The Spanish term structure of interest rates revisited: cointegration with multiple structural breaks, 1974-2010

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Abstract

In this paper we consider the possibility that a linear cointegrated regression model with multiple structural changes would provide a better empirical description of the Spanish term structure of interest rates. Our methodology is based on instability tests recently proposed in Kejriwal and Perron (2008, 2010) as well as the cointegration tests developed in Arai and Kurozumi (2007) and Kejriwal (2008). The results obtained are consistent with the existence of linear cointegration between the long and the short run Spanish interest rates. However, our empirical results also show that the cointegrating relationship has changed over time. In particular, the Kejriwal-Perron tests for testing multiple structural breaks in cointegrated regression models suggest a model of two regimes.

Keywords: Term structure of interest rates; Cointegration; Multiple Structural Breaks

JEL classification: C32, E43

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1 Introduction

The expectations hypothesis (EH) of the term structure of interest rates is one of the oldest and simplest analytical framework to study rational behaviour in financial markets. The EH of the term structure of interest rates, which basically states that the observed term structure can be used to infer market participants expectations about future interest rates, has been at the origin of an extraordinary amount of econometric analysis; see, e. g., Campbell (1995), Campbell and Shiller (1987, 1991), Engsted and Tanggaard (1994a,b), Hall et al. (1992), Hardouvelis (1994), Jondeau and Ricart (1999), Lanne (2000), Sarno et al. (2007), Thornton (2006), and Tzavalis (2003).

Understanding the term structure of interest rates has always been viewed as crucial to assess the impact of monetary policy and its transmission mechanism, to predict interest rates, exchange rates and economic activity, and to provide information about expectations of participants in financial markets. However, the term structure of interest rates is likely to be subject to variation as a result of changes in the structure of the economy, like alterations in monetary policy or in the exchange rate regime and reforms in the financial market regulation. Therefore, if the information content of the term structure of interest rates is subject to changes over time, any empirical modelling not accounting for the possible instability in this relationship can produce misleading results.

According to the EH, the long-term interest rates should reflect future shortterm changes. Specifically, long-term interest rates would be the average of future expected short rates. Hence, the EH in the context of the cointegration theory suggests that the long and short interest rates are linked through a longrun relationship with parameters (1, -1), i.e. that the interest rate spread is mean-reverting. Following the work by Campbell and Shiller (1987), a number of further contributions have arisen. These works have strived to test the EH of the term structure of interest rates applying cointegration techniques on a linear model, leading sometimes to contradictory results. A non-exhaustive list of them would include Stock and Watson (1988), Hall et al. (1992), Engsted and Tanggaard (1994a, b) and Cuthbertson (1996).

In an empirical study, Camarero and Tamarit (2002) extended the analysis on the expectations model of the term structure of interest rates addressing the question of whether the relationship is stable over time, or exhibit a structural break allowing for instability to occur at an unknown point. In their application to the Spanish economy, they found evidence of linear cointegration between long and short interest rates for the period 1980:1-1996:4, with a vector (1, -1) as predicted by the theory. Moreover, the tests for instability and structural change detected the presence of a break in 1994 when two factors that may have affected the term structure of interest rates were acting: first, the devaluations of the peseta, that happened at the end of 1982 and between 1992 and 1995; secondly, the financial changes that were introduced at the beginning of 1994 as a result of the commitments of Spain in the context of the process towards the European Monetary Union.

Camarero and Tamarit (2002) applied several methods to detect the struc-

tural changes or instability in the cointegration regressions. The first group of tests are those of the null hypothesis of no change in cointegrated models proposed by Hansen (1992). These LM test procedures are based on the fully modified estimation method (Phillips and Hansen, 1990) which has been shown to lead to tests with very poor finite sample properties (Carrion-i-Silvestre and Sansó-i-Roselló, 2006). The results reached by Quintos and Phillips (1993) also suggest that the LM tests are likely to suffer from the problem of low power in finite samples. Moreover, simulation experiments carry out by Hansen (2000) show that the LM test behaves quite poorly in the presence of structural changes in the marginal distributions of the regressors. The second group of tests were proposed by Gregory and Hansen (1996a, b) and consider the residual-based test for the null of no cointegration against the alternative of cointegration with a structural break of unknown timing. A rejection by these tests would then confirm the presence of a cointegrating relationship with a structural break. However, the value of the break associated with the minimal value of a given statistic is not, in general, a consistent estimate of the break date if a change is present. Moreover, these tests are designed to have power against the alternative of a single break in parameters and hence may have low power when the alternative involves more than one break.¹ The third group of tests are the multiple structural changes tests proposed by Bai and Perron (1998a, b) in the context of OLS recursive estimation applied to stationary variables. However, these tests are only valid for stationary variables and the interest rates series are both I(1) or non-stationary variables.

