Trade, Innovation and Productivity

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Abstract

Empirical evidence shows that trade liberalization improves productivity not just because of a selection effect but also because of productivity gains within firms. This paper proposes a trade model that allows for both channels, by adding the option to innovate. In contrast to the existing literature, the process innovation is modelled as a continuous variable and there is both a fixed and a variable cost to innovate. The interaction between the innovation and export choices is key to understand the different equilibria in the open economy and the outcomes following a trade liberalization. I calibrate the model to match the Spanish economy and explore the consequences of different trade policies. Simulations reveal that a fixed trade cost liberalization is more effective on innovation while a variable trade cost liberalization is more effective on average productivity.

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1 Introduction

The link between firm productivity and international trade has become increasingly important as an area of economic research over the last ten years. Evidence strongly supports the self selection of more productive firms into foreign market participation, but there is mixed evidence on the positive impact of export market participation on firm productivity. Motivated by these observations, I study the existence of within firm productivity gains through innovation in an open economy.

The literature has focused on models with heterogeneous firms because of their ability to match the export decisions of firms to characteristics such as productivity, size or ownership status. This literature, with early contributions by Bernard et al. (2003) and Melitz (2003), emphasizes the selection effect into export status and the reallocation of production factors and shares between firms as the source of aggregate productivity gains. However, empirical evidence points out that productivity gains occur not only between firms but also within firms(Pavcnik (2002), Trefler (2004), and De Loecker (2007)). The early literature did not identify within firm gains, because firm productivity was modeled by a random draw from a probability distribution.

The empirical literature has emphasized that the effect of trade liberalization on within firm productivity happens both along the extensive margin and the intensive margin. For example. Alvarez (2001) supports the hypothesis that exporters invest more intensively than non exporters. Likewise, Aw et al. (2008) show empirically that prior export market activity increases the probability of investing in R&D and that the interdependence between R&D activities and the exporting choice is critical to explain current investment decisions. Finally, Lileeva and Trefler (2010) show that new exporters innovate along different dimensions. Hence, evidence suggests that both the intensive and extensive margin are important in order to explain the productivity gains within firms and that the interaction of innovation, entry, exit and export decisions should be further explored.

This paper proposes a trade model with heterogeneous firms that have the option to invest in process innovation. The model follows the standard setup of Melitz (2003) with a basic difference: once a firm learns about its productivity it can decide to spend resources in process innovation to improve its technology. I am not the first to explore the effect of trade on within firm productivity gains, but I am the first to study this issue along both the intensive and the extensive margin. In contrast to Navas-Ruiz and Sala (2007) and Costantini and Melitz (2008) , I model process innovation as a continuous variable, and therefore can analyze the intensive margin. And, in contrast to Vannoorenberghe (2008), Bustos (2011) and Atkeson and Burstein (2010) innovation involves both fixed and variable costs. By having both a fixed and variable costs not all firms will be innovating and I can explore how trade liberalization affects the extensive margin and intensive margin of innovation at the same time.

The interaction between the innovation decision and the exporting decision will determine which one of the equilibria emerges. In all equilibria, firms at the high end of the distribution will export and innovate, while firms at the lower end of the distribution will not perform any of those activities. The behavior of middle productivity firms differs across equilibria. In the *low cost innovation equilibrium* trade costs are high in comparison with the cost-benefit ratio of innovation, so that middle productivity firms choose to innovate rather than enter new markets. In the *low cost trade equilibrium*, trade costs are low in relation to the cost-benefit to innovation, and firms that are productive enough choose to export rather than engage in innovation. In between these equilibria is the *intermediate equilibrium*, where firms are either very productive and can undertake both activities or do not perform any of them.

Analytically the main contribution of the paper is the ability to analyze the innovation decisions of firms through a tractable innovation policy function. A second contribution of the paper is understanding how a trade liberalization affects firms decision to innovate and export and to provide insight into the channels through which productivity gains occur. To my knowledge this is the first paper that studies along which margin within-firm productivity gains from innovation may happen.

The paper is organized as follows. In Section 2, I present the model of the economy where firms take decisions on innovation and exporting. In Section 3, I explore the equilibria determined by the interaction between the exporting and innovation choices creates. In Section 4, I calibrate the model to match the Spanish economy. In Section 5 I analyze the effects of two different trade policies on firms decisions, aggregate innovation and aggregate productivity. Section 6 concludes.

2 Model

The model is based on the monopolistic competition framework proposed by Melitz (2003). I consider a symmetric $n+1$ country world each of which use a single factor of production (labour L) to produce goods.The model is extended to allow these firms to have the opportunity to engage in process innovation.

2.1 Demand

I denote the source country by i and the destination country by j, where $i, j = 1, ..., n+1$. In each country j, there is a continuum of consumers of measure L_j. Given the set Ω of varieties supplied to the market, the consumer's preferences of country j are represented by the standard C.E.S. utility function

$$\left[\int_{\omega\in\Omega} q_{ij}^{\rho}(\omega)d\omega\right]^{\frac{1}{\rho}}$$

where $q_{ij}(\omega)$ denotes the quantity consumed of variety ω produced by firm i in country j and $\sigma = \frac{1}{1-\rho} > 1$ is the elasticity of substitution across varieties.

The market is subject to the expenditure-income constraint:

$$\int_{\omega \in \Omega} p_{ij}(\omega) q_{ij}(\omega) d\omega = R_j$$

where R_j is the total revenues obtained in country j.

Then standard utility maximization implies that the demand for each individual variety will be:

$$q_{ij}(\omega) = [p_{ij}(\omega)]^{-\sigma} \frac{R_j}{P_j^{1-\sigma}} \tag{1}$$

where $p_{ij}(\omega)$ is the price of each variety ω and $P_j = \left[\int_{\omega \in \Omega} p_{ij}(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$ denotes the price index of the economy.

2.2 Supply

There is a continuum of firms, each producing a different variety ω. Each firm draws its productivity φ from a distribution $G(\varphi)$ with support $(0, \infty)$ after paying a labor sunk cost of entry f_E. Since a firm is characterized by its productivity φ, it is equivalent to talk about variety ω or productivity φ.

Production requires only labor, which is inelastically supplied at its aggregate level L_j, and therefore can be taken as an index of country's j size.

In contrast to the Melitz model where firms use a constant returns to scale production technology, firms can affect their marginal cost through process innovation. To enter country j, firm i needs $f_{ij} > 0$ labour units and I make the standard iceberg cost assumption that $\tau_{ij} > 1$ units of the good have to be produced by firm i to deliver one unit to country j. Without loss of generality, I assume that $\tau_{ii} = 1$ and thus I denote $\tau_{ij} = \tau \; \forall i \neq j$.[1] Hence,

[1] Note that $\tau_{ij} = \tau_{ji}$ by symmetry and there is no possibility of transportation arbitrage

to produce an output $q_{ij}(\varphi)$, a firm requires $l_{ij}(\varphi)$ labor units

$$l_{ij}(\varphi) = f_{ij} + c(z_i) + \frac{q_{ij}(\varphi)}{\varphi} \frac{\tau_{ij}}{(1+z_i)^{\frac{1}{\sigma-1}}}$$

where z_i is a measure of the productivity increase from innovation that has an associated cost function $c(z_i)$.

The cost function of the innovation follows Klette and Kortum (2004), Lentz and Mortensen (2008) and Stähler et al. (2007). Firms pay a fixed cost, that can be attributed to the acquisition and implementation of the technology, plus a variable cost that depends directly on the process innovation performed by each firm. Hence the cost function $c(z_i)$ is defined as

$$c(z_i) = \begin{cases} z_i^{(\alpha+1)} + \kappa & \text{if } z > 0 \\ 0 & \text{if } z = 0 \end{cases}$$

where κ is the fixed cost required to implement the process innovation and $\alpha > 0$ measures the rate at which the marginal cost of the innovation increases, thus the higher the level of innovation the higher the cost associated with marginal increases.

Even though it can be argued that the cost of innovation can be simplified by imposing a linear variable cost, the existence of convex innovation costs is a standard feature in the literature and ensures that innovation is finite. Another simplification would be to have either a fixed cost or a variable cost but not both. Nevertheless maintaining a flexible cost function is important. For example, Vannoorenberghe (2008) assumes away a fixed innovation cost, which implies that all firms engage in process innovation. This eliminates the possibility of studying the interaction between the export and innovation decisions along the extensive margin, which is one of the purposes of this paper.

2.3 Firm's problem

Figure 1 represents the timing of the firm problem in the open economy. In a first stage, as in Melitz (2003), entering the market means paying a labor sunk cost f_E, in order to get a draw of the productivity parameter φ. In the second stage, with the knowledge of their own productivity, firms decide which activities to undertake. Since both exporting and innovation require paying a labor fixed cost, f_X and κ respectively, there will be four types of firms in the open economy. Type D firms are only active in the domestic market and do not perform innovation; Type DI firms are those active only in the domestic market that innovate; Type X firms are those active in both the domestic and the foreign market that do not perform any innovation; and Type XI firms are active in the domestic and foreign markets that engage on innovation activities. Finally, in the third stage, firms decide prices.

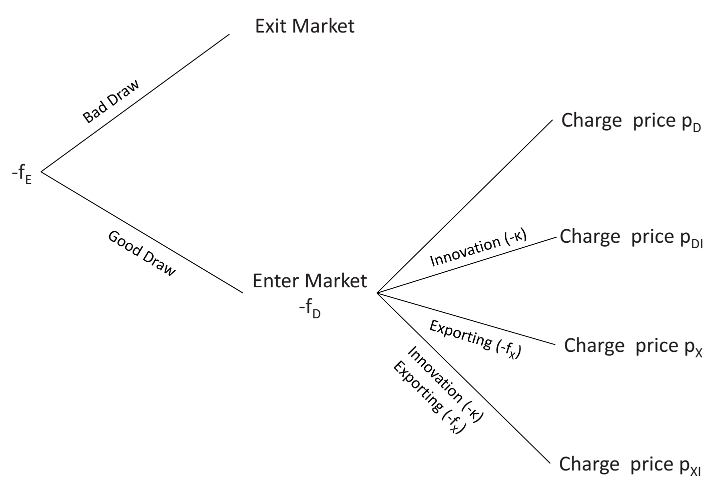


Figure 1: Timing

Given the timing, I solve the firms problem through backward induction.

Optimal Pricing Rule In the last stage of the problem the firm sets its optimal price given its innovation decision and the market conditions which are summarized by the price index P_j and R_j.

$$\max_{p_{ij}(\varphi)} p_{ij}(\varphi)\, q_{ij}(\varphi) - f_{ij} - \frac{\tau_{ij} q_{ij}(\varphi)}{\varphi\left[(1+z_i)^{\frac{1}{\sigma-1}}\right]} - c(z_i)$$

The corresponding first order condition is

$$p_{ij}(\varphi) = \left(\frac{\sigma}{\sigma-1}\right)\frac{\tau_{ij}}{\varphi}\cdot\frac{1}{(1+z_i)^{\frac{1}{\sigma-1}}} \quad \forall\, z \tag{2}$$

Optimal Innovation Decision The returns of process innovation increase with the participation in more countries. Thus, the optimal innovation rule for firm i is obtained from the first order condition of the maximization of $\sum_j \pi_{ij}(\varphi) = \sum_j [p_{ij}(\varphi)q_{ij}(\varphi) - l_{ij}(\varphi)]$ with respect to z_i, provided that the firm makes higher profits by innovating than by choosing not to innovate.

$$z_i(\varphi) = \begin{cases} \left[1+n\tau^{1-\sigma}\right]^{\frac{1}{\alpha}}\left[\frac{1}{\alpha+1}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{1}{\alpha}} & \text{if } \sum_j \pi_{ij}^{I}(\varphi) \geq \sum_j \pi_{ij}^{NI}(\varphi) \\ 0 & \text{if } \sum_j \pi_{ij}^{I}(\varphi) < \sum_j \pi_{ij}^{NI}(\varphi) \end{cases} \tag{3}$$

where $\frac{1}{\alpha}$ is the parameter that shapes the optimal innovation function and tells us how innovation rises with size, where I take the productivity parameter $\varphi^{\sigma-1}$ to be the indicator of size. If the function is linear ($\alpha = 1$), then innovation rises proportionately with size, however, if the function is concave ($\alpha > 1$), then the amount of innovation performed will rise less than proportionally with size, and if the function is convex ($0 < \alpha < 1$) the amount of innovation performed will increase more than proportionally with the productivity.

