Vertical Integration and Innovation

Xingyi LIU*

February 22, 2013

Abstract

We studied the effect of vertical integration on investment incentives and social welfare when both upstream and downstream firms make innovative investments. When only upstream investment is essential for the final product, vertical integration softens upstream competition and partially forecloses the independent upstream firm. The integrated upstream firm has excessive investment incentive and social welfare is lower under vertical integration. When downstream investment is also essential, upstream investment incentive is insufficient. Vertical integration enhances both upstream and downstream investment incentives but also strengthens the foreclosure effect. The net effect on social welfare is positive with quadratic cost function.

Key Words: Vertical Integration, Innovation, Complementarity

JEL Classification: D4, L1, L4

1 Introduction

In a number of vertically related industries, innovative investments take place at both upstream and downstream levels. Take smartphone for an example, upstream market consists of those firms that develop technology; while downstream market includes those who design and manufacture cellphones. Vertical integration is a common practice in this industry. However, there are also independent upstream developers and independent downstream manufacturers. Our purpose in this paper is to study the impact of vertical integration both on investment incentives and on social welfare.

We present a model with duopoly in both upstream and downstream markets. Our first result concerns the case when only upstream innovation is essential for the industry, i.e. downstream firms do not make investment decisions but they are the necessary channels to final consumers. In this situation, partial integration between one upstream firm and one downstream firm softens upstream competition. This is because in case that both upstream firms make successful investments, they only compete for the independent downstream firm but not for the integrated downstream firm. This leaves the integrated upstream firm positive profit

*Toulouse School of Economics, GREMAQ, 21, Allee de Brienne, 31000 Toulouse, cristinliuxy@gmail.com

when upstream firms compete a-la-Bertrand to supply downstream firms, but the independent upstream firm only gets zero profit. This upstream competition soften effect gives the integrated upstream stronger investment incentive, and crowds out the investment of the independent upstream firm. Thus vertical integration has certain foreclosure effect. However, improved investment incentive of the integrated upstream firm is not socially beneficial, instead it is excessive. The intuition is simple: given that one upstream firm has already made a successful investment, there is no social value from the investment of a second upstream firm. Nevertheless, the private gain is positive for an integrated upstream firm since she still gets positive profit from the downstream affiliate. And therefore, when only upstream innovation matters, vertical integration leads to overall over-investment and hurts social welfare.

Downstream investment changes this scenario in two ways. First, downstream competition is not always present, and thus the upstream firm cannot catch the whole profit of downstream market even when she is the only upstream innovator, since downstream innovation is also necessary for the final product. Second, when upstream firms and downstream firms bargain after the realization of the outcomes of investments, we encounter the classical hold-up problem. Under vertical integration, the upstream competition soften effect still exists, which leads to higher investment from the integrated upstream firm. However, this improved incentive may not be excessive, instead it may be welfare enhancing. This is because under vertical separation, upstream firms under-invests compared to social optimum since their private gains are lower than that of the social planner. Moreover, vertical integration solves the hold-up problem inside the integrated entity, this has two effects. First, the integrated downstream firm has higher incentive to invest and then crowds out the investment of the independent downstream firm. Second, when there is only one successful upstream firm, the profit for an integrated upstream monopolist is higher than that of a separated one, which further increases the investment incentive for an integrated upstream firm. In this sense, downstream investment amplifies the upstream foreclosure effect of vertical integration. With quadratic cost function, we show that vertical integration actually increases social welfare.

Our paper contributes to the literature on the foreclosure effect of vertical integration, which dates back to the seminal paper of Ordover, Saloner and Salop (1990). With a focus on the role of investments, most of previous works have only upstream firms invest or only downstream firms invest. Bolton and Whinston (1993) studied the effect of vertical integration on downstream investments. In their model, whether there is downstream competition ex post exogenously depends on whether there are one or two units of input available. On the contrary, competition is endogenously determined by downstream investment in our model, and vertical integration softens downstream competition. In addition, we also study how downstream investments affect upstream incentives to invest.

Chen and Sappington (2010) studied upstream investment incentive with a monopolistic upstream firm, they show that vertical integration generally enhances upstream innovation under downstream Cournot competition but may diminish upstream innovation with downstream Bertrand competition. Since there is no upstream competition, upstream foreclosure does not exist there. Our paper is a first attempt to the problem with both upstream and downstream investments.

Our paper is also related to some literatures discussing the effect of integration on innovation of complementary products. Farrell and Katz (2000) shows that integration into a complementary product market allows a monopolist to extract more rent from the market where he dominates. Schmidt (2009) studies how vertical integration of an patent holder affect the contractual terms between upstream patent holders and downstream producers. In these papers, the complementarity is between horizontally related products. In our paper, such complementarity is vertical.

The paper proceeds as follows: We present the basic setup in Section 2; Section 3 studies the case when only upstream innovation is essential for the final product; Essential downstream innovation is studied in Section 4; Section 5 presents the results with both upstream and downstream innovation; Section 6 provides some extension and discussion; Section 7 concludes. All proofs are shown in the Appendix.

2 Setup

Players and Market Interaction The industry consists of an upstream market and a downstream market. There are two upstream firms U_1 and U_2, and they compete a-la-Bertrand to supply input for two downstream firms D_1 and D_2. Each $D_j, j = 1, 2$ demands only one unit of the input. We assume that once D_j fails to trade with both $U_i, i = 1, 2$, he has no alternative source for the input. Similarly, if U_i fails to trade with both downstream firms, she has no other ways to access the final consumer market.

Technology The value of the final product depends on the quality of the input and also on the downstream development. Each upstream firm U_i can make investment in innovation, which in case of success enables her to produce an input with positive quality q; otherwise, U_i has no successful innovation and is out of the market.

We model the upstream innovation in the following way: for a given level of investment e_i, U_i succeeds in innovation with probability e_i and fails with the complementary probability $1-e_i$. The cost of investment is the same for both upstream firms, which is given by $C_U(e)$. We assume that $C_U(e)$ is increasing and convex in e. We assume that there is no marginal cost of production. Thus, the total cost for an upstream firm is the fixed cost of investment. Furthermore, there is no capacity constraint or any other shocks that may constrain the production of U_i, and each U_i can supply both downstream firms if she is willing to.

Each downstream firm D_j also makes investment in innovation, which in case of success allows him to transforms input to final product on a one-to-one basis at zero cost. As the upstream market, we model downstream investment as follows: each downstream firm D_j makes an investment d_j which succeeds with probability d_j and fails with probability $1 - d_j$. The cost of investment is $C_D(d)$, which is convex and increasing.

Payoffs For each downstream firm D_j, if he is the only active one in the downstream market, he can extract all the benefit from final consumers, which we denote by Δ; if both downstream firms are active, each D_j can only extract δ from consumers. We assume that $0 < 2\delta < \Delta$, so that competition dissipates part of the profit but not all of it. Thus the payoff for downstream firms is described in the following table,

Table 1:

D_1 D_2	A	NA
A	δ,δ	Δ,0
NA	0,Δ	0,0

where "A"and "NA"indicate whether D_j is active or not active in the downstream market. Therefore, if U_i is the only upstream innovator, the industry profit is maximized when she only sells to one of the two downstream firms. We further make the following assumptions on the cost function of upstream and downstream investments,

Assumption 1. $C'_U(1) > \Delta$; *and* $C'_U(e) > 0$, $C''_U(e) > \Delta$ *for* $e \in [0, 1]$.