In this paper, we extend the existing empirical analysis of the term structure model of interest rates in two ways. In the first place, and in order to avoid the econometric problems mentioned above, we make use of recent developments in cointegrated regression models with multiple structural changes. Specifically, we use a new approach proposed by Kejriwal and Perron (2008, 2010) to test for multiple structural changes in cointegrated regression models. These authors develop a sequential procedure that not only enables detection of parameter instability in cointegration regression models but also allows for consistency of the number of breaks present. Furthermore, we test the cointegrating relationship when multiple regime shifts are identified endogenously. In particular, the nature of the long run relationship between long and short interest rates is analyzed using the residual based test of the null hypothesis of cointegration with multiple breaks proposed in Arai and Kurozumi (2007) and Kejriwal (2008). In the second place, so as to counter a common criticism to most test of the EH of the term structure of interest rates, in this paper we use a long span of the data (1974:1-2010:2), since the econometric procedures used require a large number of observations. Through our extension we were able to obtain more robust results on the fulfillment of the term structure of interest rates than in previous analysis.²

 $^{^{1}}$ A recent example is the work of Suardi (2010), who applies the Gregory and Hansen (1996a, b) tests to the Australian term structure of interest rates.

 $^{^{2}}$ An empirical study by Esteve (2006) extends the analysis of the EH of the term structure of interest rates addressing the possibility that a nonlinear model might provide a better empirical

The rest of the paper is organized as follows. A brief description of the underlying theoretical framework is provided in section 2, the methodology and empirical results are presented in sections 3 and 4, respectively, and the main conclusions are summarized in section 5.

2 A simple model of the EH of the term structure of interest rates

The fundamental equation characterizing the EH of the term structure of interest rates states that the long-term interest rate equals an average of current and expected short-term interest rates over the life of the long-term interest rate plus a constant term, representing the time constant term/risk premium $(\phi^{(n)})$:³

$$bonds_t^{(n)} = \frac{1}{k} \sum_{i=0}^{k-1} E_t \left[crm_{t+im}^{(m)} \right] + \phi^{(n)}$$
(1)

where $bonds_t^{(n)}$ is the interest rate of long-term bonds with *n*-period, $cmr_t^{(m)}$ the short-term interest rate with *m*-period, and *k* an integer denoting n/m. Expectations formulated at time *t* for the future evolution of short-term interest rates drive the longer-term interest rate. When short-term interest rates are expected to rise, long-term interest rates will also rise.

In order to test the term structure of interest rates in the context of cointegration theory the empirical studies on the EH have commonly used a linear model such as:

$$bonds_t^{(n)} = c + \gamma cmr_t^{(m)} + \varepsilon_t \tag{2}$$

According to Campbell and Shiller (1987), $bonds_t$ and cmr_t should be nonstationary and linked through a cointegration relationship with parameters (1, -1), i.e., that the interest rate spread is mean-reverting. If the spread is stationary, then the term/risk premium is also stationary and the interest rates are driven by a common stochastic trend, preventing them from drifting too far apart from the equilibrium, so that profitable arbitrage opportunities do not persist. The rate of inflation is the most obvious candidate to represent this common trend (Domínguez and Novales (2000); Engsted and Tanggaard (1994b)). Campbell and Shiller (1991) noted that this hypothesis implies that a maturity-specific multiple of the term spread predicts future changes in the long bond yield. Thus, the expectations theory of the term structure suggests

description. In his paper, Esteve (2006) applies the methodology proposed by Hansen and Seo (2002) to test for threshold cointegration to the Spanish term structure of interest rates during the period 1980:1-2002:12. The evidence found by Esteve (2006) clearly rejects the existence of nonlinear cointegration between long and short interest rates and, therefore, a linear cointegration model may provide a more adequate empirical description for the Spanish term structure of interest rates.

 $^{^{3}}$ See Taylor (1992) and Bekaert and Hodrick (2001) for the possibility that the relationship between long and short interest rates may contain time-varying term/risk premium.

that the current interest rate spread is an optimal forecast of future changes in long-run interest rates. Thus, according to this hypothesis, market's expectations about the short-rate developments of the bond yield are reflected in the slope of the term structure with a one-to-one relation, $\gamma = 1$.

3 Methodology

3.1 A linear cointegrated regression model with multiple structural changes

Issues related to structural change have received a considerable amount of attention in the statistics and econometrics literature. Bai and Perron (1998) and Perron (2006, 2008) provide a comprehensive treatment of the problem of testing for multiple structural changes in linear regression models. Accounting for parameter shifts is crucial in cointegration analysis since it normally involves long spans of data which are more likely to be affected by structural breaks. In particular, Kejriwal and Perron (2008, 2010) provide a comprehensive treatment of the problem of testing for multiple structural changes in cointegrated systems.

More specifically, Kejriwal and Perron (2008, 2010) consider a linear model with m multiple structural changes (i.e., m + 1 regimes) such as:

$$y_t = c_j + z'_{ft}\delta_f + z'_{bt}\delta_{bj} + x'_{ft}\beta_f + x'_{bt}\beta_{bj} + u_t \qquad (t = T_{j-1} + 1, ..., T_j) \quad (3)$$

for j = 1, ..., m + 1, where $T_0 = 0$, $T_{m+1} = T$ and T is the sample size. In this model, y_t is a scalar dependent I(1) variable, $x_{ft}(p_f \times 1)$ and $x_{bt}(p_b \times 1)$ are vectors of I(0) variables while $z_{ft}(q_f \times 1)$ and $z_{bt}(q_b \times 1)$ are vectors of I(1) variables.⁴ The break points $(T_1, ..., T_m)$ are treated as unknown.

