Re-examining the risk–return relationship in Europe: linear or non-linear trade-off?

Abstract

This paper analyzes the risk–return trade-off in Europe using recent data from 11 European stock markets. After relaxing linear assumptions in the risk-return relation by introducing a new approach which considers the current state of the economy, we are able to obtain positive and significant evidence for a risk-return trade-off for low volatility states; however, this evidence turns to be lower or even non-significant during periods of high volatility. Maintaining the linear assumption over the risk-return trade-off leads to non-significant estimations for all cases analyzed. These results are robust among countries despite the conditional volatility model used. This concludes that the controversial results in previous studies may be due to strong linear assumptions when modeling the risk–return trade-off. We argue that this previous evidence can only be viewed as partial evidence that fails to cover the global behavior of the relation between return and risk.

Keywords: non-linear risk-return tradeoff, pro-cyclical risk aversion, Regime-Switching GARCH, Regime-Switching MIDAS, risk premium

1. Introduction

One of the most discussed topics in financial economics is that of establishing a relationship between return and risk. Several attempts have tried to explain the dynamics and interactions between these two fundamental variables. From a theoretical framework, one of the most cited works analyzing this risk–return trade-off is the Merton's (1973) intertemporal capital asset pricing model (ICAPM). Merton (1973) demonstrates that there is a linear relationship between conditional excess market return and its conditional variance, and its covariance with investment opportunities:

$$\mu_M - r_f = A\sigma_M^2 + BX_{M,S} \tag{1}$$

where $\mu_M - r_f$ is the excess return of the portfolio over the risk-free asset, σ_M^2 is the conditional variance of excess market returns (known as idiosyncratic portfolio risk), $X_{M,S}$ is the conditional covariance between excess market returns and the state variable that represents the investment opportunities (known as the hedge component), and A and B are the prices of these sources of risk. Assuming risk-averse investors, this model establishes a positive relation between expected return and market variance (risk).

However, despite the important role of this trade-off in the financial literature, there is no clear consensus about its empirical evidence. Campbell (1987), Glosten et. al (1993), Whitelaw (1994)9 and Brandt and Wang (2004) find a negative relation between these variables, while other authors such as (Ghysels, et. al. (2005), Leon et. al. (2007), Guo and Whitelaw (2006), Ludvigson and Ng(2007) and Lundblad (2007) find a positive trade-off.

This paper analyzes the risk–return trade-off in 11 European countries (Germany, France, Spain, the United Kingdom, Switzerland, the Netherlands, Belgium, Denmark, Finland, Sweden and Greece) and tries to shed light on the controversial results about its sign and magnitude. We use different assumptions when modeling conditional volatilities (GARCH and MIDAS approaches) and we relax the strong linear assumption (usually made in previous studies) by introducing a Markov Regime-Switching process. This non-linear methodology helps us condition our estimation upon the current state of the economy obtaining different relationships between return and risk during periods of high and low volatility. To the best of our knowledge, this is one of the first attempts using non-linear models such as Regime-Switching MIDAS (RS-MIDAS) for analyzing the risk-return relation[1].

In the theoretical framework, all the parameters (the risk prices A and B in (1)) and the variables (the sources of risk σ_M^2 and $X_{M,S}$) are allowed to be time varying. However, to make the model empirically tractable one must make several assumptions; the most common is constant risk prices (Goyal and Santa-Clara, 2003; Bali et al., 2005). It is also necessary to assume specific dynamics for the conditional second moments representing the market risk. The most used are the GARCH models (Bollerslev, 1986). Finally, the empirical model is established in a discrete time economy instead of the continuous time economy used in the equilibrium model of the theoretical approach. Another common assumption is considering one set of investment opportunities constant over time, for example by retaining market risk as the only source of risk[2]

[1] The first attempt to use these models is very recent (Guérin and Marcellino (2010)) and it is a specification with potential applications to a large class of empirical studies in applied economics and finance yet to exploit

[2] Few papers such as Scruggs (1998) and Scruggs and Glabadanidis (2003) show that the lack of empirical evidence of a risk-return trade-off is due to the omission of the hedge component. However,

(Glosten et al., 1993; Shin, 2005; Lundblad, 2007). In this paper we follow these studies and analyze the effect of market returns given one risk factor defined by the conditional market volatility.

Given the assumptions mentioned above, many papers have introduced alternative empirical models to obtain favorable evidence following the theoretical intuition. The methodology most commonly used in the empirical analysis of the risk–return trade-off is the GARCH-M approach (Engle et al., 1987). This framework is simple to implement but the results obtained are controversial. Many studies fail to identify a statistically significant intertemporal relation between risk and return of the market portfolio. (Baillie and Di Gennaro, 1990, Campbell and Hendchen, 1992). A few studies do provide evidence supporting a positive risk–return relation (Bollerslev, 1986, Guo and Neely, 2008). Several studies even find that the intertemporal relation between risk and return is negative (examples include Nelson, 1991, Glosten, Li et al., 2005). Therefore, alternative approaches to the simple GARCH-M methodology have been proposed when analyzing the risk–return trade-off. The most important frameworks developed as alternative to GARCH models try essentially to obtain different estimations for conditional volatility. Whitelaw (1994) uses an instrumental variable specification for the conditional second moments. Harrinson and Zhang (1999) use non-parametric techniques in their study in opposite to the parametric approaches used more often. Ghysels et al. (2005) propose the use of different data frequencies to estimate the mean (with lower data frequency) and the variance (with higher data frequency) equations. Despite the differences among all the models presented, they share a strong linear (monotonic) assumption in the definition of the relationship between return and risk.

However, Merton's model is not the only theoretical approach explaining the risk-return relationship. Whitelaw (2000) proposes a non-linear relationship between return and risk based on an equilibrium framework. This theoretical framework is quite different from Merton's (1973) approach because a complex, non-linear, and time-varying relationship between expected return and volatility is obtained. Similarly, Mayfield (2004) employs a methodology in which states of the world are essentially defined by volatility regimes and condition the risk-return trade-off upon these different states. Other authors also draw alternative frameworks where is not expected a monotonic risk-return-relationship (Veronesi, 2000) and even some of them (Abel (1988), Backus and Gregory (1992)) develop theoretical models that support a negative risk-return relation.

The main result obtained in our paper is that a non-linear specification is necessary to reflect the positive and significant trade-off between return and risk. When several volatility states are considered, the risk–return relationship becomes significant, even ignoring possible changes in the set of investment opportunities. When linear patterns in the risk specification (GARCH and MIDAS) are considered, no significant relationship in any market is obtained. More specifically, a positive and significant trade-off between return and risk is obtained for low volatility states when non-linear patterns are considered (RS-GARCH and RS-MIDAS models). However, for high volatility states the magnitude of this relationship becomes lower or non-significant. These results are robust for all the stock indexes analyzed and show that the lack of empirical evidence in previous studies may be due to the strong assumption of a linear risk–

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they do not find clear evidence at all. Some alternative approaches use information not only about the market portfolio but also about additional risk factors such as other asset portfolios or macroeconomic indicators, thereby extending their empirical models to a multidimensional framework (see Ludvigson and Ng, 2007, Bali, 2008).

return relationship rather than a non-linear one revealing the perils of using linear frameworks to analyze empirically this trade-off. These results shed light on the controversial results obtained in previous studies using linear models about the sign of magnitude of this relationship. They also could explain why results from linear models appear not to be robust to the sample period used in the analysis. We argue that studies using linear models analyzing a sample period corresponding to a low volatility state are more likely to find a positive risk-return tradeoff, while studies that include episodes of crisis or high volatility are more likely to find a negative or insignificant trade-off. In both cases, the conclusions can only be viewed as partial evidence since the omission of non-linearities may misrepresent the evidence obtained.

The evidence obtained supporting the investor pro-cyclical risk aversion is another interesting result of this paper. During low volatility (boom) periods the risk aversion level is higher than during high volatility (crisis) periods. Although this result may seem against the theoretical intuition claiming for higher returns under more volatile markets, there are other authors using different methodologies who reach the same conclusion (Bliss and Panigirtzoglou (2004), Kim and Lee (2008), and Rossi and Timmerman (2010)).

The principal contributions of our paper are as follows. First, we study the risk-return relation for 11 European stock markets instead of US data which is more widely used in previous studies. Second, we develop an empirical framework which let us consider a non-linear risk return trade-off by using Markov-Switching processes for different specifications of the conditional variance (RS-GARCH and RS-MIDAS). Third, we show that a positive and significant risk–return trade-off is obtained for all European markets analyzed after considering non-linearities independently of the variance specification. Besides, we obtain a positive trade-off higher in magnitude during low volatility periods than during high volatility periods where the relationship is even non-significant or negative in some cases. Finally, we show the evolution of the risk premium in Europe during the recent years, including the recent period of the global financial crisis.

The remainder of this paper is structured as follows. Section 2 describes the data used in the study and develops the methodology. Section 3 reports and analyzes the main results obtained. Finally, section 4 summarizes.

2. Data and methodology

For the empirical analysis of the paper, we employ daily stock exchange indexes from 11 European countries[3] for the period August 1990 - May 2012. The sample includes data from DAX (Germany), CAC (France), IBEX35 (Spain), FTSE100 (United Kingdom), SMI (Switzerland), AEX-Index (The Netherlands), BEL20 (Belgium), OMXC20 (Denmark), OMXH25 (Finland), OMXS30 (Sweden) and Athex20 (Greece). These data allow us to calculate daily and weekly returns for the same period[4]. All the index data are obtained from DataStream.

Few authors (e.g., Theodossiou and Lee (1995) and Li et al. (2005) Guo and Neely (2008)) have investigated the risk-return relation in international stock markets, although such a study could help resolve the puzzling results obtained from U.S. data. In this paper we comprehensively analyze the patterns followed by this relation in the main European stock markets.

[3] Some countries such as Italy are not included in the analysis because the main stock index has been changed during the sample period leading to an irregular evolution of their quotations.
[4] Although the main conclusions of the paper are reached using weekly data, we also need daily observations when estimating the conditional volatility of (RS / asymmetric) MIDAS models.

Because risk-free interest rate data are not available to all financial markets under consideration over the examined period, stock market volatility is measured based on stock returns instead of excess stock returns (which is equal to stock returns minus the risk-free interest rate). Many researchers (Baillie and DeGennarro, 1990; Nelson, 1991; Choudhry, 1996; Li et al., 2005) argue that such a practice produces little difference in estimation and inference in this line of research. All these authors claim that there is virtually no difference in either the estimated parameters or the fitted variance.

