Testing for Panel Cointegration using Common Correlated Effects Estimators*

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Abstract

Spurious regression analysis in panel data when time series are cross-section dependent is analyzed in the paper. We show that consistent estimation of the long-run average parameter is possible once cross-section dependence is controlled using cross-section averages in the spirit of the common correlated effects approach in Pesaran (2006). This result is used to design a panel cointegration test statistic. The performance of the proposal is investigated when both strong and weak cross-section dependence may be present.

Keywords: panel cointegration, cross-section dependence, common factors, spatial econometrics

JEL codes: C12, C22

1 Introduction

During the last twenty years the analysis of macroeconomic panels has experienced a vast and rapid development. This has been primarily due to two reasons: first, the easy availability of statistical information concerning panels of data where the time dimension is augmented by the use of cross-section variation (for example, across countries, or industrial sectors), and, second, the belief that combining these two sources of information would lead to better statistical inference.

The recent literature has seen many efforts, in particular to design procedures aimed at estimating long-run relationships among economic variables using macro-panel data techniques. Testing for cointegration in panel data has been a particular area of focus, since it constitutes the analysis that needs to be conducted prior to estimating long-run relationships. The early papers in this area assumed cross-section independence among the units of the panel data, a situation that is rarely found in empirical economic analyses. Cross-section dependence appears naturally when studying

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economic data due to, for instance, market integration processes, globalization of economic activity, offshoring processes or because of the presence of common shocks like oil price shocks. More recent papers have therefore devoted considerable attention to devising procedures relaxing the assumption of cross-section independence.

There may be different sources of cross-section dependence, exerting different degrees of dependence intensity depending upon the sample, time-period and variable that is analyzed. On the one hand, we may have pervasive cross-section dependence due to the presence of a dominant unit in the panel data setup, a situation that can be interpreted as if there were a common factor affecting all time series. On the other hand, cross-section dependence may be only important among some neighbours. The notion of 'neighbour' does not of course necessarily need to be defined in terms of physical contiguity, such as neighbour regions or cities, but may also be defined *inter alia* in terms of economic distance, usually, trade partnerships. This characterization of cross-section dependence has given rise to the notions of weak and strong dependence as discussed prominently by Chudik, Pesaran and Tosetti (2009).

In this paper we investigate the performance of panel cointegration tests when cross-section dependence is introduced, so that it generates weak and/or strong dependence. The paper proposes a panel cointegration test that is based on the common correlated effects (CCE) estimation procedure proposed by Pesaran (2006) to allow for the possibility of cross-section dependence. We show, drawing upon arguments developed by Phillips and Moon (1999), that in a panel spurious regression, the pooled CCE estimator provides a consistent estimate of the long-run average coefficient, which captures a statistical relationship among non-individually cointegrated variables.

The procedures that are proposed in the paper are evaluated through Monte Carlo simulations to evaluate the potential benefits of using the new proposal compared to alternative approaches existing in the literature. In this respect we compare the size and power properties of the pooled CCE-based test with our own procedure developed in Banerjee and Carrion-i-Silvestre (2006).

The paper is organized as follows. Section 2 describes the model upon which the panel cointegration test statistic proposed in the paper is based. We derive consistency results for the pooled CCE estimator under different specifications of the deterministic terms. Next, Section 3 defines the panel cointegration test statistic using the CCE estimator. The finite sample performance under different sources of cross-section dependence is investigated in Section 4. Finally, Section 6 concludes. All the proofs are contained in the appendix at the end of the paper.

2 The model

Let $Y_{i,t} = \left(y_{i,t}, x'_{i,t}\right)'$ a $(1+k)$-vector of $I(1)$ stochastic processes with the following data generating process (DGP):

$$
\begin{aligned}
Y_{i,t} &= D_{i,t} + \pi_i F_t + U_{i,t} &(1)\\
(I-L)\,F_t &= v_t &(2)\\
(I-L)\,U_{i,t} &= e_{i,t}, &(3)
\end{aligned}
$$

where $D_{i,t}$ denotes the deterministic part of the model that it is given by either the absence of deterministic elements $D_{i,t} = 0\ \forall i$ (Model 0), a vector of constant terms, $D_{i,t} = \mu_i = \left(\mu_{i,0}, \mu_{i,1}, \ldots, \mu_{i,k}\right)'$ (Model 1), or a vector linear time trends, $D_{i,t} = \delta_i\,(1,t)'$, with $\delta_i = \left(\delta'_{i,0}, \delta'_{i,1}, \ldots, \delta'_{i,k}\right)'$, $\delta_{i,j} = \left(\mu_{i,j}, \eta_{i,j}\right)'$, $j = 0, 1, \ldots, k$, (Model 2). The F_t component denotes a $(r \times 1)$-vector of common factors and π_i the $((k+1) \times r)$ matrix of factor loadings. The disturbance terms $w_{j,t}$ and $\varepsilon_{i,t}$ are assumed to be I(0) stationary processes, $i = 1, \ldots, N$, $t = 1, \ldots, T$, $j = 1, \ldots, r$.

Our analysis is based on the same set of assumptions as in Bai and Ng (2004) and Banerjee and Carrion-i-Silvestre (2006). Let $M < \infty$ be a generic positive number, not depending on T and N. Further, the Euclidean norm of a generic matrix A is defined as $\|A\| = trace\,(A'A)^{1/2}$. Then:

Assumption A: (i) for non-random π_i, $\|\pi_i\| \leq M$; for random π_i, $E\|\pi_i\|^4 \leq M$, (ii) $\frac{1}{N}\sum_{i=1}^{N} \pi'_i \pi_i \xrightarrow{p} \Sigma_\Pi$, a $(r \times r)$ positive definite matrix.

Assumption B: (i) $v_t = C(L)\,w_t$, $w_t \sim iid\,(0, \Sigma_w)$, $E\|w_t\|^4 \leq M$, and (ii) $Var\,(\Delta F_t) = \sum_{j=0}^{\infty} C_j \Sigma_w C'_j > 0$, (iii) $\sum_{j=0}^{\infty} j\,\|C_j\| < M$; and (iv) $C(1)$ has rank r_1, $0 \leq r_1 \leq r$.

Assumption C: (i) for each i, $e_{i,t} = H_i(L)\,\varepsilon_{i,t}$, $\varepsilon_{i,t} \sim iid\left(0, \sigma^2_{\varepsilon,i}\right)$, $E\,|\varepsilon_{i,t}|^8 \leq M$, $\sum_{j=0}^{\infty} j\,|H_{i,j}| < M$, $\omega_i^2 = H_i(1)^2\,\sigma^2_{\varepsilon,i} > 0$; (ii) $E\left(\varepsilon_{i,t}\varepsilon_{j,t}\right) = \tau_{i,j}$ with $\sum_{i=1}^{N} |\tau_{i,j}| \leq M$ for all j; (iii) $E\left|\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left[\varepsilon_{i,s}\varepsilon_{i,t} - E\left(\varepsilon_{i,s}\varepsilon_{i,t}\right)\right]\right|^4 \leq M$, for every (t,s).

Assumption D: The errors $\varepsilon_{i,t}$, w_t and the loadings π_i are three mutually independent groups.

Assumption E: $E\|F_0\| \leq M$, and for every $i = 1, \ldots, N$, $E\|U_{i,0}\| \leq M$.

The model specification considers the case where the stochastic regressors $x_{i,t}$ are assumed to be either cross-section independent – imposing all, but the first, rows of π_i to equal zero – or cross-section dependent with dependence driven by a set of common factors F_t. In the latter case π_i is thus taken to be unrestricted. Furthermore, it is possible to assume that the set of common factors affecting the endogenous variable $y_{i,t}$ is different from those affecting the stochastic regressors $x_{i,t}$, a situation that is covered if we define π_i to be a block-diagonal matrix.

Despite the presence of the operator $(I-L)$ in equation (2), F_t does not have to be I(1). In fact, F_t can be I(0), I(1), or a combination of both, depending on the rank of $C(1)$. If $C(1) = 0$, then F_t is I(0). If $C(1)$ is of full rank, then each component of F_t is I(1). If $C(1) \neq 0$, but not full rank, then some components of F_t are I(1) and some are I(0). The presence of cointegration among $Y_{i,t} = (y_{i,t}, x'_{i,t})'$ requires F_t to be I(0). However, allowing F_t to be I(1) is also relevant from an empirical point of view since, in this case, F_t might be capturing effects from outside the model that are not included in $Y_{i,t}$. Then, cointegration among the elements in $Y_{i,t}$ up to the inclusion of I(1) factors is possible, which will imply $H_i(1) \neq 0$, but not full rank.[1]

Panel spurious regression has been tackled in Phillips and Moon (1999). Contrary to what is found at the unit level analysis – see Granger and Newbold (1974) and Phillips (1986) – pooled estimation of the parameters affecting the stochastic regressors lead to consistent estimates of the so-called long-run average coefficient β.

Let us define the projection matrix $M_D = I_T - D\left(D'D\right)^{-1} D'$, where $D = \iota$ for Model 1 and $D = [\iota\ \tau]$ for Model 2 – it should be understood that $M_D = I_T$ for Model 0. Note that M_D removes the effect of the deterministic component on the variables of the model. Using this projection matrix we have:

$$\begin{aligned} y_i &= D_i + x_i\beta_i + F\pi_i^y + u_i \\ M_D y_i &= M_D x_i \beta_i + M_D F \pi_i^y + M_D u_i \\ y_i^D &= x_i^D \beta_i + F^D \pi_i^y + u_i^D, \end{aligned}$$

where the superscript D denotes the detrended variable, and

$$\begin{aligned} M_{\tilde{F}} y_i^D &= M_{\tilde{F}} x_i^D \beta_i + M_{\tilde{F}} u_i^D \\ y_i^* &= x_i^* \beta_i + u_i^*, \end{aligned} \tag{4}$$

with $M_{\tilde{F}} = I_T - F^D\left(F^{D\prime}F^D\right)^{-1} F^{D\prime}$, where the superscript "$*$" indicates that the corresponding variable that has been detrended and defactored. Note that at this stage, we have assumed that the common factors are observable.[2] The pooled estimator is defined as:

$$\begin{aligned} \hat{\beta} &= \left[\sum_{i=1}^{N} \left(x_i^{*\prime} x_i^*\right)\right]^{-1} \sum_{i=1}^{N} \left(x_i^{*\prime} y_i^*\right) \\ &= \left[\frac{1}{N}\sum_{i=1}^{N} T^{-2}\left(x_i^{*\prime} x_i^*\right)\right]^{-1} \frac{1}{N}\sum_{i=1}^{N} T^{-2}\left(x_i^{*\prime} y_i^*\right). \end{aligned} \tag{5}$$

Theorem 1 *Let $Y_{i,t}$ be a vector of $(1+k)$ stochastic processes with DGP given by (1)-(3). Under the assumption that $H_i(1)$ is positive definite almost surely for all i (spurious regression), the pooled estimator given in (5) converges as $(T,N)_{\text{seq}} \to \infty$ to*

$$\hat{\beta} \xrightarrow{p} \beta = \Omega_{xx}^{*-1} \Omega_{xy}^*,$$

where β denotes the long-run average regression coefficient, $\Omega_{xx}^* = \Omega_{xx} - \Omega_{xF}\Omega_{FF}^{-1}\Omega_{Fx}$ and $\Omega_{xy}^* =$

[1] Note that in this case, the common factors will be accounting for misspecification errors in the model, due, for instance, to the omission of relevant stochastic regressors.

[2] Although the projection against the deterministic component and the factors can be done in one step, proceeding in two stages facilitates the derivations below.

$\Omega_{xy} - \Omega_{xF}\Omega_{FF}^{-1}\Omega_{Fy}$. *Further, as* $(T,N)_{\text{seq}} \to \infty$

$$\begin{array}{ll} \text{for Model 0:} & \sqrt{N}\left(\hat{\beta}-\beta\right) \Rightarrow N\left(0, 4\ \Omega_{xx}^{*^{-1}}\Theta_0\Omega_{xx}^{*^{-1}}\right) \\ \text{for Model 1:} & \sqrt{N}\left(\hat{\beta}-\beta\right) \Rightarrow N\left(0, 36\ \Omega_{xx}^{*^{-1}}\Theta_1\Omega_{xx}^{*^{-1}}\right) \\ \text{for Model 2:} & \sqrt{N}\left(\hat{\beta}-\beta\right) \Rightarrow N\left(0, 225\ \Omega_{xx}^{*^{-1}}\Theta_2\Omega_{xx}^{*^{-1}}\right). \end{array}$$

The proof is outlined in the appendix. The limit distributions that are obtained in Theorem 1 base on sequential limit theory, although it is possible to show that the same result is obtained when using joint limit theory if the additional assumption that $N/T \to 0$ is imposed – see Phillips and Moon (1999).

3 Panel CCE cointegration test with cross-section dependence

So far, the cross-section dependence has been assumed to be driven by a set of observable common factors. However, in most cases this situation is infeasible from an empirical point of view, and we need to devise procedures to estimate (or proxy for) the unobserved common factors.

There are two popular approaches in the literature that address this issue. First, the Bai and Ng (2002, 2004) proposal, which uses principal components to estimate the common factors and panel information criteria to chose the number of common factors. Second, we can use the cross-sectional average method suggested in Pesaran (2006, 2007), which employs cross-section averages as convenient proxies to capture the common factors without requiring the estimation of their number. This paper looks in detail at this second approach and also establishes a comparison with testing procedures based on Bai and Ng (2002, 2004).