The general model (3) is a partial structural change model in which the coefficients of only a subset of the regressors are subject to change. In our case, we suppose that $p_f = p_b = q_f = 0$, and the estimated model is a pure structural change model with all coefficients of the I(1) regressors and constant (slope and the intercept in (2)) allowed to change across regimes:

$$y_t = c_j + z'_{bt} \delta_{bj} + u_t \qquad (t = T_{j-1} + 1, ..., T_j)$$
(4)

Generally, the assumption of strict exogeneity is too restrictive and the test statistics for testing multiple breaks are not robust to the problem of endogenous regressors. To deal with the possibility of endogenous I(1) regressors, Kejriwal and Perron (2008, 2010) propose to use the so-called dynamic OLS regression (DOLS) where leads and lags of the first-differences of the I(1) variables are added as regressors, as suggested by Saikkonen (1991) and Stock and Watson (1993):

⁴The subscript b stands for "break" and the subscript f stands for "fixed" (across regimes).

$$y_t = c_i + z'_{bt} \delta_{bj} + \sum_{j=-l_T}^{l_T} \Delta z'_{bt-j} \Pi_{bj} + u_t^*, \quad \text{if } T_{i-1} < t \le T_i$$
(5)

for i = 1, ..., k + 1, where k is the number of breaks, $T_0 = 0$ and $T_{k+1} = T$.

3.2 Structural Break Tests

In this paper we test the parameter instability in cointegration regression using the tests proposed in Kejriwal and Perron (2008, 2010). They present issues related to structural changes in cointegrated models which allow both I(1) and I(0) regressors as well as multiple breaks. They also propose a sequential procedure which permits consistent estimation of the number of breaks, as in Bai and Perron (1998).

Kejiriwal and Perron (2010) consider three types of test statistics for testing multiple breaks. First, they propose a sup *Wald* test of the null hypothesis of no structural break (m = 0) versus the alternative hypothesis that there are a fixed (arbitrary) number of breaks (m = k):

$$\sup F_T^*(k) = \sup_{\lambda \in \Lambda \varepsilon} \frac{SSR_0 - SSR_k}{\hat{\sigma}^2} \tag{6}$$

where SSR_0 denotes the sum of squared residuals under the null hypothesis of no breaks, SSR_k denotes the sum of squared residuals under the alternative hypothesis of k breaks, $\lambda = \{\lambda_1, ..., \lambda_m\}$ is the vector of breaks fractions defined by $\lambda_i = T_i/T$ for $i = 1, ..., m, T_i$, and T_i are the break dates.

Second, they consider a test of the null hypothesis of no structural break (m = 0) versus the alternative hypothesis that there is an unknown number of breaks, given some upper bound $M(1 \le m \le M)$:

$$UD\max F_T^*(M) = \max_{1 \le k \le m} F_T^*(k)$$
(7)

In addition to the tests above, Kejiriwal and Perron (2010) consider a sequential test of the null hypothesis of k breaks versus the alternative hypothesis of k + 1 breaks:

$$SEQ_T(k+1|k) = \max_{1 \le j \le k+1} \sup_{\tau \in \Lambda_{j,\varepsilon}} T\left\{SSR_T(\hat{T}_1, ..., \hat{T}_k\right\}$$
(8)

$$-\left\{SSR_{T}(\hat{T}_{1},...\hat{T}_{j-1},\tau,\hat{T}_{j},...,\hat{T}_{k}\right\}/SSR_{k+1}$$
(9)

where $\Lambda_{j,\varepsilon} = \left\{ \tau : \hat{T}_{j-1} + (\hat{T}_j - \hat{T}_{j-1})\varepsilon \leq \tau \leq \hat{T}_j - (\hat{T}_j - \hat{T}_{j-1})\varepsilon \right\}$. The model with k breaks is obtained by a global minimization of the sum of squared residuals, as in Bai and Perron (1998).

3.3 Cointegration tests with structural changes

Kejriwal and Perron (2008, 2010) show that the structural change tests may suffer from an important lack of power against spurious regression (i.e, no cointegration), i.e., these tests can reject the null of stability when the regression is really a spurious one. In this sense, tests for breaks in the long run relationship are used in conjuction with tests for the presence or absence of cointegration allowing for structural changes in the coefficients.

In this paper, we use the residual-based test of the null of cointegration with an unknown single break against the alternative of no cointegration proposed in Arai and Kurozumi (2007). These authors developed a LM test based on partial sums of residuals where the break point is obtained by minimizing the sum of squared residuals. They considered three models: i) Model 1, a level shift; ii) Model 2, a level shift with a trend; iii) and Model 3, a regime shift.

