Growth and Welfare Effects of Public Research

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Abstract

This paper analyses the growth and welfare effects of public research. It highlights the differences between public basic and applied research regarding the spillovers and crowding-out effects of each type of R&D. The results suggest that the business sector underinvests in research and that this research is too much applied oriented. This is due to the fact that the research firm does not internalize the social benefits from innovation and in particular, those of basic research which usually suffer from a long delay in time and are harder to incorporate to the firms productivity. Moreover, public basic research is unambigously growth promoting if performed in isolation, while the growth maximizing amount of public applied research would be 0. Regarding welfare, if we combine public basic and applied research, the shares of total applied and basic research should be the social planner's choice. However, calibration results suggest that utility is higher when we set applied research to 0 and find the welfare maximizing value of public basic research. The results show that there exist optimal amounts of public research that may improve the growth performance of the economy but they also show that too much public research may damage growth and that its composition regarding basic and applied components is a key aspect to take into account when designing an optimal research policy.

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1 Introduction

Some OECD countries ... have announced cuts in their annual budget provisions for R&D and tertiary education... This reduces resources for public research and private R&D activities in the short term, and could lead, over the longer term, to declines in the human resources available for innovation. ... Austria, Germany, Korea and the United States have recently increased investment in the science base, strengthening public research and human resources in order to improve future innovation and growth prospects.

OECD Science, Technology and Industry Outlook 2010.

Endogenous growth theory establishes a relationship between R&D and productivity. Both the expanding variety model in Romer (1990) and the Schumpeterian model in Aghion and Howitt (1998) identify R&D investment as the source of growth. Empirical studies like Ha and Howitt (2007) and Madsen (2008) find evidence that support this positive effect of R&D on productivity. A sepparate branch of empirical studies has documented the influence of R&D stock on the level of TFP in the OECD countries (Coe and Helpman 1995; Guellec and Van Pottelsberghe de la Potterie 2004 among others). These results endorse research policies that promote R&D investment in order to mantain long run growth. However, the effectiveness of the various instruments of research policy are not so well documented. In particular, increasing R&D performed by higher education and governement research institutes may not have an equivalent effect on productivity as business R&D has. The objective of this paper is to introduce public research in an R&D model of endogenous growth and to consider the optimal allocation of public resources to basic and applied research.

There are few papers that consider public research investment from a macroeconomic perspective. Glomm and Ravikumar (1994) and Pelloni (1997) present models in which the economy grows thanks to public research but do not allow for private research. On the contrary, Park (1998) considers both public and private research. Public research indirectly contributes to economic growth because it causes a positive external effect on the knowledge accumulation of the private sector. However, the paper is mainly concerned with open economies issues and international spillovers rather than with public research policy and does not distinguish between basic and applied research. Indeed, the difference between basic and applied research is absent from all the papers previously mentioned. Very few authors have tried to address this issue, especially in a dynamic macroeconomic context. Adams (1990) finds that the expansion of academic knowledge exerts a positive effect on productivity and technological growth, though its effects suffer a long lag of 20 years. Aghion and Howitt (1996) identifies basic research with horizontal innovation and Aghion et al (2005) find that in the early stages of the innovation process it is optimal to opt for a public innovation system. The paper by David (2000) reviews the literature focusing on the differences between basic and applied research and the need for public provision of basic knowledge. Gersbach et al (2009) combines the expanding variety growth model with a hierarchy of basic and applied research in which basic research is exclusively financed by government and applied R&D is performed solely by private firms. Cozzi and Galli (2009) introduces basic and applied research in a Schumpeterian growth model in which basic research is performed only by non-profit motivated R&D units. Basic research creates new half-ideas that will be developed later by other firms engaged in a patent race to implement the new quality of the product. The paper emphasizes the need for an incentive scheme to motivate public researchers due to its influence on aggregate innovation. Morales (2004) considers several research policy instruments in the context of a Schumpeterian growth model. The present paper recovers and deepens the public research provision analysis of the previous paper, looking for the optimal amounts of basic and applied research and considering whether public intervention can replicate this result by means of publicly funded research. It adds to previous literature in the following senses: It allows for both the private and the public sector to perform both types of research. Moreover, since it uses a schumpeterian framework, it allows for the existence of crowding out of private research as a consequence of public R&D activities.

Regarding empirical studies on the influence of R&D expenditures on productivity growth, Griliches (1986) finds evidence of the positive effects of both publicly financed R&D and basic research while Mansfield (1995) analyses the interaction between academic research and industrial innovation. Guellec and Van Pottelsberghe de la Potterie (2003) attempts to quantify the aggregate net effect of government funding on business R&D. They find that government-performed research has a negative impact on business funded R&D while the effect of university research is zero. Guellec and Van Pottelsberghe de la Potterie (2004) show that R&D performed by the public sector has a positive and significant impact on multifactor productivity growth. They also find that the elasticity of MFP with respect to public research is higher when the business R&D intensity of the economy is higher, when the share of universities in public research is higher and when the business-funding of university research is lower.

The analysis in this paper introduces public research in Aghion and Howitt (1998) R&D growth model. Public research will induce more competition and a crowding-out effect which may counteract the positive spillovers caused by the increased research activity. These opposite effects depend upon the shares of basic and applied research performed by the public sector. Basic research is not generally intended to obtain marketable innovations. Thus, succesful basic research produces new knowledge though, withouth some kind of applied research, it will not give raise to a new product or technology. This does not imply that basic research is useless for private firms since it may increase the productivity of applied efforts facilitating the absorption of spillovers from other sectors or the understanding of the knowledge base. Therefore, private firms will decide the allocation of research investment to basic and applied fields in order to maximise their probability of success. However this choice nay not be optimal for the aggregate economy since the effects of basic research on the probability of success of a firm's research project are not the same as the effects of basic research on aggregate knowledge. We have already mentioned the results of Adams (1990) regarding the long-lagged effects of basic research on firms productivity. Guellec and Van Pottelsberghe de la Potterie (2004) find that the "the long-term impact of R&D seems to be higher when it is performed by the public sector than by the business sector, probably because the former concentrates more on basic research, which is known to generate more externalities. As more uncertainty is associated with basic research, it is logically associated with a higher social return."

Even if basic research may not be essential in order to obtain a private patentable innovation, it should be considered as essential for the long-run growth of knowledge. Basic and applied research are viewed as complementary factors in a knowledge accumulation function in which basic research opens new fields that should be explored and turn usable by applied research. If no basic research is performed, applied research will be able to produce new knowledge for a while but not in the long run.

The results show that there is an optimal combination of basic and applied research determined by the contributions of both types of research to knowledge growth. The market fails to obtain the optimum because private research firms do not internalize the spillovers of research activity. In particular, they systematically invest a larger share of applied research than the socially optimal. Therefore, the role of public research should be to compensate this difference so as to approach the total shares of basic and applied research to the optimum.

The rest of the paper is divided in the following sections: section 2 presents the empirical evidence and motivation, section 3 outlines the model, section 4 analyzes the steady state and the last two sections present the results and the conclusions.

2 Empirical evidence and motivation

Business enterprise expenditure on R&D (BERD) is typically the major component of R&D activity and is normally oriented to find new products or techniques that can be exploited commercially. The applied share of BERD for the OECD average in the last decades has been 95% and for the US was a 96%. The applied component of public research is substantially smaller, particularly the applied share of higher education research with an average of 47% and a 30% for the US, but also of government R&D, with applied shares of 72% for the average and 76% for the US. These figures indicate a clearly different strategy of resource allocation to basic and applied research depending on the sector of performance. Furthermore, we observe significant differences for the applied share of publicly performed research accross countries, being China the country with larger applied shares (91% for the government and 81% for higher education) and the Slovak Republic the country with the lowest public shares of applied research (38% and 16%). Which is the right strategy? Which is the optimal share of applied research for the government and higher education sectors?