To see how the CCE procedure works, we define the average of (1) as:

$$\bar{Y}_t = \bar{D}_t + \bar{\pi}F_t + \bar{U}_t,$$

where

$$\begin{aligned} \bar{Y}_t &= \frac{1}{N}\sum_{i=1}^{N} Y_{i,t}; \quad \bar{D}_t = \bar{\delta}d_t; \quad \bar{\delta} = \frac{1}{N}\sum_{i=1}^{N}\delta_i \\ \bar{\pi} &= \frac{1}{N}\sum_{i=1}^{N}\pi_i; \quad \bar{U}_t = \frac{1}{N}\sum_{i=1}^{N} U_{i,t}, \end{aligned}$$

with $d_t = 0$ for Model 0, $d_t = 1$ and $\delta_i = \left(\mu_{i,0}, \mu_{i,1}, \ldots, \mu_{i,k}\right)'$ for Model 1, and $d_t = (1,t)'$ and $\delta_i = \left(\delta'_{i,0}, \delta'_{i,1}, \ldots, \delta'_{i,k}\right)'$, $\delta_{i,j} = \left(\mu_{i,j}, \eta_{i,j}\right)'$, $j = 0, 1, \ldots, k$, for Model 2. Let us assume that $rank\left(\bar{\pi}\right) = r \leq (1+k)$ for all N as $N \to \infty$. If this rank condition is met, we have

$$F_t = \left(\bar{\pi}'\bar{\pi}\right)^{-1}\bar{\pi}'\left(\bar{Y}_t - \bar{D}_t - \bar{U}_t\right).$$

Provided that $\bar{U}_t \overset{q.m.}{\rightarrow} 0$ as $N \rightarrow \infty$ for all t, and , $\bar{\pi} \overset{p}{\rightarrow} E(\pi_i) = \pi$ as $N \rightarrow \infty$, where $\overset{q.m.}{\rightarrow}$ denotes convergence in mean square error, we have that

$$F_t - \left(\bar{\pi}'\bar{\pi}\right)^{-1} \bar{\pi}' \left(\bar{Y}_t - \bar{D}_t\right) \overset{q.m.}{\rightarrow} 0 \text{ as } N \rightarrow \infty,$$

which indicates that, for sufficiently large N, the observable averages $\bar{h}_t = \left(\bar{D}_t, \bar{Y}_t'\right)'$ can be used to proxy the unobserved factors.

Following Holly, Pesaran and Yamagata (2010), let us specify the cross-section augmented regression:

$$y_{i,t} = D_{i,t} + x_{i,t}'\beta_i + \bar{z}_t'\varphi_i + u_{i,t}, \tag{6}$$

where $\bar{z}_t = (\bar{y}_t, \bar{x}_t')'$ collects the cross-section averages of the dependent and the stochastic regressors of the model. In order to estimate the β parameters in (6), Holly, Pesaran and Yamagata (2010) use the pooled CCE estimator (PCCE) in Pesaran (2006), which is given by:

$$\hat{\beta}_{PCCE} = \left(\sum_{i=1}^{N} x_i'\bar{M}x_i\right)^{-1} \left(\sum_{i=1}^{N} x_i'\bar{M}y_i\right), \tag{7}$$

where $x_i = [x_{i,1}\ x_{i,2}\ \ldots\ x_{i,k}]$, $x_{i,j} = (x_{i,j,1}, x_{i,j,2}, \ldots, x_{i,j,T})'$, denotes the $(T \times k)$ matrix of regressors of interest, y_i is the $(T \times 1)$ vector of the dependent variable for the i-th unit, and $\bar{M} = I - \bar{H}\left(\bar{H}'\bar{H}\right)^{-1}\bar{H}'$, $\bar{H} = [\bar{z}]$ for Model 0, $\bar{H} = [\iota\ \bar{z}]$ for Model 1 and $\bar{H} = [\iota\ \tau\ \bar{z}]$ for Model 2, with $\iota = (1, 1, \ldots, 1)'$ a vector of ones, $\tau = (1, 2, \ldots, T)'$ a linear time trend and $\bar{z} = [\bar{x}\ \bar{y}]$ the $(T \times (k+1))$ matrix of cross-section averages. One interesting feature is that the PCCE estimator is easy to compute and does not require the estimation of the factors driving the cross-section dependence, although the main drawback is that consistency has been only proved in Kapetanios, Pesaran and Yamagata (2010) under the maintained hypothesis that cointegration exists, which, in fact, is the hypothesis that we want to test. Therefore, in order to assess the validity of the testing procedure that we apply, we need to show whether the PCCE estimator is consistent under the null hypothesis of no cointegration. This result is provided in the following Theorem.

Theorem 2 *Let $Y_{i,t}$ be a vector of $(1+k)$ stochastic processes with DGP given by (1)-(3). Under the assumption that $H_i(1)$ is positive definite almost surely for all i (spurious regression) and $\operatorname{rank}(\bar{\pi}) = r \leq (1+k)$ for all N as $N \rightarrow \infty$, the pooled estimator given in (7) converges as $T, N \rightarrow \infty$ with $N/T \rightarrow 0$ to the expressions obtained in Theorem 1.*

The proof is outlined in the appendix.

Given that the consistency of the PCCE estimator under the null hypothesis of spurious regression, in the second stage we use the PCCE estimated parameters to define the variable:

$$\tilde{y}_{i,t} = y_{i,t} - x_{i,t}'\hat{\beta}_{PCCE}, \tag{8}$$

for which the following model is estimated using the OLS estimation procedure:

$$\tilde{y}_{i,t} = D_{i,t} + e_{i,t},$$

and the OLS residuals are then computed as $\hat{e}_{i,t} = \tilde{y}_{i,t} - \hat{D}_{i,t}$. The null hypothesis of no cointegration is tested analyzing the order of integration of $\hat{e}_{i,t}$ through the application of the cross-section augmented ADF cointegration (CADFC) statistic:

$$CADFC_P = N^{-1} \sum_{i=1}^{N} t_{\hat{\alpha}_{i,0}},$$

where $t_{\hat{\alpha}_{i,0}}$ denotes the pseudo t-ratio of the estimated $\alpha_{i,0}$ parameter in the regression:

$$\Delta \hat{e}_{i,t} = \alpha_{i,0} \hat{e}_{i,t-1} + \sum_{j=1}^{p} \alpha_{i,j} \Delta \hat{e}_{i,t-j} + \xi_i \bar{\hat{e}}_{t-1} + \sum_{j=0}^{p} \eta_{i,j} \Delta \bar{\hat{e}}_{t-j} + w_{i,t}. \quad (9)$$

The critical values for the individual cointegration test $\left(t_{\hat{\alpha}_{i,0}}\right)$ are reported in Tables 1 and 2 for Models 1 and 2, respectively. As for the pooled test, the critical values for the $CADFC_P$ statistic are presented in Tables 3 and 4 for Models 1 and 2, respectively. To obtain these critical values we have generated the dependent variable as $y_{i,t} = y_{i,t-1} + f_t + \varepsilon_{1,i,t}$ and a vector of k explanatory variables $x_{i,t} = x_{i,t-1} + \varepsilon_{2,i,t}$, where $\varepsilon_{i,t} = \left(\varepsilon_{1,i,t}, \varepsilon'_{2,i,t}\right)' \sim iid\ N\left(0, I_{k+1}\right)$, and $f_t \sim iid\ N\left(0,1\right)$ for $i = 1, 2, \ldots, N$, $t = -50, -49, \ldots, T$, and $y_{i,-50} = x_{i,-50} = 0$. Using these independent time series we have computed the PCCE estimator and computed the $\hat{e}_{i,t}$ residuals that are used to estimate the regression equation in (9) and compute the individual and $CADFC_P$ statistics. The simulation bases on 50,000 replications using all different combinations of $T = \{$10, 15, 20, 30, 40, 50, 100, 200, 250, 500$\}$ and $N = \{$10, 15, 20, 30, 40, 50, 100, 200$\}$, for $k = \{1, 2, 3\}$ – due to space contraints, we only report the tables of critical values for $k = 1$ and $k = 2$. Two remarks are in order. First, the critical values do not seem to depend on k as T and N get large. This is something to be expected, provided that the estimation of the long-run average parameter is consistent as $T, N \to \infty$. Second, the critical values are close to the ones computed in Pesaran (2007) when T is large, although they differ in finite samples.

4 Finite sample performance

4.1 Common factor model

Let us consider the DGP defined by:

$$y_{i,t} = x_{i,t} + F_t + u_{i,t} \tag{10}$$

$$\Delta x_{i,t} = v_{i,t} \tag{11}$$

$$F_t = \rho F_{t-1} + w_t \tag{12}$$

$$u_{i,t} = \phi_i u_{i,t-1} + \varepsilon_{i,t}, \tag{13}$$

where $r = 1$, $v_{i,t} \sim N(0,1)$, $w_t \sim N\left(0, \sigma_F^2\right)$ and $\varepsilon_{i,t} \sim N(0,1)$ are three mutually independent groups. Under the null hypothesis of no cointegration we specify $\phi_i = 1\ \forall i$, whereas under the alternative hypothesis of cointegration we have $|\phi_i| < 1$ for some i. Note that the definition of cointegration that we are testing for only focuses on the idiosyncratic component, regardless of the order of integration of the common factor. Thus, if $F_t \sim I(1)$ cointegration exists among $(y_{i,t}, x_{i,t}, F_t)$ but not between $(y_{i,t}, x_{i,t})$.

The simulations use the following setup. The empirical size is analyzed using $\phi_i = 1$, whereas the empirical power is investigated using $\phi_i = \{0.99, 0.95, 0.9\}$. As for the common factor component, we consider one common factor with autoregressive parameter given by $\rho = \{1, 0.99, 0.95\}$ with different importance, which is modelled through the following values for the variance $\sigma_F^2 = \{0.5, 1, 10\}$. Our simulations consider two different cases, depending on whether the number of common factors is equal to the true one ($r = 1$) or it is estimated using the panel BIC information criterion of Bai and Ng (2002) – throughout this paper, the maximum number of common factors is equal to six. The time dimension is set at $T = \{50, 100, 250\}$ and the cross-section dimension is $N = \{10, 20\}$. The nominal size is set at the 5% level and the critical values tabulated in the previous section are used. The simulations are performed using GAUSS with 1,000 replications.

Before presenting the results for the empirical size and power of the panel cointegration test statistic that is proposed in this paper, we have conducted a small Monte Carlo simulation to show that the consistency property obtained in Theorem 2 gives a proper approximation in finite samples. Table 5 reports the results of the mean, median and root mean square error of the $\hat{\beta}_{PCCE}$ estimator. As can be seen, the mean and the median is close to the true value of the parameter – i.e., $\beta = 1$ in (10) – with a small root mean square error, which decreases as ϕ_i and ρ move away from one. These results support the theoretical derivations that are shown in Theorem 2.

Table 6 reports the results for the PCCE estimator based procedure when the true number of common factors is imposed. Table 7 presents the corresponding results when the number of common factors is estimated. In each table we also report the results for the test statistics in Banerjee and Carrion-i-Silvestre (2006) – hereafter, BC test – which model the cross-section dependence through an approximate common factor model. If we compare the two sets of tables we realize that estimating the number of common factors does not have any effect on the properties of the principal components based test statistics – i.e., the panel BIC information criterion always selects the true number of common factors in our simulation setup.

Let us focus on the results in Table 7. As can be seen, the ADF tests statistic in Banerjee and Carrion-i-Silvestre (2006) applied to the idiosyncratic component of the model – hereafter, Z_τ statistic – has the correct size, regardless of the the value of the autoregressive parameter of the common factor (ρ) and the variance of common factor error $\left(\sigma_F^2\right)$. The $CADFC_P$ statistic has the correct size when $\rho = 1$, although we observe that the test statistic tends to be conservative (underrejects) as ρ moves away from 1 and T gets large. Note that this may be explained by the fact that, as pointed out in Bai and Ng (2010), in these latter cases the setup violates the assumption that is implicitly required by Pesaran's (2007) framework, namely that $\phi_i = \rho$ – i.e., the dynamic of the idiosyncratic component should be the same as the one driving the common factor component. As for empirical power, we observe that the $CADFC_P$ statistic only out-performs the Z_τ statistic when $\phi_i = 0.99$ and $T = 50$, although such superiority can be explained by the mild overrejection that the $CADFC_P$ statistic shows under the null hypothesis of no cointegration. Despite this, the Z_τ statistic outperforms the $CADFC_P$ statistic in the rest of cases. However, it should be pointed out that the empirical power of the two statistics is almost equivalent for large T.

So far, we have compared the panel data test statistics that are computed using the estimated idiosyncratic component. The procedure in Banerjee and Carrion-i-Silvestre (2006) also allows us to analyze the stochastic properties of the estimated common factors. The ADF statistic that is computed using the estimated common factor is reported in the columns labelled as $t_{\tilde{F}}$. As can be seen, the $t_{\tilde{F}}$ has the correct size under the null hypothesis that $\rho = 1$, with empirical power that increases, as expected, as ρ moves away from 1 and T gets large.

To sum up, the principal components-based panel cointegration test in Banerjee and Carrion-i-Silvestre (2006) shows better overall performance, with empirical size close to the nominal size and empirical power, except in one particular case, higher than those demonstrated by the CCE-based statistics. However, both approaches tend to provide the same empirical power when the time dimension is large, and the convenience of the CCE-based approach needs also to be taken into account when assessing the relative merits of these alternative testing procedures.

Finally, it could be stressed that the procedure in Banerjee and Carrion-i-Silvestre (2006) is more informative, considering that it allows to get a complete picture of the stochastic properties of all components affecting the model that is specified. From an empirical point of view, assessing the stochastic properties of the common factors is important since allows us to interpret whether $(y_{i,t}, x_{i,t})$ cointegrate alone or whether we need to consider $(y_{i,t}, x_{i,t}, F_t)$ to get a cointegrating relationship.

4.2 Spatial autocorrelation

We have followed Baltagi, Bresson and Pirotte (2007) and specify weak cross-section dependence in the panel data setup using a spatial error model. The DGP is given by

$$
\begin{aligned}
y_{i,t} &= x_{i,t} + u_{i,t} \\
\Delta x_{i,t} &= v_{i,t} \\
u_{i,t} &= \phi_i u_{i,t-1} + \xi_{i,t},
\end{aligned}
$$

where the error component can follow one of these three different spatial models:

- Spatial autoregressive (SAR) specification:

  $$\xi_t = \vartheta W_N \xi_t + \varepsilon_t = \left(I_N - \vartheta W_N\right)^{-1} \varepsilon_t,$$

  with $\xi_t = \left(\xi_{1,t}, \xi_{2,t}, \ldots, \xi_{N,t}\right)'$, W_N is an $(N \times N)$ known spatial weights matrix, ϑ is the spatial autoregressive parameter and ε_t is an $(N \times 1)$ error vector assumed to be distributed independently across cross-sectional dimension with constant variance σ_ε^2.

- Spatial moving average (SMA) specification:

  $$\xi_t = \varepsilon_t + \vartheta W_N \varepsilon_t = \left(I_N + \vartheta W_N\right) \varepsilon_t,$$

  where now ϑ is the spatial moving average parameter.