In the next subsections we develop the methodology proposed for all empirical models used to analyze the risk–return trade-off.

2.1. Standard GARCH

The first approach is the traditional GARCH-M model of Engle et al. (1987). This framework is the most used in the financial literature to study the risk–return trade-off despite the puzzled results from previous studies. In this approach the mean equation is defined as follows:

$$r_t = c + \lambda h_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, h_t) \qquad (2)$$

where r_t is the market return, h_t is the conditional variance, and ε_t represents the innovations, which are assumed to follow a normal distribution. The conditional volatility is obtained using a standard GARCH specification as in Bollerslev (1986):

$$\varepsilon_t = h_t z_t \qquad z_t \sim N(0,1) \qquad (3)$$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \qquad (4)$$

where $\hat{\alpha} + \hat{\beta} < 1$ guarantees the stationarity of the process.

We estimate this first model using the quasi maximum likelihood (QML) function of Bollerslev and Wooldridge (1992), which allows us to obtain robust estimates of standard errors:

$$L(\theta) = \sum_{t=1}^{T} \ln\left[f\left(r_t, \Omega_t; \theta\right)\right] \qquad where \qquad f\left(r_t, \Omega_t; \theta\right) = \left(2\pi h_t\right)^{-\frac{1}{2}} e^{-\frac{(\varepsilon_t)^2}{2h_t}} \qquad (5)$$

However, this approach has not presented favorable evidence on the significance of the risk aversion parameter in many previous studies, such as Baillie and De Gennaro (1990), Glosten et al. (1993), Shin (2005), and Leon et al. (2007). Some authors argue that that conditional volatility using this GARCH-M methodology has almost no explanatory power for realized returns and that could be the reasons of the non-significant results (see Lundblad, 2007). Other authors claim that the controversial results are due to wrong modeling of conditional volatility (see Ghysels et. al, 2005; Leon et al., 2007).

2.2. MIDAS regression

Recently, a new methodology has been developed to capture a significant relationship between return and risk using data from different frequencies to obtain expected returns and variances, namely the MIDAS (mixed data sampling) regression (Ghysels et al., 2005). They find evidence of a significant positive trade-off between return and risk and claim the advantages of this methodology regarding GARCH models. MIDAS models allow the estimation of smooth expected return series using low frequency data and the estimations of more variable conditional variances using higher frequency data.

We use this specification with weekly returns (r_t) combined with D daily[5] lag squared returns (R_t^2) to obtain the weekly variance.

[5] In the original specification, these summations are infinite. We truncate them at 250 daily lag squared returns to estimate the weekly variance.

The mean equation of this model is similar to Equation 2 with conditional variance as a explanatory variable for the expected returns:

$$r_t = c + \lambda VAR(r_t) + \varepsilon_t \qquad \varepsilon_t \sim N(0, VAR(r_t)) \qquad (6)$$

However, the MIDAS estimator of weekly conditional variance is not obtained through a GARCH parameterization but from a function of D lag squared daily returns (R_t^2):

$$VAR(r_t) = \sum_{d=0}^{D} \omega(k_1, k_2, d) R_{t-d}^2 \qquad (7)$$

$$\text{where} \quad \omega(k_1, k_2, d) = \frac{\exp\left(k_1 d + k_2 d^2\right)}{\sum_{i=0}^{D} \exp\left(k_1 i + k_2 i^2\right)} \qquad (8)$$

is the function which measure the impact of each lag daily return in the variance formation[6].

Assuming normality in returns $r_t \sim N\left(c + \lambda VAR(r_t), VAR(r_t)\right)$, we estimate this model by maximizing the Bollerslev–Wooldridge QML function, as in Equation 5.[7.]

2.3. Asymmetric case

The symmetric models presented above can easily be extended to the asymmetric case in which the variance responds more after negative returns than it does after positive returns (leverage effect). For the GARCH specification, we add a new variable $\eta_t = \min\left(\varepsilon_t, 0\right)$ in the variance process using the asymmetric GJR model (Glosten et al., 1993). These models are estimated in a similar way to that presented above, substituting Equations 4 for 9.

$$h_t = \omega + \alpha \varepsilon_t^2 + \beta h_{t-1} + \delta \eta_{t-1}{}^2 \qquad (9)$$

We estimate the MIDAS model for the asymmetric case substituting Equation 7 for Equation 10:

$$Var\left(r_t\right) = \theta \sum_{d=0}^{D} \omega\left(k_1^-, k_2^-, d\right) r_{t-d}^2 \cdot 1_{t-d}^- + \left(2 - \theta\right) \sum_{d=0}^{D} \omega\left(k_1^+, k_2^+, d\right) r_{t-d}^2 \cdot 1_{t-d}^+ \qquad (10)$$

where $\theta, k_1^-, k_2^-, k_1^+, k_2^+$ are the parameters to be estimated and $1_{t-d}^-, 1_{t-d}^+$ are the indicator functions for $\left\{r_{t-d} < 0\right\}$ and $\left\{r_{t-d} \geq 0\right\}$, respectively. We use Equation 5 again to estimate these models.

2.4. Non-linear models

Some authors claim that strong linear assumption for the risk-return relationship could lead to misleading results since imposing this condition may bias the evidence on this relationship. To overcome this limitation, some works develop alternative theoretical frameworks which assume a more flexible relationship between return and risk even proposing a non-monotonic relationship over time (Rossi and Timmerman, 2010).

So, in the next subsections we develop the methodologies to analyze if there is a non-linear risk-return trade-off in the European markets. An explanation for the controversial results in previous studies may lie in the wrong specification for the relationship between risk and return which follows non-linear rather than linear patterns. Therefore, an insightful extension is to

[6] Ghysels et al. (2007) develop several weight functions for the MIDAS estimator, but owing to its tractability, the Almon Lag specification is the most frequently used in the literature.

[7] Although some authors estimate this specification by using non-linear least squares, Ghysels et al. (2005) use the QML estimate in their original paper.

consider non-linearities in this trade-off against the linear framework usually implemented. In order to provide robustness to our results, we introduce non-linearites assuming two forms for the conditional volatilities. As a result, RS-GARCH and RS-MIDAS models are developed; their specifications are given below.

a) *Regime Switching GARCH model*

RS-GARCH specification is based on the model originally proposed by Hamilton (1989); it allows us to distinguish between different volatility states governed by a hidden state variable that follows a Markov process. In this model, the mean equation is not exactly as shown in Equation 2 because it is state-dependent:

$$r_{t,s_t} = c_{s_t} + \lambda h_{t,s_t} + \varepsilon_{t,s_t} \qquad \varepsilon_{t,s_t} \sim N(0, h_{t,s_t}) \qquad (11)$$

where r_{t,s_t} , h_{t,s_t} , and ε_{t,s_t} are the state-dependent returns, variances, and innovations respectively and $s_t = 1$ (state 1) or 2 (state 2).

The state-dependent innovations follow a normal distribution, with two possible variances depending on the state of the process. The state-dependent variances are modeled as in Equation 4 following a GARCH parameterization, but allowing different parameters depending on the state[8.] in this case.

$$\varepsilon_{t,s_t} = h_{t,s_t} z_t \qquad z_t \sim N(0,1) \qquad (12)$$

$$h_{t,s_t} = \omega + \alpha_{s_t} \varepsilon_{t-1}^2 + \beta_{s_t} h_{t-1} \qquad (13$$

The shifts from one state to another are governed by a hidden state variable following a Markov process with a transition matrix:

$$P = \begin{pmatrix} \Pr\left(s_t = 1 \middle| s_{t-1} = 1\right) = p & \Pr\left(s_t = 1 \middle| s_{t-1} = 2\right) = (1-q) \\ \Pr\left(s_t = 2 \middle| s_{t-1} = 1\right) = (1-p) & \Pr\left(s_t = 2 \middle| s_{t-1} = 2\right) = q \end{pmatrix} \qquad (14)$$

Because of this state-dependence, the model is econometrically intractable[9]. We must, therefore, obtain state-independent estimates of variances and innovations. We use the recombinative method presented in Gray (1996) which assuming conditional normality in each regime, uses the definition of unconditional variance in returns maintaining the nature of the GARCH process:

$$h_t = E\left[\Delta r_t^2 | \Omega_{t-1}\right] - E\left[\Delta r_t | \Omega_{t-1}\right]^2 = \pi_{1,t}\left(\mu_{t,1}^2 + h_{t,1}\right) + \left(1 - \pi_{1,t}\right)\left(\mu_{t,2}^2 + h_{t,2}\right) - \left(\pi_{1,t}\mu_{t,1} + \left(1 - \pi_{1,t}\right)\mu_{t,2}\right)^2 \quad (15)$$

In order to obtain state-independent errors we use the definition of unconditional error:

$$\varepsilon_t = \Delta r_t - \pi_{1,t}\mu_{t,1} + \left(1 - \pi_{1,t}\right)\mu_{t,2} \qquad (16)$$

where h_t and ε_t are the state-independent variances and innovation, μ_{t,s_t} is the conditional mean equation $\mu_{t,s_t} = c_{s_t} + \lambda h_{t,s_t}$ for a given state s_t and:

$\pi_{1,t} = p * P\left(s_{t-1} = 1 \middle| \Omega_{t-1}; \theta\right) + \left(1-q\right) P\left(s_{t-1} = 2 \middle| \Omega_{t-1}; \theta\right)$ (17) is the ex-ante probability

[8] Following Capiello and Fearnley (2000), to facilitate convergence the constant variance term is not allowed to switch between regimes.

[9] See e.g. Gray (1996) and Dueker (1997). The main problem is derived from the fact that state-dependence increase exponentially the size of the likelihood function.