According to OECD Science, Technology and Industry Outlook 2010, total OECD BERD has been on a strong upward trend since 1998. As a percentage of GDP, Japan and the United States had a BERD intensity of 2.7% and 2% respectively, while for the EU27 it was 1.1% in 2008. Regarding higher education and government expenditure in R&D, its share of GDP is considerably smaller though it varies greatly across countries ranging from a 0.27% of the Slovak Republic to 1.12% in Iceland. The public research intensity for Japan, the United States and EU 27 was around 0.65%. Again a question arises, are countries spending too much or too little in public research? The European Union has established that total R&D spending should reach a 3% of GDP and that public R&D should be around 1% of GDP. Are these the optimal amounts? What is the basis to propose that one third of total R&D spending should be publicly financed?

3 The model

This paper modifies the R&D-based growth model of Aghion and Howitt (1998) introducing public research on one hand and the difference between basic and applied research on the other. Since the relevant differences regard the research sector, the specification of knowledge accumulation and the public budget, the rest of the model will be briefly outlined referring the reader to the work of Aghion and Howitt for further details.

Share of applied R&D expenditure by sector of performance

	θa business	θa gov	θa hes	θa total
Australia	0,95	0,71	0,41	0,73
Austria	0,95	0,79	0,52	0,82
China	0,99	0,91	0,81	0,95
Chezq Rep.	0,97	0,37	0,47	0,76
Denmark	0,96	0,80	0,44	0,84
France	0,96	0,79	0,13	0,78
Germany	0,95	0,61	0,25	0,80
Hungary	0,95	0,48	0,55	0,72
Iceland	0,99	0,78	0,53	0,79
Ireland	0,92	0,80	0,54	0,79
Israel	0,95	0,80	0,25	0,83
Italy	0,96	0,65	0,48	0,78
Korea	0,91	0,77	0,61	0,86
Japan	0,94	0,81	0,66	0,88
Mexico	0,95	0,68	0,59	0,70
Netherlands	0,84	0,68	0,98	0,85
New Zealand	0,89	0,55	0,42	0,65
Norway	0,98	0,85	0,52	0,84
Poland	0,94	0,52	0,42	0,63
Portugal	0,97	0,92	0,54	0,78
Russia	0,97	0,58	0,66	0,86
Slovak	0,94	0,38	0,16	0,67
Spain	0,94	0,76	0,51	0,80
Sweden	0,98	0,67	0,30	0,77
Switzerland	0,90	0,96	0,17	0,71
Turkey	0,93	0,84		
UK	0,95	0,80		
US	0,96	0,76	0,30	0,84
Average	0,95	0,72	0,47	0,79

Figure 1: Source: OECD Science, Technology and Industry Outlook 2010 and author's calculations.

3.1 Consumers

There exists an infinitely lived representative consumer whose utility function is assumed to be logarithmic for the sake of simplicity. Consequently, the lifetime utility of the consumer will be given by the following expression:

$$V(C_t) = \int_0^\infty \ln(C_t) e^{-\rho t} dt, \tag{1}$$

where C_t is consumption at time t and ρ is the rate of discount.

3.2 Final good sector

The consumption good is produced in a competitive market out of labor and intermediate goods. Labor is represented by a continuous mass of individuals L, which is assumed to be inelastically supplied and normalized to 1. Intermediate goods are produced by a continuum of sectors of mass 1, being x_{it} the supply of sector i at date t. The production function is a Cobb-Douglas with constant returns on intermediate goods and efficiency units of labor

$$Y_t = L^{1-\alpha} \int_0^1 A_{it} x_{it}^{\alpha} di$$

where Y_t is final good production and A_{it} is the productivity coefficient of each sector. The evolution of each sector's productivity coefficient A_{it} is determined in the research sector. I assume equal factor intensity to simplify calculations.

3.3 Intermediate goods

Intermediate goods are used as factors of production in the final good sector. Each sector has a monopolistic structure. In order to become the monopolist producer of an intermediate good, the entrepreneur has to buy the patent of the latest version of the product. This patent gives him the right to produce the good until an innovation occurs and the monopolist is displaced by the owner of the new technology. The innovator enters in Bertrand competition with the incumbent who produces an inferior quality of the product. Since the innovator is a superior rival, the incumbent exits and cannot threaten to reenter. Therefore, the innovator can charge the unconstrained monopolist price.

The only input in the production of intermediate goods is capital. In particular, it is assumed that A_{it} units of capital are needed to produce one unit of intermediate good i at date t. This implies that more productive intermediate inputs are more capital intensive, an assumption that simplifies the analysis and has no important implications under the Cobb-Douglas conditions.

Capital is rented in a perfectly competitive market at rate R_t. Hence, the monopolist problem is

$$\max_{x_{it}} p\left(x_{it}\right) x_{it} - A_{it} R_t x_{it},$$

where $p\left(x_{it}\right) = \alpha A_{it} x_{it}^{\alpha-1}$ is the marginal product of intermediate good i.

Due to the assumption of equal factor intensity, supply of intermediate goods is equal in all sectors, $x_{it} = x_t$. Thus, the aggregate demand of capital is equal to $\int_0^1 A_{it} x_t di$. Let $A_t = \int_0^1 A_{it} di$, be the aggregate productivity coefficient. Then, equilibrium in the capital market requires demand to equal supply

$$A_t x_t = K_t,$$

or equivalently, the flow of intermediate output must be equal to capital intensity, k_t

$$x_t = k_t = \frac{K_t}{A_t}.$$

In equilibrium the flow of profits of each intermediate firm and the rental rate will thus be given by the following expressions:

$$\begin{aligned} \pi_{it} &= \alpha(1-\alpha) A_{it} k_t^{\alpha} \\ R_t &= \alpha^2 k_t^{\alpha-1}. \end{aligned} \tag{2}$$

3.4 Research sector

For each of the above intermediate sectors, there is a number of research firms competing in a patent race to get the next innovation. Innovations are produced using the same technology of the final good. Hence, it needs physical capital (embodied in the intermediate goods) apart from labor to be produced. Technology is assumed to be increasingly complex and hence further innovations will require higher investments. Accordingly, the amount invested in research in each sector N_{it} will be adjusted by a coefficient representing the aggregate state of knowledge. This coefficient will be given by $A_t^{\max}$, the productivity parameter of the leading edge technology. Hence, we may define $n_{it} = \frac{N_{it}}{A_t^{\max}}$ as the productivity adjusted level of research.

I propose an innovation production function in which each firm invests n_j effective units of output in research. The firm obtains a probability of obtaining an innovation, $h(n_j)$. The value of the innovation, v, is given by the present value of the flow of profits that would be obtained from the monopolistic exploitation of the new product or variety. That is

$$v_t = \frac{\tilde{\pi}}{r_t + p_t},$$

where p_t stands for the probability of innovation in that sector and $\tilde{\pi} = \frac{\pi}{A_t^{\max}}$. The technology for innovation that I propose is the following:

$$h(n_a, n_b) = \lambda (n_a)^c (n_a + B n_b)^{1-c},$$

where λ, c, B are all positive and $c \leq 1$. This specification considers basic and applied research investment as inputs of the innovation function with some degree of substitutability but identifying applied research as essential in order to get a profitable innovation. The choice of exponents also yields a constant returns technology that simplifies the research market analysis and does not contradict the evidence.[1]

[1] Buscar referencias

Given n_j, the optimal shares of applied and basic research are

$$n_a = \begin{cases} \frac{cB}{(B-1)} n_j = \theta_a n_j & if \quad B \geq \frac{1}{1-c} \\ n_j & if \quad B < \frac{1}{1-c} \end{cases}$$

$$n_b = \begin{cases} \frac{(B-1)-cB}{(B-1)} n_j = (1-\theta_a) n_j & if \quad B \geq \frac{1}{1-c} \\ 0 & if \quad B < \frac{1}{1-c} \end{cases} .$$

Let

$$\theta = (\theta_a)^c (\theta_a + B(1-\theta_a))^{1-c}$$
$$\theta_a = \theta = 1 \ \ if \ \ B < \frac{1}{1-c},$$

then the innovation production function is given by the following expression:

$$h(n) = \lambda \theta n,$$

where $n = \sum_j n_j$ is the total amount of research in a sector. In what follows we will assume $B \geq \frac{1}{1-c}$ so that the firm performs a positive amount of basic research as observed on average in the data.

