

PAPER OR PLASTIC? MONEY AND CREDIT AS MEANS OF PAYMENT*

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Abstract

Technological advances in electronic record-keeping have made credit cards as ubiquitous as cash as means of payment in many economies. This transformation in the payments landscape poses several new challenges for policymakers, particularly in determining how inflation, market structure, and new regulation shapes how consumers pay and what merchants accept. To help guide the discussion, we develop a simple model where both money and credit can be used as means of payment. Our framework captures the two-sided market interaction between consumers and lenders, leading to complementarities and network externalities that frequently characterize retail payment systems. In addition, limited commitment makes debt contracts self-enforcing and yields an endogenous upper bound on credit use. Our model can explain why an economy can fall into a credit crunch as inflation falls, and how hold-up problems in technological adoption can prevent retailers from accepting credit as consumers continue to coordinate on cash usage. We also discuss policy implications and the extent to which our model can reconcile some key patterns in the use of cash and credit in retail transactions.

Keywords: coexistence of money and credit, inflation, costly record-keeping, credit constraints

JEL Classification Codes: D82, D83, E40, E50

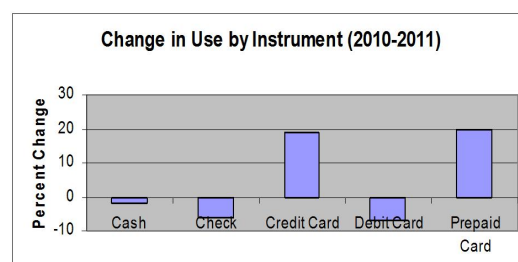
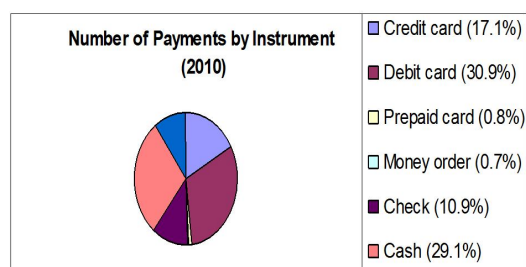
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1 Introduction

Technological improvements in electronic record-keeping have made credit cards as ubiquitous as cash as means of payment in many OECD countries. In 2011 alone, \$6.7 trillion worth of transactions were channelled through credit cards in the United States, according to the Nilson Report. In addition, consumers now have more payment instruments than ever to choose from, ranging from cash, credit cards, prepaid cards, smart money, mobile account payments, and electronic payments (“e-money”).¹ According to Gerdes, Walton, Liu, and Parke (2005), innovations in the retail landscape have generated a payments transformation, as card payments now dominate more traditional paper-based ones. For example, 48.8% of transactions recorded in the Federal Reserve Bank of Boston’s 2010 Survey of Consumer Payment Choice were conducted with payment cards, while 40.8% of transactions used paper instruments, such as cash or check. This suggests that while consumers are indeed adopting new payment instruments, they are not completely abandoning older ones.



Source: Schuh (2012), Boston FRB 2010 and 2011 Consumer Payment Choice

As consumers change the way they pay and businesses change the way they accept payments, it is increasingly important to understand how consumer demand affects merchant behavior and vice versa. In fact, the payment system is a classic example of a two-sided market where both consumers and firms must make choices that affect one other. This dynamic often generates complementarities and network externalities, which is a key characteristic of the retail payment market (BIS (2012)).² Moreover, the recent trends in retail payments raise many interesting and challenging questions

¹Evidence from Foster, Meijer, Schuh, and Zabek (2009) reveals that in 2009, the average U.S. consumer holds 5.0 of the nine most common payment instruments and used 3.8 of them during a typical month.

²Network externalities exist when the value of a good or service to a potential user increases with the number of other users using the same product. Credit cards are a good example of network good, where its adoption and use can be below the socially optimal level because consumers or firms do not internalize the benefit of their own use on others’ use. For evidence and a discussion of the empirical issues, see Gowrisankaran and Stavins (2004).

for central banks and policymakers. In particular, how does the availability of alternative means of payment, such as credit cards, affect the role of money? And if both money and credit can be used, how does policy and inflation affect the money-credit margin?

We investigate the possible substitution away from cash to electronic payments such as credit cards using a simple model where money and credit can coexist as means of payment. Our objective is to determine the impact that retail payment innovations can have on future cash usage. Understanding how agents substitute between cash and credit is a key policy concern for central banks when setting an inflation target, as well as legislators when developing new regulation on credit card fees. As it is the sole issuer of bank notes, central banks also need to understand substitution patterns to predict consumers' demand for cash.

To capture the two-sided nature of actual payment systems, our model focuses on the market interaction between consumers (buyers, or borrowers) and retailers (sellers, or lenders). A vital distinction between monetary and credit trades is that the former is *quid pro quo* and settled on the spot while the latter involves delayed settlement.³ For credit to have a role, we introduce a costly record-keeping technology that allows transactions to be recorded.⁴ A retailer that invests in this technology will thus be able to accept an IOU from a consumer. In this way, credit allow retailers to sell to illiquid consumers or to those paying with future income. Due to limited commitment and enforcement however, lenders cannot force borrowers to repay their debts. In order to motivate voluntary debt repayment, we assume that default by the borrower triggers a global punishment that banishes agents from all future credit transactions. In that case, a defaulter can only trade with money. Consequently, debt contracts must be self-enforcing and the possibility of strategic default generates an endogenous upper-bound on credit use.⁵

Our paper identifies a key channel through which monetary policy can affect payment arrangements and welfare. In particular, inflation has two effects: a higher inflation rate both lowers the rate of return on money and makes default more costly. This relaxes the credit constraint and induces agents to shift from money to credit to finance their consumption. Consequently, the economy can fall in a credit crunch as inflation falls. When monetary policy follows the Friedman rule, deflation completely crowds out credit and there is a flight to liquidity where all borrowing

³This distinction separates debit cards, which are “pay now” cards, from credit cards, or “pay later” cards. For debit cards, funds are typically debited from the cardholder’s account within a day or two of purchase, while credit cards allow consumers to access credit lines at their bank which are repaid at a future date. In this paper, we interpret credit trades as occurring with credit cards and monetary trades as those occurring with cash or debit.

⁴As is well established by now, the same frictions that render money essential make credit arrangements impossible. These frictions include imperfect record-keeping over individual trading histories, lack of commitment, and lack of enforcement (Kocherlakota and Wallace (1998)).

⁵These two key features of the model— costly record-keeping and limited commitment— are also key features of credit payments in practice. For example, Fung, Huynh, and Sabetti (2012) find that the primary determinants of credit card usage include availability of record-keeping and the ability to make payments.

and lending ceases to exist. In that case, efficient monetary policy drives out credit. Our model therefore gives rationale for why central banks often prefer low inflation targets over deflation: the latter subsidizes monetary transactions at the expense of credit, which in turn weakens the credit sector. When choosing the optimal inflation rate, the central bank must therefore trade off the cost of distorting monetary transactions with the benefit of a stronger credit system. This implies that in economies where both money and credit are used, the optimal inflation rate can be strictly positive.⁶

The channel through which monetary policy affects macroeconomic outcomes is through buyers' choice of portfolio holdings, sellers' decision to invest in the record-keeping technology, and the endogenously determined credit constraint. If sellers must invest ex-ante in a costly technology to record credit transactions, there are strategic complementarities between the seller's decision to invest and the buyer's ability to repay. When more sellers accept credit, the gain for buyers from using and redeeming credit increases, which relaxes the credit constraint. At the same time, an increase in the buyer's ability to repay raises the incentive to invest in the record-keeping technology and hence the fraction of credit trades. This complementarity leads to feedback effects that can generate multiple equilibria, including outcomes where both money and credit are used.

Moreover, this channel mimics the mechanism behind two-sided markets in actual payment systems as described by McAndrews and Zhu (2008): merchants are more willing to accept credit cards that have many cardholders, and cardholders want cards that are accepted at many establishments. Just as in our model, the payment network benefits the merchant and the consumer jointly, leading to the same kind of complementarities and network externalities highlighted in the industrial organization literature.⁷ At the same time, consumers may still coordinate on using cash due to a hold-up problem in technological adoption. Since retailers do not receive the full surplus associated with technological adoption, they fail to internalize the total benefit of accepting credit. The choice of payment instruments will therefore depend on fundamentals, as well as history and social conventions.

This potential for coordination failures also raises new concerns for policymakers. In contrast with conventional wisdom, our theory suggests that economies with similar technologies, institutions, and policies can still end up with very different payment systems, some being better in terms of social welfare than others. If for example society prefers a payment system with only credit, the government may want to introduce special policies such as information campaigns, advertisements, or even financial literacy programs that help coordinate agents on using credit. These measures

⁶Limited commitment and imperfect enforcement also leads to a positive optimal inflation rate in Antinolfi, Azariadis, and Bullard (2009) in the context of a Bewley (1980) monetary model.

⁷Chakravorti (2003) provides a theoretical survey of the industrial organization approach to credit card networks.

that enhance communication are especially relevant for many emerging economies where the lack of financial infrastructure and intermediation makes new forms of mobile credit payments particularly appealing.⁸ Hence our model not only provides policymakers a useful framework for understanding how consumers substitute between money and credit, but also makes clear the channels through which their policies affect prices, trade, and social welfare.

The remainder of the paper proceeds as follows. Section 1.1 reviews the related literature. Section 2 describes the basic environment with limited enforcement. Section 3 then determines equilibrium where an exogenous fraction of sellers accept credit. Section 4 determines the endogenous debt limit and characterizes properties of monetary and non-monetary equilibrium. Section 5 endogenizes the fraction of credit trades and discusses multiplicity. Section 6 turns to normative considerations and discusses welfare, and Section 7 relates our model with the empirical evidence on consumer payments. Finally, Section 8 concludes.

1.1 Related Literature

Within modern monetary theory, there is a strong tradition of studying the coexistence of money and credit. Shi (1996) provides the first model with bargaining, money, and credit and shows that money can coexist with credit that yields a higher rate of return. In Kocherlakota and Wallace (1998), an equilibrium with money and credit can be sustained if individual histories are made public with a lag. In another approach, Calvacanti and Wallace (1999), Williamson (1999), Williamson (2004), and Sanches and Williamson (2010) assume limited participation to allow for monetary and credit transactions.

Instead in our framework, the assumptions of anonymity, imperfect record-keeping, and limited enforcement allow credit and money to coexist. Camera and Li (2008) also study the coexistence of money and credit with limited enforcement, but in a model where fiat money is indivisible. Consequently, monetary policy has no effect on credit use or debt limits. Moreover in all these approaches, only an exogenous subset of agents can use credit while the choice of using credit is endogenous in this paper.⁹ Dong (2011) also introduces costly record-keeping, but focuses on the buyer's choice of payments used in bilateral meetings.¹⁰

⁸For example, the November 17, 2012 *Economist* article “War of the Virtual Wallets” predicts that “the biggest prize of all lies in emerging markets where a lack of financial infrastructure is hastening the rise of phone-based payments systems.”

⁹Schreft (1992) and Dotsey and Ireland (1996) introduce costs paid to financial intermediaries to endogenize the composition of trades that use money or credit. Similarly, Prescott (1987) and Freeman and Kydland (2000) feature a fixed record-keeping cost for transactions made with demand deposits. However these papers assume that money is used in the economy and does not provide microfoundations for the essentiality of money.

¹⁰Arango and Taylor (2008) and Turban (2008) find that record-keeping or other technological costs associated with accepting credit are incurred by the seller.

