Exchange Rate, Risk Premium and Factors:
What Can Term Structure of Interest Rates Tell Us about the Dynamics of the Exchange Rate?

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Abstract

In this paper, I investigate the role of expectations of the current and future status of economies in determining the dynamics of exchange rate, through the channel of the risk premium for holding a currency. The risk premium is introduced as an additional term to a best-fit time series model and is instrumented by bilateral latent factors obtained from the term structure of interest rates. Results show that it can significantly improve the baseline model in terms of in-sample goodness of fit and out-of-sample forecast accuracy in exchange rate changes. In particular, the proposed model can beat the benchmark *naive* models in out-of-sample forecasting at short-run horizons that range from one to twelve months, as measured by the root of mean squared errors or the direction of changes. The non-linearity of the risk premium in latent factors further renders state-dependent and time-varying response of change in the exchange rate to an identified monetary policy adjustment. The above findings hold for seven out of eight advanced-economy currency pairs (AUD, CAD, GBP, JPY, NOK, NZD, SEK against USD). Once it is included in the Fama regression, the risk premium can also help in solving the UIP Puzzle, which has been detected in the cases of GBP/USD and JPY/USD.

Keywords: Exchange Rate, Term Structure of Interest Rates, Risk Premium, Forecast, Monetary Policy Analysis, UIP Puzzle

JEL Classification: E43, F31, F37, G14

1 Introduction

The failures of the uncovered interest rate parity (UIP) and macro models to explain the dynamics of the exchange rate as documented in empirical literature indicate that the differentials of interest rates and traditional macroeconomic fundamentals play a very limited role in driving the changes in exchange rate. Then what else, if any, can potentially impact on the exchange rate?

Some recent phenomena observed in the exchange rate market may shed some light on the above question. During the European Sovereign Debt Crisis, the exchange rate of Euro to U.S. Dollar was found to fluctuate contemporaneously with the long-term bond yields of European peripheral countries. This can be taken as evidence that the long-term interest rates, or more accurately the whole spectrum of term structure of interest rates which contains abundant information on the present and expected future stance of an economy, may contribute to the dynamics of the exchange rate of its currency.

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Several recent researchers have already put their efforts in linking the foreign exchange rate and bond yields, through a risk premium channel. For example, Chen and Tsang (2011) take a macro-finance approach and explicitly use the differentials of latent factors extracted from cross-country yield curves as proxies for the risk premium. They find strong evidence that both financial (bond yield factors) and macro variables (output gap and inflation) are important in explaining exchange rate and *ex post* excess currency return.

Another approach in this direction is to relate term structure of interest rates to the exchange rate in a non-arbitrage joint bond-currency model. The risk premium can arise endogenously in the model and play a key role in determining the exchange rate (e.g. Backus et al., 2001; Li and Yin, 2010; Sarno et. al, 2012). Their results show that model-implied exchange rate changes can closely match the observed data, and the model-implied risk premia can produce unbiased predictions for currency excess returns.

However, previous studies which link bond and currency generally possess two shortcomings: They either (1) neglect higher order term-structure factors in the currency risk premium term by assuming it as a linear function of the factors; or (2) neglect a very important property of the currency market – the efficiency of the foreign exchange market in absorbing new information – by treating the risk premium as a determinant of the future exchange rate, rather than the current period one. This is not plausible because, due to the semi-strong efficient market hypothesis, the spot exchange rate, as the price of an asset (one unit of foreign currency), should incorporate publicly available information and change instantly to reflect the flow of new information (Frenkel and Mussa, 1985).

After taking the above shortcomings into account, the present work distinguishes itself from the previous bond-currency studies in the following ways: (1) currency risk premium is assumed (also can be proved) to depend on the first and second order of term structure factors; (2) it *partly* introduces the semi-strong market efficiency hypothesis and assumes the exchange rate instantly adjusts to reflect new information. In practice, this means that the risk premium is loaded into the current period exchange rate change rather than to act as an predictor for future changes; (3) pure out-of-sample forecast exercises are conducted, and are evaluated by using two criteria: the root of mean squared forecast errors and direction of changes; (4) it conducts monetary policy analysis through its impact on the risk premium term (expressed in factors); (5) it revisits the UIP Puzzle that results from the Fama (1984) regression, by utilizing the model-implied quantitative estimates of the unobservable risk premium.

In this paper, eight advanced-economy currency pairs (AUD, CAD, CHF, GBP, JPY, NOK, NZD, SEK against USD) are investigated with monthly observations, over the period between 1990s-2009.

The empirical model used for estimation, forecast and policy analysis is a risk-premium augmented autoregressive distributed lag (ARDL) model, i.e. an ARDL model, which includes the currency return (first difference of log exchange rate) and interest rate differential in lag terms, is augmented by a contemporaneous risk premium term that is instrumented by term structure factors in the first and second orders.

Empirical results show that:

(1) The estimated coefficients of the first and second order factors (which represent the risk premium) are jointly significant at a high critical level (1% or 5%). The inclusion of the factors (risk premium) enhances the explanatory power of the original ARDL model, if measured by the *adjusted*-R^2, from 0.106 to 0.177 (lowest increase) or from 0.001 to 0.209 (highest increase).

(2) The proposed model can generate more accurate forecasts than benchmark naive models at short-term horizons ranging from one to twelve months. The pure time-t information forecasts over the 2004.06-2009.05 period[1] show that, if measured by the root of mean squared error criterion, the model can beat the random walk model at 1- to 3-period ahead horizons for GBP, NZD, NOK and SEK, 6-10 for JPY. If the same forecasts are evaluated with the changes in the right direction criterion, the model can beat the benchmark model that issues equal probability for upward and downward changes at longer horizons and for more currencies: H=1-12 for GBP, H=1-8 for SEK, H=1-5 for NZD, H=1-12 for NOK, H= 2-12 for JPY and H=1 for CAD. The accuracy in forecasting the AUD and CAD can be greatly improved once the *realized* future data are used. However, the

[1] 2005.06 - 2009.05 for NOK and CAD due to a smaller sample size

proposed model is not very successful in predicting the CHF.

(3) Monetary policy shocks can, through their impact on the risk premium, induce time-varying and state-dependent responses of the change in exchange rate, while even the signs of responses can vary over time.

(4) The UIP Puzzle is no longer present if a risk premium term implied by the proposed model in this paper is added into the Fama (1984) regression, as evidenced by the fact that the coefficient of the interest rate differential turns from significantly negative to (in)significantly positive. Moreover, these model-implied values of risk premium are found to be better proxies for the unobservable risk premium in the UIP context than the widely used differential of spreads between long- and short-term bonds, as the puzzle still remains once the latter is added into the Fama regression. These results hold for the GBP/USD and JPY/USD.

The paper is organized as follows. Section 2 describes the model for exchange rate dynamics. Section 3 presents the data and regression results. Section 4 conducts forecasting and monetary policy analysis. Section 5 discusses robustness check and interprets the risk premium. Section 6 revisits the UIP Puzzle. Section 7 concludes.

2 Model on Exchange Rate Dynamics

2.1 Modeling the dynamics of the exchange rate

2.1.1 A risk-adjusted UIP approach to the exchange rate

Deviating from the standard uncovered interest rate parity (UIP), I assume that (a) the investors on the foreign exchange market are risk-averse, and (b) the returns on one-period holding of domestic and foreign government bonds are risky. Then the risk-adjusted UIP approach is as follows:

At time t, domestic investor firstly checks his investment opportunities. The expected return on one-period holding of domestic government bond is:

$$E_t^{RA}[1 + r_{t+1}] = 1 + i_t + \lambda_t^r$$

where i_t is the domestic risk-free interest rate, and λ_t^r is the risk premium for holding the government bond.

Similarly, the expected return on one-period holding of foreign government bond is:

$$\begin{aligned} &E_t^{RA}[(1 + (e_{t+1} - e_t))(1 + r_{t+1}^*)] \\ &= 1 + E_t^{RA}(e_{t+1} - e_t) + i_t^* + \lambda_t^{r^*} + E_t^{RA}[(e_{t+1} - e_t)r_{t+1}^*] \end{aligned}$$

where i_t^* is the foreign risk-free rate, and $\lambda_t^{r^*}$ is the risk premium for one-period holding of foreign government bond.

In equilibrium, the expected returns should be identical such that investors would be indifferent between investing in domestic and foreign bond market:

$$i_t = E_t^{RA}(e_{t+1} - e_t) + i_t^* + \lambda_t^*$$

where $\lambda_t^* = \lambda_t^{r^*} + E_t^{RA}[(e_{t+1} - e_t)r_{t+1}^*] - \lambda_t^r$ is the overall *relative* risk premium that domestic investors require to compensate the risks for investing in foreign bond market. This term makes the risk-averse investors value the foreign currency differently from the one risk-neutral investors valued in the standard UIP approach, where the equilibrium condition is: $i_t = E_t^{RN}(e_{t+1} - e_t) + i_t^*$.

To illustrate the effect of the extra term λ_t, assume at time t, both domestic and foreign risk-free interest rate (i_t and i_t^*) are 1% , and domestic investor perceive future risk for the foreign investment and require a compensation of 10% (λ_t^*), then the expected exchange rate change will be $E_t^{RA}(e_{t+1} - e_t) = -10\%$, which means that the foreign (domestic) currency is expected to depreciate (appreciate) in the consecutive period, whereas the risk-neutral investors would expect no change at all.

In the following, the λ_t^* is replaced by $-\lambda_t$ for the sake of computational convenience, where $\lambda_t \equiv -\lambda_t^*$, and represents the *relative* risk premium for holding domestic government bond in comparison with holding foreign bond.

2.1.2 'Efficient' foreign exchange market

The foreign exchange market is the largest and most liquid financial market in the world. Its average daily turnover exceeded \$3 Trillion as of April 2007 (Wang, 2008). Thus it is fair to assume that the exchange rate, like other financial asset prices, instantly changes to reflect the inflow of new information. Here I re-define the term 'market efficiency' of the foreign exchange market as the swift absorption of new information and immediate adjustment in the exchange rate.

At time t, domestic investors observe the current status and form their expectations on both domestic and foreign economies, based on which they evaluate the relative risk for holding domestic bond. Then they require λ_t as a compensation for the risk. Thus the *ex ante* expected exchange rate change is given by:

$$E_t e_{t+1} - e_{t+0} = i_t - i_t^* + \lambda_t$$

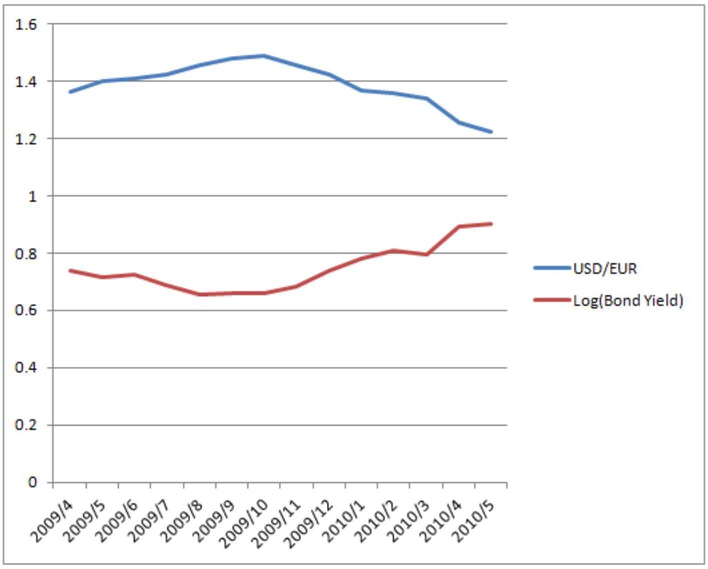
However, as the foreign exchange market is 'efficient', the new component (λ_t) of the expected depreciation will be materialized immediately, i.e.:

$$e_{t+0} - e_{t-0} = \lambda_t$$

Where e_{t+0} is the exchange rate after absorbing new information, and e_{t-0} is the pre-believed exchange rate[2].

This means the exchange rate adjusts instantly to reflect the flow of new information. In words, investors would *not* wait and price the perceived risk into the expected change of exchange rate ($E_t e_{t+1} - e_{t+0}$) until the next period. Instead, they would load the risk premium λ_t into the exchange rate in the current period ($e_{t+0} - e_{t-0}$).

Figure 1: An illustration of foreign exchange 'market efficiency'



[Referring to section 2.1.2] Immediate adjustment of the EUR/USD rate to new information. In April 2010, a downgrade of Greek government bond resulted in a contemporaneous depreciation of the Euro, which reflects the efficiency of the foreign exchange market as defined in the text.

As an illustration of foreign exchange 'market efficiency', the above chart shows that the exchange rate moves one on one with the new information that embedded in the government bond

[2]The pre-believed exchange rate can be determined by PPP, flexible price model, UIP, AR(p), ARDL, etc. Details will be discussed in section 2.3.

yields: Amid the Euro Area Sovereign Debt Crisis, the Greek government bond was downgraded to junk grade in April 2010. This news caused a sudden jump in Greek bond yields, which is accompanied by an immediate-plunge of the Euro in exchange with the U.S. Dollar.

2.1.3 The overall change of exchange rate

To complete the model, the pre-beliefs of the exchange rate e_{t-0} is assumed to be determined by economic theory, or econometrician's best forecast, or market participants' beliefs, etc. Several models are considered in the section 2.3, here I take the random walk model for the purpose of illustration:

$$e_{t-0} - e_{t-1} = 0$$

Combine the above equation with $e_{t+0} - e_{t-0} = \lambda_t$, and let $e_t = e_{t+0}$, we get the *eventual* dynamics of the exchange rate:

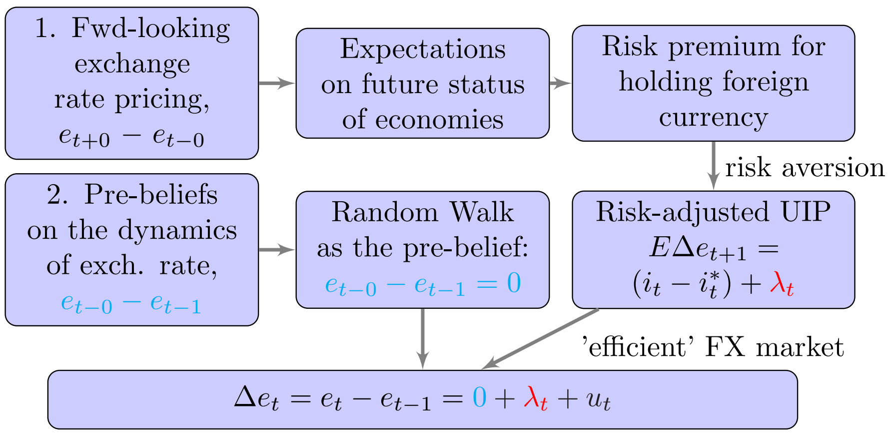
$$\Delta e_t \equiv e_t - e_{t-1} = 0 + \lambda_t + u_t$$

Where u_t is the residual that contains the component of the exchange rate change that cannot be explained by the model.

Now, one can see that the risk premium, which compensates the *relative* risk of holding domestic bond for one period, is loaded into the current period, and effectively serves as an *ex post* excess return for holding foreign assets, i.e. $\lambda_{t-1\to t}^{ex\ post} = \lambda_{t\to t+1}^{ex\ ante} \equiv \lambda_t$.

As an illustration, the following chart shows the intuition behind the complete model of the dynamics of exchange rate:

Figure 2: An illustration of intuition on the dynamics of the exchange rate



[Referring to section 2.1.3] Flow chart of modeling the dynamics of the exchange rate. The exchange rate is determined by two components: (1) the forward-looking element that reflects market's expectation on the future status of economies. (2) the pre-belief element that reflects the investors'/economists'/econometricians' beliefs on the fair value of the exchange rate at time t if no new information emerges.

2.2 Determinants of the risk premium

As the risk premium for currency holding is not observable, it would be hard to link it to other variables. However, there are some efforts in both theoretical and empirical literature that try to explain the determinants of the risk premium: Utility-based asset pricing models have explicitly shown that it depends on the growths of consumption and their risks in domestic and foreign countries (e.g. Verdelhan, 2010; Bansal ans Shaliastovich, 2008). Arbitrage-free bond-currency factor models have linked the risk premium to the factors that determines both exchange rate return and bond yields (e.g. Sarno, 2012a). Empirical work also link the risk premium with macroeconomic and policy uncertainties (e.g. Martin and Urrea, 2007).

In this paper, I will mainly focus on the framework in which the risk premium depends on bond yield factors[3]. In a similar approach, Sarno et al. (2012) estimate bond-currency model jointly and obtain factors that can fit the exchange rate data well but at a cost of fitting poorly of the bond yield data. Another shortcoming of the Sarno (2012) approach is that it is hard to interpret these factors ecnomically as they contains mixed information of both exchange rates and bond yields. In this paper, I will keep the bond yield factors intact, i.e. the factors are extracted from the affine term structure model of *interest rates*, and check how the information embedded therein can help in explaining the dynamics of the exchange rate, through the risk premium channel. This approach can be justified by the following reasons:

First, presumably the risk premium depends on the investors' expectations on the future status of the two economies, which are usually very hard to measure. But intuitively, as the term structure is a spectrum of interest rates that covers short, mid and long terms in the time dimension, it naturally incorporates the information on the current status and expected development of the economies. Thus it is reasonable to use the factors of the term structures of interest rates to represent these expectations. Second, the interest rates are also financial prices, and they move instantly with the new information on expectations of macroeconomic variables (whereas those variables themselves are usually persistent in values and measured with lags). For these two reasons, the information picked by the term structure of interest rates may also be perfect inputs that drive the currency risk premium (hence changes of exchange rate). Third, the affine model for the term structure of interest rates has its own advantages: The model can fit the bond yields data almost perfectly well, and all the interest rates as well as the time-varying market price of risk can be explicitly expressed as function of latent factors. These characters guarantee that these factors are reliable represents of the whole spectrum of interest rates, and make economic interpretation and analysis possible given the existence of structural relationship between factors and interest rates.

1. Affine term structure model of interest rates

   In standard approaches of the affine term structure model of interest rates, e.g. Ang and Piazzesi (2013), Hamilton and Wu (2010), the bond yields can be expressed as a function of latent factors.

   The observation equations are:

$$y_t = A + BF_t + \mu_t$$

   where $y_t = [y_{1,t}, y_{2,t}, \ldots, y_{n,t}]'$ is a vector of observed bond yields with various maturities, and $F_t = [f_{1,t},\ f_{2,t},,\ f_{3,t}]'$ is a vector of latent factors and is assumed to follow a VAR(1) process:

$$F_t = \tilde{c} + \tilde{\rho} F_{t-1} + \upsilon_t$$

   The closed-form expression of bond yields as a function of factors is derived from an arbitrage-free model of asset pricing[4], $P_{n+1,t} = E_t[M_{t+1}P_{n,t+1}]$, where $P_{n+1,t}$ is the price of bond that has a maturity of $n+1$; M_{t+1} is the pricing kernel and is assumed to be a function of latent factors F_t. The yield of bond that matures at time $t+n$ is given by:

$$y_{n,t} = A_n + B_n' F_t$$

[3]The focus on the currency-bond factor model is because that the utility-based model find itself difficult to match the data due to the high persistence of consumptions; in alternative approaches such as Marin and Urrea (2004), the way of choosing instruments for macroeconomic and policy uncertainties are somehow arbitrary, thus it is difficult to find valid instruments that are universal for all currencies.

[4]Details can be found in e.g. Ang and Piazzesi (2013), Hamilton and Wu (2010)

where the coefficients can be calculated recursively: $A_{n+1} = A_n + B_n(\mu - \Sigma\lambda_0) + \frac{1}{2}B_n'\Sigma\Sigma' B_n - \delta_0$ and $B_{n+1} = B_n(\phi - \Sigma\lambda_1) - \delta_1$.

