = n^{-1/2}\sum_{t=s_n+1}^n u_t u_{t-s_n} + O_p(n^{-1/2}) \tag{C.7}$$

where $E[u_t u_{t-s_n}] \to 0$ as n, $s_n \to \infty$ with $s_n = O(n^{1/2})$. Thus, by Theorem 1 in McCabe et.al. (2006) and under the additional condition on the bandwidth parameter $q_n = o(s_n)$, $q_n < s_n$, $\hat{H}_{v,n}(q_n) \to^d N(0,1)$. For the test statistic $\hat{H}_{\upsilon,n}(q_n)$ in (C.6), by defining $\hat{\sigma}_{v,n}^2 = n^{-1}\sum_{t=1}^n \hat{v}_t^2$ and $\sigma_{u,n}^2 = n^{-1}\sum_{t=1}^n u_t^2$, the sequence of squared and centered OLS residuals $\hat{v}_t^2 - \hat{\sigma}_{v,n}^2$ is decomposed as

$$\hat{v}_t^2 - \hat{\sigma}_{v,n}^2 = u_t^2 - \sigma_{u,n}^2 + n^{-2\kappa}\hat{\mathbf{\Theta}}'_{k0,n}(\mathbf{x}_{k,nt}\mathbf{x}'_{k,nt} - \mathbf{Q}_{kk,n})\hat{\mathbf{\Theta}}_{k0,n}$$
$$- 2n^{-2\kappa}\hat{\mathbf{\Theta}}'_{k0,n}\left(n^{\kappa}\mathbf{x}_{k,nt}u_t - n^{-(1-\kappa)}\sum_{j=1}^{n}\mathbf{x}_{k,nj}u_j \right)$$

so that the term in the numerator of $\hat{H}_{\upsilon,n}(q_n)$ is given by

$$n^{-3/2}\sum_{t=1}^{n} t(\hat{v}_t^2 - \hat{\sigma}_{v,n}^2) = n^{-3/2}\sum_{t=1}^{n} t(u_t^2 - \sigma_{u,n}^2)$$
$$+ n^{1/2-2\kappa}\hat{\mathbf{\Theta}}'_{k0,n}\left(n^{-1}\sum_{t=1}^{n}(t/n)\mathbf{x}_{k,nt}\mathbf{x}'_{k,nt} - \frac{n+1}{2}\mathbf{Q}_{kk,n} \right)\hat{\mathbf{\Theta}}_{k0,n}$$
$$- 2n^{1/2-2\kappa}\hat{\mathbf{\Theta}}'_{k0,n}\left(n^{-(1-\kappa)}\sum_{t=1}^{n}(t/n)\mathbf{x}_{k,nt}u_t - \frac{n+1}{2}n^{-(1-\kappa)}\sum_{j=1}^{n}\mathbf{x}_{k,nj}u_j \right)$$

Again, under the assumption of stationary cointegration with κ = 1/2 we get

$$n^{-3/2}\sum_{t=1}^{n} t(\hat{v}_t^2 - \hat{\sigma}_{v,n}^2) = n^{-3/2}\sum_{t=1}^{n} t(u_t^2 - \sigma_{u,n}^2) + O_p(n^{-1/2})$$
$$= n^{-1/2}\sum_{t=1}^{n} n^{-1}\left(t - \frac{(n+1)}{2} \right)[(u_t^2 - \sigma_u^2) - (\sigma_{u,n}^2 - \sigma_u^2)] + O_p(n^{-1/2})$$
$$= n^{-1/2}\sum_{t=1}^{n} n^{-1}\left(t - \frac{(n+1)}{2} \right)(u_t^2 - \sigma_u^2) + O_p(n^{-1/2})$$
$$= n^{-1/2}\sum_{t=1}^{n}\left(\frac{t}{n} - \frac{1}{2} \right)(u_t^2 - \sigma_u^2) + O_p(n^{-1/2}) \tag{C.8}$$

given that $-(1/2n)n^{-1/2}\sum_{t=1}^{n}(u_t^2 - \sigma_u^2) = O_p(n^{-1})$. For the main term above, we have $n^{-1/2}\sum_{t=1}^{n}(t/n - 1/2)(u_t^2 - \sigma_u^2) = \int_0^1 (r - 1/2)dB_{\upsilon,n}(r)$ where $B_{\upsilon,n}(r)$ is the partial sum process of $\upsilon_t = u_t^2 - \sigma_u^2$, that weakly converges to the Brownian motion process $B_\upsilon(r)$, so that under cointegration $n^{-3/2}\sum_{t=1}^{n} t(\hat{v}_t^2 - \hat{\sigma}_{v,n}^2) \to^d \int_0^1 (r - 1/2)dB_\upsilon(r) =^d N(0,(1/12)\omega_\upsilon^2)$. Thus, under the same condition as above for the bandwidth parameter q_n we get $\hat{H}_{\upsilon,n}(q_n) \to^d N(0,1)$.

C.2. The case of stationary cointegration under a time-varying cointegrating vector via Chebyshev time polynomials

From the estimation of the time-varying cointegrating regression in (B.1), with $m = m_0$, the true or proper order of the Chebyshev polynomial approximation, then (B.8) is now given by

$$n^{1/2+\kappa}\begin{pmatrix} \hat{\boldsymbol{\alpha}}_{k0,n} - \boldsymbol{\alpha}_{k0} \\ \hat{\mathbf{A}}_{km,n} - \mathbf{A}_{km} \end{pmatrix} = \mathbf{Q}_n^{-1}(m)n^{-(1-\kappa)}\sum_{t=1}^{n}\mathbf{X}_{nt}(m)u_t \tag{C.9}$$

so that the OLS residuals, $\hat{u}_t(m) = y_t - \hat{\boldsymbol{\beta}}'_{k,t}(m)\mathbf{x}_{k,t}$, can be written as

$$\hat{u}_t(m) = u_t - n^{-\kappa}(\mathbf{x}'_{k,nt}, \mathbf{X}'_{km,nt})'\left\{ n^{1/2+\kappa}\begin{pmatrix} \hat{\boldsymbol{\alpha}}_{k0,n} - \boldsymbol{\alpha}_{k0} \\ \hat{\mathbf{A}}_{km,n} - \mathbf{A}_{km} \end{pmatrix} \right\} \tag{C.10}$$

which implies that $\hat{u}_t(m) = u_t + O_p(n^{-1/2})$ under cointegration, and hence all the above results are verified. Also, taking the partial sum of these OLS residuals, under cointegration in the time-varying setup we obtain

$$n^{-1/2}\sum_{t=1}^{[nr]}\hat{u}_t(m) = n^{-1/2}\sum_{t=1}^{[nr]}u_t - n^{-1}\sum_{t=1}^{[nr]}(\mathbf{x}'_{k,nt},\mathbf{X}'_{km,nt})'\left\{n\begin{pmatrix}\hat{\boldsymbol{\alpha}}_{k0,n}-\boldsymbol{\alpha}_{k0}\\ \hat{\mathbf{A}}_{km,n}-\mathbf{A}_{km}\end{pmatrix}\right\}$$
$$\Leftrightarrow B_u(r) - \int_0^r \mathbf{m}'^{(m)}(s)ds\,\mathbf{Q}^{-1}(1,m)\left\{\int_0^1 \mathbf{m}^{(m)}(s)dB_u(s)+\boldsymbol{\Delta}_{ku}^{(m)}\right\} \qquad \text{(C.11)}$$

which gives

$$n^{-1/2}\sum_{t=1}^{[nr]}\hat{u}_t(m) \Leftrightarrow \omega_u W_u^{(m)}(r) \qquad \text{(C.12)}$$

with

$$W_u^{(m)}(r) = W_u(r) - \int_0^r \mathbf{m}'^{(m)}(s)ds\,\mathbf{Q}^{-1}(1,m)\int_0^1 \mathbf{m}^{(m)}(s)dW_u(s) \qquad \text{(C.13)}$$

under strict exogeneity of the stochastic integrated regressors.

Appendix D. Finite sample upper critical values for KPPS-type test by Shin (1994) and CUSUM-type test by Xiao and Phillips (2002) under time-varying cointegration via Chebyshev polynomials of order $m = 1, \ldots, 5$.

Table D.1. Upper critical values for KPSS-type and Xiao-Phillips test statistics under time-varying cointegration for models 1-3 with $k = 1$ stochastic integrated regressor

		KPSS-TYPE					XIAO-PHILLIPS				
***n* = 100**		m = 1	2	3	4	5	m = 1	2	3	4	5
Model 1	0.9	0.6498	0.5662	0.5087	0.4740	0.4463	1.4185	1.3205	1.2596	1.1943	1.1709
	0.95	0.9615	0.8315	0.7599	0.6901	0.6707	1.6608	1.5391	1.4622	1.4018	1.3589
	0.975	1.3039	1.1265	1.0431	0.9768	0.9123	1.8968	1.7434	1.6608	1.6054	1.5605
	0.99	1.8821	1.5737	1.4540	1.3644	1.2564	2.1843	2.0305	1.9133	1.8667	1.7731
		m = 1	2	3	4	5	m = 1	2	3	4	5
Model 2	0.9	0.1580	0.1320	0.1177	0.1062	0.0998	0.8841	0.8190	0.7795	0.7472	0.7207
	0.95	0.2115	0.1806	0.1635	0.1492	0.1398	0.9887	0.9202	0.8783	0.8377	0.8161
	0.975	0.2773	0.2384	0.2165	0.2018	0.1876	1.0909	1.0171	0.9713	0.9362	0.8973
	0.99	0.3611	0.3197	0.3021	0.2762	0.2598	1.2178	1.1405	1.0805	1.0533	1.0083
		m = 1	2	3	4	5	m = 1	2	3	4	5
Model 3	0.9	0.0782	0.0443	0.0305	0.0234	0.0189	0.6790	0.5476	0.4737	0.4242	0.3907
	0.95	0.0961	0.0530	0.0352	0.0264	0.0211	0.7379	0.5927	0.5122	0.4561	0.4170
	0.975	0.1155	0.0605	0.0395	0.0291	0.0231	0.7984	0.6335	0.5471	0.4832	0.4386
	0.99	0.1402	0.0720	0.0457	0.0328	0.0259	0.8702	0.6932	0.5898	0.5226	0.4763
		KPSS-TYPE					XIAO-PHILLIPS				
***n* = 150**		m = 1	2	3	4	5	m = 1	2	3	4	5
Model 1	0.9	0.6337	0.5705	0.5027	0.4672	0.4312	1.4256	1.3247	1.2595	1.1936	1.1494
	0.95	0.9223	0.8119	0.7489	0.6819	0.6316	1.6365	1.5450	1.4660	1.3993	1.3443
	0.975	1.2238	1.0990	1.0169	0.9323	0.8558	1.8550	1.7342	1.6556	1.6027	1.5291
	0.99	1.7434	1.4451	1.3573	1.2762	1.1617	2.1343	1.9848	1.9036	1.8159	1.7546
		m = 1	2	3	4	5	m = 1	2	3	4	5
Model 2	0.9	0.1612	0.1309	0.1167	0.1075	0.0994	0.8937	0.8295	0.7846	0.7498	0.7236
	0.95	0.2160	0.1832	0.1663	0.1528	0.1416	1.0010	0.9337	0.8841	0.8541	0.8235
	0.975	0.2912	0.2426	0.2266	0.2092	0.1936	1.1100	1.0356	0.9889	0.9531	0.9187
	0.99	0.3983	0.3526	0.3149	0.2954	0.2741	1.2665	1.1756	1.1214	1.0796	1.0550
		m = 1	2	3	4	5	m = 1	2	3	4	5
Model 3	0.9	0.0777	0.0435	0.0302	0.0229	0.0185	0.6859	0.5545	0.4808	0.4287	0.3941
	0.95	0.0969	0.0517	0.0347	0.0259	0.0204	0.7522	0.5982	0.5175	0.4614	0.4208
	0.975	0.1183	0.0598	0.0392	0.0287	0.0228	0.8148	0.6402	0.5517	0.4900	0.4448
	0.99	0.1418	0.0708	0.0460	0.0334	0.0258	0.8763	0.6943	0.5933	0.5277	0.4803
		KPSS-TYPE					XIAO-PHILLIPS				
***n* = 200**		m = 1	2	3	4	5	m = 1	2	3	4	5
Model 1	0.9	0.6302	0.5537	0.5031	0.4577	0.4330	1.4181	1.3122	1.2484	1.1906	1.1547
	0.95	0.9189	0.7972	0.7389	0.6815	0.6466	1.6681	1.5280	1.4539	1.3881	1.3549
	0.975	1.2404	1.0877	0.9653	0.8801	0.8597	1.8879	1.7466	1.6478	1.5685	1.5396
	0.99	1.7255	1.4892	1.3136	1.2283	1.1799	2.1272	1.9858	1.8939	1.7840	1.7214
		m = 1	2	3	4	5	m = 1	2	3	4	5
Model 2	0.9	0.1567	0.1297	0.1123	0.1031	0.0943	0.8941	0.8339	0.7851	0.7482	0.7223
	0.95	0.2123	0.1812	0.1619	0.1498	0.1391	0.9995	0.9321	0.8868	0.8519	0.8202
	0.975	0.2727	0.2365	0.2145	0.1956	0.1848	1.1034	1.0251	0.9828	0.9439	0.9126
	0.99	0.3725	0.3263	0.2992	0.2649	0.2488	1.2233	1.1536	1.0942	1.0572	1.0238
		m = 1	2	3	4	5	m = 1	2	3	4	5
Model 3	0.9	0.0764	0.0440	0.0296	0.0225	0.0183	0.6969	0.5600	0.4834	0.4323	0.3964
	0.95	0.0944	0.0513	0.0343	0.0253	0.0203	0.7566	0.6026	0.5201	0.4611	0.4243
	0.975	0.1127	0.0593	0.0389	0.0285	0.0223	0.8184	0.6462	0.5550	0.4920	0.4490
	0.99	0.1378	0.0690	0.0454	0.0328	0.0250	0.8870	0.6924	0.5899	0.5316	0.4824

Note. Model 1 indicates the specification of the time-varying cointegrating regression without intercept, while that Models 2 and 3 incorporate this component. Model 2 assumes a fixed intercept and a time-varying (TV) slope coefficient, while that Model 3 indicates that both the intercept and slope parameters are TV and given by a weighted sum of Chebyshev polynomials up to degree $m = 1, \ldots, 5$, with deterministic weights.

Appendix E.

