Welfare Effect of Consumption Taxes

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Abstract

Tax reform is a recurrent topic, but most of the prevailing proposals are fail to promote either efficiency or equity. In this paper, we consider an alternative reform, a consumption tax reform. The results show that aggregate capital, labor and consumption are improved by replacing a labor income tax with a consumption tax. Moreover, a progressive consumption tax alone can achieve a significant welfare gain, and the welfare gap between the rich and the poor is reduced.

Keywords: Incomplete markets, Consumption taxes, Welfare inequality

JEL classification: E2, D52, H21

1. Introduction

Given the current government deficit, highly unequal distribution of tax burden and extremely complexness of the tax system, a tax reform is crucial. However, most of the tax reforms, which aim at adjusting income tax codes, are at the cost of either efficiency or equity. Therefore, many political and business commentators have argued that consumption tax reforms might be the solution to the efficiency-equity trade-off.

Hall (1995) illustrates the principle of consumption taxes. That is people are taxed on what they take out of the economy, instead of what they put in. Thus, the mainly difference between consumption taxes and income taxes is that consumption taxes can exempt savings / investment from taxation. As a result, the added investment will lead to a bigger economic "pie" to be divided among households. This opinion is also shared by many other economists like Seidman (1995), Kaldor (1955) and Summer (1984b), etc..

But whether consumption taxes can contribute to reduce welfare inequality is left unknown. Corriea (2010) argues that a consumption tax can lead to a lower level of inequality. But however, her results are obtained by the assumption that all taxes are flat and under a complete market setting, hence the aggregates are not affected. Therefore, in this paper, we prove the same conclusion as Corriea (2010) by developing an alternative approach, the intuition of which can be extended to an incomplete market. The numerical results show that with incomplete market, replacing a flat labor tax with a flat consumption tax can increase

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aggregates and reduce welfare inequality. This is because the consumption tax reform encourages saving, so the interest rate decreases while the wage rate increases, and by a larger amount than the increase in consumption tax. As a result, low-asset households, who largely depend on labor income, would have an income increased more than consumption tax, so consumption increases. Whereas high-asset households, who gain their income mainly through capital returns, might experience a decrease in income. The decline in income, accompanied by an increase in consumption tax, result in a decrease in their consumption.

However, in a more realistic case, where the labor tax is progressive, the welfare inequality of levying a flat consumption tax instead of a progressive labor tax is actually enlarged. This results have been shown by some existing literatures, like Alig et al. (2001), Feenberg (1997) and Auerbach (1983) etc.. Although most of these works are under the framework of OLG model, the results of broadened welfare gap in both OLG model and Aiyagari (1994) model are straightforward to interpret. That is by eliminating a progressive labor tax, low-asset households, most of whom are also with low productivity, do not have much improvement in income, but are made to pay a higher consumption tax. Thus, the reform might make them worse off. High-asset households, who are more likely to have high labor efficiency, on the other hand, are no longer subject to a previous high labor tax. For theses households, an increase in income outweighs the increase in consumption tax, thus they are better off after the reform.

Regarding the welfare loss by switching from a progressive labor tax to a flat consumption tax, Seidman (1997) proposed UAS tax, in which he argues that consumption tax should be progressive. In principle, people should be taxed based on what they take from the economic pie, the more one takes, the less he leaves for the others. Therefore, a surcharge on top of a flat consumption tax is necessary, and this forms a progressive consumption tax. Moreover, Gentry (1997) also mentions that consumption taxes should be at least as progressive as labor taxes, but without a numerical confirmation. Thus, in this paper, we numerically implement the reform that moves from a progressive labor tax to a progressive consumption tax with the same progressivity under a balanced government budget. The welfare effect is quite significant and Gini indexes of consumption equivalent is brought down by 3%.

Though long-run consequence of a tax reform should be considered in a tax reform, short term effect is at the center of the issue. Thus, we also examine the transition path of the progressive consumption tax reform and find a appealing welfare gain in short-run; whereas the flat consumption tax reform leads in a welfare loss.

The rest of the paper is organized as follows. Section 2 presents the reform under complete markets and provides analytical solutions. Section 3 extends the previous analysis to incomplete markets. In this section, we first calibrate the model, then show the numerical results at steady states for different scenarios and during the transition paths. Section 4 considers an alternative reform. Section 5 concludes the paper.

2. Complete Markets

2.1. The Model

We first consider an economy with complete markets with no labor shock. The economy is populated with a continuum (with measure 1) of infinite lived households, who differ in their initial assets. Each agent is endowed with 1 unit of time which can be divided into work and leisure. Households do not value leisure, and the preferences over sequences of consumption take the form

$$\max_{c_t} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $\beta \in (0,1)$ is the subjective discount factor. The period utility function $u(\cdot)$ satisfies the following conditions: $u(0) = 0$; $u(\cdot)$ is continuously differentiable; $u'(\cdot) > 0$; $u'(\cdot)$ is strictly decreasing function; $\lim_{x\to\infty} u(x) = 0$; $\lim_{x\to 0} u(x) = \infty$.

The household's pre-tax labor income is $w_t l_t$, where w_t is the wage rate at period t. Labor income is subject to a flat labor income tax of τ_w. Asset income is subject to a capital income tax of τ_a, so the after-tax capital income is $\big(1 + r(1-\tau)\big)a_t$, where a_t is the asset holding at period t. Consumption is subject to a flat consumption tax of τ_c. The budget constraint at period t is

$$(1+\tau_c)c_t + a_{t+1} = (1 + r_t^\tau)a_t + (1-\tau_w)w_t$$
$$\text{where } r_t^\tau = (1-\tau_a)r_t.$$

There is a representative firm who borrows capital and labor from households to maximize profits according to

$$\max_{K_t, L_t} AF(K_t, L_t) - (r_t + \delta)K_t - wL_t$$

where δ is the depreciation rate. This maximization problem leads to

$$r_t = AF_K(K_t, L_t) - \delta$$
$$w_t = AF_L(K_t, L_t)$$

where K_t and L_t denote aggregate capital and labor at period t, F_K and F_L are first order derivatives with respect to capital and labor respectively.

The government has a constant consumption of G each period. Assuming that the government has a balanced budget, and all the revenue is collected from taxing consumption, labor income and capital income.

$$G = \tau_c C_t + \tau_a r_t K_t + \tau_w w_t L_t$$

where C_t denotes aggregate consumption at period t.

The asset and labor markets clearing requires that aggregate asset and labor provided by the households are equal to the capital and labor that required by the firm. The output market clearing condition equates the output to aggregate investment, consumption of households and the government consumption.

$$\int_S a_t(a) d\Gamma_t(a) = K_t$$
$$\int_S 1 d\Gamma_t(a) = L_t$$
$$\int_S c_t(a) d\Gamma_t(a) = C_t$$
$$C_t + K_{t+1} - (1-\delta)K_t + G = AF(K_t, L)$$

where $\Gamma_t(a)$ is the distribution over asset domain S at period t.