Closely related to our paper is Gomis-Porqueras and Sanches (2011), who also discuss the role of money and credit in a model with anonymity, limited commitment, and imperfect record-keeping. A key difference in the set-up is that they adopt a different pricing mechanism by assuming a buyer-take-all bargaining solution. By contrast, our assumption of proportional bargaining allows us to take the analysis further in two important ways. By giving the seller some bargaining power, proportional bargaining allows us to endogenize the fraction of sellers that can accept credit by allowing them to invest in a costly record-keeping technology. This also allows us to discuss hold-up problems on the seller's side which will lead to complementarities with the buyer's borrowing limit. This generates interesting multiplicities that the previous study cannot discuss.

This paper is also related to a growing strand in the industrial organization literature that examines the costs and benefits of credit cards to network participants. In particular, recent work by Wright (2003) and Rochet and Tirole (2011) models the bilateral transactions between consumers and retailers to study the effects of regulatory policies and market structure in the credit card industry. However, this literature abstracts from a critical distinction between monetary and credit transactions by ignoring the actual borrowing component of credit transactions.

By contrast, our paper is explicit about the intertemporal nature of credit transactions by allowing consumers to issue an IOU to the seller, or paying on the spot with cash. In turn, our framework can be used to determine the conditions under which consumers prefer one type of payment instrument over the other, and how this can be affected by policy. As new forms of payment develop and become increasingly prevalent, these issues are central concerns that both central banks and policymakers need to understand.

2 Environment

The basic environment is similar to Rocheteau and Wright (2005) and Nosal and Rocheteau (2011). Time is discrete and continues forever. Each period is divided into two sub-periods where economic activity will differ. In the first sub-period, agents meet pairwise and random in a decentralized market, called the *DM*. *Sellers* can produce output, $q \in \mathbb{R}_+$, but do not want to consume, while *buyers* want to consume but cannot produce. Agents' identities as buyers or sellers are permanent, exogenous, and determined at the beginning of the DM. In the second sub-period, trade occurs in a frictionless centralized market, called the *CM*, where all agents can consume a numéraire good, $x \in \mathbb{R}_+$, by supplying labor, y , one-for-one using a linear technology.

Instantaneous utility functions of buyers, (U^b) , and sellers, (U^s) , are assumed to be separable

between sub-periods and linear in the CM:

$$\begin{aligned} U^b(q, x, y) &= u(q) + x - y, \\ U^s(q, x, y) &= -c(q) + x - y. \end{aligned}$$

Functional forms for utility and cost functions in the DM, $u(q)$ and $c(q)$ respectively, are assumed to be C^2 with $u' > 0$, $u'' < 0$, $c' > 0$, $c'' > 0$, $u(0) = c(0) = c'(0) = 0$, and $u'(0) = \infty$. Also, let $q^* \equiv \{q : u'(q^*) = c'(q^*)\}$. All agents discount the future between periods, but not sub-periods, with a discount factor $\beta \in (0, 1)$.

The only asset in this economy is fiat money, which is perfectly divisible and storable. Money $m \in \mathbb{R}_+$ is valued at ϕ , the price of money in terms of the general good. Its aggregate stock in the economy, M , can grow or shrink each period at a constant gross rate $\gamma \equiv \frac{M_{t+1}}{M_t}$. Changes in the money supply are facilitated through lump-sum transfer or taxes in the CM to buyers. In the latter case, we assume that the government has sufficient enforcement so that agents will repay the lump-sum tax.¹¹

To purchase goods in the DM, both monetary and credit transactions are feasible due to the availability of a record-keeping technology that can record agent's transactions. However this technology is only available to a fraction $\Lambda \in [0, 1]$ of sellers. As a result, these sellers can accept both credit and money as payment for DM transactions. The remaining $1 - \Lambda$ sellers can only accept money.¹² For now we take the fraction of sellers with access to the record-keeping technology, and hence the fraction of credit transactions, as given. In Section 5, Λ is made endogenous by allowing sellers to invest ex-ante.

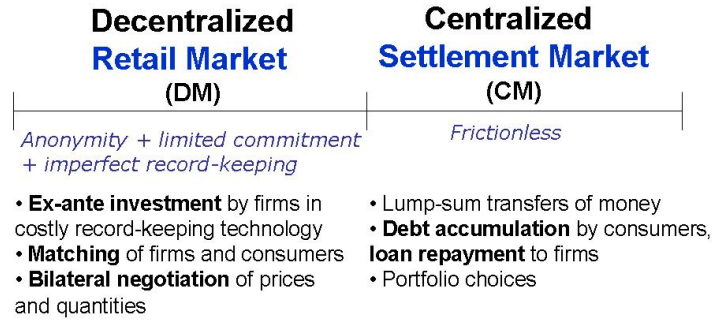
We assume that contracts written in the DM can be repaid in the subsequent CM. Buyers can issue $b \in \mathbb{R}_+$ units of one-period IOUs that we normalize to be worth one unit of the numéraire good. While the record-keeping technology can identify agents and record their transactions, enforcement may be imperfect. This leads to the possibility of strategic default by the borrower. In order to support trade in a credit economy, potential borrowers must be punished if they do not deliver on their promise to repay. We assume that punishment for default entails permanent exclusion from the credit system.¹³ In that case, a borrower who defaults can only use money for all future

¹¹While the government can never observe agents' real balances, it has the authority to impose arbitrarily harsh penalties on agents who do not pay taxes when $\gamma < 1$. Alternatively, Andolfatto (2007) considers an environment where the government's enforcement power is limited. When the payment of lump-sum taxes is voluntary and governments cannot confiscate output or force agents to work, the Friedman rule fails to be incentive feasible even though it is desirable. As a result, there is an incentive induced lower bound on the deflation rate.

¹²Alternatively, investment in this technology is infinitely costly for a fraction $1 - \Lambda$ of firms while costless for the remaining Λ firms.

¹³This is in the spirit of Kehoe and Levine (1993) where the threat of banishment from future credit transactions will motivate voluntary debt repayment. Off-equilibrium path punishments are also considered by Aiyagari and

Figure 1: **Timing of Representative Period**



transactions.

The timing of events in a typical period is summarized in Figure 1. At the beginning of the DM, a measure σ of buyers and sellers are randomly matched, where the buyer has $m \in \mathbb{R}_+$ units of money, or equivalently, $z = \phi m$ real balances. Then in each trade, buyers and sellers bargain over the terms of trade. Then in the CM, buyers produce the numéraire good, redeem their loan, and acquire money, while sellers can purchase the general good with money and can get their loan repaid. We focus on stationary equilibria where real balances are constant over time.

3 Equilibrium

The model can be solved in four steps. First, we characterize properties of agents' value functions in the CM. Using these properties, we then determine the terms of trade in the DM. Third, we determine the buyer's choice of asset holdings and the endogenous debt limit. Finally in Section 5, we determine Λ endogenously by allowing sellers to invest in the costly record-keeping technology.

Williamson (2000), Antinolfi, Azariadis, and Bullard (2007), Berentsen, Camera, and Waller (2007), Camera and Li (2008), Sanches and Williamson (2010), Gomis-Porqueras and Sanches (2011), and Li and Li (2012).

3.1 Centralized Market

In the beginning of the CM, all agents consume the general good x , supply labor y , and readjust their portfolios. Let $W^b(z, -b)$ denote the value function of a buyer who holds $z = \phi m$ units of real balances and has issued b units of IOUs in the previous DM. The buyer's maximization problem at the beginning of the CM, $W^b(z, -b)$, is

$$W^b(z, -b) = \max_{x, y, z' \geq 0} \left\{ x - y + \beta V^b(z') \right\} \quad (1)$$

$$\text{s.t. } x + b + \phi m' = y + z + T \quad (2)$$

$$z' = \phi' m' \quad (3)$$

where V^b is the buyer's continuation value in the next DM. According to (2), the buyer finances his net consumption of the general good ($x - y$), the repayment of his IOUs (b), and his following period real balances ($\gamma z'$) with his current real balances (z) and the lump-sum transfer from the government (T). Substituting $m' = z'/\phi'$ from (3) into (2), and then substituting $x - y$ from (2) into (1) yields

$$W^b(z, -b) = z - b + T + \max_{z' \geq 0} \left\{ -\gamma z' + \beta V^b(z') \right\}. \quad (4)$$

The buyer's lifetime utility in the CM is the sum of his real balances net of any IOUs to be repaid, the lump-sum transfer from the government, and his continuation value at the beginning of the next DM net of the investment in real balances. The gross rate of return of money is $\frac{\phi_{t+1}}{\phi_t} = \frac{M_t}{M_{t+1}} = \gamma^{-1}$. Hence in order to hold z' units of real balances in the following period, the buyer must acquire $\gamma z'$ units of real balances in the current period.

Notice that $W^b(z, -b)$ is linear in the buyer's current portfolio: $W^b(z, -b) = z - b + W^b(0, 0)$. In addition, the choice of real balances next period is independent of current real balances. Identically, the value function $W^s(z, b)$ of a seller who holds z units of real balances and b units of IOUs can be written:

$$W^s(z, b) = z + b + \beta V^s(0, 0)$$

where $V^s(0, 0)$ is the value function of a seller at the beginning of the following DM since they have no incentive to accumulate real balances in the DM.

3.2 Terms of Trade

We now turn to the terms of trade in the DM. Agents meet bilaterally, and bargain over the units of money or IOUs to be exchanged for goods. We adopt the proportional bargaining solution

where the buyer receives a constant share $\theta \in (0, 1)$ of the match surplus, while the seller gets the remaining share, $(1 - \theta) > 0$.

The terms of trade depend on buyers' portfolios and what sellers accept. We first suppose that the seller accepts credit. In that case, the buyer holding z units of real balances proposes an offer (q, b, d) that maximizes his expected surplus such that the seller gets a constant share $1 - \theta$ of the total surplus. To apply the pricing mechanism, notice that the surplus of a buyer who gets q for payment $d + b$ to the seller is $u(q) + W^b(z - d - b) - W^b(z) = u(q) - b - d$, by the linearity of W^b . Similarly, the surplus of a seller is $-c(q) + d + b$. The bargaining problem then becomes

$$(q, d, b) = \arg \max_{q, d, b} \{u(q) - d - b\} \quad (5)$$

$$\text{s.t. } -c(q) + d + b = \frac{1 - \theta}{\theta} [u(q) - d - b] \quad (6)$$

$$d \leq z \quad (7)$$

$$b \leq \bar{b}. \quad (8)$$

According to (5) – (8), the buyer's offer maximizes his trade surplus such that (i) the seller's payoff cannot be less than a constant share $\frac{1-\theta}{\theta}$ of the buyer's payoff, (ii) the buyer cannot transfer more money than he has, and (iii) the buyer cannot borrow more than he can repay. Condition (7) is a feasibility constraint on the amount the buyer can transfer to the seller, while condition (8) is the buyer's incentive constraint that motivates voluntary debt repayment. The threshold \bar{b} is an equilibrium object and represents the endogenous borrowing limit faced by the buyer, which is taken as given in the bargaining problem but is determined endogenously in the next section.

Combining the feasibility constraint (7) and the buyer's incentive constraint (8) then results in the payment constraint

$$d + b \leq z + \bar{b} \quad (9)$$

which says the total payment to the seller, $d + b$, cannot exceed what the buyer holds, which is $z + \bar{b}$ when the seller accepts credit. The solution to the bargaining problem will depend on whether the payment constraint, (9), binds. If (9) does not bind, then the buyer will have sufficient wealth to purchase the first-best level of output, q^* . In that case, payment to the seller will be exactly

$$d + b = (1 - \theta)u(q^*) + \theta c(q^*).$$

If (9) binds, then the buyer simply hands over what he has,

$$z + \bar{b} = (1 - \theta)u(q^c) + \theta c(q^c) \quad (10)$$

and gets in return $q^c \equiv q(z + \bar{b})$. Hence a buyer who does not have enough payment capacity will just pay with their cash on hand and borrow up to their limit in order to purchase the maximum quantity of output $q^c < q^*$.

