ESTIMATING HOSPITAL SERVICES PRODUCTION FUNCTIONS THROUGH FLEXIBLE REGRESSION MODELS

Keywords: Generalized Additive Models (GAMs), Hospital services, production function, Cobb-Douglas, Translog.

Francisco Reyes-Santías [a, b, d], Carmen Cadarso-Suárez [c, e], María Xosé Rodríguez- Álvarez [c, d, e]

[a] Departamento de Organización de Empresas y Mercadotecnia. University of Vigo, Spain
[b] Instituto Universitario de Ciencias Neurológicas. University of Santiago de Compostela, Spain
[c] Unit of Biostatistics, Dept. of Statistics and Operations Research. University of Santiago de Compostela, Spain
[d] Complexo Hospitalario Universitario de Santiago de Compostela (CHUS). Santiago de Compostela, Spain
[e] Instituto de Investigación Sanitaria de Santiago de Compostela (IDIS). Santiago de Compostela, Spain

Corresponding autor: Francisco Reyes-Santías
Post Address: Facultad de Ciencias Económicas y Empresariales, Universidad de Vigo
Lagoas-Macrosende
36200 Vigo (Spain)
Phone: 34-678487167
e-mail address: francisco.reyes.santias@sergas.es

Support received: Spanish MEC Grant MTM2008-01603 and Galician Regional Authority (Xunta de Galicia) projects INCITE08PXIB208113PR and INCITE08CSA0311918PR. M.X. Rodr´ıguez- ´Alvarez was supported by a grant [CA09/00539] from the Instituto de Salud Carlos III (Spanish Ministry of Science and Technology)

ABSTRACT

The subject of discussion is health care production functions. The parametric two-factor Cobb-Douglas and transcendental logarithmic (translog) production functions are frequently used. However, empirical and theoretical work has often questioned the validity of the parametric Cobb-Douglas and Translog as a model for the production of health care.

The aim of this study has been to compare a new flexible form of production function based on Generalized Additive Models (GAMs) with the classical Cobb-Douglas and translog forms, in estimating production factor behaviour in four different clinical and surgical services.

Using data from public hospitals in the Spanish Region of Galicia for the period 2002-2008, including the number of beds, consultants and discharges by DRG, it has been identified that Flexible GAM provides a better description of the production function and their factors than Cobb-Douglas or Translog models, thus providing a better fit in the presence of nonlinear relationships and more accurate prediction values.

1.- BACKGROUND

Production function analysis has been used by economists to study efficiency since the 1930s and is one of the econometric methods most often used by health economists (Eastaugh 1992). Talking about *Production function* refers to the physical relationship between an organization's input of productive resources and its output of goods and services per unit of time. In particular, the provision of health services involves the use of physical and human capital resources (inputs) to produce goods and services (outputs). These outputs bestow benefits upon individuals and society which may be called outcomes.

Two models are commonly used in the estimation of clinical services production functions (Rosko and Broyles 1988): the Cobb-Douglas model and the transcendental logarithmic (translog model). Cobb-Douglas has long been popular among economists because it is easy to work with and can explain the substitution between health care inputs. As Border (2004) pointed out, economists have also been somewhat well disposed toward the Cobb-Douglas function because it gives simple closed-form solutions to many economic problems. However, empirical and theoretical work has often questioned the validity of the parametric Cobb-Douglas as a model for the production of health care (López Casasnovas, 1988). In comparison with the Cobb-Douglas model, the translog function model has a number of advantages. This model adds the effects of interactions between inputs. The Cobb-Douglas model, in contrast, omits this effect, which is less realistic. Thus, most of the clinical services production function studies have used this flexible translog function form (Rosko and Broyles 1988; McGuire 1987).

Nevertheless, in some circumstances, parametric models like the Cobb-Douglas or Translog models can be very restrictive. Using these models for estimation and prediction, the functional shape of continuous inputs is "forced" to follow a linear parametric form, which frequently does not fit the data closely. The relative lack of flexibility of parametric models has led to the development of non-parametric regression techniques based on the broad family of generalized additive models (GAMs; Hastie and Tibshirani, 1990; Wood, 2006).