Definition 1 *Recursive equilibrium : Given a vector of tax rate (τ_c, τ_a, τ_w), asset domain $A = [b, \infty)$, a probability distribution $\Gamma(a)$ over the domain A, and the government consumption G, the competitive equilibrium is a value function $V(a)$, a pair of policy function $g_c(a)$ and $g_k(a)$, a vector of aggregate capital and labor (K, L), factor prices $\big(r(K,L), w(K,L)\big)$, such that:*
1. Value functions and policy functions solve household utility maximization problem:

$$\begin{aligned} V(a) &= \max_{c,a'} u(c) + \beta V(a') \\ s.t. (1+\tau_c)c + a' &= (1+r^\tau)a + (1-\tau_w)w \\ r^\tau &= (1+\tau_a)r \\ c &= g_c(a) \\ a' &= g_a(a) \\ a &\ \textit{is given}; \end{aligned}$$

2. Factor prices satisfy the condition of maximizing profit:

$$\begin{aligned} r &= AF_K(K,L) - \delta \\ w &= AF_L(K,L); \end{aligned}$$

3. The government budget constraint holds: $G = \tau_c C + \tau_a rK + \tau_w wL$, where $C = \int_A g_c(a) d\Gamma(a)$ is the aggregate consumption; 4. Market clearing:

$$\begin{aligned} \int_A a d\Gamma(a) &= K \\ \int_A 1 d\Gamma(a) &= L \\ C + K' - (1-\delta)K + G &= AF(K,L). \end{aligned}$$

2.2. Effects of tax reforms: Theoretical results

At this stage, we focus on steady states. Since the market is complete with inelastic labor, aggregate capital does not change across periods. Thus aggregate consumption can be calculated from the budget constraint, $(1+\tau_c)C = (1+r^\tau)K + (1-\tau_w)wL$, so we have

$$C = \frac{1}{1+\tau_c}\big(r^\tau K + (1-\tau_w)wL\big) \tag{1}$$

The government collects its revenue from consumption and income taxes: $G = \frac{r^\tau}{1+r^\tau}(\tau_c C + \tau_a rK + \tau_w wL)$. We can simplify this expression by substituting out C using (1), and write G in terms of aggregate capital and labor,

$$G = \frac{\tau_c + \tau_a}{1+\tau_c} rK + \frac{\tau_c + \tau_w}{1+\tau_c} wL$$

Consider a tax reform that shifts taxes from (τ_c, τ_w) to (τ_c^R, τ_w^R) with $\tau_c^R > \tau_c$, and $\tau_w^R < \tau_w$. Then the post reform government spending is

$$G^R = \frac{\tau_c^R + \tau_a}{1+\tau_c^R} r^R K^R + \frac{\tau_c^R + \tau_w^R}{1+\tau_c^R} w^R L^R$$

where $K^R = K$, $r^R = r$ and $w^R = w$ as a result of market completeness, and $L^R = L$ because of inelastic labor supply. Define $\lambda_c = \frac{1+\tau_c^R}{1+\tau_c}$ as the change in consumption tax, $\lambda_w = \frac{(1-\tau_w^R)w^R}{(1-\tau_w)w}$ as the change in wage. The assumption of a balanced government budget requires that $G^R = G$. Therefore, λ_c and λ_w satisfy the following equations:

$$\lambda_w = \lambda_c + (\lambda_c - 1)\frac{r^\tau K}{(1-\tau_w)w} \tag{2}$$

The proof see the appendix.

Proposition 1 $\lambda_w > \lambda_c$. *That is the increasing in wage is greater than that of the consumption tax after the reform.*

Proof. Because of the complete market assumption, the wage rate does not change with the reform. Therefore, the after-tax wage before and after the reform is $(1-\tau_w)w$ and $\lambda_w(1-\tau_w)w$. By assuming that $\tau_c^R > \tau_c$, $\lambda_c > 1$. Since the other factors on the right handside of equation (2) are all positive, $\lambda_w > \lambda_c$. ■

The intuition behind this is that, since consumption share of total GDP is higher than that of labor, if wage increases by 1 unit as a result of reducing labor income tax, then a less than 1 unit increase in consumption tax is required to maintain the government budget.

Proposition 2 *After the reform, aggregate welfare increases, and welfare inequality decreases. Moreover, aggregate welfare is an increasing function of λ_c, the change in consumption tax, while welfare inequality is decreasing in λ_c.*

Before going to the strict proof, we provide some intuition, which can also be applied to the explanation under incomplete markets. Since we focus on steady states, asset holdings of next period should be identical to that of current period, that is $a' = a$. Therefore, by rearranging budget constraint, pre-reform steady state consumption is,

$$c = \frac{r^\tau a}{1+\tau_c} + \frac{(1-\tau_w)wl}{1+\tau_c}$$

Since $1+\tau_c^R = \lambda_c(1+\tau_c)$, and $1+\tau_w^R = \lambda_w(1+\tau_w)$, post-reform steady state consumption can be written as

$$c = \frac{r^\tau a}{\lambda_c(1+\tau_c)} + \frac{\lambda_w}{\lambda_c}\frac{(1-\tau_w)wl}{1+\tau_c}$$

By proposition 1, $\lambda_w > \lambda_c$. Therefore, the tax reform increases the second part of consumption for all types of households by the same amount. By assuming that $\tau_c^R > \tau_c$, we have $\lambda_c > 1$. Thus the first part of consumption decreases. As a result, the direction to which consumption is changing after the reform is ambiguous. But nevertheless, whether consumption increases or decreases depends on asset level: the lower the asset holdings, the smaller the reduction in the first part of consumption, thus the more likely it is that the increase in the second part of consumption dominates, rising consumption. Since the complete market setting guarantees analytical solutions, we are able to provide a strict proof of to what extent that assets affect consumptions.

Proof. The budget constraint before the reform is

$$(1+\tau_c)c + a' = (1-\tau)a + (1-\tau_w)w, \text{ where } a \geq 0$$

where a' is the next period asset holding. Because of the market completeness, $a' = a$, so the budget constraint becomes

$$(1+\tau_c)c = r^\tau a + (1-\tau_w)w, \text{ where } a \geq 0 \tag{3}$$

The reform increases the consumption tax λ_c and wage by λ_w respectively, the post-reform budget constraint can therefore be written as

$$\lambda_c(1+\tau_c)c^R = r^\tau a + \lambda_w(1-\tau_w)w, \text{ where } a \geq 0 \tag{4}$$

where c^R denotes post-reform consumption.

If we plug (2), into the above post-reform budget (4) constraint, we have

$$\begin{aligned} \lambda_c(1+\tau_c)c^R &= r^\tau a + \lambda_c(1-\tau_w)w + (\lambda_c - 1)\frac{r^\tau K}{(1-\tau_w)wL}(1-\tau_w)w \\ &= r^\tau a + \lambda_c(1-\tau_w)w + (\lambda_c - 1)r^\tau K \end{aligned} \tag{5}$$

The comparison between the pre-reform budget constraint (3) and the post-reform budget constraint (5) shows that the reform increase both the consumption price and the after-tax labor income by λ_c. Whether post-reform consumption is higher than that of pre-reform consumption depends on whether the summation of asset income $r^\tau a$ and the last term $(\lambda_c - 1)r^\tau K$ exceeds $\lambda_c r^\tau a$.