- Spatial error component (SEC) specification:

  $$\xi_t = \varepsilon_t + \vartheta W_N \psi_t,$$

  where ε_t is an $(N \times 1)$ vector of local error components and ψ_t is an $(N \times 1)$ vector of spillover error components. The two component vectors are assumed to consist of *iid* terms with respective variances σ_ε^2 and σ_ψ^2, and are uncorrelated.

Of special interest is the SEC specification since we can relate the spatial model with the common factor model that has been investigated in the previous section. We can specify:

$$\xi_t = \varepsilon_t + \vartheta W_N \Gamma F_t,$$

where now $\psi_t = \Gamma F_t$, with $\Gamma = \left(\gamma_1', \gamma_2', \ldots, \gamma_N'\right)'$ the $(N \times r)$ matrix of loadings. Further, if we set $\vartheta = 1$ and $W_N = I_N$ we get the common factor representation used above. This allows us to specify different models depending on the degree of weak correlation that we want to allow. For instance, if the spatial weight matrix is now $V_N = I_N + W_N$ with $\psi_t = \Gamma F_t$ and $\vartheta \neq 0$, the common factors will not only affect each unit, but also their neighbours.

The simulations that are reported in this section follow the setup in Baltagi, Bresson and Pirotte (2007), who use two different values for $\vartheta = \{0.4, 0.8\}$ and the spatial weight matrix W_N given by the sparse weight matrix $W(1,1)$ that defines the '1 ahead and 1 behind' matrix with the i-th row $(1 < i < N)$ of this $N \times N$ matrix having non-zero elements in positions $i+1$ and $i-1$. Other sparse weight matrices $W(j,j)$, $j = 2, 3, \ldots, 10$, were used in Baltagi, Bresson and Pirotte (2007), although they claimed that similar qualitative results were obtained. Therefore and in order to save space, we only use the $W(1,1)$ matrix as a way to illustrate the effect of spatial dependence on the panel data cointegration tests that we consider in the paper.

Tables 10 and 11 present the results of the empirical size and power of the panel data cointegration test statistics when $N = 20$ and one common factor is imposed for the computation of the statistics in Banerjee and Carrion-i-Silvestre (2006), whereas Table 12 shows the results when the number of common factors is estimated. The same set of results is available in Tables 13 to 15 for $N = 10$.

Qualitatively similar results are obtained regardless of the number of cross-section units and the way in which we estimate the number of common factors. Let us focus on Table 12, since it is closer to the empirical practice. When the SAR specification is used, both $CADFC_P$ and $Z_\tau$statistics show size distortions (over-rejection problems), which are more important for the former statistic. As for empirical power, the $CADFC_P$ statistic reveals higher power when $\phi_i = 0.99$, although this superiority disappears for $\phi_i < 0.99$ when $T \geq 100$ for $\vartheta = 0.4$, and for $T > 100$ for $\vartheta = 0.8$. Note that this feature is similar to what has been found in the previous simulation exercise that uses the common factors setup.

When the spatial dependence is driven by an SMA specification, the size distortions shown by the two statistics is similar, with dominance of the $CADFC_P$ statistic in terms of empirical power only when $T < 100$. Thus, the Z_τ statistic encompasses $CADFC_P$ in terms of empirical power when $T \geq 100$, regardless of the value of ϑ.

The three SEC specifications that we have considered lead to similar qualitative results, which are also in line with the results obtained using the SAR and SMA spatial specifications. In general, the empirical size of the two statistics is close to the nominal one regardless of the value of T and ϑ. Only mild overrejection problems is found for the $CADFC_P$ statistic when $T = 50$. The superiority of the $CADFC_P$ statistic in terms of empirical power when $\phi_i = 0.99$ can be attributed to the mild overrejection, although both statistics show similar behaviour in terms of empirical power as ϕ_i moves away from unity and T gets large.

5 Empirical illustrations

5.1 House prices in the US

Holly, Pesaran and Yamagata (2010) analyze the long-run relationship between the logarithm of the real house price index $(p_{i,t})$ and the logarithm of the real per capita disposable income $(y_{i,t})$ for 48 US States and the District of Columbia $(N = 49)$ using annual data between 1975 and 2003

($T = 29$) – see Figures 1 and 2. The model under investigation is given by

$$p_{i,t} = \alpha_i + \beta y_{i,t} + u_{i,t},$$

where note that slope homogeneity is imposed. Unfortunately, the panel data that is used in their paper does not strictly fit the theoretical requirements of our approach since T is small and, more importantly, smaller than N. Nevertheless, the empirical evidence that is obtained in this section may be understood as an illustration of our approach. We also do not (given the dimensions of the panel) undertake a comparison with the BC test since the N dimension is too small for our needs (in order to enable the computation of the factors). However it may be seen as an advantage of the $PCCE$-based approach that a feasible test for cointegration can be constructed in the presence of cross-section dependence for reasonably small N and T – see tables for size and power properties.

We have computed the CCE test statistics proposed in this paper using up to four lags for the autoregressive correction in (9). Table 16 shows that the null hypothesis of no cointegration is rejected at the 5% level of significance in 3 ($p = 0$), 8 ($p = 1$), 13 ($p = 2$), 18 ($p = 3$) and 18 ($p = 4$) cases out of 49 – if the level of significance is set at the 10%, rejection happens in 6 ($p = 0$), 13 ($p = 1$), 19 ($p = 2$), 25 ($p = 3$) and 20 ($p = 4$) cases out of 49. Therefore, even in the most favorable situation, evidence in favor of cointegration is found for only half of the units. It would be the case that pooling the individual information will lead to better statistical inference, provided that the assumption of cross-section independence of $\hat{e}_{i,t}$, $i = 1, 2, \ldots, N$, in (9) is met. The computation of the $CADFC_P$ statistic gives $CADFC_P = -1.85$ ($p = 0$), $CADFC_P = -2.56$ ($p = 1$), $CADFC_P = -2.78$ ($p = 2$), $CADFC_P = -3.20$ ($p = 3$), and $CADFC_P = -2.87$ ($p = 4$), depending on the order of the autoregressive correction that is used. As can be seen, when we compare the values of the $CADFC_P$ statistic with the critical values in Table 3 we conclude that, except for $p = 0$, the null hypothesis of no cointegration is rejected at the 5% level of significance.

5.2 Production function

The second empirical application focuses on the estimation of a production function using the database in Banerjee, Eberhardt and Reade (2010), who, in turn, bases on the Penn World Table database (version 6.3). We define a panel data set of developed countries that includes Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Ireland, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States. The selection of these countries allows us to have a balanced panel data set covering the period between 1951 and 2007. Notice also that our data set includes almost all EU-15 countries – we have not been able to include Germany because of lack of information between 1951 and 1969 – and almost all G7 countries – the exception is Japan, for which we do not have information for the whole period. Therefore, we deal with a panel data set of dimension $T = 57$ and $N = 19$, which fits the requirement of having a panel with larger time dimension than cross-section dimension.

The model that is estimated is given by:

$$y_{i,t} = \alpha_i + \beta_1 l_{i,t} + \beta_2 k_{i,t} + u_{i,t},$$

where $y_{i,t}$ denotes the logarithm of the real GDP per capita, $l_{i,t}$ is the logarithm of the population and $k_{i,t}$ is the logarithm of the real capital stock per capita. The CCE estimation of the slope parameters equals $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2)' = (0.8, 0.78)'$.

Table 17 presents the individual CCE $t_{\hat{\alpha}_{i,0}}$ statistics, $i = 1, 2, \ldots, 19$. As can be seen, the null hypothesis of no cointegration can be rejected at the 5% level of significance in 2 ($p = 0$), 1 ($p = 1$), 2 ($p = 2$), 1 ($p = 3$) and 1 ($p = 4$) cases out of 19 – we use the critical values for $N = 20$ and $T = 50$. If the level of significance is set at the 10% level, the rejection of the null hypothesis of no cointegration happens in 3 ($p = 0$), 3 ($p = 1$), 2 ($p = 2$), 3 ($p = 3$) and 3 ($p = 4$) cases out of 19. Thus, using the individual based statistics we find little evidence against the null hypothesis of no cointegration. The individual information can be combined computing the $CADFC_P$ statistic, which produces $CADFC_P = -1.68$ ($p = 0$), $CADFC_P = -1.71$ ($p = 1$), $CADFC_P = -1.67$ ($p = 2$), $CADFC_P = -1.48$ ($p = 3$), and $CADFC_P = -1.52$ ($p = 4$), depending on the order of the autoregressive correction that is used. As can be seen, when we compare the values of the $CADFC_P$ statistic with the critical values in Table 3 for $N = 20$ and $T = 50$ we conclude that the null hypothesis of no cointegration cannot be rejected at the 5% level of significance, regardless of the order of autocorrelation that is considered.

The results of the computation of the BC test with up to six common factors are reported in Table 18. The use of the panel BIC information criterion in Bai and Ng (2002) leads to select the maximum number of factors that is allowed. Regardless of the number of common factors, the BC statistics indicates that the idiosyncratic disturbance terms are stationary. However, we cannot conclude that the variables in the vector $Y_{i,t} = (y_{i,t}, l_{i,t}, k_{i,t})'$ generate a cointegration relationship, since at least one non-stationary common factor has been detected. Therefore, cointegration is possible up to the inclusion of common factors in the model.

6 Conclusions

The paper has shown that consistent estimate of the long-run average coefficient is obtained when time series in the panel data are cross-section dependence, which is accounted for using a common factor model approach. The estimation procedure that is applied is based on the CCE approach in Pesaran (2006). Our result contributes to the literature of non-stationary panel data analysis, where consistent estimation of the parameters of the model is feasible in a spurious regression framework. The paper conducts an extensive simulation exercise to study the finite sample performance of the statistic that has been proposed in the paper, allowing for weak and strong cross-section dependence.

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A Appendix

Lemma 1 *Define the vector of stochastic processes* $V_{i,t} = \left(e'_{i,t}, v'_t\right)'$ *that satisfies the panel functional central limit theorem (CLT):*

$$T^{-1/2}\sum_{t=1}^{[rT]} V_{i,t} \Rightarrow C_i(1) W_i(r) \qquad \text{as } T \to \infty \text{ for all } i,$$

where $C_i(1)$ *is a* $((1+k+r)\times(1+k+r))$*-matrix of conditional long-run standard deviations.*

Proof: see Lemma 3 in Phillips and Moon (1999).

A.1 Proof of Theorem 1

A.1.1 No deterministic component

In this section we analyze the model specification that does not include any deterministic component, i.e., $D_{i,t} = 0\ \forall i$ in (1). For ease of exposition, we consider that all common factors in the model are I(1). This assumption will be relaxed once the main derivations are obtained.

Let $M_i(r) = (M_{y_i}(r)', M_{x_i}(r)', M_F(r)')' = C_i(1) W_i(r) = (C_{y_i}(1)', C_{x_i}(1)', C_F(1)')'W_i(r)$, where $M_i(r)$ is a randomly scaled Brownian motion with a conditional covariance matrix $C_i(1) C_i(1)'$ that has a well defined expectation provided that $\left\| EC_i(1) C_i(1)' \right\| < \infty$ as shown in Lemma 1(d) in Phillips and Moon (1999). Let us define $Z_{i,t} = \left(U'_{i,t}, F'_t\right)'$, by the continuous mapping theorem we have that as $T \to \infty$ for a fixed N

$$T^{-2}\sum_{t=1}^{T} Z_{i,t}Z'_{i,t} \Rightarrow C_i(1)\int_0^1 W_i(r) W'_i(r)\, dr\ C_i(1)' = \int_0^1 M_i(r) M'_i(r)\, dr.$$

Averaging across i

$$N^{-1}\sum_{i=1}^{N} T^{-2}\sum_{t=1}^{T} Z_{i,t}Z'_{i,t} \Rightarrow N^{-1}\sum_{i=1}^{N} C_i(1)\int_0^1 W_i(r) W'_i(r)\, dr\, C_i(1)' = N^{-1}\sum_{i=1}^{N}\int_0^1 M_i(r) M'_i(r)\, dr.$$

Lemma 4 in Phillips and Moon (1999) shows that $E\left\|\int_0^1 M_i(r) M'_i(r)\, dr\right\|^2 < \infty$, so that as $N \to \infty$ the law of strong numbers gives us

$$N^{-1}\sum_{i=1}^{N}\int_0^1 M_i(r) M'_i(r)\, dr \overset{a.s.}{\to} E\left(\int_0^1 M_i(r) M'_i(r)\, dr\right).$$

In order to compute this expectation we need to define the long-run conditional covariance matrix of $Z_{i,t} = \left(U'_{i,t}, F'_t\right)' = \left(U_{y_i,t}, U_{x_{i,1},t}, \ldots, U_{x_{i,k},t}, F'_t\right)'$ as

$$\begin{aligned}
\Omega_i &= \begin{bmatrix} \Omega_{y_i y_i} & \Omega_{y_i x_i} & \Omega_{y_i F} \\ \Omega_{x_i y_i} & \Omega_{x_i x_i} & \Omega_{x_i F} \\ \Omega_{F y_i} & \Omega_{F x_i} & \Omega_{FF} \end{bmatrix} = \begin{bmatrix} \Omega_{1i} & \Omega_{2i} \\ \Omega_{3i} & \Omega_{FF} \end{bmatrix} \\
&= C_i(1) C_i(1)' = \begin{bmatrix} C_{y_i}(1) C_{y_i}(1)' & C_{y_i}(1) C_{x_i}(1)' & C_{y_i}(1) C_F(1)' \\ C_{x_i}(1) C_{y_i}(1)' & C_{x_i}(1) C_{x_i}(1)' & C_{x_i}(1) C_F(1)' \\ C_F(1) C_{y_i}(1)' & C_F(1) C_{x_i}(1)' & C_F(1) C_F(1)' \end{bmatrix}.
\end{aligned}$$

Using these long-run conditional covariance matrices, we define the long-run average covariance matrix of $Z_{i,t}$ as:

$$\Omega = \begin{bmatrix} \Omega_{yy} & \Omega_{yx} & \Omega_{yF} \\ \Omega_{xy} & \Omega_{xx} & \Omega_{xF} \\ \Omega_{Fy} & \Omega_{Fx} & \Omega_{FF} \end{bmatrix} = E\left(C_i(1) C_i(1)'\right).$$