GAMs are useful in finding predictor-response relationships in many kinds of data without using a specific model. They combine the ability to explore many nonparametric relationships simultaneously with the distributional flexibility of Generalized Linear Models (GLM, McCullagh and Nelder 1989).

The GAM approach has shown its relevance in a broad range of different research domains in biology and medicine (e.g. Hastie and Tibshirani 1990; Wood 2006). Hardle, Muller and Sperlich (2004) present the statistical and mathematical principles of GAMs with a focus on econometrics and biometrics, showing the agreement between flexibility and simplicity of these statistical procedures. Also, the AM approach by Reyes, Cadarso and Rodriguez-Alvarez (2010) to measure production of hospitals has shown to be useful in exploring the behaviour of flexible functional forms compared with parametric models (Cobb-Douglas and Translog) applied to 14 public hospitals.

Along with the previous advantages, the existence of statistical software implementing GAM models (i.e. SAS and the free software R), allows practitioners from different applied areas to use these models in practice.

The main aim of this paper is to calculate a new flexible medical service production function, by using GAMs, and to compare the results of the new approach with the two most popular production functions used in the health care sector, the Cobb-Douglas and the Translog models. The hospital centre structured in clinical and surgical services, integrates and combines human, capital and technological resources in order to face the demand of health care services producing intermediate outputs (consultancies, surgical interventions or diagnosis test). The output of the hospital services identifies itself by means of the intensity of the input used in the treatment of each patient. It is therefore necessary to know the intensity in the use and the interaction between inputs to study the achievement of a determinate output, being understood as an intermediate output (consultancies, surgical interventions, diagnostic tests) rather than the output capital of health.

The GAM approach together with the Cobb-Douglas and Translog models are all applied to data set in the Galician hospital industry that operated in the period of 2002-2008. The following medical services have been chosen for this research: General Surgery, Traumathology, General Medicine and Gynaecology. The main aim of this selection has been to choose general and more specialized medical services both from surgerical and clinical fields.

The rest of the paper is organised as follows: Section 2 describes the characteristics of the Galician region`s hospital industry. A summary of the data set to be analyzed is presented in Section 3. Section 4 briefly describes the Cobb-Douglas and Translog production function and introduces the flexible GAM production function. The results of the data analysis and a relevant comparison of the different approaches are presented in Section 5. Finally, we conclude with a discussion Section 6.

2.- GALICIAN HOSPITAL INDUSTRY

Regarding the institutional framework the Spanish National Health Care System (NHS) was created in 1987, following the approval of the Constitution in 1978. The health service model then put in place was characterized by universal coverage, equity concerns, and financed via tax revenues. In fact, the Constitution guaranteed all citizens 'the right to health', to be provided by a universal National Health Service, comprehensive and free of charge at hospital care level.

At the beginning of the 1990s, Galicia (Spain) began a process of decentralization which gave this autonomous region control over its own health system within the general Spanish context. The Galician public health sector comprises of 10 hospitals and hospital groups, which are owned by the Public regional health service authority, the Galician Health Service (SERGAS).

In order to give an impression of the Galician hospital sector size, some quantitative characteristics show that Public hospitals comprise of about 7.446 beds, accompanied by 3.917 full-time physician equivalents. In 2008 the

number of inpatients treated in hospitals was 248.371 against 2.233.894 inpatient days, which was accompanied at the same time by 1.431.011 first-time visits and 1.105.083 emergencies produced.

3. DATA DESCRIPTION

The variables we use consist of inputs to hospital production in the form of capital and labour, and outputs from production.

We have chosen the output of Inpatient care which is measured as number of admissions standardized by means of complexity, obtaining homogeneous units of production that we call henceforth Hospitalation Production Units (HPUs), which we will calculate by multiplying the number of admissions by the complexity (weight), obtained from the Diagnostic Related Groups (DRGs) (López el al. 1996).

Following Ferrier and Valmanis (2004) hospital inputs are measured as follows: in terms of capital we use the average number of beds (Beds) in the year in each hospital and this data is available in the Health Statistics publications. Labour inputs are measured by the number of consultant full-time equivalents (FTEs) employed in each hospital and is counted in December of each year. Given the shift work nature of hospitals and prevalence of part-time employment this will give a more accurate indication of the amount of labour used to provide services than will a simple count of the number of staff employed or the overall cost of labour.