The LM test statistic (for one break), $V_1(\lambda)$, is given by:

$$\tilde{V}_1(\hat{\lambda}) = (T^{-2} \sum_{t=1}^T S_t(\hat{\lambda})^2) / \hat{\Omega}_{11}$$
(10)

where $\hat{\Omega}_{11}$ is a consistent estimate of the long run variance of u_t^* in (5), the date of break $\hat{\lambda} = (\hat{T}_1/T, ..., \hat{T}_k/T)$ and $(\hat{T}_1, ..., \hat{T}_k)$ are obtained using the dynamic algorithm proposed in Bai and Perron (2003).

The Arai and Kurozumi (2007) test may be quite restrictive since only a single structural break is considered under the null hypothesis. Hence, the test may tend to reject the null of cointegration when the true data generating process exhibits cointegration with multiple breaks. To avoid this problem, Kejiriwal (2008) has extended the Arai and Kurozumi (2007) test by incorporating multiple breaks under the null hypothesis of cointegration. The Kejiriwal (2008) test of the null of cointegration with multiple structural changes is denoted -with k breaks- as $\tilde{V}_k(\hat{\lambda})$.

4 An application to the Spanish term structure of interest rates

The data used in this paper are monthly for Spain and cover the period 1974:1 to 2010:2. The variables utilized in the empirical application are the nominal long-term interest rate, $bonds_t$ (private bonds of electric utilities before February 1978; from March 1978 to December 1992, central government bonds at more than two years; and, from January 1993, central government benchmark bond of 10 years), and the nominal short-term interest rate, cmr_t (1-month interbank market rates before December 1976; and, from January 1977, 3-month interbank market rates). Both series have been obtained from Bank of Spain (2010).

The evolution of the two series is shown in Figure 1 and there seems to be a close comovement between the two series. An informal inspection of the plot in Figure 1 suggests the possibility of several slope shifts in the trend function of both series: i) the first break in the slope of the interest rates series occurs around 1980, which coincides with the start of the period of financial deregulation and liberalization process as well as with the development of a secondary market for public debt which became the reference of long-term interest rates; ii) the second break in the slope of the interest rates series starts at the end of 1992/beginning of 1993, and can be attributed to the four devaluations suffered by the peseta (from september of 1992 until the first quarter of 1995) as well as to the financial changes that occurred in Spain as a result of its commitments to the process of forming the European Monetary Union;⁵ iii) the next slope shift in the trend function of long and short interest rates seems to occur around 1998, just before the adoption of a single currency, the euro, and a common central bank by eleven European countries, including Spain, on January 1999; iv) finally, the last shift appears after the international financial crisis that began in 2008. Overall, these changes in the trend function suggest that the long run relationship between long and short interest rates may have changed over time. In all cases, after each one of these structural changes the long run interest rate seems to react more sharply to the short run interest rate.

As a preliminary step in our analysis, we examine the time series properties of the series by testing for a unit root over the full sample. We have used a modified version of the Dickey-Fuller and Phillips-Perron tests proposed by Ng and Perron (2001), which try to solve the main problems present in these conventional tests for unit roots.

In general, the majority of the conventional unit root tests suffer from three problems. First, many tests have low power when the root of the autoregressive polynomial is close to, but less than, unit (Dejong *et al.*, 1992). Second, the majority of the tests suffer from severe size distortions when the moving-average polynomial of the first-differenced series has a large negative autoregressive root (Schwert, 1989; Perron and Ng, 1996). Third, the implementation of unit root tests often needs the selection of an autoregressive truncation lag, k; however, as discussed in Ng and Perron (1995) there is a strong association between k and the severity of size distortions and/or the extend of power loss.

Recently, Ng and Perron (2001) have proposed a methodology that solves these three problems. This method consists of a class of modified tests, called \bar{M}_{MAIC}^{GLS} , originally developed in Stock (1999) as M tests, with GLS detrending of the data as proposed in Elliot *et al.* (1996), and using the Modified Akaike Information Criteria (MAIC).⁶ Also, Ng and Perron (2001) have proposed a similar procedure to correct for the problems of the standard Augmented

⁵During this period the Spanish peseta was a full member of the Exchange Rate Mechanism (ERM) of the European Monetary System (EMS). After a previous period (1987-1992) of ever growing confidence in the ERM, the EMS saw the beginning of the successive crises from September 1992, which first took the British pound and the Italian lira out of the ERM, and the opening of a year of unprecedented turmoil in the history of EMS. This period finish with the broadening of the fluctuations bands in august 1993. The German unification shock and its consequent impact on the European economy (already in a recession), the increased financial risks due to the removal of capital controls and the surge in bank lending were seen as the main underlying causes of the crises in the EMS (Ledesma-Rodríguez et al., 2005).

⁶These tests are the $\bar{M}Z_{\alpha}^{GLS}$, $\bar{M}SB^{GLS}$ and $\bar{M}Z_{4}^{GLS}$.