The following conditions summarizes how consumption changes after the reform.

$$\text{If} \begin{cases} a < K \Longrightarrow r^\tau a + (\lambda_c - 1)r^\tau K > \lambda_c r^\tau a \Longrightarrow c^R > c, \\ a = K \Longrightarrow r^\tau a + (\lambda_c - 1)r^\tau K = \lambda_c r^\tau a \Longrightarrow c^R = c, \\ a > K \Longrightarrow r^\tau a + (\lambda_c - 1)r^\tau K < \lambda_c r^\tau a \Longrightarrow c^R < c. \end{cases}$$

That is, for households whose asset levels are below the average, consumption increases after the reform. But for households with assets that are higher than the average, the reform reduces their consumption. Since there is no uncertainty, individual welfare is an increasing function of consumption. Therefore, welfare increases for the households with assets that are lower than the average, and decreases for the households with asset that are higher than the average. Since households with low assets have higher marginal utility of consumption than households with high assets, the increase in consumption of low-asset households causes a larger welfare gain than the welfare loss by their counterparts. It follows that aggregate welfare increases, and inequality decreases. The greater the λ_c, the more the improvement in welfare, and the lower the welfare inequality. ■

3. Incomplete markets

3.1. The Model

Now we extend the analysis of complete markets to an incomplete market economy, where households face idiosyncratic labor shocks.

Households

Households are endowed with 1 unit of time each period, which they divide into consumption and leisure. The preference over sequences of consumption takes the form

$$\max_{c_t, k_{t+1}, h_t} \sum_{t=0}^{\infty} E_0 \beta^t u(c_t, l_t)$$

where c_t and l_t are consumption and labor at period t respectively, $\beta \in (0,1)$ is subjective discount factor, and E_0 denotes the conditional expectation at date 0. The period utility function $u(c)$ satisfies Inada condition: $u(0,0) = 0$; $u(\cdot,\cdot)$ is continuously differentiable in both its arguments; $u_1(\cdot,\cdot) > 0$, $u_2(\cdot,\cdot) < 0$; both $u_1(\cdot,\cdot)$ and $u_2(\cdot,\cdot)$ are strictly decreasing functions; $\lim_{x\to\infty} u(x,\cdot) = 0$, $\lim_{x\to 1} u(\cdot, x) = 0$; $\lim_{x\to 0} u(x,\cdot) = \infty$, $\lim_{x\to 0} u(\cdot, x) = \infty$.

Each period, households receive an idiosyncratic labor shock ϵ, which is *i.i.d.* across households and follows the Markov process with transition matrix $\pi(\epsilon_{t+1}|\epsilon_t)$. The household's pre-tax labor income is $w_t l_t \epsilon_t$, where w_t is wage rate of period t and l_t denotes labor supply at t. Labor income is subject to a labor income tax of τ_w.

To insure against income uncertainty, households can trade (either by lending or borrowing with a borrowing limit of b) their physical capital each period which is subject to a capital income tax τ_a. The after-tax capital income is $(1 + r^\tau)a_t = \big(1 + (1-\tau_a)r\big)a_t$, where a_t are the asset holdings of current period. The total disposable income, which consists of after-tax labor and after-tax capital income $\big(1 + (1-\tau_a)\big)a_t + (1-\tau_w)w_t l_t \epsilon$, is divided into consumption c_t and the next period asset holding a_{t+1}. The consumption is subject to a consumption tax, denoted by τ_c. The budget constraint of household is

$$\begin{aligned}(1+\tau_c)c_t + a_{t+1} &= (1+r^\tau)a_t + (1-\tau_w)w_t l_t \epsilon_t \\ a_{t+1} &\geq b \\ a_0 &\text{ is given}\end{aligned}$$

Throughout the paper, we assume household cannot borrow, which means that the borrowing limit b is 0.

Production

The representative firm produces according to a constant return to scale production function $AF(K, L) : R^2 \to R$, where K and L denote capital and labor respectively. Each period the firm borrows capital and labor from households to maximize profits according to

$$\max_{K_t, L_t} AF(K_t, L_t) - (r_t + \delta)K_t - w_t L_t$$

where δ is the depreciation rate. This maximization problem leads to the following factor prices,

$$\begin{aligned}r_t &= AF_K(K_t, L_t) - \delta \\ w_t &= AF_L(K_t, L_t)\end{aligned}$$

Government

The government has a consumption of G each period, and the revenue is collected from taxing consumption, capital and labor. Assume that the government keeps a balanced budget.

Market clearing

The asset and labor markets clearing requires that the total asset and labor supplied by the households equals the total capital and labor that the firm uses for production. The good market clearing condition equates the total output to the sum of aggregate household investment and consumption, plus government consumption.

Definition 2 *Competitive Equilibrium: Definition: Given a vector of tax (τ_c, τ_a, τ_w), a transition matrix π, initial distribution $\Gamma(a, \epsilon)$ over a Borel set consist of shocks and asset holding $S = A \times E$, where $A = [b, \infty)$ is the asset domain and E is the set of shock, competitive equilibrium is consist of a value function $V(a, \epsilon; \Gamma)$, a pair of policy function $g_c(a, \epsilon; \Gamma)$ and $g_a(a, \epsilon; \Gamma)$, an evolution in probability distribution $T(\Gamma)$, a vector of aggregate capital and labor (K, L), factor prices $\big(r(a, \epsilon), w(a, \epsilon)\big)$, such that,*

1. The value function and policy functions solve households utility maximization problem:

$$V(a, \epsilon; \Gamma) = \max_{c,a',h} u(c, h) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) V(a', \epsilon'; \Gamma')$$

$$\begin{aligned} s.t.(1 + \tau_c)c + a' &= (1 + r^\tau)a + (1 - \tau_w)wh\epsilon \\ r^\tau &= (1 + \tau_a)r \\ c &= g_c(a, \epsilon; \Gamma) \\ a' &= g_a(a, \epsilon; \Gamma) \\ \epsilon' &= \pi(\epsilon'|\epsilon)\epsilon \\ \Gamma' &= T(\Gamma) \\ a' &\geq 0 \end{aligned}$$

2. Factor prices satisfy the firm profit maximization conditions,

$$\begin{aligned} r(K, L) &= AF_K(K, L) - \delta \\ w(K, L) &= AF_L(K, L) \end{aligned}$$

3.The government budget constraint satisfies

$$G = \int_S \tau_c c + \tau_a a + \tau_w l\epsilon d\Gamma$$

4. Market clearing*:*

$$\int_S g_a(a, \epsilon; \cdot) d\Gamma = K'$$

$$\int_S l\epsilon d\Gamma = L$$

$$C + K' - (1 - \delta)K + G = AF(K, L)$$

5. Consistency*: Γ is consistent with the agents' optimal decisions, in the sense that it is generated by the optimal decision rules and by the law of motion of the shock.*

3.2. Numerical Results

This section presents the quantitative results of incomplete market models. First, we outline experiments and discuss the calibration for the benchmark economies. Then we study the effect of switching from labor income taxes to consumption taxes in different scenarios.