In a free entry market, competition would drive profits to zero so that the long-run equilibrium implies

$$\lambda \theta n_j \frac{\tilde{\pi}}{r + \lambda \theta n} - n_j = 0$$
$$\frac{\lambda \theta \tilde{\pi}}{r + \lambda \theta n} = 1.$$

Thus, the equilibrium of the market determines a total amount of investment in research n, while the number of firms remains undetermined. Introducing public research in this framework is simple. Public research would just add to the total amount of research investment contributing to the probability of reaching an innovation in the sector. Let $j(\Gamma_a, \Gamma_b)$ be the innovation technology for the public sector and Γ_a and Γ_b the public applied and basic research intensities for a given sector.[2,3] The research arbitrage condition with public research is given by the following expression:

$$\frac{\lambda \theta \tilde{\pi}}{r + \lambda \theta n + j(\Gamma_a, \Gamma_b)} = 1. \tag{3}$$

For the rest of the paper we will assume that the public sector has access to the same innovation technology of the private sector and therefore

$$j(\Gamma_a, \Gamma_b) = \lambda \Gamma_a^c (\Gamma_a + B\Gamma_b)^{1-c}.$$

Thus, the probability that an innovation occurs in a given sector will be given by $\psi(n) = \lambda \theta n + j(\Gamma_a, \Gamma_b)$. In order to express this flow probability of innovation in terms of private research, let us define $x = \theta_a + \frac{\Gamma_a}{n}$ and $y = (1-\theta_a) + \frac{\Gamma_b}{n}$ so that $\psi(n) = \lambda \chi n$, where $\chi = \theta + (x - \theta_a)\left(1 + B\frac{y-(1-\theta_a)}{x-\theta_a}\right)^{1-c}$.

[2] Let $\tilde{\Gamma}$ be the amount of output invested in research by the public sector. Then $\Gamma = \frac{\tilde{\Gamma}}{A_t^{\max}}$ is the level of research intensity.

[3] I assume that the amount of public research is the same for every intermediate sector. Since the mass of intermediate sectors is 1, aggregate and sectoral amounts coincide.

When an innovation occurs in a given sector, the productivity parameter of that sector jumps discontinuously to $A_t^{\max}$, the leading edge productivity coefficient. Thus, advances in other sectors spillover to the rest of the economy making the technology improvement induced by the next innovation more important. The evolution of $A_t^{\max}$ is determined by the evolution of the aggregate state of knowledge which in turn will be influenced by the levels of research intensity corresponding to the long-run equilibrium. I propose the following growth rate of knowledge

$$g_t = \frac{\dot{A}_t^{\max}}{A_t^{\max}} = \sigma\lambda \left(n_a + \Gamma_a\right)^{\phi} \left(n_b + \Gamma_b\right)^{1-\phi},$$

where ϕ and $1-\phi$, with $0 < \phi \leq 1$ represent the contributions of applied and basic research to knowledge growth. I use the Cobb-Douglas functional form for simplicity but also because it captures the extended idea that both types of research are necessary to guarantee long-run knowledge growth. Using the definitions of x and y introduced before we have

$$g_t = \frac{\dot{A}_t^{\max}}{A_t^{\max}} = \sigma x^{\phi} y^{1-\phi} \lambda n_t,$$

where we observe that the relative amounts of public to private research may alter the aggregate effect on knowledge growth.

3.5 Capital market

Equilibrium in the capital market requires the cost of one unit of capital to equal the rental rate, that is

$$r_t + \delta = \alpha^2 k_t^{\alpha-1}, \tag{4}$$

where we are using equation (2) and r_t, and δ are the interest rate and the depreciation rate.

3.6 Public budget

In order to close the model we need a public budget specifying how public research is financed. We allow for lump-sum taxation, thus

$$\begin{aligned} T_t &= \tilde{\Gamma} = \tilde{\Gamma}_a + \tilde{\Gamma}_b \\ \frac{T_t}{A_t^{\max}} &= \tau = \Gamma_a + \Gamma_b \end{aligned}$$

4 Dynamics

The dynamic system is formed by the following equations:

$$Case\,(0) \quad \begin{array}{l} \left\{ \begin{array}{l} \dot{k}_t = k_t^{\alpha} - c_t - \left(n_t + \Gamma_a + \Gamma_b\right)\Omega - \left(\delta + g_t\right) k_t \\ \dot{c}_t = c_t \left(r_t - \rho - g_t\right) \end{array} \right. \\ n_t = \tilde{\pi}\left(k_t\right) - \frac{r_t + j(\Gamma_a, \Gamma_b)}{\lambda\theta} \\ g_t = \frac{\dot{A}_t^{\max}}{A_t^{\max}} = g\left(n_t, \Gamma_a, \Gamma_b\right) = g\left(k_t\right) \\ r_t = R_t - \delta \end{array},$$

where we are using the research arbitrage condition (3) and the capital market equilibrium condition (**??**) to express the interest rate, r_t and research intensity, n_t as functions of capital intensity. Ω is a constant that relates $A_t^{\max}$ and A_t.[4] Given that the growth rate of technology is a function of research intensity it may also be considered as a function of capital intensity so that the relevant variables of the dynamic system are k_t and consumption per effective unit of output, c_t.This system presents saddle path stability.

5 Steady-state analysis

Given that the system is saddle path stable, we may perform comparative statics on the steady state of the economy. The long-run equilibrium will be given by the following expressions:

$$\begin{aligned} 1 &= \frac{\lambda\theta\tilde{\pi}}{g+\rho+\lambda\theta n+j\left(\Gamma_a,\Gamma_b\right)} \\ g+\rho+\delta &= \alpha^2 k^{\alpha-1}. \end{aligned}$$

A general result is that an increase in the amount of public research will reduce private research intensity. This result is due to the business-stealing effect caused by the increased competition in the patent race which reduces the value of the innovation and therefore, the incentives to invest in research. In addition, public research increases the cost of the factors of production used in R&D so that even if the business-stealing effect were not present, the crowding out of private research would still persist.

5.1 The market solution

Combine the three equations that define the steady state

$$\begin{aligned} 1 &= \frac{\lambda\theta\tilde{\pi}}{g+\rho+\lambda\theta n+j\left(\Gamma_a,\Gamma_b\right)} \\ g+\rho+\delta &= \alpha^2 k^{\alpha-1}. \\ g &= \sigma\lambda\left(n_a+\Gamma_a\right)^{\phi}\left(n_b+\Gamma_b\right)^{1-\phi} \end{aligned}$$

to obtain

$$F\left(n,\Gamma_a,\Gamma_b\right)=g-\lambda\theta\alpha\left(1-\alpha\right)\alpha^{\frac{2\alpha}{1-\alpha}}\left(g+\rho+\delta\right)^{\frac{-\alpha}{1-\alpha}}+\lambda\theta n+j\left(\Gamma_a,\Gamma_b\right)+\rho=0. \tag{5}$$

Equation (5) implicitly defines n as a function of Γ_a and Γ_b. Let us refer to this solution as n_M or the research intensity chosen by the market for any given values of public research. Given Γ_a and Γ_b we can obtain the equilibrium research intensity n_M, the growth rate g_M and the level of capital intensity, k_M.