If the seller does not have access to record-keeping or if the buyer has a recorded history of default, credit cannot be used. In that case, $b = \bar{b} = 0$ and the bargaining problem can be described by (5) – (7). If $z \geq z^* \equiv (1 - \theta)u(q^*) + \theta c(q^*)$, then the buyer has enough payment capacity to obtain q^* . Otherwise, the buyer just hands over his real balances,

$$z = (1 - \theta)u(q) + \theta c(q) \quad (11)$$

where $q \equiv q(z) < q^*$.

3.3 Decentralized Market

We next characterize agents' value functions the DM. After simplification, the expected discounted utility of a buyer holding z units of real balances at the beginning of the period is:

$$V^b(z) = \sigma(1 - \Lambda)\theta[u(q) - c(q)] + \sigma\Lambda\theta[u(q^c) - c(q^c)] + z + W^b(0, 0), \quad (12)$$

where we have used the bargaining solution and the fact that the buyer will never accumulate more balances than he would spend in the DM. According to (12), a buyer in the DM is randomly matched with a seller who does not have access to record-keeping with probability $\sigma(1 - \Lambda)$, receives θ of the match surplus, $u(q) - c(q)$, and can only pay with money. With probability $\sigma\Lambda$, a buyer matches with a seller with access to record-keeping, in which case he gets θ of $u(q^c) - c(q^c)$ and can pay with both money and credit. The last two terms result from the linearity of W^b and is the value of proceeding to the CM with one's portfolio intact.

3.4 Optimal Portfolio Choice

Next, we determine the buyer's choice of real balances. Given the linearity of W^b , the buyer's bargaining problem (5) – (7), and substituting $V^b(z)$ from (12) into (4), the buyer's choice of real

balances must satisfy:

$$\max_{z \geq 0} \{-iz + \sigma(1 - \Lambda)\theta[u(q) - c(q)] + \sigma\Lambda\theta[u(q^c) - c(q^c)]\} \quad (13)$$

where $i = \frac{\gamma - \beta}{\beta}$ is the cost of holding real balances. As a result, the buyer chooses his real balances z in order to maximize his expected surplus in the *DM*, net of the cost of holding real balances, i .

Since the objective function (13) is continuous and maximizes over a compact set, a solution exists. We further assume that $u(q^*) - z(q^*) > 0$ in order to guarantee the existence of a monetary equilibrium. The first-order condition for problem (13) when $z > 0$ is

$$i = \sigma(1 - \Lambda) \frac{\theta[u'(q) - c'(q)]}{\theta c'(q) + (1 - \theta)u'(q)} + \sigma\Lambda \frac{\theta[u'(q^c) - c'(q^c)]}{\theta c'(q^c) + (1 - \theta)u'(q^c)}. \quad (14)$$

3.5 Equilibrium with Full Enforcement

Before proceeding to the case with limited enforcement, we first make a few remarks on equilibrium when enforcement is perfect. With full enforcement, buyers are never constrained by $b \leq \bar{b}$ and can borrow as much as they want to finance consumption in the first-best, q^* . As a result, the second term on the right-hand-side of (14) equals to zero since at q^* , $u'(q^*) = c'(q^*)$. In that case, the right-hand-side is increasing with Λ , meaning that an increase in the fraction of credit trades Λ decreases $q(z)$ and hence real balances z .

4 Limited Enforcement and Credit Limits

When the government's ability to force repayment is limited, borrowers have an incentive to renege on their debts. In order to support trade in a credit economy, we assume that punishment for default entails permanent exclusion from the credit system. In that case, debt-contracts must be self-enforcing, and a borrower that defaults can no longer use credit and will need to use money in all transactions.

The borrowing limit, \bar{b} , is determined in order to satisfy the buyer's incentive constraint to voluntarily repay his debt in the CM. The buyer will repay his debt if

$$W^b(z, -b) \geq \widetilde{W}^b(z),$$

where $W^b(z, -b)$ is the value function of a buyer who chooses to repay his debt at the beginning of the CM, and $\widetilde{W}^b(z)$ is the value function of a buyer who chooses to default. By the linearity of

W^b , the value function of a buyer who repays his debt in the CM is

$$W^b(z, -b) = z - b + W^b(0, 0).$$

On the other hand, the value function of a buyer who defaults, $\widetilde{W}^b(z)$ must satisfy

$$\begin{aligned}\widetilde{W}^b(z) &= z + T + \max_{z' \geq 0} \left\{ -\gamma z' + \beta \widetilde{V}^b(z') \right\} \\ &= z + \widetilde{W}^b(0)\end{aligned}$$

since by defaulting, he is excluded from the use of credit for all future trades and can only use \widetilde{z} real balances for trades in the next DM. As a result, a buyer will repay his debt if

$$\begin{aligned}W^b(z, -b) - \widetilde{W}^b(z) &\geq 0, \\ W^b(0, 0) - \widetilde{W}^b(0) &\geq b, \\ \bar{b} &\geq b,\end{aligned}$$

where $\bar{b} \equiv W^b(0, 0) - \widetilde{W}^b(0)$ is the endogenous debt limit determined such that buyers will always repay their debt. Therefore, the repayment constraint takes the form of an upper bound on credit use. To obtain the expression for \bar{b} , we now determine $W^b(0, 0) - \widetilde{W}^b(0)$. The value functions of a buyer who does not default in the CM can be rewritten as

$$\begin{aligned}W^b(0, 0) &= T + \max_{z \geq 0} \left\{ -\gamma z + \beta V^b(z) \right\} \\ &= T + \max_{z \geq 0} \left\{ -\gamma z + \beta \left[\sigma(1 - \Lambda)\theta S(z) + \sigma\Lambda\theta S(z + \bar{b}) + z + W^b(0, 0) \right] \right\},\end{aligned}$$

where $S(z) \equiv u[q(z)] - c[q(z)]$ is the total trade surplus when sellers only accept money, and $S(z + \bar{b}) \equiv u[q(z + \bar{b})] - c[q(z + \bar{b})]$ is the total trade surplus when both money and credit are accepted. Rearranging and dividing both sides of the equality by β results in

$$\frac{(1 - \beta)}{\beta} W^b(0, 0) = \frac{T}{\beta} + \max_{z \geq 0} \left\{ -\frac{(\gamma - \beta)}{\beta} z + \sigma\theta \left[(1 - \Lambda) S(z) + \Lambda S(z + \bar{b}) \right] \right\}.$$

Similarly, the value functions of a buyer who defaults in the CM can be rewritten as

$$\begin{aligned}\widetilde{W}^b(0) &= T + \max_{\widetilde{z} \geq 0} \left\{ -\gamma \widetilde{z} + \beta \widetilde{V}^b(\widetilde{z}) \right\} \\ &= T + \max_{\widetilde{z} \geq 0} \left\{ -\gamma \widetilde{z} + \beta \left[\sigma\theta S(\widetilde{z}) + \widetilde{z} + \widetilde{W}^b(0) \right] \right\},\end{aligned}$$

where $\tilde{z} > 0$ solves

$$i = \sigma \frac{\theta[u'(q(\tilde{z})) - c'(q(\tilde{z}))]}{\theta c'(q(\tilde{z})) + (1 - \theta)u'(q(\tilde{z}))}, \quad (15)$$

since a buyer who defaults can only use money irrespective of what the seller accepts. Rearranging and dividing both sides of the equality by β results in

$$\frac{(1 - \beta)}{\beta} \widetilde{W}^b(0) = \frac{T}{\beta} + \max_{\tilde{z} \geq 0} \left\{ -\frac{(\gamma - \beta)}{\beta} \tilde{z} + \sigma \theta S(\tilde{z}) \right\}.$$

Therefore, the debt limit, \bar{b} , must satisfy

$$\frac{(1 - \beta)}{\beta} \bar{b} = \frac{(1 - \beta)}{\beta} [W^b(0, 0) - \widetilde{W}^b(0)].$$

Substituting in the expressions for $W^b(0, 0)$ and $\widetilde{W}^b(0)$ then leads to the following expression for \bar{b} :

$$r\bar{b} = \max_{z \geq 0} \{-iz + \sigma\theta [(1 - \Lambda)S(z) + \Lambda S(z + \bar{b})]\} - \max_{\tilde{z} \geq 0} \{-i\tilde{z} + \sigma\theta S(\tilde{z})\} \equiv \Omega(\bar{b}) \quad (16)$$

where $r = \frac{1 - \beta}{\beta}$ is the discount rate. From (16), we see that the debt limit depends on the availability of credit, monetary policy, frictions in the environment, and agents' patience. As agents get more patient (r falls), the borrowing limit goes up since the buyer can credibly promise to repay more.

We now define equilibrium of the economy when enforcement is limited and Λ is exogenous.

Definition 1. *Given Λ , a stationary monetary equilibrium with limited enforcement is a list $(q, q^c, z, \tilde{z}, \bar{b})$ that satisfy (10), (11), (14), (15), and (16).*

Before characterizing properties of equilibrium with the endogenous debt limit, \bar{b} , we start by establishing some key properties of $\Omega(\bar{b})$.

Lemma 1. *The function $\Omega(\bar{b}) \equiv \max_{z \geq 0} \{-iz + \sigma\theta [(1 - \Lambda)S(z) + \Lambda S(z + \bar{b})]\} - \max_{\tilde{z} \geq 0} \{-i\tilde{z} + \sigma\theta S(\tilde{z})\}$ has the following key properties:*

1. $\Omega(0) = 0$,
2. $\Omega'(\bar{b}) \geq 0$,
3. $\Omega'(0) = i\Lambda \geq 0$,
4. $\Omega'(\bar{b}) = 0$ for all $\bar{b} \geq (1 - \theta)u(q^*) + \theta c(q^*)$,
5. $\Omega(\bar{b})$ is a concave function,

6. $\Omega(\bar{b})$ is continuous for all $z > 0$.

Given Lemma 1, we can now characterize how the debt limit, \bar{b} , affects the value of money and output in the DM.

Lemma 2. *There exists thresholds for the debt limit, \bar{b}_0 and \bar{b}_1 , such that*

1. *If $\bar{b} \in [0, \bar{b}_0)$, then $z > 0$ and $q(z + \bar{b}) < q^*$*
2. *If $\bar{b} \in [\bar{b}_0, \bar{b}_1)$, then $z = 0$ and $q(\bar{b}) < q^*$,*
3. *If $\bar{b} \in [\bar{b}_1, \infty)$, then $z = 0$ and $q(\bar{b}) = q^*$.*

The value \bar{b}_0 is the threshold for the debt limit, above which money is no longer valued and solves

$$r\bar{b}_0 = \sigma\theta\Lambda S(\bar{b}_0).$$

The value \bar{b}_1 is the threshold for the debt limit, above which the buyer can borrow enough to finance consumption of the first-best, q^* , and is given by:

$$\bar{b}_1 = (1 - \theta)u(q^*) + \theta c(q^*).$$

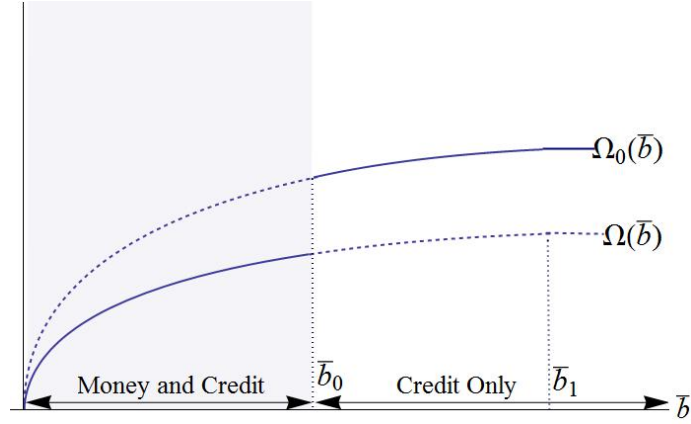
For all $\bar{b} > \bar{b}_1$, $r\bar{b} = \sigma\theta\Lambda S(q^*)$. Consequently when $z = 0$, $\bar{b}_0 \leq \bar{b}_1$.