3.2.1. Outline of Experiments and Calibration

Regarding the types of labor taxes, and labor choices, we conducted four groups of experiments. We study the effects of flat consumption tax reforms by starting from the simplest case where the labor tax is flat, and labor choice is inelastic. Then we show that the results are robust if labor supply is allowed to change. Later on, we consider a more realistic case, in which labor tax is progressive. As before, we discuss the economies with inelastic labor and elastic labor respectively. At last, we proceed to a progressive consumption tax reform in an economy, which initially has a progressive labor tax, and elastic labor supply.

Period utility form is of King et al (1988) class (KPR henceforth).

$$u(c,l) = \frac{(c(1-l)^{\gamma})^{1-\sigma}}{1-\sigma}$$

We set relative risk averse parameter $\sigma = 2$ in all benchmarks. γ calibrated such that average hour worked is 0.3 in benchmark economies. [1] The production function is Cobb-Douglas, $F(K,L) = AK^{\alpha}L^{1-\alpha}$, with $\alpha = 0.36$ to match the capital's share in production. A is normalized so that output is equal to one in the deterministic steady state of benchmark economies. We calibrate β to match capital to output ratio of 3 at the stationary equilibrium in benchmark economies. The depreciation rate δ is set to be 0.06, such that investment to output ratio is around 2. Table 1 shows the parameters in four benchmark economies.

Table 1: Parameters

Economy	Properties		Parameters		
	Labor Tax Type	Labor Choice	A	β	γ
Benchmark1	Flat	Inelastic	0.612	0.907	1.77
Benchmark2	Flat	Elastic	0.631	0.900	1.77
Benchmark3	Progressive	Inelastic	0.591	0.917	1.60
Benchmark4	Progressive	Elastic	0.675	0.912	1.60

3.2.2. Steady States

a. From a flat labor tax to a flat consumption tax, inelastic labor

Anagnostopoulos and Li (2012) shows that with KPR utility flat consumption taxes do no distort saving decisions, thus the change in aggregate capital stems from the removal of labor taxes. Consider the partial equilibrium, in which interest rate and wage remain the

[1] Parameter γ is irrelevant if labor supply is inelastic. Therefore, we first calibrate γ in the benchmark economy with elastic labor and apply it to the inelastic labor economy.

same after the reform. Eliminating labor taxes increases labor income by 37% [2], implying that the stochastic part of income takes up a larger proportion in aggregate income. As a result, stronger precautionary motives stimulate more savings from all types of households, so aggregate capital raises. Back to general equilibrium, higher aggregate capital yields higher wage rate and lower interest rate. Shown in Table 2, on one hand, a 8.55% rise in wage rate further amplifies the uncertainty of income, thus more precautionary savings are spurred; On the other hand, a 27.17% drop in interest rate depresses households savings. But nevertheless, the impact on aggregate savings of a higher wage rate dominates that of a lower interest rate. Therefore, aggregate capital increases after the reform. Since capital is below the golden rule level, the increase in capital results in the raise in the sum of consumption and government revenue. Thus, given a fixed government spending, aggregate consumption follows aggregate capital, improving by 6.81%.

The changes in aggregate variables are broken down into details, displayed in Table 6. From an income point of view, a flat consumption tax reform favors households either with low assets or with high labor efficiency. This is because eliminating the labor tax can promote wage income but reduce capital returns. Households with low assets or high labor efficiency are usually with high labor-to-capital ratio. Thus the impact on income of a higher wage is stronger than a lower interest rate. With higher income, households have tendency to increase consumption.

Anagnostopoulos and Li (2012) also proves that with KPR utility, other things equal, the ratio of households consumption under two consumption taxes is inverse to the ratio of the two taxes. That is, if the elimination of the labor tax alone, with no consumption tax, promises a household with type (a, ϵ) a consumption $c_0(a, \epsilon)$, then after a consumption tax τ_c is adjusted to balance government budget, the household's consumption $c_1(a, \epsilon)$ satisfies $c_1(a, \epsilon) = \frac{1}{1+\tau_c} c_0(a, \epsilon)$. This property can provide some intuition to the impact on consumption of a flat consumption tax reform. The Table 6 shows that the reform raises shares in aggregate consumption for households in the first four asset quintile, while reduces the share for the top quintile. Consider households with no asset and the lowest labor efficiency. The removal of the labor tax increases wage, and thus total income by 8.55%. The numerical results shows that these households still have no saving after the reform. Therefore, all the increment of income is used to promote consumption, so consumption is increased by 8.55%. However, in order for the government to maintain a balanced budget, a 28.99% consumption tax is levied. Thus the new consumption under the new consumption tax is $1/(1 + 28.99\%)$ of the consumption after removing the labor tax but without a consumption tax. We proved with a complete market, after a flat consumption tax reform, wage increases more than that of consumption tax, and this result can be carried over to incomplete market models. Thus, consumption raises by 6.63% for households with no asset and the lowest labor shock. In general, households in lower asset quintile possess some asset both before and after the reform, but the amount is quite moderate. For example, the total asset held by the first four quintile is less than 10%. Thus, even households in low asset quintile are facing the decline in capital income, a more sizeable increase in wage than consumption tax is still possible to provide them with higher levels of consumption. The opposite occurs to households

[2] In partial equilibrium, the raise in labor income comes from the decrease in labor tax, thus it is $\frac{1}{1+0.269} =$ 37%

who belong to top asset quintile. If their labor efficiency, and thus labor income, are not high enough to cancel out the negative effect on consumption of a lower capital income and a higher consumption tax, then their consumption decreases. As shown in Table 3, with consumption being more equally distributed among households, Gini index of consumption drops.

b.From a flat labor tax to a flat consumption tax, elastic labor

When labor choice is brought into the picture, all the previous results go through: aggregate capital and consumption increases, Gini index of consumption decreases, except that we need to add one more dimension to our analysis.

Since Anagnostopoulos and Li (2012) shows that a flat consumption tax do not distort labor decision for KPR class of utility, the change in labor supply comes from the removal of labor tax. The elimination of labor tax rises wage rate, and with respect to different levels of assets and labor shocks, households' reactions to the rising wage are also different. For households with same labor efficiency, the income effect and the substitution effect of higher wage on labor-leisure decisions are the identical because with the same labor shock, the increase in labor income and opportunity cost of leisure are the same. Therefore, if interest rate is unchanged, then labor supply should increase (decrease) by the same amount across households with different assets. However, encountering a decrease in interest rate, households with higher assets are also suffering from a shrinkage in capital income, resulting in a less increase or even a decrease in total income, so the budget is tightened. It follows that leisure reduces, labor supply increases. But for households with low asset holdings, they do not experience a big drop in capital income, so total income is still probably higher than pre-reform. Thus a relaxed budget might lead to higher leisure, and low labor supply. Our numerical results confirm this analysis, that Gini index of leisure decreases from 0.187 to 0.159. In general, the increase in labor supply of high-asset households outweighs the decrease of their counterpart. As a result, average hour worked rises by 0.011. Since households with high assets also tend to have high labor efficiency, the effective labor is increased by 0.068, a larger magnitude than that of average hour worked.