Proposition 1 *Private research intensity, n_M decreases when either applied or basic public research intensity are increased.*

[4] See Appendix.

Proof. Implicit differentiation yields $\frac{dn}{d\Gamma_i} = -\frac{\frac{\partial F}{\partial \Gamma_i}}{\frac{\partial F}{\partial n}} = -\frac{(1+H)\frac{\partial g}{\partial \Gamma_i} + \frac{\partial j}{\partial \Gamma_i}}{(1+H)\frac{\partial g}{\partial n} + \lambda\theta} < 0$, where $i = a, b$ and $H = \lambda\theta\alpha^{\frac{2}{1-\alpha}} (g + \rho + \delta)^{\frac{-\alpha}{1-\alpha} - 1}$. ■

Due to the crowding-out effect, n_M decreases with Γ_a and Γ_b, however, the effect on growth and welfare depends upon the actual levels of public research intensity since the crowding out may be compensated by the positive effect on knowledge accumulation.

Proposition 2 *The effect on growth of either type of public research will be positive whenever the relative impact of public research on growth is larger than the business-stealing effect it induces on private research, i.e. when*

$$\frac{\frac{\partial g}{\partial \Gamma_i}}{\frac{\partial g}{\partial n}} > \frac{\frac{\partial j}{\partial \Gamma_i}}{\lambda\theta} \tag{6}$$

Proof. The effect on growth of Γ_i, for $i = a, b$ is given by $\frac{dg}{d\Gamma_i} = \frac{\partial g}{\partial n}\frac{dn}{d\Gamma_i} + \frac{\partial g}{\partial \Gamma_i} = \frac{\lambda\theta \frac{\partial g}{\partial \Gamma_i} - \frac{\partial j}{\partial \Gamma_i}\frac{\partial g}{\partial n}}{(1+H)\frac{\partial g}{\partial n} + \lambda\theta}$. ■

The expression in equation (6) depends upon the actual levels of Γ_a and Γ_b, upon the contributions of each type of research to knowledge growth but also upon the private research intensity. The following propositions analyse each type of public research in more detail.

Proposition 3 *For public applied research the effect on growth will be positive iff*

$$\phi\theta \geq \frac{(x - \phi(x - \theta_a))(x - \theta_a) + (xc - \phi(x - \theta_a)) B (y - (1 - \theta_a))}{(x - \theta_a)^{1-c} (x - \theta_a + B(y - (1 - \theta_a)))^c},$$

where $x = \theta_a + \frac{\Gamma_a}{n}$, $y = (1 - \theta_a) + \frac{\Gamma_b}{n}$. *For* y *large enough, there exists a non-empty interval of values of* x *for which this condition is satisfied.*

$$\frac{\left(\theta_a + \frac{\Gamma_a}{n} - \phi\frac{\Gamma_a}{n}\right)\frac{\Gamma_a}{n} + \left(\left(\theta_a + \frac{\Gamma_a}{n}\right)c - \phi\frac{\Gamma_a}{n}\right) B\frac{\Gamma_b}{n}}{\frac{\Gamma_a}{n}^{1-c}\left(\frac{\Gamma_a}{n} + B\frac{\Gamma_b}{n}\right)^c} = \frac{\left(\theta_a + (1-\phi)\frac{\Gamma_a}{n}\right) + \left(c\theta_a + (c - \phi)\frac{\Gamma_a}{n}\right) B\frac{\Gamma_b}{\Gamma_a}}{\left(1 + B\frac{\Gamma_b}{\Gamma_a}\right)^c} = \frac{(\theta_a + (1-\phi)(x - \theta_a))}{(}$$

Proof. $\frac{dg}{dx} \geq 0 \iff \frac{\partial g}{\partial x}\frac{\partial h}{\partial n} + \frac{\partial g}{\partial x}\frac{\partial j}{\partial n} - \frac{\partial g}{\partial n}\frac{\partial j}{\partial x} \geq 0 \iff \phi\theta \geq \left(\frac{(x-\phi(x-\theta_a))(x-\theta_a)+(xc-\phi(x-\theta_a))B(y-(1-\theta_a))}{(x-\theta_a)^{1-c}(x-\theta_a+B(y-(1-\theta_a)))^c}\right)$. Assuming $\phi > c$, $\left(\frac{(x-\phi(x-\theta_a))(x-\theta_a)+(xc-\phi(x-\theta_a))B(y-(1-\theta_a))}{(x-\theta_a)^{1-c}(x-\theta_a+B(y-(1-\theta_a)))^c}\right)$ is decreasing up to a point $x^{\min}$ and then increasing. That minimum may yield a value of the function smaller than $\phi\theta$ if y is large enough. Therefore, there may exist an interval for x for which $\frac{dg}{dx} > 0$ given a value of y large enough. ■

The definition of $x - \theta_a$ and $y - (1 - \theta_a)$ as the amount of each type of public research with respect to private research makes it easier to explore the effects of public applied research on growth. We find that at $\Gamma_a = 0$ i.e. $x = \theta_a$, $\frac{\partial j}{\partial \Gamma_a} = +\infty$ for any values of public basic research yielding $\frac{dg}{d\Gamma_a} = -\infty$. We also observe that for $x > \theta_a$, there could exist a range of values for which public applied research would be growth promoting provided that public basic research is large enough.

Proposition 4 *The effect on growth of public basic research will be positive iff*

$$(1 - \phi)\theta \geq (x - \theta_a)^c \left(\frac{(\phi - c) By + (1 - \phi) B (1 - \theta_a) - (1 - \phi)(x - \theta_a)}{(x - \theta_a + B(y - (1 - \theta_a)))^c}\right),$$

Proof. $\frac{dg}{dy} \geq 0 \iff \frac{\partial g}{\partial y}\frac{\partial h}{\partial n}+\frac{\partial g}{\partial y}\frac{\partial j}{\partial n}-\frac{\partial g}{\partial n}\frac{\partial j}{\partial y} \geq 0 \iff (1-\phi)\,\theta \geq \left(\frac{x-\theta_a}{x-\theta_a+B(y-(1-\theta_a))}\right)^c ((\phi-c)\,By+(1-\phi)\,B\,(1-\theta_a)$
This is true for $x=\theta_a$ and for any $y \leq \frac{(1-\phi)(x-\theta_a)-(1-\phi)B(1-\theta_a)}{(\phi-c)B}$. ■

As in Morales (2004), we find that the effect on growth of public basic research is unambigously growth promoting when $\Gamma_a = 0$ (i.e. when $x=\theta_a$) because there is no business stealing in that case. Notice also that a sufficient condition to obtain positive growth effects of public basic research when public applied research is positive is to have x large enough so that $y \leq \frac{(1-\phi)(x-\theta_a)-(1-\phi)B(1-\theta_a)}{(\phi-c)B}$ may be satisfied for positive values of $y-(1-\theta_a)$, that is, of Γ_b. Intuitively, if no public applied research is performed, then public basic research is unambigously growth promoting. However, when public applied research is present, the business-stealing effect associated to it requires a large enough relative amount of public applied R&D in order to have a positive effect of public basic research.

Next section presents the centralized equilibrium solution which will serve us as reference to compare the market solution and the different research policies that could be performed.