Table E.1. Tests for cointegration in regressions with a constant term

Country		Phillips-Ouliaris		Shin's test			Hansen's Lc test	
Austria	**Maturity**	Z1	Z2	OLS	DOLS	FMOLS	OLS	FMOLS
Credits for house purchase	short-term	-6.3825	-1.8744	0.6474	0.8076	0.5515	1.2723	1.3459
	medium-term	-4.8183	-1.9922	0.5307	0.7130	0.4266	0.8897	0.8197
	long-term	-5.7736	-1.6719	0.5980	0.6579	0.4508	1.2131	1.2164
Loans for consumption	short-term	-11.5156	-2.4381	0.6312	0.3484	0.9786	1.0940	1.7496
	medium-term	-14.3695	-2.8127	0.7642	0.4645	1.1117	1.1095	1.6997
	long-term	-62.7751	-6.2313	0.3727	0.3807	0.4093	1.1879	1.4046
		Phillips-Ouliaris		Shin's test			Hansen's Lc test	
Belgium	**Maturity**	Z1	Z2	OLS	DOLS	FMOLS	OLS	FMOLS
Credits for house purchase	short-term	-6.5250	-1.8802	0.2610	0.2821	0.1581	0.4637	0.5557
	medium-term	-1.8464	-0.7870	0.8474	0.7159	0.6448	1.4394	1.4585
	long-term	-0.7415	-0.2841	0.9833	0.7165	0.7733	1.6823	1.6625
Loans for consumption	short-term	-42.0701	-4.5898	0.2809	0.1244	0.2490	0.4957	0.4827
	medium-term	-5.5874	-1.3489	0.6750	0.4822	0.4588	1.0791	1.0393
	long-term	-8.8392	-2.0473	0.6555	0.5481	0.4300	1.0369	0.9151
		Phillips-Ouliaris		Shin's test			Hansen's Lc test	
Finland	**Maturity**	Z1	Z2	OLS	DOLS	FMOLS	OLS	FMOLS
Credits for house purchase	short-term	-12.4134	-2.6497	0.2375	0.1253	1.1825	0.3322	1.3230
	medium-term	-11.6353	-2.6468	0.1824	0.3233	0.5055	0.3352	0.6597
	long-term	-10.2260	-2.4437	0.2404	0.4065	0.2209	0.5109	0.6465
Loans for consumption	short-term	-5.5559	-1.8695	0.8519	0.8629	0.8538	2.6645	2.6678
	medium-term	-15.9483	-2.9349	0.5157	0.5784	0.5231	1.7620	1.7988
	long-term	-34.4559	-4.3969	0.4664	0.4125	0.4225	1.1348	1.0669
		Phillips-Ouliaris		Shin's test			Hansen's Lc test	
France	**Maturity**	Z1	Z2	OLS	DOLS	FMOLS	OLS	FMOLS
Credits for house purchase	short-term	-5.4450	-1.7302	0.6520	0.7027	0.5391	1.1780	1.1028
	medium-term	-4.6529	-2.0505	1.0393	0.9781	0.9457	1.6481	1.5790
	long-term	-5.1125	-2.2314	1.4138	1.1090	1.3091	2.0084	1.9135
Loans for consumption	short-term	-5.5679	-1.6842	0.8056	0.5246	1.0591	0.8987	1.2386
	medium-term	-7.0907	-1.8478	0.6802	0.9136	0.6343	1.6670	1.8428
	long-term	-9.5897	-2.0800	0.3946	0.6816	0.3458	0.9527	1.0616
		Phillips-Ouliaris		Shin's test			Hansen's Lc test	
Germany	**Maturity**	Z1	Z2	OLS	DOLS	FMOLS	OLS	FMOLS
Credits for house purchase	short-term	-5.5950	-1.5905	0.7208	0.7171	0.5813	1.4477	1.4430
	medium-term	-2.7338	-1.2115	1.0691	0.9136	0.9374	1.8975	1.7886
	long-term	-2.1736	-1.0105	1.5009	1.0799	1.3955	2.2693	2.1656
Loans for consumption	short-term	-14.2619	-2.7476	0.2074	0.1528	0.2089	0.5052	0.5201
	medium-term	-7.0110	-1.8758	1.2118	1.1099	1.1428	1.7265	1.6547
	long-term	-52.7371	-5.3731	0.7695	0.4689	0.7160	1.7432	1.6599
		Phillips-Ouliaris		Shin's test			Hansen's Lc test	
Italy	**Maturity**	Z1	Z2	OLS	DOLS	FMOLS	OLS	FMOLS
Credits for house purchase	short-term	-15.2975	-2.9371	0.5788	0.1819	1.1441	0.9054	1.7087
	medium-term	-7.0463	-2.1747	0.2863	0.3009	0.3765	0.4771	0.5506
	long-term	-6.9910	-2.0575	0.3367	0.4927	0.2442	0.6847	0.7030
Loans for consumption	short-term	-8.5387	-2.0492	0.8199	0.5037	0.6531	1.2573	1.0923
	medium-term	-16.6298	-3.3439	0.6811	0.6507	0.6412	0.9118	0.9005
	long-term	-7.0453	-1.8942	0.8592	0.4995	0.8936	1.4858	1.5595
		Phillips-Ouliaris		Shin's test			Hansen's Lc test	
Spain	**Maturity**	Z1	Z2	OLS	DOLS	FMOLS	OLS	FMOLS
Credits for house purchase	short-term	-6.7332	-1.7731	0.5407	0.2375	0.5347	1.2117	1.2156
	medium-term	-4.8093	-1.4928	0.9160	0.4588	1.0536	1.6208	1.8573
	long-term	-5.4733	-1.6119	0.2969	0.2524	0.3798	0.5227	0.6040
Loans for consumption	short-term	-20.6012	-3.3605	0.2926	0.2420	0.3502	0.5322	0.5429
	medium-term	-10.0494	-2.2883	0.9345	0.6010	0.9406	2.0250	2.0797
	long-term	-5.0576	-1.5845	1.0582	0.7020	1.1811	1.5742	1.8131

Notes. Asymptotic critical values for Z1 (normalized estimation error) and Z2 (pseudo-T ratio) test statistics by Phillips and Ouliaris (1990)are given by −17.0309(10%), −20.4935(5%), −28.3218(1%), and −3.0657(10%), −3.3654(5%), −3.9618(1%), respectively. For the Shin's test, the critical values are 0.231(10%), 0.314(5%) and 0.533(1%), while that for Hansen's test for stability of the cointegration relationship are 0.450(10%), 0.575(%), and 0.898(1%).

Table E.2. Tests for cointegration in time-varying regressions with a constant term

Country		Credits for house purchase						Loans for consumption					
Austria		Model 2. Fixed intercept, TV slope			Model 3. TV intercept and slope			Model 2. Fixed intercept, TV slope			Model 3. TV intercept and slope		
Maturity		TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC
short	m = 1	0.4714	1.1253	-199.4073	0.1661	0.8553	-315.0593	0.5982	1.5331	-428.8247	0.1488	0.8408	-512.7211
	2	0.4187	1.3210	-219.7468	0.1511	0.7029	-318.9892	0.2426	1.1130	-543.3924	0.0930	0.5944	-588.1555
	3	0.4427	1.2556	-307.9701	0.0577	0.5681	-457.8922	0.1794	0.8826	-574.1671	0.0695	0.4640	-623.7697
	4	0.2579	1.0620	-323.9294	0.0429	0.4603	-522.4725	0.2413	1.1299	-592.5537	0.0274	0.3594	-672.9680
	5	0.2603	1.0481	-321.0162	0.0287	0.4339	-606.4163	0.0910	0.7109	-637.8969	0.0215	0.3707	-687.7957
	6	0.2504	1.0276	-321.0191	0.0204	0.3594	-633.7684	0.0617	0.5713	-653.3452	0.0196	0.3860	-705.2680
	7	0.2675	1.0395	-319.6615	0.0162	0.3532	-689.6035	0.0610	0.5489	-650.9108	0.0194	0.4105	-702.9163
	8	0.2431	0.9949	-341.7791	0.0152	0.3594	-698.0421	0.0645	0.5412	-648.1755	0.0158	0.2822	-758.7472
	9	0.2088	1.0207	-348.7296	0.0131	0.3365	-712.7086	0.0653	0.5971	-649.6654	0.0148	0.3881	-779.0521
	10	0.2186	0.8912	-372.7239	0.0130	0.4262	-747.0597	0.0884	0.7714	-665.0045	0.0115	0.3145	-828.7065
medium	m = 1	0.3429	0.9926	-130.0929	0.1927	0.8982	-176.9936	0.6630	1.6355	-207.8961	0.2052	0.9938	-303.4787
	2	0.2953	1.1440	-139.4117	0.1833	0.7998	-176.4340	0.2364	0.9140	-329.4380	0.1179	0.6670	-359.4704
	3	0.2600	1.1577	-317.4774	0.0660	0.5327	-394.6232	0.1316	0.7090	-381.3519	0.0833	0.5821	-406.1717
	4	0.2743	1.1749	-314.3871	0.0510	0.4892	-612.9785	0.1889	0.9660	-406.6288	0.0290	0.4985	-472.5316
	5	0.2732	1.1989	-311.7992	0.0255	0.4353	-799.7428	0.0316	0.4060	-464.1432	0.0239	0.4662	-481.5911
	6	0.2620	1.1899	-325.6590	0.0238	0.3770	-810.2076	0.0298	0.4610	-463.0120	0.0153	0.3804	-500.6645
	7	0.2575	1.1878	-322.6613	0.0165	0.4002	-900.3232	0.0274	0.4496	-461.2465	0.0148	0.3760	-497.9047
	8	0.2304	1.1461	-341.0587	0.0136	0.3409	-963.3521	0.0291	0.4419	-459.5987	0.0129	0.3475	-507.3670
	9	0.2095	1.1379	-340.7092	0.0124	0.4114	-976.6048	0.0298	0.3861	-469.3075	0.0127	0.3628	-508.7950
	10	0.2250	1.0511	-369.0754	0.0116	0.4130	-998.8054	0.0337	0.4759	-477.0555	0.0129	0.3423	-508.8845
long	m = 1	0.4831	1.1962	-196.9633	0.1413	0.6394	-320.9471	0.3862	1.3488	-310.9482	0.2614	0.9080	-327.6337
	2	0.4418	1.3392	-203.3915	0.1416	0.6369	-320.0035	0.1575	1.0023	-376.4537	0.0998	0.7131	-392.4416
	3	0.4475	1.2610	-271.2300	0.0588	0.5744	-429.1816	0.1934	1.0222	-389.8812	0.0584	0.5833	-429.6534
	4	0.2623	1.0670	-287.9771	0.0459	0.4899	-499.4448	0.1260	0.9426	-403.8452	0.0214	0.3785	-457.2409
	5	0.2635	1.0574	-284.8872	0.0367	0.4133	-585.9552	0.1425	1.0077	-400.9084	0.0193	0.3846	-458.7094
	6	0.2563	1.0427	-282.9009	0.0257	0.3260	-635.7287	0.1200	0.9389	-405.9302	0.0192	0.4155	-457.8686
	7	0.2693	1.0497	-280.8947	0.0159	0.3155	-782.6227	0.1204	0.9284	-403.1564	0.0160	0.3863	-462.6698
	8	0.2453	1.0095	-306.3026	0.0154	0.2859	-780.9741	0.1035	0.8957	-403.5228	0.0127	0.3716	-468.1772
	9	0.2099	1.0468	-314.6846	0.0126	0.3090	-805.3293	0.1135	0.9329	-404.9907	0.0107	0.2931	-478.1957
	10	0.2303	0.9351	-348.6441	0.0118	0.3785	-851.1246	0.1114	0.9243	-402.3490	0.0106	0.3163	-480.1331

Note. HQC is the estimated value of the Hannan-Quinn information criterion defined by $HQC(n,k,m) = n\log(\mathrm{SSR}(k,m)/n) + 2(k+1)(m+1)\log(\log(n))$, with $k = 1$. The critical values for testing the null of time-varying cointegration for each model and test statistic (TV-KPSS and TV-XP) are given in Table D.1

Table E.2. Tests for cointegration in time-varying regressions with a constant term

Country		Credits for house purchase						Loans for consumption					
Belgium		Model 2. Fixed intercept, TV slope			Model 3. TV intercept and slope			Model 2. Fixed intercept, TV slope			Model 3. TV intercept and slope		
Maturity		TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC
Short	m = 1	0.2568	1.1208	-133.1260	0.1626	0.8836	-146.7497	0.2416	1.2415	-97.4773	0.1584	0.9904	-102.2706
	2	0.1253	0.9646	-165.4373	0.1255	0.7202	-200.1272	0.2315	1.2297	-94.8515	0.1296	0.7901	-103.1244
	3	0.1889	1.0081	-168.4519	0.0587	0.5772	-283.4843	0.2113	1.1762	-92.0288	0.0344	0.4249	-147.2051
	4	0.1899	1.0099	-165.4053	0.0410	0.4163	-321.3643	0.2262	1.1521	-90.7213	0.0228	0.4062	-153.5944
	5	0.1841	0.9644	-196.0414	0.0299	0.4048	-462.9701	0.2099	1.1123	-104.7517	0.0214	0.4038	-151.4277
	6	0.1616	0.9732	-194.1445	0.0188	0.5781	-525.3000	0.0825	0.7882	-120.2161	0.0166	0.4458	-155.5358
	7	0.0681	0.8301	-247.4852	0.0178	0.6237	-547.8143	0.0852	0.7824	-117.5538	0.0162	0.4205	-154.4488
	8	0.0899	0.7902	-264.7787	0.0168	0.5837	-551.0344	0.0994	0.8551	-117.2232	0.0160	0.3682	-157.0986
	9	0.0985	0.8035	-262.2462	0.0143	0.5758	-580.1186	0.1012	0.8558	-114.1889	0.0162	0.3724	-164.2694
	10	0.0463	0.6743	-303.6811	0.0144	0.5518	-582.9246	0.0523	0.6316	-122.2767	0.0160	0.3630	-164.9290
medium	m = 1	0.8251	1.5545	-206.5919	0.1872	0.7844	-323.3118	0.6683	1.2957	-110.4662	0.1196	0.6266	-183.4419
	2	0.7439	1.3352	-253.9473	0.1243	0.6552	-389.1191	0.5582	1.2433	-134.6814	0.1231	0.7132	-197.0242
	3	0.5623	1.2174	-256.9891	0.0569	0.6537	-474.1649	0.3870	0.9903	-138.4764	0.0228	0.3408	-273.6929
	4	0.5008	1.1343	-257.6280	0.0352	0.3845	-567.0499	0.3496	0.9995	-137.7296	0.0206	0.3008	-278.3591
	5	0.5043	1.1101	-261.9300	0.0243	0.4074	-640.0837	0.4002	1.0056	-170.1518	0.0170	0.3420	-288.8017
	6	0.3602	1.1440	-285.8748	0.0176	0.4601	-666.0587	0.2515	0.9889	-191.0471	0.0154	0.3113	-287.6396
	7	0.2541	0.8301	-352.9870	0.0157	0.4965	-707.2772	0.1564	0.8306	-211.6488	0.0151	0.2763	-303.3373
	8	0.2598	0.8243	-350.0471	0.0147	0.4468	-715.4917	0.1712	0.8264	-209.5538	0.0153	0.3054	-301.9109
	9	0.2123	0.7981	-363.2108	0.0139	0.3727	-737.5017	0.1494	0.8049	-208.6444	0.0145	0.2908	-322.1198
	10	0.1689	0.6867	-431.7466	0.0140	0.3423	-741.1187	0.0777	0.6876	-234.7029	0.0147	0.3273	-322.7747
long	m = 1	0.9592	1.4927	-276.1265	0.1427	0.7041	-457.0393	0.6477	1.5144	-19.7231	0.1625	0.7837	-81.1802
	2	0.8537	1.5054	-290.7995	0.1412	0.7634	-461.5068	0.5376	1.2175	-75.1321	0.0831	0.6087	-134.6725
	3	0.5741	1.1827	-305.7377	0.0587	0.5030	-577.0707	0.4403	1.1901	-73.6370	0.0304	0.4194	-161.2092
	4	0.5195	1.1723	-305.4890	0.0352	0.4124	-650.6168	0.4040	1.1362	-71.5996	0.0181	0.3407	-175.4327
	5	0.5437	1.1499	-316.4177	0.0202	0.5142	-712.7704	0.4037	1.1161	-75.9181	0.0163	0.2860	-173.7036
	6	0.3939	1.1330	-342.3277	0.0174	0.5139	-715.0248	0.2934	1.1073	-83.7866	0.0162	0.2584	-172.3097
	7	0.2843	0.7726	-389.4486	0.0164	0.5020	-738.9568	0.1729	0.7282	-115.9588	0.0161	0.2562	-172.3147
	8	0.2645	0.8074	-388.3418	0.0149	0.4498	-752.1183	0.1761	0.7310	-112.9432	0.0157	0.2529	-173.2267
	9	0.2190	0.8126	-399.5928	0.0135	0.4104	-776.7271	0.1298	0.7473	-119.9519	0.0151	0.3047	-211.1517
	10	0.1824	0.7302	-469.7740	0.0133	0.4090	-794.6761	0.0583	0.6366	-148.1731	0.0146	0.3441	-216.6852

Note. HQC is the estimated value of the Hannan-Quinn information criterion defined by $HQC(n,k,m) = n\log(\mathrm{SSR}(k,m)/n) + 2(k+1)(m+1)\log(\log(n))$, with $k = 1$. The critical values for testing the null of time-varying cointegration for each model and test statistic (TV-KPSS and TV-XP) are given in Table D.1

Table E.2. Tests for cointegration in time-varying regressions with a constant term