To make the joint decision of whether to enter the foreign markets and whether to innovate or not, firms will choose the option that yields the highest profits. Since countries are symmetric we can drop the subscripts and classify firms in four types.

- Profits of a domestic non-innovator firm (Type D):

$$\pi_D = \frac{R\,(P\rho)^{\sigma-1}}{\sigma}\varphi^{\sigma-1} - f_D$$

- Profits of a domestic innovator firm (Type DI):

$$\pi_{DI} = \frac{R\,(P\rho)^{\sigma-1}}{\sigma}\varphi^{\sigma-1}\,(1 + z_D\,(\varphi)) - f_D - c\,(z_D\,(\varphi))$$

- Profits of an exporter non-innovator firm (Type X):

$$\pi_X = \left(1 + n\tau^{1-\sigma}\right)\frac{R\,(P\rho)^{\sigma-1}}{\sigma}\varphi^{\sigma-1} - nf_X - f_D$$

- Profits of an exporter innovator firm (Type XI):

$$\pi_{XI} = \left(1 + n\tau^{1-\sigma}\right)\frac{R\,(P\rho)^{\sigma-1}}{\sigma}\varphi^{\sigma-1}\,(1 + z_X\,(\varphi)) - nf_X - f_D - c\,(z_X\,(\varphi))$$

where $f_D = f_{ii}$, $f_X = f_{ij} = f_{ji}\ \forall j \neq i$, $z_D(\varphi) = \left[\frac{1}{\alpha+1}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{1}{\alpha}}$, and $z_X(\varphi) = [1 + n\tau^{1-\sigma}]^{\frac{1}{\alpha}}\left[\frac{1}{\alpha+1}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{1}{\alpha}}$.

3 Equilibrium

There will be three different equilibria that will cover the whole parameter space. First, the *low-cost innovation equilibrium*, where the activity of exporting is relatively costly in comparison to innovation and therefore only the most productive firms will carry on both activities, middle productivity firms will innovate but not export and the lower productivity firms will neither innovate nor export. Second, the *low-cost trade equilibrium,* where the activity of innovation is relatively costly in comparison to exporting and therefore only the most productive firms will carry on both activities, middle productivity firms will export but not engage in innovation and the lower productivity firms will neither innovate nor export. Thirdly, between these two equilibria there will be the *intermediate equilibrium* where firms are either very productive and can undertake both activities or do not perform any of them.

The existence of these three equilibria is consistent with the empirical evidence found both in the trade and the innovation literature. Costantini and Melitz (2008) suggest that exporting and innovation are performed by the most productive firms while domestic producers are typically less innovative and less productive, a feature common to all the equilibria. Vives (2008)provides intuition for the decisions taken by middle productivity firms in each equilibrium. If trade costs are relatively high, middle productivity firms are domestic innovators while being an exporter without innovating is not profitable. A decrease in trade costs attracts the most productive firms from the foreign country, discouraging middle productivity domestic firms to undertake innovation. The disappearance of domestic innovators as trade costs fall can be explained by this Schumpeterian effect and is also predicted by the dynamic model of Costantini and Melitz (2008). However, a fall in trade costs enables more firms to participate actively in both markets which explains the existence of exporter non-innovators when trade costs are low enough.

Different papers have identified these equilibria separately, but never all in a single model. Bustos (2011) identifies the equilibrium where there are no domestic innovators firms since it is an unprofitable choice. In Vannoorenberghe (2008) all firms innovate, therefore it is not possible to study the interaction between both decisions. Finally, Navas-Ruiz and Sala (2007) identify the two extreme equilibria, but fail to identify the intermediate equilibrium. The main contribution of the theoretical model is the identification of all the equilibria with the ability to study the transitions between them and the possible productivity gains that might occur through the intensive and extensive margins of innovation. In this section, I describe each of the two main equilibria, the effects that trade has on innovation in each case, the parameter restrictions that give rise to the different equilibriums and conclude by focusing on the interaction between exporting and innovation.

3.1 Low Cost Innovation Equilibrium

The *low cost innovation equilibrium* is characterized by exporting being less attractive than innovation. In Figure 2, I depict the profits of all types of firms as a function of productivity when trade costs are relatively high in comparison to innovation costs. The envelope line shows the type of firm that will be chosen by a firm with productivity φ as it maximizes profits. In this equilibrium, the least productive firms ($\varphi < \varphi_D$) exit, the low productivity firms ($\varphi_D < \varphi < \varphi_{DI}$) are active in the domestic market but do not innovate or export, middle productivity firms ($\varphi_{DI} < \varphi < \varphi_{XI}$) are active only on the domestic market but innovate, and the most productive firms ($\varphi > \varphi_{XI}$) are active both in the domestic and export market, and innovate. Note that there is no range of productivity level where exporting without innovating is profitable, that is, the marginal exporter is an innovator as well.

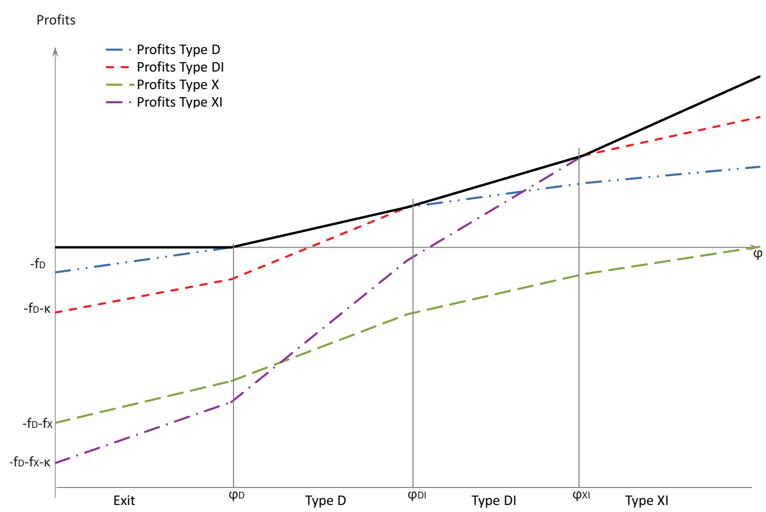


Figure 2: Low Cost Innovation Selection Path

The conditions of entry in the domestic and export markets plus the innovation condition allows to solve for the different productivity cutoffs in the *low cost innovation equilibrium.*

The Zero Profit Condition (ZPC) in the domestic market is $\pi_D\left(\varphi_D^*\right) = 0$, so that:

$$\left(\varphi_D^*\right)^{\sigma-1} = \frac{f_D}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)} \tag{4}$$

The Innovation Profit Condition (IPC) determines the productivity cutoff φ_{DI}^* which is the productivity of the firm indifferent between innovating or not while operating only on the domestic market, i.e. $\pi_{DI}\left(\varphi_{DI}^*\right) = \pi_D\left(\varphi_{DI}^*\right)$, so that:

$$\left(\varphi_{DI}^*\right)^{\sigma-1} = \frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)} \tag{5}$$

The Innovation Export Profit Condition (IXPC) determines the exporting-innovation cutoff φ_{XI}^* which is the productivity of an innovating firm indifferent between participating also on the exporting market or not.

$$\pi_{XI}\left(\varphi_{XI}\right) - \pi_{DI}\left(\varphi_{XI}\right) = 0 \tag{6}$$

Proposition 1.

The economy is in the low cost innovation equilibrium, $\varphi_{XI}^* > \varphi_{DI}^* > \varphi_D^*$, *if the following parameter restrictions hold*

1. $\tau^{\sigma-1} f_X \geq \frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}}\kappa + \left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)$
2. $\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) \geq f_D$

Proof. The formal proof can be found in the Appendix A. The proof is divided in two parts. First I show that there exist a single solution to equation (6). The non linearity present in the optimal innovation decision is the source of the complexity of finding a closed form for the cutoff φ^*_{XI}. Nevertheless, I show that selection into exporting and innovation ($\varphi^*_{XI} > \varphi^*_{DI}$) requires that condition 1 of Proposition 1 holds, that is exporting costs should be high enough relative to innovation costs. Notice that condition 2 of Proposition 1 ensures that there is selection into innovation ($\varphi^*_{DI} > \varphi^*_{D}$). Secondly, I show that equations (4) to (6) along with the Free Entry (FE) condition, which requires that the sunk entry cost equals the present value of expected profits:

$$\frac{1}{\delta}\left[\int_{\varphi^*_D}^{\varphi^*_{DI}} \pi_D(\varphi)\, dG(\varphi) + \int_{\varphi^*_{DI}}^{\varphi^*_{XI}} \pi_{DI}(\varphi)\, dG(\varphi) + \int_{\varphi^*_{XI}}^{\infty} \pi_{XI}(\varphi)\, dG(\varphi)\right] = f_E \tag{7}$$

uniquely determine the equilibrium price (P), the number of firms (M) and the distribution of active firms productivity in the economy along with the productivity cutoffs $\varphi^*_D,\ \varphi^*_{DI}$ *and* φ^*_{XI}. □

3.2 Low Cost Trade Equilibrium

The *low cost trade equilibrium* is characterized by exporting being more attractive than innovation. In Figure 3, I depict the profits of all types of firms as a function of productivity when trade cost are relatively low in comparison to innovation costs. The envelope line shows the type of firm that will be chosen by a firm with productivity φ as it maximizes profits. In this equilibrium, the least productive firms ($\varphi < \varphi_D$) exit, the low productivity firms ($\varphi_D < \varphi < \varphi_{DI}$) are active in the domestic market but do not innovate or export, middle productivity firms ($\varphi_{DI} < \varphi < \varphi_{XI}$) are active only on the domestic market but innovate, and the most productive firms ($\varphi > \varphi_{XI}$) are active both in the domestic and export market, and innovate. Note that there is no range of productivity level where innovation without exporting is profitable, that is, the marginal innovator is an exporter.

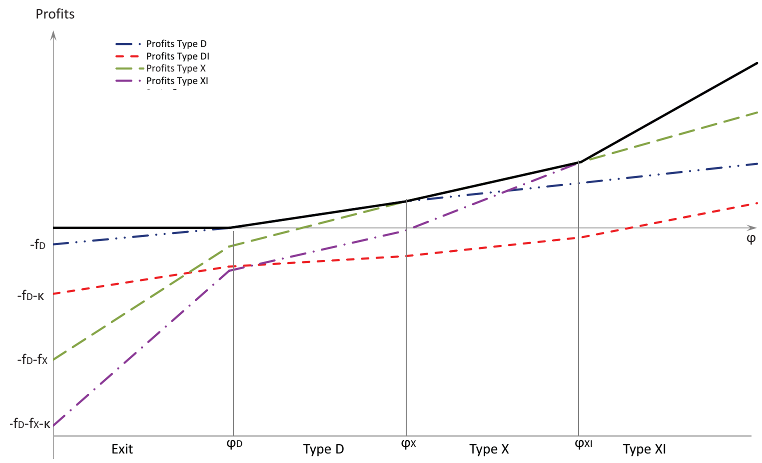


Figure 3: Low Cost Trade Selection Path

The conditions of entry in the domestic and export markets, plus the innovation conditions allows to solve the different productivity cutoffs in the *low cost trade equilibrium.*

The Zero Profit Condition (ZPC) in the domestic market[2] is $\pi_D\left(\varphi_D^*\right)=0$ so that:

$$\left(\varphi_D^*\right)^{\sigma-1}=\frac{f_D}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)} \tag{8}$$

The Exporting Profit Condition (XPC) determines the exporting-entry productivity cutoff φ_X^* which is the productivity of the firm indifferent between staying in the domestic market and participating in the export market, i.e. $\pi_X\left(\varphi_X^*\right)=\pi_D\left(\varphi_X^*\right)$:

$$\left(\varphi_X^*\right)^{\sigma-1}=\frac{f_X}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\tau^{1-\sigma}} \tag{9}$$

[2]The ZPC condition is defined theoretically in the same way in every equilibrium. However, since the aggregates in each situation are different, the entry cutoff will also be different.

The Exporting Innovation Profit Condition (XIPC) determines the innovation exporting productivity cutoff φ^*_{XI}, which is the productivity of an exporting firm indifferent between innovating or not, i.e. $\pi_{XI}\left(\varphi^*_{XI}\right) = \pi_X\left(\varphi^*_{XI}\right)$:

$$\left(\varphi^*_{XI}\right)^{\sigma-1} = \frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\left(1+n\tau^{1-\sigma}\right)} \tag{10}$$

Proposition 2.