5.2 The social planner's solution

The social planner's problem would be to choose the total amounts of basic and applied research in order to maximize consumer's utility, i.e.

$$\begin{aligned} & \max_{I_t, N_a, N_b} \int_0^\infty \ln(C_t)\, e^{-\rho t} dt \\ s.t. \quad &: \quad C_t = A_t^{1-\alpha} K_t^{\alpha} - I_t - N_{at} - N_{bt} \\ \dot{A}_t^{\max} &= \sigma\lambda\,(N_{at})^{\phi}\,(N_{bt})^{1-\phi} \\ \dot{K}_t &= I_t - \delta K_t, \end{aligned}$$

where N_{at} and N_{bt} are the total amounts of output devoted to applied and basic research respectively and I_t is capital investment. The first order conditions imply the following result:

Proposition 5 *The social planner's solution implies* $\frac{N_{bt}}{N_{at}} = \frac{1-\phi}{\phi}$. *The capital intensity,* k_S *is the solution to* $\sigma\lambda\Phi\,(1-\alpha)\,L^{1-\alpha}k^{\alpha} - \alpha L^{1-\alpha}k^{\alpha-1} + \delta = 0$ *and the growth rate* g_S *and research intensity* n_S *are determined by* $g_S = \sigma\lambda\Phi\,(1-\alpha)\,L^{1-\alpha}k_S^{\alpha} - \rho$ *and* $n_S = (1-\alpha)\,L^{1-\alpha}k_S^{\alpha} - \frac{\rho}{\sigma\lambda\Phi}$ *respectively.*

Proof. Inmediate from the solution of the social planner's problem. See appendix for details. ■

Thus, the social planner chooses the total amounts of basic and applied research so that they equal the relative contributions of each type of research to knowledge growth. If no public research is performed, the market equilibrium would yield $\frac{N_{bt}}{N_{at}} = \frac{1-\theta_a}{\theta_a}$. So for $\phi < \theta_a$, the social planner chooses a higher share of basic R&D and gets both a higher growth rate and level of welfare. Therefore, we could use public research to approach the decentralized equilibrium solution to the optimum.

A calibration exercise may help us compare the other results. Using the standard parameter values when possible and choosing the rest so as to calibrate the US economy we obtain[5]

[5] $B = \frac{\theta_a}{\theta_a - c}$; $\theta_a = 0.96$; $\phi = 0.75$; $c = 0.25$; $\alpha = 0.7$; $\rho = 0.05$; $\delta = 0.1$; $\lambda = 1.1$; $\sigma = 0.055$; $L = 1$

	Table 1		
	Market		Planner
	$\Gamma_a = \Gamma_b = 0$	$\Gamma_a = \frac{\phi n}{3}$, $\Gamma_b = \frac{(1-\phi)n}{3}$	$n_a = \phi n$, $n_b = (1-\phi)\,n$
n	1.01	0.68	2.43
g	2.65	2.85	8.37
k	30.06	28.95	38.72
$\frac{C_t}{A_t^{\max}}$	6.02	5.91	3.39
Utility	46.49	45.29	57.89

The firsts two columns of Table 1 compare the solutions of the market without public research and with an arbitrary combination of public R&D similar to the one suggested by data in most developed countries. The last column presents the social planner's solution. We observe that even though we are very far from the first best, public research can improve *both growth and welfare*. The next section looks for the optimal public research intensities.

5.3 Public research policy

In order to choose the amounts of public research the authorities may want to maximize growth, given the equilibrium relation between private research intensity and public research expressed by $F(n, \Gamma_a, \Gamma_b) = 0$.

$$\max_{\Gamma_a,\Gamma_b} g = \sigma\lambda\left(\theta_a n + \Gamma_a\right)^{\phi}\left(\left(1-\theta_a\right)n + \Gamma_b\right)^{1-\phi} \tag{7}$$

$$s.t. \quad : \quad \begin{array}{l} F(n, \Gamma_a, \Gamma_b) = g - \sigma\lambda\theta\alpha(1-\alpha)\,\alpha^{\frac{2\alpha}{1-\alpha}}L(g+\rho+\delta)^{\frac{-\alpha}{1-\alpha}} + \sigma\lambda\theta n + j(\Gamma_a,\Gamma_b) + \rho = 0 \\ C_t = L^{1-\alpha}A_t^{1-\alpha}K_t^{\alpha} - \dot{K}_t - \delta K_t - N_{at} - N_{bt} - \tilde{\Gamma}_a - \tilde{\Gamma}_b \\ \Gamma_a \geq 0; \Gamma_b \geq 0; n \geq 0; C_t \geq 0. \end{array}$$

Using the definitions of x and y we can rewrite the problem as

$$\max_{x,y} g = \sigma\lambda n x^{\phi} y^{1-\phi}$$

$$s.t. \quad : \quad \begin{array}{l} F(n,x,y) = g - \sigma\lambda\theta\alpha(1-\alpha)\,\alpha^{\frac{2\alpha}{1-\alpha}}L(g+\rho+\delta)^{-\frac{\alpha}{1-\alpha}} + \sigma\lambda\theta n + \sigma\lambda n\left(x-\theta_a\right)^{c}\left(x-\theta_a+B\left(y-\left(1-\theta_a\right)\right)\right)^{1-} \\ C_t = L^{1-\alpha}A_t^{1-\alpha}K_t^{\alpha} - \dot{K}_t - \delta K_t - N_{at} - N_{bt} - \tilde{\Gamma}_a - \tilde{\Gamma}_b \\ x - \theta_a \geq 0; y - (1-\theta_a) \geq 0; n \geq 0; C_t \geq 0. \end{array}$$

Proposition 6 *An interior candidate to the optimization problem in (7) must satisfy* $\frac{n_b+\Gamma_b}{n_a+\Gamma_a} = \frac{1-\phi}{\phi(1-c)B}\left(1 + cB\frac{\Gamma_b}{\Gamma_a}\right)$. *Alternatively, an interior solution to the problem expressed in terms of* x *and* y *must satisfy* $\frac{x}{y} = \frac{\phi}{(1-\phi)}\frac{(1-c)B}{1+cB\frac{(y-(1-\theta_a))}{(x-\theta_a)}}$. *Given* x*, this expression provides a unique value of* y *for which both first order conditions will be satisfied. Combining this expression with the non-negativity restrictions, we obtain a set of interior candidates for the maximization problem.*

Proof. FOC require $\frac{dg}{dx} = \frac{g}{n}\frac{dn}{dx} + \frac{\partial g}{\partial x} = 0$ and $\frac{dg}{dy} = \frac{g}{n}\frac{dn}{dy} + \frac{\partial g}{\partial y} = 0$. That is $\frac{\frac{\partial g}{\partial y}}{\frac{\partial j}{\partial y}} = \frac{\frac{\partial g}{\partial x}}{\frac{\partial j}{\partial x}} \iff \frac{\frac{(1-\phi)}{y}}{\frac{\partial j}{\partial y}} = \frac{\frac{\phi}{x}}{\frac{\partial j}{\partial x}} \iff \frac{\frac{(1-\phi)}{y}}{j\frac{(1-c)B}{(x-\theta_a+B(y-(1-\theta_a)))}} = \frac{\frac{\phi}{x}}{j\frac{x-\theta_a+cB(y-(1-\theta_a))}{(x-\theta_a)(x-\theta_a+B(y-(1-\theta_a)))}} \iff \frac{(1-\phi)}{\phi} = \frac{y}{x}\frac{(1-c)B}{1+cB\frac{(y-(1-\theta_a))}{(x-\theta_a)}}$. ■

The expression in proposition 6 implies that any interior candidate must satisfy $y-(1-\theta_a) = \frac{(1-\phi)}{B(\phi-c)} \frac{(x-\theta_a)\left(x-\frac{\phi(1-\theta_a)B(1}{1-\phi}\right.}{\left(x-\frac{\phi(1-c)\theta_a}{\phi-c}\right)}$ However, interior candidates may not attain a higher growth rate than corner solutions with $\Gamma_a = 0$. The calibration exercise will show that this is the case.

and that the growth maximizing combination of public R&D devotes all the available resources to basic research up to the point where there are no resources left for consumption. The intuition for this result is simple. Public basic research performed in isolation does not cause any business-stealing effect while its effect on growth is unambiguously positive. Thus, if the objective is to maximize growth, public basic research is to be raised up to the first binding restriction, in this case, the non-negativity of consumption.