The risk-free rate is given by:

$$r_t = \delta_0 + \delta_1 F_t$$

2. The currency risk premium

Due to aforementioned reasons, it is natural to assume that the *relative* risk premium for holding domestic government bond (in comparison with holding foreign bond), λ_t, depends on bond yield factors of both domestic and foreign country. Thus without lose of generosity, the risk premium is *assumed*[5] to have the following form:

$$\lambda_t = H(\tilde{F}_t) = (C + D\tilde{F}_t)'\tilde{F}_t$$

Here, $\tilde{F}_t = [F_t', F_t^{*'}]'$ is a vector that contains both domestic and foreign bond yield factors, F_t and F_t^*. $H(\tilde{F}_t)$ can be a liner function ($C \neq 0_{6\times1}$, $D = 0_{6\times6}$) or a higher order ($C \neq 0_{6\times1}$, $D \neq 0_{6\times6}$) function of the factors. In addition, I assume that the matrix D is block diagonal such that there is no cross multiplication between domestic and foreign factors in the second order terms.

In section 3, I will report results of regressions that include both linear and second order terms.

2.3 Models of the pre-believed exchange rate change: $e_{t-0} - e_{t-1}$

As there is no consensus on how the exchange rate may change intertemporally given the absence of new information, economists, econometricians, or investors may hold different beliefs on the 'fair value' of the exchange rate, or the change of it. In the following, I list six models that can potentially describe the most popular beliefs held by monetary economists and time-series econometricians.

- M1: Purchasing power parity model (PPP)

Suppose the Purchasing Power Parity (PPP) holds continuously, the nominal exchange rate is determined by the differential of price levels.

$$e_t = p_t - p_t^*$$

then the assumed intertemporal exchange rate change is:

$$e_{t-0} - e_{t-1} = \Delta p_t - \Delta p_t^* = \pi_t - \pi_t^*$$

- M2: Flexible price monetary model (FP)

Suppose the Purchasing Power Parity holds continuously, and prices can adjust without friction. When the money demand equals money supply, we have:

$$m_t - p_t = \alpha_1 y_t - \alpha_2 i_t$$

The change of exchange rate is given by:

$$e_{t-0} - e_{t-1} = \Delta p_t - \Delta p_t^* = (\Delta m_t - \Delta m_t^*) - \alpha_1(\Delta y_t - \Delta y_t^*) + \alpha_2(\Delta i_t - \Delta i_t^*)$$

[5] In the international finance literature, it can be shown that the risk premium can be modeled explicitly as function of those factors at least to the second order. However, those factors are estimated to jointly match data on exchange rate returns and yield curves for both countries. According to Sarno (2012), a good estimation of the exchange rate data is at the cost of poor match of the yield curves, thus the factors are no longer easily interpreted. As the priority of this paper is to look for the driving forces of the exchange rate, other than trying to find the best statistical estimation of the *ex ante* expected excess return and the expected exchange rate change. Thus I would prefer to keeping the original source of information unaltered, thus make it is easy to make economically meaningful and consistent explanations.

- M3: Uncovered interest parity model (UIP)

The Uncovered Interest Parity (UIP) model argues that the expected exchange rate depreciation equals to the interest rate differential:

$$e_{t-0} - e_{t-1} = i_{t-1} - i^*_{t-1}$$

- M4: Taylor rule model (TR)

Suppose the Uncovered Interest Parity (UIP) holds, and the monetary policies in both domestic and foreign countries are assumed to follow the Taylor rule:

$$i_t = \rho_1 y_t^{gap} + \rho_2 \pi_t$$

In a symmetric monetary policy setup[6], the change of exchange rate is given by:

$$e_{t-0} - e_{t-1} = \rho_1 (y_{t-1}^{gap} - y_{t-1}^{gap*}) + \rho_2 (\pi_{t-1} - \pi^*_{t-1})$$

- M5: Random walk model (RM)

The random walk model argues that the best prediction of the exchange rate is the one of the last period, thus the pre-believed exchange rate change would be zero:

$$e_{t-0} - e_{t-1} = 0$$

- M6: Autoregressive distributed lags model (ARDL)

The Autoregressive Distributed Lags (ARDL) model is a also a time-series model that is free of underlying economic theory. In this model, dependent variable is a function of lag terms of its own and some exogenous variables. In this paper, following the same spirit as the UIP, I assume interest rate (monetary policy rate) differential is the reasonable exogenous variable that drives the changes in exchange rate. The ARDL(p,q) model is:

$$e_{t-0} - e_{t-1} = \beta_0 + \beta_1 \Delta e_{t-1} + \beta_2 \Delta e_{t-2} + ... + \beta_p \Delta e_{t-p} + \gamma_1 (i_{t-1} - i^*_{t-1}) + ... + \gamma_q (i_{t-q} - i^*_{t-q})$$

where p is the lag order for exchange rate change, and q is the lag order for interest rate differential.

In the next section, I will check which of the above models can fit the data the best, thus can be used as the baseline model for the pre-belief component ($e_{t-0} - e_{t-1}$) of the overall exchange rate dynamics ($e_t - e_{t-1}$).

3 Empirical Results

3.1 Data

3.1.1 Data on exchange rates

In this paper, the US is treated as the home country, thus all exchange rates are defined as the amount of the US dollars that one unit of foreign currency can convert to at the foreign exchange market. In the analysis, monthly data are used. The following table is a summary of available time periods of the data on the exchange rates and the sources.

[6] For simplicity, the parameters of the Taylor rule for both countries are assumed to be the same. In a popular setup in the literature, the Taylor rule should be asymmetric, and the central bank of foreign country also explicitly targets the nominal exchange rate to its PPP level. As the primary goal here is not to examine how the Taylor rule components, i.e. output gap and inflation, drives the exchange rate, but how well they can explain the variation of the change in exchange rate, the assumption of symmetric rules is feasible.

Table 1: Summary of Datasets for Exchange Rate

Country	Currency	Time Period	Source
US	USD	n.a.	n.a.
Japan	JPY	1957.01-now	International Financial Statistics, IMF
Germany/EA	-	-	-
UK	GBP	1975.01-now	Bank of England
Canada	CAD	1957.01-now	International Financial Statistics, IMF
Switzerland	CHF	1957.01-now	International Financial Statistics, IMF
Norway	NOK	1960.01-now	Norges Bank
Sweden	SEK	1957.01-now	International Financial Statistics, IMF
Australia	AUD	1969.07-now	Reserve Bank of Australia
New Zealand	NZD	1957.01-now	International Financial Statistics, IMF

3.1.2 Data on macro fundamentals

As for the macroeconomic fundamentals in the pre-believed models, money base M0 is used for the money supply whenever it is available, otherwise narrow money M1 is employed as an alternative. Monthly industry production is used as proxy for the output/income. The output gap is obtained by detrended output with HP filter (lambda=129600). CPI is used for the price index and for calculating inflation, which is defined as 12-month difference of CPI[7]. All variables are transformed to log values. The data are drawn primarily from the IMF's International Financial Statistics, and supplemented by the 'Economic Indicators' of the OECD statistics.

3.1.3 Data on term structure of interest rates

Thanks to Wright's elaborate work (2011), an international panel dataset of the term structure of interest rates is publicly available and ready-to-use. The dataset contains zero-coupon non-callable government bond yields at all maturities starting from three months up to ten years with a 3-month increment, for ten major advanced economies. In the dataset, data are reported at a monthly frequency from the starting date to 2009.05. The following table gives a brief summary of the dataset:

Table 2: Summary of Datasets for Bond Yields

Country	Time Period	Source
US	1971.09-2009.05	Gürkaynak, Sack, and Jonathan H. Wright (2007)
Japan	1987.01-2009.05	Datastream and author's calculations
Germany/EA	-	-
UK	1979.01-2009.05	Nicola Anderson and John Sleath (2001)
Canada	1986.01-2009.05	Bank of Canada and BIS database
Switzerland	1988.01-2009.05	Swiss National Bank and BIS database
Norway	1998.01-2009.05	Norges Bank and BIS database
Sweden	1993.01-2009.05	Riksbank and BIS database
Australia	1987.02-2009.05	Datastream and author's calculations
New Zealand	1990.01-2009.05	Datastream and author's calculations

[7]The RPI (retail price index) is used for UK due to its availability in a longer period of time

3.1.4 Factor generation

Factors for the term structure of interest rates: In recent macroeconomics and fiance literature, the class of Gaussian affine term structure models (e.g. Duffie and Kan, 1996; Dai and Singleton, 2002) has become the workhorse in modeling the bond yields at various maturities. According to the model setup, all variables, e.g. time-varying market price of risk, bond yields and term premia are driven by the latent factors of the economy. The model is usually estimated by algorithms such as maximum likelihood or minimum chi square. In this paper, I particularly refer to the latter method that is proposed by Hamilton and Wu (2012a) due to its time-efficiency. The factors are calculated on a second step after obtaining the estimates of parameters.

3.2 Empirical Evidence

3.2.1 The sample for estimation

Although the data on exchange rates, bond yields and macroeconomic fundamentals are available for long periods for most currencies, it is not a good idea to use the full samples as exchange rate models generally suffer from parameter instability problems, especially the risk of structural change is high when a long sample is used. Take a few for example: For the case of GBP, the sample starts from 1979.01, but in 1992.09 the Bank of England changed its monetary policy from pegged pound sterling in the European Exchange Rate Mechanism to an inflation-targeting interest rate setting after the turmoil of the Black Wednesday. Thus the sample of 1993.01-2009.05 is used for GBP. Similarly, the JPY entered into a phase of zero interest rate[8] in the second half of 1995, thus the functioning of the monetary policy and economic activity is supposed to be different from previous periods. Hence the 1996.01-2009.05 period is used for the JPY. For some other reasons, the CHF and CAD are investigated for the period of 1996.01-2009.05 and 1999.01-2009.05 respectively. However, the truncation of these samples does not mean the model works badly during the periods that has been left out, instead it should be taken as a necessary step to avoid parameters instability across periods of time. To see this, a robustness check is implemented in section 5 for various time periods that includes the full and several sub samples of each currency.

3.2.2 Model selection on the pre-believed models

In this section, I investigate which of the pre-believed models listed in section 2.3 is the best one in terms of fitting the realized data on exchange rate changes. Then the best model will be selected as the baseline pre-believed model for the $e_{t-0} - e_{t-1}$ (see section 2.1.3), which will be combined with the risk adjustment component $e_{t+0} - e_{t-0} = \lambda_t$ after taking into account the new information about the future status of the economies to make a complete model for the exchange rate change.

The $Adj. - R^2$s of the models for each currency are reported in table 3:

——— insert Table 3 here ———

The model selection criteria used here is simply the $Adj. - R^2$. As the primary purpose of this paper is to investigate whether the risk premium term (expectations), as a function of bond yield factors, can explain a negligible portion of the variation in exchange rate change on top of that can be maximally explained by an economic or time-series model, at the current stage the the $Adj. - R^2$ is expected to serve as a proper criteria.

As shown in table 3, the M6: ARDL model fits the data best for five out of eight currencies in selected periods. The M2: Flexible price monetary model is the second best model and it fits the data best for the rest three currencies (CHF, JPY, NOK). Simple numerical comparison would suggest that the ARDL model should be selected as the benchmark of the pre-belief on the exchange rate change, $e_{t-0} - e_{t-1}$.

There are several other reasons for the preference of the ARDL models v.s. the flexible price model: First, later on, when the bond yield factors are added into a pre-believed model to make a complete model, the model based on flexible price model turns out to underperform the one based

[8] A phase of zero interest rate is defined as the headline monetary policy rate, 3-month interest rate, goes and has been stayed below 1%.

on the ARDL model, especially for the cases of JPY and CHF[9]. Second, data on macroeconomic fundamentals are released with time lags, and often subject to adjustment afterwards. Thus when it comes to forecasting the exchange rate, which is another important criterion to evaluate models, both the availability and quality of these data in real time would raise concerns. Furthermore, traditional fundamentals barely contribute to the forecasts for the change in exchange rate when comparing to a naive benchmark random walk model (e.g. see Engle, 2005).

Now, combining the ARDL model (M6) with the risk premium adjustment, $e_{t+0} - e_{t-0}$, the complete model used in this paper to describe the dynamics of exchange rate change is:

$$\Delta e_t = \beta_0 + \beta_1 \Delta e_{t-1} + ... + \beta_p \Delta e_{t-p} + \gamma_1 (i_{t-1} - i^*_{t-1}) + ... + \gamma_q (i_{t-q} - i^*_{t-q}) + \underbrace{(C + D\tilde{F}_t)'\tilde{F}_t}_{\lambda_t} + u_t \quad (1)$$

3.2.3 The first step when using the ARDL model: Testing a long-run relationship

Before run the regression, it is necessary to test whether there is a long-run relationship between the exchange rate change and the interest rates deferential in the ARDL model. If there is one, it indicates that in equilibrium the Δe_t is expected to change at a constant value in response to one unit change in $i_{t-1} - i^*_{t-1}$ in the long-run. If a long-run relationship is detected, it would be more proper to use the error correction (EC) representation of the ARDL model[10].

After estimating the ARDL model, the Wald test can be used for testing the null hypothesis:

$$H_0: \qquad LRC = \frac{\gamma_1 + ... + \gamma_q}{1 - \beta_1 - \beta_2 - ... - \beta_p} = 0$$

Where the LRC stands for the long-run coefficient for a potential long-run relationship $E[\Delta e_t] = LRC \cdot E[i_{t-1} - i^*_{t-1}]$. Results for tests of the null suggests that 1) in general, there is no long-run relationship in the truncated sample for all of the currencies. 2) When full samples are considered, long-run relationships are detected for the GBP and JPY. Thus in the following, unless otherwise stated, only the original ARDL model will be used for estimation, forecast and policy analysis, as the samples are mostly truncated.

3.2.4 Regression results and the relevance of risk premium (expectations).

In this paper, there are two important questions to be answered:

a) Whether the risk premium that based on expectations on the future status of economies (embodied in bond yield factors) is relevant for the dynamics of the exchange rate?

b) Analytically, whether the risk premium term is a linear function of the factors or it should also include the higher (second) order terms?

To answer the first question, one would check in the assumed function $\lambda_t = (C + D\tilde{F}_t)'\tilde{F}_t$[11], whether the estimates of vector C or the joint estimates of vector C and matrix D is significantly different from zero.

To answer the second question, one would check whether the goodness of fit is sufficiently enhanced by inclusion of the second order factor terms in comparison with the case that only first order is considered, conditional on the significance of the vector C and joint significance of the vector C and matrix D. The measurements of goodness of fit are the *adjusted*-R^2 and the Akaike Information Criteria, AIC. The former measures to what extent variation in the dependent variable can be explained by the independent variables. The higher the *adjusted*-R^2, the better the model fits the data. While the latter measures the information loss of a given model when it is used to represent the date generating process of the dependent variable, and it make a balance

[9] For the case of JPY (CHF), only 10.0% (9.73%) of variation in exchange rate change can be explained by the Flexible Price + factor model, whereas the counterpart explained by the ARDL + factor model is 20.88% (20.59%). The numbers for the NOK is moderate: 12.37% v.s. 14.32%.

[10] More details for the EC representation of the ARDL model are shown in the Appendix

[11] For simplicity, the interaction between factors across countries in matrix D are suppressed.

of the goodness of fit and the complexity of the model. For this measurement, the lower a value is, the better the model fits the data.

In the following, I provide a set of results that allow one to assess how well the $ARDL + factor$ models fit the data.

——— insert Table 4 here ———

——— insert Table 5 here ———

——— insert Table 6 here ———

——— insert Table 7 here ———

As shown in the above tables, for all of the currencies, the null hypothesis that of $C = 0_{6\times 1}$ ($D = 0_{6\times 6}$) for the $ARDL + F^{1^{st}}$ ($ARDL + F^{1^{st},2^{nd}}$) is strongly rejected. This indicates that the factors, either in a linear or non-linear form, are jointly important explanatory variables for the exchange rate, which further suggests that the presumed risk premium (expectations) indeed is a non-negligible determinant of the dynamics of the exchange rate.

It also worth noting that the goodness of fit is greatly enhanced by including the factors (risk premium) into the regression. The variation of exchange rate that can be explained by the non-linear factor model ranges from 14.32% to 35.33%[12], of which, 40%-90% is explained by the risk premium term.

The above evidence suggests that the risk premium, expressed in a function of bond yield factors, does influence the dynamics of the exchange rate, and contributes enormously to the variation of the exchange rate change. Especially, this seems to be a common feature for almost all the currencies. Thus we can take above findings as strong evidence that the risk premium (expectations) should not be neglected in exchange rate models. This view will be further supported by the forecast performance of the proposed model. So the answer to the first question is: Yes.

When it comes to the second question, whether the risk premium should be in a linear or non-linear form of the factors, it is obvious that for most currencies the second order terms of factors should be included, as 1) they are jointly significant, 2) greatly improve the goodness of fit as measured by both of the *adjusted*-R^2 and the AIC, when compared with the one which only first order terms are included[13].

The non-linearity of the risk premium in the factors indicates that the way factors could affect exchange rate dynamics is state dependent. This is also the rationale underlying the assumption $\lambda_t = (C + D\tilde{F}_t)'\tilde{F}_t$, where the $(C + D\tilde{F}_t)'$ is effectively a time-varying coefficient of the factor vector $\tilde{F}_t$. I will discuss this further in the policy analysis part of Section 4.

4 Forecasts and Policy Analysis

4.1 Exchange rate forecasting

The primary purpose of this paper is to investigate whether expectations on the future status of economies, in the form of risk premium, can help in explaining the dynamics of the exchange rate. In the previous section, the proposed model, *equation 1*, is found to fit the data much better than the baseline best-fit model. However, as it is documented in studies by Shinn (2012) and Engle (2013), while evaluating models of exchange rates by their in-sample fit remains valuable, ever since

[12] This is sufficiently higher than that has been reported in an alternative non-linear approach to explain the UIP Puzzle by Sarno, et. al. (2006), which ranges from 3.7% to 17%.

[13] One exception is the GBP: the AIC for the non-linear model is slightly bigger than that of the linear model. But as the two values are very close, and the joint significance of the second order terms are very strong, I take these results as evidence that there is no clear cut for the case of GBP during the reported period. To keep its model consistent with other currencies, the performance of non-linear model for the GBP is still examined in the following sections.

the study of Messe and Rogoff (1983), the 'Golden Rule' has been shifted towards their usefulness in forecasting exchange rates.

Following this guideline, in this subsection I will check the performance of the proposed non-linear model in forecasting the exchange rate out of sample, in comparison with its linear counterparts, and popular benchmark naive models. If this model can also be used as predictive models for the exchange rate changes/returns, it would shed further light on its ability to explain exchange rate movements over time.

Before proceeding further, it may prove worthwhile to emphasize that what I focus on is the pure time-t out-of-sample forecast, based on which the relative advantages of models will be compared[14]. That being said, unless otherwise stated, no contemporaneous values of the right-hand-side variables are used to predict future exchange rates.

4.1.1 Models for Forecasting

The models used for forecasting and comparison are again the same as the ones used in regressions:

The best-fit baseline time series model, $ARDL$:

$$\Delta e_t = \beta_0^0 + \beta_1^0 \Delta e_{t-1} + \cdots + \beta_p^0 \Delta e_{t-p} + \gamma_1^0 (i_{t-1} - i_{t-1}^*) + \cdots + \gamma_q^0 (i_{t-q} - i_{t-q}^*) + u_t^0$$

$ARDL$ model with linear factors, $ARDL + F^{1^{st}}$:

$$\Delta e_t = \beta_0^1 + \beta_1^1 \Delta e_{t-1} + \cdots + \beta_p^1 \Delta e_{t-p} + \gamma_1^1 (i_{t-1} - i_{t-1}^*) + \cdots + \gamma_q^1 (i_{t-q} - i_{t-q}^*) + C^1 \tilde{F}_t + u_t^1$$

$ARDL$ model with non-linear factors, $ARDL + F^{1^{st},2^{nd}}$:

$$\Delta e_t = \beta_0^{1,2} + \beta_1^{1,2} \Delta e_{t-1} + \cdots + \beta_p^{1,2} \Delta e_{t-p} + \gamma_1^{1,2} (i_{t-1} - i_{t-1}^*) + \cdots + \gamma_q^{1,2} (i_{t-q} - i_{t-q}^*) + (C^{1,2} + D^{1,2} \tilde{F}_t)' \tilde{F}_t + u_t^{1,2}$$

The naive benchmark models[15], e.g. the Random Walk model, RM:

$$\Delta e_t = \eta_t^{RM}$$

For the proposed model, $ARDL + F^{1^{st},2^{nd}}$, although the exchange rate change depends on the contemporaneous values of factors, it is still possible to make out-of-sample forecasts based on the information that is available till time t, i.e. the information set I_t, as the dynamics of factors is exogenously given by $F_t = \tilde{c} + \tilde{\rho} F_{t-1} + \upsilon_t$, and $F_t^* = \tilde{c}^* + \tilde{\rho}^* F_{t-1}^* + \upsilon_t^*$ for domestic and foreign government bond yields. It can be shown that the second order terms of factors satisfies $f_{i,t} \cdot f_{j,t} = F_{t-1}^{'} Q_{ij} F_{t-1}$, and $f_{i,t}^* \cdot f_{j,t}^* = F_{t-1}^{*'} Q_{ij}^* F_{t-1}^*$, where Q_{ij} and Q_{ij}^* are matrices calculated as function of $\tilde{\rho}$ and $\tilde{\rho}^*$. The interest rates can be expressed as $r_t = \delta_0 + \delta_1 F_t$ and $r_t^* = \delta_0^* + \delta_1^* F_t^*$. Based on the estimates of coefficients β, γ and C and D, and forecasts of factors in the first and second order, one can obtain the the one-period ahead forecast on exchange rate changes. Accumulating one-period ahead forecasts for certain horizons, one may further obtain multiple period ahead forecasts.