At $\bar{b} = 0$, credit is not used and the buyer can only use money. If $\bar{b} \in (0, \bar{b}_0)$, the buyer has a low borrowing limit and will hold money in order to finance consumption in the DM, but cannot borrow enough to obtain the first-best, q^* . In this region, both money and credit can be used. If $\bar{b} \in [\bar{b}_0, \bar{b}_1)$, the buyer stops holding money but still cannot borrow enough to finance q^* . In this range, money is no longer valued and equilibrium become non-monetary as credit starts to drive out money: credit is more profitable than money in the sense that the value of money decreases over time. Finally when $\bar{b} \in [\bar{b}_1, \infty)$, money is not valued and the buyer has a high enough borrowing limit to permit consumption of q^* .

For $z \geq 0$, the right side of (16) is continuous for all $\bar{b} \in [0, \bar{b}_0)$ and becomes discontinuous at \bar{b}_0 when money is no longer valued. When $z = 0$ and $\bar{b} \in [\bar{b}_0, \bar{b}_1)$, the right side of (16) is given by $\Omega_0(\bar{b})$ and becomes linear at \bar{b}_1 when the buyer can borrow enough to obtain the first-best, q^* . Furthermore, from Lemma 1, the slope of $\Omega(\bar{b})$ at $z > 0$ is $\sigma\theta\Lambda S'(z)$, which is smaller than the slope of $\Omega_0(\bar{b})$ at $z = 0$, which is $\sigma\theta\Lambda S'(0)$. Consequently, $\Omega(\bar{b})$ when $z > 0$ is less than $\Omega_0(\bar{b})$ when $z = 0$. This is shown in Figure 2 where $\Omega(\bar{b})$ lies strictly below $\Omega_0(\bar{b})$. Money is valued only in the shaded region where $\bar{b} < \bar{b}_0$, while equilibria must be non-monetary for all $\bar{b} > \bar{b}_0$.

We now turn to characterizing the different types of equilibria that can arise in the model.

Figure 2: Monetary and Credit Trades: $\Omega(\bar{b})$ and $\Omega_0(\bar{b})$



4.1 Pure Credit Equilibrium

A non-monetary equilibrium with credit exists when $\Lambda > 0$, $z = 0$, and $\bar{b} \in [b_0, \infty)$. In order for $z = 0$, the debt limit \bar{b} must satisfy

$$r\bar{b} = \sigma\theta\Lambda S(\bar{b}) \equiv \Omega_0(\bar{b}). \quad (17)$$

At $z = 0$, differentiating the right side of (17), $\Omega_0(\bar{b})$, with respect to \bar{b} at $\bar{b} = 0$ yields

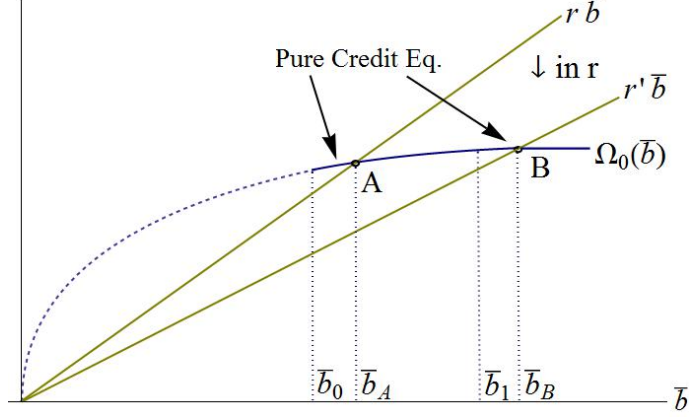
$$\begin{aligned} \left. \frac{\partial \Omega_0(\bar{b})}{\partial \bar{b}} \right|_{\bar{b}=0} &= \sigma\theta\Lambda S'(0) \\ &= \sigma\Lambda \frac{\theta}{1-\theta}. \end{aligned}$$

Consequently, only credit is used if the slope of $\Omega_0(\bar{b})$ is greater than the slope of $r\bar{b}$ at $\bar{b} = 0$:

$$\sigma\Lambda \frac{\theta}{1-\theta} > r. \quad (18)$$

An equilibrium where only credit is used exists if borrowers prefer repaying their loans to defaulting. This occurs if the measure of sellers with access to record-keeping Λ is high enough, the matching probability σ is high enough, the buyer's bargaining power θ is high enough, or agents' discount rate r is low enough. When the fraction of sellers accepting credit is exogenous, there exists a threshold for the fraction of credit trades, below which $\bar{b} = 0$ and above which $\bar{b} > 0$. From 18, this

Figure 3: Pure Credit Equilibrium: Decrease in r



threshold is given by

$$\bar{\Lambda} \equiv \frac{r(1-\theta)}{\sigma\theta}.$$

Figure 3 depicts the pure credit equilibrium and shows the effects of a decrease in r : as agents become more patient, the economy moves from point A to B , which causes the debt limit to increase from \bar{b}_A to \bar{b}_B . Indeed, as agents become more patient, the borrowing limit relaxes since buyers can credibly promise to repay more, and the quantity traded increases from $q < q^*$ to $q = q^*$. More generally, $\Omega_0(\bar{b})$ shifts up as the measure of sellers with access to record-keeping Λ increases, the matching probability σ increases, or the buyer's bargaining power θ increases. Consequently the debt limit \bar{b} will increase. If on the other hand r increases above $\sigma\Lambda\frac{\theta}{1-\theta}$, the borrowing limit is driven to zero as borrowers are not patient enough to sustain credit use. As a result, a pure credit equilibrium will cease to exist.

4.2 Pure Monetary Equilibrium

In a pure monetary equilibrium, $z > 0$ and credit is not used, $\bar{b} = 0$. At $\bar{b} = 0$, $\Omega(0) = 0$ by Lemma 1. Further, since $S(z)$ is concave and $S'(0) = \frac{1}{(1-\theta)}$ by the Envelope Theorem, real balances are positive ($z > 0$) at $\bar{b} = 0$ if and only if

$$\begin{aligned} i &= \sigma\theta S'(z), \\ i &< \sigma\theta S'(0), \\ i &< \frac{\sigma\theta}{1-\theta} \equiv \bar{i}. \end{aligned}$$

Figure 4: Pure Monetary Equilibrium

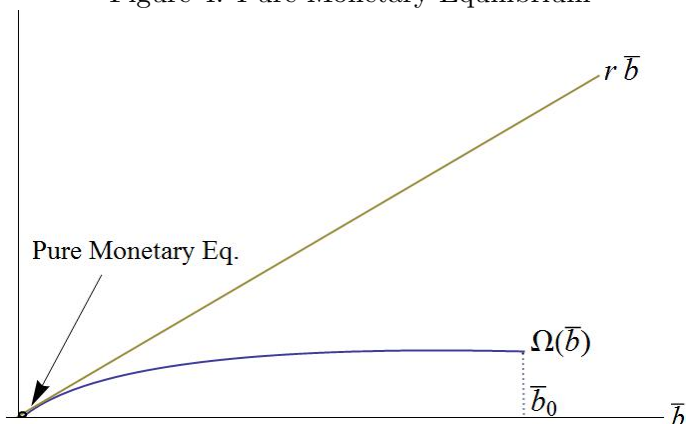


Figure 4 plots $\Omega(\bar{b})$ as a function of \bar{b} and shows that for $i < \bar{i}$, the pure monetary equilibrium will exist uniquely if $i\Lambda < r$, so that the slope of $\Omega(\bar{b})$ at $\bar{b} = 0$ is less than the slope of $r\bar{b}$, and $\Lambda < \bar{\Lambda}$, so that credit is not accepted. In that case, $\Omega(\bar{b})$ and $r\bar{b}$ intersect once at $\bar{b} = 0$, and the unique equilibrium is one where only money is used.

The next proposition characterizes how a key policy variable, the money growth rate γ , affects the existence of a pure monetary equilibrium.

Proposition 1. *In all stationary equilibria where $\gamma \geq \beta$ and $i\Lambda < r$, the following cases can arise:*

1. *At the Friedman rule, $\gamma = \beta$, a pure monetary equilibrium with $z = z^* \equiv (1 - \theta)u(q^*) + \theta c(q^*)$, $\bar{b} = 0$, and $q = q^*$ exists uniquely.*
2. *When $\gamma \in (\beta, 1)$, a pure monetary equilibrium with $z < z^*$, $\bar{b} = 0$, and $q < q^*$ exists for all $\Lambda \in [0, 1]$ and exists uniquely if $\Lambda < \bar{\Lambda}$.*
3. *When $\gamma \in (1, \bar{\gamma})$ where $\bar{\gamma} \equiv \beta(1 + \bar{i})$, a pure monetary equilibrium with $z < z^*$, $\bar{b} = 0$, and $q < q^*$ will exist so long as Λ is not too large.*

The first part of Proposition 1 is very intuitive and simply says that when $\gamma = \beta$, the rate of return on money is high enough so that there is no need to use credit. This is because when γ , or equivalently i , decreases, the expected surplus from defaulting increases which increases the incentive to renege on debt repayment. This in turn tightens the credit constraint and leads the debt limit \bar{b} to fall. In the limit where $\gamma = \beta$ or $i = 0$, money becomes costless to hold and the incentive to renege is too high to support voluntary debt repayment. Because of limited commitment, the

availability of credit dries up and the economy falls into a “credit crunch.” Efficient monetary policy drives out credit and money alone is enough to finance consumption of the first-best q^* .

However, the Friedman rule is sufficient but not necessary to permit the uniqueness of a pure monetary equilibrium. Proposition 1 also shows that so long as there is deflation, it is possible for a pure monetary equilibrium to exist for all $\Lambda \in [0, 1]$. To take the most extreme case, suppose that $\Lambda = 1$ so that all sellers accept credit. In that case, a pure monetary equilibrium will still exist so long as $i < r$ or $\gamma < 1$. Even though all sellers accept credit, buyers choose to only hold real balances. Since the economy is deflating, the incentives to renege on debt repayment increases, which decreases the cost of default and therefore tightens the credit constraint by driving the debt limit to zero. Consequently, only money is used when $\gamma < 1$ and $\Lambda = 1$.

When $\gamma \in (\beta, \bar{\gamma})$, it is possible for a money and credit equilibrium to coexist, provided that $\Lambda > \bar{\Lambda}$. However, credit is not robust in the sense that an efficient monetary policy (i.e. one that implements the Friedman rule) completely drives out credit. At the optimum, q^* , only money is used and it recovers the allocation that a social planner would choose under perfect record-keeping.