Dickey-Fuller (ADF) test, ADF_{MAIC}^{GLS} .⁷ Table 1 reports the results of Ng and Perron tests. This table shows that the nominal long-term interest rate is found to be I(1), while the null hypothesis of nonstationarity for the nominal short-term interest rate can be rejected at the 1% significance level with the $\bar{M}Z_{\alpha}^{GLS}$, $\bar{M}Z_{t}^{GLS}$ and ADF^{GLS} tests. Therefore, according to the results of these tests, the nominal short-term interest rate series could be I(1) or I(0).

A potential difficulty in assessing the time series properties of monetary and financial variables, is that they can be subject to potential structural breaks in the form of infrequent changes in the mean or the drift of the series, due to exogenous shocks or changes in the policy regime. Hence, in order to provide further evidence on the degree of integration of variables, we have also applied the Perron-Rodriguez test (Perron and Rodriguez, 2003) for a unit root in the presence of a one time change in the trend function.

Perron and Rodriguez (2003) extend the tests for a unit root analyzed by Perron and Ng (2001) to the case where a change in the trend function is allowed to occur at an unknown time, T_B . In this paper we use the method where the break date is selected minimizing the tests, as suggested by Zivot and Andrews (1992). The results are presented in Table 2. We consider the Model II where a structural change in intercept and slope is allowed to occur at an unknown time. Using the MAIC to select k, there is no evidence against the unit root for the nominal short-term interest rate series at the 5% significance level. The break date is selected at 1980:2.

An alternative method to select the break date, as used in Perron (1997), is to choose it so that the absolute value of the t-statistic on the coefficient of the change in slope is maximized. Table 3 presents the results of the tests. For nominal short-term interest rate series, there is not evidence against the unit root. The break date selected is 1979:9. Therefore, according to the results of these tests, cmr_t would be I(1).

Once the order of integration of the series has been analyzed, we estimate the long-run or cointegration relationship between $bonds_t$, and cmr_t . Given the relatively small sample size, we will estimate and test the coefficients of the cointegration equation by means of the Dynamic Ordinary Least Squares (DOLS) method from Saikkonen (1991) and Stock and Watson (1993) and following the methodology proposed by Shin (1994). This estimation method provides a robust correction to the possible presence of endogeneity in the explanatory variables, as well as serial correlation in the error terms of the OLS estimation. Furthermore, in order to overcome the problem of the low power of the classical cointegration tests in the presence of persistent roots in the residuals of the cointegration regression, Shin (1994) suggests a test where the null hypothesis is that of cointegration. In the first place, we estimate a long-run dynamic equation including the leads and lags of all the explanatory variables, the so-called DOLS regression; in our case:

⁷See Ng and Perron (2001) and Perron and Ng (1996) for a detailed description of these tests.

$$bonds_t^{(n)} = c + \gamma cmr_t^{(m)} + \sum_{j=-q}^q \gamma_j \Delta cmr_{t-j}^{(m)} + v_t$$
 (11)

Secondly, the Shin's test is based on the calculation of two LM statistics from the DOLS residuals, C_{μ} , to test for deterministic cointegration. The parameter γ is the long-run cointegrating coefficient estimated between the long and short interest rates.

The results of Table 4 show that the null of deterministic cointegration between $bonds_t$ and cmr_t is not rejected at the 1% level of significance, and the estimated value for γ is 0.77, significantly different from zero at the 1% level. But this estimate would be significantly different from one at the 1% level, according to a Wald test on the null hypothesis $\hat{\gamma} = 1$ against the alternative $\hat{\gamma} < 1$, distributed as a χ_1^2 and denoted by W_{DOLS} in Table 4. Since the estimate of the long-run cointegrating coefficient is significantly lower than one, it seems that the changes in the long-term interest rate did not adjust completely so as to compensate the behaviour of the short-term interest rates.

Accounting for parameter shifts is crucial in cointegration analysis, since this type of analysis normally involves long spans of data, which for this reason are more likely to be affected by structural breaks. In particular, our data covers thirty five years of the history of the Spanish interest rates, and during that period of time the term structure of interest rates have changed due to variations in macroeconomic and market forces, i.e., changes in the structure of the economy, in monetary policy or in the exchange rate regime, not to mention supply shocks and reforms in financial and fiscal regulations. Therefore, as we argued before, it is very relevant to allow for structural breaks in our cointegration relationship.

We now consider the tests for structural change that have been proposed in Kejiriwal and Perron (2008, 2010). Since we have used a 15% trimming, the maximum numbers of breaks we have under the alternative hypothesis is 5. Moreover, the intercept and the slope in equation (11) are permitted to change. Table 5 presents the results of stability tests as well as the number of breaks selected by the sequential procedure (SP) proposed by Bai and Perron (2003). The UD max test is significant at the 5% level, which implies that at least one break is present. The sup $F_T(1)$ test is significant at the 5% level, unlike $\sup F_T(2)$, suggesting that the data do not support a two-break model. The sequential procedure selects a single break and provide evidence against the stability of the long run relationship. Overall, the results of the Kejriwal-Perron tests suggest a model with one break estimated at 1979:6 and two regimes, 1974:1-1979:6 and 1979:7-2010:2.⁸ The break date 1979:6 is precisely estimated with since their 95% confidence interval cover only a few months before and after (1979:4-1980:11). There are some factors that may explain the placement of such structural change of the Spanish term structure of interest rates.