Assumption 2. $C'_D(1) > \Delta$; *and* $C'_D(d) > 0$, $C''_D(d) > \Delta$ *for* $d \in [0, 1]$.

The above assumptions guarantee that the profit functions are well-defined and the solutions are interior. In the special case of quadratic cost function $C_U(e) = \frac{1}{2}c_u e^2$ and $C_D(d) = \frac{1}{2}c_d d^2$, the above assumptions amount to say $c_u > \Delta$ and $c_d > \Delta$.

3 Essential Upstream Innovation

To distinguish the main forces at work, we start with the case when only upstream innovation is needed for the final product. However, the upstream firms do not have direct access to the final consumer market. Instead, downstream firms sell directly to final consumers. That is to say downstream firms do not directly contribute to the value of the final product, they specify in developing channels to final consumers. The question we try to ask is how vertical integration of upstream firm and downstream firm affects the investment incentives and social welfare.

3.1 A Simple Benchmark

As a simple benchmark, we look into the situation when there is a monopolistic upstream firm U. The game is as follows:

- *Investment Stage*: U makes investment decision e; if she fails, the game ends;
- *Bargaining Stage*: If U succeeds, she makes a public take-it-or-leave-it offer to each of the downstream firms D_i, which requires a payment from D_i to U; Each D_i decides whether to accept or reject the offer of U; if no downstream firm accepts the offer, the game ends;
- *Payoff Stage*: If any of the two downstream firms accept the offer, payment is made and input is delivered; Downstream market payoff realizes and game ends.

We assume that when downstream firm D_i is indifferent between accepting and rejecting the offer, he chooses to accept. It is obvious that when U is successful in innovation, she would make offers to downstream firms such that only one of them would accept. Since U has the right to make an offer, she would extract all downstream market profit Δ. Introducing bargaining between upstream monopolist and downstream firms does not change this result, since competition between the two downstream buyers would drive the input price up to Δ.

The problem for U is then to choose an e^* such that

$$e^* = argmax_e\{e\Delta - C_U(e)\}$$

which gives us $e^* = C_U'^{-1}(\Delta)$.

In case of quadratic cost function, we have

$$e^M = \frac{\Delta}{c_u}$$

and the corresponding social welfare is

$$W^M = e^M\Delta - C_U(e^M) = \frac{\Delta^2}{2c_u}$$

3.2 Vertical Separation

Now we move on to the case with upstream competition. We first analyze the situation when all firms remain separated. The market unfolds as follows:

- *Investment Stage*: Each U_i makes investment decision e_i; if both firms fail, the game ends;
- *Bargaining Stage*: If only one firm U_i succeeds, she acts as an upstream monopolist; if both upstream firms succeed, they make simultaneous public offers to both downstream firms; Each D_j decides whether to accept or reject the offers, and which one to accept in case of acceptance; if both downstream firms reject the offers, the game ends;
- *Payoff Stage*: If any of the two downstream firms accept any offer, the payment is made and input is delivered; Downstream market payoff realizes and game ends.

If only one upstream has successful innovation, she acts as a monopolist in the benchmark model above, i.e. she gets the whole downstream market profit Δ. When both upstream firms obtain successful innovation, they compete to sell to downstream firms a-la-Bertrand. Then it is clear that no upstream firm can do better than making an offer of zero price for one unit of input to each downstream firm D_j. The reason is straightforward, if any U_i can make an offer with positive price p_{ij} to D_j and D_j accepts the offer in equilibrium, then $U_{i'}, i' \neq i$ can make an offer with slightly lower price $p_{i'j} = p_{ij} - \epsilon$ such that D_j would accept the offer by $U_{i'}$, and then both $U_{i'}$ and D_j are better off. Therefore, the payoff for upstream firms U_i can be summarized in the following matrix,

where "S"and "F"indicate whether U_i succeeds or fails in investment. Then for each upstream firm U_i, the problem is to choose an effort level to maximize expected profit

$$e_i^* = argmax_{e_i}\{e_i(1 - e_{i'})\Delta - C_U(e_i)\}, i' \neq i$$

Table 2:

U_1 U_2	S	F
S	0,0	Δ,0
F	0,Δ	0,0

which gives us the best response of U_i, given the investment level of $U_{i'}$

$$e_i^* = {C_U'}^{-1}((1-e_{i'})\Delta), i' \neq i$$

It is easy to see that $\frac{\partial e_i^*}{\partial e_{i'}} = -\frac{\Delta}{C_U''(e_i^*)} < 0$, and thus the investments of the two upstream firms are strategic substitutes. As $U_{i'}$ increases investment, it is more likely that U_i cannot recoup benefit of his innovation and then U_i has less incentive to invest.

Moreover, from Assumption 1, it is clear that $-1 < \frac{\partial e_i^*}{\partial e_{i'}} < 0$ and then there exists a unique equilibrium in the investment game e_{VS}^*, which is given by the solution of the equation

$$e_{VS}^* \text{ solves } C_U'(e) + \Delta e = \Delta$$

With quadratic cost function, we have

$$e_{VS}^* = \frac{\Delta}{c_u + \Delta}$$

which is increasing in the benefit of innovation Δ and decreasing in the cost of innovation c_u. And the corresponding social welfare is given by

$$\begin{aligned} W^{VS} &= [1-(1-e_{VS}^*)^2]\Delta - \tfrac{1}{2}c_u {e_{VS}^*}^2 \\ &= \tfrac{\Delta^2}{c_u+\Delta} \end{aligned}$$

3.3 Vertical Integration

Suppose now one upstream firm and one downstream firm integrate, without loss of generality, we assume that U_1 and D_1 now integrate, and U_2 and D_2 remain separated. The time-line is as follows,

- *Investment Stage*: Each U_i makes investment decision e_i; if both firms fail, the game ends;
- *Bargaining Stage*: If only U_1 succeeds, she supplies the input to her own downstream affiliate; if only U_2 succeeds, she makes public offers to $D_j, j = 1, 2$; if both upstream firms succeed, they make simultaneous public offers to D_2; Each D_i decides whether to accept or reject the offers, and which one to accept in case of acceptance; if both downstream firms reject the offers, the game ends;
- *Payoff Stage*: If any of the two downstream firms accept any offer, the payment is made and input is delivered; Downstream market payoff realizes and game ends.

Compared to the case of vertical separation, a few remarks need to be made. First, if the integrated entity $U_1 - D_1$ was the only successful firm in the upstream market, U_1 is an upstream monopolist and can extract all downstream market profit Δ. We assume that in this case U_1 would only supply D_1 and D_2 is excluded in the downstream market.

Second, if the independent upstream firm U_2 is the only one who has successful upstream innovation, she acts as a monopolist in the benchmark model. We do not model any information leakage, which would give the independent downstream firm D_2 certain bargaining power. For example, U_2 may face the risk that if she supplies inputs to D_1, D_1 may leak some key information to U_1, and then U_1 would also have successful innovation and compete with U_2. This would make D_1 as an inferior downstream firm in the perspective of U_2, which grants D_2 a stronger bargaining position. This type of information leakage problem has been studied in a few other papers such as Allain et al(2011) and Chen(2011). Our focus here is not on how vertical integration affects the information flows in the industry, hence we assume away any information problem. When U_2 is the sole upstream innovator, she is still able to extract the whole downstream market profit Δ due to downstream competition.