4.3 Money and Credit Equilibrium

In an equilibrium with both money and credit, it must be that $\Lambda > 0$, $z > 0$, and $\bar{b} \in (0, \bar{b}_0)$. When $\bar{b} > 0$, z has an interior solution so long as

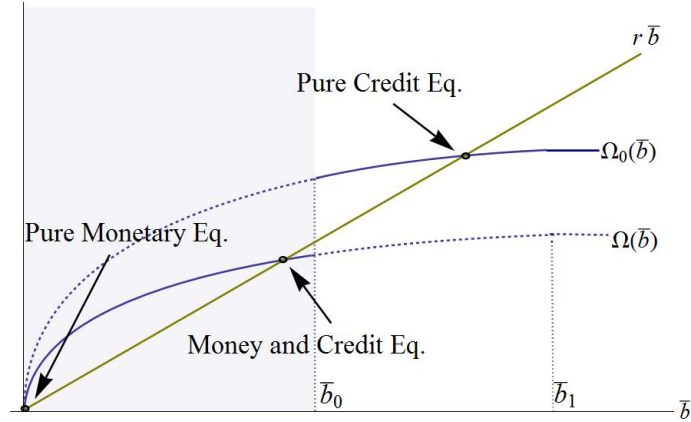
$$\begin{aligned} i &= \sigma\theta(1 - \Lambda)S'(z) + \sigma\theta\Lambda S'(z + \bar{b}), \\ i &< \sigma\theta(1 - \Lambda)S'(0) + \sigma\theta\Lambda S'(\bar{b}), \\ i &< \frac{\sigma\theta(1 - \Lambda)}{1 - \theta} + \sigma\theta\Lambda S'(\bar{b}), \\ i &< (1 - \Lambda)\bar{i} + \sigma\theta\Lambda S'(\bar{b}) \equiv \tilde{i}. \end{aligned}$$

When $i\Lambda > r$, the slope of $\Omega(\bar{b})$ from (16) is higher than the slope of $r\bar{b}$ at $\bar{b} = 0$. In that case, $z > 0$ and $\bar{b} \in (0, \bar{b}_0)$, and there exists an equilibrium such that both money and credit are used, as shown in Figure 5. Also notice that since the condition for a pure monetary equilibrium $i < \bar{i}$ is always satisfied if $\frac{r}{\Lambda} < i < \tilde{i}$, the pure monetary equilibrium will always exist whenever there is an equilibrium with both money and credit.¹⁴

Since $i\Lambda > r$ when both money and credit are used, an increase in Λ increases the right-hand-side of (16), which shifts $\Omega(\bar{b})$ up and induces an increase in \bar{b} . When more sellers accept credit, the

¹⁴This can also be seen in Figure 5 and follows directly from Lemma 1: since $\Omega(0) = 0$, the function $\Omega(\bar{b})$ will always intersect $r\bar{b}$ at the origin $\bar{b} = 0$ (i.e. a pure monetary equilibrium) whenever there is also an interior solution (i.e. a money and credit equilibrium if $i\Lambda > r$).

Figure 5: Pure Monetary, Money and Credit, and Pure Credit Equilibria



gain for buyers from using and redeeming credit increases, which relaxes the payment constraint $b \leq \bar{b}$. The increase in Λ can be high enough so that credit starts to drive money out of circulation. This can cause the economy to shift to a pure credit equilibrium where money is no longer valued.

An increase in inflation (analogously, i) generates the same qualitative effect: credit is more profitable than money in the sense that the value of money decreases over time. In this way, inflation has two effects in this model: first, is the usual effect on reducing the purchasing power of money, which reduces trade and hence welfare; second, is the effect on reducing agents' incentive to default. Intuitively, an increase in inflation relaxes the credit constraint by increasing the cost of default, since defaulters need to bring enough money to finance their consumption.

In sum, the endogenous upper bound on credit depends on the fraction of credit trades, the rate of return on money, and agents' patience. The larger the fraction of sellers that accept credit, the lower the rate of return on money, or the more patient agents become, the less likely the credit constraint $b \leq \bar{b}$ will be binding. In these cases, the buyer can credibly promise to repay more, which relaxes the credit constraint.

4.4 Multiple Equilibria

A particularly striking feature of the model is that there can be a multiplicity of equilibria even without any changes in fundamentals. The next proposition establishes the possible cases for multiple equilibria, which the remainder of this subsection discusses.

Proposition 2. *When $i > \bar{i} \equiv \frac{\sigma\theta}{1-\theta}$, equilibrium will be non-monetary and there will either be (i) autarky where neither money nor credit is used or (ii) a pure credit equilibrium. When $i < \bar{i}$, a*

pure monetary equilibrium either (iii) exists uniquely, (iv) coexists with a pure credit equilibrium, or (v) coexists with both a pure credit equilibrium and a money and credit equilibrium.

Figure 5 shows that the pure monetary equilibrium, money and credit equilibrium, and pure credit equilibrium coexist, even without any changes in fundamentals. A monetary equilibrium without credit is characterized by $\bar{b} = 0$. A monetary equilibrium with credit is possible only in the shaded region where $\bar{b} \in (0, \bar{b}_0)$ while only non-monetary equilibria are possible in the non-shaded region $\bar{b} \in [\bar{b}_0, \infty)$. Given these properties, buyers in this model will use only money for small purchases, both money and credit for intermediate purchases, and only credit for large purchases.¹⁵ This is because the buyer's total wealth and hence output traded q is increasing in the debt limit, \bar{b} .

The next proposition establishes a particularly interesting case where the equilibrium with both money and credit ceases to exist.

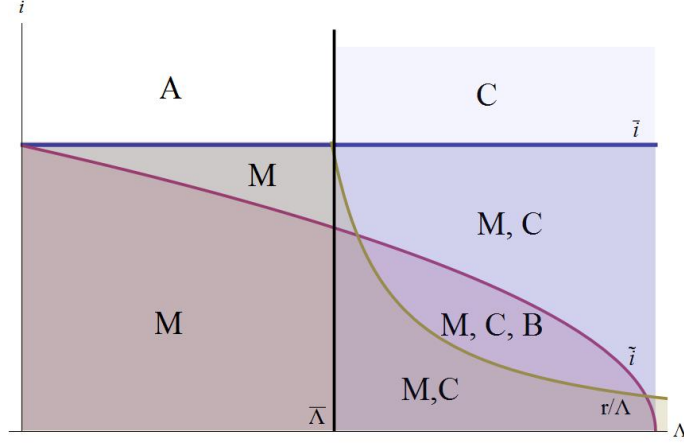
Proposition 3. *When $\Lambda = 1$, there can either be a pure credit equilibrium where $\bar{b} > 0$ and $z = 0$ or a pure monetary equilibrium where $\bar{b} = 0$ and $z > 0$, but there cannot be an equilibrium where both money and credit are used.*

PROOF. If $\Lambda = 1$ and money is valued, (14) and (15) imply that $i = \sigma\theta S'[q(z + \bar{b})] = \sigma\theta S'[q(\tilde{z})]$, or $q(z + \bar{b}) = q(\tilde{z})$. Then since $z + \bar{b} = \tilde{z} = (1 - \theta)u(q) + \theta c(q)$ from the bargaining solution, the left side of the debt limit (16) becomes $-i[z - \tilde{z}] = -i[\tilde{z} - \bar{b} - \tilde{z}] = i\bar{b}$. Consequently, (16) implies that $r\bar{b} = i\bar{b}$, or $\bar{b} = 0$. \square

Proposition 3 highlights an important dichotomy between monetary and credit trades when $\Lambda = 1$: there can be trades with credit only or trades with money only, but never trades with both money and credit. At $\Lambda = 1$, (14) and (15) implies that if money is valued, the debt limit must be zero: $z = \tilde{z} > 0$ implies $\bar{b} = 0$. Since buyers obtain the same surplus whether or not they default, there cannot exist a positive debt limit that supports voluntary debt repayment. Consequently, the debt limit is driven to zero and there cannot be a monetary equilibrium where credit is also used. This special case also points to the difficulty of getting both money and credit to be used when all trades are identical and record-keeping is costless: either only credit is used as money becomes inessential, or only money is used since the incentive to renege on debt repayment is too high. Since there can never be an equilibrium where both money and credit can be used, monetary policy has no effect on credit use.

¹⁵Similarly, our model implies retailers that accept credit cards sell more than retailers that only accept cash. This prediction is consistent with Ernst and Young (1996)'s survey of retailers in the United States that 83% of merchants said that their sales increased by accepting credit cards.

Figure 6: Multiple Equilibria



Fortunately, this dichotomy between monetary and credit trades breaks down for $\Lambda \in (0, 1)$. Figure 6 plots existence conditions for different types of equilibria in (Λ, i) -space.¹⁶ We have shown in the previous sub-sections that a pure monetary equilibrium will exist if $i < \bar{i} \equiv \frac{\sigma\theta}{1-\theta}$ while a pure credit equilibrium will exist if $\Lambda > \bar{\Lambda} \equiv \frac{r}{\bar{i}}$, assuming that $\bar{\Lambda} < 1$ or $r < \bar{i}$. For both money and credit to be used, it must be that $\frac{r}{\Lambda} < i < \tilde{i} \equiv (1 - \Lambda)\bar{i} + \sigma\theta\Lambda S'(\bar{b})$.

There may be a unique equilibrium with credit only when $i > \bar{i}$ and $\Lambda > \bar{\Lambda}$; a unique equilibrium with money only when $i < \bar{i}$ and $\Lambda < \bar{\Lambda}$; or multiple equilibria when $i < \bar{i}$ and $\Lambda > \bar{\Lambda}$. If inflation further increases above \bar{i} , money stops being valued altogether and equilibria become non-monetary; in that case, there is either autarky where neither money nor credit is used or a pure credit equilibrium where only credit is used.

Figure 6 also shows how payment systems depend not just on fundamentals but also on histories and social conventions. Suppose that inflation is initially low and the economy is in an equilibrium where a pure monetary equilibrium coexists with a pure credit equilibrium (region M, C). As inflation increases above \bar{i} , the pure monetary equilibrium disappears and only credit is used (region C). But when inflation goes back down to its initial level, it is possible that agents may still coordinate on the pure credit equilibrium. The economy therefore displays hysteresis and inertia: when there are many possible types of equilibria, social conventions and histories can dictate the equilibrium that prevails.

When agents get less patient (r increases), both the threshold for credit to be used, $\bar{\Lambda}$, and

¹⁶The types of equilibria in Figure 6 are autarky (A) where neither money nor credit is used, a pure credit (C) equilibrium where only credit is used, a pure monetary (M) equilibrium where only money is used, and a mixed equilibrium (B) where both money and credit are used.

the condition for both money and credit to be used, $\frac{r}{\Lambda}$, increases. The vertical line $\bar{\Lambda}$ in Figure 6 therefore shifts to the right while the curve $\frac{r}{\Lambda}$ shifts up, which decreases the set of pure credit equilibria and the set of equilibria where both money and credit are used. Intuitively, less patient buyers find it more difficult to credibly promise to repay their debts, which decreases their borrowing limit \bar{b} , and hence the possibility that credit will be used.

When the buyer's bargaining power θ increases, both \bar{i} and \tilde{i} increase while $\bar{\Lambda}$ decreases. In Figure 6, both the horizontal line \bar{i} and the curve \tilde{i} shifts up while the vertical line $\bar{\Lambda}$ shifts to the left. Consequently, an increase in θ decreases the possibility of autarky and increases the set of pure monetary equilibria, pure credit equilibria, and equilibria where both money and credit are used. When $\theta = 1$, buyers have all the bargaining power and the pricing mechanism corresponds to a buyer-take-all-rule; in that case, two types of equilibria remain: one where a pure monetary equilibrium coexists with a pure credit equilibrium (region M, C) and one where all three types of equilibria coexist (region M, C, B).

5 Costly Record-Keeping

We now consider the choice of accepting credit by making $\Lambda \in [0, 1]$ endogenous. In order to accept credit, sellers must invest *ex-ante* in a costly record-keeping technology that records and authenticates an *IOU* proposed by the buyer.¹⁷ The per-period cost of this investment is $\kappa > 0$, which is drawn from a cumulative distribution $F(\kappa) : \mathbb{R}_+ \rightarrow [0, 1]$. Sellers are heterogeneous according to their record-keeping cost and are indexed by κ .¹⁸ Hence for some sellers this cost will be close to zero, so that they will always accept credit, while for others this cost will be very large and they will never accept credit. The distribution of costs across sellers is known by all agents and is assumed to be continuous.