Country		Credits for house purchase						Loans for consumption					
Finland		Model 2. Fixed intercept, TV slope			Model 3. TV intercept and slope			Model 2. Fixed intercept, TV slope			Model 3. TV intercept and slope		
Maturity		TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC
Short	m = 1	0.1517	0.6966	-399.6078	0.1429	0.6844	-403.5275	0.6413	1.3403	-274.8459	0.1640	0.9684	-362.1959
	2	0.1337	0.7013	-401.9850	0.1296	0.6012	-406.9145	0.3584	1.3990	-345.0941	0.0669	0.5309	-495.7892
	3	0.0681	0.6524	-492.9469	0.0499	0.5811	-520.2213	0.3085	1.2211	-362.4214	0.0378	0.6685	-540.4210
	4	0.0509	0.7058	-500.4465	0.0371	0.4469	-543.0967	0.3321	1.2756	-361.2184	0.0285	0.5554	-546.6198
	5	0.0481	0.6469	-501.4704	0.0253	0.3897	-596.9699	0.1783	0.8125	-412.2612	0.0253	0.4716	-553.2159
	6	0.0453	0.6447	-503.3490	0.0196	0.3816	-611.3773	0.1347	0.8311	-448.9335	0.0243	0.4470	-557.5370
	7	0.0537	0.6834	-533.0356	0.0161	0.3296	-672.1877	0.1448	0.8075	-466.3018	0.0155	0.3323	-603.9686
	8	0.0490	0.6448	-570.7725	0.0142	0.3129	-697.3619	0.1329	0.7760	-465.2405	0.0154	0.3432	-601.8057
	9	0.0477	0.6417	-567.9272	0.0128	0.3168	-707.5506	0.1333	0.7757	-461.9653	0.0142	0.2984	-613.9475
	10	0.0251	0.5240	-631.5737	0.0122	0.5473	-765.5951	0.1395	0.8041	-459.9964	0.0110	0.2630	-637.6453
medium	m = 1	0.2130	0.7402	-370.3123	0.1414	0.7373	-390.2133	0.5233	1.3420	35.9286	0.0929	0.7886	-39.5770
	2	0.1656	0.8403	-382.6126	0.1237	0.5883	-394.6929	0.2519	1.2992	-7.7112	0.0609	0.9892	-64.3032
	3	0.0608	0.6908	-475.7831	0.0492	0.6106	-494.9389	0.2097	1.2392	-13.9615	0.0626	0.9954	-63.3177
	4	0.0767	0.7369	-473.7631	0.0388	0.4683	-520.8632	0.1928	1.1860	-11.9993	0.0531	0.8709	-70.6154
	5	0.0782	0.7099	-471.2626	0.0314	0.4350	-562.7376	0.0913	0.7901	-36.9571	0.0368	0.7500	-94.1883
	6	0.0746	0.7046	-469.6810	0.0229	0.3506	-596.1270	0.0845	0.7965	-36.3828	0.0244	0.6429	-147.0704
	7	0.1061	0.7443	-494.9428	0.0163	0.3253	-719.7410	0.0846	0.7646	-37.5754	0.0215	0.5890	-174.0815
	8	0.0861	0.7138	-533.8477	0.0147	0.2956	-737.2003	0.0625	0.7603	-55.9874	0.0172	0.5535	-199.2611
	9	0.0806	0.7269	-532.0614	0.0133	0.3007	-748.3447	0.0558	0.7361	-81.2319	0.0150	0.6051	-223.2643
	10	0.0702	0.6319	-612.1995	0.0127	0.5461	-827.4028	0.0579	0.6795	-88.7836	0.0125	0.6137	-246.1418
long	m = 1	0.2815	0.8457	-304.2661	0.1242	0.6529	-345.4383	0.4812	1.5381	185.4967	0.1173	0.8749	126.3500
	2	0.2336	0.9789	-313.5978	0.1145	0.5677	-345.5378	0.2365	1.2598	143.3876	0.0948	0.8646	104.9462
	3	0.1584	0.8312	-390.9413	0.0574	0.6242	-427.4373	0.2517	1.3006	146.0306	0.0743	0.8773	96.8749
	4	0.1250	0.7882	-389.1363	0.0433	0.4508	-461.8645	0.1887	1.2486	133.0501	0.0330	0.6060	46.1198
	5	0.1254	0.7840	-385.9457	0.0351	0.4290	-533.1887	0.1042	0.9268	122.1609	0.0186	0.4065	27.7893
	6	0.1228	0.7780	-383.2757	0.0254	0.3332	-572.9655	0.1011	0.8686	123.2786	0.0145	0.5146	20.3037
	7	0.1663	0.8256	-403.4467	0.0162	0.3046	-769.0209	0.1005	0.8965	121.6133	0.0137	0.5323	21.6434
	8	0.1443	0.8105	-450.2007	0.0150	0.2918	-784.4111	0.0717	0.7526	107.5369	0.0143	0.5445	20.8311
	9	0.1203	0.8546	-455.0972	0.0136	0.2828	-792.8693	0.0691	0.7612	106.3521	0.0146	0.5569	22.7175
	10	0.1318	0.7597	-513.7703	0.0128	0.4988	-883.4579	0.0732	0.8400	103.5313	0.0138	0.5409	20.9499

Note. HQC is the estimated value of the Hannan-Quinn information criterion defined by $HQC(n,k,m) = n\log(\mathrm{SSR}(k,m)/n) + 2(k+1)(m+1)\log(\log(n))$, with $k = 1$. The critical values for testing the null of time-varying cointegration for each model and test statistic (TV-KPSS and TV-XP) are given in Table D.1

Table E.2. Tests for cointegration in time-varying regressions with a constant term

Country		Credits for house purchase						Loans for consumption					
France		Model 2. Fixed intercept, TV slope			Model 3. TV intercept and slope			Model 2. Fixed intercept, TV slope			Model 3. TV intercept and slope		
Maturity		TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC
Short	m = 1	0.4402	1.0069	-238.0710	0.1946	0.9422	-306.6659	0.5387	1.5423	-80.1440	0.2608	0.9036	-146.9802
	2	0.3718	1.2077	-259.8456	0.1654	0.7971	-320.8163	0.1907	1.0538	-161.9495	0.1321	0.6989	-201.6628
	3	0.3834	1.1803	-364.6054	0.0562	0.5329	-476.7271	0.1399	0.6469	-271.8893	0.0918	0.5552	-336.7957
	4	0.3099	1.0596	-363.8852	0.0386	0.4355	-553.3542	0.0970	0.8505	-305.3118	0.0379	0.4576	-417.0800
	5	0.3111	1.0307	-370.0457	0.0275	0.4379	-603.2606	0.1551	0.8469	-324.1537	0.0278	0.4176	-459.8406
	6	0.3017	1.0245	-369.6747	0.0194	0.3697	-616.0507	0.1151	0.7389	-353.4380	0.0239	0.3870	-485.7253
	7	0.2896	1.0391	-369.2190	0.0122	0.2992	-640.6168	0.1162	0.7449	-352.6902	0.0227	0.4021	-493.3991
	8	0.2837	0.9999	-417.5418	0.0121	0.3077	-637.4841	0.0957	0.7588	-368.2403	0.0122	0.3260	-550.4998
	9	0.2329	1.0261	-436.1980	0.0118	0.2984	-636.3234	0.0985	0.7547	-366.0114	0.0097	0.3381	-562.0219
	10	0.2354	0.9320	-441.6805	0.0110	0.2589	-641.2921	0.1032	0.7688	-426.9554	0.0094	0.2919	-567.0266
medium	m = 1	0.4746	1.0806	-242.6087	0.2233	1.1196	-334.5960	0.2424	0.9821	-328.0875	0.2404	0.9644	-328.1142
	2	0.4059	1.3077	-287.1995	0.1776	0.8614	-364.2656	0.2751	0.9562	-391.8681	0.0894	0.5091	-440.1651
	3	0.4902	1.3246	-417.3118	0.0630	0.5069	-573.8839	0.2683	0.9583	-388.6569	0.0715	0.5633	-475.2030
	4	0.3484	1.1054	-422.9995	0.0480	0.4310	-701.8762	0.1688	0.9381	-441.9564	0.0585	0.5964	-492.0719
	5	0.3483	1.1487	-423.5605	0.0232	0.3319	-771.9683	0.1837	0.9747	-438.9337	0.0394	0.4496	-587.8502
	6	0.3373	1.1130	-424.5375	0.0189	0.3308	-771.3876	0.1917	1.0306	-437.0485	0.0286	0.4247	-643.9389
	7	0.3288	1.1075	-422.0236	0.0131	0.4121	-786.7734	0.1887	0.9985	-437.1250	0.0176	0.3257	-693.1846
	8	0.3078	1.0781	-450.8141	0.0131	0.3550	-785.2585	0.2000	0.9986	-435.0887	0.0151	0.3502	-695.1711
	9	0.2604	1.0967	-464.6948	0.0130	0.3744	-783.8521	0.2185	0.9963	-444.6288	0.0123	0.2962	-711.1100
	10	0.2586	1.0182	-467.6460	0.0130	0.3712	-781.6353	0.2277	0.9756	-460.0425	0.0119	0.2829	-710.0984
long	m = 1	0.5046	1.1686	-340.4669	0.2327	1.1265	-452.2350	0.1994	0.8371	-303.3133	0.2020	0.9055	-303.7111
	2	0.4512	1.3563	-403.6018	0.1640	0.8105	-503.3346	0.2529	0.9771	-344.6633	0.0717	0.5293	-391.8941
	3	0.5681	1.4050	-508.6121	0.0612	0.5072	-677.7499	0.2452	0.9814	-341.4875	0.0672	0.5406	-403.7179
	4	0.3478	1.1040	-527.4378	0.0405	0.4275	-742.6834	0.1620	0.9567	-380.9035	0.0525	0.5553	-426.9200
	5	0.3459	1.0851	-524.6387	0.0179	0.3875	-776.7483	0.1358	0.8918	-378.8631	0.0356	0.3828	-503.9608
	6	0.3388	1.0968	-522.4715	0.0194	0.3881	-775.3036	0.1579	1.0472	-385.3209	0.0278	0.4123	-562.8369
	7	0.3352	1.0973	-519.3562	0.0165	0.4988	-782.2095	0.1561	1.0197	-384.5309	0.0150	0.2897	-651.7362
	8	0.3068	1.0214	-535.6892	0.0159	0.4371	-781.5165	0.1748	1.0240	-386.0044	0.0112	0.2276	-664.6872
	9	0.2591	1.0130	-557.6636	0.0157	0.4369	-779.0287	0.1805	1.0097	-386.8335	0.0091	0.2597	-679.5562
	10	0.2556	0.9513	-556.7372	0.0160	0.4139	-777.1447	0.1957	1.0193	-425.8239	0.0091	0.2573	-691.5234

Note. HQC is the estimated value of the Hannan-Quinn information criterion defined by $HQC(n,k,m) = n\log(\mathrm{SSR}(k,m)/n) + 2(k+1)(m+1)\log(\log(n))$, with $k = 1$. The critical values for testing the null of time-varying cointegration for each model and test statistic (TV-KPSS and TV-XP) are given in Table D.1

Table E.2. Tests for cointegration in time-varying regressions with a constant term

Country		Credits for house purchase						Loans for consumption					
Germany		Model 2. Fixed intercept, TV slope			Model 3. TV intercept and slope			Model 2. Fixed intercept, TV slope			Model 3. TV intercept and slope		
Maturity		TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC
Short	m = 1	0.5406	1.2599	-263.0508	0.1431	0.7109	-410.2262	0.1986	1.1860	-157.0713	0.1986	1.1868	-157.0714
	2	0.4948	1.4051	-273.4850	0.1355	0.6748	-415.4084	0.2416	1.2818	-181.9103	0.1098	0.8064	-225.2593
	3	0.5142	1.3665	-329.6471	0.0587	0.6398	-513.0534	0.1847	1.1414	-209.8967	0.0796	0.6092	-269.1327
	4	0.2900	1.0507	-356.4809	0.0433	0.5261	-561.9621	0.2188	1.2595	-218.1050	0.0366	0.5718	-349.9416
	5	0.2889	1.0576	-353.3472	0.0279	0.4212	-653.8802	0.1054	0.9707	-272.3718	0.0265	0.5685	-404.5317
	6	0.2804	1.0353	-352.1798	0.0167	0.3238	-718.1553	0.0621	0.8284	-310.3117	0.0235	0.5139	-405.7443
	7	0.3028	1.0586	-351.5423	0.0143	0.3423	-742.7004	0.0550	0.7897	-318.5761	0.0231	0.4735	-406.7572
	8	0.2880	1.0396	-387.9432	0.0138	0.2944	-758.5945	0.0570	0.7934	-319.3635	0.0147	0.2861	-474.4178
	9	0.2453	1.0527	-399.2545	0.0129	0.3093	-762.1545	0.0568	0.7975	-316.1274	0.0104	0.3383	-494.9066
	10	0.2459	0.9488	-408.3747	0.0122	0.2939	-807.5219	0.0513	0.7476	-321.4031	0.0106	0.3082	-496.0923
medium	m = 1	0.6064	1.3791	-246.9386	0.2111	0.9679	-417.3691	0.4224	1.2874	-386.4556	0.1493	0.7955	-454.6440
	2	0.5609	1.5185	-278.3636	0.1685	0.8005	-459.6959	0.4165	1.2819	-383.1754	0.0933	0.6120	-524.5963
	3	0.6280	1.4837	-344.7926	0.0667	0.5286	-717.9006	0.2323	1.0083	-501.8537	0.0420	0.5588	-578.5874
	4	0.3772	1.1484	-385.8735	0.0452	0.4078	-851.5763	0.1245	0.7393	-551.5789	0.0238	0.5145	-617.7367
	5	0.3770	1.1530	-386.6826	0.0201	0.4257	-1005.7446	0.0760	0.6731	-553.5919	0.0247	0.5388	-621.4684
	6	0.3648	1.1330	-388.3252	0.0185	0.3497	-1007.6578	0.0772	0.5650	-553.7348	0.0173	0.5599	-638.2931
	7	0.3559	1.1323	-385.9008	0.0166	0.3409	-1050.8041	0.0643	0.4902	-573.8952	0.0160	0.5384	-641.4065
	8	0.3302	1.0605	-407.6466	0.0152	0.3412	-1076.5100	0.0741	0.5243	-573.7967	0.0144	0.5217	-650.1784
	9	0.2834	1.0852	-442.6575	0.0133	0.2789	-1098.4979	0.0765	0.5195	-575.0008	0.0125	0.4692	-662.9848
	10	0.2801	1.0149	-443.4316	0.0126	0.4046	-1190.6930	0.0843	0.6097	-578.3956	0.0124	0.4633	-673.0394
long	m = 1	0.6778	1.5454	-329.2332	0.2253	0.8458	-634.6166	0.7581	1.3597	-399.8182	0.1074	0.6192	-496.9619
	2	0.6606	1.5516	-368.0696	0.1315	0.6893	-727.6561	0.4860	1.4678	-427.0635	0.0620	0.5042	-516.0585
	3	0.6626	1.4808	-387.8647	0.0851	0.6663	-910.6462	0.3818	1.3811	-466.3692	0.0269	0.5292	-541.3408
	4	0.3733	1.1275	-466.2976	0.0465	0.5319	-1057.0795	0.3343	1.2730	-469.0670	0.0226	0.5561	-541.6830
	5	0.3729	1.1241	-463.1076	0.0317	0.4507	-1146.8539	0.1573	0.8132	-497.0725	0.0233	0.5459	-538.9567
	6	0.3737	1.1240	-459.9071	0.0196	0.3657	-1237.7063	0.1359	0.8253	-497.5209	0.0223	0.5083	-539.5657
	7	0.3597	1.1316	-459.3527	0.0179	0.3381	-1268.5282	0.1381	0.8054	-495.0973	0.0149	0.4445	-551.0601
	8	0.3299	1.0554	-477.4433	0.0165	0.3566	-1279.2273	0.1245	0.7252	-495.0609	0.0126	0.4939	-552.9516
	9	0.2804	1.0342	-510.3423	0.0149	0.3396	-1293.8650	0.1282	0.8212	-499.3882	0.0119	0.4567	-553.3320
	10	0.2780	1.0103	-507.4669	0.0151	0.4449	-1360.4082	0.1229	0.7756	-503.4376	0.0105	0.3985	-557.7708

Note. HQC is the estimated value of the Hannan-Quinn information criterion defined by $HQC(n,k,m) = n\log(\mathrm{SSR}(k,m)/n) + 2(k+1)(m+1)\log(\log(n))$, with $k = 1$. The critical values for testing the null of time-varying cointegration for each model and test statistic (TV-KPSS and TV-XP) are given in Table D.1