One-period ahead forecast exercise is carried out for all currency pairs. The convention of implementing 'rolling regressions' is adopted here[16]. Being consistent with the estimation exercises in section 3, the first estimation window starts from 1990.01, 1993.01, 1996.01, 1998.01 and 1999.01 respectively, and ends at 2004.05[17]. Thus the size of estimation window varies across currencies.

[14]The forecasts based on time-$(t+h)$ information are used as complementary evidence once necessary.

[15]Naive model may refer to difference meanings: (1) no-change model, such as the Random Walk model, which will be used as the benchmark when evaluating forecast accuracy by the criterion of RMSE. (2) equal-chance of upward and downward change, which will be used as the benchmark when evaluating forecast accuracy by the criterion of direction of changes.

[16]Exceptions is for the case of SEK, in which case the parameter instability of the rolling regression might be problematic.

[17]For NOK and CAD, it ends at 2005.05 to guarantee a thumb rule that the estimation window takes 2/3 of full sample size, and the forecast window takes the rest 1/3. Thus in these cases, there are 48 (36) period of forecast at 1- (12) horizons.

The estimation window rolls forward until the last period, 2009.05. The forecast period for the 1- (12-) period ahead horizon is 60 (48) months. The setup for forecasting each currency is summarized in the following table:

Table': Summary of setups for one-step ahead forecasts

	Sample	Window Size	Rolling	Horizon	Forecast Period	Forecast Size
AUD	90.01-09.05	173	1	1-12	2004.06-2009.05	60
NZD	90.01-09.05	173	1	1-12	2004.06-2009.05	60
GBP	93.01-09.05	137	1	1-12	2004.06-2009.05	60
SEK	93.01-09.05	Growing	0[18]	1-12	2004.06-2009.05	60
CHF	96.01-09.05	101	1	1-12	2004.06-2009.05	60
JPY	96.01-09.05	101	1	1-12	2004.06-2009.05	60
NOK	98.01-09.05	90	1	1-12	2005.06-2009.05	48
CAD	99.01-09.05	78	1	1-12	2005.06-2009.05	48

Note: The regime of forecasting considered here is the pure time-t information forecast $\widehat{\Delta e_{t+h}}|I_t$.

4.1.2 Forecast Comparison

The following tables report the results of out-of-sample forecast for all currencies at horizons (H) from one month to twelve months.

——— insert Table 8 here ———

——— insert Table 9 here ———

——— insert Table 10 here ———

——— insert Table 11 here ———

I evaluate predictive performance in two ways. First, it is the root of mean squared errors (RMSE), which measures the distance that forecasts deviate from the true values. A smaller value indicates a better performance of the model. Inferences are based on the Clark and West (2005) test[19]. Second, the direction of change, which is the probability of predictions in the correct direction, defined as the ratio of the number of predictions in the correct direction over the total number of predictions. A value above 50% indicates a better performance in forecasting than a naive model that assigns equal probabilities of the downward and upward changes. Inferences for this criterion are based on the Diebold and Mariano (1995) sign test[20].

- The RMSE criterion

As is shown in the left panel of each table, where the entries are the values of RMSE, the proposed $ARDL+F^{1^{st},2^{nd}}$ model is the best forecast model when comparing to the $ARDL$ and the random walk model[21] for the following cases: H=1-3 for GBP, H=1 for NZD, H=6-10 for JPY, H=1 for NOK and H=1-2 for SEK.

These findings indicate that for five out of eight floating exchange rates, 1) The predictive power of the baseline model, $ARDL$, can be greatly improved by adding the factors into the model, which can be taken as evidence that the factors (risk premium) are proper predictors of the exchange rate. 2) The $ARDL+F^{1^{st},2^{nd}}$ model can properly capture the features of the dynamics of exchange

[19] See Appendix

[20] See Appendix

[21] The comparison with the baseline model, $ARDL$, allows us to evaluate the role of risk premium term (expressed in factors) in predicting the exchange rate, while the comparison with the naive model, the random walk, is a standard approach in the literature.

rate in the short run, as it can outperform the random walk model on out-of-sample forecasts. In practice, the apparently naive model, the random walk[22], is regarded as the 'toughest' benchmark model for forecasting the exchange rate, as it is very hard for alternative models to outperform it when measured by the RMSE, especially in the short run (see, e.g. Rogoff, 1983). Thus a model that can beat the random walk in forecasting can be admitted as a proper model of the exchange rate.

However, as has been pointed out by Cheung et al. (2005), the evaluation based on the RMSE criterion has its own shortcomings: The RMSE only measures the distance that forecasts deviate from the true values, but ignores the underlying economic contents, i.e. whether the forecasts are in the right direction. Studies powered by simulation exercise have already shown that even if forecasts are in the wrong direction for *all* periods, it can still beat the benchmark random walk model if measured by the RMSE criterion.

To avoid drawing improper conclusions based on spurious evidence, I also evaluate the same forecasts by the criterion of the direction of change, i,e. the probability of changes in the right direction, which is usually recognized as a economic measure of the forecast accuracy.

- The direction of change criterion

As is shown in the right panel of each table, where the entries are the probabilities of changes that have been predicted in the right direction, the proposed $ARDL{+}F^{1^{st},2^{nd}}$ model can outperform the $ARDL$ and the benchmark naive model that issues equal probabilities on upward and downward changes, in the following cases: H=3-10 for GBP, H=1, 4, 5 for SEK, H=1-3 for NZD, H=3-12 for JPY, H=3-12 for NOK. If supplemented by the forecasts based on time-$(t+h)$ information, more cases can be included: H =1-12 for AUD, and H=3, 9, 12 for CAD.

Above findings have once again shown that the factors are proper predictors for the exchange rate, and the $ARDL{+}F^{1^{st},2^{nd}}$ model can properly capture the features of the dynamics of exchange rate. Moreover, under the direction of change criterion, the evidence is even stronger: The $ARDL{+}F^{1^{st},2^{nd}}$ model outperforms the ARDL and the benchmark naive model in a broader range of horizons and for a bigger set of currencies pairs (seven out of eight when counts on the time-$(t+h)$ forecasts).

——— insert Table 12 here ———

Discussion 1:

Under the RMSE criterion, the forecast performance of the $ARDL + F^{1^{st},2^{nd}}$ model for the AUD, CAD and CHF are unexpectedly poor. The reasons might be: 1) the forecasts of the factors deviate from their true values to a large extent, or 2) this model is not a good one for these currencies, or 3) if the model is a valid one, its parameters may be highly unstable. To see whether it is because of the properties of the model (reason 2 or 3), I use the observed value of the factors, i.e. utilizing time-$(t+h)$ information, and repeat the forecast exercises. The results turn out that, as shown in table 12, if measured by the RMSE, the $ARDL + F^{1^{st},2^{nd}}$ model outperforms the RM model at horizons of H=1-6, 8-9 for AUD; and if measured by the change in right direction, the $ARDL + F^{1^{st},2^{nd}}$ model outperforms the RM model at H=1-12 horizons for AUD, H=3, 9, 12 for the CAD. Although the factor models still perform poorly in forecasting for the case of CAD when measured by the RMSE criterion, but once measured by the changes in the right direction criterion, it can still outperform the baseline ARDL model and the benchmark naive model.

Based on above observations, we can conclude that for the cases of AUD and CAD, the mild forecast performance is very likely due to the poor forecast of the bond yield factors (reason 1), rather than the failure of the model itself.

However, the forecast performances of both $ARDL + F^{1^{st},2^{nd}}$ and $ARDL + F^{1^{st}}$ model for the CHF are very poor even if the time-$(t+h)$ information is utilized. This indicates that either the factor models are not proper models (reason 2) for the CHF in forecasting or the parameters are highly unstable (reason 3) even if the this can be partly relieved by the rolling estimation.

[22]A random walk model claims that the the best prediction for the exchange rate in the next period is its value in the current period.

Discussion 2:

Under the direction of change criterion, the $ARDL + F^{1^{st},2^{nd}}$ model is more successful in significantly outperforming the $ARDL$ and naive models than it is under the RMSE criterion. This indicates that the non-linear factor terms bring important information about the direction of changes, despite the fact that it may also bring extra deviation from the true values. Accordingly, when it comes to the question that which criterion we should rely on, we can see that the widely used RMSE criterion alone may not be a good measure of forecast accuracy, as it is merely a statistical measure which overlooks the underlying economic contents, thus may lead to misleading conclusions. In this case, for a more meaningful evaluation of forecasts, it would be suggested that conclusions based on the RMSE criterion should be at least double-checked (if not replaced) by the ones based on the direction of change criterion.

4.1.3 Conclusion for the forecast exercises

The above forecast exercises show evidence that based on the pure time-t out-of-sample forecast (supplemented by the time-$(t+h)$ information forecast), the proposed factor model $ARDL+F^{1^{st},2^{nd}}$ can outperform the baseline time series model $ARDL$ and naive models at short-run horizons that ranges from one month to twelve months for multiple currencies[23], when both of the RMSE and the direction of change criteria are used for evaluating the accuracy of forecasts.

Above findings suggest that the risk premium (which reflects expectations), as a nonlinear function of the bond yield factors, play an important role in capturing the actual dynamics of the exchange rate, as firstly evidenced by the goodness of fit of the proposed factor model, and then further strengthened by its forecast accuracy.

4.2 Policy Analysis

It is of great interest to learn how the exchange rate responds to a monetary policy adjustment. In the present context, it is possible to conduct such a policy analysis as the 3-month government bond yields for the advanced economies, 1) are nearly the same as the 3-month money market interest rates, which are usually taken as proxies for the monetary policy rates, and 2) can be expressed in terms of the latent factors *analytically* thanks to the convenience of the arbitrage-free affine terms structure model of the bond yields. Accordingly, as will be shown below, an unexpected monetary policy adjustment can be transformed to (or expressed in) shocks to the latent factors. That being said, the monetary adjustment is treated as actually being induced by the same underlying source of shocks [24] as the unexpected adjustments in latent factors are, but it loads the shocks in a certain way. The Cholesky decomposition is employed for identifying the shocks on short-, mid- and long-term interest rates, of which the short-term one is relevant for monetary policy analysis.

4.2.1 Transforming policy shocks to factor shocks

Recall that in the affine term structure models, yields can be expressed as $y_t = A + BF_t + \mu_t$. In standard approaches, the policy rates ($y_t = (i_t^{3m}, i_t^{12m}, i_t^{10y})'$) can be assumed to be observed without measurement errors; and the factors follow a VAR(1) process: $F_t = \tilde{c} + \tilde{\rho}F_{t-1} + \upsilon_t$, where $\tilde{c}$ is assumed to be zero (e.g. see Hamilton and Wu, 2010).

Substituting factors with yields in the VAR(1) model for factors, we get: $B^{-1}(y_t - A) = \tilde{\rho}B^{-1}(y_{t-1} - A) + \upsilon_t$.

Rearranging above equation, we have: $y_t = B(A - \tilde{\rho}B^{-1}A) + B\tilde{\rho}B^{-1}y_{t-1} + B\upsilon_t$

This is the reduced-form VAR(1) model for the yields, with residual term $\eta_t = B\upsilon_t$, and $var(\eta_t) = B'B$.

Given that the interest rates in vector y_t are short-, mid- and long-term bond yields, it is possible to identify the shocks with a Cholesky decomposition: $\eta_t = C\epsilon_t$, where C is a lower triangular matrix and satisfies $C'C = var(\eta_t)$; ϵ_t is a vector that contains the identified shocks to short-, mid- and long-term rates, respectively. The intuition behind the identification is that

[23]For most of the cases, $ARDL + F^{1^{st},2^{nd}}$ can also outperform the $ARDL + F^{1^{st}}$ model.

[24]All economic shocks that may affect the whole spectrum of bond yields.

contemporaneously, longer maturity bond yields tend to be affected by the shorter maturity bond yields, but not *vice versa*[25].

By $Bv_t = C\epsilon_t$, we can get $v_t = B^{-1}C\epsilon_t$. This equation shows the way that how a shock on the short-/mid-/long-term interest rate (element in ϵ_t) can be transformed into shocks to the factors. In words, this transformation allows us express economically meaningful shocks as functions of the un-identified statistical shocks. In the following, the monetary policy analysis is based on this transformation.

4.2.2 Counter-factual study on monetary policy shocks

In the following, I carry out a counter-factual perturbation study in which, at each point of time, I allow a further one standard deviation change in the short-term interest rates (3-month) on top of the one that has already realized in the data, and see how much further change in the exchange rate will be induced. This exercise is similar to but not exactly the same as the impulse response analysis[26].

The responses are calculated based on equation 1:

$$\Delta e_t = \beta_0 + \beta_1 \Delta e_{t-1} + ... + \beta_p \Delta e_{t-p} + \gamma_1 (i_{t-1} - i^*_{t-1}) + ... + \gamma_q (i_{t-q} - i^*_{t-q}) + \underbrace{(C + D\tilde{F}_t)'\tilde{F}_t}_{\lambda_t} + u_t$$

Given the presence of non-linear factor terms, the exchange rate change responses to a policy perturbation (analogous to a shock) tends to be state-dependent and time-varying. As an illustration, given a shock vector v_t to the factors, the $t = 0$ change in the second order term $\Delta(f_{i,t|v_t} \cdot f_{j,t|v_t})$ depends on the realized state variables $f_{i,t}$ and $f_{j,t}$:

$\Delta(f_{i,t|v_t} \cdot f_{j,t|v_t}) = (f_{i,t} + v_{i,t}) \cdot (f_{j,t} + v_{j,t}) - f_{i,t} \cdot f_{j,t} = (f_{i,t} \cdot v_{j,t} + f_{j,t} \cdot v_{j,t}) + v_{i,t} \cdot v_{j,t}$

This is the key difference between the non-linear and linear analyses, as the latter[27] generates homogenous initial responses across all periods in real time, while the former gives a unique responses at each point of time, depending on the state of the economy that reflected by the bond yield factors, F_t.

4.2.3 Results

The **concurrent** responses (similar to the elasticity) of exchange rate change following a one-standard-error monetary policy adjustment are calculated for each point of time in history. The monetary policy is proxied with the 3-month government bond yield. In the following, Figure 3 and 4 depict the exchange-rate-change responses to a one-standard-deviation change in the U.S. and foreign monetary policy rates[28].

Exchange rate responses to extra US monetary policy adjustment

——— insert Figure 3 here ———

Exchange rate responses to extra foreign monetary policy adjustment

——— insert Figure 4 here ———

[25]However, the yield on longer maturity bond can still impact the its shorter maturity counterpart with a time lag, as they follow a VAR(1) process.

[26]As for the impulse response, a shock is defined as the deviation from its expected equilibrium value. In the present context, I investigate the effect of a perturbation, which is defined as deviation from its realized value. The latter is similar as a policy-rate elasticity of exchange rate.

[27]Conventional linear analysis such as VAR or VECM.

[28]Here I treat the interest rates across countries as independent to each other. In an extended version of this paper, the interaction between interest rates will be considered explicitly.

We can have the following two observations from the above figures: 1) An unexpected tightening of monetary policy (increase in interest rate) in a country can result in instant appreciation or depreciation in the value of its currency. For example, an increase of 0.547% in the GBP monetary policy rate in most of the time drives the GBP to appreciate. But during the most chaotic period in the second half of 2008, the same amount of increase results in a significant depreciation of the GBP. 2) The amount of instant response of exchange rate change may vary with time. For example, the appreciation of USD versus GBP, induced by an increase of 0.539% in the USD monetary policy rate, can range from 0% to 17%. These two findings reflects the fact that the instant response of exchange rate change to a monetary policy 'shock', tends to be time-varying and state-dependent, which is in contrast with the homogenous responses given by most exchange rate models. This in turn reflects the importance of the non-linearity of the risk premium (exchange rate change) in factors, as the time-varying and state-dependent response arises from the second order terms.

5 Robustness check and interpretation of the risk premium

5.1 Robustness check

5.1.1 Robustness at different time horizons

To check whether the risk premium term in equation (1) is only significant at the reported sample periods or it is so in a more general sense, I redo the regressions for all currencies at four alternative sample periods: 1) Full Sample, which takes all available periods into account. 2) Early Days, which starts from the very beginning of a sample and stops around the mid of the sample. 3) Common, which includes the common periods for all currencies, i.e. 1998.01-2009.05. 4) No-Crisis, which contains all the common periods but excludes the recent financial crisis period, i.e. 1998.01-2008.01. The results are reported in the following table.

—– insert Table 13 here —–

There are two criteria for the robustness check: (i) significance of the risk premium (expressed in factors) in determining the exchange rate change, and (ii) the gain in goodness of fit, in comparison with the $ARDL$ model, as measured by the AIC[29].

In the Full Sample case, most currencies get a 'y', which indicates that the above two criteria are satisfied. Exceptions are the CHF, JPY and CAD. An 'n' for the CHF, JPY and CAD means that the gains in goodness of fit are negligible in their full samples, thus the risk premium term (factors) are redundant in these cases. However, if one splits the full samples and check the resulted sub-samples named as Early Days and the Reported periods, she can see that for CHF and CAD, the two criteria are met separately in each of the sub-samples. This indicates that the redundancy of factors in the full sample cases may arise from the instability of parameters across sub-samples. In contrast, we still get an '~n' for the JPY during Early Days, which indicates that it is only a recent phenomenon for the proposed $ARDL+F^{1st,2nd}$ model to well explain the dynamics of JPY.

In the Common sample between 1998.01 and 2009.05 for all currencies, we can see that the proposed $ARDL + F^{1st,2nd}$ model works very well for most currencies. Less strong evidence is obtained for the AUD, NZD and CAD. A '~y' for these currencies shows that the goodness of fit as measured by the AIC for the $ARDL + F^{1st,2nd}$ model is significantly better than the one for $ARDL$ model, but *not* better than its counterpart for the $ARDL + F^{1st}$ model. This indicates that a risk premium, which includes the first order factors *only,* can perform as good as the one counts on the second order factors, thus it may *not* be necessary to go to the second order at this period of time for these currencies.

Furthermore, conditional on the Common sample for all currencies, I explicitly exclude the recent financial crisis period during 2008.02-2009.05, to check whether the above findings are merely a result of crisis-time phenomenon or they hold in a more general sense. Except for the AUD and NOK, results for the other currencies are still robust. Less satisfactorily, a 'y-' for the NOK

[29]Here I use the AIC as the measurement of goodness of fit, because it also punishes the attempt to add more regressors into the model. Thus it is more informative than the $adj. - R^2$, as the latter generally increases when more regressors are included in a regression.

means that although the two criteria are met, the goodness of fit as measured by the AIC for the $ARDL + F^{1st,2nd}$ model is just as good as the $ARDL$ model. In this sense, the inclusion of the risk premium to the second order only brings moderate gain in fitting the data. However, when the AIC is complemented by the $adj. - R^2$, the gain is found to be substantial.

For the AUD, neither $ARDL + F^{1st,2nd}$ nor $ARDL + F^{1st}$ works well at the no-crisis time and in early days, which indicates that the factor models can properly capture the dynamics of AUD *only* when the recent crisis periods are included.