At the beginning of each period before trades occur, sellers choose whether or not to invest in the costly record-keeping technology. When making this decision, sellers take as given buyer's choice of real balances, z , and the debt limit, \bar{b} . The seller's problem is given by

$$\max\{-\kappa + \sigma(1 - \theta)S(z + \bar{b}), \sigma(1 - \theta)S(z)\}. \quad (19)$$

According to (19), if the seller decides to invest, he incurs the disutility cost $\kappa > 0$ that allows him

¹⁷This cost can also reflect issues of fraud and information problems that currently permeate the credit industry. In fact, the credit card industry is facing serious challenges in the form of credit card fraud, identity theft, and the need to secure confidential information. Besides being a costly drain on banks and retailers that accept credit, these problems may erode consumer confidence in the credit card industry.

¹⁸Arango and Taylor (2008) find that merchants perceive cash as the least costly form of payment while credit cards stand out as the most costly due relatively high processing fees.

to extend a loan to the buyer. In that case, the seller extracts a constant fraction $(1 - \theta)$ of the total surplus, $S(z + \bar{b}) \equiv u[q(z + \bar{b})] - c[q(z + \bar{b})]$. If the seller does not invest, then he can only accept money, and gets $(1 - \theta)$ of $S(z) \equiv u[q(z)] - c[q(z)]$. Since total surplus is increasing in the buyer's total wealth $z + \bar{b}$, $S(z + \bar{b}) > S(z)$. Further, $S(z + \bar{b})$ and hence the first term in the seller's maximization problem (19) increases with \bar{b} , and both terms of (19) increase with z .

There exists a threshold for the record-keeping cost, $\bar{\kappa}$ below which the seller invests in the record-keeping technology and above which they do not invest. From (19), this threshold is given by

$$\bar{\kappa} \equiv \sigma(1 - \theta)[S(z + \bar{b}) - S(z)], \quad (20)$$

and gives the seller's expected benefit of accepting credit. Since $S(z + \bar{b})$ increases with \bar{b} , the seller's expected benefit $\bar{\kappa}$ increases with \bar{b} . Given κ , let $\lambda(\kappa) \in [0, 1]$ denote an individual seller's decision to invest. This decision problem is given by

$$\lambda(\kappa) = \begin{cases} 1 & \text{if } \kappa < \bar{\kappa} \\ [0, 1] & \text{if } \kappa = \bar{\kappa} \\ 0 & \text{if } \kappa > \bar{\kappa} \end{cases} \quad (21)$$

Condition (21) simply says that all sellers with $\kappa < \bar{\kappa}$ will invest in the costly record-keeping technology, since the benefit exceeds the cost; sellers with $\kappa > \bar{\kappa}$ do not invest; and any seller with $\kappa = \bar{\kappa}$ will invest with an arbitrary probability since they are indifferent.

Consequently, since $F(\kappa)$ is continuous, the aggregate measure of sellers that invest is

$$\Lambda \equiv \int_0^{\infty} \lambda(\kappa) dF(\kappa) = F(\bar{\kappa}). \quad (22)$$

That is, the measure of sellers that invest is given by the measure of sellers with $\kappa \leq \bar{\kappa}$.

Definition 2. *A stationary monetary equilibrium with limited enforcement and endogenous Λ is a list $(q, q^c, z, \bar{z}, \bar{b}, \Lambda)$ that satisfy (10), (11), (14), (15), (16), and (22).*

To determine equilibrium when Λ is endogenous, we first determine buyers' choice of real balances and how much they want to borrow, given sellers' investment decisions Λ . Next we determine sellers' investment decision Λ , given buyers' choice of real balances and decision to repay their debts. These decisions are then depicted as reaction functions for buyers and sellers, respectively.

5.1 Buyers' Reaction Function

Given sellers' investment decisions Λ , buyers must decide how much money to hold and how much to borrow. Indeed, the buyer's choice of real balances, z , and the amount borrowed, \bar{b} , are each functions of the measure of sellers accepting credit, Λ .

When $\Lambda \in [0, \frac{r}{i})$, buyers only use money, and we show in Section 4.2 that a pure monetary equilibrium with $z > 0$ and $\bar{b} = 0$ will exist uniquely. When $\Lambda \in (\frac{r}{i}, 1]$, there can either be a money and credit equilibrium with $z > 0$ and $\bar{b} \in (0, \bar{b}_0)$, or a pure credit equilibrium with $z = 0$ and $\bar{b} \in (\bar{b}_0, \bar{b}_1)$. However when $\Lambda = 1$, it follows from Proposition 3 that the only equilibrium with a positive debt limit is a pure credit equilibrium with $z = 0$ and $\bar{b} = \bar{b}_1$.

The following lemma establishes some key properties of the buyer's reaction function.

Lemma 3. *The debt limit \bar{b} is a strictly increasing and convex function of Λ .*

At \bar{b}_0 , the corresponding fraction of sellers that accept credit is defined as $\Lambda_0 \equiv \frac{r\bar{b}_0}{\sigma\theta S(\bar{b}_0)}$. Clearly when $\Lambda \in [0, \frac{r}{i})$, credit is not used: $\bar{b} = 0$. When $\Lambda \in (\frac{r}{i}, \Lambda_0)$, the debt limit becomes positive with $\bar{b} \in (0, \bar{b}_0)$. Finally when $\Lambda \in (\Lambda_0, 1)$, money is no longer valued and the debt limit is $\bar{b} \in (\bar{b}_0, \bar{b}_1)$. The buyer's reaction function is depicted in Figures 7 and 8, which each show the buyer's choice of \bar{b} for a given Λ .

5.2 Sellers' Reaction Function

Given buyers' decision to hold money and whether to repay their debts, sellers must decide whether or not to invest in the costly technology to record credit arrangements. Indeed, sellers' investment decisions, Λ , depend on the buyer's choice of real balances z , and the amount borrowed, \bar{b} .

When $\bar{b} = 0$, the debt limit is zero and buyers only use money for DM trades. Clearly from (20), the seller's expected benefit of accepting credit at $\bar{b} = 0$ is zero: $\bar{\kappa} = 0$. Consequently, no sellers will invest and $\Lambda = 0$. When $\bar{b} = \bar{b}_1$, money is no longer valued and buyers only use credit. In that case, the seller's expected benefit of accepting credit is at its maximum: $\bar{\kappa}_{\max} \equiv \sigma(1 - \theta)S^*$ where $S^* \equiv S(\bar{b}_1)$. With no loss in generality, we assume that all sellers have a record-keeping cost smaller than this maximum value, $\bar{\kappa}_{\max}$. Consequently, all sellers will invest at $\bar{b} = \bar{b}_1$, in which case $\Lambda = 1$.¹⁹

The equilibrium measure of sellers who invest when $\bar{b} \in (0, \bar{b}_1)$ will depend on the shape of the seller's reaction function. In what follows, we assume that sellers' record-keeping costs are

¹⁹Without assuming an upper bound on the record-keeping cost, it is also possible to have a pure credit equilibrium with $\Lambda < 1$. In that case, the seller's reaction function in Figure 7 can intersect with the buyer's at $\bar{b} \in (\bar{b}_0, \bar{b}_1)$ so that total surplus in the pure credit equilibrium will be less than S^* . This distinction does not matter for our qualitative results, but will matter for welfare, as shown in Section 7.

Figure 7: Buyers' and Sellers' Reaction Functions: Case 1 ($\Lambda_0 < \Lambda_s$)

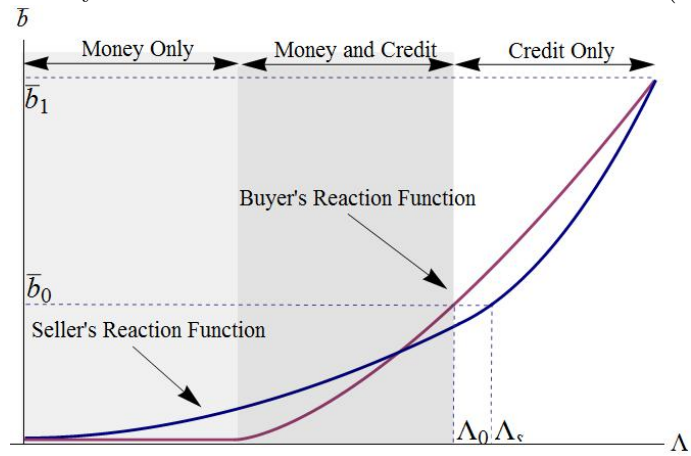
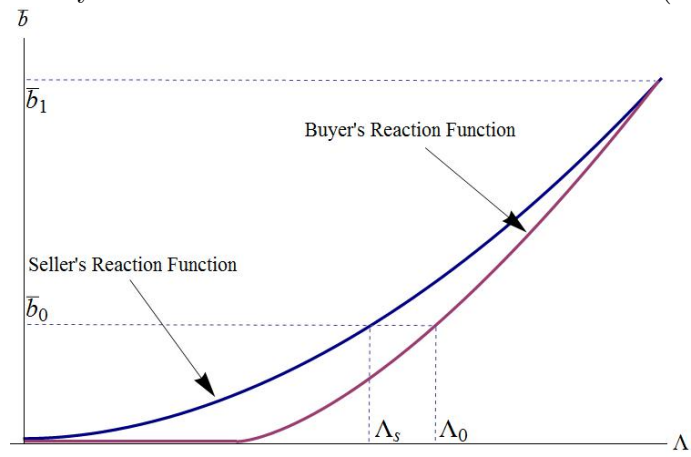


Figure 8: Buyers' and Sellers' Reaction Functions: Case 2 ($\Lambda_s < \Lambda_0$)



drawn from a uniform distribution of κ over $[0, \bar{\kappa}_{max}]$.²⁰ The following lemma establishes that the aggregate measure of sellers who invest, Λ , is an increasing function of debt limit \bar{b} for all $\bar{b} \in [0, \bar{b}_1]$.

Lemma 4. *The fraction of sellers who invest Λ is a strictly increasing function of the credit limit \bar{b} .*

Proposition 4. *When $\Lambda_0 < \Lambda_s$, there is a pure monetary equilibrium ($\Lambda = 0$), a pure credit equilibrium ($\Lambda = 1$), and a money and credit equilibrium ($\Lambda < \Lambda_0 \in (0, 1)$). When $\Lambda_0 > \Lambda_s$, there can only be a pure monetary equilibrium ($\Lambda = 0$) and a pure credit equilibrium ($\Lambda = 1$).*

Two cases have to be distinguished according to the position of Λ_s relative to Λ_0 . In Figure 7, $\Lambda_0 < \Lambda_s$, and the buyers' and sellers' reaction functions intersect three times, corresponding to three different types of equilibria: a pure monetary equilibrium where only money is used ($\Lambda = 0$), a money and credit equilibrium where a fraction $\Lambda < \Lambda_0$ of sellers accept both money and credit while the remaining $(1 - \Lambda)$ sellers only accept money, and a pure credit equilibrium where money is not valued and all sellers only accept credit ($\Lambda = 1$). In contrast, Figure 8 shows the other case with $\Lambda_0 > \Lambda_s$. Here, buyers' and sellers' reaction functions do not have any interior intersection. Therefore, there are only two types of equilibria: a pure monetary equilibrium and a pure credit one.

6 Welfare

We now turn to examining some of the model's normative implications and begin by comparing the different types of equilibria in terms of social welfare. Society's welfare is measured as the sum of buyers' and sellers' utilities in the DM: $\mathcal{W} \equiv (1 - \beta)V^b(z) + (1 - \beta)V^s(0)$. This is given by

$$\mathcal{W} \equiv \sigma[\Lambda S(z + \bar{b}) + (1 - \Lambda)S(z)] - k \quad (23)$$

where $k \equiv \int_0^{\bar{\kappa}} \kappa dF(\kappa)$ is defined as the average aggregate record-keeping cost across all sellers. Consider the case where $\Lambda_0 < \Lambda_s$, in which case there are three types of equilibria, as illustrated in Figure 7. There can be a pure monetary equilibrium with $\bar{b} = 0$, $z > 0$, and $\Lambda = 0$; a pure credit equilibrium with $\bar{b} = \bar{b}_1$, $z = 0$, and $\Lambda = 1$; and finally a money and credit equilibrium with $\bar{b} \in (0, \bar{b}_0)$ and $z > 0$ where a fraction $\Lambda \in (0, 1)$ of sellers accept both money and credit while the remaining $(1 - \Lambda)$ sellers only accept money. The table below summarizes social welfare across these types of equilibria.