Comparing to the previous case, where labor supply is fixed, a higher effective labor further amplifies the stochastic part of income, therefore, precautionary savings increases by more.

c.From a progressive labor tax to a flat consumption tax, inelastic labor

In this experiment, we start from a benchmark economy, which has a progressive labor tax. The functional form of labor tax is adopted from Gouveia and Strauss (1994), who estimated the functional form of US income tax code.

$$T = \kappa_0(y - (y^{-\kappa_1} + \kappa_2)^{-1/\kappa_1})$$

where y is labor income. Parameters κ_0 and κ_1 govern the average tax rate and the progressivity respectively, and κ_2 is used to balanced government budget. In Gouveia and Strauss (1994), the parameter were estimated using data for period 1979 to 1989. Because the inequality of wage is increasingly large since 1970, Gouveia and Strauss (1994) is not able to capture the entire feature of wage gap. Therefore, in this paper we adopt the values of parameters from Anagnostopoulos et al. (2010), who estimate $\kappa_0 = 0.414$, $\kappa_1 = 0.888$, and $\kappa_2 = 1.34$ following the same procedure as Guvenen et al. (2012), using PSID data covering the time period from 1983 to 2003.

The results of aggregate variables are displayed in Table 4. The impact on aggregates of removing a progressive labor tax is similar to that of removing a flat labor tax. Because higher wage creates greater uncertainty over income, higher precautionary capital is accumulated to insure against a more volatile income.

But on individual level, the results are reversed. Table 8 exhibits the distributions of variables over asset quintile. After the reform, the share in aggregate consumption decrease for the first four asset quintile, but increases for the households in top 20% of asset distribution, as well as the top groups. Consequently, Gini index of consumption increases. The explanation is straightforward: low-asset households are not benefit much from the elimination of labor tax because most of them also possess low labor efficiency, but paying a high consumption tax makes them worse off. In contrast, high-asset households are exempt from the pre-reform high labor tax, and are only subject to a relatively low consumption tax. Therefore, they are in favor of the flat consumption tax reform.

d. From a progressive labor tax to a flat consumption tax, elastic labor

When labor supply is allowed to vary, the results of moving from a progressive labor tax to a flat consumption tax is consistent with the results obtained by a fixed labor; and the analysis of the effects on aggregate labor is analogous to that in b, where the labor tax is flat. Regardless of the type of the labor tax, as long as it is removed, wage rate increases and interest rate decreases. As a result, labor supply decreases for households with low assets, but increases for their counterpart with high asset possession. Thus Gini index of labor decreases. But the decrease in inequality of labor supply is by less amount when initial labor tax is progressive as oppose to initially a flat labor tax. This is because under a progressive labor tax region, low-asset households do not pay a labor tax as high as under a flat labor tax region, the elimination of the progressive labor tax causes a less improvement in labor income, so the income effect plays a less dominant role as compared to the elimination of a flat labor tax. The same argument can be applied to households with high assets: the removal of a progressive labor tax indicates a more sizeable improvement in labor income. Thus, income effect has stronger impact on their labor decision. As a result, the extent to which the poor reduces labor supply, and the amount by which the rich increases labor supply are both lower when initial labor tax is progressive. But in general, the increase in labor supply the the rich outweighs the decrease by the poor, aggregate labor increases after the reform. In comparison to the case with fixed labor, higher aggregate labor results in a greater boost in aggregate capital accumulation.

e. From a progressive labor tax to a progressive consumption tax, elastic labor

It has been mentioned in some existing literature that moving from a progressive labor tax to a flat consumption tax can broaden welfare gap, and hurts the poor. For example Feenberg (1997) uses variety sources of data, conducts different types of experiments and concludes that under a flat consumption tax low income households bear much higher tax burden than high-income ones. Regarding this feature of the flat consumption tax reform, we conduct the experiment of a progressive consumption tax reform.

The progressive consumption tax was first discussed by Senators Domenici, Kerry and Nunn in USA tax (Unlimited Savings Allowance) in 1995. And it was officially proposed by Maya MacGuineas, president of the Committee for a Responsible Federal Budget, as replacement for the income tax, 2004. The principle of a progressive consumption tax adheres to that of a flat consumption tax; That is people should pay tax on what they take out of the economic pie, the more one takes out, the less he leaves for the others, therefore, a higher surcharge he is subject to. Over decades, progressive consumption tax reforms not only receive strong political supports, but also academic advocate. One of the most fervent proponent of progressive consumption taxes is Robert Frank. Frank (2005) illustrates that a progressive consumption tax can efficiently boost saving rate and reduce inequality. Likewise, Gentry (1997) shows that low-income households bears higher tax burden under a flat consumption tax, therefore, consumption tax should be progressive with at least the same progressivity as current income taxes.

Thus, based on this assertion and with respect to the fact that there is lack of literature discussing the optimal progressivity of consumption taxes, we apply the same functional form and progressivity as the labor tax to the consumption tax. Besides, we set κ_2 in consumption tax function the same as that in labor tax function, and adjust κ_0 to balance government budget.

The aggregate variables are shown in Table 5. As before, the elimination of a labor tax amplifies the stochastic part of income, such that aggregate capital increases. But a progressive consumption tax distorts savings, thus the increase in aggregate capital also comes from the disproportional consumption tax rate. The lagrangian multiplier of the budget constraints of a flat consumption tax is $\frac{u_c(c,l)}{1+\tau_c}$, and that of a progressive consumption tax is $\frac{u_c(c,l)}{1+T'_c(c)}$, where $T'_c(c)$ is the marginal consumption tax rate with respect to consumption c. Suppose that the progressive consumption tax reform yields the same equilibrium as the previous flat consumption tax reform and consider low-asset households with the consumption c such that $T'_c(c) < \tau_c$. That is the marginal cost of saving under a progressive consumption tax region is higher than that under a flat consumption tax region. Intuitively, when facing a progressive consumption tax, households with low assets tend to reduce savings, whereas high-asset households save more.