Another interesting result in Proposition 6 is that if public research mimics the behaviour of the firm regarding the shares of basic and applied R&D as in the third column of Table 1, then an effective unit of output invested in research by the public sector crowds out exactly one unit of private research and the resulting rate of growth and level of welfare of this equilibrium would be identical to the ones without public research.

Notice that we have imposed $\theta_a > \phi$. This implies that the share of applied research in the private sector is larger than the contribution of applied research to productivity growth. Estimates of θ_a are provided in section 2 with an average of 0.95. Up to my knowledge there are no direct estimates of ϕ, however, several empirical studies on the effect of basic research on productivity suggest that it is larger than perceived by private firms due to its long-term effects and generation of spillovers.[6] Thus, private firms would be underinvesting in basic research, which suggests that $(1-\theta_a)$ is smaller than $(1-\phi)$, the contribution of basic research to productivity and therefore, $\theta_a > \phi$. Consequently, in what follows we will consider only this case even though the model could be used for the opposite assumption.

When the objective function is welfare rather than growth, the policy maker problem would be the following:

$$\max_{\Gamma_a,\Gamma_b} \int_0^\infty e^{-\rho t} \ln(C_t)\,dt = \int_0^\infty e^{-\rho t} \left(\ln(c) + \ln\left(A_0 e^{gt}\right)\right) dt = \frac{\ln A_0}{\rho} + \frac{\ln c}{\rho} + \frac{g}{\rho^2}, \tag{8}$$

where $c = \frac{C_t}{A_t^{\max}}$ is consumption intensity at the steady state.

$$\frac{x}{y} = \frac{\phi}{1-\phi}.$$

Under this condition, welfare increases as public basic research (Γ_b or y) increases. This also implies that Γ_a must increase and that n will decrease. In fact, the simulation exercise shows that at some point welfare reaches a plateau and ceases to increase even if we continue increasing public R&D. In fact, global research intensity remains practically unchanged suggesting that what matters is not whether research is public or private but whether the appropriate shares of basic and applied R&D are kept.

$\Gamma_a = 0$ and $\Gamma_b > 0$ does not get larger welfare than the interior candidates even though it would get larger growth rates.

[6] See for example Adams (1990), OECD (2003) and Guellec and Van Pottelsberghe de la Potterie (2004).

Proposition 7 *Any interior welfare maximazing solution must satisfy* $\frac{n_a+\Gamma_a}{n_b+\Gamma_b} = \frac{\phi}{1-\phi}$*. Under this condition, welfare is increased as public applied research approaches 0. For* $\Gamma_a = 0$*, the optimal amount of public basic research is* $\Gamma_b = \Gamma_b^*$ *where* $\Gamma_b^* = \arg\max_{\Gamma_b}\left(\frac{\ln A_0}{\rho} + \frac{\ln c}{\rho} + \frac{g}{\rho^2}\right)$.

Proof. See appendix. ∎

The result established in Proposition 7 is quite intuitive given what we obtained in Proposition 6. In order to maximize welfare we set public applied research to 0 and increase public basic R&D up to the point at which marginally increasing Γ_b causes an effect on growth that is just compensated by the reduction in consumption. For the calibration values used in Table 1, the welfare maximising policy would be the fourth column of the following table:

	Table 2			
	Market		**Planner**	**ORP**
	$\Gamma_a = \Gamma_b = 0$	$\Gamma_a = \frac{\phi n}{3}$, $\Gamma_b = \frac{(1-\phi)n}{3}$	$n_a = \phi n$, $n_b = (1-\phi)\,n$	$\Gamma_a = 0$, $\Gamma_b = 0.175$
n	1.01	0.68	2.43	0.76
g	2.65	2.85	8.37	3.21
k	30.06	28.95	38.72	27.1
c	6.02	5.91	3.39	5.56
Utility	46.49	45.29	57.89	47.14

6 Conclusions

This paper analyses the growth and welfare effects of public research. It highlights the differences between public basic research and applied research regarding the spillovers and crowding out effects of each type of R&D. The results suggest that the business sector underinvests in research and that this research is too much applied oriented. This is due to the fact that the research firm does not internalize the social benefits from innovation and in particular, those of basic research which usually suffer from a long delay in time and are harder to incorporate to the firms productivity. Thus, a social planner would choose a higher research intensity and a higher weight of basic research in total R&D expenditure. Moreover, public basic research is unambigously growth promoting if performed in isolation, while the growth maximizing amount of public applied research would be 0. Regarding welfare, if we combine public basic and applied research, the shares of total applied and basic research should be the social planner's choice. However, calibration results suggest that utility is higher when we set applied research to 0 and find the welfare maximizing value of public basic research. The results show that there exist optimal amounts of public research that may improve the growth performance of the economy but they also show that too much public research may damage growth and that its composition regarding basic and applied components is a key aspect to take into account when designing an optimal research policy.

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7 Appendix

7.1 Relationship between $A_t^{\max}$ and A_t

Following Aghion and Howitt (1998) one can show that the ratio $\frac{A_t^{\max}}{A_t}$ will converge monotonically to a constant Ω. Each innovation replaces A_{it} with the leading edge $A_t^{\max}$. Since innovations occur at the rate $\psi_t = \chi\lambda n_t$ per product and the average change across innovating sectors is $A_t^{\max} - A_t$, we have

$$\dot{A}_t = \psi_t \left(A_t^{\max} - A_t\right).$$

Since growth in the leading edge is given by $\frac{\dot{A}_t^{\max}}{A_t^{\max}} = \sigma x^{\phi} y^{1-\phi}\lambda n_t = \frac{\sigma x^{\phi} y^{1-\phi}}{\chi}\psi_t$, the ratio $\Omega_t = \frac{A_t^{\max}}{A_t}$ evolves according to

$$\frac{\dot{\Omega}_t}{\Omega_t} = \frac{\sigma x^{\phi} y^{1-\phi}}{\chi}\psi_t - \psi_t \left(\Omega_t - 1\right).$$

As long as ψ_t is bounded above 0, Ω_t will converge asymptotically to $\Omega = 1 + \frac{\sigma x^{\phi} y^{1-\phi}}{\chi}$.

Proof. Proof of Proposition 5.

The corresponding Hamiltonian is

$$H\left(I_t, N_a, N_b\right) = \ln\left(A_t^{1-\alpha} K_t^{\alpha} - I_t - N_{at} - N_{bt}\right) e^{-\rho t} + \nu_t \left(I_t - \delta K_t\right) + \varepsilon_t \sigma\lambda \left(N_{at}\right)^{\phi} \left(N_{bt}\right)^{1-\phi}$$

and the first order conditions:

$$\begin{aligned}
\frac{\partial H}{\partial I_t} &= \frac{-e^{-\rho t}}{C_t} + \nu_t = 0 &(9)\\
\frac{\partial H}{\partial K_t} &= \frac{\alpha e^{-\rho t} A_t^{1-\alpha} K_t^{\alpha-1}}{C_t} - \delta \nu_t = -\dot{\nu}_t &(10)\\
\frac{\partial H}{\partial N_{at}} &= \frac{-e^{-\rho t}}{C_t} + \varepsilon_t \sigma \lambda \phi \left(\frac{N_{bt}}{N_{at}}\right)^{1-\phi} = 0 &(11)\\
\frac{\partial H}{\partial N_{bt}} &= \frac{-e^{-\rho t}}{C_t} + \varepsilon_t \sigma \lambda (1-\phi) \left(\frac{N_{bt}}{N_{at}}\right)^{-\phi} = 0 &(12)\\
\frac{\partial H}{\partial A_t} &= \frac{e^{-\rho t}(1-\alpha) A_t^{-\alpha} K_t^{\alpha}}{\Omega C_t} = -\dot{\varepsilon}_t &(13)
\end{aligned}$$