Where $\pi_{2,t} = 1 - \pi_{1,t}$ (18)

And the ex-post (filtered) probabilities are defined as:

$$P\left(s_t = k \middle| \Omega_t;\theta\right) = \frac{\pi_{s_t=k,t}\, f\left(r_t \middle| s_t = k, \Omega_t;\theta\right)}{\sum_{k=1}^{2} \pi_{s_t=k,t}\, f\left(r_t \middle| s_t = k, \Omega_t;\theta\right)} \qquad (19) \quad \textit{for } k = 1, 2$$

We estimate this model, maximizing the QML function of Bollerslev–Wooldridge (1992), weighted by the filtered probability of being in each state:

$$L(\theta) = \sum_{t=1}^{T} \ln\left[\sum_{k=1}^{2} \pi_{s_t=k,t}\, f\left(r_t, \Omega_t;\theta\right)\right] \quad \textit{where} \quad f\left(r_t \middle| s_t, \Omega_t;\theta\right) = \left(2\pi h_{t,s_t}\right)^{-\frac{1}{2}} e^{-\frac{\left(\varepsilon_{t,s_t}\right)^2}{2h_{t,s_t}}} \qquad (20)$$

b) <u>Regime Switching MIDAS model</u>

Further, the Markov Switching MIDAS incorporates regime-switching in the parameters of the mixed data sampling models (MIDAS) and allows for the use of mixed-frequency data in Markov-Switching models. The reason to introduce this model is to see the role of non-linearities in an alternative variance specification to GARCH modeling.

The modeling of a Regime-Switching MIDAS model for our purposes analyzing the risk-return trade-off is drawn as follows. We define the state-dependent mean equation using weekly returns ($r_{s_t,t}$) which are explained by a state-dependent constant and a time-varying state-dependent conditional variance using D daily[10] lag squared returns (R_t^2); therefore, the mean equation of this model is:

$$r_{s_t,t} = c_{s_t} + \lambda_{s_t} VAR(r_{s_t,t}) + \varepsilon_{t,s_t} \qquad \varepsilon_{t,s_t} \sim N(0, VAR(r_{s_t,t})) \qquad (21)$$

In this case, although the MIDAS estimator of weekly conditional variance is again a function of D lag squared daily returns (R_t^2), we let the weight parameters to switch among states:

$$VAR(r_{t,s_t}) = \sum_{d=0}^{D} \omega\left(k_{1,s_t}, k_{2,s_t}, d\right) R_{t-d}^2 \qquad (22)$$

where $\omega\left(k_{1,s_t}, k_{2,s_t}, d\right) = \dfrac{\exp\left(k_{1,s_t} d + k_{2,s_t} d^2\right)}{\sum_{i=0}^{D} \exp\left(k_{1,s_t} i + k_{2,s_t} i^2\right)}$ (23) is the state-dependent weight function for a given state $s_t = 1,2$.

To estimate this model we need the help of Bayesian inference in a similar way explained in the RS-GARCH model. Setting that the transition probability matrix, the ex-ante probabilities and the ex-post (filtered) probabilities are defined as in equation (17), (18) and (19) respectively, we can estimate this model by maximizing the following QML function.

$$L(\theta) = \sum_{t=1}^{T} \ln\left[\sum_{k=1}^{2} \pi_{s_t=k,t}\, f\left(r_t, \Omega_t;\theta\right)\right] \quad \textit{where} \quad f\left(r_t \middle| s_t, \Omega_t;\theta\right) = \left(2\pi \cdot VAR(R_{t,s_t})\right)^{-\frac{1}{2}} e^{-\frac{\left(\varepsilon_{t,s_t}\right)^2}{2VAR(R_{t,s_t})}} \qquad (24)$$

[10] In the original specification, these summations are infinite. We truncate them at 250 daily lag squared returns to estimate the weekly variance.

3. Empirical results

This section displays the estimations of the conditional mean and volatility of stock returns using the models from the previous section and it discusses the relationship between return and risk in Europe. First, we show the main results using our proposed linear models: GARCH and MIDAS. Second, we analyze if the introduction of an asymmetric effect on volatility has any impact when analyzing the risk-returns trade-off. Third, we provide a discussion about the results obtained after relaxing the linear assumption on the risk–return trade-off through the use of Regime-Switching models. Finally, we study the evolution of the market risk premium in all the stock markets considered over the examined period (during the last twenty years).

3.1 Estimations for linear models

We initially discuss the results of the models presented in section 2.1 (GARCH) and 2.2 (MIDAS). Although these two models argue for a linear relationship between return and risk, they are quite different in their construction and estimation methodologies for the conditional variance as it is explained in section 2. The main results for these models in all the stock markets considered are shown in Table 1.

[INSERT TABLE 1]

The results for the GARCH model (left side of table 1) are qualitatively similar to those presented in part of the literature (Glosten et al., 1993; Shin, 2005; Leon et al., 2007). The results indicate a non-significant relationship between return and risk suggesting there is no linear relation between market return and market risk. This confirms the puzzled results of previous studies which are incapable to provide clear evidence of this trade-off. Furthermore, the variance parameters present the typical patterns reported in the literature with a high persistence of the GARCH term (the persistence varies between 95% and 99.9% depending on the country). This fact has led some authors (Lameroux and Lastrapes, 1990; Marcucci, 2005) to consider this as a sign for the existence of different regimes for the variance process. They suggest that if these regime shifts are ignored, GARCH models tend to overestimate persistence in periods of financial instability and underestimate it in calm periods.

Further, the right side of Table 1 shows the results obtained using the MIDAS methodology. The main difference between this and previous models is the way obtaining conditional volatilities using different data frequencies used to obtain expected returns (weekly data) and variances (daily data), as explained in the previous section. The risk aversion coefficient is again non-significant for this kind of models. The results fail to provide a positive and significant relationship between return and risk as it would be expected. Our results are different from previous studies using this methodology which obtain favorable evidence using this methodology. These differences may be due to the use of mixed daily and weekly data, whereas most studies use mixed daily (variance) and monthly (returns) data, see Ghysels et al. (2005) and Leon et al. (2007). However, the consideration of the last financial crisis period (post-2008) in the empirical analysis could blur the evidence of a monotonic risk-return trade-off in a linear framework. The variance estimations using this specification also indicate a high degree of persistence because a great number of daily lags are needed to accurately estimate the variance. In almost all countries, the impact of lagged squared returns superior to 30 days into the variance formation represents more than the 40% of the total volatility. That means, 40% of volatility is explained by returns occurred one month before the current observation.

So, neither standard GARCH models nor standard MIDAS are able to show a significant trade-off between return and risk in Europe.

3.2 Estimations for asymmetric case

We now discuss the results of the models presented in section 2.3 (Assymetric GARCH and MIDAS). There is evidence in the literature (Nelson, 1991; Glosten et al., 1993) of a different effect of shocks of different sign in the volatility formation. Negative shocks usually present a greater impact on volatility known as leverage effect. The inclusion of this effect on the conditional volatility estimation could have effect on the sign and significance of the risk-return trade-off observed in Europe. The estimation for the asymmetric models including the leverage effect for all the stock markets considered are shown in Table 2.

[INSERT TABLE 2]

The results for the asymmetric GARCH model (left side of table 2) are similar to those for the symmetric case. The results show again a non-significant relationship between return and risk. The only country with a significant trade-off is Greece which exhibits a negative relation between return and risk. The variance parameters present again a high level of persistence but it is slightly lower than the standard case (the persistence varies between 90% and 99.9% depending on the country).

The estimations for the asymmetric MIDAS also fail to show a significant risk-return trade-off for most of the countries. Using this model we find the abnormal case of Greece with a negative and significant risk-return trade-off. Regarding the variance persistence, this specification let us distinguish what is the impact of positive and negative shock and how long they persist. It is very difficult to give a general interpretation of the patterns followed by positive or negative shocks for all countries since each market volatility seems to follow idiosyncratic patterns. However, in most of the countries negative shocks are less persistent than positive shocks. These findings could be in line with those of Marcucci (2005) since during periods of market jitters, there is an increase of the number of negative shocks, and this increase in the number of innovation reduces their impact over time.

So, asymmetric GARCH and MIDAS models are not able to show a significant risk-return relationship for Europe during the sample period analyzed confirming the incapability of linear models and questioning the theoretical framework supporting them. Therefore, it seems important to go beyond this setup and relax the strong assumption of a linear risk-return trade-off.

3.3. Estimation for non-linear models

The results reported in the previous sections do not support the linear assumption of the risk-return trade-off. Even Merton (1980) remarks that this relationship does not have to be linear. For this reason, we introduce in the previous models a Regime-Switching process which relaxes the linear assumption taken when analyzing the risk-return trade-off by conditioning our results to states of high and low volatility. Next we show the results for our two volatility specifications in the case we introduce a non-linear risk-return trade-off; i.e. we discuss the results obtained from RS-GARCH and RS-MIDAS specifications.

a) Regime-Switching GARCH model

Table 3 presents the estimations for the non-linear model assuming a GARCH process in the volatility formation (RS-GARCH). The results for this model let us shed light to the dynamics followed by the risk-return relation. In particular, we can associate state 1 with low volatility periods and state 2 with high volatility periods using the medians of the estimated volatility in each state[11]. For $s_t = 1$, corresponding to the low volatility state, there is a significant positive relationship between return and risk for almost all countries (at 1% for Germany, France, Spain, UK, Switzerland, Belgium, Denmark and Sweden; at 5% for Finland and at 10% for the Netherlands). The only country with no significant relationship between return and risk is Greece.

[INSERT TABLE 3]

However, when we look at the results for the state $s_t = 2$ we obtain less evidence for a positive and significant relationship between return and risk. Only for Germany, Spain and Sweden the relationship is significant at 5% and in Denmark and France at 10%. In the rest of the countries the trade-off between these two variables is not significant during high volatility states. Besides, and even more interesting, the risk aversion coefficient during high volatility states ($\lambda_{s_t=2}$) is lower than it is for the low volatility regime. This finding is not consistent with the spirit of the theoretical linear models that suggest that higher volatility should be compensated with higher returns. However, some papers such as Lundblad (2007), Kim and Lee (2008), and Rossi and Timmerman (2010) report the same evidence. This fact indicates that in high volatility periods the investor's risk aversion is lower. One potential explanation for this result may be based on the existence of a different risk price depending on the volatility regime. An investment considered too risky in calm periods (low volatility) is less risky when there is a period of market instability with more uncertainty and any investment involving risk. This finding could also be explained by investors' characteristics in high volatility states. In these periods, more risk-averse investors leave the market, letting less risk-averse investors adjust the price of risk according to their less demanding preferences (Bliss and Panigirtzoglou (2004)). Authors such as Kim and Lee (2008) find a pro-cyclical behaviour for investor risk aversion. During low volatility periods investors are more risk averse and during high volatility periods investors have more will to accept risk. However, a recent study developed by Rossi and Timmerman (2010) shows that the risk-return trade-off may follow non-monotonic patterns. These authors claim that at low-medium levels of conditional volatility there is a positive risk-return trade-off but this relationship gets inverted at high levels of volatility. Our results seems to support these studies observing a strong evidence of a positive risk-return trade-off during periods of low volatility but the evidence is different during periods of high volatility.