Even though the discrepancy of capital holding is widened, which is indicated by the increase in Gini index of wealth, the progressive consumption tax enables households at the low ends of both wealth distribution and labor efficiency distribution higher levels of consumption. The first order condition under the progressive consumption tax is

$$\frac{u_c(c,n)}{1+T'_c(c)} = \beta E(1+r^\tau)\frac{u_c(c',n')}{1+T'_c(c')} \tag{6}$$

If the equilibrium allocations are the same as that under a flat consumption tax, and suppose that households with low assets and low labor efficiency and with the consumption such that $T'_c(c) < \tau_c$. Then the marginal cost of saving, the left hand side of (6), is likely to exceed the marginal benefit,the left hand side of (6). This is because with a better shock tomorrow, consumption increases, implying $T'_c(c)$ is higher, which might exceeds τ_c. Therefore, in order for the Euler equation to hold, households should increase current consumption. In contrast, households with high assets and labor efficiency are more likely to have marginal cost of saving less than the marginal benefit, which suggests them to reduce consumptions. As a result, consumption is more equally distributed among households. Although Gini index of consumption is slightly higher after the progressive consumption tax reform, it is much lower than that of a flat consumption tax reform. If we take into account the increase in aggregate consumption, then households in low quintile have higher levels of consumption than pre-reform.

A higher marginal cost of saving for households with low assets and low labor efficiency also indicates a higher level of leisure. The intratemporal substitution between leisure and consumption is given by

$$\frac{u_1(c, 1-n)}{1+T_c'(c)} = -\frac{u_2(1-n)}{w\epsilon}$$

a higher marginal utility of consumption under progressive consumption tax region than a flat consumption tax region suggests a higher disutility of labor. Therefore, low income households increase leisure by reducing labor supply, and high income households perform the opposite, so Gini index of labor decreases. Aggregate labor is dominated by households with high labor efficiency, so total efficient labor increases by 6.25%.

3.2.3. Transition

In this section, we present the transition paths of switching from a progressive labor tax to a flat consumption tax and to a progressive consumption tax respectively. In both reforms, we eliminate labor income tax once and for all. Then we adjust τ_c in the flat consumption tax case to maintain government spending; similarly we rebalance government budget by changing κ_0 in the progressive consumption tax case, while keeping κ_1 and κ_2 fixed. In both case, average consumption tax rate has an immediate jump after the labor tax is eliminated. This is because both capital and labor remain at the low levels, in order to balance government budget, a higher-than-steady-state consumption tax is imposed. As more and more capital and consumption is accumulated, the consumption tax rate gradually decreases to the new equilibrium level. Because of the monotonic decrease in consumption tax rate, by Euler equation, interest rate keeps on dropping, which implies that aggregate capital experiences a smooth increase throughout the transition. Labor income increases right after the reform, substitution effect dominates income effect results in a sudden increase in labor supply. With the increase in wage income, substitution effect becomes less dominant, hence aggregate labor drops and eventually converges to the new equilibrium level. Aggregate consumption follows the same pattern as capital, except an abrupt drop at the beginning because of the unexpected raise in consumption taxes.

Besides the transition path of aggregate variables, we also present the welfare effect on individual levels. After the flat consumption tax reform, all the households with the lowest shock suffer from a welfare loss, because the increase in wage income cannot compensate the increase in consumption tax. Since interest rate is dragged down by a larger capital accumulation, the higher the asset, the greater the loss in capital income. And this transfers to a more sizeable welfare loss of households with high asset holdings. Because of the welfare loss by the biggest bulk of population, aggregate welfare decreases by 11.53%. As compared to the flat consumption tax reform, a progressive consumption tax reform causes a welfare gain for 42% of households with lowest labor efficiency. And there are more households with other two levels of labor efficiency experience larger welfare gain. Consequently, aggregate welfare increases by 3.32%.

3.2.4. Extension

From a capital tax to a consumption tax

This section considers an alternative reform, shifting capital income taxes to a consumption tax. The results are similar to that of replacing labor income taxes. A flat consumption tax reform benefits households with high assets, but hurts the poor. This is because the immediate effect of removing capital income taxes is the increase in capital returns. Therefore, the higher the assets, the greater the increase in income. On contrary, the poor cannot benefit from the elimination of capital tax because they have very little possession of capital. Thus, charging the poor with a high consumption tax makes them worse off. However, if a progressive consumption tax is used in place of capital taxes, then the consequence is reversed. Households with low assets end up with higher consumption, and thus more likely higher welfare; while the rich are worse off. This is because waiving capital taxes encourage savings, thus wage rate is also promoted. And the increase in wage favors households with low assets, because their main resource of income is labor. Since these households also have low levels of consumption, paying a consumption tax at a rate which is not compatible to their increased income, their consumption increases as a result. Therefore, a progressive consumption tax reform can effectively increase welfare, and reduce welfare inequality.

4. Conclusion

In this paper we study the effects of consumption tax reforms. In general, regardless of the types of labor tax and consumption tax, replacing a labor income tax by a consumption tax increases aggregate variables. There are two interpretations for this, first is that because higher wage increases the uncertainly of income, precautionary savings are stimulated; The other explanation is that financing government spending through a consumption tax exempt saving from taxation, therefore more capital is accumulated. This conclusion is in line with the argument by the advocates of consumption taxes. But with respect to welfare inequality, the results vary across experiments according to different assumptions about labor tax. If labor tax is flat before the reform, then a flat consumption tax can reduce the Gini index of consumption and leisure, and thus the inequality of welfare. Because the asset poor households have a wage income increases by a larger amount than consumption tax, and they are not subject to a decrease in capital income; whereas asset rich households, the benefit of higher wage income is cancel out by the shrinkage in the return of capital. However, if the labor tax is progressive, then imposing a flat consumption tax instead of of a progressive labor tax hurts the poor, because the poor are not benefit from much from the removal of labor tax large enough to guarantee a higher consumption and leisure. Thus welfare inequality increases. Because of this negative impact of a flat consumption tax reform on welfare, we consider a progressive consumption tax reform. Because of the fact that there is lack of studies of the optimal progressivity of consumption taxes, we apply the same progressivity as labor taxes to consumption taxes. The numerical results exhibits a sizeable welfare improvement of the progressive consumption tax reform. Our future work involves discussion of the optimal consumption tax rate and its progressivity.

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Table 2: Steady State of a

	Parameters								
Economy	τ_c	τ_w	r	w	K	H	L	K/Y	C
Benchmark	0	0.269	6.00	0.551	4.32	0.30	1.67	3.00	0.830
Reform	0.290	0	4.37	0.599	5.43	0.30	1.67	3.47	0.886

Table 3: Steady State of b

	Parameters								
Economy	τ_c	τ_w	r	w	K	H	L	K/Y	C
Benchmark	0	0.269	5.99	0.578	5.09	0.30	1.88	3.00	0.975
Reform	0.272	0.00	4.25	0.631	6.74	0.310	1.94	3.51	1.10

Table 4: Steady State of c

	Parameters								
Economy	τ_c	$(\kappa_l 0, \kappa_l 1, \kappa_l 2)$	r	w	K	H	L	K/Y	C
Benchmark	0.00	(0.414, 0.888, 1.34)	6.00	0.522	4.09	0.300	1.67	3.00	0.763
Reform	0.322	(0.00, -, -)	3.25	0.604	6.14	0.300	1.67	3.89	0.854