Workload statistics were collected, as panel data, from the Regional Ministry's Information System, from all 10 public hospitals and include, for the period 2002-2008, the following:

- Aggregated number of beds per hospital service
- Aggregated number of FTEs per hospital service
- Aggregated number of inpatient admissions and discharges (by DRG) per hospital service

4. THE STATISTICAL MODELS

4.1. The classic approach

The ***Cobb-Douglas production function*** proposed by Charles W. Cobb and Paul H. Douglas (1928), although anticipated by Knut Wicksell (1901, 1923) and, some have argued, J.H. von Thünen (1863), takes the following form:

$$Q = \alpha L^{\beta_1} K^{\beta_2},$$

where *Q*, *L* and *K* are output, labour and capital respectively, and α, β_1 and β_2 are constants.

A problem with the original specification of the functional relationship is the omission of technical change. The need to take account of technical change in estimation was noted by Handsaker and Douglas (1937) and Williams (1945). A standard procedure for introducing the possibility of technical change is to include a time trend (*T*). This captures observed changes in technology although it is assumed exogenous to the estimated specification.

$$Q = \alpha(T) L^{\beta_1} K^{\beta_2}$$

where $\alpha(T) = \alpha e^{\varphi T}$, and α and φ are constants. φ is a measure of the proportionate change in output per time period when input levels are held constant (i.e. the proportionate change in *Q* that happens as a result of technical progress). This formulation implies that technical change is exogenous and disembodied.

The former equation is usually estimated as follows:

$$\ln Q = \ln(\alpha) + \varphi T + \beta_1 \ln(L) + \beta_2 \ln(K) + \varepsilon,$$

where ε is an error term that follows a normal distribution. The log-linear specification means that the estimates of β_1 and β_2 are elasticities.

An alternative to the Cobb-Douglas production function is the ***translog production function*** (Christensen, Jorgenson and Lau, 1973). As for the Cobb-Douglas equation, in case the effects of technical-technological progress are assumed to be neutral as according to Hicks, the form of translog production function for two factors is simplified as follows:

$$\ln Q = \beta_0 + \beta_1 \ln(L) + \beta_2 \ln(K) + \beta_3 \ln(L)\ln(K) + \beta_4 T + \varepsilon,$$

where Q stands for the aggregate output, T is time, K fixed capital and L is labour. β means the function parameters.

For each of the service considered, the Cobb-Douglas and Translog models have the following form:

- Cobb-Douglas:

$$\ln UPHs = \beta_0 + \beta_1 \ln(FTEs) + \beta_2 \ln(Beds) + \beta_4 Year + \varepsilon. (1)$$

- Translog:

$$\ln UPHs = \beta_0 + \beta_1 \ln(FTEs) + \beta_2 \ln(Beds) + \beta_3 \ln(FTEs)\ln(Beds) + \beta_4 Year + \varepsilon. (2)$$

4.2 .The AM approach

In our application, the response variable is continuous. In such a case, the Generalized Additive Model (GAM) is usually referred to in statistical literature as Additive Model (AM). Thus, from now on the flexible model considered will be simply denoted as AM.

For each of the service considered, the AM model has the following form:

$$\ln UPHs = \beta_0 + \beta_1 Year + f_1(\ln FTEs) + f_2(\ln Beds) + f_3(\ln FTEs, \ln Beds) + \varepsilon, (3)$$

where f_1 and f_2 are unknown smooth functions of the number of beds (log scale) and the number of physicians (log scale) respectively, f_3 is an unknown smooth function representing the possible interaction between the number of beds and the number of physicians (both in log scale), and ε is the error term, following a normal distribution with zero mean.

With regard to the estimation of the model (3), penalized thin plate splines (Wood, 2004, 2006a, 2006b) were used to represent the smooth functions f_1, f_2 and f_3 and the mixed model representation of a penalized AM was considered (Wood, 2004). Within this configuration, smoothing parameters can be estimated via Restricted (or Residual) Maximum Likelihood (REML, see e.g Ruppert, Wand and Carroll, 2003).