The persistence of the GARCH term during low volatility states is higher than the observed for high volatility states. This fact confirms the evidence from the literature (Marcucci, 2005). He concludes that in high volatility periods there are a higher number of shocks affecting the variance formation and reducing their impact over time. Further, the persistence is overestimated in high volatility periods if RS is ignored (Marcucci, 2005).

In addition, the expected duration[12] for the low volatility state is approximately 12 weeks, about four times higher than the high volatility state. Figure 1 shows the smoothed probabilities[13] of being in state 1 for the sample period.

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[11] For brevity, the table containing this information is not displayed in this version of the paper but is available upon request.

[12] We obtain the expected duration of being in each state s_t =1,2 as $\frac{1}{1-p}$ *and* $\frac{1}{1-q}$, respectively.

[INSERT FIGURE 1]

Interestingly, the smooth probabilities of being in low/high volatility states are very close with the economic cycles (boom / crisis) during the sample period and could be associated with them. For example, in figure 1 we can observe clearly the downturn in worldwide economic activity around 2001 and the effect of the last financial crisis in 2008 with the high volatility regime becoming the most important during these periods. So, our results also provide further evidence for those papers claiming a pro-cyclical risk aversion depending on the economic cycle (more risk-aversion during boom periods, less risk aversion during crisis periods).

b) Regime-Switching MIDAS model

In the previous section we are successful finding a positive risk-return relationship during low volatility states under a non-linear specification with GARCH variances. Here, we provide robustness for the claim of a non-linear relation between return and risk by using an alternative variance model to GARCH.

Table 4 displays the estimations for the RS-MIDAS model for all countries considered. Supporting the results of the last section, during low volatility states the relation between return and risk is positive and significant for all countries considered. Also, the value for the risk aversion coefficient is higher than the obtained for high volatility periods. The results for high volatility periods are very different between countries. For Germany, Spain, UK and Switzerland we obtain positive and significant estimation for the risk-return trade-off in high volatility states (but of a lower magnitude than during low volatility states). For markets such as France, Belgium, Denmark and Finland the relationship is not significant. Further, for the rest of the markets (the Netherlands, Sweden and Greece) the trade-off during high volatility periods is negative.

The results for this family of models seem to support again the interpretations about the existence of a pro-cyclical risk aversion in the investor behavior. The investors trading during high volatility (crisis) periods are less risk averse than the investors trading during low volatility (boom) periods. They also support the existence of a non-monotonic (and non-linear) relationship between return and risk depending on the state of the economy. During low volatility periods a positive and significant trade-off is observed, but this relationship turn to be different during high volatility periods.

3.4. Risk premium evolution in Europe

In this last section we analyze the evolution followed by the market risk premium during the last years in each European market. The risk premium demanded by the investors is given by the non-diversifiable risk existing in the market.

Figure 2 shows the risk premium evolution in Europe during the sample period. The market risk premium is measured by the time-varying variable h_t in our models. For the Regime-Switching models, we obtain the independent estimation for the conditional variance at each period t through a weighted average using the filter probabilities $\left(h_t = \pi_{1,t} h_{t,1} + \left(1 - \pi_{1,t}\right) h_{t,2}\right)$

[13] The smoothed probability is defined as the probability of being in each state considering the entire information set $P\left(s_t = 1 \middle| \Omega_T; \theta\right) = P\left(s_t = 1 \middle| \Omega_t; \theta\right)\left[p \frac{P\left(s_{t+1} = 1 \middle| \Omega_T; \theta\right)}{P\left(s_{t+1} = 1 \middle| \Omega_t; \theta\right)}\right] + \left[(1-p)\frac{P\left(s_{t+1} = 2 \middle| \Omega_T; \theta\right)}{P\left(s_{t+1} = 2 \middle| \Omega_t; \theta\right)}\right]$

where $\pi_{1,t}$ is the ex-ante probability of being in the state 1 and h_{t,s_t} $for\ s_t = 1,2$ are the state-dependent variances.

[INSERT FIGURE 2]

These figures represent the weekly market risk premiums (expressed in basic points) for all the sample period, the different methodologies and the countries considered. The figures show similar patterns for the risk premium evolution. It seems that risk exposure is similar for all methodologies with slightly differences between them. However, the evolution is quite different depending on the country analyzed. Although all markets have been hit by similar crisis, it seems that in some of them (Finland at the beginnings 2000s, Greece over last years) the effect was worse than in the rest. However, all the countries share a huge increase of the demanded risk premium in the recent years coinciding with the last financial crisis of high instability.

Table 5 shows the median[14] of the estimated weekly risk premiums series for all the European stock market indexes considered. Almost all the obtained risk premiums vary between 2%-4% depending on the country. Only in the cases of Greece and Finland the demanded premiums exhibits higher values (around 5.5% for Finland and 7% for Greece). The differences in the risk premiums among methodologies are slightly. Most of these premiums are similar than the 3% to 5% obtained in other studies for US data (Bali, 2008).

[INSERT TABLE 4]

So, although the evolution of the market risk premium in Europe has followed similar patterns during the last years for most of the countries analyzed, there are certain differences among countries (due to idiosyncratic characteristics of each market) which lead to different levels in the demanded risk premium. Especially countries such as Finland or Greece seem to departure from the 'standard' European risk premium.

4. Conclusion

This study proposes an answer to the well-know controversy about the empirical evidence of the relation between return and risk. From the basis of the theoretical works explaining this trade-off, the interaction between these two variables is tested empirically using 11 European stock markets under two main different frameworks. The first framework considers a linear relation between return and risk while the second one relaxes this assumption and allows for non-linear dynamics.

Considering linear empirical models to analyze this relationship leads to non-significant evidence of this basic trade-off. However, when this strong linear assumption is relaxed, we are able to identify a significant relationship between expected return and risk. One of our claims is that the risk-return trade-off presents different patterns depending on the state of the economy. The dynamics of this relation observed during low volatility states (which supports the theoretical intuition) are different from those observed during high volatility states. This fact leads to a non-monotonic relation over time which is totally against the linear assumption made in many previous studies.

One of the main results of the study also provides a relationship between volatility regimes and risk aversion level. The risk aversion level in stock markets tends to be higher in low volatility states and lower in high volatility states. The investor profile in each context may have an influence on this lower risk aversion coefficient during high volatility periods. More risk averse investors leave the market during periods of market turmoil while less risk averse investors remain trading under these circumstances. Besides, high volatility regimes correspond to

[14] We use the median instead of the mean as a proxy for the average non-diversifiable risk in each period because it is less affected by outliers.

periods of recession or low expansion in the country's economy, whereas low volatility regimes correspond with periods of economic expansion which let us link the findings in this paper with others that support a procyclical risk aversion of investors in developed markets.

Above all, these results highlight the perils of strong linear assumption when analyzing the interactions between return and risk and claim that previous studies using linear models were likely to fail on the attempt to capture the global behavior between these two variables.

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APPENDIX A

All the models are estimated by maximum likelihood. The computations of the Regime-Switching models are carried out using the Optimization Library FMINCON of Matlab R2010b selecting the BFGS algorithm.

Although Regime-Switching GARCH models and Regime-Switching MIDAS models are different (as it is discussed in section 2.4 and 2.5), once the parameterization of the variance equation is defined in each case, the algorithm to implement is similar for both specifications.

Let Φ be the vector of parameters of the different models; $\pi_{s_t=k,t} = \Pr\left(s_t = k \middle| \Omega_{t-1}\right)$ the ex-ante probability of being in the state k (where Ω_{t-1} is the information set up to t-1); $P\left(s_t = k \middle| \Omega_t;\theta\right)$ the filtered probability of being in the state k; and $f\left(r_t \middle| s_t, \Omega_t;\theta\right) = \left(2\pi h_{t,s_t}\right)^{-\frac{1}{2}} e^{-\frac{\left(\varepsilon_{t,s_t}\right)^2}{2h_{t,s_t}}}$ the state- dependent likelihood vector (where the main difference between GARCH[15] and MIDAS specification is in the parameterization of the time-varying variance h_{t,s_t}).

The algorithm we used is described by the following steps:

1) Give initial values for the parameters of the model and the ex-ante probabilities:

$$\Phi^0,\ \pi_{s_t=k,t=1} = \Pr\left(s_{t=1} = k \middle| \Omega_{t=0}\right)$$

2) Implement Hamilton (1989) filtering procedure using this first observation.

$$P\left(s_{t=1} = k \middle| \Omega_{t=1};\Phi^0\right) = \frac{\pi_{s_t=k,t=1} f\left(r_{t=1} \middle| s_{t=1} = k, \Omega_{t=1};\Phi^0\right)}{\sum_{k=1}^{2} \pi_{s_t=k,t=1} f\left(r_{t=1} \middle| s_{t=1} = k, \Omega_{t=1};\Phi^0\right)} \qquad \textit{for } k = 1, 2$$

3) Compute the value of the log-likelihood function for t=1

$$\ln\left[\sum_{k=1}^{2} \pi_{s_t=k,t=1} f\left(r_{t=1}, \Omega_{t=1};\Phi^0\right)\right]$$

4) Repeat steps 2 and step 3 for all observations and compute the log-likelihood function until t=T

$$L\left(\Phi^0\right) = \sum_{t=1}^{T} \ln\left[\sum_{k=1}^{2} \pi_{s_t=k,t} f\left(r_t, \Omega_t;\Phi^0\right)\right]$$

5) Maximize the log-likelihood function to obtain an update version of the vector of parameters Φ^j:

$$L\left(\Phi^j\right) = \sum_{t=1}^{T} \ln\left[\sum_{k=1}^{2} \pi_{s_t=k,t} f\left(r_t, \Omega_t;\Phi^j\right)\right]$$

6) Iterate steps 2-5 with the updated parameters until achieving convergence.

Hamilton (1989) claims that this algorithm is a special case of the EM algorithm: the expectation (E) step corresponds to step 2 and the Maximization (M) step to step 3. During the Expectation step the algorithm is able to guess the values for the latent variable given the data and the updated parameters while the values of the parameters that maximize the log-likelihood function are driven in the Maximization step.