Table E.2. Tests for cointegration in time-varying regressions with a constant term

Country		Credits for house purchase						Loans for consumption					
Italy		Model 2. Fixed intercept, TV slope			Model 3. TV intercept and slope			Model 2. Fixed intercept, TV slope			Model 3. TV intercept and slope		
Maturity		TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC
Short	m = 1	0.2802	0.9778	-194.5226	0.1595	0.7701	-215.0331	0.4067	1.5528	42.5701	0.1610	1.0974	-37.0996
	2	0.3025	1.1184	-196.0750	0.1153	0.6826	-240.4917	0.4678	1.5089	37.2301	0.0999	0.8245	-115.2457
	3	0.3830	1.3485	-212.7152	0.0481	0.5565	-306.6385	0.4470	1.3977	12.0736	0.0464	0.6361	-157.3353
	4	0.1041	0.7016	-288.5038	0.0370	0.4794	-338.0063	0.2389	1.2799	-10.3322	0.0293	0.5061	-207.7046
	5	0.0982	0.6891	-286.4423	0.0271	0.5148	-375.0906	0.2657	1.2473	-21.7182	0.0228	0.5080	-222.6727
	6	0.0990	0.7002	-283.2679	0.0207	0.3987	-386.5916	0.2561	1.2278	-20.3237	0.0193	0.4783	-253.6854
	7	0.0836	0.7169	-305.7507	0.0134	0.5225	-413.6370	0.2547	1.2270	-17.1350	0.0141	0.4010	-281.0538
	8	0.0872	0.7345	-302.7312	0.0134	0.5198	-410.6964	0.2272	1.1926	-35.7853	0.0141	0.4110	-280.3427
	9	0.0600	0.5954	-305.8175	0.0131	0.5623	-411.9126	0.2072	1.1843	-34.5542	0.0141	0.3758	-285.5941
	10	0.0684	0.6535	-324.5373	0.0123	0.5288	-416.7729	0.2074	1.0959	-44.7271	0.0134	0.4028	-304.9342
medium	m = 1	0.2960	0.8668	-339.5221	0.1721	0.8181	-366.1071	0.1522	0.6891	-223.0528	0.1188	0.5970	-229.5852
	2	0.2584	0.9701	-344.8618	0.1640	0.7285	-371.7882	0.1059	0.7447	-232.6272	0.0948	0.5807	-234.7863
	3	0.1786	0.8178	-480.9173	0.0539	0.5111	-571.6701	0.0710	0.6136	-259.7445	0.0674	0.5328	-261.1392
	4	0.1879	0.8452	-477.8016	0.0422	0.4699	-623.0589	0.0879	0.7749	-260.8696	0.0439	0.4577	-290.6180
	5	0.1774	0.8767	-485.6871	0.0312	0.4510	-703.6536	0.0897	0.8024	-258.0814	0.0257	0.5479	-364.8322
	6	0.1631	0.8866	-505.2286	0.0242	0.3436	-741.0667	0.0911	0.7844	-256.0464	0.0180	0.5188	-396.2947
	7	0.1955	0.9088	-511.4706	0.0189	0.3440	-845.2505	0.1182	0.7981	-283.7627	0.0146	0.4468	-405.5350
	8	0.1705	0.8597	-532.2535	0.0162	0.3623	-859.3016	0.0800	0.7534	-319.3018	0.0125	0.4617	-420.1216
	9	0.1429	0.8926	-537.5601	0.0139	0.4285	-877.5955	0.0734	0.7468	-316.3463	0.0122	0.4348	-428.4904
	10	0.1472	0.7661	-578.6857	0.0145	0.5867	-898.5156	0.0736	0.7317	-315.0783	0.0118	0.4273	-429.8371
long	m = 1	0.3606	0.9876	-323.9181	0.1692	0.8378	-375.2126	0.4759	1.5996	-171.4382	0.1618	0.8439	-267.2800
	2	0.3134	1.1514	-333.5626	0.1603	0.7388	-378.8605	0.4860	1.6289	-168.6964	0.0930	0.7485	-314.3514
	3	0.2689	0.9922	-448.2277	0.0574	0.5334	-533.4669	0.4686	1.6176	-167.1982	0.0786	0.6142	-322.2929
	4	0.2331	0.9330	-445.8383	0.0437	0.4897	-606.2877	0.2081	1.1244	-237.2550	0.0421	0.4321	-403.5871
	5	0.2280	0.9911	-445.9371	0.0292	0.4107	-705.5798	0.2075	1.1093	-235.2090	0.0258	0.4004	-471.9399
	6	0.2160	0.9712	-453.6742	0.0215	0.3740	-742.6775	0.1885	1.0532	-250.8479	0.0153	0.3124	-492.4075
	7	0.2437	0.9954	-455.5953	0.0166	0.3093	-815.1000	0.1718	0.9789	-295.0498	0.0132	0.3630	-506.2128
	8	0.2217	0.9545	-485.3006	0.0152	0.3141	-832.2697	0.1851	1.0190	-292.8965	0.0128	0.3778	-504.3291
	9	0.1907	0.9750	-490.9297	0.0137	0.3187	-846.1873	0.1628	0.9712	-293.8814	0.0124	0.3394	-504.4999
	10	0.2102	0.8525	-530.6802	0.0130	0.3507	-965.1140	0.1569	0.9246	-301.7068	0.0121	0.3340	-502.7054

Note. HQC is the estimated value of the Hannan-Quinn information criterion defined by $HQC(n,k,m) = n\log(\mathrm{SSR}(k,m)/n) + 2(k+1)(m+1)\log(\log(n))$, with $k = 1$. The critical values for testing the null of time-varying cointegration for each model and test statistic (TV-KPSS and TV-XP) are given in Table D.1

Table E.2. Tests for cointegration in time-varying regressions with a constant term

Country		Credits for house purchase						Loans for consumption					
Spain		Model 2. Fixed intercept, TV slope			Model 3. TV intercept and slope			Model 2. Fixed intercept, TV slope			Model 3. TV intercept and slope		
Maturity		TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC	TV-KPSS	TV-XP	HQC
Short	m = 1	0.4763	1.6033	153.4968	0.1509	1.2560	75.4896	0.1686	1.3567	100.4133	0.1530	1.1965	98.2164
	2	0.4444	1.5590	151.2504	0.1014	1.0853	31.8462	0.1675	1.3325	103.6219	0.1016	1.0090	68.9558
	3	0.4461	1.5602	154.4370	0.0519	0.8584	-27.4212	0.2418	1.4299	73.6749	0.0690	0.8305	17.1809
	4	0.2023	1.4190	69.6855	0.0249	0.7776	-108.9168	0.1591	1.2637	43.2160	0.0403	0.7018	-22.8707
	5	0.1945	1.4262	42.4642	0.0190	0.7441	-122.1923	0.1387	1.2245	45.8605	0.0292	0.6441	-64.9522
	6	0.2173	1.4403	38.9835	0.0182	0.7410	-120.9006	0.1398	1.2227	49.1330	0.0207	0.6589	-84.6986
	7	0.2255	1.4359	41.2766	0.0175	0.7435	-118.5768	0.1432	1.2116	45.6756	0.0173	0.6444	-97.6375
	8	0.1966	1.3786	36.4177	0.0154	0.7543	-130.6157	0.1421	1.2089	48.9553	0.0133	0.6075	-126.0661
	9	0.1283	1.2606	10.5970	0.0142	0.7435	-139.8945	0.1539	1.2284	45.2467	0.0112	0.6389	-147.0973
	10	0.1322	1.2512	10.9373	0.0131	0.7139	-154.2280	0.1547	1.2551	28.5957	0.0110	0.6255	-146.0748
medium	m = 1	0.5271	1.5232	-156.3752	0.1926	0.8862	-269.6413	0.9036	1.6065	4.9044	0.1247	0.9803	-239.7878
	2	0.5328	1.5418	-153.3290	0.1038	0.5689	-345.0776	0.5926	1.7264	-60.4893	0.0495	0.6638	-292.6962
	3	0.5722	1.6028	-153.5588	0.0531	0.5344	-414.2205	0.4884	1.7058	-87.9281	0.0464	0.5787	-293.6000
	4	0.2735	1.1419	-254.1531	0.0394	0.4700	-433.2878	0.4235	1.5961	-101.5690	0.0513	0.6349	-297.2366
	5	0.2745	1.1412	-250.9993	0.0302	0.5222	-500.1599	0.1942	1.1403	-195.8849	0.0321	0.4799	-328.7717
	6	0.2586	1.0469	-259.4659	0.0250	0.4395	-508.9778	0.1852	1.0924	-195.9008	0.0177	0.3777	-367.9185
	7	0.2464	1.0657	-281.8676	0.0149	0.4523	-591.5133	0.1870	1.1479	-197.6463	0.0143	0.3372	-373.8513
	8	0.2389	1.0493	-279.0116	0.0139	0.4341	-593.6545	0.1536	0.9654	-222.3737	0.0111	0.3662	-391.2608
	9	0.1926	1.0561	-292.4057	0.0132	0.4601	-598.7943	0.1829	1.0225	-256.9852	0.0112	0.3648	-393.2404
	10	0.1914	1.0457	-289.2714	0.0118	0.5039	-617.8109	0.1945	1.0651	-256.6258	0.0108	0.3583	-392.6574
long	m = 1	0.3089	1.0349	-160.8607	0.1341	0.6301	-202.0708	0.8825	1.7309	-40.5667	0.2653	1.0116	-265.6833
	2	0.2882	1.0822	-159.6850	0.1342	0.6340	-205.0324	0.4724	1.2642	-230.4716	0.1180	0.7441	-394.8664
	3	0.2298	0.9455	-238.8652	0.0661	0.5752	-305.2267	0.3164	1.0266	-337.0869	0.0765	0.6193	-454.0129
	4	0.1983	0.9090	-236.4764	0.0499	0.4996	-367.9295	0.2962	0.9933	-334.9990	0.0402	0.4601	-505.1551
	5	0.1937	0.9599	-235.4341	0.0387	0.3818	-542.1726	0.0946	0.6834	-439.8094	0.0306	0.4000	-529.8187
	6	0.1854	0.9452	-236.0680	0.0285	0.3511	-584.4334	0.0834	0.6270	-448.9268	0.0196	0.2860	-555.3038
	7	0.2264	0.9880	-245.5564	0.0185	0.4625	-864.5941	0.0681	0.5057	-467.4264	0.0127	0.4681	-572.2603
	8	0.2013	0.9597	-278.1756	0.0169	0.3916	-894.0956	0.0766	0.5325	-465.4136	0.0128	0.4736	-571.6939
	9	0.1656	1.0032	-286.8308	0.0143	0.2753	-941.0499	0.0767	0.5257	-462.1921	0.0129	0.4684	-570.1543
	10	0.1819	0.8950	-325.1211	0.0128	0.4760	-1039.3717	0.0914	0.6530	-480.9014	0.0124	0.4665	-567.9922

Note. HQC is the estimated value of the Hannan-Quinn information criterion defined by $HQC(n,k,m) = n\log(\mathrm{SSR}(k,m)/n) + 2(k+1)(m+1)\log(\log(n))$, with $k = 1$. The critical values for testing the null of time-varying cointegration for each model and test statistic (TV-KPSS and TV-XP) are given in Table D.1

Table E.3. Tests for cointegration in time-varying regressions with a constant term

Country		Credits for house purchase				Loans for consumption			
Austria		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope	
Maturity		MLH1	MLH2	MLH1	MLH2	MLH1	MLH2	MLH1	MLH2
Short	m = 1	0.0762	3.1098	-2.3777	2.6764	0.1764	2.4989	0.3895	1.1703
	2	0.5783	2.3863	-1.4094	2.6828	2.9735	0.9157	1.6157	-0.2775
	3	1.3021	0.5404	1.0589	-0.8693	2.2477	0.2391	0.7875	-1.8420
	4	1.5352	-0.7375	0.9439	-1.3911	3.2874	-0.2678	1.9788	-2.4817
	5	1.4762	-0.7110	1.2300	-1.8583	2.3686	-1.6127	1.9920	-2.5040
	6	1.5407	-0.8832	0.0828	-2.4531	2.4466	-2.1322	2.5683	-2.6128
	7	1.4436	-0.9103	0.5756	-2.1515	2.5262	-2.1315	2.7209	-2.5471
	8	1.0211	-1.4203	-0.0395	-1.4942	2.6134	-2.0967	2.3180	-2.8293
	9	0.9379	-1.5166	-0.2829	-0.7443	2.5080	-2.1497	3.4206	-2.3249
	10	0.2056	-2.1308	-0.1751	-0.5576	3.0401	-1.8715	3.7103	-1.7241
medium	m = 1	-1.7834	3.2455	-3.2488	3.1400	-1.0101	3.1276	0.0483	2.4419
	2	-0.5804	2.9578	-2.7400	3.2374	1.9757	2.1193	1.1019	1.7613
	3	1.2679	-0.1431	0.0379	-1.0212	-0.8235	1.0578	0.1455	-0.2077
	4	1.2543	-0.0672	0.5666	-1.4545	1.1227	-0.0582	1.5166	-1.8539
	5	1.4069	-0.0888	-0.8203	-2.9120	1.3094	-2.5052	1.5151	-2.1151
	6	1.2049	-0.6260	-1.8856	-3.0111	1.3262	-2.4413	1.8552	-1.8326
	7	1.2321	-0.6297	-1.2069	-2.5863	1.3485	-2.5820	1.8050	-1.7754
	8	0.9358	-1.2036	-0.3971	-2.1565	1.5780	-2.5702	1.5907	-1.7065
	9	0.8795	-1.2954	0.2065	-1.4753	1.3504	-2.9601	1.5600	-1.5021
	10	0.3647	-1.8375	0.5838	-1.3131	1.6483	-2.5250	1.8284	-1.1661
long	m = 1	0.5922	2.7063	-1.4264	2.1125	0.5755	3.2470	1.8481	2.7393
	2	0.7596	2.2717	-1.2452	2.2627	2.1189	1.5919	1.7322	1.4783
	3	1.4204	0.7343	1.4316	-0.5525	2.1752	1.3054	2.6764	0.2324
	4	1.6538	-0.5737	1.8814	-0.9836	2.6718	-0.0642	2.2864	-0.2554
	5	1.6185	-0.5631	1.4006	-1.6771	2.7500	0.0630	2.2625	0.0440
	6	1.6499	-0.6507	-0.0392	-2.7641	3.0099	-0.5223	2.4922	0.1854
	7	1.5666	-0.6840	-0.6376	-0.3854	2.9340	-0.5955	2.3506	0.3490
	8	1.1098	-1.2599	-0.3752	-0.4005	2.8855	-0.7530	2.1118	0.3456
	9	1.0984	-1.3147	0.1581	1.6648	2.9684	-0.5345	2.1242	0.6846
	10	0.4971	-1.9745	0.2305	1.5549	2.8776	-0.5050	2.1450	1.0481

Table E.3. Tests for cointegration in time-varying regressions with a constant term