Now, we can compute

$$\begin{aligned}
E\left(\int_0^1 M_i(r) M'_i(r)\, dr\right) &= E\left(C_i(1) E\left(\int_0^1 W_i(r) W'_i(r)\, dr\right) C_i(1)'\right) \\
&= E\left(C_i(1) \frac{1}{2} I_{(1+k+r)} C_i(1)'\right) \\
&= \frac{1}{2}\Omega.
\end{aligned}$$

Therefore,

$$N^{-1} \sum_{i=1}^{N} \int_0^1 M_i(r) M'_i(r)\, dr \overset{a.s.}{\to} \frac{1}{2}\Omega.$$

We have defined the pooled estimator as

$$\hat{\beta} = \left[\frac{1}{N} \sum_{i=1}^{N} T^{-2}\left(x_i^{*\prime} x_i^*\right)\right]^{-1} \frac{1}{N} \sum_{i=1}^{N} T^{-2}\left(x_i^{*\prime} y_i^*\right),$$

where, in this case, $x_i^* = M_F x_i$. Note that

$$\begin{aligned}
T^{-2} x_i^{*\prime} x_i^* &= T^{-2} x'_i M_F x_i \\
&= T^{-2} x'_i x_i - T^{-2} x'_i F \left(T^{-2} F' F\right)^{-1} T^{-2} F' x_i,
\end{aligned}$$

so that, in the limit

$$\begin{aligned} T^{-2}x_i^{*\prime}x_i^* &\Rightarrow C_{x_i}(1)\int_0^1 W_{i,x_i}(r)\,W_{i,x_i}'(r)\,drC_{x_i}(1)' - \left[\left(C_{x_i}(1)\int_0^1 W_{i,x_i}(r)\,W_F'(r)\,drC_F(1)'\right)\right. \\ &\left.\left(C_F(1)\int_0^1 W_F(r)\,W_F'(r)\,drC_F(1)'\right)^{-1}\left(C_F(1)\int_0^1 W_F(r)\,W_{i,x_i}'(r)\,drC_{x_i}(1)'\right)\right]. \end{aligned}$$

Using the elements obtained above,

$$N^{-1}\sum_{i=1}^N T^{-2}x_i^{*\prime}x_i^* \overset{a.s.}{\to} \frac{1}{2}\left(\Omega_{xx}-\Omega_{xF}\Omega_{FF}^{-1}\Omega_{Fx}\right)=\frac{1}{2}\Omega_{xx}^*.$$

Similarly,

$$\begin{aligned} T^{-2}x_i^{*\prime}y_i^* &= T^{-2}x_i'M_F y_i \\ &= T^{-2}x_i'y_i - T^{-2}x_i'F\left(T^{-2}F'F\right)^{-1}T^{-2}F'y_i, \end{aligned}$$

so that

$$N^{-1}\sum_{i=1}^N T^{-2}x_i^{*\prime}y_i^* \overset{a.s.}{\to} \frac{1}{2}\left(\Omega_{xy}-\Omega_{xF}\Omega_{FF}^{-1}\Omega_{Fy}\right)=\frac{1}{2}\Omega_{xy}^*.$$

Finally,

$$\hat{\beta}\overset{p}{\to}\Omega_{xx}^{*-1}\Omega_{xy}^*=\beta.$$

In order to obtain the limiting distribution of $\hat{\beta}$ we rescale the bias by $\sqrt{N}$ so that

$$\sqrt{N}\left(\hat{\beta}-\beta\right)=\frac{1}{\sqrt{N}}\left[\frac{1}{N}\sum_{i=1}^N T^{-2}\left(x_i^{*\prime}x_i^*\right)\right]^{-1}\left(\sum_{i=1}^N\left(T^{-2}\left(x_i^{*\prime}y_i^*\right)-T^{-2}\left(x_i^{*\prime}x_i^*\right)\beta\right)\right).$$

It can be seen that

$$\begin{aligned} E\left(T^{-2}\left(x_i^{*\prime}y_i^*\right)-T^{-2}\left(x_i^{*\prime}x_i^*\right)\beta\right) &= E\left(\frac{1}{2}\left(\Omega_{x_iy_i}-\Omega_{x_iF}\Omega_{FF}^{-1}\Omega_{Fy_i}\right)-\frac{1}{2}\left(\Omega_{x_ix_i}-\Omega_{x_iF}\Omega_{FF}^{-1}\Omega_{Fx_i}\right)\beta\right) \\ &= 0, \end{aligned}$$

so that the estimator is $\sqrt{N}$-consistent. The variance of the estimator can be obtained as in Phillips and Moon (1999)

$$
\begin{aligned}
& E\left(vec\left(\int_0^1 M^*_{y_i^*}(r)\, M^{*\prime}_{x_i^*}(r)\, dr - \beta' \int_0^1 M^*_{x_i^*}(r)\, M^{*\prime}_{x_i^*}(r)\, dr\right) \times \right. \\
& \left. vec\left(\int_0^1 M^*_{y_i^*}(r)\, M^{*\prime}_{x_i^*}(r)\, dr - \beta' \int_0^1 M^*_{x_i^*}(r)\, M^{*\prime}_{x_i^*}(r)\, dr\right)'\right) \\
= & \frac{1}{6} E\left(\Omega^*_{x_i x_i} \otimes \left(\Omega^*_{y_i y_i} - \beta' \Omega^*_{x_i y_i} - \Omega^*_{y_i x_i}\beta + \beta' \Omega^*_{x_i x_i}\beta\right)\right) \\
& + \frac{1}{6} E\left(\left(\Omega^*_{x_i y_i} - \Omega^*_{x_i x_i}\beta\right) \otimes \left(\Omega^*_{y_i x_i} - \beta' \Omega^*_{x_i x_i}\right) K_{1,k}\right) \\
& + \frac{1}{4} E\left(vec\left(\Omega^*_{y_i x_i} - \beta' \Omega^*_{x_i x_i}\right) vec\left(\Omega^*_{y_i x_i} - \beta' \Omega^*_{x_i x_i}\right)'\right) \\
= & \Theta_0,
\end{aligned}
$$

where $K_{1,k}$ is the $((1 * k) \times (1 * k))$ commutation matrix. From the multivariate Lindeberg-Levy CLT, we have that as $N \to \infty$

$$
\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left(T^{-2}\left(x_i^{*\prime} y_i^*\right) - T^{-2}\left(x_i^{*\prime} x_i^*\right)\beta\right) \Rightarrow N\left(0, \Theta_0\right),
$$

and, therefore,

$$
\sqrt{N}\left(\hat{\beta} - \beta\right) \Rightarrow N\left(0, 4\ \Omega^{*^{-1}}_{xx} \Theta_0 \Omega^{*^{-1}}_{xx}\right).
$$

A.1.2 Constant term

In this section we consider the deterministic specification given by Model 1 through the definition of $D_{i,t} = \mu_i = \left(\mu_{i,0}, \mu_{i,1}, \ldots, \mu_{i,k}\right)'$. Using the projection matrix $M_D = I - D\left(D'D\right)^{-1} D'$, where $D = \iota$ denotes a vector of ones, we define $\tilde{Z}_{i,t} = \left(\tilde{U}'_{i,t}, \tilde{F}'_t\right)'$, where $\tilde{U}_i = M_D U_i$ and $\tilde{F} = M_D F$ are the OLS detrended variables. By the continuous mapping theorem we have that as $T \to \infty$ for a fixed N

$$
T^{-2} \sum_{t=1}^{T} \tilde{Z}_{i,t} \tilde{Z}'_{i,t} \Rightarrow C_i(1) \int_0^1 W_i^*(r)\, W_i^{*\prime}(r)\, dr\ C_i(1)' = \int_0^1 M_i^*(r)\, M_i^{*\prime}(r)\, dr,
$$

where $W_i^*(r) = W_i(r) - \int_0^1 W_i(s)\, ds$ and $M_i^*(r) = M_i(r) - \int_0^1 M_i(s)\, ds$ are demeaned Brownian motion vectors. In this case,

$$
\begin{aligned}
E\left(\int_0^1 M_i^*(r)\, M_i^{*\prime}(r)\, dr\right) &= E\left(C_i(1)\, E\left(\int_0^1 W_i^*(r)\, W_i^{*\prime}(r)\, dr\right) C_i(1)'\right) \\
&= E\left(C_i(1)\, \frac{1}{6} I_{(1+k+r)} C_i(1)'\right) \\
&= \frac{1}{6}\Omega.
\end{aligned}
$$

The developments carried out in the previous section follow here replacing $W_i(r)$ by $W_i^*(r)$, giving $N^{-1}\sum_{i=1}^{N} T^{-2} x_i^{*\prime} x_i^* \xrightarrow{a.s.} \frac{1}{6}\left(\Omega_{xx} - \Omega_{xF}\Omega_{FF}^{-1}\Omega_{Fx}\right) = \frac{1}{6}\Omega_{xx}^*$ and $N^{-1}\sum_{i=1}^{N} T^{-2} x_i^{*\prime} y_i^* \xrightarrow{a.s.} \frac{1}{6}(\Omega_{xy} - \Omega_{xF}\Omega_{FF}^{-1}\Omega_{Fy}) = \frac{1}{6}\Omega_{xy}^*$. Therefore, $\hat{\beta} \xrightarrow{p} \left(\Omega_{xx} - \Omega_{xF}\Omega_{FF}^{-1}\Omega_{Fx}\right)^{-1}\left(\Omega_{xy} - \Omega_{xF}\Omega_{FF}^{-1}\Omega_{Fy}\right) = \Omega_{xx}^{*^{-1}}\Omega_{xy}^* = \beta$, as above.

In order to obtain the limiting distribution of $\hat{\beta}$ we rescale the bias by $\sqrt{N}$ so that

$$\sqrt{N}\left(\hat{\beta} - \beta\right) = \frac{1}{\sqrt{N}}\left[\frac{1}{N}\sum_{i=1}^{N} T^{-2}\left(x_i^{*\prime} x_i^*\right)\right]^{-1}\left(\sum_{i=1}^{N}\left(T^{-2}\left(x_i^{*\prime} y_i^*\right) - T^{-2}\left(x_i^{*\prime} x_i^*\right)\beta\right)\right).$$

Note that $E\left(T^{-2}\left(x_i^{*\prime} y_i^*\right) - T^{-2}\left(x_i^{*\prime} x_i^*\right)\beta\right) = 0$ and, as in Phillips and Moon (1999),

$$\begin{aligned}
& E\left(vec\left(\int_0^1 M_{y_i^*}^*(r)\, M_{x_i^*}^{*\prime}(r)\, dr - \beta'\int_0^1 M_{x_i^*}^*(r)\, M_{x_i^*}^{*\prime}(r)\, dr\right) \times \right. \\
& \left. vec\left(\int_0^1 M_{y_i^*}^*(r)\, M_{x_i^*}^{*\prime}(r)\, dr - \beta'\int_0^1 M_{x_i^*}^*(r)\, M_{x_i^*}^{*\prime}(r)\, dr\right)'\right) \\
= \; & \frac{1}{90} E\left(\Omega_{x_i x_i}^* \otimes \left(\Omega_{y_i y_i}^* - \beta'\Omega_{x_i y_i}^* - \Omega_{y_i x_i}^*\beta + \beta'\Omega_{x_i x_i}^*\beta\right)\right) \\
& + \frac{1}{90} E\left(\left(\Omega_{x_i y_i}^* - \Omega_{x_i x_i}^*\beta\right) \otimes \left(\Omega_{y_i x_i}^* - \beta'\Omega_{x_i x_i}^*\right) K_{1,k}\right) \\
& + \frac{1}{36} E\left(vec\left(\Omega_{y_i x_i}^* - \beta'\Omega_{x_i x_i}^*\right) vec\left(\Omega_{y_i x_i}^* - \beta'\Omega_{x_i x_i}^*\right)'\right) \\
= \; & \Theta_1,
\end{aligned}$$

so that as $N \to \infty$

$$\sqrt{N}\left(\hat{\beta} - \beta\right) \Rightarrow N\left(0, 36\ \Omega_{xx}^{*^{-1}}\Theta_1\Omega_{xx}^{*^{-1}}\right).$$

A.1.3 Time trend

In this section we consider the deterministic specification given by Model 2, i.e., $D_{i,t} = (1,t)\,[\delta_{i,0}, \delta_{i,1}, \ldots, \delta_{i,k}]$, with $\delta_{i,j} = \left(\mu_{i,j}, \eta_{i,j}\right)'$, $j = 0, 1, \ldots, k$. Using the projection matrix $M_D = I - D\left(D'D\right)^{-1}D'$, where $D = [\iota\ \tau]$ with ι a vector of ones and $\tau = (1, 2, \ldots, T)'$. We define $\tilde{Z}_{i,t} = \left(\tilde{U}_{i,t}', \tilde{F}_t'\right)'$, where $\tilde{U}_i = M_D U_i$ and $\tilde{F} = M_D F$ are the OLS detrended variables. By the continuous mapping theorem we have that as $T \to \infty$ for a fixed N

$$T^{-2}\sum_{t=1}^{T}\tilde{Z}_{i,t}\tilde{Z}_{i,t}' \Rightarrow C_i(1)\int_0^1 W_i^*(r)\, W_i^{*\prime}(r)\, dr\ C_i(1)' = \int_0^1 M_i^*(r)\, M_i^{*\prime}(r)\, dr,$$

where $W_i^*(r) = W_i(r) - (4 - 6r)\int_0^1 W_i(s)\, ds - (-6 + 12r)\int_0^1 sW_i(s)\, ds$ and $M_i^*(r) = M_i(r) - (4 - 6r)\int_0^1 M_i(s)\, ds - (-6 + 12r)\int_0^1 sM_i(s)\, ds$ are detrended Brownian motion vectors. In this case,

$$\begin{aligned} E\left(\int_0^1 M_i^*(r)\, M_i^{*\prime}(r)\, dr\right) &= E\left(C_i(1)\, E\left(\int_0^1 W_i^*(r)\, W_i^{*\prime}(r)\, dr\right) C_i(1)'\right) \\ &= E\left(C_i(1)\, \frac{1}{15} I_{(1+k+r)} C_i(1)'\right) \\ &= \frac{1}{15}\Omega. \end{aligned}$$

As above, it can be shown that $N^{-1}\sum_{i=1}^{N} T^{-2} x_i^{*\prime} x_i^* \overset{a.s.}{\to} \frac{1}{15}\left(\Omega_{xx} - \Omega_{xF}\Omega_{FF}^{-1}\Omega_{Fx}\right) = \frac{1}{15}\Omega_{xx}^*$ and $N^{-1}\sum_{i=1}^{N} T^{-2} x_i^{*\prime} y_i^* \overset{a.s.}{\to} \frac{1}{15}\left(\Omega_{xy} - \Omega_{xF}\Omega_{FF}^{-1}\Omega_{Fy}\right) = \frac{1}{15}\Omega_{xy}^*$, so that, $\hat{\beta} \overset{p}{\to} \left(\Omega_{xx} - \Omega_{xF}\Omega_{FF}^{-1}\Omega_{Fx}\right)^{-1}$ $\left(\Omega_{xy} - \Omega_{xF}\Omega_{FF}^{-1}\Omega_{Fy}\right) = \Omega_{xx}^{*^{-1}}\Omega_{xy}^* = \beta$.