⁸Note that this result is very similar to the change selected for the nominal short-term rate series when we apply the Perron-Rodriguez test for a unit root in the presence of a one time change in the trend function (Table 3).

In the first place, we have to realize that until the 1980s most financial transactions in Spain were channelled through the banking system, which in itself was strongly regulated. In addition to reserve and investment requirements, most interest rates were administered. The numerous regulatory changes introduced during the 1980s produced the development of several financial markets, including the interbank (connected to short interest rates), the market for public debt and the stock market (both of them associated to long interest rates). In fact, it is only after 1980 when the Spanish financial sector experimented a serious deregulation process. This liberalization was in line with the various regulations and new financial directives of the EU.

Secondly, until the 1980s the deficit of the public sector were financed mostly via credits from the Bank of Spain (seigniorage). It is only after 1982 that budget deficits were increasingly financed in a more orthodox way using market mechanisms, i.e., through the issuing of public debt, which allowed the development of the secondary market for public debt and the use of central government bonds as reference of the long-term interest rates.

As a consequence of all of these structural changes, it was to be expected that the long run interest rate would react more sharply to the short run interest rate, due to the changes suffered by the Spanish financial system.

Even though the above stability tests also reject the null coefficient stability when the regression is a spurious one, we still need to confirm the presence of cointegration among the variables. For this reason we use the residual based test of the null of cointegration with an unknown single break proposed in Arai and Kurozumi (2007), $\tilde{V}_1(\hat{\lambda})$. Arai and Kurozumi (2007) show, in the single break case, that the limit distribution of the test statistic, $\tilde{V}_1(\hat{\lambda})$, depend only on the timing of the estimated break fraction $\hat{\lambda}$ and the number of I(1) regressors m. In our case, critical values are obtained for $\hat{\lambda} = 0.15$ and m = 1 by simulation using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of *i.i.d.* N(0, 1) random variables. Since we are interested in the stability of the short-term-long-term interest rate coefficient, γ , we consider only model 3 that permits the slope shift as well a level shift. Table 6 shows the results of the Arai-Kurozumi cointegration test with a single break. Again, the level of trimming used is 15%. The results show that the test $\tilde{V}_1(\hat{\lambda})$ cannot reject the null of cointegration with a structural break at 1979:6.

To compare the coefficients obtained from a break model with those reported from a model without any structural break, we proceed to estimate the cointegration equation (11) for the two sub-samples, and the results are shown in the last two columns of Table 4. First, the estimates show that the slope estimated is insignificant in the first regime and the estimated parameter is very small (0.09). Second, the coefficient estimated for the second regime is significant and higher (0.83) than the full sample estimate of 0.77. Therefore, our results do not fully support the expectations hypothesis of the term structure of interest rates – there is not a long-run relationship and the long-term interest rate does not react to short-term interest rate changes– prior to 1979:6, when the interbank market (related to short-term interest rates) was strongly regulated and the secondary market for public debt (associated to long-term interest rates) simply did not exist. On the other hand, after 1979:6, our results do not uphold completely the expectations hypothesis of the term structure of interest rates: there is a long-run relationship although changes in the long-term interest rate do not adjust fully to compensate the behaviour of the short-term interest rates. It seems that the changes experimented by the financial sector of the Spanish economy modified the agents expectations.⁹

The results obtain for both periods suggest that ignoring shifts may understate the long-run cointegration relationship between the long and short interest rates. Therefore, if the information content of the term structure of interest rates is subject to changes over time, any empirical modelling not accounting for the possible instability in this relationship can produce misleading results.

Our results contrast with the empirical evidence presented by Camarero and Tamarit (2002) in two fundamental ways.¹⁰ Firstly, the value of the short-run interest parameter in the full sample is different. In their case, they found a long-run parameter near to 1, revealing a very strong cointegration between these two variables. Instead, we have found a short-term-long-term interest rate coefficient smaller (0.77), implying that the long-term interest rates. In our case, the results only support a weak version of the expectations hypothesis of the term structure of interest rates. Secondly, Camarero and Tamarit (2002) detected the presence of a single break in 1994 while we have placed a single break in 1979, a date which was not included in their sample data.

5 Conclusions

To account for parameter shifts is crucial in cointegration analysis, since this type of analysis normally involves long spans of data, which for very reason are more likely to be affected by structural breaks. In this paper we consider the possibility that a linear cointegrated regression model with multiple structural changes would provide a better empirical description of the term structure model of interest rates. Our methodology is based on instability tests recently proposed in Kejriwal and Perron (2008, 2010) as well as the cointegration tests developed in Arai and Kurozumi (2007) and Kejriwal (2008). These methods are applied to test the Spanish term structure of interest rates during the period 1974:1-2010:2.

The results obtained in our results are consistent with the existence of linear

⁹Although the approach used in this paper is frequently adopted in empirical analysis of the term structure of interest rates, it is subject to at least one word of caution, i.e., we are testing for whether the risk premia are constant. As noted in Camarero and Tamarit (2002) the structural changes detected in equation (11) can be interpreted in terms of shifts in the term risk premia. Threfore, if we want to detect the underlying causes explaining those changes another type of analysis should be used, though this is outside the scope of this paper.