Country		Credits for house purchase				Loans for consumption			
Belgium		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope	
Maturity		MLH1	MLH2	MLH1	MLH2	MLH1	MLH2	MLH1	MLH2
Short	m = 1	-1.3169	1.9465	-1.5859	2.1552	2.7233	0.9239	2.8948	1.1221
	2	-0.8237	0.4279	0.2991	1.3168	2.7159	0.9175	2.2571	0.9697
	3	-0.9750	0.2504	-2.1025	-1.2330	2.6695	0.9719	1.7971	-1.2138
	4	-0.9780	0.2446	-2.2097	-2.3039	2.2909	0.7816	1.6998	-1.3621
	5	-1.5586	-1.1040	1.8899	-1.5284	2.0839	-0.1865	1.5465	-1.4514
	6	-1.4680	-1.0266	2.1412	-1.1202	2.1814	-0.7749	1.7071	-1.3755
	7	-1.0838	-1.4963	2.4468	0.2537	2.1082	-1.0114	1.4811	-1.2189
	8	-1.5465	-2.3552	2.2513	0.1601	2.0805	-1.3338	0.9329	-0.7360
	9	-1.6877	-2.3472	2.1034	0.4349	2.0626	-1.3376	1.1452	-0.5702
	10	-0.9909	-2.4491	2.5200	0.1643	1.9970	-1.2408	1.0601	-0.0789
medium	m = 1	-0.5091	2.4556	-2.7982	2.7087	0.3676	1.9549	0.2703	1.3663
	2	0.5076	1.0455	0.2691	1.1726	1.0493	1.1881	2.0889	0.9857
	3	0.7858	0.3826	-2.6009	-1.3000	1.2518	0.7359	0.6411	-2.9304
	4	0.6392	0.4881	-1.3153	-2.5456	1.5718	0.9851	1.7091	-2.9131
	5	0.0600	-0.1940	1.8023	-2.2424	1.0096	-1.1453	2.1779	-2.9220
	6	-0.4897	-0.3211	2.1765	-2.0940	0.6225	-1.1869	2.3372	-3.0280
	7	-0.2927	-1.8613	2.6139	-0.1622	1.3329	-1.9752	2.9430	-3.2105
	8	-0.3612	-1.9100	2.5967	0.0208	1.2284	-2.1039	2.8609	-3.1253
	9	0.4311	-1.9633	2.4947	0.6457	1.5109	-1.9948	2.3955	-2.3378
	10	-0.4971	-1.3186	3.0253	0.5194	1.8391	-2.2646	2.5173	-1.9999
long	m = 1	0.5937	2.2972	0.4161	2.3747	-0.9774	2.4186	-2.3120	1.8017
	2	0.9958	1.7312	1.4046	1.9974	0.4484	0.4261	0.4006	-0.2596
	3	1.2993	0.6689	-0.7614	-0.8629	0.5948	0.0833	-0.3556	-1.8766
	4	1.3382	0.8587	0.8074	-1.5270	0.5286	0.0925	-0.0551	-2.0075
	5	0.7070	-0.2040	2.6267	-1.2437	-0.0166	-0.6682	-0.1228	-2.2094
	6	-0.1361	-0.4701	2.4474	-1.5458	-0.4232	-0.7661	-0.0286	-2.0533
	7	-0.4369	-2.1061	2.3917	-1.3702	-0.2515	-2.2411	0.2647	-2.2098
	8	-0.1538	-1.9776	2.1039	-0.9089	-0.3022	-2.2217	0.3883	-1.8617
	9	0.6906	-1.9433	1.2547	1.5817	0.4589	-2.1877	0.9515	0.4679
	10	0.2043	-0.8069	2.0035	1.7529	-0.1320	-1.1941	0.6205	0.9078

Table E.3. Tests for cointegration in time-varying regressions with a constant term

Country		Credits for house purchase				Loans for consumption			
Finland		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope	
Maturity		MLH1	MLH2	MLH1	MLH2	MLH1	MLH2	MLH1	MLH2
Short	m = 1	-0.9041	1.8543	-0.9622	1.9462	0.2099	2.7118	-1.7709	1.1091
	2	-0.3195	1.7968	-0.7521	1.5795	3.0368	2.3588	-0.7881	-0.2417
	3	0.7244	-0.8710	0.8883	-1.0677	1.8431	2.2149	-1.9490	-1.4068
	4	0.4050	-1.0586	0.4393	-1.4698	2.1181	2.1626	-1.5902	-1.9167
	5	0.3215	-1.1776	1.0438	-1.9494	-0.0605	0.7083	-1.6698	-2.1376
	6	0.7566	-1.1857	-0.1141	-2.0753	1.5604	-1.0840	-1.7538	-2.0848
	7	0.0782	-1.6985	0.5427	-1.8229	1.1629	-1.7252	-2.2780	-2.5867
	8	0.5467	-1.9732	-0.2823	-1.4019	1.1214	-2.0660	-2.5528	-2.4671
	9	0.3572	-2.0306	-0.5677	-0.8661	1.1213	-2.0734	-1.8680	-2.1883
	10	0.3074	-2.6609	0.3053	0.1448	1.2004	-1.9804	-0.9911	-0.7924
medium	m = 1	-0.9071	2.0278	-1.3874	1.6886	2.3415	2.2818	3.1797	-0.4083
	2	-0.0429	1.5686	-0.5926	1.3852	2.7571	1.1340	3.4250	-0.7564
	3	0.9682	-1.3239	1.2672	-1.0727	2.8677	0.9671	3.4321	-0.8212
	4	0.9130	-1.2692	1.2289	-1.6384	2.8213	0.8534	3.2719	-1.3233
	5	0.8903	-1.2936	1.7231	-2.0489	3.0442	-0.4769	2.8295	-2.0164
	6	1.1100	-1.2785	-0.2932	-2.5136	3.0934	-0.6245	2.5370	-2.0293
	7	0.6188	-1.7594	0.3227	-1.9745	3.0048	-0.6980	2.5304	-1.7945
	8	0.7518	-2.0792	-0.4942	-1.5443	2.7631	-1.3346	2.4978	-1.6075
	9	1.0984	-1.9687	-0.5656	-1.0005	2.6180	-1.7982	2.6373	-1.3193
	10	0.4382	-2.9126	-0.0305	0.3740	2.6535	-1.8309	2.7150	-0.8854
long	m = 1	-0.1723	2.3429	-1.0893	1.5713	2.2337	2.1683	2.6780	0.8059
	2	0.3612	1.7307	-0.4025	1.4295	2.5777	1.5700	2.6217	0.5315
	3	1.1808	-0.8605	1.4943	-0.6171	2.5820	1.5488	2.5705	-0.0197
	4	1.2099	-1.0467	1.3068	-1.4825	2.6450	0.8594	2.4880	-1.7265
	5	1.2063	-1.0447	1.8975	-2.3256	2.7385	0.2379	2.4492	-2.0135
	6	1.3152	-1.0155	-0.3991	-2.8706	2.8069	0.1812	2.4156	-1.5109
	7	0.8678	-1.5097	-0.4189	-2.5928	2.7168	-0.1315	2.4090	-1.3611
	8	0.5951	-2.0130	-1.4562	-2.1699	2.7892	-0.9524	2.4924	-1.4764
	9	0.9979	-1.9566	-0.9128	-1.7054	2.7805	-0.9846	2.5561	-1.4574
	10	0.5870	-2.6346	-0.3595	0.4248	2.8304	-0.6447	2.4655	-1.3822

Table E.3. Tests for cointegration in time-varying regressions with a constant term

Country		Credits for house purchase				Loans for consumption			
France		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope	
Maturity		MLH1	MLH2	MLH1	MLH2	MLH1	MLH2	MLH1	MLH2
Short	m = 1	-0.8863	3.5097	-3.0622	2.2503	-1.0607	3.2590	0.7945	2.6622
	2	0.2739	2.7636	-2.3760	2.5333	1.3275	1.7955	0.5686	2.1450
	3	1.7457	0.6348	-0.0724	-0.7228	-1.3778	1.2877	-0.5649	0.7657
	4	1.6092	0.2619	-1.0162	-0.9825	-0.1820	-0.5691	0.7966	-1.1967
	5	1.9190	-0.0365	-2.3410	-0.7417	0.9692	-0.2679	0.8359	-1.2043
	6	1.8305	-0.0305	-1.7045	-1.1627	1.8261	-1.3431	0.4390	-1.0104
	7	2.0875	0.0596	-2.3541	0.4058	1.5950	-1.3153	1.9091	-0.8390
	8	1.6297	-0.7187	-2.3361	0.4090	1.8918	-1.6653	-0.6586	0.9381
	9	1.2227	-1.1716	-2.1469	0.3019	1.5896	-1.6611	-0.3609	1.3780
	10	0.9485	-1.2458	-2.1472	2.0929	1.1764	-2.3554	-0.1870	1.1058
medium	m = 1	-1.1466	3.6965	-3.4241	2.5606	1.6398	3.0662	1.6571	3.0556
	2	0.4952	2.8499	-2.9039	2.9759	2.1614	1.9203	3.1853	0.8489
	3	2.0608	0.9461	1.1804	-0.9513	2.2029	1.8569	2.5679	0.4391
	4	2.0079	0.1184	0.7782	-0.6821	2.7422	-1.3484	3.1404	-0.4585
	5	2.2390	-0.0926	0.7671	-0.8064	2.7210	-1.2172	1.3340	-1.5905
	6	2.2011	-0.2723	0.4560	-1.1161	2.8169	-1.0676	-0.0471	-1.4924
	7	2.3076	-0.2543	0.5562	-0.3186	2.7225	-1.1417	0.6272	-1.6312
	8	1.8859	-1.0513	0.6293	-0.6426	2.5766	-1.2145	0.2601	-1.5144
	9	1.2768	-1.4519	0.6730	-0.3547	2.3455	-1.4482	0.5103	-0.5869
	10	0.8893	-1.7416	0.6783	-0.3003	1.8906	-1.9507	0.4193	-0.4419
long	m = 1	-0.3176	3.6517	-3.1651	2.5221	2.2435	2.8683	2.1809	2.8914
	2	1.0036	2.6956	-2.1914	2.7941	2.7374	2.1657	3.2440	1.0316
	3	2.0407	1.3408	1.1622	-0.7377	2.7941	2.1023	2.4664	0.7358
	4	2.0613	-0.1982	-0.0112	-1.5241	2.8020	0.8774	2.6471	0.3093
	5	2.0988	-0.2765	0.0766	-1.7635	2.8808	0.6220	0.4443	-1.3615
	6	2.1004	-0.3363	0.1341	-1.5295	3.1379	0.4734	-1.0929	-0.5923
	7	2.1293	-0.3189	0.2525	-0.8556	2.9975	0.4541	-0.3080	1.2488
	8	1.6809	-1.0386	0.1916	-0.9943	2.8371	0.3867	-0.4403	1.5122
	9	0.7709	-1.3879	0.2164	-1.0239	2.6491	0.2742	-0.3280	1.6241
	10	0.5145	-1.5386	0.2798	-1.0603	2.2829	0.3088	0.1222	1.3623

Table E.3. Tests for cointegration in time-varying regressions with a constant term

Country		Credits for house purchase				Loans for consumption			
Germany		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope	
Maturity		MLH1	MLH2	MLH1	MLH2	MLH1	MLH2	MLH1	MLH2
Short	m = 1	1.1599	2.8423	-1.6190	2.1808	3.9533	2.2624	3.9551	2.2626
	2	1.2406	2.2226	-0.9874	2.4130	4.1369	2.5404	3.1016	1.8614
	3	2.0064	1.1504	1.2281	-0.4002	3.4240	2.0180	2.6345	0.3045
	4	2.1631	-0.3112	0.7910	-1.0814	3.5087	1.2234	3.8646	-1.3678
	5	2.1842	-0.3344	1.9719	-1.6918	3.7565	-0.1239	4.1793	-1.5863
	6	2.2777	-0.4315	0.1414	-1.7638	3.3351	-1.1016	4.0200	-1.7304
	7	2.1055	-0.4465	0.6459	-1.2767	3.4449	-1.5037	3.8049	-1.7430
	8	1.6934	-1.0545	-0.1037	-0.3471	3.7937	-1.6159	3.1965	-0.4513
	9	1.3553	-1.2607	0.9794	0.0047	3.7732	-1.6133	3.3714	0.4591
	10	0.7233	-1.6616	0.4534	1.9441	3.8673	-1.6700	3.4927	0.4055
medium	m = 1	1.7858	3.5347	-2.7063	2.6510	-0.3592	2.6825	0.6939	1.0756
	2	1.9499	2.5500	-2.2570	2.8594	-0.3014	2.6798	-1.1100	-1.1363
	3	2.6486	1.9446	2.1828	-0.6300	-0.1217	-0.6292	-0.0868	-2.1616
	4	2.3513	0.3581	0.0280	-1.6996	-0.4091	-2.1033	-0.2201	-2.4970
	5	2.4910	0.1421	-1.2141	-3.0768	-0.2806	-2.2353	-0.0245	-2.4095
	6	2.4961	-0.0443	-0.8837	-3.3023	-0.0848	-2.2467	-0.1902	-3.1815
	7	2.5836	-0.0285	-0.0402	-3.3573	-0.0746	-2.9955	-0.0035	-3.3036
	8	2.1903	-0.7104	-1.4065	-3.2884	0.1024	-3.1783	0.2066	-3.4546
	9	1.7149	-1.2239	1.0416	-3.4058	-0.0868	-3.5366	0.1949	-3.5595
	10	1.4331	-1.4189	-0.0809	-1.5531	0.5347	-3.6501	0.3127	-2.8163
long	m = 1	3.0167	3.4150	-1.0135	2.9537	0.5023	1.9849	-0.1749	-2.3289
	2	2.3863	2.5956	1.8565	1.7285	2.2642	0.8870	-0.0869	-2.6288
	3	2.2435	2.4711	3.1105	1.0634	2.1272	-0.0942	0.1293	-2.8691
	4	2.2891	0.0383	2.2069	-2.2451	1.7918	-0.4072	0.0934	-2.8516
	5	2.2902	0.0242	0.1259	-2.8502	1.1834	-2.2860	0.1477	-2.8178
	6	2.2930	0.0314	-0.0765	-3.1271	1.2965	-2.4235	0.0599	-3.0390
	7	2.4464	0.0675	0.4024	-2.7029	1.2393	-2.4675	-0.1306	-3.0517
	8	2.1133	-0.7125	-0.0554	-2.6871	1.0171	-2.3850	-0.1019	-2.8881
	9	1.3884	-1.3038	1.6477	-2.6176	1.2673	-2.6764	0.0374	-2.9827
	10	1.2892	-1.3608	2.0950	-0.4221	0.8696	-2.6840	-0.0013	-2.8689

Table E.3. Tests for cointegration in time-varying regressions with a constant term

Country		Credits for house purchase				Loans for consumption			
Italy		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope	
Maturity		MLH1	MLH2	MLH1	MLH2	MLH1	MLH2	MLH1	MLH2
Short	m = 1	0.2352	2.6407	-0.5564	2.1319	0.6830	1.7780	1.0961	1.3306
	2	1.5134	2.6089	0.3305	1.6733	0.8736	1.9033	0.1229	0.0423
	3	2.4778	2.1060	2.9467	-0.2929	1.1262	1.0104	0.9065	-1.7232
	4	2.6626	-0.1653	3.3084	-0.3508	2.1501	-0.6173	-0.2170	-2.3715
	5	2.7862	-0.3586	3.3179	-0.2251	1.6433	-0.5519	0.1712	-2.3786
	6	2.8273	-0.3332	3.2375	-0.3719	1.5445	-0.7480	-0.3403	-2.1131
	7	3.1769	-1.3305	3.2273	1.2630	1.5545	-0.7533	-0.9984	-1.6403
	8	3.2286	-1.3455	3.2290	1.2778	1.2551	-1.3349	-1.0009	-1.5930
	9	2.7829	-1.4801	3.3995	1.1723	1.1345	-1.4355	-1.2301	-1.9676
	10	3.0295	-0.8708	3.4775	1.4883	0.7093	-1.9754	-1.3762	-1.3133
medium	m = 1	-1.4120	3.1224	-2.4365	2.7494	0.8874	1.6777	0.4779	1.7719
	2	-0.6191	2.7321	-1.6929	2.9524	1.8491	1.1043	1.5125	1.1618
	3	0.1059	-0.5540	0.4734	-1.2120	2.1769	0.2501	1.9092	0.3134
	4	0.1454	-0.5041	0.9641	-1.6300	2.4783	-0.0036	0.8094	-1.9176
	5	0.7035	-0.8242	2.3431	-2.2884	2.5616	-0.0493	0.8760	-2.5254
	6	1.1122	-1.1389	-0.2671	-3.0108	2.4758	-0.1241	1.0295	-2.6097
	7	0.9294	-1.2539	0.1775	-2.2658	1.9941	-0.7861	1.5256	-2.4736
	8	0.5997	-1.6201	-0.8733	-2.1173	2.3561	-1.9093	1.0213	-2.0026
	9	0.8609	-1.6471	-0.5275	-0.9928	2.3292	-1.9973	1.3098	-1.8651
	10	0.4488	-2.4974	-0.3313	-0.9798	2.5681	-2.0092	1.1993	-1.2574
long	m = 1	-0.8372	2.9962	-2.5358	2.6903	2.0548	2.5951	2.0405	1.7536
	2	-0.0626	2.4866	-1.7401	2.9189	2.2214	2.6103	3.0306	0.8425
	3	1.0862	-0.3579	0.6800	-1.0563	2.1786	2.5652	2.2553	0.6118
	4	1.0588	-0.5555	0.8799	-1.4390	2.6146	0.7637	0.1812	-1.6626
	5	1.3192	-0.6903	1.5210	-2.2236	2.8314	0.7268	0.2701	-2.6669
	6	1.5481	-0.9282	-0.2050	-2.5307	1.8697	-0.1802	0.8808	-2.9016
	7	1.3724	-0.9769	0.4580	-2.7978	1.4004	-1.6407	0.9522	-2.7334
	8	0.9883	-1.4243	-0.6439	-2.5592	1.5140	-1.5793	1.1026	-2.7503
	9	1.1371	-1.4857	-0.0614	-2.3025	1.3860	-1.5958	1.2143	-2.7191
	10	0.4296	-2.1051	-0.0176	0.3828	1.0399	-1.9461	1.2101	-2.5587