Conditions (11) and (12) together imply $\frac{N_{bt}}{N_{at}} = \frac{1-\phi}{\phi}$. From condition (9) we get $\frac{\dot{\nu}_t}{\nu_t} = -\rho - \frac{\dot{C}_t}{C_t}$ and substituting in (10) we get $\frac{\dot{C}_t}{C_t} = \alpha k_t^{\alpha-1} - \delta - \rho$. From (12) and (13) we get $\frac{-e^{-\rho t}}{C_t} = -\varepsilon_t \sigma \lambda \phi \left(\frac{1-\phi}{\phi}\right)^{1-\phi}$ and $\frac{\dot{C}_t}{C_t} = \frac{\sigma\lambda\phi\left(\frac{1-\phi}{\phi}\right)^{1-\phi}(1-\alpha)}{\Omega} k_t^{\alpha} - \rho$. Thus,

$$S(k) = \frac{\sigma\lambda\phi\left(\frac{1-\phi}{\phi}\right)^{1-\phi}(1-\alpha)}{\Omega} k_t^{\alpha} - \alpha k_t^{\alpha-1} + \delta = 0, \tag{14}$$

implicitly defines k_S, the level of capital intensity that the social planner would choose. Then, $g_S = \frac{\sigma\lambda\phi\left(\frac{1-\phi}{\phi}\right)^{1-\phi}(1-\alpha)}{\Omega} k_S^{\alpha} - \rho$ is the growth rate corresponding to the social planner's solution and since $g_S = \sigma\lambda\phi\left(\frac{1-\phi}{\phi}\right)^{1-\phi} n_S$, we can obtain n_S, the research intensity associated to k_S as

$$n_S = \frac{(1-\alpha)}{\Omega} k_S^{\alpha} - \frac{\rho}{\sigma\lambda\Phi\phi}.$$

■

Proof. Proof of Proposition 6.

The FOC require

$$\begin{aligned}
\frac{dg}{d\Gamma_a} &= \frac{\partial g}{\partial \Gamma_a} + \frac{\partial g}{\partial n}\frac{dn}{d\Gamma_a} \leq 0 \text{ with equality if } \Gamma_a > 0 &(15)\\
\frac{dg}{d\Gamma_b} &= \frac{\partial g}{\partial \Gamma_b} + \frac{\partial g}{\partial n}\frac{dn}{d\Gamma_b} \leq 0 \text{ with equality if } \Gamma_b > 0 &(16)\\
\Gamma_a &\geq 0 \text{ with equality if } \frac{dg(0,\Gamma_b)}{d\Gamma_a} < 0\\
\Gamma_b &\geq 0 \text{ with equality if } \frac{dg(\Gamma_a,0)}{d\Gamma_b} < 0
\end{aligned}$$

Combining conditions (15) and (16) with equality and using the expressions for $\frac{dn}{d\Gamma_a}$ and $\frac{dn}{d\Gamma_b}$ useed in the proof of proposition 1, we find that any interior candidate must satisfy the following condition:

$$\begin{aligned}
\frac{n_b+\Gamma_b}{n_a+\Gamma_a} &= \frac{1-\phi}{\phi(1-c)B}\left(1 + cB\frac{\Gamma_b}{\Gamma_a}\right)\\
\Gamma_a &> 0; \text{ and } \Gamma_b > 0
\end{aligned}$$

$\lim_{\Gamma_a \to 0} \frac{dg}{d\Gamma_a} = -\infty$ due to the business stealing effect of public applied R&D, which at that point would be $-\infty$.

Regarding public basic research, $\frac{dg(0,\Gamma_b)}{d\Gamma_b} > 0$ because for $\Gamma_a = 0$, there is no business stealing while basic research contributes to knowledge growth. Thus, if we were not constrained by the financing of public R&D, it would be optimal to increase basic research without bound. Given that public research is financed by lump-sum taxes, the bound is the non-negativity of consumption, that is $\left(\rho + \left(1-\alpha^2\right)(\delta+g)\right)\alpha^{\frac{2\alpha}{1-\alpha}} L(\rho+\delta+g)^{\frac{-1}{1-\alpha}} - n - \Gamma_b \geq 0$ ■

Proof. Proof of Proposition 7..

the FOC require

$$\frac{d\left(\ln c + \frac{g}{\rho}\right)}{d\Gamma_a} \leq 0 \text{ with equality if } \Gamma_a > 0 \tag{17}$$

$$\frac{d\left(\ln c + \frac{g}{\rho}\right)}{d\Gamma_b} \leq 0 \text{ with equality if } \Gamma_b > 0 \tag{18}$$

$$\Gamma_a \geq 0 \text{ with equality if } \left.\frac{d\left(\ln c + \frac{g}{\rho}\right)}{d\Gamma_a}\right|_{\Gamma_a=0} < 0$$

$$\Gamma_b \geq 0 \text{ with equality if } \left.\frac{d\left(\ln c + \frac{g}{\rho}\right)}{d\Gamma_b}\right|_{\Gamma_b=0} < 0$$

For $\Gamma_a = 0$ we get a limit indetermination since $\lim_{\Gamma_a \to 0} \frac{dc}{d\Gamma_a} = +\infty$ while $\lim_{\Gamma_a \to 0} \frac{dg}{d\Gamma_a} = -\infty$ and they are infinities of the same order. Intuitively, marginally increasing Γ_a from 0 induces an infinite crowding out of private R&D which harms growth but that reduction of private investment allows for larger consumption.

The first two conditions are satisfied for $\frac{n_a+\Gamma_a}{n_b+\Gamma_b} = \frac{\phi}{1-\phi}$, that is, the social planner's share of applied and basic research. Thus, the following equations determine the optimal relationship that Γ_a and Γ_b must satisfy:

$$n = \frac{\phi\Gamma_b - (1-\phi)\Gamma_a}{(\theta_a - \phi)} \tag{19}$$

$$F(n,\Gamma_a,\Gamma_b) = \sigma\lambda\theta\alpha(1-\alpha)L^{1-\alpha}k^\alpha - g - \rho - \sigma\lambda\theta n - j(\Gamma_a,\Gamma_b) = 0 \tag{20}$$

$$\Gamma_b \geq \frac{(1-\phi)\Gamma_a}{\phi(\theta_a-\phi)} \text{ if } \theta_a > \phi \text{ and } \Gamma_b \leq \frac{(1-\phi)\Gamma_a}{\phi(\phi-\theta_a)} \text{ if } \theta_a < \phi.$$

If $\theta_a = \phi$, the optimal amounts of public research are 0 since the economy would be at its first best already.

Let $\Gamma_b = \Gamma(\Gamma_a)$ represent that optimal relation. The objective function would now be

$$\begin{aligned} u(\Gamma_a) &= \ln\left(\left(\rho + \left(1-\alpha^2\right)(\delta + g(\Gamma_a))\right)\alpha^{\frac{2\alpha}{1-\alpha}} L(\rho+\delta+g(\Gamma_a))^{\frac{-1}{1-\alpha}} - n(\Gamma_a) - \Gamma_a - \Gamma_b(\Gamma_a)\right) + \frac{g(\Gamma_a)}{\rho} \\ &= \ln\left(\left(\rho + \left(1-\alpha^2\right)(\delta+g)\right)\alpha^{\frac{2\alpha}{1-\alpha}} L(\rho+\delta+g)^{\frac{-1}{1-\alpha}} - \frac{g}{\sigma\lambda\Phi\phi}\right) + \frac{g}{\rho} \end{aligned}$$

$\frac{dU}{d\Gamma_a} = \left(\frac{\frac{dC}{dg}}{C} + \frac{1}{\rho}\right)\frac{dg}{d\Gamma_a}$.