All the statistical analysis was performed using R software, version 2.9.1 (R development core team, 2009). AMs were fitted using mgcv package (Wood, 2006).

5. RESULTS

In this section, in Table I the results are presented of each estimated model, for the selected medical specialties. We evaluate the models based on the AIC (Akaike Information Criterion; Akaike, 1974) and the economic interpretation for an output change due to changes in input factors. We also present for each model the value of the corrected R^2.

Following the estimation results for the *General Surgery* specialty based on the three models calculations we could see for the classic models, that both inputs, Beds and FTEs, are significant (p<0.001) although technical change and the interaction between the two factors are not (p = 0.364). On the other hand, for the flexible AM, the findings show significant effects of FTEs variable (p<0.001) and its interaction with Capital factor (p<0.001) but they are not significant for the Capital factor itself (p = 0.330) or for technical change (p = 0.441).

The findings for the CD model indicate a better fit than those for the Translog and the AM, as is shown by AIC value while R^2 (CD=96.32, Translog=96.30, AM=98.30) prefers the flexible AM. This is a good example of using AM as a tool for checking the behaviour of existing parametric models. In this case we can be confident with Cobb-Douglas estimations.

Unlike the previous findings, for the Traumatology specialty, while the variables Beds and FTEs are significant ($p<0.001$) for the CD and the AM regressions, Translog and AM regression models are both able to detect a significant interaction between Beds and FTEs inputs ($p<0.001$). However, no significant effects for year variable, as a proxy of changes in technology of production, were detected in the three regression models (CD $p=0.479$, Translog $p=0.661$, AM $p=0.100$). This outcome indicates that technical change in production is neutral in relation with output. Paying attention to AIC (CD = -26.684, Translog = -32.835, AM = -70.332) and R^2 values (CD = 91.46, Translog = 92.58, AM = 98.90) we could observe a higher explanation power from the AM rather than for the classic ones.

Figure 1(a) shows poor gains in the output as a consequence of a continuous incremental gain in the labour factor, meanwhile, the capital factor presents small increments. Nevertheless, this relationship is not extensive to other surgical specialties, like Traumatology, as is shown in Figure 1(b), where a different productivity behaviour can be seen with losses until a certain level of added Labour resources make a turning point in the output increments. It could be said that the study of the production function for a specialty like Traumatology suggests the need for a flexible approach, like the AM presented in this work.

Findings for the *Internal Medicine* specialty indicate a significant effect of the Capital variable for the three models ($p<0.001$) but not for neither Labour variable nor technical change. Likewise, interaction between both factors is significant under the flexible AM ($p<0.001$) but not for Translog model ($p=0.715$).

Based on the AIC values, the CD (AIC=-17.789) provides a better fit than the AM (AIC=-13.113) and Translog (AIC=-15.939) models. However, the R^2 (CD=95.88, Translog=95.80, AM=97.90) shows a better fit for the AM regression model.

Seeing the estimation results for the Gynaecology specialty, it is interesting to observe that Capital factor variable (Beds) is significant for the three models (CD $p<0.001$, Translog $p<0.001$, AM $p<0.001$), although Labour factor (FTEs) is not significant under neither the CD (p= 0.112) nor the AM (p = 0.194). Furthermore, the effect of interaction between input factors is captured only by Translog model (p = 0.027). Even more, none of the models is able to show that changes in production technology, captured by time trends (Year), would affect the output (CD p = 0.721, Translog p = 0.605, AM p = 0.620).

The goodness-of-fit measured by the R^2 (CD=97.13, Translog=97.38, AM=97.40) as well as the AIC (CD=-24.068, Translog=-27.669, AM=-18.825) show a more satisfactory fit of the Translog compared with Cobb-Douglas and AM. This shows another good example of using AM as a tool for evaluating the behaviour of existing parametric models.

The productivity path of the *Internal Medicine* specialty is shown in Figure 1(c) representing output gains as a linear relationship between output and Beds input with a very small effect on Labour input. This behaviour coincides with that found for other medical specialties like *Gynaecology* which could be deduced from Figure 1(d).