To sum up, we can conclude that despite of a few exceptions, the $ARDL + F^{1st,2nd}$ model fits data the best and the second order factors in the risk premium term are not to redundant, as suggested by the multi-currency and multi-period evidence.

5.1.2 Robustness under the specification with macroeconomic fundamentals

In section 3.2.2, the $ARDL$ model is selected as the best-fit pre-believed exchange rate model when comparing to monetary economic models of the exchange rate. Accordingly, all the conventional macro fundamentals are absent in the working model of this paper, the $ARDL + F^{1st,2nd}$ model. A natural question to ask is that, whether the significance of the factors is still robust once these macro fundamentals are included? Or put differently, whether the impacts of factors on exchange rate are merely perfect substitutes for the ones of macro fundamentals? In this subsection, I will check the robustness of the reported results by putting back all the relevant macroeconomic variables that have been used in section 3.2.2.

The results are reported in the following tables.

——— insert Table 14 here ———

——— insert Table 15 here ———

——— insert Table 16 here ———

——— insert Table 17 here ———

As one can see, for all of the currencies the significance of the risk premium (factors) is still very *strong* even if the macroeconomic fundamentals are included in the regression. But the marginal gain in goodness of fit from the inclusion of the fundamentals differs across currencies: for the AUD, GBP, NOK and NZD, no gain at all; for the CAD, CHF, JPY and SEK, the marginal gains are moderate. However, it is worth noting that for the latter four currencies, the macroeconomic fundamentals are jointly significant, which suggests that these variables should not be jointly excluded in a *fully* specified model for the exchange rate.

As for the significance of an individual macroeconomic fundamental, a mixed picture is obtained: The differential of money supplies is relevant for the JPY and CAD; the contemporaneous interest rate differential is significant in the cases of AUD, CHF and SEK; for the Taylor rule components, the lagged output gap differential is relevant for the NZD and JPY, while the lagged inflation differential is important for the CAD and JPY.

Discussion: Above results suggests that macroeconomic fundamentals, either individually or jointly, still play a role in explaining the dynamics of exchange rate. However, this role are relative minor as 1) if judged by increments of the goodness of fit (measured by the *adjusted*-R^2 and the AIC), the macro fundamentals bring very little information into the dynamics of the exchange rate; 2) both the values and significance of the coefficients of the factor terms *barely* change after including the macro fundamentals (i.e. the main results reported so far can still hold even when the macro variables are put back). As the primary goal of this paper is to investigate whether the information of expectation embedded in the term structure of interests rates are relevant for the exchange rate dynamics, rather than to find the best specification of a exchange rate model, it is safe to stick to the working model - $ARDL+F^{1st,2nd}$- in the whole paper, even if, to a subtle extent, there is a risk of model mis-specification and biased estimation of the coefficients of factors. In the following section, the $\Delta(i_t - i_t^*)$ will be included when necessary for certain currencies such as the AUD, CHF and SEK.

5.2 Interpretation of the risk premium

In the previous section, we have seen plenty of evidence that suggests the risk premium, which is non-linear in factors, plays an important role in determining the dynamics of exchange rate. However, even though the existence of a link between the risk premium and factors has been verified, we still have limited understanding of what exactly underlies the risk premium, as the factors are merely statistical variables and have no economic meanings. Some researchers, e.g. Chen and Tsang (2013), Dewachter and Lyrio (2006) show that the 'level' factor can be linked to inflation expectations, and the 'curvature' factor can be connected with output growth expectations or business cycle. But those variables themselves are weak explanatory variables for the risk premium and the dynamics of the exchange rate as can be evidenced in the robustness check section, which is also in line with the failure of monetary exchange rate models that have been documented in many empirical studies. In this section, I will take an alternative approach, which utilizes the intrinsic connection between factors and macro economic variables, to understand the economic determinants of the currency risk premium.

5.2.1 Transforming factors to expectations and uncertainties:

One special feature that attracted little attention in previous currency-bond studies is that, latent factors can be further 'transformed' to economically meaningful variables which reflect market expectations regarding the current and future status of the economy. Those variables, such as expected short-term interest rate and term premium can be obtained in a closed-form of observed bond yields and estimated latent factors.

After applying some simple algebra, the expected short-term risk-free rate can be obtained:

$$E[r_{t+i}] = \delta_0 + \delta_1 E_t[F_{t+i}] = \delta_0 + f(\delta_0,\, \delta_1,\, \tilde{c}, \tilde{\rho}) + \delta_1 \tilde{\rho}^i F_t$$

where the $f(\delta_0,\, \delta_1,\, \tilde{c}, \tilde{\rho})$ is a constant and is a function of the structural parameters.

The term premium for holding longer term bonds is given by:

$$\tau_{n,t} = y_{nt} - \frac{1}{n} E_t \Sigma_{i=1}^n r_{t+i}$$

Where y_{nt} is the observed yield for bond that matures at time $t+n$.

The Figure 5 in the next page depicts the estimated risk-free rates for the UK range from 3 month to 10 year in the future, at each point of time from 1979.01 to 2009.05.

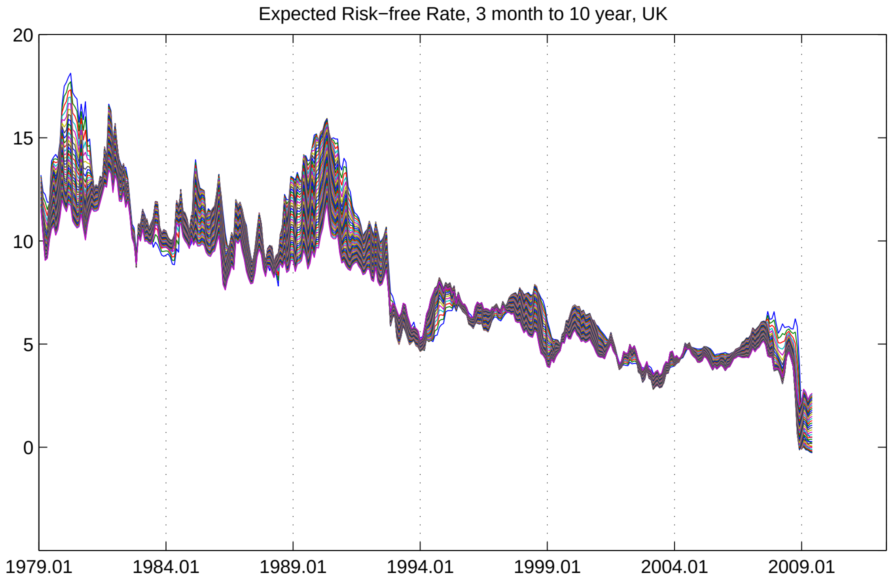
5.2.2 Model-implied expectations and uncertainties vs. the exchange rate dynamics

As one can see from the figure 5, at a certain point of time, values of expected short-term rates vary with forecast horizons. Most likely, both the level and the volatility of these expectations may capture information about investors' perception of the prospect of economic activity. For example, harmonized expected rates (less volatile) may have different implications for the exchange rate compared with dispersed rates (more volatile). The term premia may also convey a certain type of information about the future.

In the following, I treat the means and variances of the expected risk-free rate and term premium for both domestic and foreign countries as determinants of the currency risk premium, then add them to the $ARDL$ model and check whether they can impact the dynamics of the exchange rate.

Redefine $\lambda_t = \tilde{C}'\tilde{\bar{R}} + \tilde{D}'\tilde{\Sigma}$, where $\tilde{\bar{R}} = [\bar{r}^*_{t,..,t+m|t},\ \bar{r}_{t,..,t+m|t},\ \bar{tp}^*_{t,..,t+m|t},\ \bar{tp}_{t,..,t+m|t}]'$ is a vector that contains the means of expected risk-free rate and term premium for both domestic and foreign countries, $\tilde{\Sigma} = [\sigma^*_r,\ \sigma_r,\ \sigma^*_{tp},\ \sigma_{tp}]'$ is the corresponding variance vector, $\tilde{C}$ and $\tilde{D}$ are matrices of coefficients.

Figure 5: Distribution of expected risk-free interest rates of the GBP



[Referring to section 5.2.1] Figure 5: Expected short-term interest rates for the UK. At each point of time from 1979.01 to 2009.05, there are 40 expected rates that range from 3 month to 10 year in the future. The rates are calculated from the latent term structure factors.

Now the *equation* 1 becomes:

$$\Delta e_t = \beta_0 + \beta_1 \Delta e_{t-1} + ... + \beta_p \Delta e_{t-p} + \gamma_1 (i_{t-1} - i^*_{t-1}) + ... + \gamma_q (i_{t-q} - i^*_{t-q}) + \underbrace{(\tilde{C}' \tilde{\tilde{R}} + \tilde{D}' \tilde{\Sigma})}_{\lambda_t} + u_t \quad (2)$$

The regression results of *equation* 2 are given as follows:

——— insert Table 18 here ———

——— insert Table 19 here ———

——— insert Table 20 here ———

——— insert Table 21 here ———

As shown above, the risk premium that expressed in the factor-transformed variables still significantly influences the dynamics of the exchange rate: for AUD, JPY and SEK (define as **G1** currencies), only the *means* of expected risk-free rate and term premium for both domestic and/or foreign countries are found to be important; for the GBP, NOK and CHF (define as **G2** currencies), both the *means* and the *variances* are found to be key determinants; but for the NZD and CAD (define as **G3** currencies), those variables are not *jointly* relevant.

When one looks into the G1 and G2 currencies and asks how the *first moments* of expectations influence the value of a currency, she may find that 1) the signs of the coefficient of the same variable for domestic and foreign countries are opposite to each other, which, very intuitively, means the same variable has the opposite impact on the exchange rate. This feature can allow us to conveniently focus only on one side of the exchange rate determinants. 2) the higher mean of expected risk-free interest rate, the stronger its currency. This is reflected by the positive coefficient of the $\bar{r}^*_{t,..,t+m|t}$, which suggests that, *ceteris paribus*, a higher mean of expected foreign risk-free rate in the future reduces the risk premium for holding its currency, resulting in a stronger foreign currency (in short: the higher expected interest rate, the stronger currency). 3) except for the JPY and SEK, the higher mean of term premium, the stronger the currency. This is somehow surprising: It is odd for the mean of foreign term premium for holding foreign government bond, $\bar{tp}^*_{t,..,t+m|t}$, to play the same role as the mean of expected risk-free rates, as the former is generally perceived as a measure of the riskiness of a country's economy, i.e. it is usually believed that a higher term premium should correspond to a weaker currency (e.g. see Chen and Tasng, 2011).

When it comes to the second moment (the uncertainty) of the expectations, the effect on the exchange rate change do not exhibit a common pattern. However, there is a general rule that the higher the variance of expected foreign (domestic) risk-free rate, the stronger the foreign (domestic) currency, as reflected by the positive (negative) coefficient of the $\sigma^*_r(\sigma_r)$. The exceptions are the JPY and CHF, in which a higher variance of expected JPY (CHF) risk-free rate is related to a weaker Yen (Franc). This finding suggests that for the exchange rates for most currencies, a more harmonized (dispersed) expectation on the future course of risk-free interest rate are taken as a signal of more (less) risk for holding the currency, when the rest variables are controlled[30].

Among these currencies, the values of GBP and NOK in G2, CAD in G3[31] are found to be negatively correlated with their variances (uncertainties) of the bond term premium, σ^*_{tp}. This can be interpreted as that when the investors in foreign bond market hold dispersed opinion on how much more compensation (term premia) should be charged on top of the risk-free rate over the horizons up to 10 years, then the domestic[32] investors will take this as a strong signal that the foreign economy is under risk in the future, and then price the foreign currency at a lower value. This finding suggests that it is the variance (uncertainty) of the bond term premium, rather than the level or mean of it, that is perceived as a sign of riskiness of the prospect of an economy by foreign exchange market investors, as evidenced by the GBP, NOK and CAD.

6 Implications

What I have obtained so far can be understood in two ways: First, the risk premium (which represents expectations) is an important determinant of the exchange rate dynamics, thus should not be omitted in a dynamic exchange rate model. Second, it is possible to quantify the risk premium, which is usually unobservable, with the help of bond yield factors or factor-transformed economic variables.

Accordingly, these two findings can have two implications: 1) Substituting the dynamic exchange rate component (e.g UIP) in multivariate empirical models or new open economic models (esp. in the DSGE context) with the $ARDL + F^{1^{st},2^{nd}}$ model, combining this with the dynamics of factors, one may obtain more realistic moments, forecasts, or impulse responses of the exchange rate and related macroeconomic variables. 2) To utilize the numerical estimate of the risk premium and test exchange rate models that otherwise can hardly be tested if the risk premium is unobservable or unmeasurable.

In the following, I will focus on the second implication and test whether the UIP Puzzle can arise from the omission of a risk premium term, which is suggested by many studies of exchange rate. Due to its complexity, the first implication will be discussed in an extended version of this paper or very likely, a separate paper.

[30]One possible explanation to this finding is that the the observed risk-free rates of most currencies are higher than the one of the USD in most of time in history, while the observed risk-free rate of the JPY and CHF are nearly always lower than the one of the USD in the post 1996 period. This difference may cause the domestic (the U.S.) investors to hold different beliefs on the perceived higher uncertainties of the expected foreign risk-free rate: It is taken as a sign of lower risk for holding higher interest-rate currencies (GBP, AUD, etc.), but higher risk for holding lower interest-rate currencies (JPY and CHF). These beliefs in turn results in stronger and weaker foreign currencies, respectively.

[31]For the case of CAD, the individual coefficients of the variances are mostly significant despite of the weak joint significance.

[32]Also foreign investors hold the same believes as the domestic investors as we assume symmetric information and believes among investors.

Risk premium and the UIP Puzzle

6.1 Risk-premium augmented Fama regression

Despite its popularity in the international macroeconomic/fiance literature, empirically the UIP is severely violated as the estimate of β in the regression $\Delta e_t = \alpha + \beta(i_{t-1} - i^*_{t-1}) + u_t$ (Fama regression[33]) deviates significantly from unity and usually has a negative value. This is documented as the UIP Puzzle in the literature. Among many other hypotheses, Fama (1984) argues that this puzzle may be caused by the omission of a time-varying risk premium term that led to a biased estimate of β. In this section, I will focus on the risk-premium solution to the UIP Puzzle [34], and check whether the Puzzle can be mitigated by the inclusion of a time-varying risk premium term. Three proxies that can represent the unobservable risk premium, $\hat{\lambda}_t$, are considered:

(1)

$$\hat{\lambda}_t^{tp} = tp_t - tp_t^*$$

i.e. the bilateral differential of bond term premia between domestic and foreign country. Where the term premium is defined as the spread between long- and short-term government bond yields, $tp_t = i_t^{10y} - i_t^{3m}$.

(2)

$$\hat{\lambda}_t^F = (\hat{C} + \hat{D}\tilde{F}_t)'\tilde{F}_t$$

i.e. the estimate of risk premium that implied by the $ARDL + F^{1st,2nd}$ model, which is a function of factors from the terms structure of interest rates.

(3)

$$\hat{\lambda}_t^V = \hat{\tilde{C}}'\tilde{\tilde{R}} + \hat{\tilde{D}}'\tilde{\Sigma}$$

i.e. the estimate of risk premium that implied by the $ARDL + \tilde{\tilde{R}}, \tilde{\Sigma}$ model, which is a function of mean and variance of expected future short-term interest rates and term premia.

The implied risk premia $\hat{\lambda}_t^F$ and $\hat{\lambda}_t^V$ are constructed using the coefficient estimates obtained from the regressions in previous sections.

An augmented Fama regression can be obtained by adding a risk premium term to the original Fama regression:

$$\Delta e_t = \alpha' + \beta'(i_{t-1} - i^*_{t-1}) + \gamma'\hat{\lambda}_t + u'_t \tag{3}$$

Then one can check whether the risk premium can help in solving the UIP Puzzle by testing whether the $\widehat{\beta'}$ is still significantly negative.

6.2 Results

Among the currencies considered in this paper, only the GBP and JPY are found to significantly violate the UIP condition[35], as evidenced by the negative and significant estimate of β. Thus in the following, I will only focus on the cases of GBP and JPY.

As shown in the following table, for the case of GBP the interest rate differential, $i_{t-1} - i^*_{t-1}$, is negatively related to the change of exchange rate, Δe_t, for the period of 1979.01-2009.05. However, once the risk premium estimate, $\hat{\lambda}_t^F$ ($\hat{\lambda}_t^V$), that obtained from the $ARDL{+}F^{1st,2nd}$ model ($ARDL + \tilde{\tilde{R}}, \tilde{\Sigma}$ model) is added to the Fama regression, the coefficient estimate of the interest rate differential, $\widehat{\beta'}$, is no longer negative, but turns to insignificantly positive. This can be taken as an evidence that the UIP Puzzle can possibly be solved with the help of a risk premium term.

[33] An alternative version of the Fama regression is $\Delta e_t = \alpha + \beta(f_{t-1} - e_{t-1}) + u_t$, where f_{t-1} and e_{t-1} are the forward and spot exchange rates.

[34] Alternative solutions could be irrational or biased expectation of future exchange rate.

[35] While for the other currencies, negative values of $\hat{\beta}$ can also be obtained, but they are not significantly different from zero. Thus these cases are not recognized as significant violation of the UIP condition.

However, when the currency risk premium is proxied with the differential of term premia, $tp_t - tp_t^*$, the $\widehat{\beta'}$ still possesses a negative sign. This indicates that the $tp_t - tp_t^*$ may not be a good proxy for the currency risk premium. Or put differently, it fails to capture the information on the relative riskiness for currency holding. This finding also suggests that the traditional wisdom in the international economics literature that the $tp_t - tp_t^*$ is a proxy for the relative cross-country riskiness should be applied with caution.

Table 22: Comparison of Fama and risk-premium augmented Fama regression, GBP

Δe_t	UIP	UIP + λ_t^{tp}	UIP $+\lambda_t^F$	UIP $+\lambda_t^V$
c	-4.636251*	-4.583003	6.886364***	2.430201
$i_{t-1} - i_{t-1}^*$	-1.761350**	-1.396426	0.538926	0.615436
$\hat{\lambda}_t$		0.583589	0.990817***	1.023784***
$adj - R^2$	0.009248	0.006809	0.159325	0.087091
AIC	10.04270	10.04788	9.881162	9.963594

[Referring to section 6.2] Fama regression (UIP) vs risk-premium augmented Fama regression for the GBP over the period from 1979.01 to 2009.05. The first column reports the results for the Fama regression. The rest columns report the results for risk-premium augmented Fama regression, in which a risk premium term is added to the Fama regression. Details of proxies for the risk premium λ_t^{tp}, λ_t^F and λ_t^V can be found in section 6.1.

Similar results are obtained for JPY over the period of 1985.01-2009.05. One difference with the case of GBP is that, the risk premium term, $\hat{\lambda}_t$, is calculated using the factors to the first order or only the first moments (i.e. the means) of the expected risk-free rates and term premia, i.e. the $\hat{\lambda}_t$ is constructed out of the $ARDL + F^{1st}$ model or the $ARDL + \tilde{\bar{R}}$ model for JPY. The reason for omitting higher order factors or moments is that, for the JPY during the full sample period, these terms barely contribute to explaining the variation of the exchange rate[36].

Table 23: Comparison of Fama and risk-premium augmented Fama regression, GBP

Δe_t	UIP	UIP + λ_t^{tp}	UIP $+\lambda_t^{F^{1st}}$	UIP $+\hat{\lambda}_t^V$
c	11.35734***	5.856104	-44.97101**	15.90314***
$i_{t-1} - i_{t-1}^*$	-2.656034**	-1.097689	8.895920**	7.476455*
$\hat{\lambda}_t$		2.494553	0.652328***	0.624643**
$adj - R^2$	0.016855	0.015353	0.048130	0.043769
AIC	10.14960	10.15453	10.12067	10.12524

[Referring to section 6.2] Fama regression (UIP) vs risk-premium augmented Fama regression for the JPY over the period from 1985.01 to 2009.05. The inputs of the table are the same as the ones for the GBP. The only difference is that the estimated risk premium in the third and fourth columns are calculated using the factors to the first order and only the first moments (the means) of the expected risk-free rates and term premia. Results and conclusions are similar as the ones for GBP.