The economy is in the low cost trade equilibrium, $\varphi^*_{XI} > \varphi^*_X > \varphi^*_D$, if the following parameter restrictions hold

$$\frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{\left(1+n\tau^{1-\sigma}\right)} \geq \tau^{\sigma-1} f_X \geq f_D$$

Proof. Selection into exporting and innovation $\left(\varphi^*_{XI} > \varphi^*_X\right)$ requires innovation costs to be high enough relative to trade costs and selection into exporting $\left(\varphi^*_X > \varphi^*_D\right)$ requires trade costs to be high enough relative to production costs. Equations (8) *to* (10) along with the Free Entry (FE) condition, which requires that the sunk entry cost equals the present value of expected profits:

$$\frac{1}{\delta}\left[\int_{\varphi^*_D}^{\varphi^*_X} \pi_D(\varphi)\, dG(\varphi) + \int_{\varphi^*_X}^{\varphi^*_{XI}} \pi_X(\varphi)\, dG(\varphi) + \int_{\varphi^*_{XI}}^{\infty} \pi_{XI}(\varphi)\, dG(\varphi)\right] = f_E \tag{11}$$

uniquely determine the equilibrium price (P), the number of firms (M) and the distribution of active firms productivity in the economy along with the productivity cutoffs φ^*_{XI}, φ^*_X φ^*_{XI}. See Appendix B for a formal proof. □

3.3 Discussion

The firm productivity distribution varies along the parameter space according to the relation between trade costs and the relative innovation costs. This is especially relevant for firms with an intermediate level of productivity, as their decisions will be most sensitive to these costs. In particular, in the *low cost innovation equilibrium*, when trade costs are high enough, they are domestic innovators. In the *low cost trade equilibrium*, when trade costs are low enough in relation to innovation costs, middle productivity firms will be exporters and the most productive of them will export and innovate. In between these two equilibria, there is the *intermediate equilibrium*, where trade costs are not relatively high enough for firms to be domestic innovators nor low enough for firms to be exporters non- innovators. That is, middle productivity firms are either exporter innovators or domestic firms. These choices are the ones that determine the parameter restrictions associated to each equilibrium. Furthermore, notice that the three equilibria cover the whole parameter space, and therefore the firm productivity distribution and the effects of opening up to trade of an economy can be determined always. Table 1 summarizes all the possible equilibria in the open economy and the parameter restrictions associated to each one.

Equilibrium	Conditions
Low Cost Innovation Equilibrium	$\tau^{\sigma-1} f_X \geq \frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}}\kappa + \left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)$ & $\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) \geq f_D$
Intermediate Equilibrium	$\frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}}\kappa + \left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) \geq \tau^{\sigma-1} f_X$ & $\tau^{\sigma-1} f_X \geq \frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} \geq f_D$
Low Cost Trade Equilibrium	$\frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} \geq \tau^{\sigma-1} f_X \geq f_D$

Table 1: Equilibria in the Open Economy

Furthermore, the model has implications on the aggregate productivity level. Firstly, trade induces the exit of the less productive firms and the reallocation of market shares towards the more productive firms, rising the industry average productivity in the long run. This is the selection effect described in Melitz (2003). And secondly, trade has indirect effects on the average productivity through innovation. Moving from the *low cost innovation equilibrium* to the *low cost trade equilibrium*, the cost of exporting relative to the cost of innovation decreases, therefore the effect trade has on innovation will be differentiated according to the level of transportation costs. On the one hand, there is an effect through the intensive margin of innovation. The innovation intensity increases with the participation in foreign markets and thus, the effect will be larger in the *low cost trade equilibrium* where the economy is more open. On the other hand, there is an effect through the extensive margin of innovation. In Crespo Rodríguez (2011), it is shown that the impact on average productivity through the extensive margin will be negative in the *low cost innovation equilibrium*, undetermined in the *intermediate equilibrium* and can be positive in the *low cost trade equilibrium*. In the empirical analysis we will decompose the change in productivity due to trade costs into these components and quantify their relevance.

4 Calibration

The model is calibrated to match the Spanish economy in 2008 using the EFIGE survey collected within the project 'EFIGE - European Firms in a Global Economy: internal policies for external competitiveness'. This survey, conducted during the year 2009, contains both qualitative and quantitative information on a sample of 2,800 spanish firms and 12,000 firms covering 6 other European economies (Austria, France, Germany, Hungary, Italy and the UK).

Similar to Helpman et al. (2004) and Chaney (2008), I assume the productivity is distributed according to a Pareto with a probability density function

$$g(\varphi) = \frac{\theta}{\varphi^{\theta+1}}$$

where $\varphi \in [1, \infty)$ and θ is the curvature parameter.

In accordance to the model considered, I estimate by maximum likelihood the curvature parameter associated to the spanish distribution of firms according to employees, $\tilde{\theta} = \theta/(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right) \approx 1.15$. The elasticity of substitution is set to be consistent with empirical estimates provided by Broda and Weinstein (2006), who estimate over 30,000 import elasticities. The medians reported vary from 2.2 to 4.8 depending on the level of aggregation, thus I set $\sigma = 3$ which lies within the estimated values. The innovation parameter is $\alpha = 0.9$. This value is consistent with the estimate of Rubini (2009), who sets the elasticity of productivity to resources devoted to innovation to match a 5% gain in labor productivity in Canada due to the tariff reduction in the U.S.-Canada Free Trade Agreement between 1980 and 1996. Hence, I set $\theta = 5$ so that the distribution of firms according to employees in equilibrium is close to Pareto shaped.

I choose parameters such that the model equilibrium displays the key aggregate and firm level patterns for the data. Variable trade costs are set initially to $\tau = 1.35$ following Costantini and Melitz (2008). Since the entry and exit of firms is determined by the death shock and the sunk cost of entry, I set them to $\delta = 0.025$ and $f_E = 2$, following Bernard et al. (2007).

The remaining parameters are calibrated jointly to match the number of innovators in the economy, the aggregate export volume and the percentage of skilled workers in the labor force. The innovation fixed cost is $\kappa = 1.8$, so that $\sim 26\%$ of the firms innovate, $f_D = 1.55$ so that for Firms on average fixed labor cost is around $\sim 16\%$ of total labor costs, and $f_X = 15$ so that exporters sales represent a $\sim 77\%$ of total sales. The calibrated parameters are in Table 2.

Parameter	Empirical Evidence[3]	Value
δ	Death rate [B et al]	0.025
f_E	Entry rate [B et al]	2
θ	Spanish distribution of firms 2008 [Target]	5
σ	Estimated demand elasticity for imports [BW]	3
α	Innovation elasticity [R]	0.9
κ	Proportion of innovators [Target]	1.1
f_D	Average fixed labor cost [Target]	1.7
f_X	Exporter sales [Target]	15
τ	Average tariffs [CM]	1.35

Table 2: Calibration

5 Empirical Analysis

In this section I analyze the impact of a bilateral[4] trade liberalization on export and innovation decisions, aggregate innovation and aggregate productivity. I rely on comparative statics and numerical simulations to study the implications of two trade policies: a decrease in transport costs and a decrease in the fixed costs of exporting.

5.1 Firm's Decision

A reduction in transport costs increases export revenues inducing more firms to enter the export market. While revenues from the export market increase, all firms loose a portion of their domestic sales. Hence, the market shares loss induces the least productive firms to exit the economy and reduces the incentives to innovate of firms serving only the domestic market. The decrease from τ to $\widehat{\tau} < \tau$ shifts up the ZCP and IPC curves, inducing an increase in the cutoff productivity levels $\widehat{\varphi}_D > \varphi_D$ and $\widehat{\varphi}_{DI} > \varphi_{DI}$. However, since

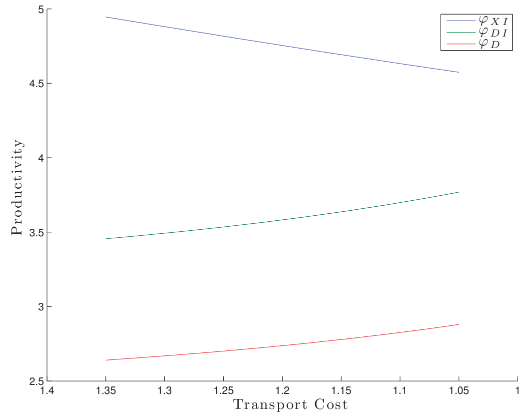
[3]The reference to the empirical literature is as follows: [B et al] for Bernard et al. (2007); [BW] for Broda and Weinstein (2006); [CM] for Costantini and Melitz (2008)); [R] Rubini (2009); [Target] Outcome of calibration;

[4]I assume that trade is done between two symmetric countries

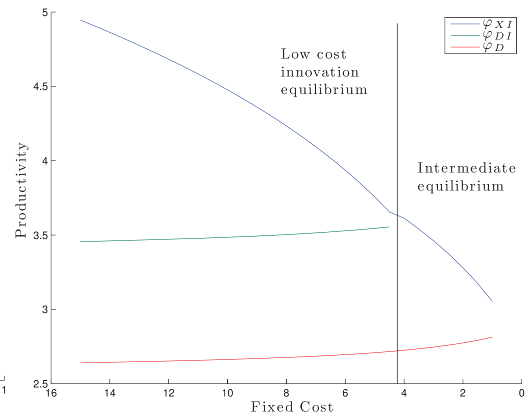
exporting is now easier, the exporter innovator cutoff $\widehat{\varphi}_{SI}$ will be below its previous level φ_{XI}.

A decrease in the fixed export market entry cost f_X induces similar changes in the cutoff levels as the decrease in τ. A decrease from f_X to $\widehat{f}_X < f_X$ induces the least productive firms to exit so that $\widehat{\varphi}_D > \varphi_D$, reduces the incentives to innovate for domestic firms so that $\widehat{\varphi}_{DI} > \varphi_{DI}$, and the increased exposure to trade induces new firms to enter the export market so that $\widehat{\varphi}_{XI} < \varphi_{XI}$. Although there is a reallocation of market shares towards more productive firms, the decrease in fixed export costs does not induce an increase in export revenues in the same way as a reduction in transport costs, since only new exporters increase their market shares.

Figure 4 represents the evolution of the productivity cutoffs under the discussed trade policies. In panel (a) I consider a reduction of 30% in transport costs, from $\tau = 1.35$ to $\tau = 1.05$ as in Costantini and Melitz (2008). In panel (b), a reduction from$f_X = 15$ to $f_X = 1$ is considered. A decrease in fixed costs does not induce an increase in the export revenues of previous exporters, only new exporters increase their market shares. On the one hand, this implies that the selection effect has less impact than in a drop in transport costs, as can be seen from the evolution ofφ_D and φ_{DI}. On the other hand, it implies that the incentives to export are much larger, to the point that a large drop in fixed costs can induce a change in the firm productivity distribution. With low enough fixed costs, being a domestic innovator is no longer profitable and the Spanish economy would go from being in the *low cost innovation equilibrium* to the *intermediate equilibrium*.



(a) Decrease in τ



(b) Decrease in f_X

Figure 4: Evolution Productivity Cutoffs

5.2 Innovation

The decisions of exporting and innovation are endogenous, therefore any liberalization that induces the exit of some firms and a reallocation of market shares has an impact on the evolution of the aggregate innovation of the economy, where aggregate innovation is measured as[5]:

$$Z(\varphi) = \int_0^\infty z(\varphi)\, dG(\varphi) \tag{12}$$

There are two dimensions through which a trade liberalization can affect the aggregate innovation. First, through the amount of resources each firm dedicates to innovation, that is, the intensive margin of innovation. And second, through the number of innovators, that is, the extensive margin of innovation. I examine how the evolution of the extensive and intensive margin of innovation determine the evolution of the aggregate innovation of the economy under a trade liberalization and how the choice in trade policy matters for this evolution.

[5]See Appendix C for a complete development of the aggregate innovation

5.2.1 Lowering transportation cost

The optimal innovation function before and after a trade liberalization is shown in Figure 5. A decrease in transport cost increases the incentives of domestic innovators to enter the export market and these new exporter innovators will increase their innovation. Also the exporters innovators before the liberalization will innovate more intensively after a trade liberalization since their innovation depends directly on transport costs (see equation (3)). Only the remaining domestic innovators will innovate less intensively since they loose market shares due to the trade liberalization. Overall, the expected effect on aggregate innovation through the intensive margin is positive.