6. DISCUSSION AND CONCLUSIONS

This paper studies the use of flexible Additive Models to calculate medical and surgical hospital specialty service production functions. The results of the new approach have been compared with the two most popular production functions used in the health care sector, the Cobb-Douglas and the Translog models.

Following Morikawa (2010) in this study we tried to control the possible case-mix bias related to the different level of hospitals by using regionally aggregated data at Regional level for the clinical and surgical services production functions. As well as in the present study, Jensen and Morrisey (1986), use a set of flexible functional form production functions and, adjusting for hospital case - mix, examine the output contribution of physicians and other inputs and the influence that physicians in different specialties have on the productivity of other physicians, as well as on other labour and capital factors. Authors find that physicians have numerous significant effects on production and conclude that physicians are an important input that should not be ignored in empirical cost and production function studies for hospitals.

Understanding the production process for physician services can be very useful for policy makers. For example, there has been a great deal of discussion regarding physicians and nursing shortages. The production function approach allows us to examine the possible effect of such a shortage on other inputs (Thurston and Libby, 2002).

It could be interesting to remember the classic study by Hellinger (1975) which pointed out that if we are incorrectly specifying the form of the production function, estimates of economies of scale and substitution relationships between inputs will be incorrect. The author concluded that restrictions imposed by the Cobb-Douglas function would be too severe to describe the hospital production function. As shown in this study, in practical situations, the AM could represent a flexible exploratory and diagnostic tool, that allow us to evaluate which one could be the more appropriate production function model.

The results presented in this work suggest that flexible AM is a promising technique for research and application areas in health economics due to its performance in checking the behaviour of existing parametric models. Moreover, results allow characterizing the domains in which our approach may be effective like those related to demand, costs and utility functions in health care.

7. ACKNOWLEDGEMENTS

The authors would like to express their gratitude for the support received in the form of the Spanish MEC Grant MTM2008-01603 and Galician Regional Authority (Xunta de Galicia) projects INCITE08PXIB208113PR and INCITE08CSA0311918PR. M.X. Rodríguez-Alvarez was supported by a grant [CA09/00539] from the Instituto de Salud Carlos III (Spanish Ministry of Science and Technology).

TABLES

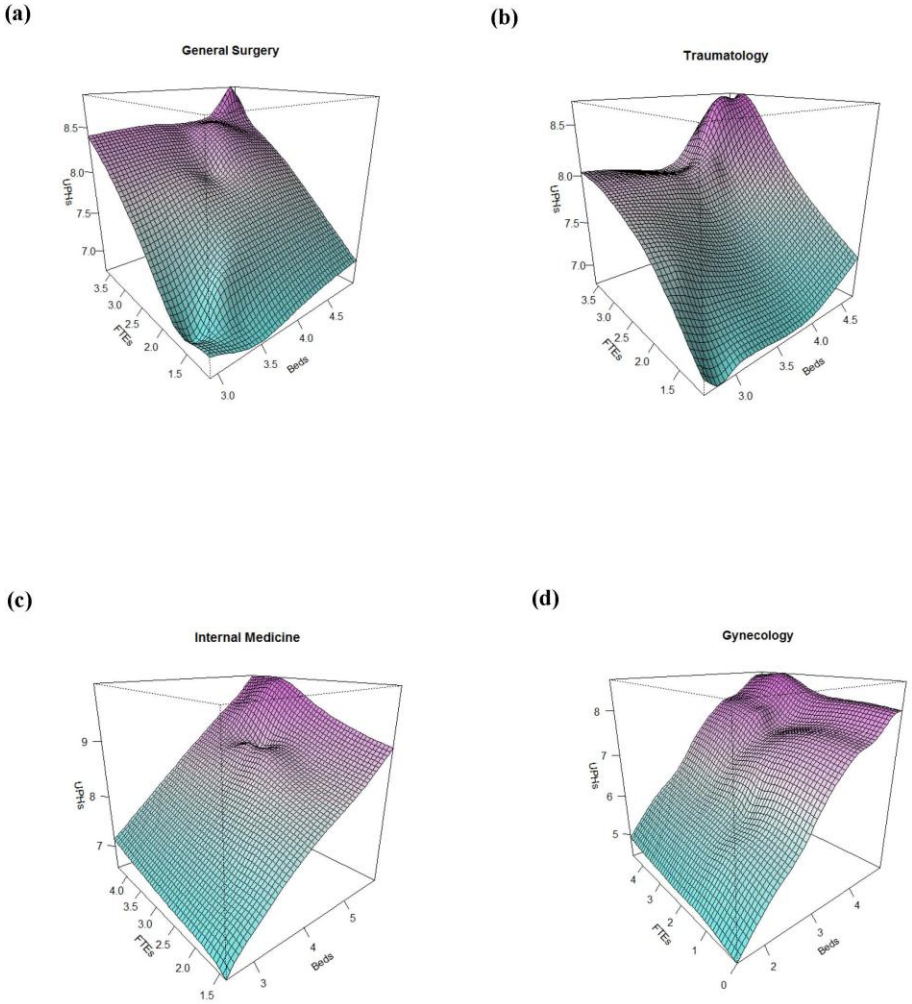
Table I. Cobb-Douglas, Translog and Flexible AM models estimates for Services.