[36]That is to say, the $ARDL+F^{1st}$ $(ARDL+\tilde{\bar{R}}\ )$ model performs as good as the $ARDL+F^{1st,2nd}$ $(ARDL+\tilde{\bar{R}}, \tilde{\Sigma})$ model in fitting the exchange rate data, as measured by the AIC. Thus it is not necessary to keep the higher order (moment) terms in the regression. One can see this in the robustness check section.

In conclusion, the above results for the GBP and JPY suggest that the UIP Puzzle is very likely caused by improperly omitting a risk premium term in the Fama regression. Once the risk premium term is included, the Puzzle is no longer exists. But one pre-condition for potentially solving the UIP Puzzle is that a valid proxy must be selected for the unobservable currency risk premium, that being said, it should properly capture the information on the relative riskiness for holding domestic (foreign) currency.

6.3 Discussion: the relation between $i_{t-1} - i^*_{t-1}$ and Δe_t, revisited

Above results indicate that the risk-premium augmented UIP ($\Delta e_t = (i_{t-1} - i^*_{t-1}) + \lambda_t + u_t$) is a better model than the UIP ($\Delta e_t = (i_{t-1} - i^*_{t-1}) + u_t$) in explaining the actual dynamics of the exchange rate, especially from the perspective of the structural relationship between the exchange rate change and the interest rate differential. The positive value of the $\widehat{\beta'}$ indicates that higher interest rate currency tends to depreciate once the currency risk premium is controlled. This is in line with what the original UIP condition predicts.

However, a crucial question to be answered is that, whether the above finding is in line or in contradiction with the *hard* empirical evidence– 'higher interest rate currency tends to appreciate'? which is suggested by the negative value of $\hat{\beta}$ from the original Fama regression (see table 22 and 23).

Table 24: Correlation between interest rate differentials and estimates of risk premia

Corr.	GBP (79.01-09.05)				JPY (85.01-09.05)		
	$\hat{\lambda}_t^{tp}$	$\hat{\lambda}_t^{F}$	$\hat{\lambda}_t^{V(F)}$		$\hat{\lambda}_t^{tp}$	$\hat{\lambda}_t^{F}$	$\hat{\lambda}_t^{V(F)}$
$i_{t-1} - i^*_{t-1}$	-0.7964	-0.3441	-0.4624		-0.8870	-0.9578	-0.9525
$i_t - i^*_t$	-0.8423	-0.4092	-0.5244		-0.8811	-0.9743	-0.9707

First of all, the negative value of $\hat{\beta}$ should be interpreted with caution: It merely means that the overall unconditional correlation[37] between the interest rate differential, $i_{t-1} - i^*_{t-1}$, and the change of exchange rate, Δe_t, is negative.

Then a closer look at the risk-premium augmented UIP ($\Delta e_t = (i_{t-1} - i^*_{t-1}) + \lambda_t + u_t$) may bring us the fact that the correlation between the interest rate differential, $i_{t-1} - i^*_{t-1}$, and the change of exchange rate, Δe_t, may arise through two channels: First, the direct and structural channel, i.e. the differential in returns on one-period holding of currencies, $i_{t-1} - i^*_{t-1}$; Second, the indirect channel, i.e. the one through the correlation between $i_{t-1} - i^*_{t-1}$ and the risk premium term, $\hat{\lambda}_t$. The correlation from the second channel may be just a statistical relationship between the two without any structural economic connection. As is shown in Table 24, for both of the GBP and JPY the statistical correlation (or maybe more precisely, the linear projection of a non-linear correlation) between $\hat{\lambda}_t^F$ ($\hat{\lambda}_t^V$) and $i_{t-1} - i^*_{t-1}$ are found to be negative[38]. Accordingly, the risk-premium augmented UIP indicates that the overall correlation between $i_{t-1} - i^*_{t-1}$ and Δe_t is actually a combination of two correlations: a positive structural one and a negative statistical one. When the negative one dominants the positive one, the overall correlation tends to be negative.

Thus from a statistical perspective, an answer to the above question is that a positive value of $\widehat{\beta'}$ is not in contradiction with the negative value of $\hat{\beta}$, because they measure a fraction of and the overall of the correlation between the $i_{t-1} - i^*_{t-1}$ and Δe_t, respectively.

[37] Which is a measure of reduced-form rather than a structural-form of linear relationship between two variables.

[38] One possible reason for this negative correlation is that the $i_{t-1} - i^*_{t-1}$ is highly and positively correlated with $i_t - i^*_t$ (0.9469 for GBP and 0.9919 for JPY), and the $i_t - i^*_t$ is negatively correlated with the $\hat{\lambda}_t$. The latter correlation is probably a result of the linear projection of a *non-linear* correlation between $i_t - i^*_t$ and $\hat{\lambda}_t$, which may arise from the fact that the $\hat{\lambda}_t$ is non-linear in factors while the interest rates are linear functions of factors.

7 Concluding Remarks

The objective of the paper has been to investigate whether the expected status of economies plays a role in explaining the dynamics of exchange rates, with a focus on the forward-looking information that is embedded in the term structure of interest rates, through a risk premium channel for holding domestic (foreign) currency. The unobservable risk premium is in particular assumed to be linked non-linearly to bond yield factors, and serves as a determinant of the exchange rate dynamics.

I have applied this setup to nominal dollar exchange rates of eight advanced-economy currencies (AUD, CAD, CHF, GBP, JPY, NOK, NZD and SEK), between the 1990s and 2009. The empirical results generally show the outstanding performance of the proposed model in terms of in-sample goodness of fit and out-of-sample forecast accuracy in the exchange rate changes, which in turn reflect the importance of the risk premium in explaining the dynamics of the exchange rate, and also its non-linearity in bond yield factors. In particular, I find evidence that the exchange rate is predictable at the short-term horizons, ranging from one month to twelve months for all the currencies studied in this paper except the CHF. Although the best forecast horizons may vary across currencies, the proposed model can basically generate better forecast than the benchmark models that either claim no-change in exchange rate value or equal probability of change in direction.

Another result worth noting is that, due to its non-linearity in factors via the risk premium channel, the exchange rate reacts to monetary policy shocks in a time-varying and state-dependent manner. I show evidence that given a certain amount of monetary policy shock, the concurrent response of the exchange rate change will no longer be homogenous as is the case with linear models, but may change over time in the amount and even the direction, depending on the perceived present and future status of the economies.

An important implication of being able to quantify the unobservable risk premium is that one can utilize the model-implied quantity of risk premium to test exchange rate models that has been otherwise impossible. I find empirical evidence for the GBP and JPY, in which cases the UIP condition is significantly violated, adding a risk premium term into the Fama regression can help in solving the UIP Puzzle. This in turn suggests that, everything else being equal, the higher interest rate currency tends to depreciate. However, this does not contradict the empirical findings that high interest rate currency tends to appreciate, as the interest rate differential is a relatively weak determinant of the exchange rate dynamics, and its negative correlation (not necessarily causal) with the risk premium is the underlying reason for the stylized but puzzling findings.

Still, many open questions and directions for future research remain. Firstly, the information about expectations that are embedded in other variables may also be relevant for the exchange rate dynamics and thus can be included in an exchange rate model. Such variables could include commodity price, policy uncertainty or financial soundness. These variables may further be allowed to interact with (instead of being orthogonal to) term structure factors. Secondly, using a Bayesian model averaging approach to combine the proposed model with other well-performing models, one can obtain a best forecast model for the exchange rate of a specific currency. Thirdly, it may prove to be a fruitful research area if one substitutes the dynamic exchange rate component (e.g UIP) in multivariate econometric models or now open economic models (especially in the DSGE context) with the proposed model in this paper, in order to obtain more realistic moments, forecasts, or impulse responses for the exchange rate and related macroeconomic variables.

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Appendix

Clark and West (2006) test

The null hypothesis is that the random walk model, $\Delta e_t = \epsilon_t$, is the correct model for the time seriesΔe_t, while the alternative hypothesis is that the proposed model, $\Delta e_t = X_t'\beta + \epsilon_t$, is the correct model. In words, $\beta = 0$ in the null model, $\beta \neq 0$ in the alternative model.

For one-period-ahead forecasts, the test statistics is defined as the difference between mean squared prediction errors, adjusted by an additional term, $s_1 \equiv \sigma_1^2 - (\sigma_2^2 - adj.) = \hat{\sigma}_1^2 - [\hat{\sigma}_2^2 - P^{-1}\Sigma_{t=0}^{p-1}(\hat{x}_{t+1}\hat{\beta}_t)^2]$, where the $\hat{\sigma}_1^2 = P^{-1}\Sigma_{t=0}^{p-1}\Delta e_{t+1}^2$, and $\hat{\sigma}_2^2 = P^{-1}\Sigma_{t=0}^{p-1}(\Delta e_{t+1} - \hat{x}_{t+1}\hat{\beta}_t)^2$; the subscript t for $\hat{\beta}_t$ is due to a rolling estimation of the coefficients; $\hat{x}_{t+1}$ is the estimate of x_{t+1} due to out-of-sample forecast; P is the number of one-period-ahead forecasts. It can be proved that the test statistics, s_1, follows $\sqrt{P}s_1 \sim N(0, V_1)$, where in finite samples, $s_1 = P^{-1}\Sigma_{t=0}^{p-1}\hat{s}_{1,t}$, and $\hat{V}_1 = P^{-1}\Sigma_{t=0}^{p-1}(\hat{s}_{1,t} - s_1)^2$, where $\hat{s}_{1,t} = 2\Delta e_{t+1}\hat{x}_{t+1}\hat{\beta}_t$.

For multiple-period-ahead forecasts, the adjustment term will be $adj. = (P-h+1)^{-1}\Sigma_{t=0}^{p-h}[(\hat{x}_{t+1} + \hat{x}_{t+2} + ... + \hat{x}_{t+h})\hat{\beta}_t]^2$, and the test statistics, s_h, follows $\sqrt{P}s_h \sim N(0, V_h)$. In finite samples, $s_h = (P - h + 1)^{-1}\Sigma_{t=0}^{p-h}\hat{s}_{h,t}$, and $\hat{V} = (P - h + 1)^{-1}\Sigma_{t=0}^{p-h}(\hat{s}_{h,t} - s_h)^2$, where $\hat{s}_{h,t} = 2\Delta e_{t+h}(\hat{x}_{t+1} + \hat{x}_{t+2} + ... + \hat{x}_{t+h})\hat{\beta}_t$.

The difference between the Clark and West (2006) test and the Diebold and Mariano (1995) test is the adjustment term, $adj. \equiv P^{-1}\Sigma_{t=0}^{p-1}(\hat{x}_{t+1}\hat{\beta}_t)^2$. Clark and West (2006) argue that the mean of the Diebold and Mariano (1995) test $s \equiv \hat{\sigma}_1^2 - \hat{\sigma}_2^2$ is negative rather than zero, thus the $\sqrt{P}s \sim N(0, V)$ provides a poor approximation to the actual finite sample distribution of $\hat{\sigma}_1^2 - \hat{\sigma}_2^2$. This argument is supported by evidence obtained from extensive simulations. The reason for the negative mean is that in $\hat{\sigma}_1^2 - \hat{\sigma}_2^2 = P^{-1}\Sigma_{t=0}^{p-1}2\Delta e_{t+1}\hat{x}_{t+1}\hat{\beta}_t - [P^{-1}\Sigma_{t=0}^{p-1}(\hat{x}_{t+1}\hat{\beta}_t)^2]$, the former term (relates to the mean of the $\hat{\beta}_t$) tends to be zero and the latter term (relates to the variance of the $\hat{\beta}_t$) tends to be positive if the null is true, i.e. $\hat{\beta}_t = 0$. The intuition behind it is that the mean squared prediction errors of the alternative model, $\hat{\sigma}_2^2$, is pushed upwards by the noise term, $adj.$. Thus the Clark and West (2006) test should be employed in this case.

Diebold and Mariano (1995) sign test

The null hypothesis is that no forecasting performance of the proposed model, i.e. under the null, the probability of changes in the right direction predicted by the proposed model is not bigger than the naive model.

The number of correct predictions in a sample of size P follows the binomial distribution with parameters P and $1/2$ under the null. Thus the test statistics can be defined as $s_2 = \Sigma_{t=1}^{p}I_+(d_t)$, where $I_+(d_t)$ is a state-contingent function that takes value 1 when the predicted direction is correct, i.e., $d_t > 0$. The asymptotic distribution of the standardized statistics $s_{2a} = \frac{s_2 - 0.5P}{\sqrt{0.25P}}$ is standard normal, $N(0, 1)$.

Tables and Figures

Table 3: Evaluation on how well pre-believed models can fit the data

	Models					
	M1 (PPP)	M2 (FP)	M3 (UIP)	M4 (TR)	M5 (RW)	M6 (ARDL)
AUD						
90.01-09.05	0.002847	0.030202	-0.004322	0.010868	-	**0.031807** (3,1)
CAD						
99.01-09.05	0.009290	0.092047	0.063539	0.039803	-	**0.179898** (6,5)
CHF						
96.01-09.05	-0.005115	**0.040054**	0.009539	0.025011	-	0.019749(8,3)
						0.205917*
GBP						
93.01-06.04[39]	-0.001037	0.030668	-0.003049	-0.005328	-	**0.055758** (7,2)
JPY						
96.01-09.05	0.015983	**0.063908**	0.002030	-0.003745	-	0.000779 (6,5)
						0.208859*
NOK						
98.01-09.05	0.018394	**0.033015**	-0.006952	-0.008798	-	0.013185 (1,1)
NZD						
90.01-09.05	0.021886	0.010411	-0.004327	0.014445	-	**0.130208 (3,1)**
SEK						
97.01-09.05[40]	0.024112	0.040760	-0.000447	0.000592	-	**0.150437 (2,1)**

[Referring to section 3.2.2] Model selection of the pre-believed models of exchange rate changes, $e_{t-0} - e_{t-1}$. The M1-M6 are defined in section 2.3. For each currency, $Adj. - R^2$s are report for all models. (p,q) in the last column for M6 indicates the lag orders in the ARDL model that selected according to the AIC. The numbers in bold represent the highest value of $Adj. - R^2$ for each currency. For CHF and JPY, the number with a * is the $Adj. - R^2$ after including the factors in the regression, the ARDL+factor model has a higher $Adj. - R^2$ than its counterparts.

Table 4: Regression results of linear and non-linear models for the exchange rate. GBP and SEK

	GBP (93.01-09.05)			SEK (93.01-09.05)		
Δe_t	$ARDL$	$ARDL + F^{1^{st}}$	$ARDL + F^{1^{st},2^{nd}}$	$ARDL$	$ARDL + F^{1^{st}}$	$ARDL + F^{1^{st},2^{nd}}$
$constant$	5.597804	-8.181222	-14.49341*	-0.206939	-25.03893*	-49.62301***
Δe_{t-1}	0.062807	-0.096913	-0.158929***	0.442030***	0.373680***	0.278751***
Δe_{t-2}				-0.172505**	-0.191793***	-0.261047***
Δe_{t-3}						
Δe_{t-4}						
Δe_{t-5}						
Δe_{t-6}						
Δe_{t-7}						
Δe_{t-8}						
$i_{t-1} - i^*_{t-1}$	3.365628**	9.746557**	12.94162**	-9.641009	-0.301176	-11.04477
$i_{t-2} - i^*_{t-2}$				9.287479	3.620617	12.75431
$i_{t-3} - i^*_{t-3}$						
$i_{t-4} - i^*_{t-4}$						
$i_{t-5} - i^*_{t-5}$						
$f^*_{1,t}$		3.270911***	4.093740*		2.660618**	7.518157***
$f^*_{2,t}$		7.008977***	4.170401**		2.687876***	4.857086**
$f^*_{3,t}$		-3.626806***	0.704236		-0.411540	-1.730068
$f_{1,t}$		4.481000**	-3.269649		5.494941**	-7.572669
$f_{2,t}$		-3.940776***	-7.816412*		-3.925754**	-9.500255***
$f_{3,t}$		-4.513902**	-12.49997**		-1.519771	-8.983435**
$f^*_{1,t} \cdot f^*_{1,t}$			0.027558			-0.236658
$f^*_{1,t} \cdot f^*_{2,t}$			0.532879			-0.085270
$f^*_{1,t} \cdot f^*_{3,t}$			-0.433638			0.050497
$f^*_{2,t} \cdot f^*_{2,t}$			-0.063122			-0.170399
$f^*_{2,t} \cdot f^*_{3,t}$			1.091648**			0.025376
$f^*_{3,t} \cdot f^*_{3,t}$			-0.352737			0.026191
$f_{1,t} \cdot f_{1,t}$			-2.414802*			-2.175842
$f_{1,t} \cdot f_{2,t}$			-1.090505			-2.725851***
$f_{1,t} \cdot f_{3,t}$			-1.936353*			-2.457503*
$f_{2,t} \cdot f_{2,t}$			-0.349361			0.152370
$f_{2,t} \cdot f_{3,t}$			-1.209779***			-1.405457***
$f_{3,t} \cdot f_{3,t}$			-0.473133			-0.853322*
$Adj. - R^2$	0.014807	0.154524	0.188009	0.162717	0.208140	0.260814
AIC	9.577392	9.453952	9.469410	9.598535	9.571922	9.558223
Prob $\chi^2_{1^{st}}$	-	0.0000***	0.0017 ***	-	0.0505***	0.0001***
Prob$\chi^2_{2^{nd}}$	-	-	0.0000***	-	-	0.0000***
Prob$\chi^2_{1^{st},\,2^{nd}}$	-	-	0.0000***	-	-	0.0000***

[Referring to section 3.2.4] Comparison of models on how well they can fit the data, for the GBP and SEK during the period of 1993.01-2009.05. The notations used in the tables are $ARDL$: Autoregressive distributed lags model discussed in section 2.3; $ARDL+F^{1^{st}}$: ARDL model augmented with factors in linear form; $ARDL + F^{1^{st},2^{nd}}$: ARDL model augemented with factor in non-linear form (including the second order factors). The $Adj. - R^2$ and AIC are reported for each model and each currency. Especially, the nulls on the $C = 0_{6\times1}$, $D = 0_{6\times6}$, C and D are jointly zero are tested respectively, probabilities of the χ^2 statistics are reported. The '***', '**', '*' denote significance at 1%, 5% and 10% level respectively. All the inferences are adjusted with the Newey-West autocorrelation and heteroskedasticity consistant standard errors. The lag orders of the $ARDL$ model are selected by the AIC.

Table 5: Regression results of linear and non-linear models for the exchange rate. AUD and NZD

	AUD (90.01-09.05)				NZD (90.01-09.05)		
Δe_t	$ARDL$	$ARDL + F^{1^{st}}$	$ARDL + F^{1^{st},2^{nd}}$		$ARDL$	$ARDL + F^{1^{st}}$	$ARDL + F^{1^{st},2^{nd}}$
$constant$	0.495515	-25.98079**	-9.012043		0.456397	-2.480006	-9.903086
Δe_{t-1}	0.182373*	0.107767	0.020697		0.339334***	0.284310***	0.190668***
Δe_{t-2}	-0.083594	-0.131261**	-0.141496				
Δe_{t-3}	0.146119***	0.082473	0.025153**				
Δe_{t-4}							
Δe_{t-5}							
Δe_{t-6}							
Δe_{t-7}							
Δe_{t-8}							
$i_{t-1} - i^*_{t-1}$	0.153410	0.400601	4.021980		0.113527	0.941270	-3.291492
$i_{t-2} - i^*_{t-2}$							
$i_{t-3} - i^*_{t-3}$							
$i_{t-4} - i^*_{t-4}$							
$i_{t-5} - i^*_{t-5}$							
$f^*_{1,t}$		1.220156	0.063879			3.034814*	1.963514
$f^*_{2,t}$		-10.46787**	-19.53336			1.237132	2.208666
$f^*_{3,t}$		-6.311557	-16.36752**			-1.791633	-5.420379***
$f_{1,t}$		8.545139**	4.477018**			4.472535**	0.699866
$f_{2,t}$		-5.376314**	-1.689460			-0.543845	-1.381435
$f_{3,t}$		1.211274	-1.367353			-0.150251	-3.379949
$f^*_{1,t} \cdot f^*_{1,t}$			-0.147412				0.194297
$f^*_{1,t} \cdot f^*_{2,t}$			-0.577908				-0.223674
$f^*_{1,t} \cdot f^*_{3,t}$			-0.064977				0.084312
$f^*_{2,t} \cdot f^*_{2,t}$			-4.105370***				-0.382260**
$f^*_{2,t} \cdot f^*_{3,t}$			-5.037975***				0.716588**
$f^*_{3,t} \cdot f^*_{3,t}$			-1.412484***				0.079214
$f_{1,t} \cdot f_{1,t}$			1.696218				0.189727
$f_{1,t} \cdot f_{2,t}$			-1.884283***				-1.995192***
$f_{1,t} \cdot f_{3,t}$			1.670596				0.409905
$f_{2,t} \cdot f_{2,t}$			0.774623**				0.131581
$f_{2,t} \cdot f_{3,t}$			0.096388				-1.061442***
$f_{3,t} \cdot f_{3,t}$			-0.182368				-0.482494
$Adj. - R^2$	0.031807	0.104885	0.214802		0.105866	0.128891	0.177333
AIC	10.08084	10.02720	9.943615		9.646137	9.645118	9.635847
Prob χ^2_{1st}	-	0.0010 ***	0.0325 **		-	0.0472 **	0.0003 **
Probχ^2_{2nd}	-	-	0.0000 ***		-	-	0.0001 ***
Prob$\chi^2_{1^{st},\,2^{nd}}$	-	-	0.0000 ***		-	-	0.0000 ***

[Referring to section 3.2.4] Comparison of models on how well they can fit the data, for the AUD and NZD during the period of 1990.01-2009.05. The notations used here are the same as in the previous table.