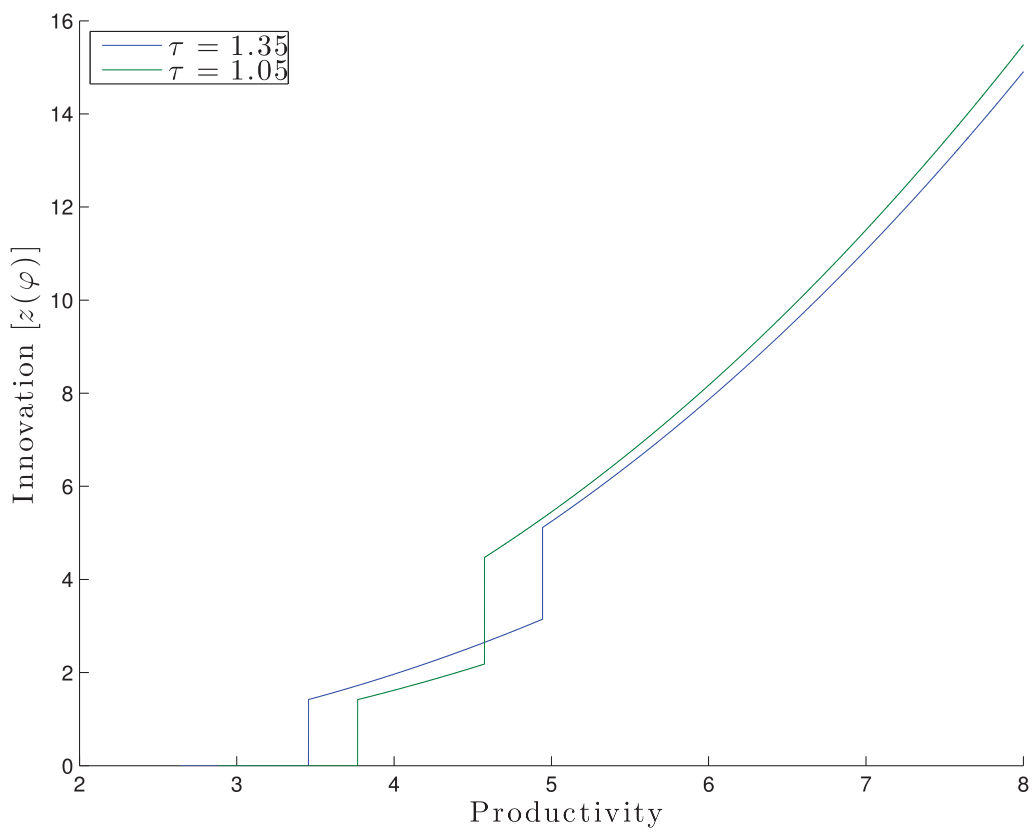


Figure 5: Intensive Innovation

Figure 6 shows the extensive margin of innovation and the share of innovators. The number of firms active in the market with $\tau = 1.35$ has been normalized to 1, therefore the share and the number of innovators are the same before the trade liberalization. A trade liberalization implies that more domestic innovators will become exporter innovators while it is harder to be a domestic innovator. Therefore, the mass of innovators $\widehat{M_I} = \widehat{m}_{DI} + \widehat{m}_{XI}$ is reduced with respect to M_I, the number of innovators when $\tau > \widehat{\tau}$, and keeps decreasing as the economy opens. However, the share of innovators remains constant throughout the liberalization, because the number of firms that exit is exactly equal to the number of firms that stop performing innovation due to the loss of market shares in the domestic market.

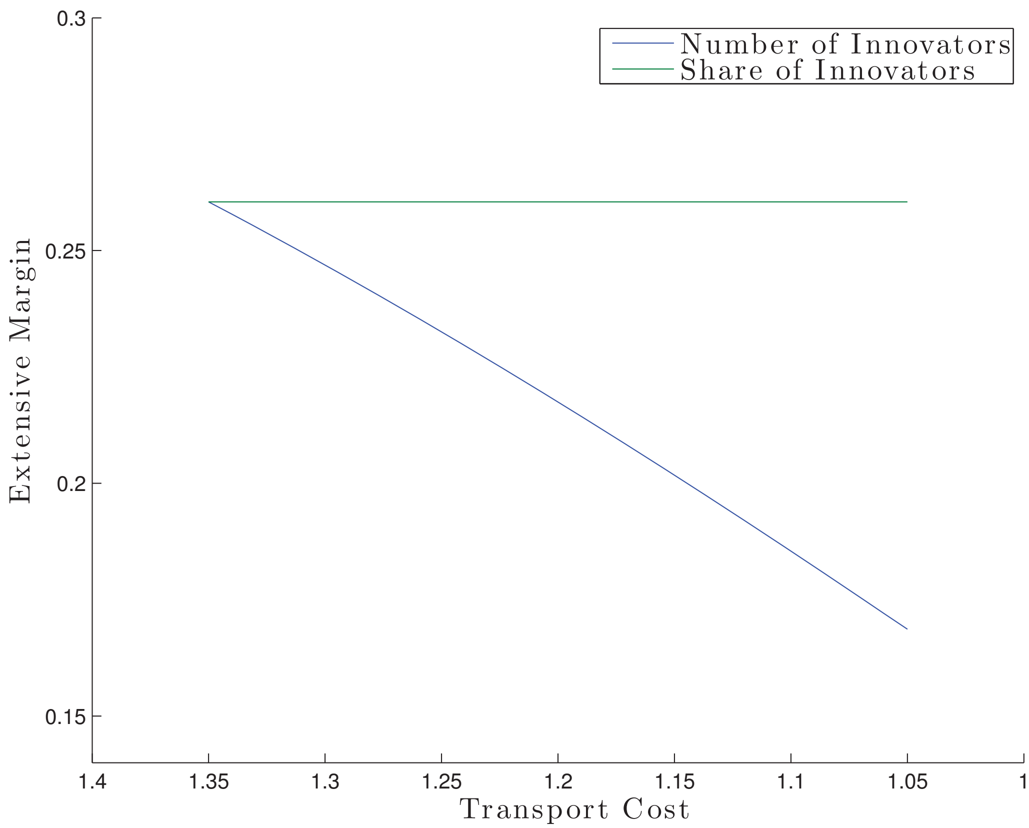


Figure 6: Extensive Innovation

A trade liberalization has opposite effects on the intensive and extensive margin of innovation. Figure 7 depicts the variation of the economys aggregate innovation with respect to the aggregate innovation before trade liberalization. The simulation of a 30% decrease in transport costs reveals almost a 14% decrease in the aggregate innovation. The downward sloping trend signals that negative effects through the extensive margin dominate the positive effects through the intensive margin brought by a trade liberalization.

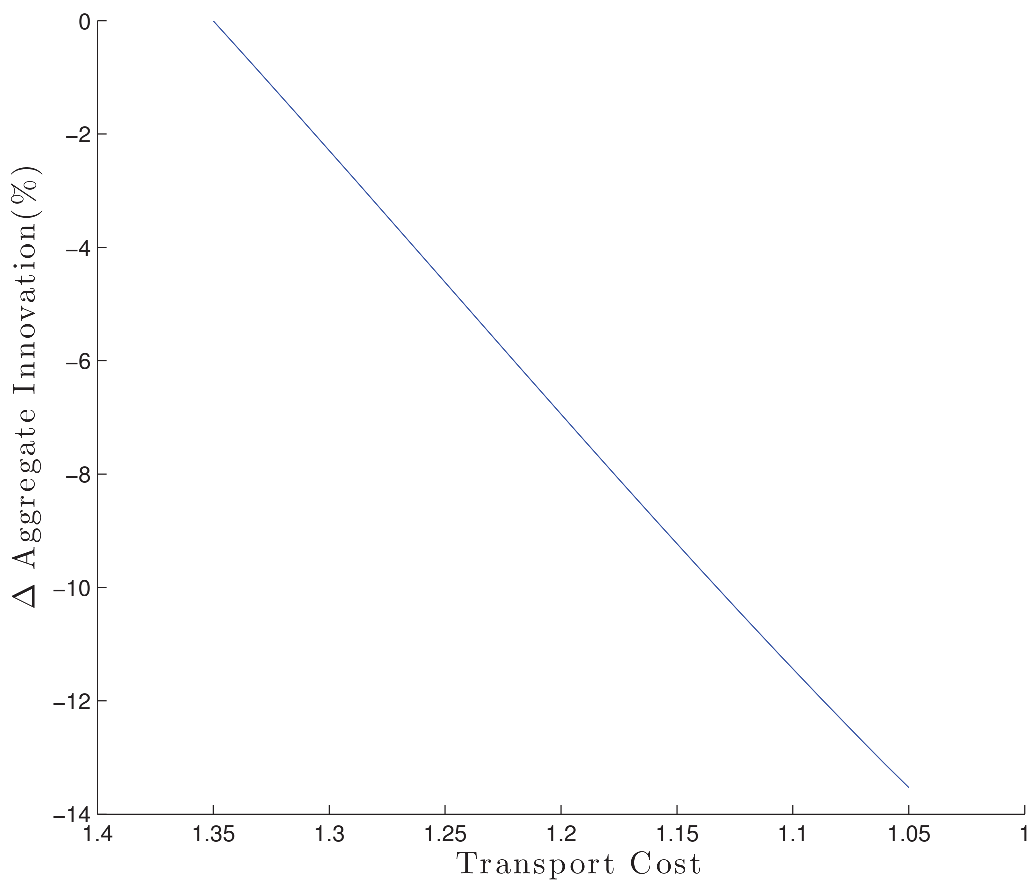


Figure 7: Aggregate Innovation

5.2.2 Lowering fixed exporting cost

Figure 8 shows the optimal innovation function before the liberalization, after a mild drop in fixed exporting costs and after a large drop in fixed costs. Similarly to a decrease in transport costs, with a decrease in fixed costs the expected effect on aggregate innovation through the intensive margin is positive. However, since the exporting fixed cost does not have a direct impact on innovation intensity, the main impact of a drop in f_X on the innovation intensity comes from new exporter innovators. The more open the economy, the larger this effect will be.

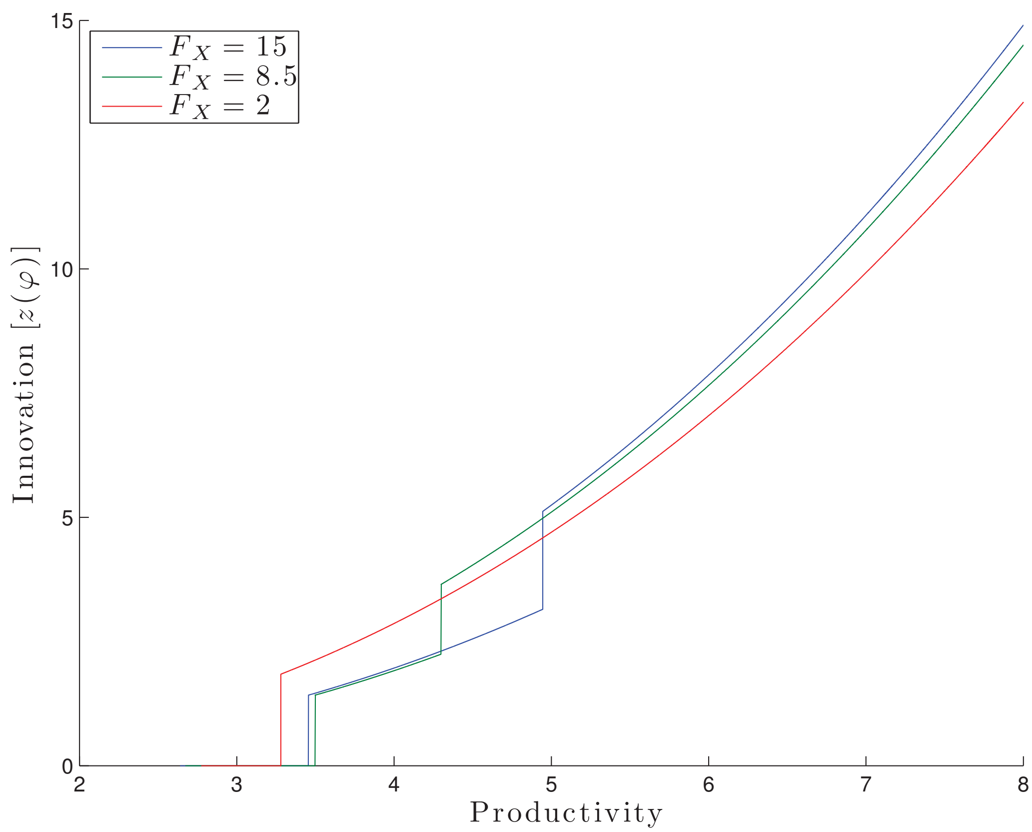


Figure 8: Intensive Innovation

Figure 9 shows the extensive margin of innovation and the share of innovators. The number of firms active in the market with $f_X = 15$ has been normalized to 1, therefore the share and the number of innovators are the same before the trade liberalization. A trade liberalization implies that more domestic innovators will become exporter innovators while it is harder to be a domestic innovator. Therefore, the mass of innovators is reduced with respect to M_I , the number of innovators when $f_X > \widehat{f_X}$, and keeps decreasing as the economy opens. However, once there are no longer domestic innovators, the economy shifts to the *intermediate equilibrium*, and a decrease in export fixed cost induces firms that did not innovate or export to undertake both activities and the number of innovators increases.