Table 5: Steady State of d

	Parameters								
Economy	$(\kappa_l 0, \kappa_l 1, \kappa_l 2)$	$(\kappa_l 0, \kappa_l 1, \kappa_l 2)$	r	w	K	H	L	K/Y	C
Benchmark	(0.00, -, -)	(0.414,0.888,1.34)	5.98	0.643	5.81	0.300	1.92	3.00	1.04
FCT	(0.319, -, -)	(0.00, -, -)	3.07	0.753	9.80	0.330	2.10	3.97	1.33
PCT	(0.449,0.888, 1.34)	(0.00, -, -)	3.28	0.743	9.16	0.316	2.04	3.88	1.27

Table 6: Distribution of a

Distribution of Wealth

	Gini	Quintile					Top Groups		
Eco		1st	2nd	3rd	4th	5th	Top 5%	Top 2%	Top 1%
Ben	0.834	2.80E-03	2.81E-03	2.17	5.86	91.97	47.7	24.0	13.4
FCT	0.855	2.88E-03	2.88E-03	1.76	4.09	94.14	51.25	25.82	14.36

Distribution of Consumption

	Gini	Quintile					Top Groups		
Eco		1st	2nd	3rd	4th	5th	Top 5%	Top 2%	Top 1%
Ben	0.789	3.00	3.00	13.00	16.64	64.35	29.17	14.19	7.81
FCT	0.810	3.24	3.24	13.08	17.14	63.29	29.21	14.16	7.75

Table 7: Distribution of b

Distribution of Wealth									
	Gini	Quintile					Top Groups		
Eco		1st	2nd	3rd	4th	5th	Top 5%	Top 2%	Top 1%
Ben	0.834	0	0	2.47	5.71	91.81	43.32	19.96	10.48
FCT	0.855	0	0	1.95	3.95	94.09	47.30	21.89	11.49
Distribution of Labor									
	Gini	Quintile					Top Groups		
Eco		1st	2nd	3rd	4th	5th	Top 5%	Top 2%	Top 1%
Ben	0.187	24.2	24.2	22.7	20.1	8.78	3.41	1.64	0.875
FCT	0.159	23.29	23.29	22.42	21.26	9.74	3.64	1.72	0.916
Distribution of Consumption									
	Gini	Quintile					Top Groups		
Eco		1st	2nd	3rd	4th	5th	Top 5%	Top 2%	Top 1%
Ben	0.780	3.23	3.23	13.95	16.69	62.89	29.23	13.84	7.36
FCT	0.804	3.37	3.37	13.73	17.54	61.99	30.02	14.16	7.51

Table 8: Distribution of c

Distribution of Wealth									
	Gini	Quintile					Top Groups		
Eco		1st	2nd	3rd	4th	5th	Top 5%	Top 2%	Top 1%
Ben	0.826	0	0	2.28	7.39	90.32	45.35	22.86	12.85
FCT	0.849	0	0	1.95	4.71	93.34	50.07	25.29	14.11
Distribution of Consumption									
	Gini	Quintile					Top Groups		
Eco		1st	2nd	3rd	4th	5th	Top 5%	Top 2%	Top 1%
Ben	0.560	3.80	3.82	14.47	18.22	59.69	25.78	12.48	6.88
FCT	0.589	3.30	3.30	13.53	17.51	62.35	28.18	13.64	7.47

Table 9: Distribution of d

Distribution of Wealth									
	Gini	Quintile					Top Groups		
Eco		1st	2nd	3rd	4th	5th	Top 5%	Top 2%	Top 1%
Ben	0.825	0	0	2.01	6.63	91.35	42.22	19.66	10.44
FCT	0.843	0	0	2.07	4.43	93.50	46.67	21.82	11.54
PCT	0.845	0	0	1.86	3.99	93.98	47.17	22.05	11.66
Distribution of Labor									
	Gini	Quintile					Top Groups		
Eco		1st	2nd	3rd	4th	5th	Top 5%	Top 2%	Top 1%
Ben	0.185	24.15	24.15	22.88	19.08	9.75	3.29	1.56	0.839
FCT	0.164	23.36	23.36	22.38	20.94	9.96	3.58	1.73	0.926
PCT	0.165	23.30	23.30	22.47	20.96	9.97	3.69	1.78	0.952
Distribution of Consumption									
	Gini	Quintile					Top Groups		
Eco		1st	2nd	3rd	4th	5th	Top 5%	Top 2%	Top 1%
Ben	0.569	4.03	4.03	14.87	17.14	59.93	26.31	12.43	6.66
FCT	0.600	3.40	3.40	14.00	17.59	61.61	29.01	13.85	7.40
PCT	0.579	3.90	3.90	14.34	17.85	60.00	28.02	13.32	7.10

Appendix

Proof of equation (2).

Proof. The government budget before the reform is

$$G = \frac{\tau_c + \tau_a}{1+\tau_c} rK + \frac{\tau_c + \tau_w}{1+\tau_c} wL$$

And that after the reform is

$$G^R = \frac{\tau_c^R + \tau_a}{1+\tau_c^R} r^R K^R + \frac{\tau_c^R + \tau_w^R}{1+\tau_c^R} w^R L^R$$

The assumption of a balanced government budget requires that $G^R = G$. Therefore,

$$\frac{\tau_c + \tau_a}{1+\tau_c} rK + \frac{\tau_c + \tau_w}{1+\tau_c} wL = \frac{\tau_c^R + \tau_a}{1+\tau_c^R} r^R K^R + \frac{\tau_c^R + \tau_w^R}{1+\tau_c^R} w^R L^R \tag{7}$$

Since we assume complete market and inelastic labor supply, $K^R = K$, $r^R = r$ and $w^R = w$ and $L^R = L$. Equation (7) becomes

$$\begin{aligned}
\left(\frac{\tau_c + \tau_a}{1+\tau_c} - \frac{\tau_c^R + \tau_a}{1+\tau_c^R}\right) rK &= \left(\frac{\tau_c^R + \tau_w^R}{1+\tau_c^R} - \frac{\tau_c + \tau_w}{1+\tau_c}\right) wL \\
\tau_a(\tau_c - \tau_c^R) rK &= [(\tau_c^R - \tau_c) + (\tau_w^R - \tau_w) + (\tau_c \tau_w^R - \tau_c^R \tau_w)] wL \\
&= \tau_c^R(1-\tau_w) - \tau_c(1-\tau_w^R) + (1-\tau_w) - (1-\tau_w^R) \\
\tau_a[(1+\tau_c) - (1-\tau_c^R)] rK &= [(1+\tau_c^R)(1-\tau_w) - (1+\tau_c)(1-\tau_w^R)] wL
\end{aligned}$$