Third, when both upstream firms succeed in investment, they only compete to supply the independent downstream firm D_2 while D_1 is out of the reach of U_2. Therefore, upstream competition is softened compared to vertical separation. We term this effect as the "*outlet effect*". This is the key force in play when only upstream innovation is essential for the final product. Upstream competition drives the input price for D_2 down to zero, and thus the profit for U_2 is zero while U_1 can still catch part of downstream market profit, which is δ since now both D_1 and D_2 are active in the downstream market.

In sum, the payoff matrix for $U_1 - D_1$ and U_2 is given by

Table 3:

$U_1 - D_1$ U_2	S	F
S	δ,0	Δ,0
F	0,Δ	0,0

Then the upstream firms choose e^1_{VI} and e^2_{VI} such that

$$e^1_{VI} = argmax_{e_1}\{e_1(1-e_2)\Delta + e_1 e_2 \delta - C_U(e_1)\}$$

and

$$e^2_{VI} = argmax_{e_2}\{e_2(1-e_1)\Delta - C_U(e_2)\}$$

Under Assumption 1, it is easy to see that $\frac{\partial e^1_{VI}}{\partial e_2} = -\frac{\Delta-\delta}{C''_U(e^1_{VI})} \in (-1,0)$, and $\frac{\partial e^2_{VI}}{\partial e_1} = -\frac{\Delta}{C''_U(e^2_{VI})} \in (-1,0)$. Hence the upstream investment game under vertical integration has a unique solution which solves the two best response functions,

$$\begin{cases} C'_U(e^1_{VI}) & = \Delta(1-e^2_{VI}) + \delta e^2_{VI} \\ C'_U(e^2_{VI}) & = \Delta(1-e^1_{VI}) \end{cases} \tag{1}$$

Compared to the situation under vertical separation, we have the following proposition,

Proposition 1. *The integrated upstream firm invests more than the independent upstream firm. Indeed, we have* $e^1_{VI} > e^*_{VS} > e^2_{VI}$.

Proof. See Appendix A.1. □

The proposition is shown in Figure 1. The equilibrium investment is determined by the intersection of the two best response curves $BR^1(e_2)$ and $BR^2(e_1)$. The best response function for the independent upstream firm U_2 is not affected by integration. However, integration of U_1 and D_1 leads to a clockwise rotation of the best response function of U_1, which clearly shows that the integrated upstream firm U_1 invests more than under vertical separation while the independent upstream firm U_2 invests less. With quadratic cost function, the upstream investments are given by

$$e^1_{VI} = \frac{\Delta[c_u - (\Delta - \delta)]}{c_u^2 - \Delta(\Delta - \delta)} \text{ and } e^2_{VI} = \frac{\Delta[c_u - \Delta]}{c_u^2 - \Delta(\Delta - \delta)}$$

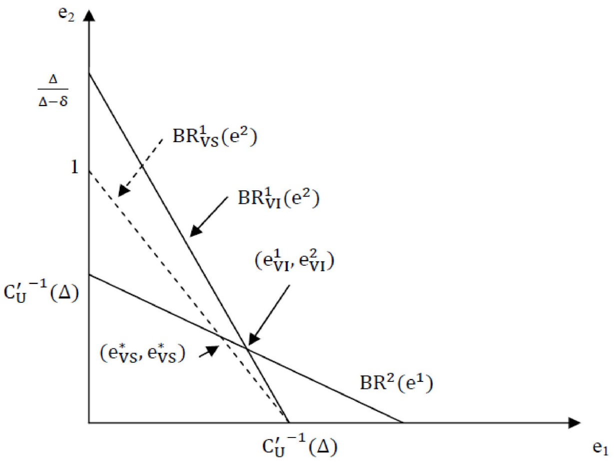

Figure 1: Equilibrium Upstream Investment under Vertical Separation and Vertical Integration

Proposition 1 shows that vertical integration has some foreclosure effect on the independent upstream firm. This originates from the outlet effect: under vertical integration, when both upstream firms make successful investment, they only compete for the independent downstream firm; and thus the integrated upstream firm is still able to recoup part of her investment, which increases her incentive to invest. Since upstream investments are strategic substitutes, the independent upstream firm has to invest less because there is a larger chance that the integrated firm would have successful innovation.

The asymmetric level of investment resulting from vertical integration has two effects on welfare: first, the total cost of investments increases as the two upstream firms become more asymmetric. The total cost $\frac{1}{2}c_u e_1^2 + \frac{1}{2}c_u e_2^2$ is convex, and thus for a given level of total investment the total cost is maximized when two upstream firms invest equal amount. And hence vertical integration would reduce social welfare. Second, the probability that the society would have successful innovation increases as the two upstream firms become more asymmetric. The probability that no innovation would happen, which is $(1 - e_1)(1 - e_2)$, attains maximum when $e_1 = e_2$ for a given level of total investment. Therefore, vertical integration would increase social welfare. However, it can be easily shown that when only upstream innovation matters for the final product, vertical integration leads to lower social welfare than vertical separation.

Actually, the upstream investment corresponds to social optimum under vertical separation. The social welfare is given by $W = (1-(1-e_1)(1-e_2))\Delta - \frac{1}{2}c_u e_1^2 - \frac{1}{2}c_u e_2^2$. Thus the social optimal investment is the solution to the following equation,

$$\begin{cases} C_U'(e^1) & = \Delta(1-e^2) \\ C_U'(e^2) & = \Delta(1-e^1) \end{cases}$$

which corresponds to the equilibrium condition under vertical separation. Thus, social welfare attains maximum under separation, and vertical integration necessarily reduce welfare. This is due to the fact that: the social value of a successful investment from a second upstream firm is only positive when she is the only one who succeeds in upstream innovation, which corresponds to the private incentive of upstream firms when they are vertically separated. Vertical integration leads to excessive investment of the integrated upstream firm; however, since the independent upstream firm responses by cutting investment on a less than one-to-one basis (the slope of the best response curve is between -1 and 0), vertical integration leads to overall over-investment and therefore reduce social welfare.

Proposition 2. *When only upstream innovation is essential, social welfare is lower under vertical integration than vertical separation.*

4 Essential Downstream Innovation

Now we turn to the case when only downstream innovation is essential for the final product, i.e. the input is sort of generic product which cannot be used directly by the final consumers. Downstream firms need to make investments which enables them to transform the input into final product in case of success. As before, we start with the benchmark case when the upstream market is monopolized by one firm U.

4.1 Monopolistic Upstream Market

4.1.1 Vertical Separation

When all firms are separated, the time-line is as follows,

- *Investment Stage*: Each D_j makes investment decision d_j; if both firms fail, the game ends;
- *Bargaining Stage*: If only one downstream firm D_j succeeds, he bilaterally bargains with U over the price of the input; if both downstream firms succeed, they simultaneously bargain with the upstream firm U; If no agreement is reached, the game ends;
- *Payoff Stage*: If any agreement is reached, payment is made and input is delivered; Downstream market payoff realizes and game ends.

We make two remarks here. First, the downstream investment is not contractible. The upstream monopolist cannot write a contract with downstream firms before any downstream investment happens. The payoff to each firm is realized through ex post bargaining, which we follow the classic property right literatures. However, the outcome of downstream investment is observable to all firms.