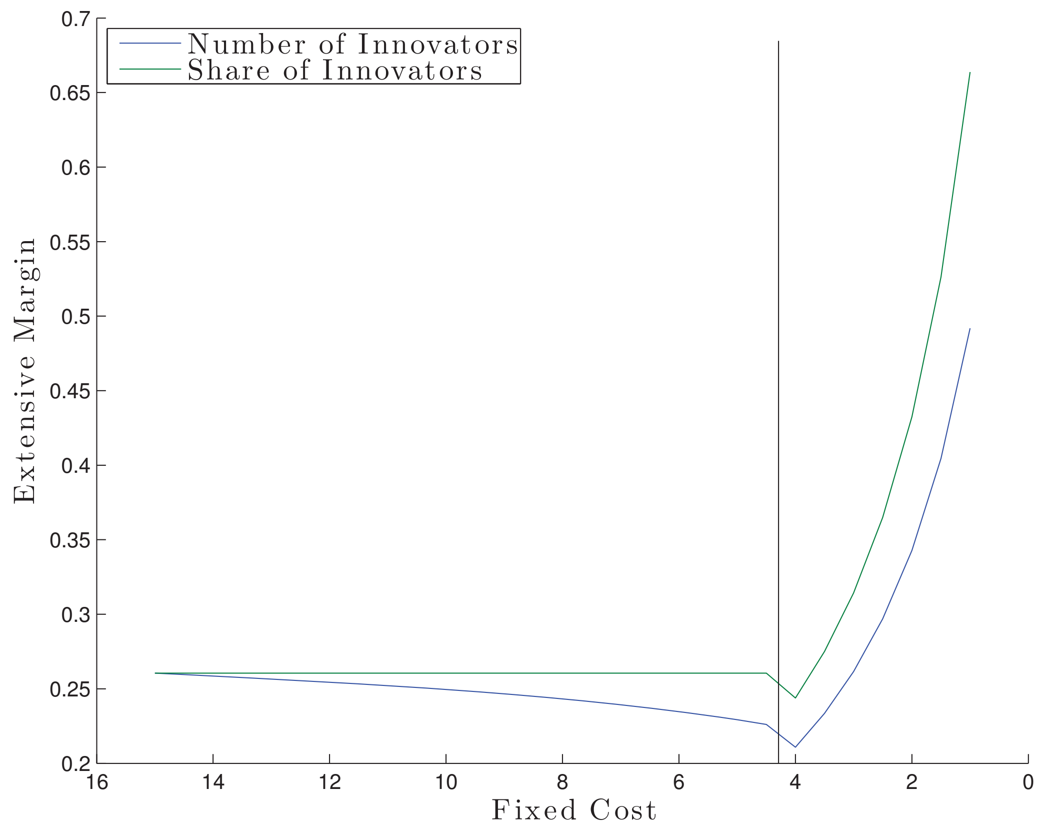


Figure 9: Extensive Innovation

The number of innovators follows a U shape as the economy shifts from one equilibrium to another due to a decrease in fixed export costs. The share of innovators remains constant throughout the *low cost innovation equilibrium* as it happened with a drop in transport costs but once domestic innovators disappear and trade induces firms to start exporting and innovating, it increases.

The impact of a decrease in the fixed cost of exporting on the extensive margin of innovation is U-shaped while it is positive on the intensive margin of innovation. Figure 10 depicts the variation of the economys aggregate innovation with respect to the aggregate innovation before trade liberalization.

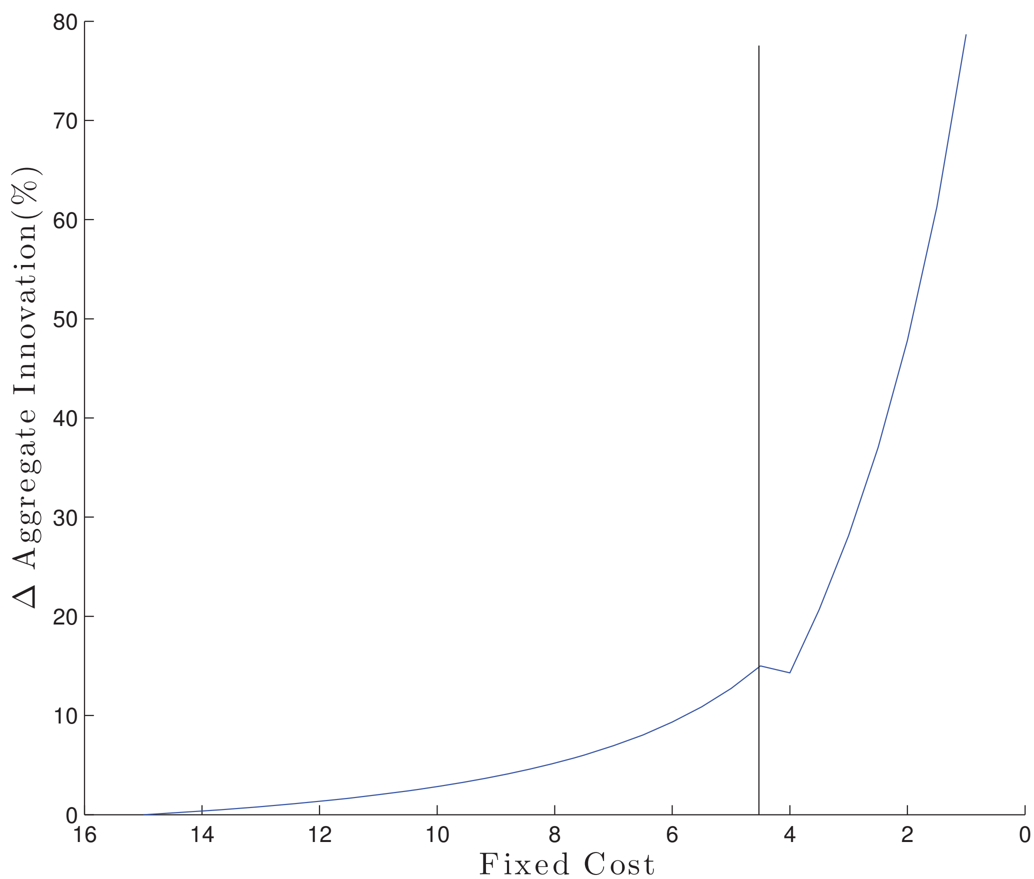


Figure 10: Aggregate Innovation

The simulation reveals that a decrease from $f_X = 15$ to $f_X = 6$ implies a 10% increase in the aggregate innovation, and that a decrease from $f_X = 15$ to $f_X = 2$ implies up to a 45% increase. The upward sloping trend signals that effects through the intensive margin overpower the negative effects through the extensive margin in its downward sloping section.

5.2.3 Discussion

I have considered the effects on innovation of two trade policies. Even though the effects of a transport costs liberalization and fixed costs liberalization go in the same direction, increasing the intensive margin of innovation and decreasing the extensive margin of innovation (while in the *low cost innovation equilibrium*), the behavior of aggregate innovation goes in opposite directions in both cases. In a transport costs liberalization the negative effects through the extensive margin overpower the positive effects through the intensive margin, even though the transport costs directly affect the intensive margin. In a fixed cost liberalization exactly the opposite happens and there are no direct effects on the intensive margin that would boost the effect of the intensive margin on aggregate innovation, hence the real difference is made by new exporter innovators. In a fixed cost liberalization, the incentives to become an exporter innovator are much larger than in a transport cost liberalization and therefore the positive effects through the intensive margin of innovation are larger. If the objective of policy makers were to increase aggregate innovation, trade policy should be focused in lowering the fixed costs of exporting and potentiating firms to enter the export market, rather than lowering transportation costs.

5.3 Aggregate Productivity

The output of the economy can be expressed as a function of the number of workers in the economy, their productivity and the elasticity of substitution

$$Q = \left(\frac{\sigma - 1}{\sigma}\right) \left[M\left(\Psi_D + \tau^{(1-\sigma)}\Psi_X + \Psi_I\right)\right]^{\frac{1}{\sigma-1}} L \tag{13}$$

where $\Psi_D = \int_{\varphi_D}^{\infty} \varphi^{(\sigma-1)}\mu(\varphi)d\varphi$, $\Psi_X = \int_{\varphi_X}^{\infty} \varphi^{(\sigma-1)}\mu(\varphi)d\varphi$ and Ψ_I is the productivity from innovation activities[6].

The term $\left[M\left(\Psi_D + \tau^{(1-\sigma)}\Psi_X + \Psi_I\right)\right]^{\frac{1}{\sigma-1}}$ can be interpreted as the aggregate productivity of the economy and the average productivity per firm is

$$\Psi = \frac{\left[M\left(\Psi_D + \tau^{(1-\sigma)}\Psi_X + \Psi_I\right)\right]^{\frac{1}{\sigma-1}}}{M} \tag{14}$$

In this section I analyze the effect on aggregate productivity of the two trade policies considered. Figure 11 depicts the variation of total and average productivity of the economy during a reduction of 30% in transport costs, from $\tau = 1.35$to $\tau = 1.05$. The simulation reveals that such liberalization would imply almost a 10% increase in total productivity and almost a 70% increase in the average productivity of the economy. Figure 12 depicts the variation of the total and average productivity of the economy during a reduction of fixed export costs, from $f_X = 15$ to $f_X = 1$. The simulation reveals that such liberalization would imply a 6.5% increase in total productivity and almost a 45% increase in the average productivity of the economy.

There is a large difference in average productivity gains while the difference in total productivity is smaller. The reason behind the difference in average productivity is the selection effect which is larger in a transport cost liberalization, that is, more firms exit the economy. If the objective of policy makers was to increase the average productivity of the economy, a

[6] See Appendix C for a formal derivation of the Aggregate Productivity in equilibrium

trade policy should be focused in lowering the transport costs of the economy. The evolution of total productivity is quite similar under both policies, however a transport cost liberalization increases more the gap between firms at the top and the bottom of the distribution, since there is a reallocation of market shares from firms that exit towards those at the top. In a fixed cost liberalization, while this effect exists is much smaller since there are less firms exiting the economy, and most of the gains come from new exporter innovators.