In order to obtain the limiting distribution of $\hat{\beta}$ we rescale the bias by $\sqrt{N}$ so that

$$\sqrt{N}\left(\hat{\beta} - \beta\right) = \frac{1}{\sqrt{N}}\left[\frac{1}{N}\sum_{i=1}^{N} T^{-2}\left(x_i^{*\prime} x_i^*\right)\right]^{-1}\left(\sum_{i=1}^{N}\left(T^{-2}\left(x_i^{*\prime} y_i^*\right) - T^{-2}\left(x_i^{*\prime} x_i^*\right)\beta\right)\right).$$

Note that $E\left(T^{-2}\left(x_i^{*\prime} y_i^*\right) - T^{-2}\left(x_i^{*\prime} x_i^*\right)\beta\right) = 0$ and, as in Phillips and Moon (2000),

$$\begin{aligned} & E\left(vec\left(\int_0^1 M_{y_i^*}^*(r)\, M_{x_i^*}^{*\prime}(r)\, dr - \beta' \int_0^1 M_{x_i^*}^*(r)\, M_{x_i^*}^{*\prime}(r)\, dr\right) \times \right. \\ & \left. vec\left(\int_0^1 M_{y_i^*}^*(r)\, M_{x_i^*}^{*\prime}(r)\, dr - \beta' \int_0^1 M_{x_i^*}^*(r)\, M_{x_i^*}^{*\prime}(r)\, dr\right)'\right) \\ = \; & \frac{11}{12600} E\left(\Omega_{x_i x_i}^* \otimes \left(\Omega_{y_i y_i}^* - \beta'\Omega_{x_i y_i}^* - \Omega_{y_i x_i}^*\beta + \beta'\Omega_{x_i x_i}^*\beta\right)\right) \\ & + \frac{11}{12600} E\left(\left(\Omega_{x_i y_i}^* - \Omega_{x_i x_i}^*\beta\right) \otimes \left(\Omega_{y_i x_i}^* - \beta'\Omega_{x_i x_i}^*\right) K_{1,k}\right) \\ & + \frac{1}{225} E\left(vec\left(\Omega_{y_i x_i}^* - \beta'\Omega_{x_i x_i}^*\right) vec\left(\Omega_{y_i x_i}^* - \beta'\Omega_{x_i x_i}^*\right)'\right) \\ = \; & \Theta_3, \end{aligned}$$

so that as $N \to \infty$

$$\sqrt{N}\left(\hat{\beta} - \beta\right) \Rightarrow N\left(0, 225\ \Omega_{xx}^{*^{-1}}\Theta_2\Omega_{xx}^{*^{-1}}\right).$$

A.2 Proof of Theorem 2

Let us define the projection matrix $\bar{M} = I - \bar{H}\left(\bar{H}'\bar{H}\right)^{-1}\bar{H}'$, where $\bar{H} = [\bar{z}]$ for Model 0, $\bar{H} = [\iota\ \bar{z}]$ for Model 1 and $\bar{H} = [\iota\ \tau\ \bar{z}]$ for Model 2, where $\bar{z} = [\bar{x}\ \bar{y}]$ the $(T \times (k+1))$ matrix of cross-section averages. Further, $M_g = I - G\left(G'G\right)^{-1} G'$ and $M_q = I - Q\left(Q'Q\right)^{+} Q'$, where in the case of Model 0 $G = F$ denotes the $(T \times r)$ matrix of unobserved factors, $Q = G\bar{P}$ with $\bar{P} = \bar{\pi}$, and A^{+} is the Moore-Penrose inverse of matrix A. For Model 1, $G = [\iota\ F]$, and

$$\bar{P} = \begin{bmatrix} 1 & \bar{\delta} \\ 0 & \bar{\pi} \end{bmatrix}.$$

Finally, for Model 2 we have $G = [\iota\ \tau\ F]$, and

$$\bar{P} = \begin{bmatrix} I_2 & \bar{\delta} \\ 0 & \bar{\pi} \end{bmatrix}.$$

The pooled estimator is computed as:

$$\begin{aligned} \hat{\beta} &= \left[\frac{1}{N} \sum_{i=1}^{N} T^{-2} \left(x_i^{*\prime} x_i^* \right) \right]^{-1} \frac{1}{N} \sum_{i=1}^{N} T^{-2} \left(x_i^{*\prime} y_i^* \right) \\ &= \left[\frac{1}{N} \sum_{i=1}^{N} T^{-2} \left(x_i^{\prime} \bar{M} x_i \right) \right]^{-1} \frac{1}{N} \sum_{i=1}^{N} T^{-2} \left(x_i^{\prime} \bar{M} y_i \right). \end{aligned}$$

From Lemmas 3 and 4 in Kapetanios, Pesaran and Yamagata (2010), when the rank condition $rank\,(\bar{\pi}) = r \leq (1+k)$ for all N as $N \to \infty$ holds:

$$\begin{aligned} T^{-2} \left(x_i^{\prime} \bar{M} x_i \right) &= T^{-2} \left(x_i^{\prime} M_g x_i \right) + O_p \left(\frac{1}{\sqrt{N}} \right) \\ T^{-2} \left(x_i^{\prime} \bar{M} y_i \right) &= T^{-2} \left(x_i^{\prime} M_g y_i \right) + O_p \left(\frac{1}{\sqrt{N}} \right), \end{aligned}$$

uniformly over i. Therefore, we have

$$\hat{\beta} = \left[\frac{1}{N} \sum_{i=1}^{N} T^{-2} \left(x_i^{\prime} M_g x_i \right) \right]^{-1} \frac{1}{N} \sum_{i=1}^{N} T^{-2} \left(x_i^{\prime} M_g y_i \right) + O_p \left(\frac{1}{\sqrt{N}} \right),$$

so that as $T, N \to \infty$ with $N/T \to 0$

$$\hat{\beta} \xrightarrow{p} \beta = \Omega_{xx}^{*-1} \Omega_{xy}^{*},$$

a result that was already established in Theorem 1.

Table 1: Critical values at the 10 and 5% level of significance for the individual statistic. Model 1

Critical values at the 5% level of significance

	$k=1$								$k=2$							
$T\backslash N$	10	15	20	30	40	50	100	200	10	15	20	30	40	50	100	200
10	-4.37	-4.32	-4.35	-4.29	-4.32	-4.34	-4.30	-4.32	-4.39	-4.34	-4.35	-4.33	-4.32	-4.27	-4.30	-4.31
15	-3.82	-3.83	-3.82	-3.79	-3.79	-3.79	-3.78	-3.78	-3.86	-3.86	-3.82	-3.79	-3.79	-3.81	-3.80	-3.79
20	-3.65	-3.62	-3.60	-3.61	-3.62	-3.61	-3.62	-3.62	-3.65	-3.66	-3.65	-3.63	-3.61	-3.62	-3.61	-3.62
30	-3.49	-3.47	-3.49	-3.46	-3.46	-3.46	-3.45	-3.44	-3.50	-3.49	-3.48	-3.47	-3.48	-3.47	-3.45	-3.44
40	-3.41	-3.40	-3.41	-3.40	-3.39	-3.38	-3.40	-3.39	-3.42	-3.41	-3.40	-3.39	-3.39	-3.38	-3.40	-3.41
50	-3.39	-3.37	-3.37	-3.36	-3.34	-3.36	-3.36	-3.35	-3.39	-3.38	-3.36	-3.39	-3.36	-3.34	-3.35	-3.34
100	-3.28	-3.30	-3.28	-3.29	-3.28	-3.29	-3.28	-3.28	-3.34	-3.33	-3.30	-3.30	-3.31	-3.31	-3.29	-3.28
200	-3.27	-3.26	-3.26	-3.25	-3.26	-3.25	-3.24	-3.25	-3.29	-3.27	-3.28	-3.26	-3.25	-3.26	-3.26	-3.25
250	-3.26	-3.24	-3.25	-3.26	-3.24	-3.24	-3.25	-3.24	-3.29	-3.26	-3.27	-3.27	-3.24	-3.25	-3.25	-3.25
500	-3.26	-3.24	-3.25	-3.25	-3.22	-3.22	-3.22	-3.24	-3.27	-3.26	-3.25	-3.23	-3.22	-3.24	-3.23	-3.23

Critical values at the 10% level of significance

	$k=1$								$k=2$							
$T\backslash N$	10	15	20	30	40	50	100	200	10	15	20	30	40	50	100	200
10	-3.68	-3.64	-3.66	-3.64	-3.63	-3.65	-3.63	-3.63	-3.70	-3.67	-3.66	-3.63	-3.63	-3.61	-3.63	-3.64
15	-3.34	-3.33	-3.32	-3.31	-3.30	-3.30	-3.29	-3.28	-3.35	-3.35	-3.33	-3.31	-3.30	-3.30	-3.30	-3.28
20	-3.21	-3.19	-3.18	-3.18	-3.19	-3.19	-3.18	-3.17	-3.22	-3.22	-3.20	-3.20	-3.18	-3.18	-3.18	-3.19
30	-3.10	-3.08	-3.10	-3.08	-3.07	-3.09	-3.07	-3.06	-3.13	-3.10	-3.10	-3.09	-3.09	-3.09	-3.07	-3.06
40	-3.05	-3.04	-3.05	-3.04	-3.03	-3.02	-3.03	-3.03	-3.08	-3.06	-3.04	-3.03	-3.03	-3.02	-3.03	-3.04
50	-3.05	-3.02	-3.01	-3.01	-3.00	-3.02	-3.00	-3.00	-3.05	-3.03	-3.01	-3.03	-3.02	-3.00	-3.00	-3.00
100	-2.96	-2.97	-2.96	-2.96	-2.96	-2.96	-2.96	-2.96	-3.01	-3.00	-2.98	-2.97	-2.97	-2.98	-2.96	-2.95
200	-2.97	-2.95	-2.95	-2.93	-2.94	-2.92	-2.93	-2.93	-2.98	-2.97	-2.95	-2.94	-2.94	-2.95	-2.94	-2.92
250	-2.95	-2.94	-2.93	-2.94	-2.93	-2.92	-2.94	-2.93	-2.98	-2.95	-2.95	-2.94	-2.93	-2.93	-2.95	-2.94
500	-2.96	-2.93	-2.94	-2.93	-2.92	-2.92	-2.93	-2.92	-2.97	-2.95	-2.95	-2.93	-2.93	-2.94	-2.91	-2.92

Notes: We have generated the dependent variable as $y_{i,t} = y_{i,t-1} + f_t + \varepsilon_{1,i,t}$ and a vector of k explanatory variables $x_{i,t} = x_{i,t-1} + \varepsilon_{2,i,t}$, where $\varepsilon_{i,t} = \left(\varepsilon_{1,i,t}, \varepsilon'_{2,i,t}\right)' \sim iid\ N(0, I_{k+1})$, and $f_t \sim iid\ N(0,1)$ for $i = 1, 2, \ldots, N$, $t = -50, -49, \ldots, T$, and $y_{i,-50} = x_{i,-50} = 0$. The simulations are based on 50,000 replications.

Table 2: Critical values at the 10 and 5% level of significance for the individual statistic. Model 2

Critical values at the 5% level of significance																
	$k=1$								$k=2$							
$T\backslash N$	10	15	20	30	40	50	100	200	10	15	20	30	40	50	100	200
10	-5.47	-5.45	-5.46	-5.46	-5.42	-5.45	-5.43	-5.40	-5.52	-5.48	-5.45	-5.44	-5.43	-5.36	-5.42	-5.42
15	-4.66	-4.65	-4.65	-4.63	-4.62	-4.61	-4.63	-4.60	-4.69	-4.67	-4.64	-4.66	-4.61	-4.59	-4.61	-4.60
20	-4.35	-4.33	-4.34	-4.33	-4.31	-4.31	-4.31	-4.30	-4.36	-4.37	-4.36	-4.35	-4.34	-4.33	-4.32	-4.31
30	-4.11	-4.10	-4.08	-4.09	-4.07	-4.07	-4.07	-4.08	-4.12	-4.10	-4.09	-4.08	-4.08	-4.08	-4.08	-4.07
40	-3.99	-3.98	-3.96	-3.97	-3.98	-3.97	-3.97	-3.98	-4.02	-3.99	-4.00	-3.97	-3.97	-3.98	-3.97	-3.98
50	-3.93	-3.93	-3.91	-3.92	-3.90	-3.90	-3.92	-3.89	-3.95	-3.94	-3.91	-3.93	-3.92	-3.92	-3.92	-3.92
100	-3.83	-3.81	-3.81	-3.79	-3.79	-3.79	-3.80	-3.79	-3.82	-3.84	-3.82	-3.81	-3.81	-3.79	-3.81	-3.78
200	-3.76	-3.74	-3.76	-3.74	-3.76	-3.74	-3.75	-3.74	-3.79	-3.77	-3.76	-3.76	-3.76	-3.75	-3.74	-3.72
250	-3.74	-3.76	-3.73	-3.74	-3.74	-3.74	-3.73	-3.73	-3.77	-3.77	-3.76	-3.73	-3.73	-3.73	-3.73	-3.74
500	-3.73	-3.71	-3.72	-3.72	-3.71	-3.72	-3.71	-3.71	-3.75	-3.73	-3.73	-3.71	-3.72	-3.71	-3.70	-3.70