Second, in the previous section, when only upstream innovation is essential for the final product, downstream competition is always present. Therefore, as long as the upstream monopolist makes successful innovation, she is able to extract the whole downstream market profit. When downstream innovation is needed for the final product, downstream competition is not guaranteed. When only one downstream firm obtains successful innovation, there is in fact an upstream monopolist and a downstream monopolist. We assume in this case that they enter a bilateral bargaining. The solution we use is Nash Bargaining with equally split bargaining power. Hence, when the market is characterized by successive monopolist, the upstream firm and the downstream firm each gets $\frac{\Delta}{2}$. However, when both downstream firms succeed, downstream competition is again present, and the upstream monopolist is able to catch the whole downstream market profit Δ and each downstream ends with zero profit. There is a major difference between our model and the model of Bolton and Whinston(1993): in their model, downstream competition is exogenously determined by whether there are one or two units of input available; however, in the present model, ex ante downstream competition always exists and ex post downstream competition is endogenously determined by the investment of each downstream firm.

The payoff matrix for the two downstream firms can be summarized as

Table 4:

D_1 D_2	S	F
S	0,0	$\frac{\Delta}{2}$,0
F	0,$\frac{\Delta}{2}$	0,0

Then for each downstream firm, they choose d_j^* such that

$$d_j^* = argmax_{d_j}\{d_j(1-d_{j'})\frac{\Delta}{2} - C_D(d_j)\}, j' \neq j$$

The same reasoning as the previous section shows that under Assumption 2, there exists a unique equilibrium in the investment stage ($d_1 = d_2 = d_{VS}^M$) which is given by the solution to the following equation

$$C_D'(d_{VS}^M) = (1-d_{VS}^M)\frac{\Delta}{2}$$

In the quadratic cost case, the downstream investment is given by

$$d_{VS}^M = \frac{\Delta}{2c_d+\Delta}$$

and the social welfare is

$$\begin{aligned} W_{VS}^M &= [1-(1-d_{VS}^M)^2]\Delta - c_d {d_{VS}^M}^2 \\ &= (\tfrac{\Delta}{2c_d+\Delta})^2(\Delta+3c_d) \end{aligned}$$

4.1.2 Vertical Integration

Now suppose that the upstream monopolist U integrates with one of the two downstream firms, without loss of generality, assume U and D_1 integrates. The game is basically the same as vertical separation except in the bargaining stage. First, when the independent downstream firm D_2 is the only one who succeeds in downstream innovation, he bilaterally bargains with U as before. Nash bargaining gives each party half of the downstream market profit $\frac{\Delta}{2}$; Second, when D_1 is the sole innovator in downstream market, the integrated entity $U - D_1$ retains the whole downstream market profit Δ. And thus D_1 is able to get the whole benefit of his investment. As in the classic literature of property rights, vertical integration *solves the hold-up problem.* Third, when both downstream firms succeed, there is no actual downstream competition. The upstream monopolist only supplies her downstream affiliates. Therefore, vertical integration has this *downstream competition soften effects.*

Then under vertical integration, the problem for $U - D_1$ and D_2 is to choose d_1^* and d_2^* such that

$$d_1^* = argmax_{d_1}\{d_1\Delta + (1-d_1)d_2\frac{\Delta}{2} - C_D(d_1)\}$$

and

$$d_2^* = argmax_{d_2}\{d_2(1-d_1)\frac{\Delta}{2}) - C_D(d_2)\}$$

The unique equilibrium (d_{VI}^1, d_{VI}^2) in the investment stage under Assumption 2 is given by the solution to the following equations

$$\begin{aligned} C_D'(d_{VI}^1) &= \Delta - \tfrac{1}{2}\Delta d_{VI}^2 \\ C_D'(d_{VI}^2) &= \tfrac{\Delta}{2} - \tfrac{1}{2}\Delta d_{VI}^1 \end{aligned} \tag{2}$$

Proposition 3. *The integrated downstream firm invests more than the independent downstream firm, indeed we have $d_{VI}^1 > d_{VS}^M > d_{VI}^2$.*

Proof. The proof is similar to the proof of Proposition 1, we thus omit the detail proof here. □

Proposition 3 can be shown in the Figure 2. The best response function for the independent downstream firm D_2 is not affected by vertical integration. However, the best response function is pushed outward by vertical integration. This comes from the two effects mentioned above, the integrated downstream firm is able to catch all the benefit of his innovation whenever he is successful, no matter whether the independent downstream firm succeeds or not. And since downstream investments are strategic substitutes, higher investment incentive from the integrated downstream firm crowds out the incentive of the independent firm.

With quadratic cost function, the downstream investments are given by

$$d_{VI}^1 = \frac{\Delta(4c_d - \Delta)}{4c_d^2 - \Delta^2} \text{ and } d_{VI}^2 = \frac{2\Delta(c_d - \Delta)}{4c_d^2 - \Delta^2}$$

Proposition 4. *If only downstream innovation is essential and cost function is quadratic, when there is a monopolistic upstream firm, social welfare is higher under vertical integration than vertical separation.*

Proof. See Appendix A.2. □

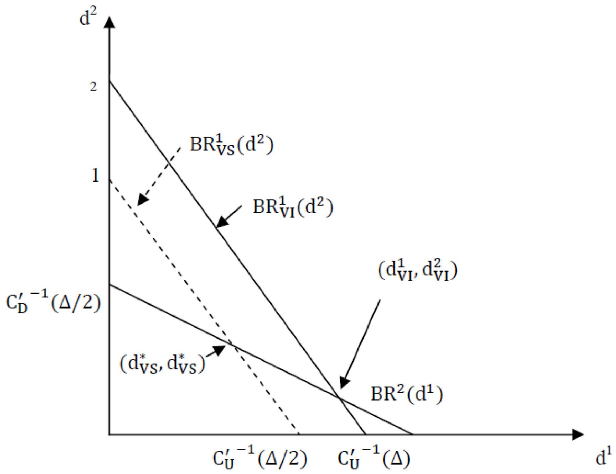

Figure 2: Equilibrium Downstream Investment under Vertical Separation and Vertical Integration

Vertical integration has two effects on social welfare: first, the total level of investment increases as $d^1_{VI} + d^2_{VI} = \frac{3\Delta}{2c_d+\Delta} > \frac{2\Delta}{2c_d+\Delta} = 2d^M_{VS}$, which comes from the fact that the slope of best response function is between -1 and 0; moreover, the asymmetric level of investment under vertical integration makes the cost of investment even larger since total cost function is convex. Second, the probability that the market has successful innovation is also higher: on one hand, asymmetric level of investment increases this probability; on the other hand, downstream hold-up problem is solved for the integrated downstream firm which further increases the level of investment and increases social welfare.

Notice that under vertical separation, there is serious under-investment problem: each downstream firm can only catch half of the benefit from his innovation $\frac{\Delta}{2}$ even when he is the sole innovator in the downstream market, while the social benefit in this case is Δ. Under vertical integration, the insufficient incentive problem persists for the independent downstream firm; however, the integrated downstream firm has excessive investment incentive. This results from the fact that the integrated downstream firm can catch the whole benefit of his innovation even when the other downstream firm already made a successful innovation, in which case the social value for the investment of the integrated firm is zero.