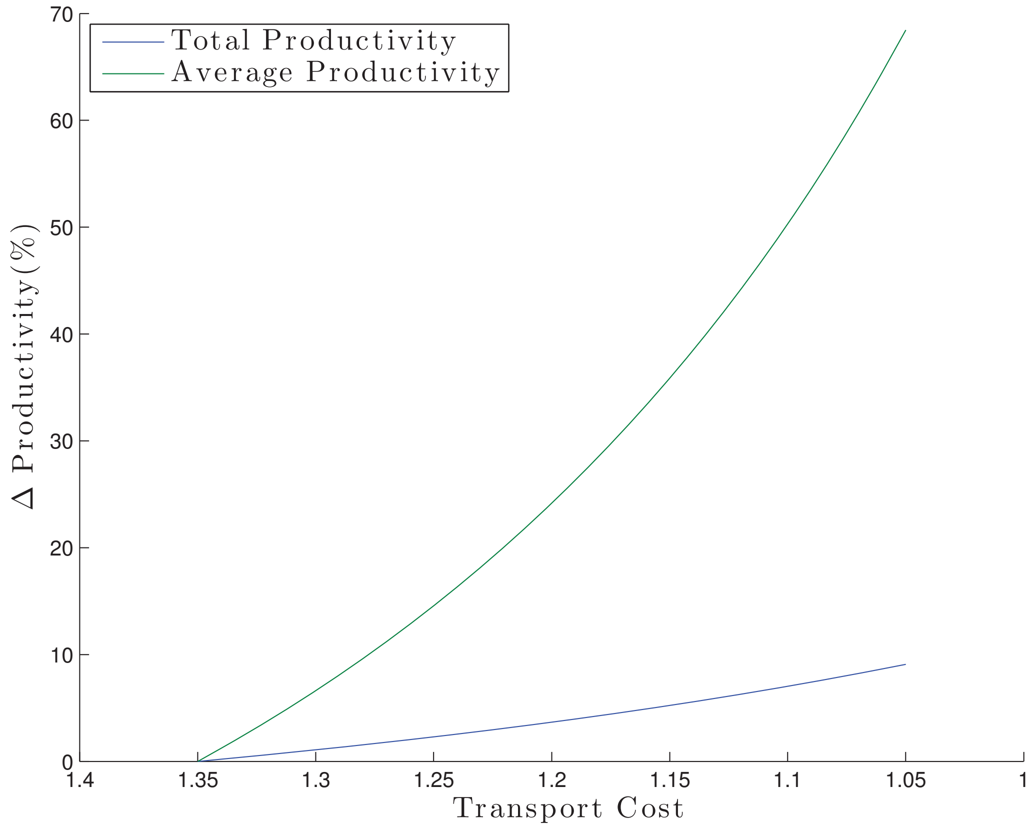


Figure 11: Evolution Productivity Cutoffs

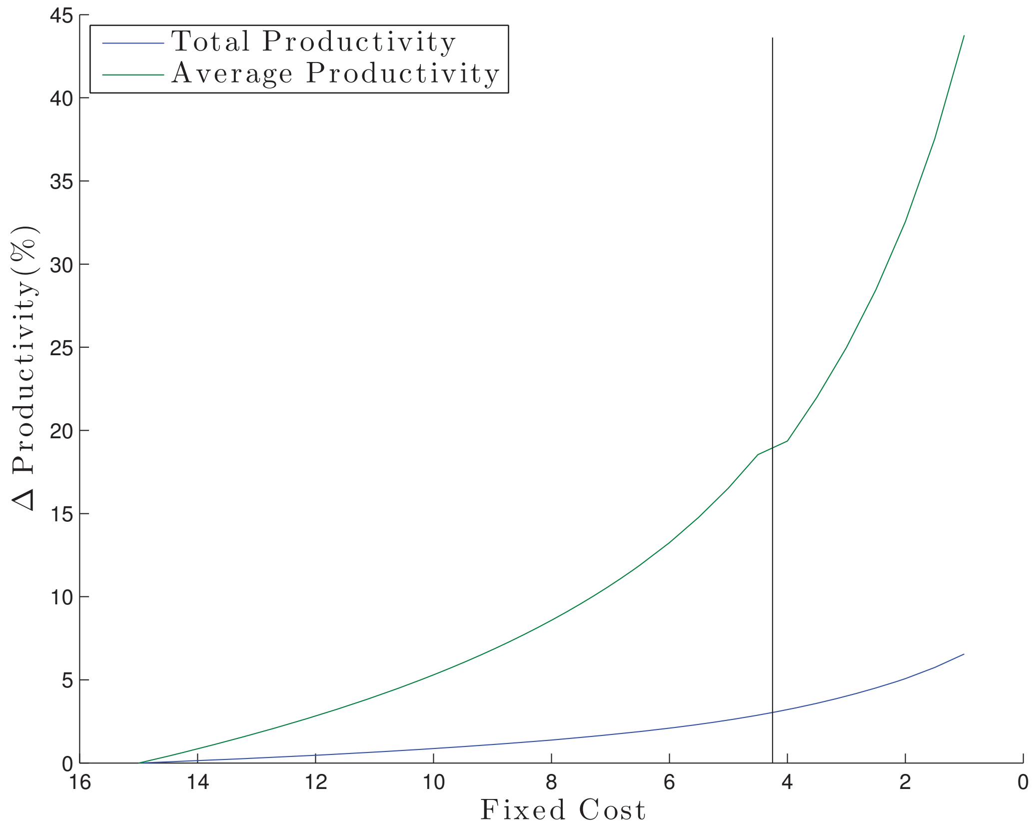


Figure 12: Evolution Productivity Cutoffs

6 Conclusions

This paper proposes a trade model with heterogeneous firms that can influence their productivity through process innovation in order to account not only for productivity gains due to the reallocation of shares between firms but also for productivity gains within the firm.

The model has implications on the aggregate productivity level. Firstly, trade induces the exit of the less productive firms and the reallocation of market shares towards the more productive firms, rising the industry average productivity in the long run. And secondly, trade has indirect effects on the average productivity through innovation. In the open economy exist three different equilibrium and it is the interaction of the innovation and the export decisions determines which one of the equilibria emerges. The cost of exporting relative to the cost of innovation decreases as the economy moves-from the low cost innovation equilibrium to the low cost trade equilibrium, and therefore the effect trade has through innovation will be differentiated according to the level of exporting costs. Thus, in the empirical section of the paper, I calibrate the model to match the Spanish economy and explore the effect of two different trade policies on firm decisions, aggregate innovation and aggregate productivity. Simulations reveal that policy makers should focus on reducing the fixed costs of exporting if their objective is to increase innovation or the number of innovators in the economy while a decrease on transport costs would be more effective to increase the average productivity in the economy.

Current research is focused on the study of the effects a trade liberalization has on the productivity gap between firms at the top and bottom of the distribution. And the decomposition of productivity gains into the selection effect, the extensive margin of innovation effect and the extensive margin of innovation, to better understand the role they play, and therefore which policy is more adequate. Further research will focus on the study of these policies in other european countries and at the sectorial level in the Spanish economy.

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Appendix A - Low Cost Innovation Economy

Productivity distribution and weighted averages

Let us denote by $\mu_D(\varphi)$, $\mu_{DI}(\varphi)$ and $\mu_{XI}(\varphi)$ respectively, the productivity distribution of domestic producers, active innovators and active innovators and exporters prior to innovation.

$$\mu_D(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_{DI})-G(\varphi_D)} & ,\varphi_{DI} > \varphi \geq \varphi_D \\ 0 & ,otherwise \end{cases}$$

$$\mu_{DI}(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_{XI})-G(\varphi_{DI})} & ,\varphi_{XI} \geq \varphi \geq \varphi_{DI} \\ 0 & ,otherwise \end{cases}$$

$$\mu_{XI}(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_{XI})} & ,\varphi \geq \varphi_{XI} \\ 0 & ,otherwise \end{cases}$$

The distributions $\mu_D(\varphi)$, $\mu_{DI}(\varphi)$ and $\mu_{XI}(\varphi)$ are not affected by the simultaneous entry and exit since the successful entrants and failing incumbents draw their productivity level from the common distribution $\mu(\varphi)$.

Let $\widetilde{\varphi} = \left[\int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)\, d\varphi\right]^{\frac{1}{(\sigma-1)}}$ and $\widetilde{\varphi}_X = \left[\int_{\varphi_{XI}}^{\infty} \varphi^{\sigma-1}\mu_{XI}(\varphi)\, d\varphi\right]^{\frac{1}{(\sigma-1)}}$ denote the average productivity levels of, respectively, all firms and exporting firms only prior to innovation. Then the weighted productivity average that reflects the combined market share of all firms can be defined as

$$\widetilde{\varphi}_t = \left\{\frac{1}{M_t}\left[M\widetilde{\varphi}^{\sigma-1} + nM_X\left(\tau^{-1}\widetilde{\varphi}_X\right)^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}$$

And let $\widetilde{\varphi}_{DI} = \left[\int_{\varphi_{DI}}^{\infty} \left(\varphi^{\sigma-1}\right)^{\frac{(\alpha+1)}{\alpha}} \mu_{DI}(\varphi)\, d\varphi\right]^{\frac{\alpha}{(\alpha+1)}\frac{1}{(\sigma-1)}}$ and $\widetilde{\varphi}_{XI}$ represent the average productivity the domestic innovators and exporter innovators get from innovation. Then the weighted productivity average that reflects the combined market share of innovation can be defined as

$$\widetilde{\varphi}_t^I = \left\{\frac{1}{M_t^I}\left[M_I\left(\widetilde{\varphi}_{DI}\right)^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)} + m_{XI}\left(\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right)\left(\widetilde{\varphi}_{XI}\right)^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\right]\right\}^{\left(\frac{\alpha}{\alpha+1}\right)\left(\frac{1}{\sigma-1}\right)}$$

Aggregate Variables

Denote by m_{XI}, m_{DI} and m_D respectively the mass of active innovators and exporters, active innovators but non-exporters and non-innovators and non-exporters present in the economy,

$$
\begin{aligned}
m_{XI} &= \frac{1 - G\left(\varphi_{XI}\right)}{1 - G\left(\varphi_D\right)} M \\
m_{DI} &= \frac{G\left(\varphi_{XI}\right) - G\left(\varphi_{DI}\right)}{1 - G\left(\varphi_D\right)} M \\
m_D &= \frac{G\left(\varphi_{DI}\right) - G\left(\varphi_D\right)}{1 - G\left(\varphi_D\right)} M
\end{aligned}
$$

with M being the mass of incumbent firms in the economy, $M_I = m_{DI} + m_{XI}$ the number of firms that perform innovation activities and $M_X = m_{XI}$ the number of firms performing exporting activities. The total number of varieties sold in the economy (by symmetry) will be $M_t = M + nM_X$, and the total number of varieties coming from innovators will be $M_t^I = M_I + nM_X$. It can be shown that the aggregates will take the following expressions

- Aggregate Price Index

$$
P^{1-\sigma} = M_t \left[p_D\left(\widetilde{\varphi}_t\right)\right]^{1-\sigma} + M_t^I \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left[p_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)\right]^{1-\sigma}
$$

- Aggregate Production

$$
Q^{\rho} = M_t \left[q_D\left(\widetilde{\varphi}_t\right)\right]^{\rho} + M_t^I \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left[q_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)\right]^{\rho}
$$

- Aggregate Revenue

$$R = M_t r_D\left(\widetilde{\varphi}_t\right) + M_t^I \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)$$

- Aggregate Profits

$$\begin{aligned} \Pi &= M_t \frac{r_D\left(\widetilde{\varphi}_t\right)}{\sigma} + M_t^I \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \frac{r_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)}{\sigma} \\ &\quad - M f_D - n M_X f_X - M_I \kappa - M_t^I \left(\frac{\kappa}{\alpha}\right) \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{\alpha+1}{\alpha}} \left(\widetilde{\varphi}_t^I\right)^{\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)} \end{aligned} \tag{A.1}$$

Low Cost Innovation Equilibrium

Proof of Proposition 1, part II

If there are sufficiently high fixed export cost, there exist a single cutoff φ_{XI}^* that solves equation (6)

Proof. The proof is divided in three sections

First, I show that the LHS of equation (6) is positive with respect to the productivity parameter. $\pi_{XI}\left(\varphi_{XI}\right) - \pi_{DI}\left(\varphi_{XI}\right) \geq 0$

$$\left[\left(1 + n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right] \alpha \left(\tfrac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} \left[\left(\tfrac{R(P\rho)^{\sigma-1}}{\sigma}\right) \varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}} + n\tau^{1-\sigma} \left(\tfrac{R(P\rho)^{\sigma-1}}{\sigma}\right) \varphi^{\sigma-1} - n f_x \geq 0$$

$$C_1 \left(\varphi^{\sigma-1}\right)^{\frac{\alpha+1}{\alpha}} + C_2 \varphi^{\sigma-1} - n f_x \geq 0$$

$$\frac{\partial LHS}{\partial \varphi} = C_1 \left(\frac{\alpha+1}{\alpha}\right) (\sigma - 1) \varphi^{\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)-1} + C_2 (\sigma - 1) \varphi^{\sigma-2} > 0$$

Secondly, I show that $\pi_{XI}\left(\varphi_{DI}\right) - \pi_{DI}\left(\varphi_{DI}\right) < 0$, otherwise the firm would choose to export and innovate instead of being indifferent between innovating or not while staying in the domestic market.

$$\pi_{XI}(\varphi_{DI}) - \pi_{DI}(\varphi_{DI}) < 0$$
$$\left[\left(1 + n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right]\kappa + n\tau^{1-\sigma}\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) - nf_X < 0$$

Thus, for f_X large enough, that is for

$$f_X > \left[\left(1 + n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right]\frac{\kappa}{n} + \tau^{1-\sigma}\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)$$

it holds that $\pi_{XI}(\varphi_{DI}) - \pi_{DI}(\varphi_{DI}) < 0$

Finally, I show that the difference between the profits of the exporting and non-exporting strategies while innovation goes to infinite as the productivity of the firm is larger.