Table E.3. Tests for cointegration in time-varying regressions with a constant term

Country		Credits for house purchase				Loans for consumption			
Spain		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope	
Maturity		MLH1	MLH2	MLH1	MLH2	MLH1	MLH2	MLH1	MLH2
Short	m = 1	3.2643	1.9082	3.4003	0.7352	1.7510	0.8582	1.6404	0.9119
	2	3.0483	1.6481	3.2072	0.1411	1.7263	0.8819	1.8899	0.8615
	3	3.0488	1.6369	2.8333	-1.1324	1.8251	0.4561	1.0908	-0.4891
	4	3.8438	0.5688	2.4460	-1.9081	2.3780	-0.9340	2.0370	-1.4006
	5	3.8539	0.3515	2.3338	-1.8015	2.3899	-1.0505	1.6200	-2.1775
	6	3.7967	0.3262	2.3371	-1.7374	2.3740	-1.0446	1.5880	-2.3158
	7	3.7445	0.2691	2.3445	-1.8096	2.3717	-1.2427	1.5974	-2.2982
	8	3.6495	-0.1216	2.2570	-1.7973	2.3732	-1.2478	1.1714	-1.9359
	9	3.0930	-1.0450	2.2054	-1.7274	2.2264	-1.2973	1.4020	-1.6999
	10	3.0292	-1.1062	2.0747	-1.5526	2.0792	-1.6275	1.3236	-1.5448
medium	m = 1	1.7169	3.1186	-1.3433	2.1844	0.9787	3.8182	1.7281	0.5790
	2	1.9120	3.1230	1.0045	0.5684	3.9309	3.1217	2.7110	-0.1700
	3	2.1005	3.0586	2.0343	-0.8299	3.1338	2.7722	2.7702	-0.3680
	4	2.5769	0.1177	1.9231	-1.7307	2.6125	2.6771	3.3296	-0.1066
	5	2.5335	0.1461	2.8168	-2.4034	2.8947	0.5050	2.7920	-0.9638
	6	2.0641	-0.6991	2.4539	-2.9633	3.1404	0.1842	2.6291	-1.1173
	7	2.2540	-1.5245	2.5533	-1.0138	3.5293	0.3045	2.7208	-0.9685
	8	2.1830	-1.5953	2.4960	-0.5307	2.7910	-1.0755	2.6475	-0.1151
	9	1.6614	-1.7428	2.7301	-0.7714	3.0776	-1.4384	2.8012	-0.3648
	10	1.6418	-1.8017	2.3603	0.3303	3.2479	-1.2047	2.8613	-0.0316
long	m = 1	0.0529	2.4166	-1.0542	2.1952	-2.3582	3.6021	0.5022	2.7735
	2	0.2508	2.2059	-1.0045	2.4139	1.8417	2.8127	1.0773	1.8056
	3	1.1340	-0.3533	1.4238	-0.2032	-1.6431	0.8457	0.8623	-0.3956
	4	1.1199	-0.5238	1.2777	-1.1002	-1.9685	0.9304	1.7768	-1.7310
	5	1.2404	-0.5745	1.5069	-2.4393	-1.9224	-2.7135	1.2779	-2.3893
	6	1.4459	-0.5887	0.8167	-3.5286	-1.4575	-2.7219	0.4386	-1.8441
	7	1.1801	-0.9424	-2.1542	-2.2989	-1.3516	-3.1679	0.7111	-1.5044
	8	0.8177	-1.5887	-1.3599	-2.9130	-1.3048	-3.0859	0.6497	-1.5813
	9	0.9323	-1.7443	1.3282	-3.7687	-1.3739	-3.0687	0.7281	-1.5706
	10	0.8198	-2.1637	2.1453	0.6857	-0.5750	-2.6784	0.6649	-1.4643

Appendix F. *Time-varying long-run interest rate pass-through estimates*

Figure F.1. Time-varying slope estimates for models 1 (left) and 2 (right): Credit for house purchase, short-term interest rate. Austria

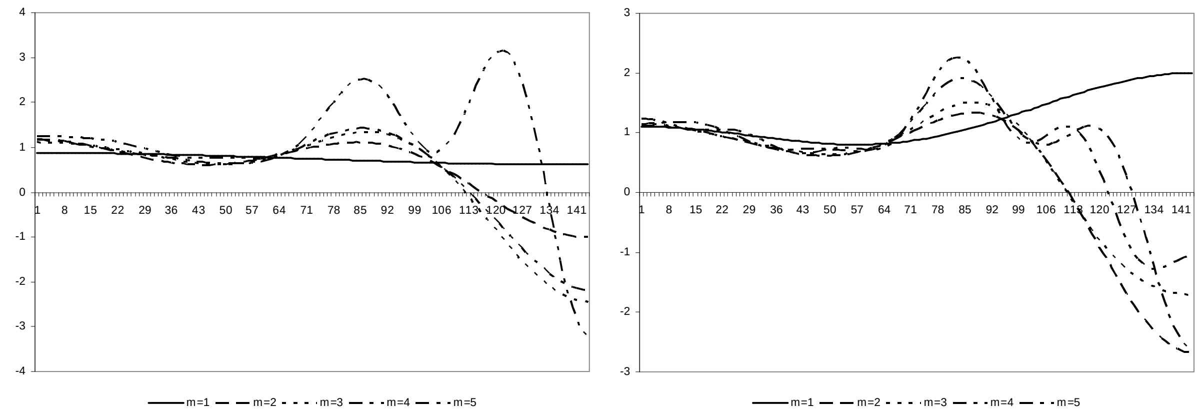


Table F.1. Sample averages and variances of the time-varying estimates of β

Country Austria		Credits for house purchase				Loans for consumption			
		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope	
Maturity									
up to 1 year	m = 1	0.7427	0.0934	1.2139	0.3984	0.4774	0.0144	0.7026	0.3384
	2	0.5451	0.6425	0.3293	1.1724	0.4867	0.1314	0.3886	0.1957
	3	0.3687	1.1039	0.4688	0.9680	0.5063	0.2273	0.6278	0.4707
	4	0.4396	1.0150	0.7521	0.9447	0.6470	0.5076	0.6626	0.5358
	5	1.1495	1.1724	0.7636	0.8677	0.6559	0.5067	0.7293	0.5816
	6	0.6152	0.4923	0.8290	0.5770	0.5957	0.0924	0.7189	0.2680
	7	0.5329	0.4131	0.7670	0.7188	0.4845	0.1894	0.4975	0.1969
	8	0.5129	0.5141	0.5908	0.4325	0.5717	0.2602	0.6451	0.2149
	9	0.4123	0.5082	0.3215	0.7144	0.6620	0.2222	0.6172	0.2708
	10	0.4742	0.4888	0.1109	0.4798	0.5868	0.3066	0.4275	0.4727
over 1 and up 5 years	m = 1	0.1766	0.1541	0.6220	0.3265	0.5789	0.0544	1.0032	0.6615
	2	-0.4690	1.3384	-0.4521	1.2982	0.5193	0.2368	0.3287	0.4347
	3	-0.5112	1.3981	-0.2784	1.1106	0.5457	0.3540	0.4773	0.2550
	4	-0.2687	1.1049	0.0187	0.9024	0.5206	0.3136	0.5659	0.3882
	5	0.2327	0.8782	-0.1917	1.2108	0.5475	0.3340	0.6417	0.4765
	6	0.0616	0.3746	0.2703	0.4821	0.8087	0.1529	1.0090	0.4118
	7	-0.3562	1.1959	0.1228	0.5654	0.5343	0.1804	0.5673	0.4141
	8	-0.0207	0.2919	0.0279	0.2346	0.6663	0.1630	0.7632	0.1369
	9	-0.0257	0.4086	0.0333	0.2425	0.7554	0.1402	0.8146	0.4062
	10	0.1106	0.1358	0.0159	0.2249	0.7683	0.2702	0.8710	0.4258
over 5 years	m = 1	0.6508	0.0691	0.9620	0.2573	0.4044	0.0670	0.6584	0.3134
	2	0.3177	0.7302	0.0679	1.3440	0.4122	0.1104	0.2653	0.3170
	3	0.0961	1.2949	0.1664	1.1965	0.2835	0.2669	0.1250	0.5897
	4	0.1394	1.2436	0.5174	1.1486	0.1546	0.5356	0.2338	0.4305
	5	0.9963	1.4081	0.4970	1.0734	0.2206	0.4635	0.2483	0.4557
	6	0.5194	0.5349	0.5481	0.4526	0.4844	0.1392	0.6354	0.2902
	7	0.2769	0.4441	0.5503	0.7864	0.2898	0.1059	0.3916	0.1658
	8	0.2691	0.5324	0.4373	0.4393	0.4531	0.2349	0.6111	0.3453
	9	0.1863	0.5754	0.1639	0.5451	0.5442	0.3645	0.6697	0.6032
	10	0.3615	0.3385	0.0760	0.3852	0.5489	0.4541	0.6621	0.7135

Note. For the results of the estimation of each model, the first column contains the estimated value of the fixed long-run IRPT computed as the sample average of $\hat{\beta}_t(m)$, $t = 1, \ldots, n$, $\bar{\beta}_n(m) = n^{-1}\sum_{t=1}^{n}\hat{\beta}_t(m) = \hat{b}_{0,n}$, and the second column indicates the sample variability computed as $\hat{\sigma}_n(m) = \sqrt{n^{-1}\sum_{t=1}^{n}(\hat{\beta}_t(m) - \bar{\beta}_n(m))^2} = \sqrt{\sum_{j=1}^{m}\hat{b}_{j,n}^2}$.

Figure F.2. Time-varying slope estimates for models 1 (left) and 2 (right): Credit for house purchase, short-term interest rate. Belgium

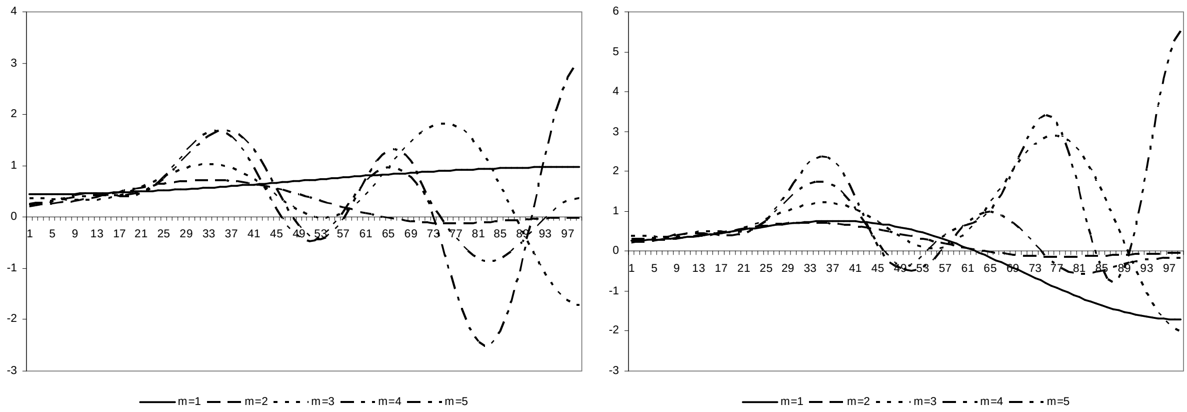


Table F.2. Sample averages and variances of the time-varying estimates of β

Country		**Credits for house purchase**				**Loans for consumption**			
Belgium		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope	
Maturity									
up to 1 year	m = 1	0.7110	0.1904	-0.0740	0.8411	0.6493	0.2095	0.5372	0.1047
	2	0.2675	0.3034	0.2593	0.3076	0.4414	0.1774	0.1688	0.5315
	3	0.5031	0.7851	0.7810	1.0556	0.4323	1.2322	2.1233	3.1019
	4	0.3666	1.1811	0.3504	0.6416	2.1804	3.3491	2.1684	3.8852
	5	0.3344	0.6597	1.1689	1.3265	2.1626	3.8470	3.1183	4.2729
	6	0.5744	0.2126	0.3998	0.4276	0.5502	0.2256	0.3092	0.1232
	7	0.4939	0.8812	0.3017	0.7514	0.1440	0.5934	0.4985	0.2574
	8	0.4771	0.4069	0.3432	0.4241	0.5294	0.2381	0.5268	0.9357
	9	0.1467	0.4333	0.0587	0.6167	0.1077	0.7605	-0.1276	2.7949
	10	-0.1455	0.8024	-0.1812	0.8268	-1.0432	4.0070	-0.3520	3.6559
over 1 and up 5 years	m = 1	0.4330	0.1553	-0.1912	0.6656	0.5262	0.0369	-0.2617	0.9771
	2	-0.4101	1.0439	-0.2385	0.8637	-0.6847	1.7054	-0.4372	1.4392
	3	-0.1623	0.8695	0.7420	1.3017	-0.1507	1.6420	1.2133	2.4966
	4	0.4601	0.7371	0.4593	0.7264	0.9188	1.5127	0.9144	1.6362
	5	0.5152	0.8553	1.1129	0.9236	0.9572	1.8794	1.8750	2.0253
	6	0.2143	0.1908	-0.0653	0.2783	0.2178	0.0870	-0.1315	0.4364
	7	0.1542	0.4118	0.1579	0.2086	-0.2188	0.3772	-0.0943	0.3056
	8	0.1888	0.1744	0.1921	0.2015	0.1592	0.2853	0.1729	0.4475
	9	0.0665	0.2573	0.0150	0.3845	-0.4230	1.5253	-0.3028	1.7940
	10	-0.0599	0.4621	-0.0126	0.4522	-0.9289	2.5817	-1.3046	2.8438
over 5 years	m = 1	0.3262	0.0959	0.0445	0.2790	0.6469	0.0865	-1.0490	2.0969
	2	-0.2587	0.8088	-0.1415	0.6777	-1.3250	2.5668	-1.0965	2.3265
	3	-0.0603	0.7180	0.6476	1.1207	-0.9013	2.3533	0.6292	2.6933
	4	0.4600	0.5900	0.4626	0.5943	0.0285	2.1009	0.0271	2.0779
	5	0.5061	0.6984	1.0073	0.8369	0.1751	2.3105	1.6563	1.9357
	6	0.1567	0.1234	0.0891	0.0607	0.1876	0.1610	-0.7368	1.2257
	7	0.1367	0.3083	0.0906	0.2345	-0.2105	0.5293	-0.3697	0.3554
	8	0.1367	0.1944	0.1114	0.1946	-0.2476	0.3868	-0.0012	0.3290
	9	0.0258	0.1600	-0.0190	0.2810	-0.4783	1.2792	-0.6813	0.8556
	10	-0.0893	0.3592	-0.0035	0.3490	-2.1829	2.8107	-1.0095	2.5822

Note. For the results of the estimation of each model, the first column contains the estimated value of the fixed long-run IRPT computed as the sample average of $\hat{\beta}_t(m)$, $t = 1, \ldots, n$, $\overline{\beta}_n(m) = n^{-1}\sum_{t=1}^{n}\hat{\beta}_t(m) = \hat{b}_{0,n}$, and the second column indicates the sample variability computed as $\hat{\sigma}_n(m) = \sqrt{n^{-1}\sum_{t=1}^{n}(\hat{\beta}_t(m) - \overline{\beta}_n(m))^2} = \sqrt{\sum_{j=1}^{m}\hat{b}_{j,n}^2}$.