Model	Effects	Coefficients	df	P-value	R2(x100%)	AIC
GENERAL SURGERY						
	Beds	0.458	1.00	<0.001		
CD	FTEs	0.483	1.00	<0.001	96.32	-47.184
	Year	-0.004	1.00	0.789		
	Beds	0.645	1.00	<0.001		
	FTEs	0.656	1.00	<0.001		
Trans Log	Beds X FTEs	-0,054	1.00	0.364	96.30	-46.109
	Year	-0.002	1.00	0.789		
	s(Beds)	-	1.00	0.330		
	s(FTEs)	-	1.00	<0.001		
AM	s(Beds,FTEs)	-	12.88	<0.001	98.30	-46.246
	Year	-0.009	1.00	0.441		
INTERNAL MEDICINE						
	Beds	0.868	1.00	<0.001		
CD	FTEs	0.126	1.00	0.109	95.88	-17.789
	Year	0.013	1.00	0.510		
	Beds	0.831	1.00	<0.001		
	FTEs	0.056	1.00	0.784		
Trans Log	Beds X FTEs	0.016	1.00	0.715	95.80	-15.939
	Year	0.012	1.00	0.530		
	s(Beds)	-	1.00	<0.001		
	s(FTEs)	-	1.00	0.262		
AM	s(Beds,FTEs)	-	12.16.00	<0.001	97.90	-13.113
	Year	0.028	1.00	0.091		
TRAUMATHOLOGY						
	Beds	0.392	1.00	<0.001		
CD	FTEs	0.377	1.00	<0.001	91.46	-26.684
	Year	0.013	1.00	0.479		
	Beds	-0.322	1.00	0.247		
	FTEs	-0.230	1.00	0.328		
Trans Log	Beds X FTEs	0.213	1.00	<0.001	92.58	-32.835
	Year	0.007	1.00	0.661		
	s(Beds)	-	2.58	<0.001		
	s(FTEs)	-	1.00	<0.001		
AM	s(Beds,FTEs)	-	16.03	<0.001	98.90	-70.332
	Year	0.014	1.00	0.100		
GYNAECOLOGY						
	Beds	1.061	1.00	<0.001		
CD	FTEs	0.127	1.00	0.112	97.13	-24.068
	Year	0.007	1.00	0.721		
	Beds	1.268	1.00	<0.001		
	FTEs	0.335	1.00	<0.001		
Trans Log	Beds X FTEs	-0.068	1.00	0.027	97.38	-27.669
	Year	0.010	1.00	0.605		
	s(Beds)	-	1.90	<0.001		
	s(FTEs)	-	1.00	0.194		

AM	s(Beds,FTEs)	-	0.00	0.970	97.40	-18.825
	Year	0.009	1.00	0.620		

FIGURES

Figure 1. Productivity Growth flexible models by medical services. Variables (UPHs, FTEs, Beds) are expressed in Logarithm Scale.



REFERENCES

1. Akaike, Hirotugu. 1974. A new look at the statistical model identification. IEEE Transactions on Automatic Control 19 (6): 716–723. doi:10.1109/TAC.1974.1100705. MR0423716.