If $\varphi \to \infty$,then $\pi_{XI}(z(\varphi)) - \pi_{DI}(z(\varphi)) \to \infty$, since by definition $\pi_{XI}(z_x(\varphi)) > \pi_{XI}(z(\varphi))$ then it must be that $\pi_{XI}(z_X(\varphi)) - \pi_{DI}(z(\varphi)) \to \infty$ as $\varphi \to \infty$

$$\begin{aligned}
\pi_{XI}(j(\varphi)) - \pi_{DI}(j(\varphi)) &= n\tau^{1-\sigma}[1+z]\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1} - nf_X \\
&= n\tau^{1-\sigma}\left(\frac{1}{\alpha+1}\right)^{\frac{1}{\alpha}}\left[\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}} \\
&\quad + n\tau^{1-\sigma}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1} - nf_X
\end{aligned}$$

$$\begin{aligned}
\lim_{\varphi\to\infty}\left[\pi_{XI}(j(\varphi)) - \pi_{DI}(j(\varphi))\right] &= \lim_{\varphi\to\infty}\left[C_4\left[\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}} + C_5\varphi^{\sigma-1} - C_6\right] \\
&= \lim_{\varphi\to\infty}\left[C_4\left[\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}\right] + \lim_{\varphi\to\infty}\left[C_5\varphi^{\sigma-1}\right] - \lim_{\varphi\to\infty}(C_6) \to \infty
\end{aligned}$$

□

Proof of Proposition 1, part I

Equations (4) to (6) along with the Free Entry condition (7) completely determine the equilibrium and the productivity cutoffs can be uniquely determined and allow me to rearrange the FE conveniently for the characterizing of the equilibrium as a function of φ_D^*

$$\delta f_E = \left[1 - G\left(\varphi_D^*\right)\right]\overline{\pi}$$

$$\begin{aligned}\delta f_E \; = \; & f_D j_1\left(\varphi_D^*\right) + n\tau^{1-\sigma} f_D j_2\left(\varphi_X^*\left(\varphi_D^*\right)\right) - \left[1 - G\left(\varphi_{XI}^*\right)\right] n f_X \\ & - \left[1 - G\left(\varphi_{DI}^*\right)\right]\kappa + \alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} f_D^{\frac{\alpha+1}{\alpha}} j_3\left(\varphi_D^*\right) \\ & + \alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} f_D^{\frac{\alpha+1}{\alpha}}\left[\left(1 + n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right] j_4\left(\varphi_D^*\right)\end{aligned} \quad \text{(A.2)}$$

where $j_1\left(\varphi_D^*\right) = \frac{\left[\left(\widetilde{\varphi}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1} - 1\right]}{\left[1-G\left(\varphi_D^*\right)\right]}$, $j_2\left(\varphi_D^*\right) = \frac{\left(\widetilde{\varphi}_x\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}}{\left[1-G\left(\varphi_{XI}^*\right)\right]}$,
$j_3\left(\varphi_D^*\right) = \frac{\left[\left(\widetilde{\varphi}_{DI}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}}{\left[1-G\left(\varphi_{DI}^*\right)\right]}$ and $j_4\left(\varphi_D^*\right) = \frac{\left[\left(\widetilde{\varphi}_{XI}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}}{\left[1-G\left(\varphi_{XI}^*\right)\right]}$

Proof. Assume the parameter restrictions $\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) \geq f_D$ and $\tau^{\sigma-1} f_X \geq \frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right]}{n\tau^{1-\sigma}}\kappa + \left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)$ hold, then the Low Cost Innovation Equilibrium exists and is unique. I shall proof that the RHS of equation ($A.2$) is decreasing in φ_D^* on the domain (φ_D^*, ∞), so that φ_D^* is uniquely determined by the intersection of the latter curve with the flat line δf_E in the (φ_D^*, ∞) space.

Let $k_1\left(\varphi_D^*\right) = \left[\left(\widetilde{\varphi}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1} - 1\right]$, then

$$k_1'\left(\varphi_D^*\right) = \frac{g\left(\varphi_D^*\right)}{1 - G\left(\varphi_D^*\right)} k_1\left(\varphi_D^*\right) - \frac{(\sigma-1)\left[k_1\left(\varphi_D^*\right) + 1\right]}{\varphi_D^*}$$

Similarly, $k_3(\varphi_D^*) = \left[(\widetilde{\varphi}_{DI}(\varphi_D^*)/\varphi_D^*)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}$, thus

$$k_3'(\varphi_D^*) = \Lambda^{\frac{1}{\sigma-1}} \frac{g(\varphi_I^*)}{1-G(\varphi_I^*)} \left[k_2(\varphi_D^*) - \Lambda^{\frac{\alpha+1}{\alpha}}\right] - \left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\frac{k_2(\varphi_D^*)}{\varphi_D^*}$$

where $\frac{\partial \varphi_{DI}^*}{\partial \varphi_D^*} = \left[\frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{f_D}\right]^{\frac{1}{\sigma-1}} = \Lambda^{\frac{1}{\sigma-1}}$

Now, define $j_1(\varphi_D^*) = [1-G(\varphi_D^*)]\, k_1(\varphi_D^*)$, and $j_2(\varphi_D^*) = [1-G(\varphi_{DI}^*)]\, k_2(\varphi_D^*)$ which are non-negative.

Then the derivative and elasticity of $j_1(\varphi_D^*)$ and $j_3(\varphi_D^*)$ are

$$j_1'(\varphi_D^*) = -\frac{(\sigma-1)[k_1(\varphi_D^*)+1]}{\varphi_D^*}[1-G(\varphi_D^*)] < 0$$
$$\frac{j_1'(\varphi_D^*)\cdot\varphi_D^*}{j_1(\varphi_D^*)} = \underbrace{-(\sigma-1)\left[1+\frac{1}{k_1(\varphi_D^*)}\right]}_{<0\text{and bounded away of it}} < -(\sigma-1)$$

and

$$j_3'(\varphi_D^*) = -g(\varphi_{DI}^*)\,\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}} - \theta(\alpha+1)(\sigma-1)\frac{k_3(\varphi_D^*)}{\varphi_D^*}[1-G(\varphi_{DI}^*)] < 0$$
$$\frac{j_3'(\varphi_D^*)\cdot\varphi_D^*}{j_3(\varphi_D^*)} = \underbrace{-\frac{g(\varphi_{DI}^*)}{[1-G(\varphi_{DI}^*)]}\frac{\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}}}{k_2(\varphi_D^*)}\varphi_D^* - \beta(\sigma-1)}_{<0\text{and bounded away of it}} < -\beta(\sigma-1)$$

Thus, $j_1(\varphi_D^*)$ and $j_3(\varphi_D^*)$ must be decreasing to zero as φ goes to infinite. Furthermore, it must be that $\lim_{\varphi_D^*\to 0} j_1(\varphi_D^*) = \infty$ since $\lim_{\varphi_D^*\to 0} k_1(\varphi_D^*) = \infty$.and $\lim_{\varphi_D^*\to 0} j_3(\varphi_D^*) = \infty$ since $\lim_{\varphi_D^*\to 0} k_3(\varphi_D^*) = \infty$

Since $j_1(\varphi_D^*)$ and $j_3(\varphi_D^*)$, it follows that $j_2(\varphi_D^*)$ and $j_4(\varphi_D^*)$ do also monotonically decrease from infinite to zero on the $(0,\infty)$ parameter space.

Therefore, the RHS of (A.2) is a monotonic decreasing function from infinity to zero on the space $(0,\infty)$ that cuts the FE flat line from above identifying a unique cutoff level φ_D^*. □

Appendix B - Low Cost Trade Economy

Productivity distribution and weighted averages

Let us denote by $\mu_D(\varphi)$, $\mu_X(\varphi)$ and $\mu_{XI}(\varphi)$ respectively, the productivity distribution of domestic producers, exporters and innovators exporters.

$$\mu_D(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_X)-G(\varphi_D)} & ,\varphi_X > \varphi \geq \varphi_D \\ 0 & ,otherwise \end{cases}$$
$$\mu_X(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_{XI})-G(\varphi_X)} & ,\varphi_{XI} \geq \varphi \geq \varphi_X \\ 0 & ,otherwise \end{cases}$$
$$\mu_{XI}(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_{XI})} & ,\varphi \geq \varphi_{XI} \\ 0 & ,otherwise \end{cases}$$

The distributions $\mu_D(\varphi)$, $\mu_X(\varphi)$ and $\mu_{XI}(\varphi)$ are not affected by the simultaneous entry and exit since the successful entrants and failing incumbents draw their productivity level from the common distribution $\mu(\varphi)$.

Let $\widetilde{\varphi} = \left[\int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)\,d\varphi\right]^{\frac{1}{(\sigma-1)}}$ and $\widetilde{\varphi}_X = \left[\int_{\varphi_{XI}}^{\infty} \varphi^{\sigma-1}\mu_{XI}(\varphi)\,d\varphi\right]^{\frac{1}{(\sigma-1)}}$ denote the average productivity levels of, respectively, all firms and exporting firms only prior to innovation. Then the weighted productivity average that reflects the combined market share of all firms can be defined as

$$\widetilde{\varphi}_t = \left\{\frac{1}{M_t}\left[M\widetilde{\varphi}^{\sigma-1} + nM_X\left(\tau^{-1}\widetilde{\varphi}_X\right)^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}$$

And let $\widetilde{\varphi}_{XI} = \left[\int_{\varphi_{XI}}^{\infty} \left(\varphi^{\sigma-1}\right)^{\frac{(\alpha+1)}{\alpha}} \mu_{XI}(\varphi)\,d\varphi\right]^{\frac{\alpha}{(\alpha+1)}\frac{1}{(\sigma-1)}}$ represent the average productivity the innovators get from innovation.

Aggregate Variables

Denote by m_{XI}, m_X and m_D respectively the mass of active innovators and exporters, only exporters and non-innovators non-exporters present in the economy,

$$
\begin{aligned}
m_{XI} &= \frac{1-G(\varphi_{XI})}{1-G(\varphi_D)}M \\
m_X &= \frac{G(\varphi_{XI})-G(\varphi_X)}{1-G(\varphi_D)}M \\
m_D &= \frac{G(\varphi_X)-G(\varphi_D)}{1-G(\varphi_D)}M
\end{aligned}
$$

with M being the mass of incumbent firms in the economy, $M_I = m_{XI}$ the number of firms that perform innovation activities and $M_X = m_X + m_{XI}$ the number of firms performing exporting activities. The total number of varieties sold in the economy (by symmetry) will be $M_t = M + nM_X$.
It can be shown that the aggregates will take the following expressions

- Aggregate Price Index

$$P^{1-\sigma} = M_t \left[p_D(\widetilde{\varphi}_t)\right]^{1-\sigma} + m_{XI}\left(1+n\tau^{1-\sigma}\right)\left[\frac{\kappa}{\alpha}\right]^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left[p_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)\right]^{1-\sigma}$$

- Aggregate Production

$$Q^{\rho} = M_t \left[q_D(\widetilde{\varphi}_t)\right]^{\rho} + m_{XI}\left(1+n\tau^{1-\sigma}\right)\left[\frac{\kappa}{\alpha}\right]^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left[q_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)\right]^{\rho}$$

- Aggregate Revenue

$$R = M_t r_D(\widetilde{\varphi}_t) + m_{XI}\left(1+n\tau^{1-\sigma}\right)\left[\frac{\kappa}{\alpha}\right]^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)$$

- Aggregate Profits

$$\Pi = M_t \frac{r_D(\widetilde{\varphi}_t)}{\sigma} + m_{XI}\left(1+n\tau^{1-\sigma}\right)\left[\frac{\kappa}{\alpha}\right]^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \frac{r_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)}{\sigma} \tag{B.1}$$
$$-Mf_D - nM_X f_X - m_{XI}\kappa - m_{XI}\left(1+n\tau^{1-\sigma}\right)\left(\frac{\kappa}{\alpha}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{\alpha+1}{\alpha}}\left(\widetilde{\varphi}_{XI}^{\sigma-1}\right)^{\frac{\alpha+1}{\alpha}}$$

Low Cost Trade Economy Equilibrium

Proof of Proposition 2

Equations (8) to (10) along with the Free Entry condition (11)completely determine the equilibrium and the productivity cutoffs can be uniquely determined and I can rearrange the FE conveniently for the characterizing of the equilibrium as a function of φ_D^*

$$\delta f_E = \left[1 - G\left(\varphi_D^*\right)\right]\overline{\pi}$$

$$\delta f_E = f_D l_1\left(\varphi_D^*\right) + n f_X l_2\left(\varphi_X^*\left(\varphi_D^*\right)\right) \tag{B.2}$$
$$+\alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}}\left[f_D\left(1+\tau^{1-\sigma}\right)\right]^{\left(\frac{\alpha+1}{\alpha}\right)} l_3\left(\varphi_D^*\right) - \left[1-G\left(\varphi_{XI}^*\right)\right]\kappa$$

where $j_1\left(\varphi_D^*\right) = \frac{\left[\left(\widetilde{\varphi}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}-1\right]}{\left[1-G\left(\varphi_D^*\right)\right]}$, $j_2\left(\varphi_X^*\left(\varphi_D^*\right)\right) = \frac{\left[\left(\widetilde{\varphi}\left(\varphi_X^*\right)/\varphi_X^*\right)^{\sigma-1}-1\right]}{\left[1-G\left(\varphi_X^*\right)\right]}$ and $j_3\left(\varphi_D^*\right) = \frac{\left[\left(\widetilde{\varphi}_{XI}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}\right]^{\left(\frac{\alpha+1}{\alpha}\right)}}{\left[1-G\left(\varphi_{XI}^*\right)\right]}$

Proof. Assume the parameter restriction $\frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} \geq \tau^{\sigma-1}f_X \geq f_D$ holds, then the Low Cost Trade Equilibrium exists and is unique. I shall proof that the RHS of equation (9) is decreasing in φ_D^* on the domain (φ_D^*, ∞), so that φ_D^* is uniquely determined by the intersection of the latter curve with the flat line δf_E in the (φ_D^*, ∞) space.