[15] Due to the recursive nature of the GARCH parameterization, we use a recombinative method to obtain independent variances and errors for each t. See section 2.4 for further details.

TABLE 1. Estimated parameters for GARCH and MIDAS models for all the European stock markets considered

	GARCH-M			***MIDAS***				
Parameter *(t-stat)*	c	λ_1	Persistence	c	λ_1	*% days 1–5*	*% days 10–30*	*% days >30*
Germany	0.0027** *(0.0277)*	0.0098 *(0.4911)*	0.9534	0.0028** *(0.0155)*	-0.0466 *(0.4934)*	**15,60%**	**36,34%**	**48,06%**
France	0.0008 *(0.5786)*	0.0141 *(0.4427)*	0.9744	0.0012 *(0.3359)*	-0.0359 *(0.6158)*	**14.74%**	**38.14%**	**47.12%**
Spain	0.0019 *(0.1546)*	0.0075 *(0.6498)*	0.9884	0.0028** *(0.0155)*	-0.0993 *(0.1679)*	**14.64%**	**36.62%**	**48.74%**
UK	0.0011 *(0.2639)*	0.0214 *(0.3132)*	0.9727	0.0016* *(0.0689)*	-0.0599 *(0.4561)*	**14.58%**	**38.08%**	**47.34%**
Switzerland	0.0019** *(0.0428)*	0.0277 *(0.1886)*	0.9660	0.0019 *(0.0428)*	-0.0059 *(0.4939)*	**19.73%**	**40.38%**	**39.89%**
Netherlands	0.0023** *(0.0101)*	0.0060 *(0.6717)*	0.9957	0.0019 *(0.0268)*	-0.0606 *(0.3396)*	**16.34%**	**36.47%**	**47.19%**
Belgium	0.0015 *(0.1329)*	0.0017 *(0.5026)*	0.9509	0.0017 *(0.0571)*	-0.0923 *(0.2653)*	**15.79%**	**37.19%**	**47.01%**
Denmark	0.0026** *(0.0271)*	-0.0049 (*0.8110)*	0.9509	0.0028 *(0.0116)*	-0.1088 *(0.2418)*	**14.92%**	**35.46%**	**49.62%**
Finland	0.0025 *(0.1185)*	0.0402 *(0.9759)*	0.9847	0.0005 *(0.7268)*	0.0228 *(0.6761)*	**12.75%**	**35.93%**	**51.32%**
Sweden	0.0027* *(0.0586)*	0.0088 *(0.5733)*	0.9605	0.0025 *(0.0339)*	-0.0639 *(0.3935)*	**18.62%**	**41.97%**	**39.41%**
Greece	0.0029* *(0.0609)*	-0.0175 *(0.1320)*	0.9907	0.0031 *(0.0427)*	-0.1287** *(0.0349)*	**4.74%**	**22.06%**	**73.20%**

This table shows the estimated parameters for the standard GARCH and standard MIDAS models presented in the paper (p-values in parentheses). ***, ** and * *represents significance at 1%, 5% and 10% respectively.*

TABLE 2. Estimated parameters for asymmetric models

This table shows the estimated parameters for the asymmetric GARCH and asymmetric MIDAS models presented in

	Asymmetric -GARCH-M					***Asymmetric-MIDAS***				
	c	λ_1	Persist.			c	λ_1	*% days 1–5*	*% days 10–30*	*% days >30*
Germany	0.0021* *(0.0696)*	-0.0036 *(0.8032)*	0.9217	∅	**I −**	0.0023** *(0.0352)*	-0.0579 *(0.3856)*	**52.74%**	**46.89%**	**0.37%**
					I +			**4.20%**	**20.04%**	**75.76%**
France	0.0007 *(0.5697)*	-0.0015 *(0.9318)*	0.9542	∅	**I −**	0.0011 *(0.3358)*	-0.0355 *(0.6264)*	**43.95%**	**52.01%**	**4.04%**
					I +			**5.19%**	**24.08%**	**70.73%**
Spain	0.0026** *(0.0359)*	-0.0161 *(0.3211)*	0.9729	∅	**I −**	0.0027** *(0.0204)*	-0.1011 *(0.1738)*	**85.46%**	**15.54%**	**0.00%**
					I +			**11.87%**	**32.09%**	**56.04%**
UK	0.0013 *(0.1485)*	-0.0096 *(0.6185)*	0.9637	∅	**I −**	0.0016* *(0.0603)*	-0.0646 *(0.4365)*	**45.48%**	**53.64%**	**0.88%**
					I +			**3.62%**	**19.28%**	**77.10%**
Switzerl.	0.0020** *(0.0263)*	-0.0075 *(0.6622)*	0.9174	∅	**I −**	0.0023** (0.0164)	-0.0883 (0.3232)	**23.84%**	**48.40%**	**27.35%**
					I +			**0.01%**	**0.12%**	**99.87%**
Netherl.	0.0020** *(0.0199)*	-0.0097 *(0.5008)*	0.9777	∅	**I −**	0.0022*** *(0.0040)*	-0.0928 *(0.1614)*	**46.90%**	**48.39%**	**4.71%**
					I +			**3.54%**	**16.75%**	**79.71%**
Belgium	*0.0017** *(0.0850)*	-0.0128 (0.4802)	0.9256	∅	**I −**	0.0020** *(0.0309)*	-0.1109 *(0.1820)*	**18.69%**	**43.49%**	**37.81%**
					I +			**0.01%**	**0.01%**	**99.98%**
Denmark	0.0028** *(0.0274)*	-0.0165 (*0.4436)*	0.9581	∅	**I −**	0.0031*** *(0.0041)*	-0.1283 *(0.1516)*	**11.45%**	**32.94%**	**55.61%**
					I +			**33.24%**	**16.98%**	**49.79%**
Finland	0.0026* *(0.0975)*	-0.0060 *(0.6437)*	0.9447	∅	**I −**	0.0011 *(0.4408)*	0.0057 *(0.9117)*	**11.57%**	**32.45%**	**55.97%**
					I +			**5.97%**	**46.69%**	**47.33%**
Sweden	0.0028** *(0.0335)*	-0.0072 *(0.6209)*	0.9447	∅	**I −**	0.0029** *(0.0150)*	-0.0866 *(0.2729)*	**29.97%**	**58.08%**	**11.95%**
					I +			**6.05%**	**23.38%**	**70.57%**
Greece	0.0033** *(0.0419)*	-0.0246** *(0.0402)*	0.9907	∅	**I −**	0.0029* *(0.0548)*	-0.1235** *(0.0499)*	**4.98%**	**24.44%**	**70.58%**
					I +			**4.91%**	**21.06%**	**74.03%**

the paper (p-values in parentheses). ***, ** and * *represents significance at 1%, 5% and 10% respectively.*

TABLE 3. Estimated parameters for RS-GARCH models

*This table shows the estimated parameters for the RS-GARCH modes presented in the paper (p-values in parentheses). ***, ** and * represents significance at 1%, 5% and 10% respectively.*

	RS-GARCH-M					
Parameter *(t-stat)*	***State k=1***			***State k=2***		
	c	λ_1	Persist	c	λ_1	Persist.
Germany	-0.0124*** *(0.0022)*	0.6479*** (0.0000)	0.9016	-0.0124* *(0.0022)*	0.0520** *(0.0471)*	0.2109
France	-0.0102** *(0.0313)*	0.3451*** *(0.0034)*	0.8935	-0.0102** *(0.0313)*	0.0577* *(0.0603)*	0.2911
Spain	-0.0201*** *(0.0086)*	0.6378*** *(0.0014)*	0.8644	-0.0201*** *(0.0086)*	0.1027** *(0.0263)*	0.2032
UK	-0.0084** *(0.0109)*	0.5527*** *(0.0044)*	0.8640	-0.0084** *(0.0109)*	0.0760 *(0.1452)*	0.2663
Switzerland	-0.0124*** *(0.0029)*	0.7763*** *(0.0000)*	0.8912	-0.0124*** *(0.0029)*	0.0638 *(0.1004)*	0.2037
Netherlands	-0.0017 *(0.4870)*	0.2101* *(0.0921)*	0.6989	-0.0017 *(0.4870)*	0.0295 *(0.1484)*	0.4210
Belgium	-0.0081** *(0.0146)*	0.4995*** *(0.0002)*	0.8112	-0.0081** *(0.0146)*	0.0391* *(0.0659)*	0.2373
Denmark	-0.0063 *(0.0683)*	0.3222*** *(0.0027)*	0.8191	-0.0063 *(0.3758)*	0.0215* *(0.0683)*	0.3970
Finland	-0.0078 *(0.2052)*	0.1971** *(0.0625)*	0.8575	-0.0078 *(0.2052)*	0.0264 *(0.2698)*	0.2377
Sweden	-0.0265*** *(0.0039)*	0.7364*** *(0.0015)*	0.9191	-0.0265*** *(0.0039)*	0.1058** *(0.0466)*	0.1162
Greece	-0.0029 *(0.5480)*	0.0783 *(0.3164)*	0.6609	-0.0029 *(0.5480)*	0.0043 *(0.8178)*	0.1436