Table 6: Regression results of linear and non-linear models for the exchange rate. JPY and CHF

	JPY (96.01-09.05)			CHF (96.01-09.05)		
Δe_t	$ARDL$	$ARDL+F^{1^{st}}$	$ARDL+F^{1^{st},2^{nd}}$	$ARDL$	$ARDL+F^{1^{st}}$	$ARDL+F^{1^{st},2^{nd}}$
$constant$	13.85854*	-121.1385***	-52.20926	8.031115	-83.15247***	-82.39075***
Δe_{t-1}	-0.029807	-0.101853	-0.193388***	-0.028994	-0.165121	-0.300112**
Δe_{t-2}	0.052889	0.003827	-0.067417	-0.033358	-0.161828**	-0.258148***
Δe_{t-3}	0.051886	0.021780	-0.037383	-0.002256	-0.153842**	-0.257753***
Δe_{t-4}	-0.126041	-0.111242	-0.165177**	-0.158438**	-0.291547***	-0.396227***
Δe_{t-5}	-0.141809*	-0.152784*	-0.259870***	0.018951	-0.092576	-0.159252*
Δe_{t-6}	-0.064520	-0.075488	-0.167027**	-0.058431	-0.115879	-0.179641*
Δe_{t-7}				-0.094092	-0.132712	-0.179531**
Δe_{t-8}				-0.059452	-0.080268	-0.167273**
$i_{t-1}-i^*_{t-1}$	-9.630588	44.89281*	35.47422*	4.285005	22.00541	8.868533
$i_{t-2}-i^*_{t-2}$	23.60070	5.797371	16.46784	-38.05468	-42.19050	-32.33669
$i_{t-3}-i^*_{t-3}$	-25.51819	-25.96010	-32.99534	30.00714	38.54862**	28.35007
$i_{t-4}-i^*_{t-4}$	25.93667	32.16958	41.19106			
$i_{t-5}-i^*_{t-5}$	-17.95750	-31.65515*	-42.98444*			
$f^*_{1,t}$		-0.531379	-4.033030		2.084810	8.407364****
$f^*_{2,t}$		20.48985***	15.08973*		-2.549035	1.650923
$f^*_{3,t}$		-2.082991	-7.516744		1.762246	0.058317
$f_{1,t}$		-1.303154	-16.00488*		10.04627***	-15.12762*
$f_{2,t}$		-3.162448	6.227034		-7.958053***	-22.60410***
$f_{3,t}$		-18.77178***	-20.36388***		-8.498777**	-23.82422***
$f^*_{1,t}\cdot f^*_{1,t}$			-0.583863**			-0.133832
$f^*_{1,t}\cdot f^*_{2,t}$			-0.699953			-0.047795
$f^*_{1,t}\cdot f^*_{3,t}$			-6.755596***			1.314841*
$f^*_{2,t}\cdot f^*_{2,t}$			5.604472			0.745474
$f^*_{2,t}\cdot f^*_{3,t}$			-1.572984			1.908858
$f^*_{3,t}\cdot f^*_{3,t}$			-3.972497			0.232905
$f_{1,t}\cdot f_{1,t}$			-2.902119			0.429247
$f_{1,t}\cdot f_{2,t}$			-4.063325***			-6.051722***
$f_{1,t}\cdot f_{3,t}$			-2.550657			0.194466
$f_{2,t}\cdot f_{2,t}$			0.731746			-0.722317
$f_{2,t}\cdot f_{3,t}$			-0.478346			-3.976959***
$f_{3,t}\cdot f_{3,t}$			-1.215976*			-0.935704
$Adj.-R^2$	0.000779	0.133813	0.208859	0.019749	0.117667	0.205917
AIC	10.13415	10.02471	9.995506	10.14179	10.06998	10.02602
Prob $\chi^2_{1^{st}}$	-	0.0000 ***	0.0116 **	-	0.0007 ***	0.0000 ***
Prob$\chi^2_{2^{nd}}$	-	-	0.0000 ***	-	-	0.0000 ***
Prob$\chi^2_{1^{st},2^{nd}}$	-	-	0.0000 ***	-	-	0.0000 ***

[Referring to section 3.2.4] Comparison of models on how well they can fit the data, for the JPY and CHF during the period of 1996.01-2009.05. The notations used here are the same as in the previous table.

Table 7: Regression results of linear and non-linear models for the exchange rate. NOK and CAD

	NOK (98.01-09.05)				CAD (99.01-09.05)		
Δe_t	$ARDL$	$ARDL + F^{1^{st}}$	$ARDL + F^{1^{st},2^{nd}}$		$ARDL$	$ARDL + F^{1^{st}}$	$ARDL + F^{1^{st},2^{nd}}$
$constant$	1.632000	-3.828102	-37.57228		2.285305	-82.81479***	-116.6104***
Δe_{t-1}	0.165972	0.061508	-0.098720		0.307091***	0.266602***	0.080569
Δe_{t-2}					0.057781	0.024263	-0.080258
Δe_{t-3}					-0.119025	-0.138867	-0.227276**
Δe_{t-4}					0.239679**	0.208942**	0.093451
Δe_{t-5}					-0.014012	0.018919	0.027341
Δe_{t-6}					-0.249555**	-0.256982***	-0.211037**
Δe_{t-7}							
Δe_{t-8}							
$i_{t-1} - i^*_{t-1}$	0.272399	-5.784154	-4.018731		1.161302	11.67200	4.789903
$i_{t-2} - i^*_{t-2}$					-17.99161	-18.94077	-15.21391
$i_{t-3} - i^*_{t-3}$					23.36692*	25.26767*	29.69881**
$i_{t-4} - i^*_{t-4}$					21.97614**	20.13731	22.42479*
$i_{t-5} - i^*_{t-5}$					-29.29930***	-35.11951***	-45.57523***
$f^*_{1,t}$		-2.272233	-5.305696			5.348997***	9.151849*
$f^*_{2,t}$		-5.509398**	-2.565921			8.252400***	15.79980**
$f^*_{3,t}$		0.879457	2.502539			-0.768644	-12.03708*
$f_{1,t}$		9.566488***	-10.71442			4.870551*	1.576491
$f_{2,t}$		-0.808787	-7.723462			-9.146808***	-20.39541**
$f_{3,t}$		6.677506	-7.351171			-3.182890	-8.469554
$f^*_{1,t} \cdot f^*_{1,t}$			0.296081				0.129863
$f^*_{1,t} \cdot f^*_{2,t}$			1.545800*				-0.292719
$f^*_{1,t} \cdot f^*_{3,t}$			0.967347				-0.892027
$f^*_{2,t} \cdot f^*_{2,t}$			-0.104119				1.994429**
$f^*_{2,t} \cdot f^*_{3,t}$			-1.976031*				-3.373804**
$f^*_{3,t} \cdot f^*_{3,t}$			1.456566**				3.230163***
$f_{1,t} \cdot f_{1,t}$			-2.162639				-6.520114***
$f_{1,t} \cdot f_{2,t}$			-4.216904***				0.455725
$f_{1,t} \cdot f_{3,t}$			-2.493055				-6.356443***
$f_{2,t} \cdot f_{2,t}$			-0.025452				-0.869032
$f_{2,t} \cdot f_{3,t}$			-2.264375**				0.226405
$f_{3,t} \cdot f_{3,t}$			-1.427416*				-2.455694***
$Adj. - R^2$	0.013185	0.074797	0.143231		0.179898	0.241420	0.353113
AIC	10.10530	10.08262	10.08251		9.187586	9.151046	9.064817
Prob $\chi^2_{1^{st}}$	-	0.0261**	0.2768		-	0.0251**	0.0037 ***
Prob$\chi^2_{2^{nd}}$	-	-	0.0477 **		-	-	0.0019 ***
Prob$\chi^2_{1^{st},\,2^{nd}}$	-	-	0.0086 ***		-	-	0.0020 ***

[Referring to section 3.2.4] Comparison of models on how well they can fit the data, for the NOK and CAD during the period of 1998.01-2009.05 and 1999.01-2009.05. The notations used here are the same as in the previous table.

Table 8: Forecast evaluation and comparison. GBP and SEK

GBP Forecasts, $\widehat{\Delta e_{t+h}}|I_t$

RMSE	*ARDL*	... + $F^{1^{st}}$	... + $F^{1^{st},2^{nd}}$	*RW*		**Direct.**	*ARDL*	... + $F^{1^{st}}$	... + $F^{1^{st},2^{nd}}$	*Naive*
1	0.0291	0.026*	0.0262**	0.0289		1	0.5333	0.5833	0.55	0.5
2	0.0468	0.0408*	0.0425*	0.0458		2	0.5085	0.5932	0.5763	0.5
3	0.0641	0.0569*	0.06*	0.0615		3	0.431	0.6034	0.7069***	0.5
4	0.0805	0.073	0.0781	0.076		4	0.386	0.6316*	0.6842***	0.5
5	0.097	0.0907	0.0995	0.0902		5	0.3393	0.625*	0.6786***	0.5
6	0.1108	0.1071	0.1066	0.1011		6	0.2909	0.5818	0.7091***	0.5
7	0.1235	0.1207	0.1231	0.111		7	0.2593	0.5556	0.6852***	0.5
8	0.1344	0.1289	0.128	0.119		8	0.2075	0.4906	0.6792***	0.5
9	0.1438	0.1391	0.1391	0.1255		9	0.1731	0.4231	0.6731**	0.5
10	0.1519	0.1479	0.1512	0.1308		10	0.1765	0.3529	0.6275*	0.5
11	0.1573	0.1532	0.1603	0.1343		11	0.16	0.34	0.6	0.5
12	0.1627	0.1578	0.1682	0.1379		12	0.1224	0.2449	0.551	0.5

SEK Forecasts, $\widehat{\Delta e_{t+h}}|I_t$

RMSE	*ARDL*	... + $F^{1^{st}}$	... + $F^{1^{st},2^{nd}}$	*RW*		**Direct.**	*ARDL*	... + $F^{1^{st}}$	... + $F^{1^{st},2^{nd}}$	*Naive*
1	0.0302***	0.0296***	0.0289***	0.0323		1	0.5862	0.6207*	0.6379**	0.5
2	0.0567	0.0559	0.0541***	0.0546		2	0.5088	0.5263	0.5789	0.5
3	0.075	0.0753	0.0728	0.0708		3	0.5	0.4821	0.5893	0.5
4	0.0892	0.0914	0.0877	0.0854		4	0.4545	0.5818	0.6909***	0.5
5	0.1048	0.1103	0.1097	0.1008		5	0.4444	0.5741	0.6667**	0.5
6	0.1206	0.1311	0.1357	0.1149		6	0.3962	0.434	0.6038	0.5
7	0.1345	0.1499	0.156	0.1264		7	0.3462	0.3269	0.5962	0.5
8	0.145	0.1652	0.1721	0.1343		8	0.3529	0.2941	0.5294	0.5
9	0.1525	0.1791	0.1914	0.1389		9	0.28	0.22	0.48	0.5
10	0.1577	0.1898	0.2081	0.1415		10	0.2653	0.1429	0.4082	0.5
11	0.1607	0.197	0.2201	0.1424		11	0.3333	0.1458	0.3333	0.5
12	0.1645	0.2029	0.2297	0.1429		12	0.3404	0.1277	0.3191	0.5

[Referring to section 4.1.2] Evaluation and comparison of models on how well they can forecast the exchange rate change/return, for the GBP and SEK during the sample period of 1993.01-2009.05. Rolling estimation for the GBP with a window size of 137. Recursive estimation for the SEK. Forecast window 2004.06-2009.05. The notations used in the tables are *ARDL*: Autoregressive distributed lags model discussed in section 4.1.1; ... + $F^{1^{st}} \equiv ARDL + F^{1^{st}}$: ARDL model augmented with factors in linear form; ... + $F^{1^{st},2^{nd}} \equiv ARDL + F^{1^{st},2^{nd}}$: ARDL model augmented with factor in non-linear form (including the second order factors). *RM*: Random walk model; *Naive* : Model predicts equal probability of change in direction. 1-12 in the left column are the respective forecast horizons. **RMSE**: root of mean squared forecast errors. **Direct.**: the probability of changes that have been predicted in the right direction. Clark and West (2006) test is used for RMSE. Diebold and Mariano test (1995) is used for Direction of Change. ’***’,’*’,’*’ indicate being significant at 1%, 5% and 10% level.

Table 9: Forecast evaluation and comparison. AUD and NZD

AUD Forecasts, $\widehat{\Delta e_{t+h}}|I_t$

RMSE	*ARDL*	$... + F^{1^{st}}$	$... + F^{1^{st},2^{nd}}$	*RW*		**Direct...**	*ARDL*	$... + F^{1^{st}}$	$... + F^{1^{st},2^{nd}}$	*Naive*
1	0.0437	0.046	0.0506	0.0423		1	0.5167	0.5833	*0.55*	0.5
2	0.0724	0.0726	0.077	0.0658		2	0.4407	0.4915	0.5254	0.5
3	0.0911	0.0892	0.0935	0.0828		3	0.5	0.3448	0.569	0.5
4	0.1119	0.1083	0.1098	0.0989		4	0.4211	0.3684	0.5263	0.5
5	0.1309	0.1275	0.1264	0.1129		5	0.375	0.3571	0.4464	0.5
6	0.1506	0.1455	0.1385	0.1262		6	0.3455	0.2364	0.3455	0.5
7	0.1638	0.1628	0.1547	0.1336		7	0.3519	0.1852	0.3148	0.5
8	0.1706	0.175	0.1689	0.1393		8	0.3396	0.2453	0.283	0.5
9	0.1802	0.1862	0.1813	0.1435		9	0.3077	0.1731	0.2308	0.5
10	0.1862	0.1933	0.1916	0.1447		10	0.3137	0.2157	0.1961	0.5
11	0.1913	0.1992	0.2021	0.1461		11	0.36	0.26	0.2	0.5
12	0.1953	0.2038	0.21	0.1474		12	0.3265	0.2653	0.2041	0.5

NZD Forecasts, $\widehat{\Delta e_{t+h}}|I_t$

RMSE	*ARDL*	$... + F^{1^{st}}$	$... + F^{1^{st},2^{nd}}$	*RW*		**Direct.**	*ARDL*	$... + F^{1^{st}}$	$... + F^{1^{st},2^{nd}}$	*Naive*
1	**0.0325****	**0.032****	***0.0324*****	0.0341		1	0.6	**0.6667*****	**0.6667*****	0.5
2	0.0577	0.0563	0.0578	0.0571		2	**0.661****	**0.6441****	**0.7119*****	0.5
3	0.0759	0.0735	0.0759	0.0736		3	0.5345	**0.6207***	**0.7069*****	0.5
4	0.0944	0.0898	0.0923	0.0904		4	0.4561	0.4912	0.5965	0.5
5	0.1171	0.1109	0.1157	0.1084		5	0.4107	0.4821	0.5536	0.5
6	0.1372	0.1297	0.1387	0.1233		6	0.3818	0.4364	0.5091	0.5
7	0.1547	0.1445	0.153	0.1357		7	0.4074	0.463	0.537	0.5
8	0.1706	0.1597	0.1705	0.146		8	0.3774	0.4151	0.5094	0.5
9	0.1844	0.1735	0.1872	0.1543		9	0.3462	0.3654	0.4615	0.5
10	0.1958	0.1838	0.2015	0.1604		10	0.3529	0.3529	0.4118	0.5
11	0.2037	0.1919	0.213	0.1641		11	0.32	0.34	0.4	0.5
12	0.2105	0.1994	0.2227	0.1671		12	0.3061	0.3673	0.4082	0.5

[Referring to section 4.1.2] Evaluation and comparison of models on how well they can forecast the exchange rate change/return, for the AUD and NZD during the sample period of 1990.01-2009.05. Rolling estimation for the AUD and NZD with a window size of 173. Forecast window 2004.06-2009.05. The notations used here are the same as in the previous table. Clark and West (2006) test is used for RMSE. Diebold and Mariano test (1995) is used for Direction of Change. ’***’,’*’,’*’ indicate being significant at 1%, 5% and 10% level.

Table 10: Forecast evaluation and comparison. JPY and CHF

JPY Forecasts, $\widehat{\Delta e_{t+h}}|I_t$

RMSE	$ARDL$	... + $F^{1^{st}}$	... + $F^{2^{nd}}$	RW		**Direct.**	$ARDL$	... + $F^{1^{st}}$	... + $F^{2^{nd}}$	$Naive$
1	0.0379	0.0398	0.0394	0.0274		1	0.4167	0.55	0.4833	0.5
2	0.0478	0.0541	0.0537	0.0404		2	0.3898	0.4746	0.5424	0.5
3	0.0564	0.06	0.0629	0.0517		3	0.4828	0.5172	**0.6379****	0.5
4	0.0637	0.0657	0.0684	0.0591		4	0.5439	0.5439	**0.6316****	0.5
5	0.069	0.0662	0.0701	0.0646		5	0.4821	0.6071	**0.6964*****	0.5
6	0.0727	**0.0662*****	**0.0636*****	0.0679		6	0.5091	**0.6727****	**0.7636*****	0.5
7	0.0761	**0.0696*****	**0.0624*****	0.0713		7	0.5	**0.7778*****	**0.8704*****	0.5
8	0.0807	0.0757	**0.0669*****	0.0749		8	0.4906	**0.7736*****	**0.8491*****	0.5
9	0.0859	0.083	**0.0703*****	0.0785		9	0.5192	**0.8077*****	**0.8846*****	0.5
10	0.0921	0.0934	**0.0783*****	0.0817		10	0.4902	**0.8039*****	**0.8235*****	0.5
11	0.1012	0.1006	0.091	0.0867		11	0.48	**0.8*****	**0.78*****	0.5
12	0.1104	0.1065	0.102	0.0919		12	0.3878	**0.7551*****	**0.7551*****	0.5

CHF Forecasts, $\widehat{\Delta e_{t+h}}|I_t$

RMSE	$ARDL$	... + $F^{1^{st}}$	... + $F^{1^{st},2^{nd}}$	RW		**Direct.**	$ARDL$	... + $F^{1^{st}}$	... + $F^{1^{st},2^{nd}}$	$Naive$
1	0.0364	0.038	0.0394	0.0339		1	0.4333	0.45	0.4333	0.5
2	0.0505	0.056	0.057	0.0439		2	0.5932	0.5254	*0.5763*	0.5
3	0.0604	0.0711	0.0734	0.0509		3	0.5	0.3448	0.4138	0.5
4	0.0724	0.0846	0.0859	0.0603		4	0.5439	0.4386	*0.5263*	0.5
5	0.0819	0.0945	0.0949	0.0675		5	0.5	0.4286	0.5357	0.5
6	0.0915	0.1038	0.1039	0.075		6	0.4909	0.4364	0.5091	0.5
7	0.1006	0.1143	0.1103	0.0804		7	0.4259	0.3519	0.5185	0.5
8	0.1082	0.124	0.1199	0.0838		8	0.3962	0.3019	0.4717	0.5
9	0.1127	0.1311	0.126	0.0854		9	0.4423	0.3462	0.5385	0.5
10	0.1181	0.1395	0.1336	0.0877		10	0.3529	0.2941	0.4902	0.5
11	0.1221	0.1458	0.1432	0.0893		11	0.3	0.24	0.44	0.5
12	0.1249	0.1496	0.1488	0.0911		12	0.3469	0.2653	0.4898	0.5

[Referring to section 4.1.2] Evaluation and comparison of models on how well they can forecast the exchange rate change/return, for the JPY and CHF during the sample period of 1996.01-2009.05. Rolling estimation with a window size of 101. Forecast window 2004.06-2009.05. The notations used in the tables: $ARDL$: Autoregressive distributed lags model discussed in section 4.1.1; $...+F^{1^{st}} \equiv ARDL+F^{1^{st}}$: ARDL model augmented with factors in linear form; $... + F^{1^{st},2^{nd}} \equiv ARDL + F^{1^{st},2^{nd}}$: ARDL model augmented with factor in non-linear form (including the second order factors)[a]. RM: Random walk model; $Naive$: Model predicts equal probability of change in direction. 1-12 in the left column are the respective forecast horizons. **RMSE**: root of mean squared forecast errors. **Direct.**: the probability of changes that have been predicted in the right direction. Clark and West (2006) test is used for RMSE. Diebold and Mariano test (1995) is used for Direction of Change. '***','*','*' indicate being significant at 1%, 5% and 10% level.