 $^{^{10}}$ It should be noted that the data used by Camarero and Tamarit (2002) only cover the years going from 1980 to 1996 while in our study we have data for the 1974-2010 period. Since the number of observations available is in our analysis much greater, we were also able to potencially detect more (and different) structural changes.

cointegration between the long and the short run Spanish interest rates, with a vector (1, -0.77). Thus, the cointegration vector is not (1, -1), as predicted by the theory. However, our empirical results show also that the cointegrating relationship has changed over time. In particular, the Kejriwal-Perron tests for testing multiple structural breaks in cointegrated regression models would suggest a model of two regimes, with the date of the break estimated at 1979:6. The break date of 1979:6 is precisely estimated since the 95% confidence interval cover only a few months before and after (1979:4-1980:11). In addition, the Arai-Kurozumi-Kejriwal cointegration test with a single structural break cannot reject the null of cointegration with a structural break at 1979:6.

There are several factors that contribute to explain the date of such structural change of the Spanish term structure of interest rates. First, until the 1980s, most financial transactions in Spain were channelled through the banking system, which was strongly regulated. In addition to reserve and investment requirements, most interest rates (both short-term and long-term) were administered. The numerous regulatory changes introduced during those years allowed for the development of several important financial markets, including the interbank market (connected to short-term interest rates), the market for public debt and the stock market (the latter closely associated to the fixing of longterm interest rates). In fact, in the 1980s, the Spanish financial sector began a serious deregulation process. This liberalization implemented was in line with the various regulations and new financial directives of the EU.

Secondly, until the 1980s public deficits were financed mostly via credits from the Bank of Spain (seigniorage). It is only after 1982 that budget deficits were increasingly financed through the issuing of debt. This change allow for the development of a secondary market for public debt and the consequent employment of central government bonds as the reference for long-term interest rates.

Summing up, the results obtained in this study only support a weak version of the expectations hypothesis of the term structure of interest rates for the Spanish economy. In fact, our empirical results uphold a long-run relationship between the long and short interest rates, but this cointegration relationship is not stable. Moreover, the estimate value of the short-term interest rate parameter is significantly lower than one (in the full sample and second regime), implying that changes in the long-term interest rate do not fully adjust to compensate the behaviour of the short-term interest rates.

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Table 1 Ng-Perron tests for a unit roots

I(2) vs. $I(1)$		Case: $p =$	$= 0, \bar{c} = -7.0$	
Variable	$\bar{M}Z^{GLS}_{\alpha}$	$\bar{M}Z_t^{GLS}$	$\bar{M}SB^{GLS}$	ADF^{GLS}
$\Delta bonds_t$	-124.1***	-7.87***	0.063	-8.96***
Δcmr_t	-181.3^{***}	-9.51^{***}	0.052	-11.65^{***}
I(1) vs. $I(0)$		Case: $p =$	$1, \bar{c} = -13.5$	
				~~~~
Variable	$\bar{M}Z^{GLS}_{\alpha}$	$\bar{M}Z_t^{GLS}$	$\bar{M}SB^{GLS}$	$ADF^{GLS}$
$bonds_t$	-3.32	-1.26	0.380***	-1.26
	$-28.15^{***}$	-3.73***	0.132	-3.77***

Notes:

 a  A *, ** and *** denote significance at the 10%, 5% and 1% levels, respectively.

^b The MAIC information criteria is used to select the autoregressive truncation lag, k, as proposed in Perron and Ng (1996). The critical values are taken from Ng and Perron (2001), table 1:

Critical values:	Case:	$p=0, \bar{c}$	= -7.0	Case:	$p=1, \overline{c}$ =	= -13.5
	10%	5%	1%	10%	5%	1%
$MZ^{GLS}_{\alpha}$	-5.7	-8.1	-13.8	-14.2	-17.3	-23.8
$\bar{M}SB^{GLS}$	0.275	0.233	0.174	0.185	0.168	0.143
$\bar{M}Z_t^{GLS}, ADF^{GLS}$	-1.62	-1.98	-2.58	-2.62	-2.91	-3.42

#### Table 2 $\,$

Perron and Rodríguez^{a,b} tests for a unit root with one time change in the trend function choosing the break point minimizing the tests

I(1) vs. $I(0)$				Case:	p =	$1, \bar{c} = -2$	2.5			
			$M_{MAI}^{GLS}$	$_{C}$ tests						
Variable	$MZ^{GLS}_{\alpha}$	k	$T_B$	$MZ_t^{GLS}$	k	$T_B$	$ADF^{GLS}$	k	$T_B$	$\hat{lpha}$
$cmr_t$	-26.6*	16	1980:2	-3.64*	16	1980:2	-3.42	16	1980:2	0.9

Notes:

 a  A *, ** and *** denote significance at the 10%, 5% and 1% levels, respectively.