TABLE 4. Estimated parameters for RS-MIDAS

RS-MIDAS		c	λ_1	*% days 1–5*	*% days 10–30*	*% days >30*
Germany	**St=1**	0.0018*** *(0.000)*	0.1205*** *(0.000)*	**53.64%**	**46.21%**	**0.14%**
	St=2		0.0544*** *(0.000)*	**0**	**0.14%**	**99.86%**
France	**St=1**	0.0012 *(0.1148)*	0.2112*** *(0.0001)*	**18.53%**	**47.12%**	**34.35%**
	St=2		0.0245 *(0.1185)*	**0.01%**	**0.01%**	**99.98%**
Spain	**St=1**	0.0026*** *(0.000)*	0.1582*** *(0.0001)*	**49.20%**	**49.79%**	**1.01%**
	St=2		0.0905*** *(0.0000)*	**0.08%**	**1.65%**	**98.27%**
UK	**St=1**	0.0015*** *(0.000)*	0.0744** *(0.0265)*	**44.86%**	**52.62%**	**2.52%**
	St=2		0.0674*** *(0.000)*	**0.01%**	**0.01%**	**99.98%**
Switzerland	**St=1**	-0.0031*** *(0.000)*	0.0310*** *(0.000)*	**49.13%**	**50.16%**	**0.71%**
	St=2		0.0196*** *(0.0000)*	**0.01%**	**0.01%**	**99.98%**
Netherlands	**St=1**	-0.0147*** *(0.0008)*	0.0413*** *(0.0000)*	**52.15%**	**44.75%**	**3.10%**
	St=2		-0.6947*** *(0.0000)*	**0.06%**	**3.17%**	**96.78%**
Belgium	**St=1**	0.0012*** *(0.000)*	0.2577*** *(0.0000)*	**58.09%**	**41.55%**	**0.37%**
	St=2		-0.0172 *(0.5071)*	**1.15%**	**6.98%**	**91.87%**
Denmark	**St=1**	0.0127*** *(0.0000)*	1.7785*** *(0.0012)*	**8.19%**	**20.29%**	**71.52%**
	St=2		0.0406 *(0.4794)*	**14.61%**	**35.89%**	**49.50%**
Finland	**St=1**	0.0108*** *(0.000)*	1.6041*** *(0.0000)*	**16.57%**	**34.25%**	**49.17%**
	St=2		0.0440 *(0.2786)*	**26.23%**	**54.33%**	**19.44%**
Sweden	**St=1**	0.0075*** *(0.0000)*	1.1174*** *(0.0000)*	**17.33%**	**57.62%**	**25.06%**
	St=2		-0.0134* *(0.0575)*	**24.95%**	**57.46%**	**17.59%**
Greece	**St=1**	0.0046*** *(0.0002)*	0.3702*** *(0.0000)*	**2.14%**	**12.52%**	**85.34%**
	St=2		-0.2601*** *(0.0000)*	**6.67%**	**30.69%**	**62.64%**

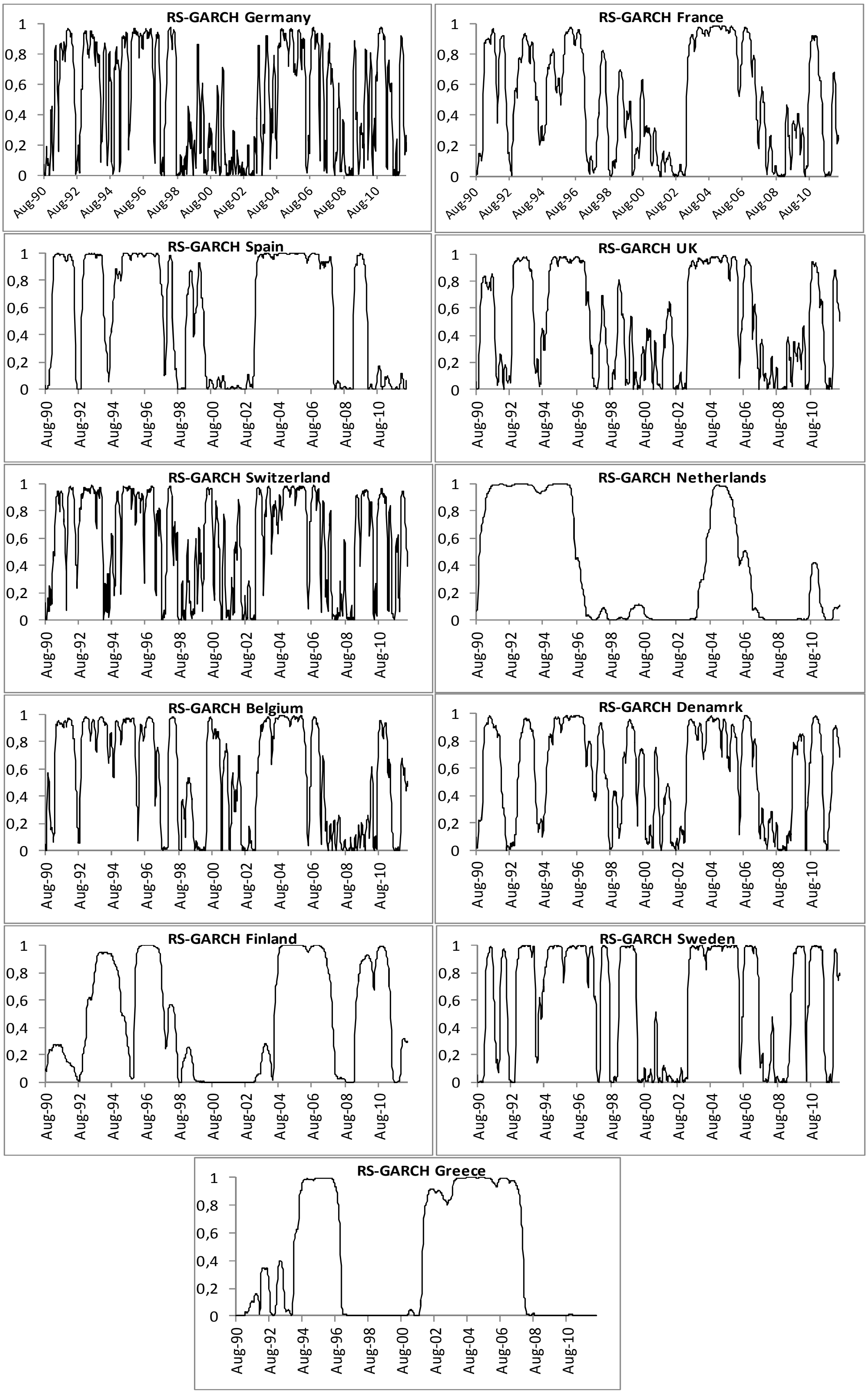
This table shows the estimated parameters for the RS-MIDAS modes presented in the paper (p-values in parentheses). ***, ** and * *represents significance at 1%, 5% and 10% respectively.*

TABLE 5. Annualized market risk premium for Europe

This table shows the average risk premium (using the median of the series) for each country considered using the different models proposed.

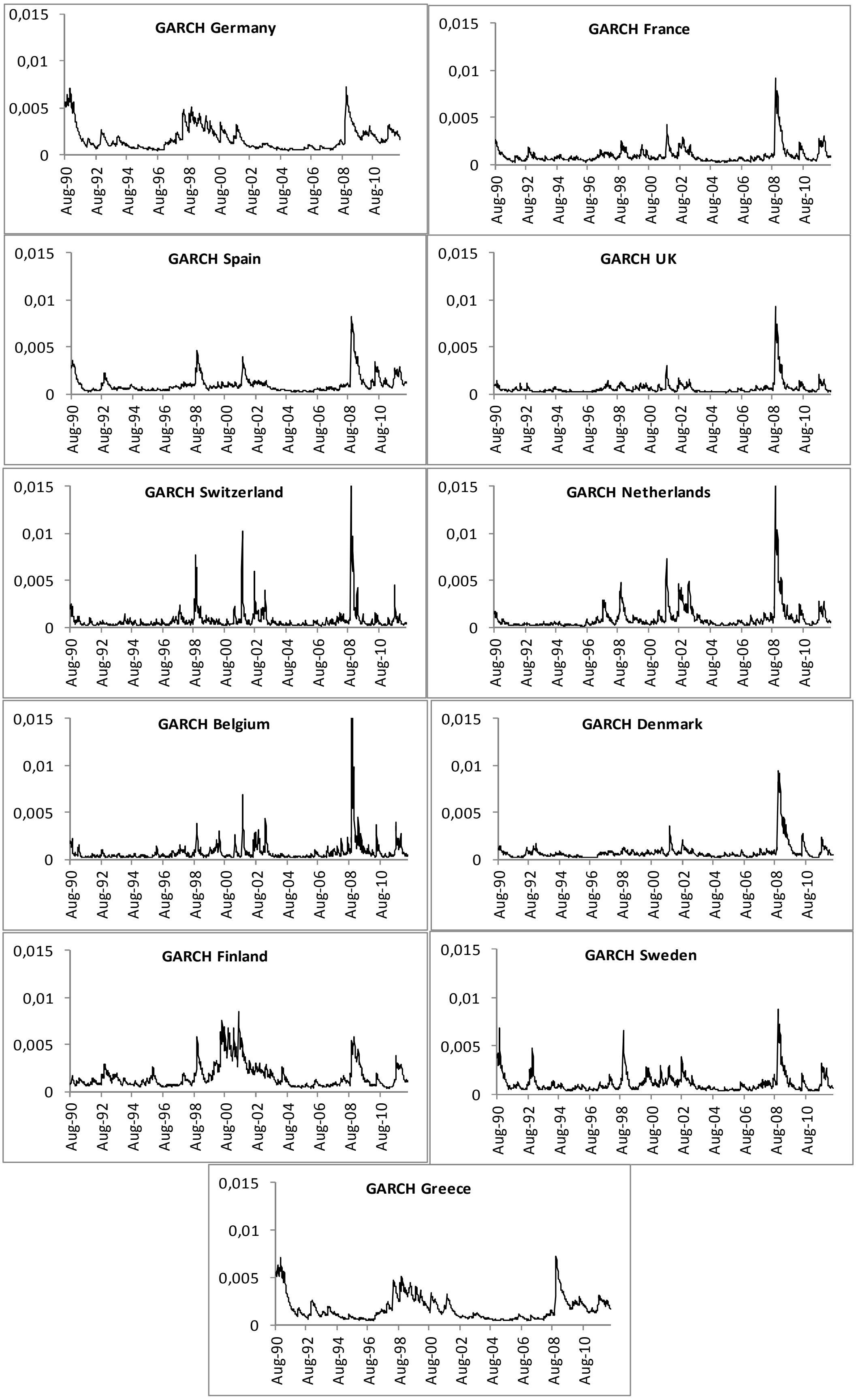
Average Risk premium						
	GARCH	*Asymmetric GARCH*	*RS-GARCH*	*MIDAS*	*Asymmetric MIDAS*	*RS-MIDAS*
Germany	3.882%	3.771%	3.684%	3.636%	3.692%	4.267%
France	3.812%	3.606%	3.552%	3.746%	3.823%	4.029%
Spain	3.981%	3.554%	3.246%	3.985%	3.993%	4.482%
UK	2.309%	2.057%	2.075%	2.277%	2.432%	2.424%
Switzerland	2.425%	2.130%	2.119%	2.307%	2.388%	2.219%
Netherlands	2.900%	2.627%	3.021%	2.891%	2.936%	2.577%
Belgium	2.422%	2.316%	2.067%	2.289%	2.243%	2.623%
Denmark	3.017%	2.838%	2.620%	2.619%	2.634%	2.600%
Finland	5.932%	5.816%	5.859%	4.877%	4.788%	4.830%
Sweden	4.132%	3.893%	3.463%	3.272%	3.486%	2.976%
Greece	6.881%	6.888%	8.054%	7.293%	7.315%	7.080%