[a]Note that the forecasts by $ARDL + F^{2^{nd}}$ model are reported for JPY

Table 11: Forecast evaluation and comparison. NOK and CAD

NOK Forecasts, $\widehat{\Delta e_{t+h}}|I_t$

RMSE	*ARDL*	$... + F^{1^{st}}$	$... + F^{1^{st},2^{nd}}$	*RW*		**Direct.**	*ARDL*	$... + F^{1^{st}}$	$... + F^{1^{st},2^{nd}}$	*Naive*
1	0.0357	0.0367	**0.0338***	0.0353		1	0.4894	0.4681	0.5532	0.5
2	0.0627	0.0651	0.0600	0.0595		2	0.5	0.5	0.6087	0.5
3	0.0865	0.0912	0.0871	0.0799		3	0.4667	*0.5556*	**0.7111*****	0.5
4	0.1072	0.1115	0.1125	0.0972		4	0.5	*0.5909*	**0.6818****	0.5
5	0.1264	0.1309	0.1383	0.1123		5	0.4884	*0.5581*	**0.6744****	0.5
6	0.1424	0.1485	0.1645	0.1237		6	0.4524	*0.5476*	**0.6429***	0.5
7	0.1566	0.1552	0.1819	0.1327		7	0.4146	*0.5366*	**0.7073****	0.5
8	0.1696	0.1483	0.1856	0.1405		8	0.35	0.5	**0.7500*****	0.5
9	0.1808	0.1595	0.2018	0.1463		9	0.3077	0.4359	**0.7436*****	0.5
10	0.1887	0.1696	0.2247	0.1499		10	0.2368	0.2895	**0.7368*****	0.5
11	0.1946	0.1778	0.2471	0.1525		11	0.2432	0.2162	**0.7297****	0.5
12	0.1995	0.1863	0.2698	0.1548		12	0.2222	0.25	**0.7222****	0.5

CAD Forecasts, $\widehat{\Delta e_{t+h}}|I_t$

RMSE	*ARDL*	$... + F^{1^{st}}$	$... + F^{1^{st},2^{nd}}$	*RW*		**Direct.**	*ARDL*	$... + F^{1^{st}}$	$... + F^{1^{st},2^{nd}}$	*Naive*
1	**0.0265****	0.0279	0.0321	0.0275		1	*0.6042*	**0.6458****	*0.6042*	0.5
2	0.0459	0.0486	0.0581	0.0437		2	0.4894	0.5957	0.4681	0.5
3	0.0614	0.0675	0.0811	0.0552		3	0.3696	0.5435	0.413	0.5
4	0.075	0.0834	0.0996	0.0663		4	0.3556	0.4889	0.4222	0.5
5	0.0905	0.1	0.1181	0.0787		5	0.2727	0.5	0.4545	0.5
6	0.1071	0.1218	0.1406	0.0902		6	0.2326	0.3721	0.4186	0.5
7	0.119	0.1394	0.1615	0.0994		7	0.1429	0.381	0.4048	0.5
8	0.1273	0.1453	0.1662	0.1067		8	0.1463	0.4146	0.4146	0.5
9	0.1352	0.1553	0.1852	0.1124		9	0.15	0.4	0.45	0.5
10	0.1424	0.1658	0.2064	0.1171		10	0.2051	0.359	0.3846	0.5
11	0.1482	0.1742	0.229	0.1212		11	0.1579	0.3158	0.4211	0.5
12	0.153	0.1803	0.2493	0.125		12	0.1892	0.2703	0.4865	0.5

[Referring to section 4.1.2] Evaluation and comparison of models on how well they can forecast the exchange rate change/return, for the NOK and CAD during the sample periods of 1998.01-2009.05 and 1999.01-2009.05. Rolling estimation for the NOK (CAD) with a window size of 90 (78). Forecast window 2005.06-2009.05. The notations used here are the same as in the previous table. Clark and West (2006) test is used for RMSE. Diebold and Mariano test (1995) is used for Direction of Change. '***','*','*' indicate being significant at 1%, 5% and 10% level.

Table 12: Forecast evaluation and comparison. AUD and CAD

AUD Forecasts, $\widehat{\Delta e_{t+h}}|I_{t+h}$

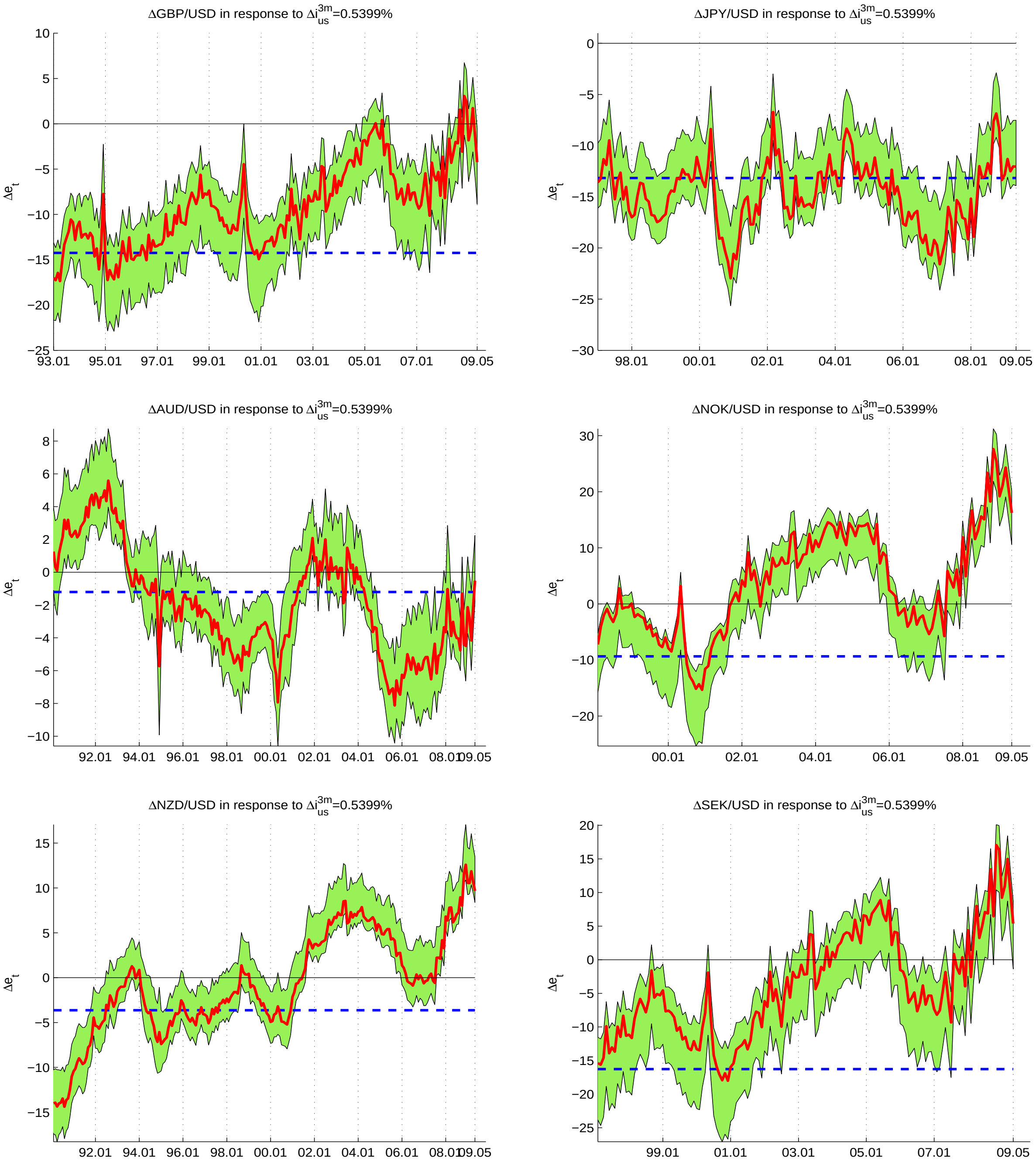
RMSE	*ARDL*	... + $F^{1^{st}}$	... + $F^{1^{st},2^{nd}}$	*RW*		**Direct.**	*ARDL*	... + $F^{1^{st}}$	... + $F^{1^{st},2^{nd}}$	*Naive*
1	0.0437	0.038***	0.0414***	0.0423		1	0.5167	0.6333	0.6667***	0.5
2	0.0729	0.0595***	0.0599**	0.0658		2	0.5254	0.5932	0.6441**	0.5
3	0.0925	0.0744	0.0748***	0.0828		3	0.4483	0.6207*	0.7069***	0.5
4	0.1141	0.0916	0.0874***	0.0989		4	0.4737	0.6667**	0.8421***	0.5
5	0.1342	0.1078	0.1014***	0.1129		5	0.3929	0.6607**	0.8036***	0.5
6	0.1547	0.1244	0.1202***	0.1262		6	0.4	0.5818	0.8***	0.5
7	0.1688	0.137	0.1394	0.1336		7	0.4074	0.5185	0.8333***	0.5
8	0.1773	0.1467	0.1214***	0.1393		8	0.4151	0.5094	0.7925***	0.5
9	0.1882	0.1574	0.1268***	0.1435		9	0.3846	0.4808	0.8077***	0.5
10	0.1961	0.1666	0.1335	0.1447		10	0.3922	0.451	0.8039***	0.5
11	0.2026	0.1751	0.1471	0.1461		11	0.42	0.48	0.8***	0.5
12	0.2076	0.1834	0.1619	0.1474		12	0.4286	0.5306	0.7551***	0.5

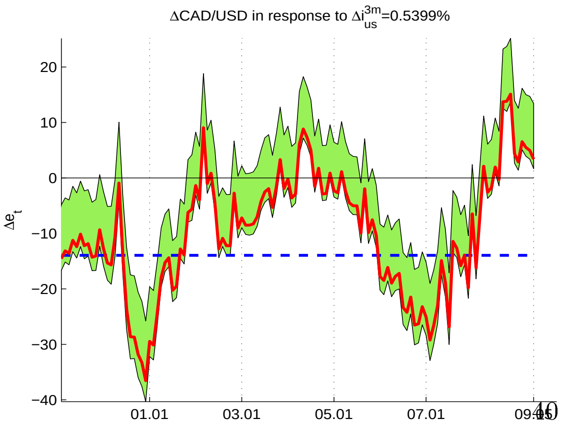
CAD Forecasts, $\widehat{\Delta e_{t+h}}|I_{t+h}$

RMSE	*ARDL*	... + $F^{1^{st}}$	... + $F^{1^{st},2^{nd}}$	*RW*		**Direct.**	*ARDL*	... + $F^{1^{st}}$	... + $F^{1^{st},2^{nd}}$	*Naive*
1	0.0278	0.0308	0.0393	0.0275		1	0.5833	0.625*	0.5	0.5
2	0.0407**	0.0515	0.0742	0.0437		2	0.6383*	0.5957	0.5532	0.5
3	0.0555	0.0698	0.1126	0.0552		3	0.587	0.6522**	0.6304**	0.5
4	0.0678	0.0876	0.1586	0.0663		4	0.4444	0.6222*	0.5333	0.5
5	0.0791	0.1011	0.2116	0.0787		5	0.4773	0.6818**	0.6136	0.5
6	0.091	0.1104	0.2509	0.0902		6	0.4419	0.6744**	0.5814	0.5
7	0.1013	0.1158	0.2838	0.0994		7	0.4524	0.7143***	0.5952	0.5
8	0.1109	0.1144	0.3113	0.1067		8	0.4878	0.7317***	0.6098	0.5
9	0.1197	0.1249	0.3578	0.1124		9	0.425	0.75***	0.625*	0.5
10	0.1279	0.1359	0.3931	0.1171		10	0.4615	0.7179***	0.5897	0.5
11	0.1338	0.1516	0.4311	0.1212		11	0.4474	0.7105***	0.6053	0.5
12	0.1394	0.1691	0.4718	0.125		12	0.4865	0.7568***	0.7027**	0.5

[Referring to section 4.1.2] Evaluation and comparison of models on how well they can forecast the exchange rate change/return, for the AUD during the sample period of 1990.01-2009.05, and for the CAD during 1999.01-2009.05. **Time-($t+h$) information is used for forecasting**, i.e. the contemporaneous values for the righ-hand-side variables are used. Rolling estimation with window size 173 (78) for the AUD (CAD). Forecast window 2004.06-2009.05 for the AUD and 2005.06-2009.05 for the CAD. The notations used here are the same as in the previous table. Clark and West (2006) test is used for RMSE. Diebold and Mariano test (1995) is used for Direction of Change. '***','*','*' indicate being significant at 1%, 5% and 10% level.

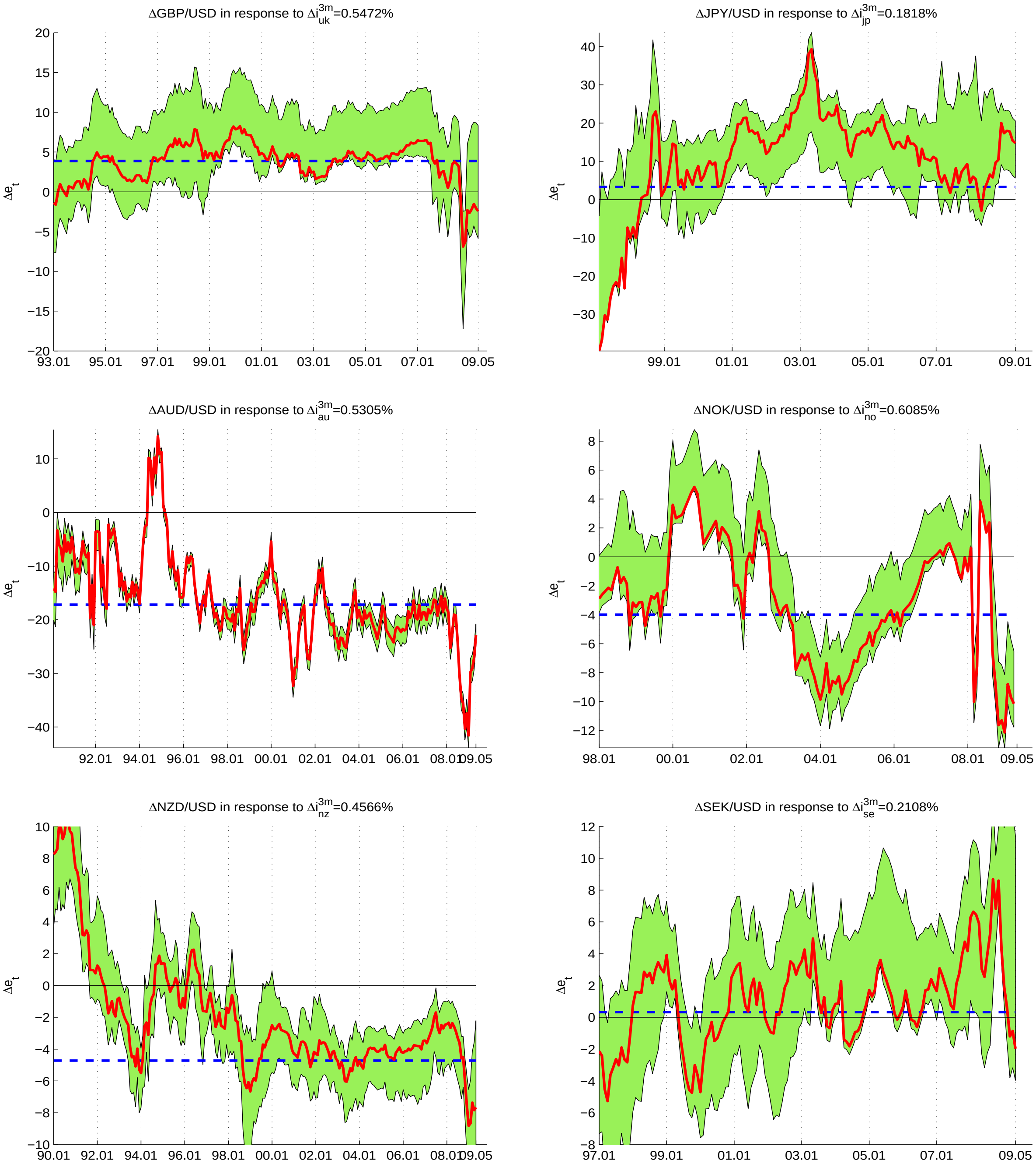
Figure 3: Exchange rate change in response to U.S. monetary policy change

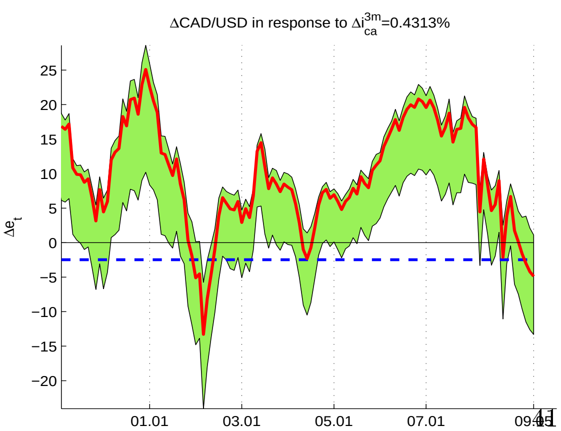




[Referring to section 4.2.3] Note: The **concurrent** responses (bold red line) of exchange rate change following a one-standard-error domestic (the U.S.) monetary policy adjustment on top of the realized value at each point of time in history. The green shadow region indicates the bootstrapped 90% confidence interval of the responses. The blue dashed line is the contribution by the first order factor terms (excluding the one from non-linear factor terms).

Figure 4: Exchange rate change in response to foreign monetary policy change





[Referring to section 4.2.3] Note: The **concurrent** responses (bold red line) of exchange rate change following a one-standard-error foreign monetary policy adjustment on top of the realized value at each point of time in history. The green shadow region indicates the bootstrapped 90% confidence interval of the responses. The blue dashed line is the contribution by the first order factor terms (excluding the one from non-linear factor terms).