Critical values at the 10% level of significance																
	$k=1$								$k=2$							
$T\backslash N$	10	15	20	30	40	50	100	200	10	15	20	30	40	50	100	200
10	-4.66	-4.65	-4.66	-4.62	-4.60	-4.64	-4.64	-4.60	-4.69	-4.67	-4.65	-4.65	-4.65	-4.59	-4.62	-4.64
15	-4.10	-4.09	-4.09	-4.07	-4.08	-4.06	-4.08	-4.06	-4.13	-4.11	-4.10	-4.09	-4.07	-4.06	-4.07	-4.06
20	-3.87	-3.86	-3.87	-3.86	-3.85	-3.86	-3.85	-3.84	-3.90	-3.91	-3.88	-3.88	-3.87	-3.85	-3.85	-3.87
30	-3.71	-3.70	-3.68	-3.70	-3.68	-3.66	-3.68	-3.68	-3.74	-3.70	-3.69	-3.69	-3.69	-3.69	-3.69	-3.68
40	-3.63	-3.62	-3.61	-3.61	-3.61	-3.60	-3.60	-3.61	-3.65	-3.63	-3.62	-3.60	-3.60	-3.61	-3.60	-3.61
50	-3.58	-3.58	-3.56	-3.57	-3.55	-3.56	-3.55	-3.55	-3.60	-3.59	-3.56	-3.58	-3.57	-3.57	-3.56	-3.56
100	-3.50	-3.49	-3.49	-3.48	-3.47	-3.47	-3.48	-3.47	-3.51	-3.51	-3.50	-3.49	-3.48	-3.48	-3.48	-3.47
200	-3.45	-3.44	-3.45	-3.45	-3.45	-3.44	-3.44	-3.44	-3.48	-3.47	-3.46	-3.44	-3.45	-3.44	-3.43	-3.43
250	-3.44	-3.44	-3.43	-3.44	-3.44	-3.43	-3.42	-3.42	-3.47	-3.46	-3.45	-3.43	-3.43	-3.42	-3.42	-3.44
500	-3.42	-3.42	-3.42	-3.41	-3.41	-3.42	-3.41	-3.42	-3.45	-3.44	-3.43	-3.41	-3.41	-3.41	-3.41	-3.40

Notes: We have generated the dependent variable as $y_{i,t} = y_{i,t-1} + f_t + \varepsilon_{1,i,t}$ and a vector of k explanatory variables $x_{i,t} = x_{i,t-1} + \varepsilon_{2,i,t}$, where $\varepsilon_{i,t} = \left(\varepsilon_{1,i,t}, \varepsilon'_{2,i,t}\right)' \sim iid\ N\left(0, I_{k+1}\right)$, and $f_t \sim iid\ N\left(0,1\right)$ for $i = 1, 2, \ldots, N$, $t = -50, -49, \ldots, T$, and $y_{i,-50} = x_{i,-50} = 0$. The simulations are based on 50,000 replications.

Table 3: Critical values at the 10 and 5% level of significance for the panel statistic. Model 1

Critical values at the 5% level of significance

	k = 1								k = 2							
$T \backslash N$	10	15	20	30	40	50	100	200	10	15	20	30	40	50	100	200
10	-2.92	-2.80	-2.74	-2.65	-2.62	-2.60	-2.54	-2.51	-2.95	-2.81	-2.74	-2.67	-2.62	-2.59	-2.54	-2.51
15	-2.65	-2.54	-2.48	-2.41	-2.37	-2.36	-2.32	-2.29	-2.67	-2.55	-2.49	-2.41	-2.39	-2.36	-2.32	-2.29
20	-2.55	-2.44	-2.39	-2.33	-2.30	-2.28	-2.24	-2.22	-2.57	-2.46	-2.40	-2.34	-2.30	-2.29	-2.24	-2.22
30	-2.47	-2.38	-2.32	-2.27	-2.24	-2.22	-2.18	-2.17	-2.50	-2.39	-2.33	-2.27	-2.24	-2.23	-2.19	-2.17
40	-2.44	-2.34	-2.30	-2.24	-2.21	-2.19	-2.16	-2.14	-2.47	-2.36	-2.31	-2.25	-2.22	-2.20	-2.16	-2.14
50	-2.43	-2.33	-2.28	-2.23	-2.20	-2.18	-2.14	-2.12	-2.45	-2.35	-2.29	-2.24	-2.20	-2.18	-2.14	-2.13
100	-2.39	-2.30	-2.24	-2.20	-2.17	-2.15	-2.12	-2.10	-2.42	-2.31	-2.26	-2.20	-2.17	-2.16	-2.12	-2.10
200	-2.37	-2.28	-2.24	-2.18	-2.16	-2.14	-2.11	-2.09	-2.40	-2.30	-2.24	-2.19	-2.16	-2.14	-2.11	-2.09
250	-2.37	-2.28	-2.23	-2.18	-2.15	-2.14	-2.10	-2.08	-2.40	-2.30	-2.24	-2.18	-2.16	-2.14	-2.10	-2.08
500	-2.37	-2.27	-2.23	-2.17	-2.15	-2.13	-2.10	-2.08	-2.39	-2.30	-2.24	-2.18	-2.15	-2.13	-2.10	-2.08

Critical values at the 10% level of significance

	k = 1								k = 2							
$T \backslash N$	10	15	20	30	40	50	100	200	10	15	20	30	40	50	100	200
10	-2.72	-2.62	-2.58	-2.52	-2.49	-2.48	-2.43	-2.42	-2.75	-2.64	-2.59	-2.53	-2.50	-2.48	-2.44	-2.42
15	-2.49	-2.40	-2.36	-2.31	-2.28	-2.27	-2.24	-2.23	-2.51	-2.42	-2.37	-2.31	-2.29	-2.27	-2.24	-2.23
20	-2.41	-2.33	-2.28	-2.24	-2.22	-2.20	-2.17	-2.16	-2.43	-2.34	-2.30	-2.25	-2.22	-2.20	-2.18	-2.16
30	-2.34	-2.27	-2.22	-2.18	-2.16	-2.15	-2.12	-2.11	-2.38	-2.28	-2.24	-2.19	-2.16	-2.16	-2.12	-2.11
40	-2.32	-2.24	-2.20	-2.16	-2.14	-2.12	-2.10	-2.08	-2.35	-2.26	-2.21	-2.16	-2.14	-2.13	-2.10	-2.08
50	-2.31	-2.23	-2.19	-2.14	-2.12	-2.11	-2.08	-2.07	-2.33	-2.24	-2.20	-2.15	-2.13	-2.11	-2.08	-2.07
100	-2.27	-2.20	-2.16	-2.12	-2.10	-2.08	-2.06	-2.04	-2.30	-2.21	-2.17	-2.13	-2.10	-2.09	-2.06	-2.05
200	-2.26	-2.18	-2.15	-2.10	-2.09	-2.07	-2.05	-2.03	-2.29	-2.20	-2.16	-2.11	-2.09	-2.08	-2.05	-2.04
250	-2.26	-2.18	-2.14	-2.11	-2.09	-2.07	-2.04	-2.03	-2.28	-2.20	-2.15	-2.11	-2.09	-2.07	-2.05	-2.03
500	-2.25	-2.18	-2.14	-2.10	-2.08	-2.07	-2.04	-2.03	-2.28	-2.20	-2.15	-2.10	-2.08	-2.07	-2.04	-2.03

Notes: We have generated the dependent variable as $y_{i,t} = y_{i,t-1} + f_t + \varepsilon_{1,i,t}$ and a vector of k explanatory variables $x_{i,t} = x_{i,t-1} + \varepsilon_{2,i,t}$, where $\varepsilon_{i,t} = \left(\varepsilon_{1,i,t}, \varepsilon'_{2,i,t}\right)' \sim iid\ N(0, I_{k+1})$, and $f_t \sim iid\ N(0,1)$ for $i = 1, 2, \ldots, N$, $t = -50, -49, \ldots, T$, and $y_{i,-50} = x_{i,-50} = 0$. The simulations are based on 50,000 replications.

Table 4: Critical values at the 10 and 5% level of significance for the panel statistic. Model 2

Critical values at the 5% level of significance

	$k=1$								$k=2$							
$T \backslash N$	10	15	20	30	40	50	100	200	10	15	20	30	40	50	100	200
10	-3.88	-3.74	-3.66	-3.59	-3.54	-3.52	-3.48	-3.45	-3.89	-3.75	-3.68	-3.60	-3.55	-3.53	-3.47	-3.45
15	-3.39	-3.27	-3.21	-3.14	-3.11	-3.08	-3.04	-3.01	-3.41	-3.28	-3.22	-3.15	-3.11	-3.09	-3.04	-3.01
20	-3.21	-3.11	-3.04	-2.99	-2.95	-2.93	-2.88	-2.86	-3.24	-3.12	-3.05	-2.99	-2.96	-2.93	-2.89	-2.86
30	-3.07	-2.97	-2.91	-2.86	-2.82	-2.80	-2.76	-2.74	-3.10	-2.99	-2.93	-2.87	-2.83	-2.81	-2.76	-2.74
40	-3.01	-2.91	-2.86	-2.80	-2.77	-2.75	-2.71	-2.68	-3.04	-2.93	-2.87	-2.81	-2.77	-2.76	-2.71	-2.68
50	-2.98	-2.88	-2.83	-2.77	-2.74	-2.72	-2.68	-2.65	-3.00	-2.90	-2.84	-2.78	-2.74	-2.72	-2.68	-2.66
100	-2.91	-2.82	-2.77	-2.71	-2.68	-2.66	-2.62	-2.60	-2.94	-2.84	-2.79	-2.72	-2.69	-2.67	-2.63	-2.60
200	-2.89	-2.79	-2.74	-2.69	-2.66	-2.64	-2.60	-2.57	-2.91	-2.81	-2.76	-2.69	-2.67	-2.64	-2.60	-2.58
250	-2.88	-2.79	-2.74	-2.69	-2.65	-2.63	-2.59	-2.57	-2.91	-2.80	-2.75	-2.69	-2.66	-2.64	-2.60	-2.57
500	-2.87	-2.78	-2.73	-2.67	-2.64	-2.62	-2.58	-2.56	-2.89	-2.79	-2.74	-2.68	-2.65	-2.63	-2.58	-2.56

Critical values at the 10% level of significance

	$k=1$								$k=2$							
$T \backslash N$	10	15	20	30	40	50	100	200	10	15	20	30	40	50	100	200
10	-3.64	-3.54	-3.48	-3.42	-3.39	-3.37	-3.34	-3.32	-3.65	-3.55	-3.50	-3.44	-3.40	-3.38	-3.34	-3.32
15	-3.22	-3.13	-3.09	-3.03	-3.01	-2.99	-2.95	-2.93	-3.24	-3.15	-3.09	-3.04	-3.01	-2.99	-2.96	-2.94
20	-3.07	-2.99	-2.94	-2.89	-2.87	-2.85	-2.82	-2.80	-3.10	-3.00	-2.95	-2.90	-2.87	-2.86	-2.82	-2.80
30	-2.95	-2.87	-2.82	-2.78	-2.75	-2.73	-2.70	-2.69	-2.97	-2.88	-2.83	-2.79	-2.76	-2.74	-2.71	-2.68
40	-2.89	-2.82	-2.78	-2.73	-2.70	-2.68	-2.65	-2.64	-2.92	-2.83	-2.79	-2.74	-2.71	-2.69	-2.65	-2.63
50	-2.86	-2.79	-2.74	-2.70	-2.67	-2.66	-2.63	-2.61	-2.89	-2.80	-2.76	-2.71	-2.68	-2.66	-2.63	-2.61
100	-2.81	-2.73	-2.69	-2.65	-2.62	-2.61	-2.58	-2.55	-2.83	-2.75	-2.71	-2.66	-2.63	-2.61	-2.58	-2.56
200	-2.78	-2.71	-2.67	-2.62	-2.60	-2.58	-2.55	-2.53	-2.81	-2.73	-2.68	-2.63	-2.61	-2.59	-2.55	-2.53
250	-2.78	-2.70	-2.66	-2.62	-2.59	-2.58	-2.55	-2.53	-2.80	-2.72	-2.67	-2.63	-2.60	-2.58	-2.55	-2.53
500	-2.77	-2.69	-2.65	-2.61	-2.59	-2.57	-2.53	-2.52	-2.79	-2.71	-2.67	-2.62	-2.59	-2.57	-2.54	-2.52

Notes: We have generated the dependent variable as $y_{i,t} = y_{i,t-1} + f_t + \varepsilon_{1,i,t}$ and a vector of k explanatory variables $x_{i,t} = x_{i,t-1} + \varepsilon_{2,i,t}$, where $\varepsilon_{i,t} = \left(\varepsilon_{1,i,t}, \varepsilon'_{2,i,t}\right)' \sim iid\ N(0, I_{k+1})$, and $f_t \sim iid\ N(0,1)$ for $i = 1, 2, \ldots, N$, $t = -50, -49, \ldots, T$, and $y_{i,-50} = x_{i,-50} = 0$. The simulations are based on 50,000 replications.