²⁰Our results go through more generally, with any continuous distribution $F(\kappa) : \mathbb{R}_+ \rightarrow [0, 1]$. Here, the assumption of the uniform distribution just permits us to determine the value of Λ at \bar{b}_0 .

Table 1: **Welfare Across Equilibria**

EQUILIBRIUM	WELFARE
Pure Monetary	$\mathcal{W}_m = \sigma S(z)$
Pure Credit	$\mathcal{W}_c = \sigma S^* - k$
Money and Credit	$\mathcal{W}_{mc} = \sigma[\Lambda S(z + \bar{b}) + (1 - \Lambda)S(z)] - k$

Which equilibria dominates in terms of welfare depends on the record-keeping cost. For the pure credit equilibrium to dominate the pure monetary equilibrium, the aggregate record-keeping cost must be low enough; that is, $\mathcal{W}_c > \mathcal{W}_m$ if $k < \sigma[S^* - S(z)]$. Even with a low-record keeping cost however, it is still possible for the welfare-dominated monetary equilibrium to prevail due to a rent-sharing externality: since sellers must incur the full cost of adopting the record-keeping technology but only obtains a fraction $(1 - \theta)$ of the total surplus, they fail to internalize the full benefit of accepting credit. Consequently, there can be coordination failures and excess inertia in the decision to accept credit, in which case the economy can end up in the Pareto-inferior monetary equilibrium.

On the other hand, society will prefer the pure monetary equilibrium if the monetary authority implements the Friedman rule. When $i = 0$, $\mathcal{W}_m = \sigma S^*$ and $\mathcal{W}_c = \sigma S^* - k$: so long as $k > 0$, the pure monetary equilibrium dominates the pure credit equilibrium in terms of social welfare. In this case, the pure monetary equilibrium leads to the first-best allocation and saves society on record-keeping costs.

In addition, the model feature an important channel for monetary policy and inflation to affect both monetary and credit allocations. When the economy only uses money, it is not surprising that social welfare is decreasing in the inflation rate: $\frac{d\mathcal{W}_m}{di} = \sigma S'(z) \frac{dz}{di} < 0$ since from (14), $\frac{dz}{di} = \sigma \theta S''(z) < 0$. Since inflation is a tax on money holdings, an increase in inflation will reduce the purchasing power of money, and hence output traded and welfare.

In the pure credit equilibrium, increasing inflation has no effect on welfare since at \bar{b}_1 , total surplus is at its maximum, S^* .²¹

[TO BE COMPLETED.]

7 Discussion

In this section, we discuss relevant policy issues and the extent to which our model is able to reconcile some of the empirical evidence on the use of money and credit in retail transactions.

²¹As mentioned previously, without assuming an upper bound on the distribution of sellers' record-keeping costs, it is possible to have a pure credit equilibrium with $\Lambda < 1$, in which case $\bar{b} \in (\bar{b}_0, \bar{b}_1)$ and total surplus is $S(\bar{b}) < S^*$.

Understanding the substitution between cash and other payment instruments such as credit cards is a vital policy concern as this mechanism is crucial for policymakers when developing regulation on surcharges and interchange fees to implement the socially optimal payment system. As it is the sole issuer of bank notes, central banks also need to understand substitution patterns to predict cash demand.

7.1 Entrenchment of Cash

Despite the increased availability of credit cards, many studies find that cash still remains the dominant means of payment, particularly for small transactions. This pattern has been documented in the United States by Klee (2008), Canada by Arango, Huynh, and Sabetti (2011), Netherlands by Bolt, Jonker, and van Renselaar (2011), and Australia by Simon, Smith, and West (2011). In our model, cash both saves society on record-keeping costs and allows agents to engage in anonymous transactions. Hence in low inflation economies, agents may prefer to use cash for small transactions in order to avoid the fixed costs associated with credit. In addition, since cash transactions are anonymous, consumers may favor money over credit even in legal transactions, such as for vice goods like cigarettes and alcohol.²²

While consumers are adopting new instruments such as credit cards and electronic payments, they are not necessarily discarding older ones such as cash (Schuh (2012)). Even with falling costs in electronic record-keeping, our model predicts that agents may still coordinate on using cash due to a hold-up externality in technological adoption. Since retailers do not receive the full surplus associated with technological adoption, they fail to internalize the total benefit of adopting credit. Consequently, there may be inertia in the adoption of new forms of payment.

This can explain why some merchants have been slow to adopt new technologies for accepting credit. For example, Gerdes (2008) reports that the vast majority of card payments made within the United States are still being made using magnetic stripe technology even though advanced chip-based technology on “smart cards” are available. Adoption of new technologies remains limited because merchants have not extensively adopted terminals that can read them. As we discuss more below, policymakers must therefore remember to take into account this sluggish response when designing policies or regulations geared towards merchant behavior.

²²Gomis-Porqueras, Peralta-Alva, and Waller (2011) formalize this idea in a model with money, credit, and illegal activity to quantify the size of the informal sector.

7.2 The Credit Crunch and Deflation

The recent recession from 2008 and 2009 in the United States provides a good recent example of our theory in practice. During this period, the economy went from an average inflation rate of 3.85% to deflation at -0.34% per year. Similarly, short-term interest rates available to consumers through the rate on one-month certificates of deposit, fell from about 5% before the recession to nearly zero. These changes in economic conditions also lead to changes in payment behavior. For the United States, Foster, Meijer, Chuh, and Zabek (2011) find that between 2008 and 2009, cash payments increased by 26.9%, cash holdings increased by 25.5%, while credit card payments decreased by 21.9%.²³

This shift in consumer behavior during the 2008–2009 recession can be explained by our theory. According to Proposition 1, the economy can fall in a credit crunch where all borrowing and lending ceases to exist as inflation falls. In particular, deflation completely crowds out credit and there is a flight towards a more liquid means of payment such as money. In turn, the debt limit is driven to zero and credit ceases to exist. As the evidence suggests, since both the use of cash and credit is indexed to inflation, agents treat the two as substitutes as economic conditions change. Similarly, Kahn, Senhadji, and Smith (2006) find that for countries with low inflation, increases in inflation decreases cash use and increases credit use.

7.3 Hysteresis and Coordination Failures

Under certain conditions, our theory predicts that two economies with similar technologies, constraints, and policies can still end up with very different payment systems. Indeed, Proposition 2 and Figure 6 show that multiple equilibria can arise even without any changes in fundamentals. To take one example, suppose there are two countries, A and B. In each country, inflation is initially very low and all agents coordinate on an equilibrium where only money is used. Now suppose that both countries experience a temporary period of high inflation. If inflation increases above some threshold, the pure monetary equilibrium ceases to exist and agents turn to using credit to avoid the inflation tax. Now suppose that in both countries, inflation goes back down to its initial low level. It is possible that agents in country A still coordinate on using credit while agents in country B go back to using only money. Since both outcomes are consistent with economic fundamentals, which equilibrium an economy ends up in will depend in large part on the beliefs and expectations of market participants.

²³Besides deflation, the authors also discuss other factors that may possibly explain this shift in consumer payments including changes in government regulations toward credit and debit cards, and changes in consumers' assessment about the security of electronic payments. Our model has a role for these factors as well, as we discuss below.

The reason why these coordination failures can arise is due to the two-sided nature of the payment system and the beliefs of market participants: buyers have to choose their portfolio and sellers have to choose which payment instrument to accept. As is evident from agents' upward-sloping reaction functions in Figure 7, what the seller accepts affects what the buyer holds and vice versa. Coordination failures such as the kind described above can therefore arise since the maximum the buyer can borrow and the measure of sellers that adopt the record-keeping technology are complements.²⁴

Consequently, economies with similar fundamentals can still end up with drastically different payment systems, some being better from society's perspective than others. This therefore provides rationale for policy interventions that help individuals coordinate on the better outcome. For instance, the normative analysis in Section 6 implies that a pure credit equilibrium dominates a pure monetary one in terms of social welfare if inflation is high enough or the record-keeping cost is low enough. In that case, policymakers may want to introduce mechanisms that help coordinate agents on using credit, such as information campaigns, advertisements, or even financial literacy programs.

7.4 Payment Regulations and Merchant Fees

Besides monetary policy intervention through changes in the nominal interest rate or inflation rate, government policies that affect the provision of credit cards through fees and regulations also affects consumer demand for different payment instruments. For instance, new restrictions were recently imposed on banks and payment card networks in the United States that lead to declines in credit card acceptance by merchants as well as decreases in credit usage by consumers. According to Foster, Meijer, Chuh, and Zabek (2011), businesses pay merchant discount fees on credit cards to their banks that amount to as much as 2% of total sales. Regulations that increased these fees decreased the number of merchants that choose to accept credit, and have lead some merchants to attempt at steering consumers toward lower cost payment instruments such as cash.²⁵

Our theory suggests that since policy affects both retailers and consumers jointly, regulations of this kind may have multiplier effects. For instance, suppose that policymakers pass a new law that decreases credit card fees for merchants. In the model, a fall in κ gives merchants stronger incentives to invest in the technology that allows for credit. At the same time, when more sellers accept credit, the gain for buyers from using and redeeming credit also increases. This relaxes the

²⁴Coordination failure frequently arise in economies with frictions, and show up in many different contexts such as goods markets and labor markets, as in Diamond (1982), Blanchard and Summer (1987), and discussed in Rocheteau and Tasci (2007).

²⁵Schuh and Stavins (2012) find that these costs significantly affect debit card use. Similarly, the recent increase in the cost of credit cards issued by some banks may reduce consumers' reliance on card payments for transactions.

borrowing constraint and permits buyers to use more credit relative to cash for transactions, which in turn amplifies the initial policy change. Through this feedback effect, policy changes such as the wave of new regulation imposed on payment card networks in 2008 and 2009 may end up inducing sizable shifts in consumer behavior.²⁶

7.5 Payment Systems and Market Structure

The U.S. payment card system differs from the rest of the world. There are two main characteristics of the U.S. payment system: the issuer market is competitive while the market power of retailer markets varies by industry. It is therefore important to understand how differences in market structure across countries affects the incentives of private agents. In fact, a key recent debate in the U.S. is how increasing competitiveness in the credit card industry changes households' incentives to use credit and retailers' decision to accept credit.²⁷

While our model admittedly abstracts from the issuer market, the framework can still shed light on how market structure in retail markets impact the adoption of payment instruments by both firms and consumers. In particular, the bargaining power parameter θ in this model is an important determinant for the adoption of credit. $(1 - \theta)$ can therefore be interpreted as the merchant's pricing power, or the degree of competitiveness the merchant faces, which is directly related to the price mark-up: having greater competition in goods markets implies $\theta \rightarrow 1$ and a mark-up close to 1. When the acceptability of credit is endogenous, the expected benefit of investing in costly record-keeping is a decreasing function of θ .²⁸ In the limit where $\theta \rightarrow 1$, sellers no longer have any pricing power and hence no longer invest. Then since by Lemma 4, $\bar{\kappa}$ is strictly increasing in \bar{b} , the debt limit is a decreasing function of θ : $\frac{d\bar{\kappa}}{d\theta} < 0$. Consequently, increased competitiveness in the retailer market (an increase in θ) reduces the equilibrium measure of sellers that invest and hence the fraction of credit transactions.