Table 13: Robustness Check of Significance of the Risk Premium

	Reported		Full Sample		Early Days		Common		No-Crisis	
AUD	**90.01-09.05**	y	**87.02-09.05**	y	90.01-99.12	**n**	98.01-09.05	~y	98.01-08.01	**n**
NZD	**90.01-09.05**	y	**90.01-09.05**	y	90.01-99.12	y	98.01-09.05	~y	98.01-08.01	y
GBP	**93.01-09.05**	~y	**79.01-09.05**[41]	y	79.01-95.12	y	98.01-09.05	y	98.01-08.01	y
SEK	**93.01-09.05**	y	**93.01-09.05**	y	93.01-00.12	y	98.01-09.05	y	98.01-08.01	y
CHF	**96.01-09.05**	y	**88.01-09.05**	**~n**	88.01-95.12	y	98.01-09.05	y	98.01-08.01	y
JPY	**96.01-09.05**	y	**85.01-09.05**	**~n**	85.01-95.12	**~n**	98.01-09.05	y	98.01-08.01	y
NOK	**98.01-09.05’**	y	**98.01-09.05**	y	n.a.		98.01-09.05	y	98.01-08.01	y-
CAD	**99.01-09.05**	y	**86.01-09.05**	**n**	86.01-97.12	y	98.01-09.05	~y	98.01-08.01	y

[Referring to section 5.1] Robustness check of the proposed $ARDL + F^{1st,2nd}$ model of the exchange rate dynamics, *equation 1,* for all currencies at different time horizons. The definition of ’Full Sample’, ’Early Days’, ’Common’ and ’No-Crisis’ are shown in the text. There are two criteria for the robustness check: 1) significance of the risk premium term (factors) in the equation. 2) the AIC (goodness of fit) for the $ARDL + F^{1st,2nd}$ model is smaller (better) than the $ARDL$ model. In above table, an ’y’ indicates the two criteria for robustness are met. An ’y-’ means a relatively weaker ’y’ in terms of the criterion 2). The ’~n’ mean only the criterion 1) is met. ’n’ means neither of the criteria is met. ’~y’ indicates that although the criterion 1) is met, but the goodness of fit for the $ARDL + F^{1st,2nd}$ as measured by the AIC is very close and may a bit worse than the $ARDL + F^{1st}$ model. In this case, the second order terms are not strongly significant.

Table 14: Robustness check in the presence of macro fundamentals. GBP and SEK

	GBP (93.01-06.04)			SEK (97.01-09.05)	
Δe_t	$... + F^{1^{st},2^{nd}}$	$... + F^{1^{st},2^{nd}} + Macro$		$... + F^{1^{st},2^{nd}}$	$... + F^{1^{st},2^{nd}} + Macro$
...	...	...		...	...
$\pi_t - \pi_t^*$		4.037740			-1.932342
$\Delta(m_t - m_t^*)$		-1.628168			-0.523363
$\Delta(y_t - y_t^*)$		-0.226431			0.108401
$\Delta(i_t - i_t^*)$		-16.56216			42.11352***
$y_{t-1}^{gap} - y_{t-1}^{gap*}$		-0.194011			-0.457220
$\pi_{t-1} - \pi_{t-1}^*$		-0.066324			-3.555682
$Adjusted - R^2$	0.256646	0.246244		0.344932	0.375426
AIC	9.243196	9.285574		9.550694	9.531860
Prob χ^2_{macro}	-	0.6890		-	0.0085 ***
Prob $\chi^2_{Factor12}$	0.0000***	0.0000 ***		0.0000***	0.0000 ***

Table 15: Robustness check in the presence of macro fundamentals. AUD and NZD

	AUD (90.01-09.05)			NZD (90.01-09.05)	
Δe_t	$... + F^{1^{st},2^{nd}}$	$... + F^{1^{st},2^{nd}} + Macro$		$... + F^{1^{st},2^{nd}}$	$... + F^{1^{st},2^{nd}} + Macro$
...	...	...		...	...
$\pi_t - \pi_t^*$		3.728521			8.584208
$\Delta(m_t - m_t^*)$		0.497165			-0.443546
$\Delta(y_t - y_t^*)$		-0.976477			-0.534847
$\Delta(i_t - i_t^*)$		-21.11592*			5.920638
$y_{t-1}^{gap} - y_{t-1}^{gap*}$		-0.563279			-1.009491*
$\pi_{t-1} - \pi_{t-1}^*$		-3.573037			-2.152234
$Adjusted - R^2$	0.214802	0.216439		0.177333	0.216439
AIC	9.943615	9.964044		9.635847	9.964044
Prob χ^2_{macro}	-	0.3973		-	0.2181
Prob $\chi^2_{Factor12}$	0.0000***	0.0000 ***		0.0000***	0.0000 ***

[Referring to section 5.1.1] Notes: Robustness check when including the conventional macro fundamentals in the working model of this paper. This can be accomplished by checking the joint significance of the coefficients of macro fundamentals and factors for the $ARDL + F^{1^{st},2^{nd}} + Macro$ model. The notations used in this table are: $... + F^{1^{st},2^{nd}} \equiv ARDL + F^{1^{st},2^{nd}}$: ARDL model augmented with factor in non-linear form (including the second order factors). $... + F^{1^{st},2^{nd}} + Macro \equiv ARDL + F^{1^{st},2^{nd}} + Macro$: ARDL model augemented with factors in non-linear form and traditional macro fundamentals.

Table 16: Robustness check in the presence of macro fundamentals. JPY and CHF

	JPY (96.01-09.05)			CHF (96.01-09.05)	
Δe_t	$... + F^{1^{st},2^{nd}}$	$... + F^{1^{st},2^{nd}} + Macro$		$... + F^{1^{st},2^{nd}}$	$... + F^{1^{st},2^{nd}} + Macro$
...	...	...		...	...
$\pi_t - \pi_t^*$		-2.167285			-0.252127
$\Delta(m_t - m_t^*)$		2.505537**			-1.646169
$\Delta(y_t - y_t^*)$		-0.537623			-0.941849
$\Delta(i_t - i_t^*)$		-3.826421			33.64608**
$y_{t-1}^{gap} - y_{t-1}^{gap*}$		-1.304823*			0.618132
$\pi_{t-1} - \pi_{t-1}^*$		-7.741083**			-8.552289
$Adjusted - R^2$	0.208859	0.257215		0.205917	0.244046
AIC	9.995506	9.960087		10.02602	10.00446
Prob χ^2_{macro}	-	0.0027 ***		-	0.0159 **
Prob $\chi^2_{Factor12}$	0.0000***	0.0000 ***		0.0000***	0.0000 ***

Table 17: Robustness check in the presence of macro fundamentals. NOK and CAD

	NOK (98.01-09.05)			CAD (99.01-09.05)	
Δe_t	$... + F^{1^{st},2^{nd}}$	$... + F^{1^{st},2^{nd}} + Macro$		$... + F^{1^{st},2^{nd}}$	$... + F^{1^{st},2^{nd}} + Macro$
...	...	...		...	...
$\pi_t - \pi_t^*$		0.556385			0.500650
$\Delta(m_t - m_t^*)$		-0.345970			-2.122777***
$\Delta(y_t - y_t^*)$		-0.238424			0.223045
$\Delta(i_t - i_t^*)$		-1.422903			-8.393160
$y_{t-1}^{gap} - y_{t-1}^{gap*}$		-0.434642			-0.080269
$\pi_{t-1} - \pi_{t-1}^*$		-6.341001*			-7.942008**
$Adjusted - R^2$	0.143231	0.129621		0.353113	0.379620
AIC	10.08251	10.13276		9.064817	9.053737
Prob χ^2_{macro}	-	0.4904		-	0.0556 *
Prob $\chi^2_{Factor12}$	0.000***	0.0000 ***		0.0004 ***	0.0011 ***

[Referring to section 5.1.1] Notes: Robustness check when including the conventional macro fundamentals in the working model of this paper. This can be accomplished by checking the joint significance of the coefficients of macro fundamentals and factors for the $ARDL + F^{1^{st},2^{nd}} + Macro$ model. The notations used in this table are: $... + F^{1^{st},2^{nd}} \equiv ARDL + F^{1^{st},2^{nd}}$: ARDL model augmented with factor in non-linear form (including the second order factors). $... + F^{1^{st},2^{nd}} + Macro \equiv ARDL + F^{1^{st},2^{nd}} + Macro$: ARDL model augemented with factors in non-linear form and traditional macro fundamentals.

Table 18: Regression results of exchange rate models with expectations and uncertainties.

GBP and SEK

	GBP (93.01-09.05)			SEK (93.01-09.05)		
Δe_t	$ARDL$	$ARDL+\tilde{\bar{R}}$	$ARDL+\tilde{\bar{R}},\tilde{\Sigma}$	$ARDL$	$ARDL+\tilde{\bar{R}}$	$ARDL+\tilde{\bar{R}},\tilde{\Sigma}$
$constant$	5.051606	53.47841***	38.09224*	-0.212793	52.58799***	80.20335**
Δe_{t-1}	0.072882	-0.073139	-0.131747**	0.450109	0.387825***	0.370036***
Δe_{t-2}				-0.170434	-0.189277***	-0.190543***
Δe_{t-3}						
Δe_{t-4}						
Δe_{t-5}						
Δe_{t-6}						
Δe_{t-7}						
Δe_{t-8}						
$i_t - i_t^*$				7.361294	23.77210**	29.10528**
$i_{t-1} - i_{t-1}^*$	3.318426	18.11328***	13.60867***	-20.22149	-24.16103**	-28.97631**
$i_{t-2} - i_{t-2}^*$				12.65682	10.01266	12.11554
$i_{t-3} - i_{t-3}^*$						
$i_{t-4} - i_{t-4}^*$						
$i_{t-5} - i_{t-5}^*$						
$\bar{r}^*_{t,..,\,t+m\|t}$		16.54400***	13.59611***		22.89243***	24.87222**
$\bar{r}_{t,..,\,t+m\|t}$		-27.00200***	-21.41308***		-24.08511***	-32.19437***
$\bar{tp}^*_{t,..,\,t+m\|t}$		-0.066681	14.36035**		-21.59628	-20.51940
$\bar{tp}_{t,..,\,t+m\|t}$		-7.555211	-8.098500		-14.50292**	-17.98192**
σ_r^*			46.51901**			3.058705
σ_r			-12.19932			-156.2141*
σ_{tp}^*			-62.84368***			3.303434
σ_{tp}			4.751782			23.50929
$Adj.-R^2$	0.014807	0.154524	0.188009	0.161825	0.225891	0.228619
AIC	9.577392	9.453952	9.469410	9.604529	9.544460	9.559916
Prob $\chi^2_{\tilde{\bar{R}}}$	-	0.0000***	0.0017 ***	-	0.0002 ***	0.0153 **
Prob$\chi^2_{\tilde{\Sigma}}$	-	-	0.0000***	-	-	0.1090
Prob$\chi^2_{\tilde{\bar{R}},\,\tilde{\Sigma}}$	-	-	0.0000***	-	-	0.0023 ***

[Referring to section 5.2.2] Comparison of models that with expectation variables, on how well they can fit the data for the GBP and SEK during the period of 1993.01-2009.05. The notations used in the tables are $ARDL$: Autoregressive distributed lags model discussed in section 2.3; $ARDL+\tilde{\bar{R}}$: ARDL model augmented with means of excpectations as defined in text; $ARDL+\tilde{\bar{R}},\tilde{\Sigma}$: ARDL model augemented with means and variances of expectations. The $Adj.-R^2$ and AIC are reported for each model and each currency. Especially, the nulls on the $\tilde{C} = 0_{6\times1}$, $\tilde{D} = 0_{6\times6}$, $\tilde{C}$ and $\tilde{D}$ are jointly zero are tested respectively, probabilities of the χ^2 statistics are reported. The '***', '**', '*' denote significance at 1%, 5% and 10% level respectively. All the inferences are adjusted with the Newey-West autocorrelation and heteroskedasticity consistant standard errors. The lag orders of the $ARDL$ model are selected by the AIC.

Table 19: Regression results of exchange rate models with expectations and uncertainties.

AUD and NZD

	AUD (90.01-09.05)				NZD (90.01-09.05)		
Δe_t	$ARDL$	$ARDL + \tilde{\bar{R}}$	$ARDL + \tilde{\bar{R}}, \tilde{\Sigma}$		$ARDL$	$ARDL + \tilde{\bar{R}}$	$ARDL + \tilde{\bar{R}}, \tilde{\Sigma}$
$constant$	-0.987931	18.80270	74.91103*		0.456397	-3.212269	46.18975
Δe_{t-1}	0.148618	0.135738	0.119893		0.339334***	0.306045***	0.277770***
Δe_{t-2}	-0.093554	-0.108402	-0.107434				
Δe_{t-3}	0.137384	0.117418*	0.123672*				
Δe_{t-4}							
Δe_{t-5}							
Δe_{t-6}							
Δe_{t-7}							
Δe_{t-8}							
$i_t - i_t^*$	-23.07489***	-3.533960	-2.121386				
$i_{t-1} - i_{t-1}^*$	22.31848***	11.73386	13.02233		0.113527	4.559638	4.379514
$i_{t-2} - i_{t-2}^*$							
$i_{t-3} - i_{t-3}^*$							
$i_{t-4} - i_{t-4}^*$							
$i_{t-5} - i_{t-5}^*$							
$\bar{r}^*_{t,..,t+m\|t}$		17.96603*	16.14432*			10.16583**	4.808765
$\bar{r}_{t,..,t+m\|t}$		-22.61981**	-33.14268**			-10.32528**	-14.61201**
$\bar{tp}^*_{t,..,t+m\|t}$		15.64105*	29.22874**			5.224399	8.144440
$\bar{tp}_{t,..,t+m\|t}$		-19.74823*	-19.00202*			-5.122769	-4.007528
σ_r^*			36.29833*				18.98951*
σ_r			-119.9203				-126.9446
σ_{tp}^*			-25.04547				-20.13576
σ_{tp}			-7.169475				6.151540
$Adj. - R^2$	0.031807	0.077604	0.090584		0.105866	0.113188	0.110665
AIC	10.08084	10.05313	10.05519		9.646137	9.654705	9.674024
Prob $\chi^2_{\tilde{\bar{R}}}$	-	0.0415 ***	0.0916 *		-	0.1490	0.2411
Prob$\chi^2_{\tilde{\Sigma}}$	-	-	0.1839		-	-	0.4195
Prob$\chi^2_{\tilde{\bar{R}}, \tilde{\Sigma}}$	-	-	0.1032		-	-	0.1095

[Referring to section 5.2.2] Comparison of models that with expectation variables, on how well they can fit the data for the AUD and NZD during the period of 1990.01-2009.05. The notations used here are the same as in the previous table.

Table 20: Regression results of exchange rate models with expectations and uncertainties.

JPY and CHF

	JPY (96.01-09.05)				CHF (96.01-09.05)		
Δe_t	$ARDL$	$ARDL + \tilde{\bar{R}}$	$ARDL + \tilde{\bar{R}}, \tilde{\Sigma}$		$ARDL$	$ARDL + \tilde{\bar{R}}$	$ARDL + \tilde{\bar{R}}, \tilde{\Sigma}$
$constant$	13.85854**	17.82083	44.59713*		7.828034	51.47989***	118.5425***
Δe_{t-1}	-0.029807	-0.076892	-0.085555		-0.025616	-0.156617	-0.214426*
Δe_{t-2}	0.052889	0.034277	0.016529		-0.031518	-0.158794**	-0.214310***
Δe_{t-3}	0.051886	0.044973	0.030635		-0.000585	-0.134005*	-0.202867***
Δe_{t-4}	-0.126041*	-0.096892	-0.105065		-0.156540	-0.270170***	-0.326389***
Δe_{t-5}	-0.141809*	-0.139827	-0.157501*		0.024588	-0.055232	-0.105350
Δe_{t-6}	-0.064520	-0.062969	-0.082306		-0.060323	-0.118015	-0.155489
Δe_{t-7}					-0.087961	-0.093332	-0.111345
Δe_{t-8}					-0.058058	-0.057113	-0.076118
$i_t - i_t^*$					4.263529	35.48326***	25.74151**
$i_{t-1} - i_{t-1}^*$	-9.630588	51.67464**	49.25467**		-1.056968	-0.859017	-3.139582
$i_{t-2} - i_{t-2}^*$	23.60070	-0.395568	4.635691		-36.80506	-38.18719	-33.03191
$i_{t-3} - i_{t-3}^*$	-25.51819	-26.30274	-26.81562		29.96227	37.43218**	33.17237**
$i_{t-4} - i_{t-4}^*$	25.93667	30.85263	32.59334				
$i_{t-5} - i_{t-5}^*$	-17.95750	-32.02627	-34.78036				
$\bar{r}^*_{t,..,\,t+m\|t}$		62.69724***	50.75497**			45.39369***	38.03451***
$\bar{r}_{t,..,\,t+m\|t}$		-36.77281***	-43.54631***			-58.72329***	-59.89601***
$\bar{tp}^*_{t,..,\,t+m\|t}$		7.572569	11.52264			43.99960***	51.34943***
$\bar{tp}_{t,..,\,t+m\|t}$		15.58203*	15.22046			-18.40315	-38.60343***
σ_r^*			-3474.719**				-155.8028
σ_r			-35.99504				-297.7682**
σ_{tp}^*			30.71830				-4.972996
σ_{tp}			-6.579628				40.16002
$Adj. - R^2$	0.069476	0.116267	0.118840		0.013888	0.142123	0.159350
AIC	10.13415	10.03381	10.05261		10.15344	10.03642	10.03765
Prob $\chi^2_{\tilde{\bar{R}}}$	-	0.0000 ***	0.0002 ***			0.0000 ***	0.0000 ***
Prob$\chi^2_{\tilde{\Sigma}}$	-	-	0.1378				0.0851 **
Prob$\chi^2_{\tilde{\bar{R}},\,\tilde{\Sigma}}$	-	-	0.0000 ***				0.0001 ***

[Referring to section 5.2.2] Comparison of models that with expectation variables, on how well they can fit the data for the JPY and CHF during the period of 1996.01-2009.05. The notations used here are the same as in the previous table.

Table 21: Regression results of exchange rate models with expectations and uncertainties.

NOK and CAD

	NOK (98.01-09.05)				CAD (99.01-09.05)		
Δe_t	$ARDL$	$ARDL+\tilde{\bar{R}}$	$ARDL+\tilde{\bar{R}},\tilde{\Sigma}$		$ARDL$	$ARDL+\tilde{\bar{R}}$	$ARDL+\tilde{\bar{R}},\tilde{\Sigma}$
$constant$	1.632000	-27.26461	-45.67158		2.285305	24.82102*	57.18412**
Δe_{t-1}	0.165972	0.139623	0.044861		0.307091***	0.289726***	0.242445***
Δe_{t-2}					0.057781	0.031145	0.026617
Δe_{t-3}					-0.119025	-0.135985	-0.162611
Δe_{t-4}					0.239679**	0.209022**	0.216185**
Δe_{t-5}					-0.014012	-0.010851	0.046324
Δe_{t-6}					-0.249555**	-0.252537**	-0.227411**
Δe_{t-7}							
Δe_{t-8}							
$i_{t-1}-i^*_{t-1}$	0.272399	2.268678	-7.562408		1.161302	9.260196	5.507445
$i_{t-2}-i^*_{t-2}$					-17.99161	-16.79831	-16.63913
$i_{t-3}-i^*_{t-3}$					23.36692*	24.72350**	24.80259**
$i_{t-4}-i^*_{t-4}$					21.97614**	20.20368**	18.47577*
$i_{t-5}-i^*_{t-5}$					-29.29930***	-33.94504***	-37.20897***
$\bar{r}^*_{t,..,\,t+m\|t}$		13.99852	9.824741			15.49376*	12.89360
$\bar{r}_{t,..,\,t+m\|t}$		-9.026967**	-4.066482			-17.73629*	-35.55993**
$\bar{tp}^*_{t,..,\,t+m\|t}$		36.07518	137.1314*			10.35198	56.40581*
$\bar{tp}_{t,..,\,t+m\|t}$		-12.72682*	-26.67426***			-20.41900	-18.31073
σ^*_r			16.96689*				316.8675**
σ_r			-290.1149**				-207.5318*
σ^*_{tp}			-242.0556*				-48.34425*
σ_{tp}			61.12793*				-33.68473
$Adj.-R^2$	0.013185	0.013034	0.065297		0.179898	0.198723	0.269768
AIC	10.10530	10.13355	10.10628		9.187586	9.192323	9.126091
Prob $\chi^2_{\tilde{\bar{R}}}$	-	0.1576	0.0467 **		-	0.2289	0.1198
Prob$\chi^2_{\tilde{\Sigma}}$	-	-	0.0527 *		-	-	0.1605
Prob$\chi^2_{\tilde{\bar{R}},\,\tilde{\Sigma}}$	-	-	0.0105 **		-	-	0.1292

[Referring to section 5.2.2] Comparison of models that with expectation variables, on how well they can fit the data for the NOK and CAD during the period of 1998.01-2009.05 and 1999.01-2009.05, respectively. The notations used here are the same as in the previous table.