Table 5: Mean, median and root mean square error of the $\hat{\beta}_{PCCE}$ for $N = 20$

ϕ_i	ρ	T	Mean	Median	RMSE	ϕ_i	ρ	T	Mean	Median	RMSE
1.000	1.000	50	0.999	0.999	0.092	0.950	1.000	50	0.998	0.998	0.086
1.000	1.000	100	1.000	1.000	0.092	0.950	1.000	100	1.000	1.001	0.073
1.000	1.000	250	0.999	1.001	0.092	0.950	1.000	250	0.999	1.000	0.050
1.000	0.990	50	0.999	1.000	0.092	0.950	0.990	50	0.998	0.998	0.086
1.000	0.990	100	1.000	1.001	0.092	0.950	0.990	100	1.000	1.000	0.074
1.000	0.990	250	0.999	1.000	0.093	0.950	0.990	250	0.999	1.001	0.050
1.000	0.950	50	0.999	0.999	0.092	0.950	0.950	50	0.998	0.998	0.086
1.000	0.950	100	1.000	1.001	0.094	0.950	0.950	100	1.000	1.002	0.074
1.000	0.950	250	0.999	1.001	0.095	0.950	0.950	250	0.999	1.001	0.050
1.000	0.900	50	0.999	0.999	0.094	0.950	0.900	50	0.998	0.998	0.087
1.000	0.900	100	1.000	1.001	0.095	0.950	0.900	100	1.000	1.001	0.075
1.000	0.900	250	0.999	1.000	0.096	0.950	0.900	250	0.999	1.001	0.050
0.990	1.000	50	0.999	0.999	0.092	0.900	1.000	50	0.998	0.998	0.076
0.990	1.000	100	1.001	1.000	0.090	0.900	1.000	100	1.000	1.000	0.057
0.990	1.000	250	0.999	1.000	0.084	0.900	1.000	250	1.000	1.000	0.032
0.990	0.990	50	0.999	0.999	0.092	0.900	0.990	50	0.998	0.999	0.076
0.990	0.990	100	1.001	1.001	0.091	0.900	0.990	100	1.000	1.000	0.057
0.990	0.990	250	0.999	0.999	0.085	0.900	0.990	250	1.000	1.000	0.032
0.990	0.950	50	0.999	0.999	0.092	0.900	0.950	50	0.998	0.999	0.076
0.990	0.950	100	1.001	1.002	0.092	0.900	0.950	100	1.000	1.000	0.057
0.990	0.950	250	0.999	1.000	0.086	0.900	0.950	250	1.000	1.000	0.032
0.990	0.900	50	0.999	0.998	0.094	0.900	0.900	50	0.998	0.999	0.076
0.990	0.900	100	1.001	1.002	0.093	0.900	0.900	100	1.000	1.000	0.057
0.990	0.900	250	0.999	1.000	0.087	0.900	0.900	250	1.000	1.000	0.031

Table 6: Panel cointegration test using PCCE estimator, $N = 20$, imposing one common factor ($r = 1$)

			$\sigma_F^2 = 0.5$			$\sigma_F^2 = 1$			$\sigma_F^2 = 10$		
ϕ_i	ρ	T	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$
1	1	50	0.056	0.062	0.046	0.061	0.061	0.047	0.059	0.061	0.046
1	1	100	0.059	0.052	0.048	0.058	0.053	0.050	0.057	0.055	0.049
1	1	250	0.053	0.056	0.045	0.053	0.055	0.043	0.056	0.055	0.043
1	0.99	50	0.056	0.063	0.047	0.060	0.061	0.047	0.060	0.061	0.045
1	0.99	100	0.057	0.057	0.045	0.059	0.056	0.045	0.056	0.058	0.045
1	0.99	250	0.054	0.061	0.038	0.053	0.061	0.038	0.054	0.063	0.037
1	0.95	50	0.054	0.071	0.040	0.056	0.074	0.040	0.057	0.073	0.039
1	0.95	100	0.057	0.090	0.028	0.058	0.092	0.029	0.054	0.093	0.030
1	0.95	250	0.052	0.246	0.013	0.053	0.257	0.012	0.053	0.273	0.012
1	0.9	50	0.055	0.100	0.028	0.056	0.102	0.027	0.053	0.100	0.026
1	0.9	100	0.056	0.192	0.017	0.058	0.200	0.016	0.054	0.211	0.015
1	0.9	250	0.051	0.740	0.007	0.051	0.788	0.006	0.052	0.830	0.005
0.99	1	50	0.061	0.061	0.048	0.064	0.061	0.048	0.061	0.061	0.045
0.99	1	100	0.078	0.053	0.058	0.078	0.053	0.058	0.076	0.055	0.058
0.99	1	250	0.263	0.055	0.107	0.261	0.056	0.107	0.236	0.055	0.106
0.99	0.99	50	0.062	0.062	0.050	0.064	0.061	0.049	0.061	0.061	0.046
0.99	0.99	100	0.077	0.055	0.054	0.079	0.056	0.055	0.077	0.058	0.055
0.99	0.99	250	0.261	0.062	0.095	0.26	0.060	0.096	0.238	0.063	0.093
0.99	0.95	50	0.058	0.072	0.041	0.061	0.073	0.041	0.058	0.073	0.040
0.99	0.95	100	0.077	0.091	0.034	0.077	0.092	0.034	0.074	0.093	0.035
0.99	0.95	250	0.259	0.246	0.041	0.261	0.258	0.038	0.232	0.273	0.035
0.99	0.9	50	0.055	0.099	0.029	0.057	0.102	0.028	0.054	0.101	0.027
0.99	0.9	100	0.077	0.193	0.020	0.077	0.202	0.019	0.075	0.210	0.021
0.99	0.9	250	0.256	0.756	0.026	0.259	0.798	0.022	0.231	0.831	0.019
0.95	1	50	0.223	0.061	0.103	0.217	0.061	0.103	0.164	0.060	0.102
0.95	1	100	0.840	0.054	0.361	0.83	0.054	0.358	0.683	0.055	0.359
0.95	1	250	1	0.055	1.000	1	0.054	1.000	1	0.054	1.000
0.95	0.99	50	0.224	0.063	0.103	0.218	0.062	0.101	0.165	0.061	0.100
0.95	0.99	100	0.840	0.056	0.353	0.832	0.058	0.354	0.681	0.059	0.349
0.95	0.99	250	1	0.066	1.000	1	0.064	1.000	1	0.063	1.000
0.95	0.95	50	0.219	0.072	0.083	0.214	0.073	0.084	0.165	0.074	0.085
0.95	0.95	100	0.842	0.093	0.304	0.83	0.093	0.300	0.671	0.093	0.294
0.95	0.95	250	1	0.270	0.999	1	0.271	0.999	1	0.273	0.999
0.95	0.9	50	0.216	0.101	0.068	0.209	0.101	0.065	0.161	0.1	0.065
0.95	0.9	100	0.838	0.204	0.237	0.826	0.206	0.233	0.663	0.21	0.234
0.95	0.9	250	1	0.82	0.998	1	0.830	0.998	1	0.833	0.999
0.9	1	50	0.764	0.062	0.318	0.742	0.061	0.318	0.537	0.059	0.317
0.9	1	100	1	0.057	0.973	1	0.057	0.974	0.992	0.055	0.975
0.9	1	250	1	0.056	1.000	1	0.056	1.000	1	0.053	1.000
0.9	0.99	50	0.763	0.063	0.319	0.739	0.063	0.319	0.535	0.063	0.320
0.9	0.99	100	1	0.058	0.972	1	0.059	0.973	0.992	0.061	0.973
0.9	0.99	250	1	0.066	1.000	1	0.065	1.000	1	0.064	1.000
0.9	0.95	50	0.762	0.073	0.297	0.736	0.074	0.296	0.528	0.075	0.296
0.9	0.95	100	1	0.096	0.966	1	0.094	0.966	0.991	0.093	0.966
0.9	0.95	250	1	0.284	1.000	1	0.280	1.000	1	0.275	1.000
0.9	0.9	50	0.758	0.105	0.259	0.732	0.102	0.258	0.521	0.101	0.254
0.9	0.9	100	1	0.215	0.950	1	0.209	0.950	0.992	0.212	0.951
0.9	0.9	250	1	0.847	1.000	1	0.842	1.000	1	0.834	1.000

Table 7: Panel cointegration test using PCCE estimator, $N = 20$, estimating the number of common factors ($r_{max} = 6$)

			$\sigma_F^2 = 0.5$			$\sigma_F^2 = 1$			$\sigma_F^2 = 10$		
ϕ_i	ρ	T	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$
1	1	50	0.056	0.062	0.046	0.061	0.061	0.047	0.060	0.034	0.046
1	1	100	0.059	0.052	0.048	0.058	0.053	0.050	0.057	0.049	0.049
1	1	250	0.053	0.056	0.045	0.053	0.055	0.043	0.056	0.054	0.043
1	0.99	50	0.056	0.063	0.047	0.060	0.061	0.047	0.061	0.034	0.045
1	0.99	100	0.057	0.057	0.045	0.059	0.056	0.045	0.057	0.052	0.045
1	0.99	250	0.054	0.061	0.038	0.053	0.061	0.038	0.054	0.062	0.037
1	0.95	50	0.054	0.071	0.040	0.056	0.074	0.040	0.058	0.038	0.039
1	0.95	100	0.057	0.090	0.028	0.058	0.092	0.029	0.054	0.083	0.030
1	0.95	250	0.052	0.246	0.013	0.053	0.257	0.012	0.053	0.272	0.012
1	0.9	50	0.055	0.100	0.028	0.056	0.102	0.027	0.056	0.052	0.026
1	0.9	100	0.056	0.192	0.017	0.058	0.200	0.016	0.055	0.187	0.015
1	0.9	250	0.051	0.740	0.007	0.051	0.788	0.006	0.051	0.825	0.005
0.99	1	50	0.061	0.061	0.048	0.064	0.061	0.048	0.063	0.034	0.045
0.99	1	100	0.078	0.053	0.058	0.078	0.053	0.058	0.077	0.050	0.058
0.99	1	250	0.263	0.055	0.107	0.261	0.056	0.107	0.236	0.054	0.106
0.99	0.99	50	0.062	0.062	0.050	0.064	0.061	0.049	0.064	0.034	0.046
0.99	0.99	100	0.077	0.055	0.054	0.079	0.056	0.055	0.077	0.052	0.055
0.99	0.99	250	0.261	0.062	0.095	0.260	0.060	0.096	0.238	0.062	0.093
0.99	0.95	50	0.058	0.072	0.041	0.061	0.073	0.041	0.061	0.038	0.040
0.99	0.95	100	0.077	0.091	0.034	0.077	0.092	0.034	0.075	0.083	0.035
0.99	0.95	250	0.259	0.246	0.041	0.261	0.258	0.038	0.232	0.271	0.035
0.99	0.9	50	0.055	0.099	0.029	0.057	0.102	0.028	0.060	0.054	0.027
0.99	0.9	100	0.077	0.193	0.020	0.077	0.202	0.019	0.076	0.187	0.021
0.99	0.9	250	0.256	0.756	0.026	0.259	0.798	0.022	0.231	0.826	0.019
0.95	1	50	0.223	0.061	0.103	0.217	0.060	0.103	0.163	0.035	0.102
0.95	1	100	0.840	0.054	0.361	0.830	0.054	0.358	0.684	0.051	0.359
0.95	1	250	1	0.055	1.000	1	0.054	1.000	1	0.054	1.000
0.95	0.99	50	0.224	0.063	0.103	0.218	0.062	0.101	0.164	0.035	0.100
0.95	0.99	100	0.840	0.056	0.353	0.832	0.058	0.354	0.682	0.054	0.349
0.95	0.99	250	1	0.066	1.000	1	0.064	1.000	1	0.062	1.000
0.95	0.95	50	0.219	0.072	0.083	0.214	0.073	0.084	0.164	0.041	0.085
0.95	0.95	100	0.842	0.093	0.304	0.830	0.093	0.300	0.673	0.084	0.294
0.95	0.95	250	1	0.270	0.999	1	0.271	0.999	1	0.271	0.999
0.95	0.9	50	0.216	0.101	0.068	0.209	0.101	0.065	0.157	0.054	0.065
0.95	0.9	100	0.838	0.204	0.237	0.826	0.206	0.233	0.664	0.187	0.234
0.95	0.9	250	1	0.820	0.998	1	0.83	0.998	1	0.828	0.999
0.9	1	50	0.764	0.062	0.318	0.742	0.061	0.318	0.524	0.035	0.317
0.9	1	100	1	0.057	0.973	1	0.057	0.974	0.992	0.051	0.975
0.9	1	250	1	0.056	1.000	1	0.056	1.000	1	0.052	1.000
0.9	0.99	50	0.763	0.063	0.319	0.739	0.063	0.319	0.522	0.036	0.320
0.9	0.99	100	1	0.058	0.972	1	0.059	0.973	0.992	0.055	0.973
0.9	0.99	250	1	0.066	1.000	1	0.065	1.000	1	0.063	1.000
0.9	0.95	50	0.762	0.073	0.297	0.736	0.074	0.296	0.516	0.042	0.296
0.9	0.95	100	1	0.096	0.966	1	0.094	0.966	0.991	0.085	0.966
0.9	0.95	250	1	0.284	1.000	1	0.280	1.000	1	0.274	1.000
0.9	0.9	50	0.758	0.105	0.259	0.731	0.102	0.258	0.504	0.057	0.254
0.9	0.9	100	1	0.215	0.950	1	0.209	0.950	0.991	0.191	0.951
0.9	0.9	250	1	0.847	1.000	1	0.842	1.000	1	0.830	1.000

Table 8: Panel cointegration test using PCCE estimator, $N = 10$, imposing one common factor ($r = 1$)