$\frac{dg}{d\Gamma_a} \leq 0$. $\frac{dg}{d\Gamma_a} = \sigma\lambda\Phi\frac{\phi}{(\theta_a-\phi)}\left(\theta_a\frac{d\Gamma_b}{d\Gamma_a} - (1-\theta_a)\right)$

$\left(\theta_a\frac{d\Gamma_b}{d\Gamma_a} - (1-\theta_a)\right) = -\theta_a\frac{\left[\left(\left(\frac{\alpha}{1-\alpha}\right)H(\rho+g+\delta)^{\frac{-\alpha}{1-\alpha}-1}+1\right)\frac{\partial g}{\partial\Gamma_a}+\sigma\lambda\theta\frac{\partial n}{\partial\Gamma_a}+\frac{\partial j}{\partial\Gamma_a}\right]}{\left[\left(\left(\frac{\alpha}{1-\alpha}\right)H(\rho+g+\delta)^{\frac{-\alpha}{1-\alpha}-1}+1\right)\frac{\partial g}{\partial\Gamma_b}+\sigma\lambda\theta\frac{\partial n}{\partial\Gamma_b}+\frac{\partial j}{\partial\Gamma_b}\right]} - (1-\theta_a) =$

$$= -\left[\frac{\theta_a\left[\left(\left(\frac{\alpha}{1-\alpha}\right)H(\rho+g+\delta)^{\frac{-\alpha}{1-\alpha}-1}+1\right)\frac{\partial g}{\partial\Gamma_a}+\sigma\lambda\theta\frac{\partial n}{\partial\Gamma_a}+\frac{\partial j}{\partial\Gamma_a}\right]+(1-\theta_a)\left[\left(\left(\frac{\alpha}{1-\alpha}\right)H(\rho+g+\delta)^{\frac{-\alpha}{1-\alpha}-1}+1\right)\frac{\partial g}{\partial\Gamma_b}+\sigma\lambda\theta\frac{\partial n}{\partial\Gamma_b}+\frac{\partial j}{\partial\Gamma_b}\right]}{\left[\left(\left(\frac{\alpha}{1-\alpha}\right)H(\rho+g+\delta)^{\frac{-\alpha}{1-\alpha}-1}+1\right)\frac{\partial g}{\partial\Gamma_b}+\sigma\lambda\theta\frac{\partial n}{\partial\Gamma_b}+\frac{\partial j}{\partial\Gamma_b}\right]}\right]$$

$$\theta_a\left[\left(\left(\frac{\alpha}{1-\alpha}\right)H\left(\rho+g+\delta\right)^{\frac{-\alpha}{1-\alpha}-1}+1\right)\frac{\partial g}{\partial\Gamma_a}+\sigma\lambda\theta\frac{\partial n}{\partial\Gamma_a}+\frac{\partial j}{\partial\Gamma_a}\right]+$$
$$+\left(1-\theta_a\right)\left[\left(\left(\frac{\alpha}{1-\alpha}\right)H\left(\rho+g+\delta\right)^{\frac{-\alpha}{1-\alpha}-1}+1\right)\frac{\partial g}{\partial\Gamma_b}+\sigma\lambda\theta\frac{\partial n}{\partial\Gamma_b}+\frac{\partial j}{\partial\Gamma_b}\right]=$$
$$=\theta_a\left(\left(\frac{\alpha}{1-\alpha}\right)H\left(\rho+g+\delta\right)^{\frac{-\alpha}{1-\alpha}-1}+1\right)\frac{\partial g}{\partial\Gamma_a}+\theta_a\sigma\lambda\theta\frac{\partial n}{\partial\Gamma_a}+\theta_a\frac{\partial j}{\partial\Gamma_a}+$$
$$+\left(1-\theta_a\right)\left(\left(\frac{\alpha}{1-\alpha}\right)H\left(\rho+g+\delta\right)^{\frac{-\alpha}{1-\alpha}-1}+1\right)\frac{\partial g}{\partial\Gamma_b}+\left(1-\theta_a\right)\sigma\lambda\theta\frac{\partial n}{\partial\Gamma_b}+\left(1-\theta_a\right)\frac{\partial j}{\partial\Gamma_b}=$$
$$=\left(\left(\frac{\alpha}{1-\alpha}\right)H\left(\rho+g+\delta\right)^{\frac{-\alpha}{1-\alpha}-1}+1\right)\left(\theta_a\frac{\partial g}{\partial\Gamma_a}+\left(1-\theta_a\right)\frac{\partial g}{\partial\Gamma_b}\right)+\sigma\lambda\theta\left(\theta_a\frac{\partial n}{\partial\Gamma_a}+\left(1-\theta_a\right)\frac{\partial g}{\partial\Gamma_b}\right)+\theta_a\frac{\partial j}{\partial\Gamma_a}+\left(1-\theta_a\right)\frac{\partial j}{\partial\Gamma_b}=$$

$$=-\sigma\lambda\theta+\sigma\lambda\left(1+B\frac{\Gamma_b}{\Gamma_a}\right)^{-c}\left(\theta_a\left(1+cB\frac{\Gamma_b}{\Gamma_a}\right)+\left(1-\theta_a\right)\left(1-c\right)B\right)=$$
$$=-\sigma\lambda\left(\theta-\left(1+B\frac{\Gamma_b}{\Gamma_a}\right)^{-c}\left(\theta_a\left(1+cB\frac{\Gamma_b}{\Gamma_a}\right)+\left(1-\theta_a\right)\left(1-c\right)B\right)\right)$$

The function $\left(1+B\frac{\Gamma_b}{\Gamma_a}\right)^{-c}\left(\theta_a\left(1+cB\frac{\Gamma_b}{\Gamma_a}\right)+\left(1-\theta_a\right)\left(1-c\right)B\right)$ has a global minimum at $\frac{\Gamma_b}{\Gamma_a}=\frac{1-\theta_a}{\theta_a}$. Its value at that point is θ. Therefore, for $\theta_a>\phi$, $\frac{dg}{d\Gamma_a}\leq 0$.

Thus, the sign of this derivative depends upon the sign of $\left(\frac{\frac{dC}{dg}}{C}+\frac{1}{\rho}\right)$. From the first order condition $\frac{d\left(\ln c+\frac{g}{\rho}\right)}{d\Gamma_a}=0$ we know that $\left(\frac{dc}{dg}+\frac{c}{\rho}\right)=\frac{\frac{dn}{d\Gamma_a}+1}{\left(\frac{dg}{dn}\frac{dn}{d\Gamma_a}+\frac{\partial g}{\partial\Gamma_a}\right)}$. The right hand expression is a positive constant when evaluated at $\frac{n_a+\Gamma_a}{n_b+\Gamma_b}=\frac{\phi}{1-\phi}$, therefore, $\frac{dU}{d\Gamma_a}=\left(\frac{\frac{dc}{dg}}{c}+\frac{1}{\rho}\right)\frac{dg}{d\Gamma_a}=\frac{1}{c}\left(\frac{dc}{dg}+\frac{c}{\rho}\right)\frac{dg}{d\Gamma_a}=\frac{1}{\sigma\lambda\phi\Phi c}\frac{dg}{d\Gamma_a}$. Thus, $\frac{dU}{d\Gamma_a}\leq 0$ for any value of Γ_a. Utility has an inflexion point at $\frac{\Gamma_b}{\Gamma_a}=\frac{1-\theta_a}{\theta_a}$, the point where $\frac{dg}{d\Gamma_a}=0$, and it is maximized at $\Gamma_a=0$ ■