Compare to the previous section, the welfare effect of vertical integration is reversed. In the previous section, vertical integration only softens upstream competition and results in over-investment; in the present section, vertical integration softens downstream competition and leads to excessive investment incentive for the integrated downstream firm; however, vertical integration also partially solves the hold-up problem. In a sense, the cost of under-investment resulting from hold-up problem is more serious than that resulting from excessive competition.

4.2 Upstream Competition

When there is also upstream competition, the downstream incentives to invest may change. We start with the case when there are two upstream firms. The two upstream firms compete a-la-Bertrand to supply the downstream firms, and all firms are separated. The time-line is the same as before,

- *Investment Stage*: Each D_j makes investment decision d_j; if both firms fail, the game ends;
- *Bargaining Stage*: If only one downstream firm D_j succeeds, he bilaterally bargains with each U_i over the price of the input; if both downstream firms succeed, they simultaneously bargain with both upstream firms; If no agreement is reached, the game ends;
- *Payoff Stage*: If any agreement is reached, payment is made and input is delivered; Downstream market payoff realizes and game ends.

Clearly, the presence of upstream competition drives the input price down to zero. And therefore, each downstream firm can retain the whole benefit of his innovation Δ in case when he is the sole innovator in downstream market. When both downstream firms are successful, competition dissipates part of the profit and each D_j gets δ. Thus the payoff to downstream firms is described as Table 1.

When U_1 and D_1 integrates (without loss of generality), no firm can do better than when they are separated. When only D_1 succeeds, he gets Δ; when only D_2 succeeds, upstream competition leads to zero price for the input. When both downstream firms succeed, the integrated entity $U - D_1$ retrieves δ from its downstream affiliates; while competition leads to zero input price for D_2. Therefore, the payoff to $U - D_1$ and D_2 is again given by Table 1.

Proposition 5. *When only downstream innovation is essential, in the presence of upstream competition, social welfare is the same under vertical integration and vertical separation.*

Compared to the case when only upstream innovation is essential, vertical integration has no effect on social welfare when only downstream innovation matters. This is simply because upstream competition leads to zero input price, which does not alter the nature of downstream competition. In this situation, the downstream investment is given by

$$C'_D(d_D^j) = \Delta - (\Delta - \delta)d_D^j$$

With quadratic cost function, we have

$$d_D^j = \frac{\Delta}{c_d + \Delta - \delta}$$

and the corresponding social welfare is given by

$$W^D = \frac{\Delta^2}{(c_d + \Delta - \delta)^2}(c_d + \Delta - 2\delta)$$

Notice that with competing upstream firms, downstream investment exceeds that of social optimum. This is because the private gain for a downstream firm when the other one already made successful innovation is δ, while the social value is zero.

5 Both Upstream and Downstream Investments Are Essential

Based on the analysis of the previous two sections, we now study the situation when both upstream and downstream investments are essential for the value of the final product. This situation fits a number of settings: upstream firms may be manufactures who need to make investment in order to develop high quality product, while downstream retailers need to make investment in services in order to improve relationship with consumers; upstream firms may consist of patent holders who make investment to develop new ideas, while downstream developers invests to develop the ideas into final products. There is a strong complementarity between upstream and downstream innovation, in the sense that there is value for the final product without either upstream innovation or downstream innovation.

5.1 Vertical Separation

We start with the situation when all firms remain separated. The time-line is as follows,

- *Upstream Investment Stage*: Each U_i makes investment decision e_i; if both firms fail, the game ends;
- *Downstream Investment Stage*: Each D_j makes investment decision d_j; if both firms fail, the game ends;
- *Bargaining Stage*: The successful upstream firm(s) and successful downstream firm(s) bargain over the price of the input;
- *payoff Stage*: Payments are made and inputs are delivered if any agreement is reached; downstream market realizes and game ends.

In this setting, we assume that the downstream firms invest after observing the outcomes of upstream investment. This assumption is made both for tractability and reducing socially wasteful downstream investments when no upstream firm succeeds in investment. As before, when there is only one successful upstream firm and one successful downstream firm, they enter a Nash Bargaining with equal bargaining power.

When only one upstream firm succeeds, the subgame goes as in Section 4.1.1, where the social welfare is W^M_{VS}. The continuation payoff for the upstream monopolist U_i is then

$$\pi_{VS} = 2d^M_{VS}(1 - d^M_{VS})\frac{\Delta}{2}$$

When both upstream firms obtain successful innovation, the subgame goes as Section 4.2, where each upstream firm gets zero profit and the social welfare is given by W^D. Therefore, the payoff matrix for the upstream firms at the investment stage is given by

Under Assumption 1, there is a unique equilibrium (e_{VS}, e_{VS}) in the investment game which is given by the solution to the following equation

$$C'_U(e_{VS}) = (1 - e_{VS})\pi_{VS}$$

Table 5:

U_1 U_2	S	F
S	0,0	π_{VS},0
F	0,π_{VS}	0,0

With quadratic cost function, we have

$$e_{VS} = \frac{\pi_{VS}}{c_u + \pi_{VS}}$$

And the corresponding social welfare is given by

$$W_{VS} = e_{VS}^2 W^D + 2e_{VS}(1 - e_{VS})W_{VS}^M - c_u e_{VS}^2$$

5.2 Vertical Integration

Suppose now that U_1 and D_1 integrate. The time-line is the same as vertical integration. When the independent upstream firm U_2 is the only who succeeds in upstream innovation, the subgame goes as Section 4.1.1, where the profit for U_2 is π_{VS} and the social welfare is W_{VS}^M. However, even though U_1 does not have a successful innovation, she can still get positive profit from her downstream affiliate when D_1 is the sole downstream innovator. This profit is given by

$$\pi_{VI}^F = d_{VS}^M(1 - d_{VS}^M)\frac{\Delta}{2} - C_D(d_{VS}^M)$$

When the integrated upstream firm U_1 is the sole upstream innovator, the subgame goes as Section 4.1.2. The social welfare is W_{VI}^M and the profit for the upstream monopolist U_1 is given by

$$\pi_{VI}^S = d_{VI}^1\Delta + (1 - d_{VI}^1)d_{VI}^2\frac{\Delta}{2} - C_D(d_{VI}^1)$$

It is easy to show the following lemma

Lemma 1. *With quadratic cost function, the profit for the integrated upstream monopolist is higher than the separated one, i.e.* $\pi_{VI}^S > \pi_{VS}$.

Proof. See Appendix A.3. □

The profit is higher for the integrated upstream firm due to two effects: first, hold-up problem is eliminated inside the integrated entity, which increases the investment of the integrated downstream firm; second, higher investment from the integrated downstream firm crowds out investment of the independent downstream firm, which further increases the profit of the integrated entity.

When both upstream firms make successful investment, the subgame goes as Section 4.2. However, the profit for the two upstream firms are different: the independent upstream firm U_2 gets zero profit due to competition from U_1; while the integrated firm can still get positive profit by supplying her downstream affiliates. This profit is equal to

$$\pi_D^{int} = d_D^1(1 - d_D^2)\Delta + d_D^1 d_D^2\delta - C_D(d_D^1)$$

Table 6:

$U_1 - D_1$ U_2	S	F
S	π_D^{int},0	π_{VI}^S,0
F	π_{VI}^F,π_{VS}	0,0

Then the payoff matrix for upstream firms in the investment stage is given by

Then the unique equilibrium in the upstream investment stage is given by

$$\begin{cases} C_U'(e_{VI}^1) & = \pi_{VI}^S - (\pi_{VI}^S + \pi_{VI}^F - \pi_D^{int})e_{VI}^2 \\ C_U'(e_{VI}^2) & = \pi_{VS} - \pi_{VS}e_{VI}^1 \end{cases} \tag{3}$$

then the following proposition can be easily shown

Proposition 6. *With quadratic cost function, the integrated upstream firm invests more than the independent upstream firm, indeed we have* $e_{VI}^1 > e_{VS} > e_{VI}^2$.