Figure F.3. Time-varying slope estimates for models 1 (left) and 2 (right): Credit for house purchase, short-term interest rate. Finland

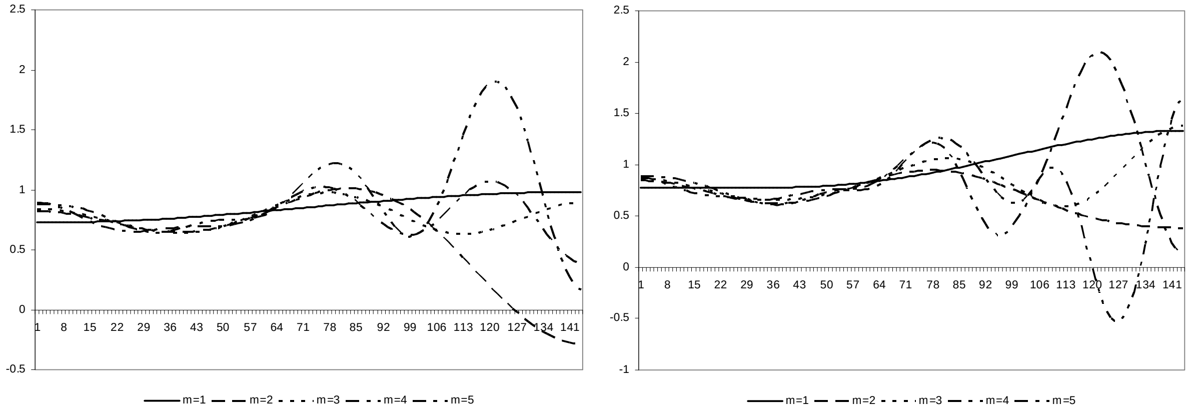


Table F.3. Sample averages and variances of the time-varying estimates of β

Country Finland		Credits for house purchase				Loans for consumption			
		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope	
Maturity									
up to 1 year	m = 1	0.8538	0.0918	0.9719	0.2040	0.5437	0.2443	0.8323	0.2486
	2	0.6129	0.3697	0.6990	0.1704	0.5356	0.2103	0.4793	0.2963
	3	0.7795	0.1070	0.8466	0.1930	0.7140	0.5115	1.0367	1.1734
	4	0.7845	0.1563	0.9691	0.4179	0.8753	0.8773	0.8234	0.8021
	5	0.9347	0.3681	0.6698	0.4116	0.8241	0.8040	0.8578	0.8282
	6	0.8398	0.1563	0.9011	0.2868	0.7281	0.4106	0.8870	0.1243
	7	0.7638	0.3347	0.8792	0.2996	0.5533	0.1945	0.4845	0.3604
	8	0.8370	0.1892	0.8933	0.2758	0.5661	0.1936	0.7606	0.1474
	9	0.6907	0.3117	0.5397	0.6736	0.8952	0.2878	0.8724	0.2743
	10	0.6545	0.4211	0.2075	0.6558	0.8273	0.3171	0.5822	0.3648
over 1 and up 5 years	m = 1	0.8373	0.0650	1.0514	0.2747	0.0821	0.2384	0.6479	0.6314
	2	0.6643	0.3467	0.6953	0.2730	0.1249	0.2425	0.2434	0.2908
	3	0.7315	0.2161	0.7740	0.1824	0.7058	1.5743	0.9570	2.1013
	4	0.7085	0.3264	0.9218	0.5128	0.7152	1.6572	0.1963	1.1829
	5	0.9992	0.6070	0.6730	0.4641	0.1142	1.7625	0.3886	2.2598
	6	0.8032	0.2215	0.9457	0.3467	0.4946	0.6106	0.7144	0.1328
	7	0.7710	0.3064	0.8952	0.3861	0.4534	0.1233	0.1312	0.6767
	8	0.7685	0.2535	0.8587	0.3966	-0.0809	0.7842	-0.2072	0.5752
	9	0.5851	0.4228	0.4347	0.6854	0.1123	0.9289	0.4798	1.3837
	10	0.5588	0.4455	0.1377	0.6296	0.5086	1.8286	-0.1121	2.6484
over 5 years	m = 1	0.8592	0.0357	1.1026	0.2776	-0.3916	0.0489	0.4533	1.1832
	2	0.6416	0.4467	0.5965	0.5569	0.6638	1.4240	1.2967	2.5676
	3	0.6039	0.5443	0.6367	0.5006	1.7964	3.9688	2.1463	4.6879
	4	0.5560	0.6979	0.8710	0.8320	1.7503	3.9772	1.0237	3.1232
	5	1.1155	1.0737	0.7355	0.6493	0.9708	3.2565	1.3201	3.8457
	6	0.7997	0.3088	0.9315	0.3607	0.1734	0.5587	0.5430	0.6015
	7	0.7062	0.3635	0.8497	0.4992	1.4581	1.2427	0.6348	0.7913
	8	0.6502	0.2869	0.7902	0.4980	0.2519	1.3500	0.6168	1.1086
	9	0.4553	0.4149	0.3116	0.6161	0.8018	1.1269	0.6139	1.3762
	10	0.4507	0.4821	0.1030	0.5658	0.5184	1.5773	-0.1193	1.8280

Note. For the results of the estimation of each model, the first column contains the estimated value of the fixed long-run IRPT computed as the sample average of $\hat{\beta}_t(m)$, t = 1, …, n, $\overline{\beta}_n(m) = n^{-1}\sum_{t=1}^{n}\hat{\beta}_t(m) = \hat{b}_{0,n}$, and the second column indicates the sample variability computed as $\hat{\sigma}_n(m) = \sqrt{n^{-1}\sum_{t=1}^{n}(\hat{\beta}_t(m) - \overline{\beta}_n(m))^2} = \sqrt{\sum_{j=1}^{m}\hat{b}_{j,n}^2}$.

Figure F.4. Time-varying slope estimates for models 1 (left) and 2 (right): Credit for house purchase, short-term interest rate. France

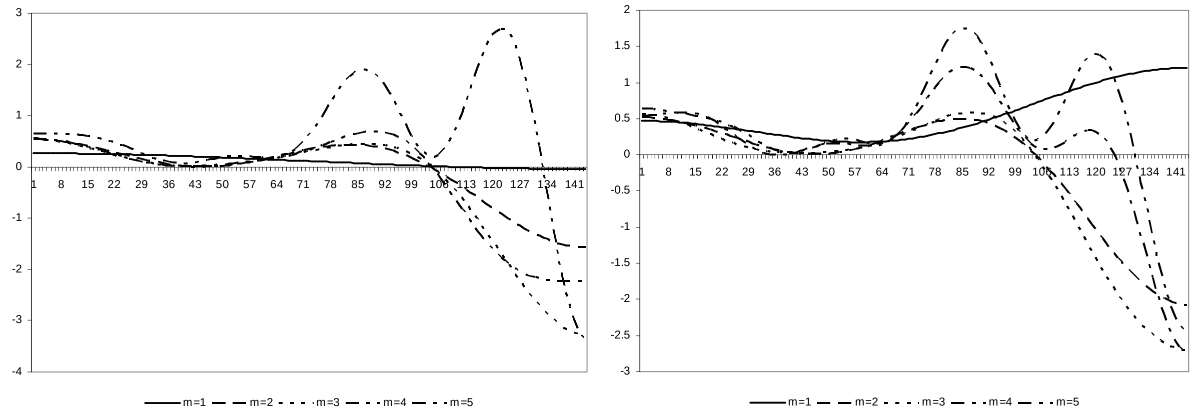


Table F.4. Sample averages and variances of the time-varying estimates of β

Country		**Credits for house purchase**				**Loans for consumption**			
France		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope	
Maturity									
up to 1 year	m = 1	0.1128	0.1088	0.5361	0.3384	0.3742	0.1637	0.8993	0.9035
	2	-0.0803	0.6176	-0.1460	0.7769	-0.1253	0.5161	-0.4150	1.0249
	3	-0.3367	1.1094	-0.2573	0.9938	-0.2322	0.5430	-0.6081	1.2634
	4	-0.2272	0.9497	0.1322	0.8135	-0.5323	1.1314	-0.3301	0.9029
	5	0.5715	1.1034	0.4053	0.8276	-0.3370	0.9004	-0.0551	0.8272
	6	0.0205	0.3155	0.3358	0.3776	0.6585	0.0929	0.8509	0.4289
	7	-0.2024	0.6433	0.0636	0.3727	-0.1986	0.6687	-0.1530	1.0426
	8	0.0933	0.2184	0.1909	0.1803	0.0824	0.2347	0.2875	0.3466
	9	0.0590	0.2429	0.0566	0.2527	0.3918	0.3744	0.4326	0.8868
	10	0.1751	0.2450	0.2151	0.3051	0.3548	0.7289	0.2916	0.6425
over 1 and up 5 years	m = 1	0.0559	0.2412	0.6108	0.3728	0.2651	0.1661	0.5048	0.2251
	2	0.0111	0.5762	-0.0912	0.8211	0.5218	0.2433	0.2686	0.3132
	3	-0.1907	0.9933	-0.1108	0.8793	0.2829	0.2746	0.3422	0.1756
	4	-0.0982	0.8609	0.1452	0.7475	0.2732	0.2787	0.2309	0.3492
	5	0.4940	0.8890	0.3668	0.7179	0.2113	0.4986	0.3412	0.6271
	6	-0.0455	0.2250	0.3646	0.4348	0.2683	0.1689	0.5015	0.4798
	7	-0.0871	0.5074	0.1366	0.3813	0.2188	0.1837	0.1230	0.2103
	8	0.0661	0.2233	0.0881	0.2042	0.1457	0.9309	0.2432	0.8074
	9	0.0265	0.2759	0.0087	0.3246	0.3337	0.5807	0.3660	0.5465
	10	-0.0250	0.4236	-0.0701	0.4046	0.3135	0.6031	0.3052	0.6536
over 5 years	m = 1	0.0203	0.3127	0.4719	0.2437	0.2308	0.0980	0.4466	0.2356
	2	0.0974	0.4008	-0.0173	0.6656	0.4698	0.2617	0.2144	0.2961
	3	-0.0402	0.7047	-0.0120	0.6641	0.1781	0.3992	0.3603	0.1364
	4	-0.0087	0.6588	0.1298	0.5874	0.2916	0.1716	0.1966	0.3242
	5	0.4409	0.6904	0.3845	0.6019	0.1837	0.4323	0.4059	0.7329
	6	-0.0570	0.0422	0.2738	0.2663	0.2179	0.0864	0.4353	0.4997
	7	-0.0078	0.3037	0.0963	0.2369	0.1787	0.2877	-0.0019	0.3102
	8	0.0165	0.1318	-0.0028	0.1516	0.0414	1.0648	0.3599	1.0209
	9	-0.0528	0.2241	-0.1212	0.2745	0.5212	0.6841	0.6092	0.6669
	10	-0.0786	0.2627	-0.0412	0.2560	0.5980	0.7053	0.7183	0.8222

Note. For the results of the estimation of each model, the first column contains the estimated value of the fixed long-run IRPT computed as the sample average of $\hat{\beta}_t(m)$, $t = 1, \ldots, n$, $\overline{\beta}_n(m) = n^{-1}\sum_{t=1}^{n}\hat{\beta}_t(m) = \hat{b}_{0,n}$, and the second column indicates the sample variability computed as $\hat{\sigma}_n(m) = \sqrt{n^{-1}\sum_{t=1}^{n}(\hat{\beta}_t(m) - \overline{\beta}_n(m))^2} = \sqrt{\sum_{j=1}^{m}\hat{b}_{j,n}^2}$.

Figure F.5. Time-varying slope estimates for models 1 (left) and 2 (right): Credit for house purchase, short-term interest rate. Germany

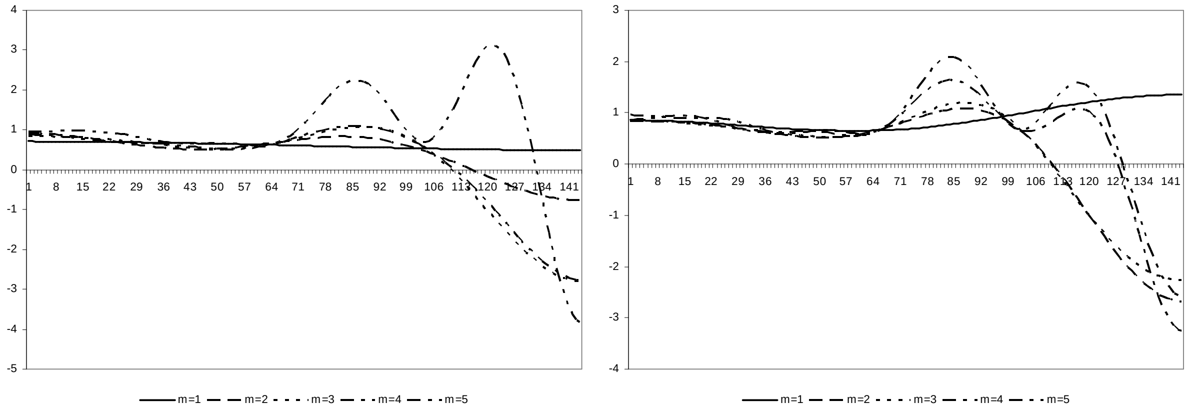


Table F.5. Sample averages and variances of the time-varying estimates of β

Country		**Credits for house purchase**				**Loans for consumption**			
Germany		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope	
Maturity									
up to 1 year	m = 1	0.6015	0.0795	0.8939	0.2304	0.3609	0.0216	0.0980	0.3992
	2	0.4234	0.4959	0.1774	1.1021	-0.4686	1.0659	-0.6679	1.4305
	3	0.1598	1.1329	0.2313	1.0309	-0.3098	0.7120	0.1728	1.0810
	4	0.2004	1.0885	0.5421	1.0273	0.3524	1.3766	0.4580	1.5599
	5	0.9438	1.2609	0.7202	0.9592	0.4562	1.5472	0.3287	1.5950
	6	0.4915	0.4262	0.5768	0.3470	0.3605	0.0213	0.1458	0.5611
	7	0.3270	0.2969	0.4913	0.4831	-0.4908	0.9519	-0.2124	0.8765
	8	0.4018	0.2652	0.5040	0.3771	0.0590	0.6579	0.2172	0.6872
	9	0.4071	0.2635	0.3103	0.4632	0.2924	0.5986	0.4031	0.6957
	10	0.4512	0.5805	0.1923	0.3894	0.4842	0.5753	0.4727	0.5576
over 1 and up 5 years	m = 1	0.1948	0.1984	0.6703	0.3250	0.1421	0.1826	0.1389	0.1866
	2	0.1775	0.4568	-0.0993	1.1329	0.7079	0.5527	0.5281	0.2439
	3	-0.2160	1.3383	-0.1183	1.1953	0.4819	0.1853	0.5491	0.2489
	4	-0.1033	1.1707	0.1434	1.0461	0.4210	0.2305	0.3750	0.2959
	5	0.7282	1.2662	0.6174	1.0894	0.3830	0.2398	0.3280	0.2483
	6	0.0736	0.3583	0.3838	0.3816	0.0197	0.0721	0.1591	0.3349
	7	0.0203	0.3372	0.1418	0.2510	0.4790	0.5307	0.3270	0.1883
	8	0.1252	0.1455	0.1328	0.1354	0.3175	0.3582	0.2503	0.4303
	9	0.0921	0.1770	-0.0017	0.2509	0.2166	0.3878	0.1972	0.5014
	10	0.0852	0.2248	-0.0038	0.1881	0.1457	0.6199	0.2444	0.7853
over 5 years	m = 1	0.0717	0.3206	0.4606	0.1985	0.2484	0.0037	0.1093	0.2046
	2	0.2367	0.2606	-0.0680	0.9710	0.4389	0.2174	0.3678	0.1007
	3	-0.0738	0.9811	-0.0765	0.9852	0.2317	0.3604	0.1497	0.5305
	4	-0.0549	0.9453	0.1228	0.8500	0.1794	0.4748	0.2373	0.3888
	5	0.5704	0.9772	0.5450	0.9354	0.2502	0.3897	0.3212	0.3207
	6	-0.0308	0.1506	0.1828	0.2124	0.1127	0.1187	0.0897	0.0248
	7	0.0079	0.0350	0.0317	0.1537	0.3278	0.2034	0.2755	0.1044
	8	0.0358	0.1129	0.0539	0.0862	0.2633	0.0824	0.4189	0.2234
	9	0.0506	0.0939	0.0147	0.1139	0.5245	0.2535	0.5467	0.2155
	10	0.0495	0.0920	0.0079	0.0910	0.5657	0.3778	0.5035	0.2853

Note. For the results of the estimation of each model, the first column contains the estimated value of the fixed long-run IRPT computed as the sample average of $\hat{\beta}_t(m)$, t = 1, …, n, $\overline{\beta}_n(m) = n^{-1}\sum_{t=1}^{n}\hat{\beta}_t(m) = \hat{b}_{0,n}$, and the second column indicates the sample variability computed as $\hat{\sigma}_n(m) = \sqrt{n^{-1}\sum_{t=1}^{n}(\hat{\beta}_t(m) - \overline{\beta}_n(m))^2} = \sqrt{\sum_{j=1}^{m}\hat{b}_{j,n}^2}$.