Let $k_1(\varphi_D^*) = \left[(\widetilde{\varphi}(\varphi_D^*)/\varphi_D^*)^{\sigma-1} - 1\right]$, then

$$k_1'(\varphi_D^*) = \frac{g(\varphi_D^*)}{1 - G(\varphi_D^*)} k_1(\varphi_D^*) - \frac{(\sigma-1)[k_1(\varphi_D^*)+1]}{\varphi_D^*}$$

Similarly, $k_3(\varphi_D^*) = \left[(\widetilde{\varphi}_{DI}(\varphi_D^*)/\varphi_D^*)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}$, thus

$$k_3'(\varphi_D^*) = \Lambda^{\frac{1}{\sigma-1}} \frac{g(\varphi_I^*)}{1 - G(\varphi_I^*)} \left[k_2(\varphi_D^*) - \Lambda^{\frac{\alpha+1}{\alpha}}\right] - \left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\frac{k_2(\varphi_D^*)}{\varphi_D^*}$$

where $\frac{\partial \varphi_{DI}^*}{\partial \varphi_D^*} = \left[\frac{\left(\frac{\kappa}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{f_D}\right]^{\frac{1}{\sigma-1}} = \Lambda^{\frac{1}{\sigma-1}}$

Now, define $j_1(\varphi_D^*) = [1 - G(\varphi_D^*)]\, k_1(\varphi_D^*)$, and $j_2(\varphi_D^*) = [1 - G(\varphi_{DI}^*)]\, k_2(\varphi_D^*)$ which are non-negative.

Then the derivative and elasticity of $j_1(\varphi_D^*)$ and $j_3(\varphi_D^*)$ are

$$j_1'(\varphi_D^*) = -\frac{(\sigma-1)[k_1(\varphi_D^*)+1]}{\varphi_D^*}[1 - G(\varphi_D^*)] < 0$$

$$\frac{j_1'(\varphi_D^*) \cdot \varphi_D^*}{j_1(\varphi_D^*)} = \underbrace{-(\sigma-1)\left[1 + \frac{1}{k_1(\varphi_D^*)}\right]}_{<0 \text{and bounded away of it}} < -(\sigma-1)$$

and

$$j_3'(\varphi_D^*) = -g(\varphi_{DI}^*)\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}} - \theta(\alpha+1)(\sigma-1)\frac{k_3(\varphi_D^*)}{\varphi_D^*}[1 - G(\varphi_{DI}^*)] < 0$$

$$\frac{j_3'(\varphi_D^*) \cdot \varphi_D^*}{j_3(\varphi_D^*)} = \underbrace{-\frac{g(\varphi_{DI}^*)}{[1 - G(\varphi_{DI}^*)]}\frac{\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}}}{k_2(\varphi_D^*)}\varphi_D^* - \beta(\sigma-1)}_{<0 \text{and bounded away of it}} < -\beta(\sigma-1)$$

Thus, $j_1(\varphi_D^*)$ and $j_3(\varphi_D^*)$ must be decreasing to zero as φ goes to infinite. Furthermore, it must be that $\lim_{\varphi_D^* \to 0} j_1(\varphi_D^*) = \infty$ since $\lim_{\varphi_D^* \to 0} k_1(\varphi_D^*) = \infty$.and $\lim_{\varphi_D^* \to 0} j_3(\varphi_D^*) = \infty$ since $\lim_{\varphi_D^* \to 0} k_3(\varphi_D^*) = \infty$

Since $j_1(\varphi_D^*)$ and $j_3(\varphi_D^*)$ are decreasing from infinity to zero on $(0,\infty)$,from the closed economy case, it follows that $j_2(\varphi_X^*(\varphi_D^*))$ does also monotonically decrease from infinite to zero on the $(0,\infty)$ parameter space.

Therefore, the RHS of $(B.2)$ is a monotonic decreasing function from infinity to zero on the space $(0,\infty)$ that cuts the FE flat line from above identifying a unique cutoff level φ_D^*. □

Appendix C - Aggregates

Aggregate Innovation

The general expression of aggregate innovation is given by equation (12), since the distribution of innovators changes in the open economy according to the relationship between export costs and innovation costs, for each case aggregate innovation can be expressed as follows

Low cost innovation equilibrium:

$$\begin{aligned} Z(\varphi) &= \int_{\varphi_{DI}}^{\varphi_{XI}} z_D(\varphi)\, m_{DI}\mu_{DI}\varphi d\varphi + \int_{\varphi_{XI}}^{\infty} z_X(\varphi)\, m_{XI}\mu_{XI}\varphi d\varphi \\ &= M_t^I \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(\widetilde{\varphi}_t^I\right)^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} \end{aligned} \tag{C.1}$$

Intermediate equilibrium:

$$\begin{aligned} Z(\varphi) &= \int_{\varphi_{XI}}^{\infty} z_X(\varphi)\, m_{XI}\mu_{XI}\varphi d\varphi \\ &= m_{XI}\left[\left(1+n\tau^{1-\sigma}\right)\frac{f_D}{\alpha+1}\right]^{\frac{1}{\alpha}} \left(\frac{1}{\varphi_D^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(\widetilde{\varphi}_{XI}\right)^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} \end{aligned} \tag{C.2}$$

Low cost trade equilibrium:

$$\begin{aligned} Z(\varphi) &= \int_{\varphi_{XI}}^{\infty} z_X(\varphi)\, m_{XI}\mu_{XI}\varphi d\varphi \\ &= m_{XI}\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left(\widetilde{\varphi}_{XI}\right)^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} \end{aligned} \tag{C.3}$$

Aggregate Productivity

In what follows I show that the output of the economy can be expressed as a function of the number of workers in the economy, their productivity and the elasticity of substitution and that equation (13) is the general form of such expression in the open economy. For the proof we use the facts that in equilibrium $L = R$, that the budget constraint is $PQ = R$ and the price rule given by equation (2).

Low cost innovation equilibrium:

$$\begin{aligned} R &= M_t r_D\left(\widetilde{\varphi}_t\right) + M_t^I\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right) \\ &= M\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} QP^{\sigma}\left\{\int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi\right. \\ &+ \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\int_{\varphi_{DI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \\ &+ \left.\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi\right\} \end{aligned}$$

Then,

$$
\begin{aligned}
L &= \left(\frac{\sigma}{\sigma-1}\right) Q M^{\frac{1}{1-\sigma}} \left\{ \int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma} \int_{\varphi_X}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi \right. \\
&+ \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \int_{\varphi_{DI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \\
&+ \left. \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right] \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \right\}^{\frac{1}{1-\sigma}}
\end{aligned}
$$

And

$$
\begin{aligned}
Q &= \left(\frac{\sigma-1}{\sigma}\right) \left[M\left(\Psi_D + n\tau^{(1-\sigma)}\Psi_X + \Psi_I\right)\right]^{\frac{1}{\sigma-1}} L \\
\Psi_I &= \left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(\Psi_{DI} + \left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right]\Psi_{XI}\right)
\end{aligned}
$$

where $\Psi_D = \int_{\varphi_D}^{\infty} \varphi^{(\sigma-1)}\mu(\varphi)d\varphi$, $\Psi_X = \int_{\varphi_X}^{\infty} \varphi^{(\sigma-1)}\mu(\varphi)d\varphi$, $\Psi_{DI} = \int_{\varphi_{CI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$ and $\Psi_{XI} = \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$

Intermediate equilibrium:

$$
\begin{aligned}
R &= M_t r_D\left(\widetilde{\varphi}_t\right) + m_{XI}\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \left(\frac{f_D}{\alpha+1}\right)^{\frac{1}{\alpha}} \left(\frac{1}{\varphi_D^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\left(\widetilde{\varphi}_{XI}\right)^{\frac{\alpha+1}{\alpha}}\right) \\
&= M\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} Q P^{\sigma} \left\{ \int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma} \int_{\varphi_X}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi \right. \\
&+ \left. \left(\frac{1}{\varphi_D^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \right\}
\end{aligned}
$$

Then,

$$\begin{aligned} L &= \left(\frac{\sigma}{\sigma-1}\right) QM^{\frac{1}{1-\sigma}} \left\{ \int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi \right. \\ &+ \left. \left(\frac{f_D}{\alpha+1}\right)^{\frac{1}{\alpha}} \left(\frac{1}{\varphi_D^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(1+\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \right\}^{\frac{1}{1-\sigma}} \end{aligned}$$

And

$$\begin{aligned} Q &= \left(\frac{\sigma-1}{\sigma}\right) \left[M\left(\Psi_D + n\tau^{(1-\sigma)}\Psi_X + \Psi_I\right)\right]^{\frac{1}{\sigma-1}} L \\ \Psi_I &= \left(\frac{f_D}{\alpha+1}\right)^{\frac{1}{\alpha}} \left(\frac{1}{\varphi_D^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(1+\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \Psi_{XI} \end{aligned}$$

where $\Psi_D = \int_{\varphi_D}^{\infty} \varphi^{(\sigma-1)}\mu(\varphi)d\varphi$, $\Psi_X = \int_{\varphi_X}^{\infty} \varphi^{(\sigma-1)}\mu(\varphi)d\varphi$ and $\Psi_{XI} = \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$

Low cost trade equilibrium:

$$\begin{aligned} R &= M_t r_D\left(\widetilde{\varphi}_t\right) + m_{XI}\left(1+n\tau^{1-\sigma}\right)\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right) \\ &= M\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} QP^{\sigma} \left\{ \int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi \right. \\ &+ \left. \left(1+n\tau^{1-\sigma}\right)\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \right\} \end{aligned}$$

Then,

$$\begin{aligned} L &= \left(\frac{\sigma}{\sigma-1}\right) QM^{\frac{1}{1-\sigma}} \left\{ \int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi \right. \\ &+ \left. \left(1+n\tau^{1-\sigma}\right)\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \right\}^{\frac{1}{1-\sigma}} \end{aligned}$$

And

$$\begin{aligned} Q &= \left(\frac{\sigma-1}{\sigma}\right)\left[M\left(\Psi_D + n\tau^{(1-\sigma)}\Psi_X + \Psi_I\right)\right]^{\frac{1}{\sigma-1}} L \\ \Psi_I &= \left(1+n\tau^{1-\sigma}\right)\left(\frac{\kappa}{\alpha}\right)^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \Psi_{XI} \end{aligned}$$

where $\Psi_D = \int_{\varphi_D}^{\infty} \varphi^{(\sigma-1)}\mu(\varphi)d\varphi$, $\Psi_X = \int_{\varphi_X}^{\infty} \varphi^{(\sigma-1)}\mu(\varphi)d\varphi$ and $\Psi_{XI} = \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$