Proof. See Appendix A.4. □

The effect of vertical integration is three-fold: first, the upstream competition soften effect is still present, when both upstream firms have successful innovation, they only compete for the independent downstream firm. This increases the investment incentive for the integrated upstream firm and decreases the incentive of the independent upstream firm.

Second, the elimination of hold-up problem and downstream competition soften effect further increases the investment incentive of the integrated upstream firm and decreases that of the independent upstream firm, which results from the fact that $\pi_{VI}^S > \pi_{VS}$. This second effect is only present when downstream innovation is also essential for the product; in this sense, downstream innovation amplifies the foreclosure effect of vertical integration on the independent upstream firm. However, such increase in investment may not be "excessive". When both upstream innovation and downstream innovation are essential for the final product, each party is not able to catch the whole benefit of their investments. Vertical integration partially solves this problem inside the integrated entity, which aligns private incentive to invest and social incentive. In this sense, downstream innovation weakens the excessive upstream investment incentive under vertical integration.

Third, the combination of upstream innovation and downstream innovation gives rise to an additional effect. This originates from the fact that the integrated upstream firm obtains positive profit even when she fails in upstream investment, since the downstream subordinate D_1 still gets $\frac{\Delta}{2}$ when he is the only successful downstream innovator. This tends to reduce the investment incentive of the integrated upstream firm and increase that of the independent upstream firm. Proposition 5 shows that this third effect is dominated by the first two effects, and the overall effect of vertical integration is a result of foreclosure over the independent upstream firm.

The foreclosure effect resulting from vertical integration has different impact on welfare, compared to the case when only upstream innovation is essential. When only upstream innovation is essential, foreclosure leads to socially excessive upstream investment. When downstream innovation is also essential, upstream investments become insufficient, vertical integration pushes upstream investments towards the social optimum level. Furthermore, since downstream investment is excessive when upstream competition is present ex post, the foreclosure effect would actually reduce the ex post probability of upstream competition and thus limit the extent of excessive downstream investment. In addition, the elimination of hold-up problem increases both upstream and downstream investment incentives and increases social welfare. Although the asymmetric levels of investment lead to higher cost due to the convexity of the cost function, the following proposition shows that the overall effect of vertical integration on social welfare is positive. With quadratic cost function, we have

$$e_{VI}^1 = \frac{c_u \pi_{VI}^S - (\pi_{VI}^S + \pi_{VI}^F - \pi_D^{int})\pi_{VS}}{c_u^2 - (\pi_{VI}^S + \pi_{VI}^F - \pi_D^{int})\pi_{VS}} \text{ and } e_{VI}^2 = \frac{c_u \pi_{VS} - \pi_{VI}^S \pi_{VS}}{c_u^2 - (\pi_{VI}^S + \pi_{VI}^F - \pi_D^{int})\pi_{VS}}$$

The corresponding social welfare is given by

$$W_{VI} = e_{VI}^1 e_{VI}^2 W_D + e_{VI}^1 (1 - e_{VI}^2) W_{VI}^M + e_{VI}^2 (1 - e_{VI}^1) W_{VS}^M - \frac{1}{2} c_u {e_{VI}^1}^2 - \frac{1}{2} c_u {e_{VI}^2}^2$$

Proposition 7. *With quadratic cost function, the social welfare is higher under vertical integration than vertical separation.*

5.3 Counter-Strategy of Rival

The integration of U_1 and D_1 hurts both the independent upstream firm U_1 and the independent downstream firm D_2. Then U_2 may also have incentive to integrate with the independent downstream firm D_2. There are indeed gains from such integration: first, when U_2 is the only upstream innovator, she can obtain π_{VI}^S instead of $\pi_{VS} < \pi_{VI}^S$; second, when both upstream firms succeed, upstream competition is totally eliminated and U_2 is able to get positive profit. The payoff matrix under full integration is

Table 7:

$U_1 - D_1$ $U_2 - D_2$	S	F
S	π_D^{int},π_D^{int}	π_{VI}^S,π_{VI}^F
F	π_{VI}^F,π_{VI}^S	0,0

And then the upstream investment is determined by

$$C_U'(e_{FI}) = (1 - e_{FI})\pi_{VI}^S + e_{FI}(\pi_D^{int} - \pi_{VI}^F)$$

Denote the social welfare under full integration as W_{FI}, we have the following proposition,

Proposition 8. *With quadratic cost function, social welfare is higher under full integration than partial integration.*

The above proposition comes from two effects: first, hold-up problem is now total resolved inside each integrated entity, which increases welfare; second, the excessive upstream investment incentive is weakened for $U_1 - D_1$; However, notice that there is still "excessive" investment incentive for the upstream firms under full integration: the private value of investment for each upstream firm is positive when the other one has already made successful innovation, while the social value is zero. Nevertheless, this "excessive" investment incentive increases upstream investment towards the social optimal level. The above result shows that the total effect is positive on social welfare.

6 Discussion and Extension

6.1 Information Disclosure by Upstream Firms

In our model, we assume that a final product necessitates both upstream and downstream innovation; but upstream innovation and downstream innovation are independent in the sense that the downstream innovation does not require any information or actual delivery of upstream innovation. All that downstream firms need to know is whether there is successful upstream innovation or not. In this subsection, we relax this assumption and assume that downstream firms need information about the upstream innovation in order to make any investment. We focus on the quadratic cost function case.

We modify the game as follows,

- *Upstream Investment Stage*: Each U_i makes investment decision e_i; if both firms fail, the game ends;
- *Information Disclosure Stage*: The successful upstream firm decides whether to disclose the information about the innovation to both downstream firms or only one of them;
- *Downstream Investment Stage*: Each D_j makes investment decision d_j if he receives information from the upstream firm; if both firms fail, the game ends;
- *Bargaining Stage*: The successful upstream firm(s) and successful downstream firm(s) bargain over the price of the input;
- *Payoff Stage*: Payments are made and inputs are delivered if any agreement is reached; downstream market realizes and game ends.

In the game above, downstream investment needs information about the upstream innovation but not the actual delivery of the input. In other words, if both upstream firms have successful innovation, the downstream firm is free to choose any upstream supplier no matter from where he gets the necessary information for investment. We assume that when the upstream firm is indifferent between disclose information and not disclose any information, she chooses to disclose. Then the subgame is the same as in Section ? if both upstream firms make successful investment under either vertical separation or vertical integration. Because even though the integrated upstream firm may refrain from disclose information to the independent downstream firm, the upstream competitor would disclose such information.

When it turns out to be the case that there is a monopolistic upstream innovator, the incentive to disclose may differ depending on whether the upstream monopolist is vertically integrated or not. Under vertical separation, the profit for the upstream monopolist is π_{VS} if she discloses the information to both downstream firms; when she only discloses to one downstream firm, it is easy to know that downstream investment is $d = \frac{\Delta}{2c_d}$, and the profit for the upstream monopolist is $\pi^1_{VS} = \frac{\Delta^2}{4c_d}$.