Figure F.6. Time-varying slope estimates for models 1 (left) and 2 (right): Credit for house purchase, short-term interest rate. Italy

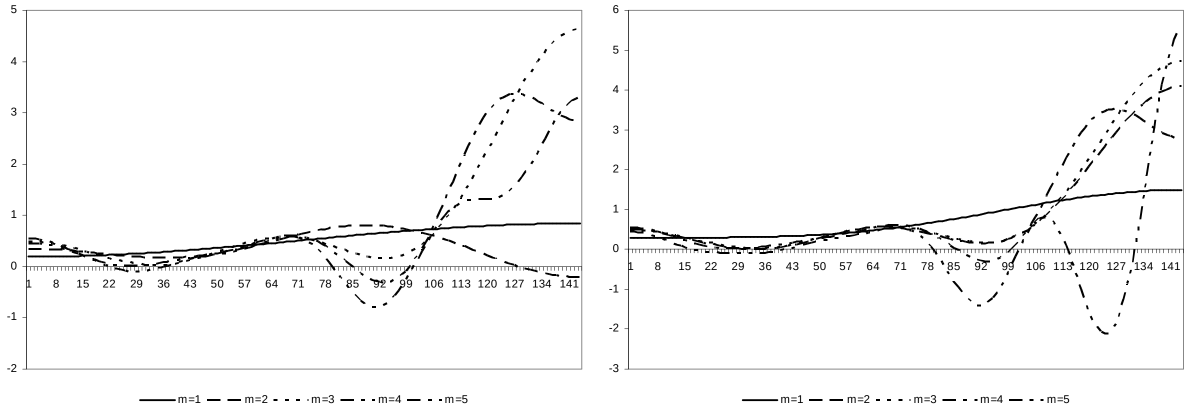


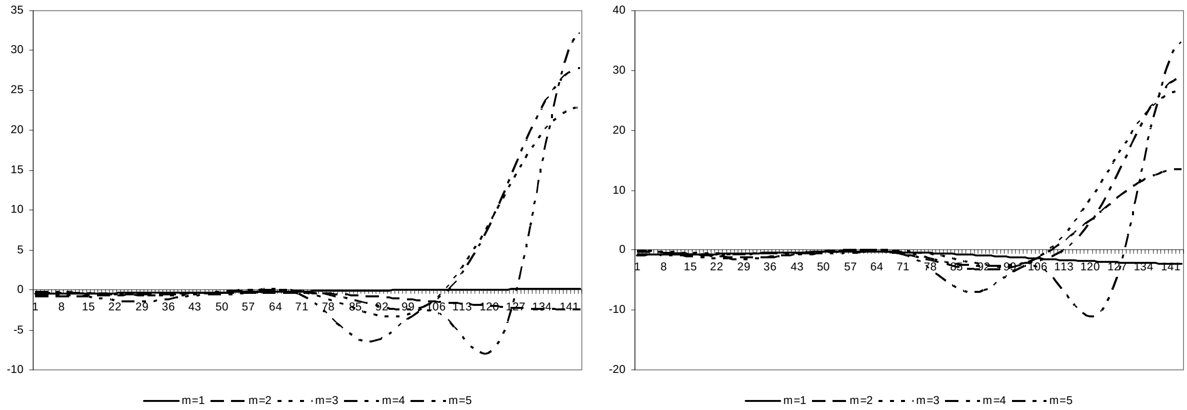
Table F.6. Sample averages and variances of the time-varying estimates of β

Country		**Credits for house purchase**				**Loans for consumption**			
Italy		Model 2. Fixed		Model 3. TV		Model 2. Fixed		Model 3. TV	
Maturity		intercept, TV slope		intercept and slope		intercept, TV slope		intercept and slope	
up to 1 year	m = 1	0.5170	0.2306	0.7395	0.4399	0.8559	0.5814	0.1812	1.2204
	2	0.3595	0.2812	0.9124	1.1888	-0.8255	2.6540	-1.5751	4.4121
	3	1.0003	1.3441	1.0117	1.3620	-0.7700	3.0694	-0.5567	2.8105
	4	0.8925	1.1851	0.9219	1.2165	-0.5485	2.8033	0.2864	2.2557
	5	0.5613	0.9229	0.1556	1.3266	0.8007	2.2440	-0.0241	2.5254
	6	0.5806	0.0615	0.9142	0.4368	0.5968	0.6091	-1.0041	0.8514
	7	0.7930	0.4351	1.0910	1.0336	-0.9640	1.3133	-0.2936	1.3668
	8	0.6539	0.7317	0.7534	0.6845	0.1967	1.3462	0.7491	1.4209
	9	0.4626	0.7427	0.5598	0.7529	1.0183	1.7266	1.2505	1.9212
	10	0.9224	0.9809	0.8883	0.9081	0.7296	1.8278	-0.2724	1.6555
over 1 and up 5 years	m = 1	0.5635	0.0845	0.7425	0.2576	-0.0159	0.2650	0.3098	0.1666
	2	0.2451	0.5247	0.2536	0.5043	-0.0967	0.5184	0.0277	0.2724
	3	0.1142	0.7447	0.2598	0.5565	0.0863	0.2276	0.0107	0.2820
	4	0.2254	0.6031	0.3820	0.5638	-0.1318	0.7547	0.2964	1.1394
	5	0.5703	0.6410	0.3218	0.5847	0.3651	1.2178	0.2447	0.9835
	6	0.5202	0.2834	0.6701	0.2517	-0.0492	0.1121	0.2419	0.1564
	7	0.3941	0.5636	0.5536	0.2527	-0.1146	0.5890	0.0711	0.7224
	8	0.3990	0.2246	0.4391	0.3464	0.2537	0.4506	0.5944	0.8704
	9	0.2921	0.2099	0.1386	0.2476	0.5510	0.6816	0.7225	1.1017
	10	0.2937	0.2469	0.1202	0.2960	0.3966	0.7995	0.0356	1.2861
over 5 years	m = 1	0.6258	0.0162	0.8558	0.2467	0.0865	0.2686	0.1624	0.3371
	2	0.3646	0.5152	0.3364	0.5841	0.2889	0.5044	0.9182	2.0114
	3	0.2502	0.7332	0.3685	0.5774	1.0239	2.1976	0.7057	1.7475
	4	0.3379	0.6217	0.5574	0.5895	0.5308	1.6820	0.6032	1.7721
	5	0.7831	0.7355	0.4947	0.5720	0.2976	1.4300	0.6032	1.9395
	6	0.5648	0.2962	0.7094	0.2874	0.2183	0.3370	0.5663	0.2048
	7	0.4432	0.4750	0.6500	0.4182	0.3473	0.1257	0.3476	0.6790
	8	0.4876	0.2205	0.5436	0.2835	0.3395	0.3172	0.5172	0.4956
	9	0.4072	0.2369	0.2667	0.3989	0.4158	0.5570	0.4169	0.5338
	10	0.3935	0.3006	0.0801	0.4532	0.1785	0.5235	0.0194	0.5506

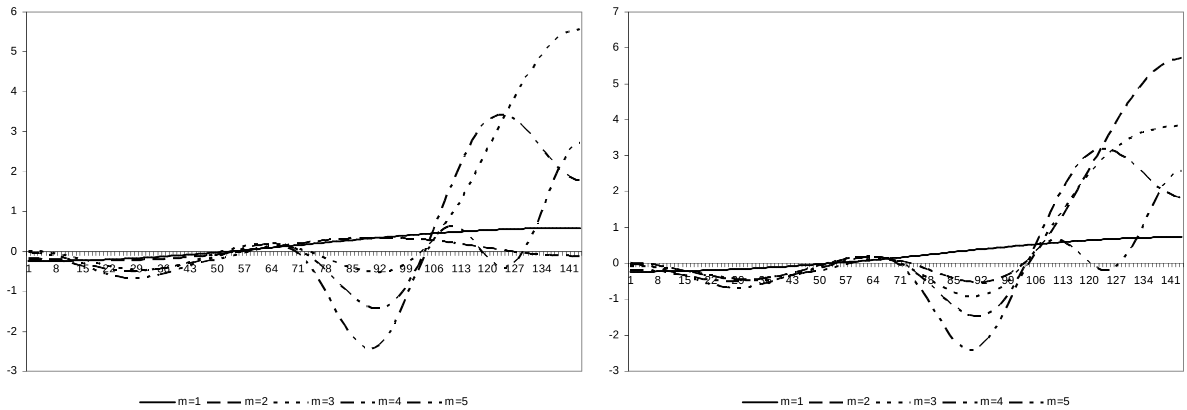
Note. For the results of the estimation of each model, the first column contains the estimated value of the fixed long-run IRPT computed as the sample average of $\hat{\beta}_t(m)$, $t = 1, \ldots, n$, $\overline{\beta}_n(m) = n^{-1}\sum_{t=1}^{n}\hat{\beta}_t(m) = \hat{b}_{0,n}$, and the second column indicates the sample variability computed as $\hat{\sigma}_n(m) = \sqrt{n^{-1}\sum_{t=1}^{n}(\hat{\beta}_t(m) - \overline{\beta}_n(m))^2} = \sqrt{\sum_{j=1}^{m}\hat{b}_{j,n}^2}$.

Figure F.7. Time-varying slope estimates for models 1 (left) and 2 (right):

Credit for house purchase, short-term interest rate. Spain



Credit for house purchase, medium-term interest rate. Spain



Credit for house purchase, long-term interest rate. Spain

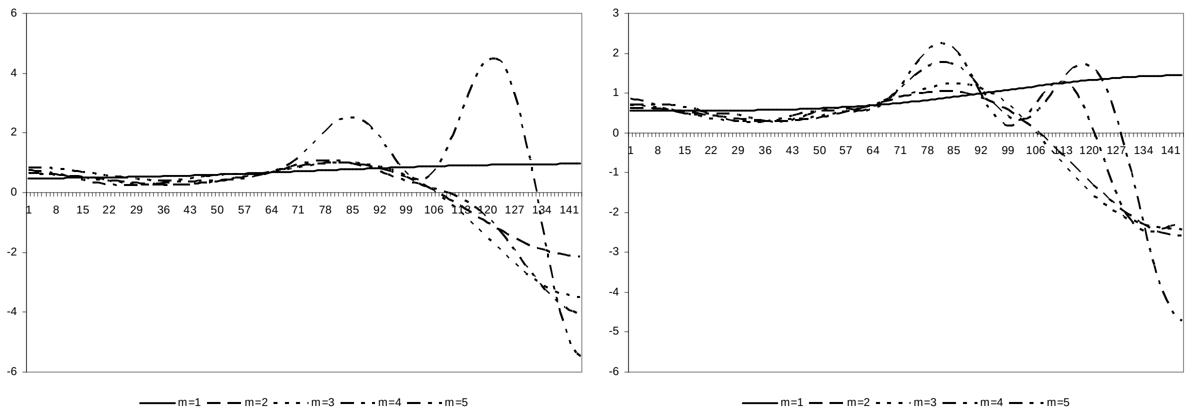


Table F.7. Sample averages and variances of the time-varying estimates of β

Country		**Credits for house purchase**				**Loans for consumption**			
Spain		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope		Model 2. Fixed intercept, TV slope		Model 3. TV intercept and slope	
Maturity									
up to 1 year	m = 1	-0.1665	0.1858	-0.9629	0.6513	0.8666	0.1944	0.8358	0.1585
	2	-1.0048	0.7110	1.0536	4.5748	-0.4519	1.4481	-1.1799	2.8262
	3	2.5481	7.2290	3.0556	8.0456	-1.2648	3.0664	-1.2362	3.0089
	4	3.1275	8.2302	2.4914	8.0208	-0.8612	2.3897	-0.8435	2.3686
	5	0.0726	8.1450	-0.3822	8.7138	-0.8995	2.4940	-0.3772	2.3905
	6	0.2112	1.5503	-0.3432	0.7669	0.7738	0.2782	0.7135	0.2889
	7	0.6296	0.8857	1.8250	3.2194	-1.0427	1.4681	-0.8365	1.9969
	8	2.7121	3.9149	2.5203	4.1632	-0.4317	1.7908	0.0188	1.5883
	9	2.6660	4.0504	1.2418	4.8943	0.2656	1.0040	1.0180	2.0216
	10	2.5162	4.0665	1.7296	3.8697	0.6059	2.2501	0.5834	2.0728
over 1 and up 5 years	m = 1	0.1726	0.2971	0.2197	0.3390	-0.1053	0.0286	0.5050	0.9086
	2	0.0310	0.2059	0.7799	1.8446	-0.2825	0.3319	0.0543	0.6037
	3	0.7586	1.8069	0.5186	1.4623	0.6663	2.3548	0.8444	2.7287
	4	0.3893	1.3821	0.3455	1.3326	1.0088	3.0406	0.6341	2.4504
	5	-0.2793	0.9874	-0.2539	0.9750	0.5753	2.4121	0.6639	2.5326
	6	0.3196	0.3784	0.6183	0.2595	0.4520	0.4744	0.5639	0.0552
	7	0.2294	0.4993	0.2970	0.2171	0.4213	0.1410	0.5564	0.1141
	8	0.0029	0.4845	0.0673	0.5340	0.4424	0.7882	0.5744	0.5964
	9	-0.2279	0.4909	-0.1112	0.5558	0.6703	0.4319	0.8998	0.8439
	10	0.0611	0.5669	0.0729	0.3895	0.8022	0.6998	0.6835	0.6744
over 5 years	m = 1	0.7209	0.1697	0.8850	0.3241	-0.1030	0.1206	0.6718	1.2262
	2	0.0918	0.9207	0.0333	1.0635	-0.1662	0.4570	-0.1202	0.4796
	3	-0.1111	1.3132	0.0361	1.1137	0.2121	1.2034	0.0392	0.8749
	4	-0.0648	1.3132	0.4076	1.3457	0.2047	1.1653	0.2440	1.2326
	5	0.9416	1.7688	0.3646	1.1495	0.2426	1.2266	0.3762	1.3419
	6	0.6230	0.6193	0.6322	0.3910	0.3784	0.3138	0.6593	0.4948
	7	0.2250	0.8173	0.5883	0.9575	0.1825	0.1284	0.2742	0.2077
	8	0.1625	0.4445	0.3545	0.4949	0.3944	0.6345	0.5389	0.5837
	9	0.0122	0.5192	0.0341	0.4064	0.6545	0.5303	0.5922	0.6004
	10	0.2341	0.2087	0.0590	0.3538	0.5774	0.6266	0.5153	0.6365

Note. For the results of the estimation of each model, the first column contains the estimated value of the fixed long-run IRPT computed as the sample average of $\hat{\beta}_t(m)$, t = 1, …, n, $\bar{\beta}_n(m) = n^{-1}\sum_{t=1}^{n}\hat{\beta}_t(m) = \hat{b}_{0,n}$, and the second column indicates the sample variability computed as $\hat{\sigma}_n(m) = \sqrt{n^{-1}\sum_{t=1}^{n}(\hat{\beta}_t(m) - \bar{\beta}_n(m))^2} = \sqrt{\sum_{j=1}^{m}\hat{b}_{j,n}^2}$.