2. Border, KC. 2004. On the Cobb-Douglas Production Function. California Institute of Technology. Division of the Humanities and Social Sciences. http://www.hss.caltech.edu/~kcb/Ec121a/Notes/CobbDouglas.pdf

3. Christensen L R, Jorgenson D W, Lau L J. 1973. Transcendental Logarithmic Production Frontiers. The Review of Economics and Statistics, Vol. 55, No. 1 Feb., pp. 28-45

4. Cobb, C W, Douglas, P H. 1928. A Theory of Production. The American Economic Review, vol.18, Supplement, March 1928, pp. 139-165.

5. Eastaugh, S. 1992. Health economics: Efficiency, Quality, and Equity. Westport Conn. Auburn House Pub. Co.

6. Ferrier, G., & Valmanis, V. 2004 Do mergers improve hospital productivity?. Journal of the Operational Research Society, 55: 1,071-1,080.

7. Handsaker M L, Douglas P H. 1937. The Theory of Marginal Productivity Tested by Data for Manufacturing in Victoria, Quarterly Journal of Economics, 52, 1-36.

8. Hastie TJ, Tibshirani RJ. 1990. Generalized Additive Models. London: Chapman and Hall.

9. Härdle W., Müller M., Sperlich S., Werwatz A. 2004. Nonparametric and Semiparametric Models. Berlin: Springer.

10. Hellinger, F. 1975. Specification of a hospital production function. Applied Economics, 7, 149-160.

11. Horowitz, JL. 2009. Semiparametric and Nonparametric Methods in Econometrics, Springer-Verlag.

12. Jensen GA, Morrisey MA. 1986. The Role of Physicians in Hospital Production. Review of Economics and Statistics, 63, nº 3, 432-442.

13. Knut Wicksell. 1900. Om gränsproduktiviteten såsom grundval för den nationalekonomiska fördelningen. Ekon Tidskrift

14. Lin, X. and Zhang, D. 1999. Inference in generalized additive mixed models using smoothing splines. J.R. Statist. Soc. B. 61, 381-400.

15. Lopez Casasnovas G, Wagstaff A. 1988. La combinación de los factores productivos en el hospital: una aproximación a la función de producción. Investigaciones Económicas (Segunda época). Vol. 12, nº 2: 305-327

16. López Rois F J, Mateo R, Gómez J R, Ramón C, Pereiras M. 1996. Methodological criteria for drawing up a contract-programme or singular sector-based agreement of specialized care using HPUs. Secretaría Xeral SERGAS. Consellería de Sanidade e Servicios Sociais. Xunta de Galicia. Santiago de Compostela.

17. McCullagh P, Nelder JA. 1989. Generalized Linear Models. London: Chapman & Hall.

18. McGuire, A. 1987. The Measurement of Hospital Efficiency. Social Science and Medicine 24(9): 719–24.

19. Morikawa, M. 2010. Economies of Scale and Hospital Productivity: An Empirical Analysis of Medical Area Level Panel Data. RIETI Discussion Paper Series 10-E-050.

20. Reyes Santías, F. Cadarso-Suárez, C. Rodríguez-Álvarez, M.X. 2010. Estimating hospital production functions through flexible regression models. Mathematical and Computer Modelling. doi:10.1016/j.mcm.2010.11.087.

21. Rosko M D, Broyles R W. 1988. The Economics of Health Care: A Reference Handbook. New York. Westport, CT: Greenwood Press, Inc.

22. Thünen, J. H. von. 1863. The Isolated State. London. Pergamon.

23. Thurston, N., Libby, A. 2002. A production function for physician services revisited. The Review of Economics and Statistics, February 2002, 84(1): 184–191.

24. Williams J. 1945. Professor Douglas' Production Function. Economic Record, 25, 55-64.

25. Wood, S.N. 2004 Stable and efficient multiple smoothing parameter estimation for generalized additive models. Journal of the American Statistical Association. 99:673- 686.

26. Wood, S.N. 2006a Low rank scale invariant tensor product smooths for generalized additive mixed models. Biometrics. 62(4):1025-1036

27. Wood S.N. 2006b Generalized Additive Models: An Introduction with R. Chapman and Hall/CRC Press.