This could explain why a country like the U.S., which has relatively more competitive goods markets than the Euro Area, adopt electronic payments at a smaller scale than retailers in Europe have. For example, the December 10, 2012 edition of *Time* magazine notes that the U.S. currently lags behind Europe in credit card technology: "Chip-and-PIN is the standard everywhere but here. Now we're sub-standard— just like our cell-phone network." Consequently, our theory predicts that

²⁶For instance, the Credit Cardholder's Bill of Rights and Regulation AA were introduced in 2008 to protect consumers from unexpected rate increases on pre-existing credit card balances and limit the fees that reduce the availability of credit. See Foster, Meijer, Chuh, and Zabek (2011) for more discussion on these legislative reforms.

²⁷Since the 1980s, credit card associations such as Visa and Mastercard have been able to control the use of and access to their networks to the advantage of their bank members. Recently however, Akers, Golter, Lamm, and Sol (2005) report that the credit card industry has become increasingly competitive.

²⁸To verify, derive $\bar{\kappa} = \sigma(1 - \theta)[S(z + \bar{b}) - S(z)]$ to obtain $\frac{d\bar{\kappa}}{d\theta} = -\sigma[S(z + \bar{b}) - S(z)] < 0$.

economies like Europe with less concentrated markets are more likely to adopt new electronic payments than countries where the retail market is more competitive, such as the United States.²⁹

7.6 Payment Innovations

In many countries, there are now new mobile payments in the form of text or SMS messages that authorizes a non-bank third party, such as a cellular phone carrier, to make a payment for the consumer. Since this new type of mobile payment may involve short-term credit extended by a cellular carrier or another third party, it serves the same role as credit cards that allow consumers to “buy now and pay later” (Foster, Meijer, Chuh, and Zabek (2011)).

Perhaps the largest potential gain from these payment innovations lies in emerging markets, where the lack of financial infrastructure makes mobile payments especially appealing (The Economist, 2012). For example, phone-based payments such as M-Pesa currency serve Kenya and several other markets. Large credit card companies such as Visa and Mastercard have set up joint ventures with many payment services in emerging countries such as Fundamo, which specialises in payment services for the un-banked and under-banked, and Telefonica, which aims to boost mobile payments across Latin America. The results from our paper suggests that in order for these innovations to be successful, there should be more information campaigns that promote their use in order to successfully coordinate agents on making the substitution away from cash.

8 Conclusion

As many economies now feature new forms of payment such as credit cards, smart cards, and electronic money, it is increasingly important for policymakers to understand how consumers substitute between cash and competing media of exchange. For that purpose, we investigate the choice of payment instruments in a simple model featuring costly record-keeping and limited commitment. Inflation triggers agents to substitute from money to credit for two reasons: a higher inflation rate both lowers the rate of return on money and makes default more costly, which relaxes agents’ borrowing limits. Consequently, the optimal inflation rate can be strictly positive in economies with both money and credit and hence provides rationale for why the Friedman rule is rarely implemented in practice. The normative implications of our theory are therefore consistent with Antinolfi, Azariadis, and Bullard (2009) and the inflation targets specified by many of the world’s central banks.

²⁹Bayoumi, Laxton, and Presenti (2004) estimate an average price mark-up of 1.30 for the Euro Area and 1.16 for the U.S., which reflects a more competitive goods market in the latter. Stavins (2003) presents a similar market structure argument regarding the market for electronic check products.

Our model also highlights a strategic complementarity between consumers' credit limit and retailers' decision to invest. Multiple equilibria and coordination failures can therefore arise due to the two-sided market nature of payment systems. This poses new challenges for policymakers and we discuss some mechanisms such as information campaigns and financial literacy programs that can help coordinate society on the socially preferred outcome. Our theory also suggests that future research should include more empirical studies on consumer preferences and merchant adoption to help policymakers enact the most beneficial regulations, laws, and educational programs that protect and support both consumer's and firm's payment choices.

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Appendix

Proof of Lemma 1

First, we show that $\Omega(0) = 0$. If $\bar{b} = 0$, it must be that $z = \tilde{z}$ since

$$-iz + \sigma\theta S(z) = -i\tilde{z} + \sigma\theta S(\tilde{z}).$$

As a result, $\Omega(0) = 0$. Next, to verify that $\Omega(\bar{b})$ is increasing in \bar{b} , differentiate $\Omega(\bar{b})$ with respect to \bar{b} to obtain

$$\begin{aligned} \frac{\partial\Omega(\bar{b})}{\partial\bar{b}} &= \sigma\theta\Lambda S'(z + \bar{b}) \\ &= \sigma\theta\Lambda \frac{u'(q^c) - c'(q^c)}{(1 - \theta)u'(q^c) + \theta c'(q^c)} > 0, \end{aligned}$$

where $q^c \equiv q(z + \bar{b})$ is output traded when the seller accepts both money and credit and $z > 0$ satisfies

$$i = \sigma\theta [(1 - \Lambda)S'(z) + \Lambda S'(z + \bar{b})].$$

The slope of $\Omega(\bar{b})$ with respect to \bar{b} is strictly positive for all $\bar{b} < (1 - \theta)u(q^*) + \theta c(q^*)$ and becomes zero when $\bar{b} > (1 - \theta)u(q^*) + \theta c(q^*)$. Further, the slope of $\Omega(\bar{b})$ at $\bar{b} = 0$ is

$$\begin{aligned} \left. \frac{\partial\Omega(\bar{b})}{\partial\bar{b}} \right|_{\bar{b}=0} &= \sigma\theta\Lambda S'(z) \\ &= i\Lambda \geq 0. \end{aligned}$$

A necessary and sufficient condition for an equilibrium with a positive debt limit is that the slope of $r\bar{b}$ from (16) is less than the slope of $\Omega(\bar{b})$ at $\bar{b} = 0$. Consequently an equilibrium with $\bar{b} > 0$ will exist so long as $r < \Omega'(\bar{b}) = i\Lambda$.

Next, $\Omega(\bar{b})$ is concave since total surplus $S(z + \bar{b})$ as a function of the buyer's liquid wealth, $z + \bar{b}$, is concave and strictly concave if $z + \bar{b} < \theta c(q^*) + (1 - \theta)u(q^*)$. Consequently,

$$\begin{aligned} \frac{\partial^2\Omega(\bar{b})}{\partial\bar{b}^2} &= \frac{\partial[\sigma\theta\Lambda S'(z + \bar{b})]}{\partial\bar{b}} \\ &= \sigma\theta\Lambda S''(z + \bar{b}) \leq 0, \end{aligned}$$

since $S''(z + \bar{b}) \leq 0$. Finally, $\Omega(\bar{b})$ is continuous for all $z > 0$, and for $i > 0$, the solution to (14) and (15) must lie in the interval $[0, z^*]$. Then from the Theorem of the Maximum, there exists a

solution to (14) and (15), and consequently (16). \square

Proof of Lemma 3

First we differentiate (16) to obtain

$$r d\bar{b} = \sigma\theta [-S(z)d\Lambda + S(z + \bar{b})d\Lambda + \Lambda S'(z + \bar{b})d\bar{b}],$$

$$\frac{d\bar{b}}{d\Lambda} = \frac{\sigma\theta [S(z + \bar{b}) - S(z)]}{r - \sigma\theta\Lambda S'(z + \bar{b})} > 0,$$

since $S(z + \bar{b}) > S(z)$ and $r > \sigma\theta\Lambda S'(\bar{b}_0)$. Next, we verify that \bar{b} is a convex function of Λ :

$$\frac{d^2\bar{b}}{d\Lambda^2} = \frac{-(\sigma\theta)^2 [S(z + \bar{b}) - S(z)] S''(z + \bar{b})}{[r - \sigma\theta\Lambda S'(z + \bar{b})]^2} > 0$$

since $S''(z + \bar{b}) \leq 0$. Therefore, $\left. \frac{d^2\bar{b}}{d\Lambda^2} \right|_{\bar{b} < \bar{b}_0} < \left. \frac{d^2\bar{b}}{d\Lambda^2} \right|_{\bar{b}_0}$. \square

Proof of Lemma 4

We first establish sellers' investment decisions at $\bar{b} = \bar{b}_0$ where money is not valued. For a given $\bar{b} = \bar{b}_0$, the fraction of sellers who invest in the record-keeping technology depends on the expected benefit of accepting credit when money is not used in trade by buyers:

$$\bar{\kappa}_0 \equiv \sigma(1 - \theta)S(\bar{b}_0).$$

Clearly, a seller will invest so long as the expected benefit of doing so outweighs the cost: $\bar{\kappa}_0 > \kappa$. Assuming a uniform distribution of κ over $[0, \bar{\kappa}_{\max}]$, the aggregate measure of sellers who invest at \bar{b}_0 is given by $\Lambda_s = \frac{S(\bar{b}_0)}{S^*}$, where $S^* \equiv S(\bar{b}_1)$ is the maximum surplus obtained when $q = q^*$.

For all $\bar{b} \in (\bar{b}_0, \bar{b}_1)$, the expected benefit of investing is

$$\bar{\kappa} = \sigma(1 - \theta)S(\bar{b})$$

where $\bar{\kappa} > \bar{\kappa}_0$. Then since Λ is increasing in $\bar{\kappa}$, it follows that $\Lambda > \Lambda_s$. Moreover, the expected benefit of investing is a strictly increasing function of the debt limit, \bar{b} : $\left. \frac{d\bar{\kappa}}{d\bar{b}} \right|_{\bar{b} > \bar{b}_0} > 0$ and $\left. \frac{d^2\bar{\kappa}}{d\bar{b}^2} \right|_{\bar{b} > \bar{b}_0} < 0$. To verify, differentiate $\bar{\kappa}$ to obtain

$$\left. \frac{d\bar{\kappa}}{d\bar{b}} \right|_{\bar{b} > \bar{b}_0} = \sigma(1 - \theta)S'(\bar{b}) > 0$$

since $S'(\bar{b}) > 0$. Then differentiating again, it is easy to see that

$$\left. \frac{d^2 \bar{\kappa}}{d\bar{b}^2} \right|_{\bar{b} > \bar{b}_0} = \sigma(1 - \theta)S''(\bar{b}) \leq 0$$

since $S'(\bar{b}) \leq 0$. Next, for all $\bar{b} \in (0, \bar{b}_0)$, the expected benefit of investing is

$$\bar{\kappa} = \sigma(1 - \theta)[S(z + \bar{b}) - S(z)].$$

Consequently, $\bar{\kappa} < \bar{\kappa}_0$ and $\Lambda < \Lambda_s$. In this region, the expected benefit of investing is also an increasing function of the debt limit, \bar{b} . To verify, we differentiate to obtain

$$\left. \frac{d\bar{\kappa}}{d\bar{b}} \right|_{\bar{b} < \bar{b}_0} = \sigma(1 - \theta) \left\{ S'(z + \bar{b}) + [S'(z + \bar{b}) - S'(z)] \frac{dz}{d\bar{b}} \right\}.$$

Then using the first order condition (14),

$$i = \sigma\theta [(1 - \Lambda)S'(z) + \Lambda S'(z + \bar{b})],$$

we obtain

$$\frac{dz}{d\bar{b}} = -\frac{1}{1 + \frac{(1-\Lambda)S''(z)}{\Lambda S''(z+\bar{b})}} < 0$$

since $S''(z) \leq 0$ and $S''(z + \bar{b}) \leq 0$. Consequently, $\left. \frac{d\bar{\kappa}}{d\bar{b}} \right|_{\bar{b} < \bar{b}_0} > 0$.³⁰ \square

³⁰We do not determine $\left. \frac{d^2 \bar{\kappa}}{d\bar{b}^2} \right|_{\bar{b} < \bar{b}_0}$ when $\bar{b} < \bar{b}_0$ since this will not affect our results and will depend on third derivative $S'''(z + \bar{b})$, which we do not know the sign of.