FIGURE 1. Smoothed probability for low volatility state in Europe



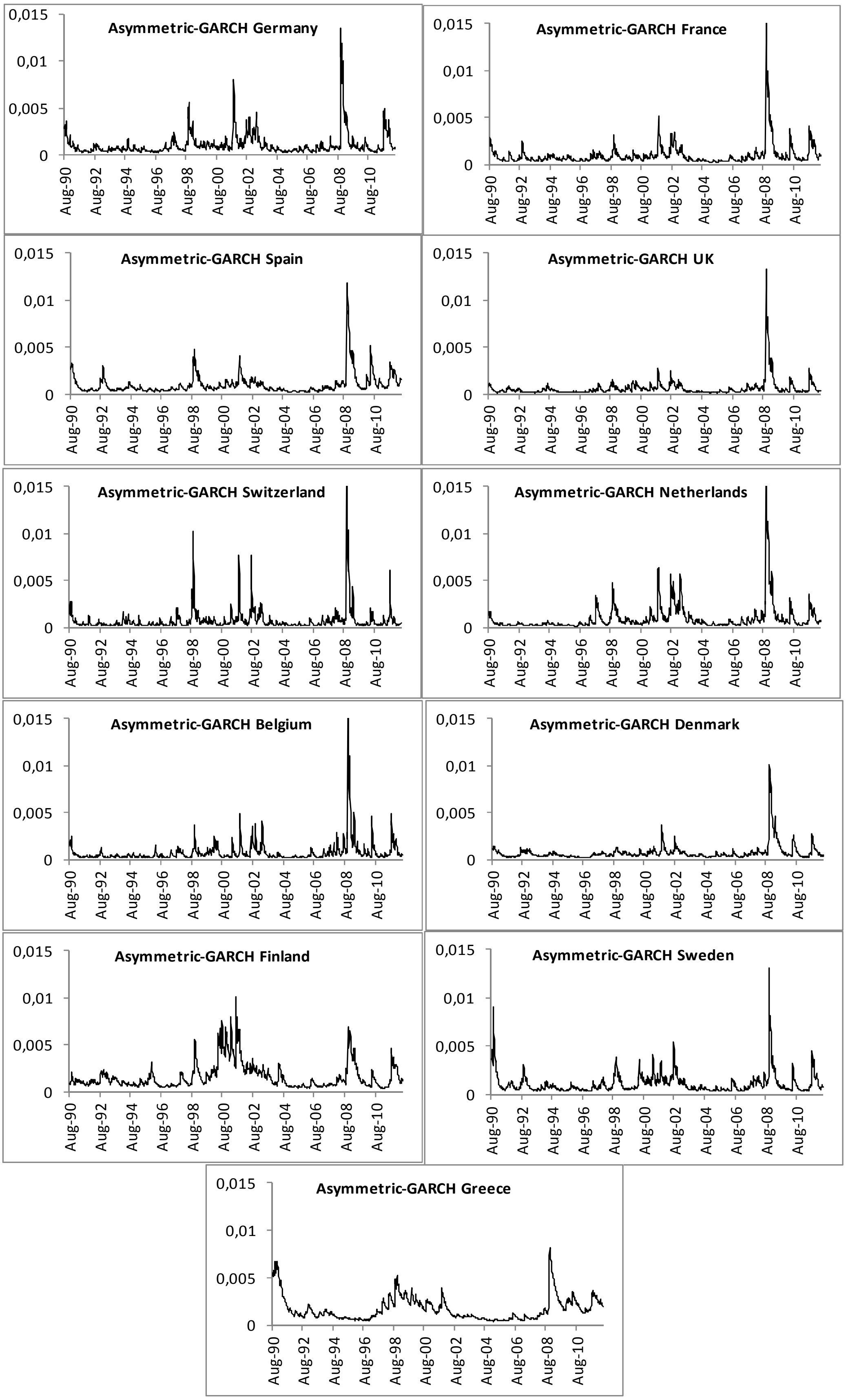
This figure represents the probability of being in a low volatility state in each European country

FIGURE 2.A Risk premium evolution in Europe using the GARCH model



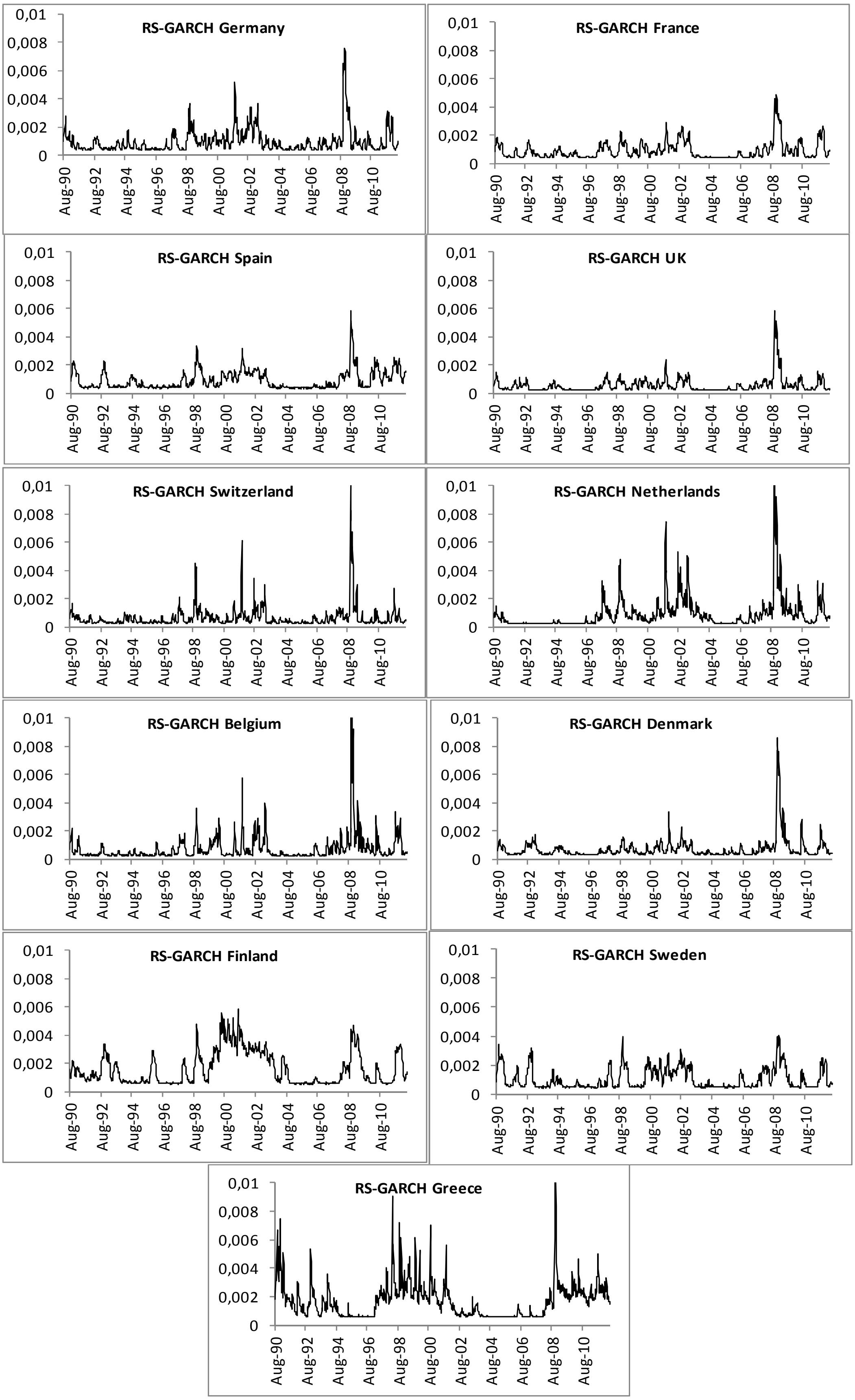
These figures show the risk premium evolution in all European countries for the standard GARCH specification