When the upstream monopolist is vertically integrated with one of the two downstream firms, the profit is π^S_{VI} if she also discloses the information to the independent downstream firm. When she refrains from disclosing the information, downstream investment is given by $d = \frac{\Delta}{c_d}$ and the profit for the integrated upstream monopolist is $\pi^1_{VI} = \frac{\Delta}{2c_d}$. Suppose there is a cost K related to disclosing information to a second downstream firm. Such cost may be related to the risk of information leakage, where it is only a private cost but not social cost; or the cost may directly concern how to convey the information correctly to the downstream firms, then it is also a social cost. To show our main insight, we assume such cost is only a private cost. The next proposition shows that an integrated upstream firm has less incentive to disclose information to both downstream firms.

Proposition 9. *There exists a range of value $K \in (\underline{K}, \bar{K})$ such that the separated upstream monopolist discloses information to both downstream firms, while the integrated upstream monopolist does not disclose to the independent downstream firm.*

Proof. See Appendix A.5. □

Under vertical separation, the upstream monopolist has more incentive to disclose information to a second downstream firm. First, due to our assumptions on the cost function, downstream competition does not lower the total level of downstream investment; second, when both downstream firms obtain successful innovation, the payoff for the upstream monopolist is now Δ rather than $\frac{\Delta}{2}$. However, under vertical integration, by disclosing information to the independent downstream firm, the integrated upstream firm has to balance the benefit of lowering investment cost (since now she makes less investment) and the loss of profit when the independent downstream firm is the only downstream innovator (since now the integrated firm can only get $\frac{\Delta}{2}$ rather than Δ).

With respect to welfare, if $K \in (\underline{K}, \bar{K})$, the social welfare is the same as in the previous section under vertical separation. However, the result is different under vertical integration when the integrated upstream firm is the only upstream innovator. Since in this case, the integrated upstream firm would choose to only disclose information to her downstream affiliate. However, it can be easily shown that if downstream investment cost is relatively high, the welfare under vertical integration is lower than that under vertical separation if there is only one upstream innovator.

Proposition 10. *When there is only one upstream innovator, if $K \in (\underline{K}, \bar{K})$, there exists a $\bar{c}_d > \Delta$ such that social welfare is higher under vertical separation than vertical integration if downstream investment cost is large enough, i.e. if $c_d > \bar{c}_d$.*

Proof. See Appendix A.6. □

Therefore, when downstream investment cost is low, the benefit from the elimination of hold-up problem inside the integrated entity outweighs the cost from excluding the independent downstream firm; however, when downstream cost is high, vertical separation delivers higher social welfare since now promoting investment incentive becomes the main issue. Taken into consideration upstream innovation as well, the result still holds. Social welfare is higher under vertical integration if downstream investment cost is low, and higher under vertical separation otherwise.

6.2 Ex ante Bargaining

In the discussion above, we assume that bargain between upstream firms and downstream firms happens after all outcomes of investments have already realized and observed by all firms. The main insights in our paper still hold if bargain happens in an ex ante stage, i.e. upstream firms bargain with downstream firms after the outcomes of upstream investments realized but before downstream firms make any investment. This would be the case if downstream innovation needs the actual delivery of the input. In this situation, hold-up problem does not exist for downstream firms; however vertical integration still affects the investment incentives of upstream firms.

First, the upstream competition soften effect still exists. When both upstream firms obtain successful innovation, they only compete for the independent downstream firm. Thus the integrated upstream firm gets positive profit while the independent upstream firm gets zero profit.

Second, when only one of the two upstream firms succeeds, the integrated upstream monopolist is able to catch a larger part of downstream market profit even though now the bargaining does not affect downstream investment incentives. This results from two effects: on one hand, the integrated upstream monopolist only bargains with the independent downstream firm; on the other hand, the integrated upstream firm holds a stronger bargaining position than the independent upstream firm, since the outside option for the integrated upstream firm is higher. As before, this two factors tend to induce overall over-investment and reduce social welfare. Since there is no gain from the elimination of hold-up problem, the net effect of vertical integration on social welfare would generally be negative.

Third, when the upstream monopolist faces the choice between selling to both downstream firms and selling to only one of them, efficient bargaining implies that a separated upstream monopolist has the same incentive as an integrated upstream one. They would sell to both downstream firms only if downstream market profit is higher when there are two firms than when there is only one, since now vertical integration has no impact on downstream investment anymore. However, this result depends on the exact form of bargaining. For instance, if the upstream monopolist have all the bargaining power, i.e. she makes take-it-or-leave-it offers to each downstream firm, then if the offer is not publicly observable or the offer is sequentially made to downstream firms, lack of commitment problem may lead to inefficient trading under vertical separation. There would be too much trading since the upstream monopolist cannot commit to trade with only one downstream firm. However, such problem does not exist under vertical integration, because the integrated upstream monopolist will always sell to the downstream affiliate. And thus she would only trade with the independent downstream firm when it is efficient to have both downstream firms active.

Furthermore, whether downstream market profit is higher when both downstream firms are active or when only one firm is active depends on the downstream investment cost. Such downstream investment cost may be affected by upstream innovation; for instance, successful upstream innovation decreases downstream investment cost. When the outcome of upstream innovation is not observable, or when information about such cost is only known for the upstream innovator, the upstream innovator may be able to signal her success by restricting trading with downstream firms.

7 Conclusion

In this paper, we studied the effect of vertical integration on the investment incentives of upstream and downstream firms. When only upstream innovation is essential for the industry, vertical integration leads to overall over-investment and decreases social welfare. The foreclosure effect of vertical integration is strengthened with essential downstream innovation. However, vertical integration also promotes both upstream and downstream investment. The overall impact of vertical integration on social welfare turns out to be positive. Our results suggest that when evaluating the impact of vertical integration, especially in industries with intensive innovation, the exact nature between upstream and downstream investment may be a key point in the decision. Studying the impact of vertical integration in a more general bargaining environment, or in the presence of other forms of complementarity between upstream and downstream innovation might be interesting avenues for future research.

8 Appendix

8.1 Proof of Proposition 1

Rewrite Equation (1) as

$$\begin{cases} C_U'(e^1) & = \Delta(1-e^2) + \alpha e^2 \\ C_U'(e^2) & = \Delta(1-e^1) \end{cases}$$

When $\alpha = 0$, the solution corresponds to the investment level under vertical separation; when $\alpha = \delta$, it is the solution under vertical integration. It is clear to see that

$$\frac{\partial e^1}{\partial \alpha} = \frac{e^2}{C_U''(e^1)}$$

which is always positive under Assumption 1. Therefore, we must have $e^1_{VI} > e^*_{VS}$. Furthermore, we have $C_U'(e^2_{VI}) = \Delta(1-e^1_{VI}) < \Delta(1-e^*_{VS}) = C_U'(e^*_{VS})$, and thus $e^2_{VI} < e^*_{VS}$.