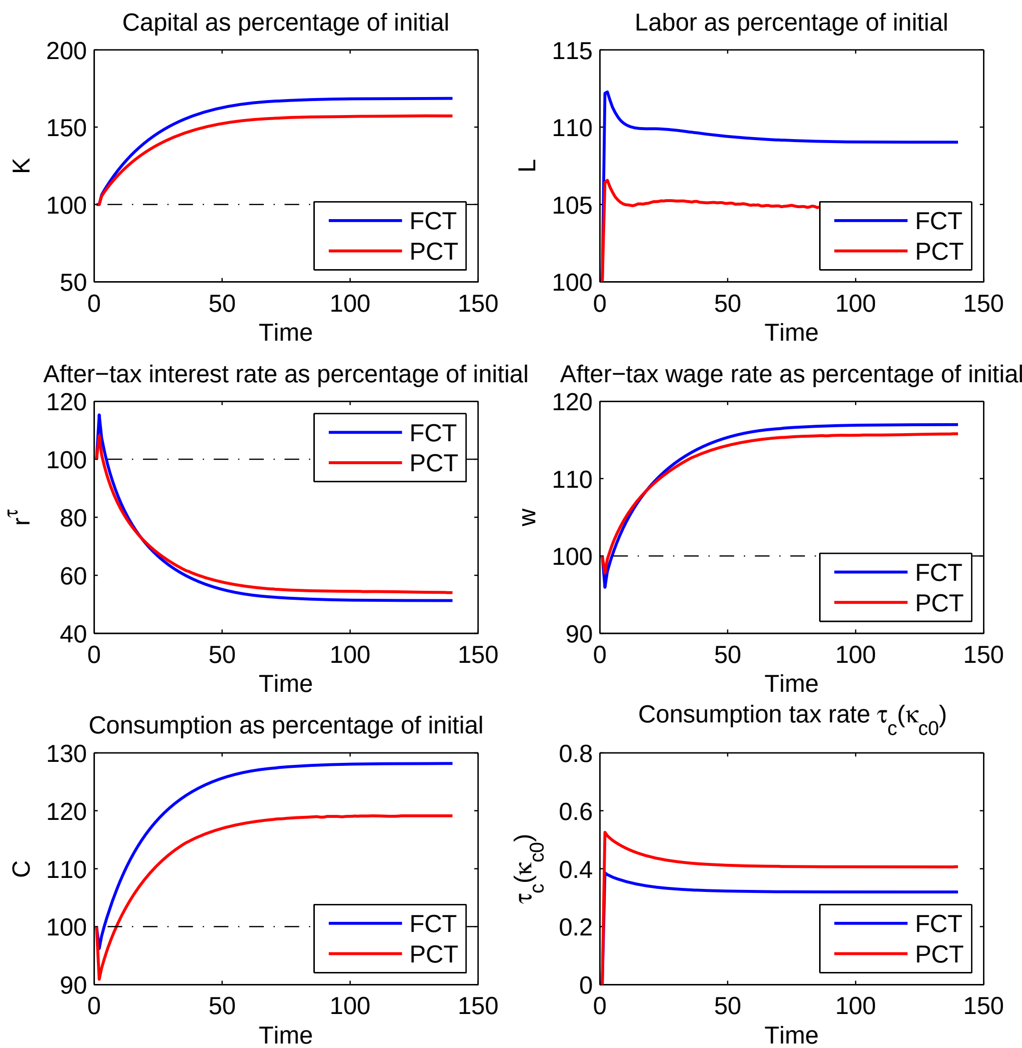
Capital as percentage of initial
K
Time
FCT
PCT
Labor as percentage of initial
L
Time
FCT
PCT
After−tax interest rate as percentage of initial
r^{τ}
Time
FCT
PCT
After−tax wage rate as percentage of initial
w
Time
FCT
PCT
Consumption as percentage of initial
C
Time
FCT
PCT
Consumption tax rate $\tau_c(\kappa_{c0})$
$\tau_c(\kappa_{c0})$
Time
FCT
PCT

Figure 1:

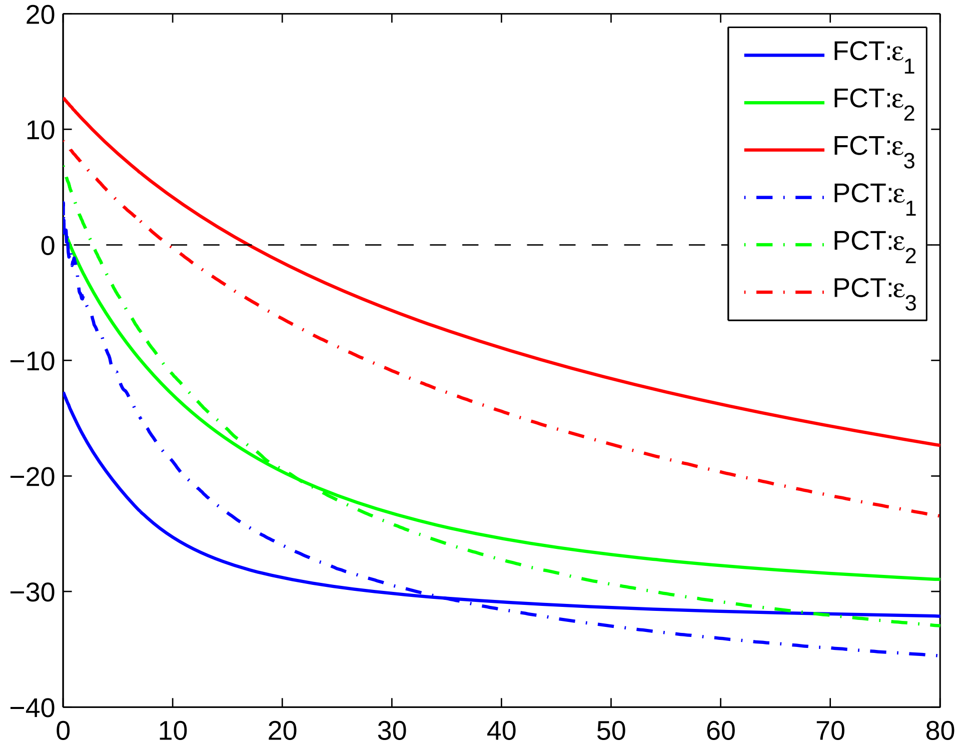
20
10
0
−10
−20
−30
−40
0
10
20
30
40
50
60
70
80
FCT:ε_1
FCT:ε_2
FCT:ε_3
PCT:ε_1
PCT:ε_2
PCT:ε_3

Figure 2:

Define $\lambda_c = \frac{1+\tau_c^R}{1+\tau_c}$ and divide both side by $1 + \tau_c$, then

$$\tau_a(1-\lambda_c)rK = [\lambda_c(1-\tau_w) - (1-\tau_w^R)]wL$$

Define $\lambda_w = \frac{(1-\tau_w^R)w^R}{(1-\tau_w)w}$, then

$$\tau_a(1-\lambda_c)rK = (\lambda_c - \lambda_w)(1-\tau_w)wL$$
$$(1-\lambda_c)\frac{\tau_a rK}{(1-\tau_w)wL} = \lambda_c - \lambda_w$$

Therefore,

$$\lambda_w = \lambda_c + (\lambda_c - 1)\frac{\tau_a rK}{(1-\tau_w)wL}$$

■