^b The *MAIC* information criteria is used to select the autoregressive truncation lag, k, as proposed Perron and Rodriguez (2003). We impose a minimal value k = 1. The critical values are taken from Perron and Rodriguez (2003), table 1 (a), Model II, T = 200:

Critical values:	Case:	$p = 1, \bar{c}$	= -22.5
	10%	5%	1%
$\bar{M}Z^{GLS}_{\alpha}$	-23.7	-27.1	-33.5
$\bar{M}Z_t^{\alpha}$	-3.42	-3.63	-4.07
$ADF^{GLS}$	-3.62	-3.90	-4.48

#### Table 3

Perron and  $\operatorname{Rodr} guez^{a,b}$  tests for a unit root with one time change in the trend function choosing the break point maximizing  $\mid t_{\hat{\beta}_2} \mid$ 

I(1) vs. $I(0)$		Case:	$p = 1, \bar{c} = -$	-22.5		
	$\bar{M}_{MAI}^{GLS}$	$_C$ tests				
Variable	$MZ^{GLS}_{\alpha}$	$MZ_t^{GLS}$	$ADF^{GLS}$	k	$T_B$	$\hat{\alpha}$
$cmr_t$	-15.9	-2.82	-3.06	16	1979:9	0.93

Notes:  a  A *, ** and *** denote significance at the 10%, 5% and 1% levels, respectively. tively.

b The *MAIC* information criteria is used to select the autoregressive truncation lag, k, as proposed Perron and Rodriguez (2003). We impose a minimal value k = 1. The critical values are taken from Perron and Rodriguez (2003), table 1 (b), Model II, T = 200:

Critical values:	Case:	$p=1, \bar{c}$	= -22.5
	10%	5%	1%
$ \begin{array}{l} \bar{M}Z^{GLS}_{\alpha} \\ \bar{M}Z^{GLS}_t \\ ADF^{GLS} \end{array} $	-3.24	-24.5 -3.47 -3.67	-3.91

Table 4		
Estimation of long-run relationships:	${\it Stock-Watson-Shin}$	$\operatorname{cointegration}$ tests

Parameter	Full sample	First regime	Second regime
estimates	1974:1-2010:2	1974:1-1979:6	1979:7-2010:2
с	2.20	10.15	1.86
	(2.88)	(7.51)	(4.15)
$\gamma$	0.77	0.09	0.83
	(11.4)	(1.04)	(20.1)
$R^2$	0.98	0.99	0.99
$C_{\mu}$	0.114	0.131	0.087
W _{DOLS}	$12.53^{*}$		18.12*

Notes:

 $^a\ t$  -statistics are in brackets. Standard Errors are adjusted for long-run variance. The long-run variance of the cointegrating regression residual is estimated using the Barlett window which is approximately equal to  $INT(T^{1/2})$ as proposed in Newey and West (1987). ^b We choose  $q = INT(T^{1/3})$  as proposed in Stock and Watson (1993). ^c  $C_{\mu}$  and  $C_{\tau}$  are LM statistics for cointegration using the DOLS residuals

from deterministic and stochastic cointegration, respectively, as proposed in Shin (1994). A *, ** and *** denote significance at the 10%, 5% and 1% levels, respectively.

^d The critical value for a  $\chi_1^2$  at 5%: 3.84.

^e The critical values are taken from Shin (1994), table 1, from m = 1:

Crit	ical valu	les:	
	10%	5%	1%
$C_{\mu}$	0.231	0.314	0.533

Table 5
Kejriwal-Perron tests for testing multiple structural breaks
in cointegrated regression models: equation (5) and $(11)^{a,b}$

		. ,	× ,		
	Specifications				
$y_t = \{bonds_t\}$	$z_t = \{1, cmr_t\}$				
q = 2	p = 0	h = 64	M = 5		
	Test				
$\sup F_T(1)$	$\sup F_T(2)$	$\sup F_T(3)$	$\sup F_T(4)$		
$12.58^{**}$	8.41	6.39	5.02		
$\sup F_T(5)$	$WD \max$				
4.17	$12.58^{**}$				
Number of Breaks	B	reak dates			
Selected	$estimates^c$				
1	$\hat{T}_1$ : 1979:	6 [1979:4-198	80:11]		

Notes:

^a  $y_t$ ,  $z_t$ , q, p, h, and M denote the dependent variable, the regressors, the number of I(1) variables (and the intercept) allowed to change across regimes, the number of I(0) variables, the minimum number of observations in each segment, and the maximum number of breaks, respectively.

 $b^*$ , **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. The critical values are taken from Kejriwal and Perron (2010), Table 1, nontrending case,  $q_b = 1$ .

 c  In parentheses, reported are the 95% confidence intervals for the break dates.

Table 6

Arai-Kurozumi-Kejriwal cointegration tests with a single structural break: equation (5) and (11)

11
979:6

Notes:

 a  *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

 b  Critical values are obtained by simulation using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of *i.i.d.* N(0,1) random variables:

Critical values:	10%	5%	1%
$\widetilde{V}_k(\hat{\lambda})$	0.168	0.223	0.425