8.2 Proof of Proposition 4

Remember that the social welfare under separation is given by

$$W_{VS}^{M} = (\frac{\Delta}{2c_d + \Delta})^2(\Delta + 3c_d)$$

under vertical integration, the social welfare is

$$W_{VI}^{M} = [1 - (1 - d_{VI}^{1})(1 - d_{VI}^{2})] - \frac{1}{2}c_d {d_{VI}^{1}}^2 - \frac{1}{2}c_d {d_{VI}^{2}}^2$$

after simplification, we have

$$W_{VI}^{M} = \frac{\Delta^2}{(4c_d^2 - \Delta^2)^2}(14\Delta^3 - 12c_d^2\Delta + \frac{3}{2}c_d\Delta^2 + \Delta^3)$$

Then

$$W_{VI}^{M} - W_{VS}^{M} = \frac{\Delta^2}{(4c_d^2 - \Delta^2)^2}(2c_d^3 - 4c_d^2\Delta + \frac{5}{2}c_d\Delta^2)$$

which is always positive when $c_d > \Delta$.

8.3 Proof of Lemma 1

With quadratic cost function, we have

$$d_{VS}^{M} = \frac{\Delta}{2c_d + \Delta}$$

under vertical separation; and under vertical integration we have

$$d_{VI}^{1} = \frac{\Delta(4c_d - \Delta)}{4c_d^2 - \Delta^2} \text{ and } d_{VI}^{2} = \frac{2\Delta(c_d - \Delta)}{4c_d^2 - \Delta^2}$$

Thus,

$$\pi_{VS} = \frac{\Delta^2}{2c_d + \Delta}$$

and

$$\pi_{VI} = \frac{\Delta^2}{(4c_d^2 - \Delta^2)^2}(12c_d^3 - 8c_d^2\Delta - \frac{1}{2}c_d\Delta^2 + \Delta^3)$$

Therefore, after simplification, we have

$$\pi_{VI} - \pi_{VS} = \frac{\Delta^2}{(4c_d^2 - \Delta^2)^2}(4c_d^3 + \frac{3}{2}c_d\Delta^2 - 4c_d^2\Delta)$$

which is always positive.

8.4 Proof of Proposition 6

The proof is the same as Proposition 1, the only thing we need to show is that $\pi_{VI}^{F} < \pi_{D}^{int}$. With quadratic cost function, we have

$$\pi_{VI}^{F} = \frac{1}{2}c_d(\frac{\Delta}{2c_d + \Delta})^2$$

and

$$\pi_{D}^{int} = \frac{1}{2}c_d(\frac{\Delta}{c_d + \Delta - \delta})^2$$

Clearly we have $\pi_{VI}^{F} < \pi_{D}^{int}$ since $2c_d + \Delta > c_d + \Delta - \delta$.

8.5 Proof of Proposition 9

It suffices to show that $\pi_{VS} - \pi_{VS}^1 > \pi_{VI}^S - \pi_{VI}^1$, which is equivalent to show $\pi_{VI}^1 - \pi_{VS}^1 > \pi_{VI}^S - \pi_{VS}$. With quadratic cost function, we have

$$\pi_{VI}^1 - \pi_{VS}^1 = \frac{\Delta^2}{4c_d}$$

and

$$\pi_{VI}^S - \pi_{VS} = \frac{\Delta^2}{(4c_d^2 - \Delta^2)^2}(4c_d^3 + \frac{3}{2}c_d\Delta^2 - 4c_d^2\Delta)$$

After simplification, we have

$$(\pi_{VI}^1 - \pi_{VS}^1) - (\pi_{VI}^S - \pi_{VS}) = \frac{\Delta^2}{4c_d(4c_d^2 - \Delta^2)^2}(16c_d^3\Delta + \Delta^4 - 14c_d^2\Delta^2)$$

which is always positive since $c_d > \Delta$.

Let $\underline{K} = \pi_{VI}^S - \pi_{VI}^1$ and $\bar{K} = \pi_{VS} - \pi_{VS}^1$, the for $K \in (\underline{K}, \bar{K})$, we have $\pi_{VS} - \pi_{VS}^1 - K > 0$, while $\pi_{VI}^S - \pi_{VI}^1 - K < 0$.

8.6 Proof of Proposition 10

With quadratic cost function, when only one upstream makes successful innovation, if she is vertically separated, the welfare is

$$W_{VS}^M = (\frac{\Delta}{2c_d + \Delta})^2(\Delta + 3c_d)$$

If she is vertically integrated, then only the integrated downstream firm makes investment, which is given by $d_1 = \frac{\Delta}{c_d}$, and thus the social welfare is

$$W_{VI}^1 = \frac{\Delta^2}{2c_d}$$

Then we have

$$W_{VS}^M - W_{VI}^1 = \frac{\Delta^2}{4c_d(2c_d + \Delta)^2}(2c_d^2 - 2c_d\Delta - \Delta^2)$$

It is clear that $2c_d^2 - 2c_d\Delta - \Delta^2$ is increasing in c_d, and it is negative when $c_d = \Delta$ and positive for c_d big enough. Therefore, there exists a $\bar{c_d}$ such that when $c_d < \bar{c_d}$, $W_{VS}^M < W_{VI}^1$; and when $c_d > \bar{c_d}$, $W_{VS}^M > W_{VI}^1$.

References

[1] M.L. Allain, C. Chambolle, and P. Rey. Vertical integration, innovation and foreclosure. *working paper*, 2011.

[2] James Anton and Dennis Yao. The sale of ideas: Strategic disclosure, peoperty rights, and contracting. *The Review of Economic Studies*, 69(3):513–231, 2002.

[3] James Anton and Dennis Yao. Little patents and big secrets: Managing intellectual property. *The RAND Journal of Economics*, 35(1):1–22, 2004.

[4] F. Berges-Sennou, P. Bontems, and V. Requillart. Economics of private labels: A survey of literature. *Journal of Agricultural & Food Industrial Organization*, 2, 2004.

[5] Patrick Bolton and Michael D. Whinston. Incomplete contracts, vertical integration, and supply assurance. *The Review of Economic Studies*, 60(1):121–148, Jan. 1993.

[6] Y. Chen. On vertical mergers and their competitive effects. *The RAND Journal of Economics*, 32(4):667–685, 2001.

[7] Y. Chen and D.M. Sappington. Innovation in vertically related markets. *Journal of Industrial Economics*, 58(2):373–401, 2010.

[8] Yongmin Chen. Refusal to deal, intellectual property rights, and antitrust. 2011. working paper.

[9] J.P. Choi and S.S. Yi. Vertical foreclosure with the choice of input specifications. *The RAND Journal of Economics*, 31(4):717–743, 2000.

[10] Joseph Farrell and Michael L. Katz. Innovation, rent extraction, and integration in systems markets. *The Journal of Industrial Economics*, 48(4):413–432, Dec. 2000.

[11] R. Inderst and C. Wey. Buyer power and supplier incentives. *European Economic Review*, 51:647–667, 2007.

[12] C. Kohler and C. Rammer. Buyer power and suppliers' incentives to innovate. *working paper*, 2010.

[13] V. Nocke and L. White. Do vertical mergers facilitate upstream collusion? *The American Economic Review*, 97(4):1321–1339, 2007.

[14] J.A. Ordover, G. Saloner, and S.C. Salop. Equilibrium vertical foreclosure. *The American Economic Review*, 80(1):127–142, 1990.

[15] M.H. Riordan. Anticompetitive vertical integration by a dominant firm. *The American Economic Review*, 88(5):1232–1248, 1998.

[16] M. Salinger. Vertical mergers and market foreclosure. *The Quarterly Journal of Economics*, 103(2):345–356, 1988.

[17] Klaus M. Schmidt. Complementary patents and market structure. *Discussion Paper*, 2009.