FIGURE 2.B Risk premium evolution in Europe using the asymmetric GARCH model



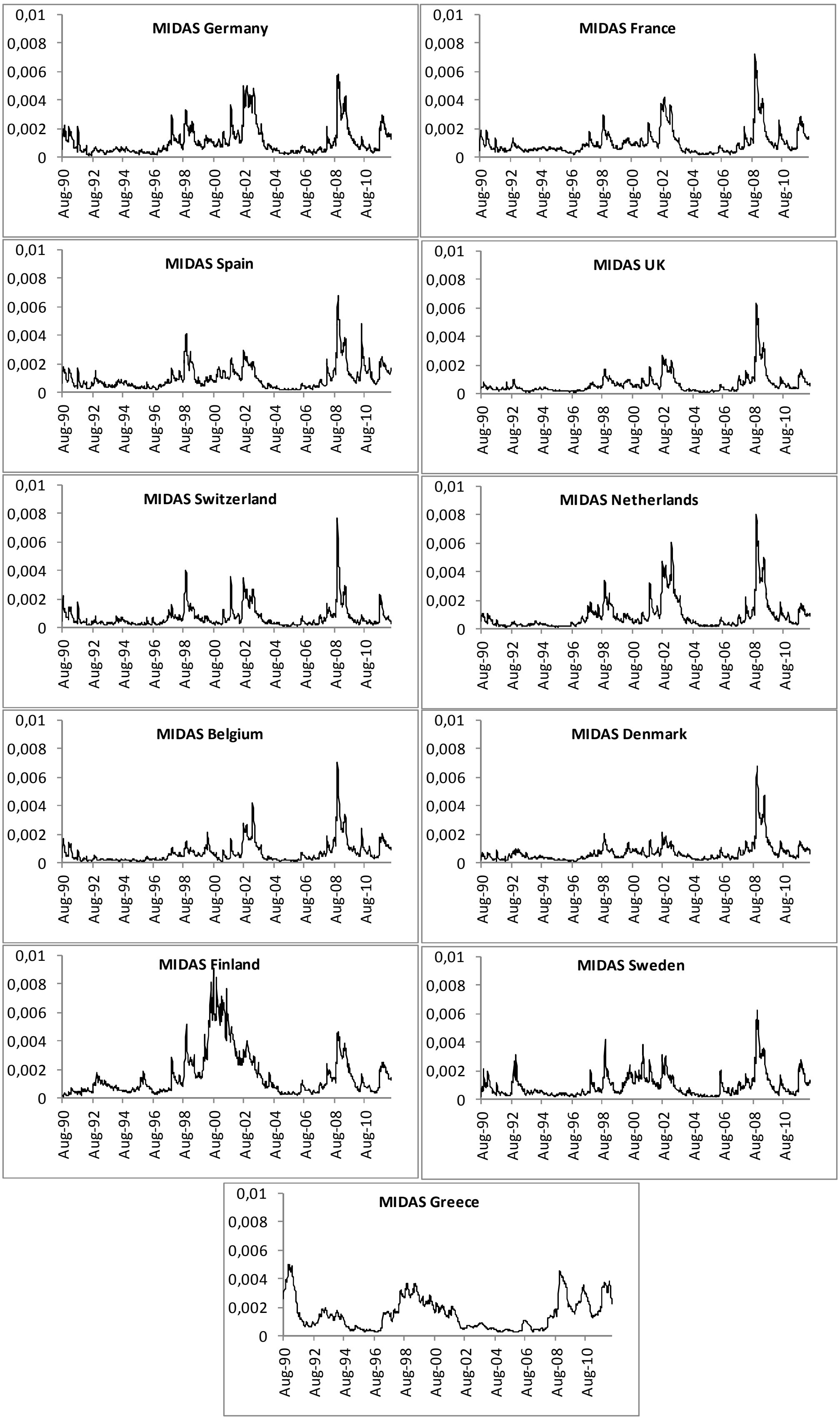
These figures show the risk premium evolution in all European countries for the asymmetric GARCH specification

FIGURE 2.C Risk premium evolution in Europe using the RS- GARCH model



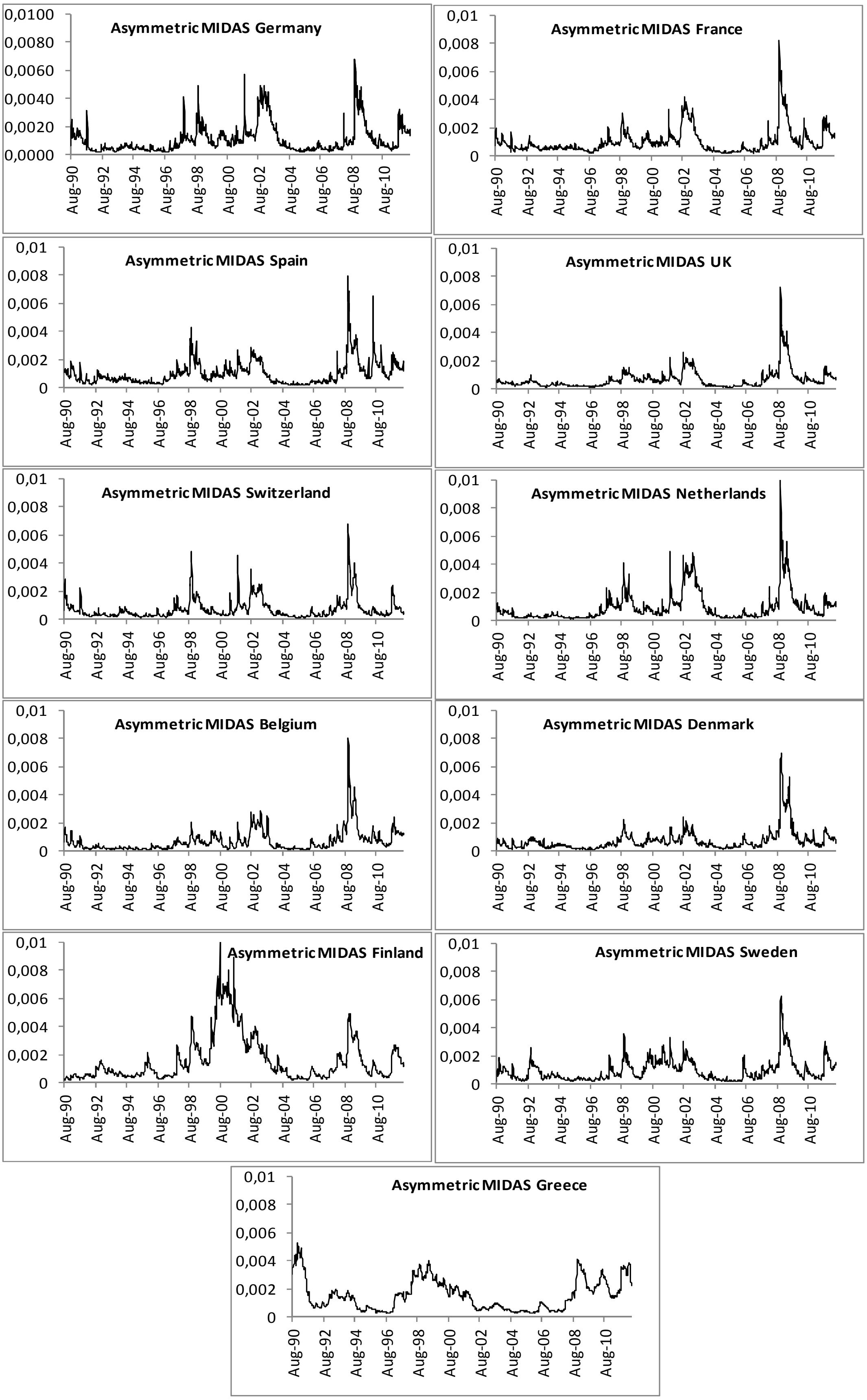
These figures show the risk premium evolution in all European countries for the RS- GARCH specification

FIGURE 2.D Risk premium evolution in Europe using the MIDAS model



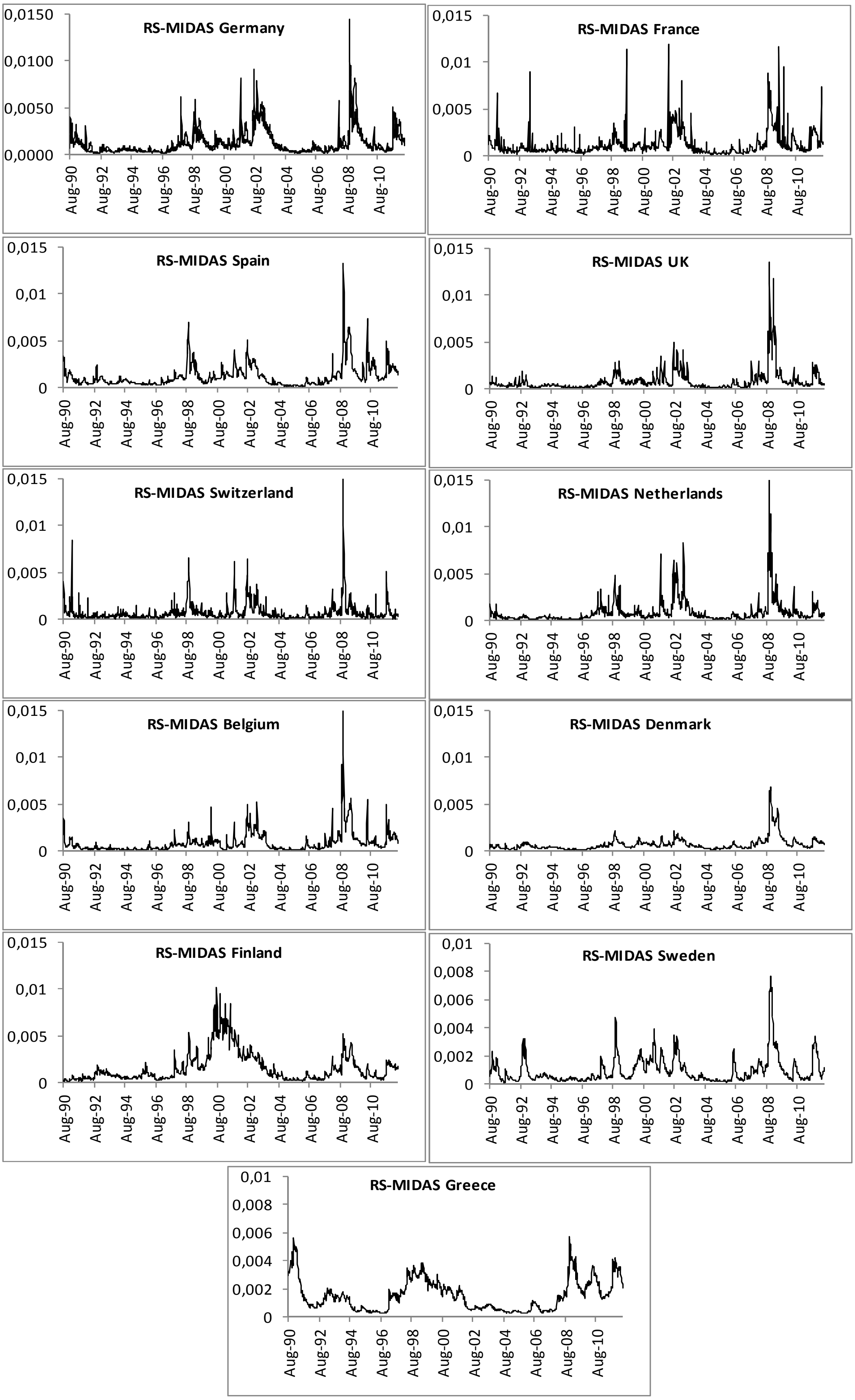
These figures show the risk premium evolution in all European countries for the MIDAS model

FIGURE 2.E Risk premium evolution in Europe using the asymmetric MIDAS model



These figures show the risk premium evolution in all European countries for the asymmetric MIDAS model

FIGURE 2.F Risk premium evolution in Europe using the RS-MIDAS model



These figures show the risk premium evolution in all European countries for the RS-MIDAS model