			$\sigma_F^2 = 0.5$			$\sigma_F^2 = 1$			$\sigma_F^2 = 10$		
ϕ_i	ρ	T	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$
1	1	50	0.060	0.063	0.049	0.062	0.061	0.051	0.067	0.061	0.054
1	1	100	0.069	0.055	0.050	0.065	0.051	0.051	0.068	0.054	0.048
1	1	250	0.065	0.050	0.047	0.066	0.051	0.045	0.064	0.053	0.049
1	0.99	50	0.060	0.061	0.049	0.060	0.060	0.053	0.067	0.060	0.054
1	0.99	100	0.069	0.054	0.050	0.066	0.054	0.051	0.069	0.054	0.048
1	0.99	250	0.064	0.061	0.042	0.065	0.064	0.042	0.064	0.066	0.041
1	0.95	50	0.059	0.069	0.046	0.061	0.069	0.047	0.063	0.072	0.048
1	0.95	100	0.065	0.082	0.043	0.064	0.083	0.040	0.066	0.088	0.039
1	0.95	250	0.064	0.208	0.029	0.064	0.235	0.025	0.063	0.271	0.021
1	0.9	50	0.058	0.097	0.041	0.059	0.095	0.040	0.062	0.098	0.038
1	0.9	100	0.064	0.167	0.033	0.061	0.185	0.030	0.064	0.197	0.029
1	0.9	250	0.064	0.512	0.029	0.065	0.642	0.021	0.064	0.814	0.012
0.99	1	50	0.068	0.064	0.052	0.069	0.061	0.055	0.068	0.061	0.055
0.99	1	100	0.089	0.054	0.055	0.088	0.052	0.057	0.085	0.053	0.054
0.99	1	250	0.186	0.053	0.081	0.186	0.053	0.085	0.180	0.052	0.086
0.99	0.99	50	0.068	0.062	0.054	0.069	0.060	0.057	0.068	0.059	0.056
0.99	0.99	100	0.088	0.057	0.053	0.087	0.054	0.054	0.084	0.053	0.052
0.99	0.99	250	0.185	0.064	0.079	0.185	0.065	0.078	0.177	0.066	0.076
0.99	0.95	50	0.067	0.069	0.050	0.066	0.069	0.050	0.067	0.071	0.050
0.99	0.95	100	0.084	0.083	0.045	0.085	0.083	0.044	0.084	0.087	0.043
0.99	0.95	250	0.182	0.220	0.057	0.181	0.244	0.050	0.175	0.275	0.044
0.99	0.9	50	0.065	0.097	0.044	0.064	0.096	0.043	0.065	0.098	0.041
0.99	0.9	100	0.084	0.166	0.035	0.083	0.185	0.032	0.083	0.198	0.032
0.99	0.9	250	0.181	0.552	0.053	0.179	0.668	0.044	0.175	0.813	0.029
0.95	1	50	0.176	0.063	0.087	0.176	0.063	0.089	0.151	0.060	0.098
0.95	1	100	0.586	0.053	0.212	0.579	0.050	0.215	0.509	0.053	0.222
0.95	1	250	1	0.060	0.947	1	0.056	0.950	0.998	0.052	0.956
0.95	0.99	50	0.176	0.062	0.088	0.175	0.061	0.088	0.149	0.060	0.095
0.95	0.99	100	0.583	0.055	0.208	0.580	0.052	0.215	0.509	0.053	0.219
0.95	0.99	250	1	0.073	0.942	1	0.069	0.944	0.998	0.063	0.949
0.95	0.95	50	0.172	0.072	0.082	0.171	0.071	0.085	0.148	0.069	0.084
0.95	0.95	100	0.586	0.092	0.189	0.577	0.086	0.187	0.511	0.089	0.190
0.95	0.95	250	1	0.288	0.933	1	0.288	0.931	0.998	0.279	0.930
0.95	0.9	50	0.171	0.099	0.073	0.170	0.100	0.071	0.144	0.098	0.069
0.95	0.9	100	0.586	0.186	0.165	0.577	0.194	0.160	0.509	0.201	0.154
0.95	0.9	250	1	0.743	0.932	1	0.783	0.923	0.999	0.824	0.909
0.9	1	50	0.515	0.069	0.207	0.509	0.065	0.213	0.421	0.059	0.225
0.9	1	100	0.991	0.056	0.757	0.99	0.053	0.765	0.957	0.053	0.776
0.9	1	250	1	0.071	1.000	1	0.061	1.000	1	0.054	1.000
0.9	0.99	50	0.513	0.067	0.206	0.510	0.066	0.213	0.422	0.059	0.224
0.9	0.99	100	0.992	0.059	0.756	0.989	0.057	0.764	0.956	0.053	0.773
0.9	0.99	250	1	0.088	1.000	1	0.076	1.000	1	0.064	1.000
0.9	0.95	50	0.516	0.078	0.197	0.509	0.074	0.200	0.420	0.070	0.210
0.9	0.95	100	0.992	0.099	0.734	0.990	0.092	0.739	0.956	0.090	0.748
0.9	0.95	250	1	0.338	1.000	1	0.311	1.000	1	0.282	1.000
0.9	0.9	50	0.518	0.106	0.182	0.506	0.103	0.183	0.417	0.099	0.185
0.9	0.9	100	0.993	0.208	0.714	0.990	0.203	0.710	0.954	0.203	0.708
0.9	0.9	250	1	0.837	1.000	1	0.833	1.000	1	0.832	1.000

Table 9: Panel cointegration test using PCCE estimator, $N = 10$, estimating the number of common factors

			$\sigma_F^2 = 0.5$			$\sigma_F^2 = 1$			$\sigma_F^2 = 10$		
ϕ_i	ρ	T	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$
1	1	50	0.118	0.009	0.049	0.124	0.010	0.051	0.130	0.004	0.054
1	1	100	0.075	0.050	0.050	0.072	0.046	0.051	0.090	0.036	0.048
1	1	250	0.065	0.050	0.047	0.066	0.051	0.045	0.064	0.053	0.049
1	0.99	50	0.120	0.009	0.049	0.123	0.010	0.053	0.128	0.004	0.054
1	0.99	100	0.074	0.049	0.050	0.072	0.048	0.051	0.091	0.036	0.048
1	0.99	250	0.064	0.061	0.042	0.065	0.064	0.042	0.064	0.065	0.041
1	0.95	50	0.120	0.010	0.046	0.120	0.010	0.047	0.125	0.004	0.048
1	0.95	100	0.070	0.075	0.043	0.070	0.076	0.040	0.089	0.060	0.039
1	0.95	250	0.064	0.208	0.029	0.064	0.235	0.025	0.063	0.270	0.021
1	0.9	50	0.121	0.014	0.041	0.118	0.013	0.040	0.121	0.005	0.038
1	0.9	100	0.070	0.151	0.033	0.069	0.167	0.030	0.088	0.127	0.029
1	0.9	250	0.064	0.512	0.029	0.065	0.642	0.021	0.064	0.808	0.012
0.99	1	50	0.122	0.009	0.052	0.127	0.010	0.055	0.137	0.004	0.055
0.99	1	100	0.098	0.049	0.055	0.096	0.047	0.057	0.106	0.035	0.054
0.99	1	250	0.186	0.053	0.081	0.186	0.053	0.085	0.181	0.052	0.086
0.99	0.99	50	0.121	0.009	0.054	0.125	0.009	0.057	0.138	0.005	0.056
0.99	0.99	100	0.096	0.052	0.053	0.095	0.048	0.054	0.106	0.035	0.052
0.99	0.99	250	0.185	0.064	0.079	0.185	0.065	0.078	0.178	0.065	0.076
0.99	0.95	50	0.122	0.010	0.050	0.121	0.009	0.050	0.134	0.003	0.050
0.99	0.95	100	0.092	0.075	0.045	0.090	0.075	0.044	0.106	0.059	0.043
0.99	0.95	250	0.182	0.220	0.057	0.181	0.244	0.050	0.176	0.274	0.044
0.99	0.9	50	0.119	0.014	0.044	0.121	0.014	0.043	0.129	0.005	0.041
0.99	0.9	100	0.091	0.150	0.035	0.089	0.167	0.032	0.104	0.128	0.032
0.99	0.9	250	0.181	0.552	0.053	0.179	0.668	0.044	0.175	0.807	0.029
0.95	1	50	0.178	0.009	0.087	0.179	0.010	0.089	0.183	0.004	0.098
0.95	1	100	0.568	0.049	0.212	0.560	0.044	0.215	0.484	0.035	0.222
0.95	1	250	1.000	0.060	0.947	1.000	0.056	0.950	0.998	0.052	0.956
0.95	0.99	50	0.178	0.009	0.088	0.179	0.008	0.088	0.182	0.005	0.095
0.95	0.99	100	0.566	0.050	0.208	0.561	0.047	0.215	0.483	0.035	0.219
0.95	0.99	250	1.000	0.073	0.942	1.000	0.069	0.944	0.999	0.063	0.949
0.95	0.95	50	0.174	0.011	0.082	0.177	0.010	0.085	0.183	0.004	0.084
0.95	0.95	100	0.565	0.082	0.189	0.556	0.078	0.187	0.484	0.061	0.190
0.95	0.95	250	1.000	0.288	0.933	1.000	0.288	0.931	0.999	0.278	0.930
0.95	0.9	50	0.170	0.014	0.073	0.172	0.014	0.071	0.179	0.005	0.069
0.95	0.9	100	0.565	0.168	0.165	0.557	0.175	0.160	0.482	0.131	0.154
0.95	0.9	250	1.000	0.743	0.932	1.000	0.783	0.923	0.999	0.818	0.909
0.9	1	50	0.342	0.010	0.207	0.338	0.010	0.213	0.309	0.004	0.225
0.9	1	100	0.973	0.050	0.757	0.971	0.046	0.765	0.904	0.035	0.776
0.9	1	250	1.000	0.071	1.000	1.000	0.061	1.000	1.000	0.053	1.000
0.9	0.99	50	0.340	0.010	0.206	0.336	0.009	0.213	0.308	0.005	0.224
0.9	0.99	100	0.973	0.052	0.756	0.970	0.050	0.764	0.901	0.034	0.773
0.9	0.99	250	1.000	0.088	1.000	1.000	0.076	1.000	1.000	0.063	1.000
0.9	0.95	50	0.339	0.011	0.197	0.336	0.010	0.200	0.305	0.004	0.210
0.9	0.95	100	0.974	0.087	0.734	0.971	0.081	0.739	0.901	0.060	0.748
0.9	0.95	250	1.000	0.338	1.000	1.000	0.311	1.000	1.000	0.281	1.000
0.9	0.9	50	0.343	0.015	0.182	0.335	0.014	0.183	0.302	0.005	0.185
0.9	0.9	100	0.975	0.185	0.714	0.970	0.181	0.710	0.903	0.131	0.708
0.9	0.9	250	1.000	0.837	1.000	1.000	0.833	1.000	1.000	0.826	1.000

Table 10: Empirical size and power of the panel cointegration test using PCCE estimator with spatial dependence, imposing one common factor ($r = 1$) with $N = 20$. SAR and SMA specifications

SAR

		$\vartheta = 0.4$			$\vartheta = 0.8$		
ϕ_i	T	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$
1	50	0.136	0.059	0.137	0.277	0.058	0.239
1	100	0.148	0.051	0.153	0.288	0.054	0.259
1	250	0.134	0.052	0.146	0.268	0.051	0.254
0.99	50	0.139	0.060	0.139	0.277	0.059	0.242
0.99	100	0.175	0.054	0.157	0.311	0.054	0.268
0.99	250	0.281	0.068	0.207	0.361	0.063	0.311
0.95	50	0.261	0.072	0.196	0.360	0.073	0.288
0.95	100	0.585	0.095	0.380	0.537	0.095	0.425
0.95	250	0.994	0.300	0.965	0.896	0.307	0.870
0.90	50	0.567	0.106	0.352	0.509	0.107	0.389
0.90	100	0.962	0.229	0.848	0.822	0.229	0.742
0.90	250	1.000	0.876	1.000	0.995	0.871	1.000

SMA

		$\vartheta = 0.4$			$\vartheta = 0.8$		
ϕ_i	T	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$
1	50	0.092	0.070	0.086	0.113	0.067	0.103
1	100	0.085	0.057	0.088	0.104	0.062	0.108
1	250	0.080	0.048	0.093	0.101	0.047	0.113
0.99	50	0.102	0.073	0.089	0.116	0.070	0.105
0.99	100	0.113	0.062	0.098	0.131	0.065	0.121
0.99	250	0.237	0.063	0.149	0.254	0.060	0.170
0.95	50	0.217	0.088	0.137	0.237	0.084	0.155
0.95	100	0.586	0.119	0.320	0.579	0.115	0.333
0.95	250	0.999	0.355	0.991	0.999	0.346	0.984
0.90	50	0.574	0.130	0.306	0.567	0.129	0.318
0.90	100	0.991	0.287	0.914	0.984	0.277	0.891
0.90	250	1.000	0.924	1.000	1.000	0.913	1.000

Table 11: Empirical size and power of the panel cointegration test using PCCE estimator with spatial dependence, imposing one common factor ($r = 1$) with $N = 20$. SEC specifications

SEC1

		$\vartheta = 0.4$			$\vartheta = 0.8$		
ϕ_i	T	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$
1	50	0.059	0.063	0.047	0.058	0.062	0.052
1	100	0.053	0.056	0.047	0.053	0.056	0.047
1	250	0.042	0.055	0.050	0.042	0.051	0.050
0.99	50	0.061	0.066	0.049	0.063	0.062	0.053
0.99	100	0.072	0.057	0.058	0.071	0.061	0.055
0.99	250	0.173	0.063	0.092	0.172	0.066	0.095
0.95	50	0.180	0.088	0.087	0.178	0.089	0.085
0.95	100	0.628	0.125	0.289	0.626	0.119	0.292
0.95	250	1.000	0.421	0.999	1.000	0.415	0.999
0.90	50	0.597	0.152	0.271	0.598	0.152	0.271
0.90	100	0.999	0.351	0.958	0.999	0.344	0.957
0.90	250	1.000	0.964	1.000	1.000	0.964	1.000

SEC2

		$\vartheta = 0.4$			$\vartheta = 0.8$		
ϕ_i	T	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$
1	50	0.061	0.061	0.051	0.064	0.069	0.057
1	100	0.048	0.053	0.048	0.057	0.056	0.058
1	250	0.041	0.049	0.050	0.051	0.052	0.059
0.99	50	0.063	0.061	0.050	0.071	0.069	0.057
0.99	100	0.072	0.059	0.056	0.080	0.058	0.068
0.99	250	0.178	0.062	0.098	0.186	0.063	0.108
0.95	50	0.174	0.081	0.090	0.187	0.087	0.105
0.95	100	0.635	0.124	0.293	0.630	0.117	0.312
0.95	250	1.000	0.406	1.000	1.000	0.373	0.999
0.90	50	0.601	0.154	0.271	0.601	0.141	0.286
0.90	100	0.999	0.337	0.956	0.998	0.313	0.947
0.90	250	1.000	0.955	1.000	1.000	0.939	1.000

SEC3

		$\vartheta = 0.4$			$\vartheta = 0.8$		
ϕ_i	T	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$
1	50	0.064	0.068	0.061	0.072	0.063	0.065
1	100	0.067	0.055	0.067	0.074	0.059	0.081
1	250	0.057	0.050	0.065	0.060	0.052	0.073
0.99	50	0.071	0.072	0.063	0.072	0.063	0.067
0.99	100	0.087	0.058	0.077	0.096	0.060	0.095
0.99	250	0.197	0.061	0.117	0.202	0.064	0.123
0.95	50	0.183	0.085	0.114	0.185	0.080	0.120
0.95	100	0.621	0.118	0.325	0.615	0.111	0.333
0.95	250	1.000	0.371	0.998	1.000	0.357	0.995
0.90	50	0.597	0.137	0.292	0.582	0.133	0.298
0.90	100	0.998	0.298	0.938	0.998	0.293	0.929
0.90	250	1.000	0.934	1.000	1.000	0.935	1.000

Table 12: Empirical size and power of the panel cointegration test using PCCE estimator with spatial dependence, with estimated number of common factors using panel BIC information criterion ($r_{max} = 6$) with $N = 20$

		SAR				SMA			
		$\vartheta = 0.4$		$\vartheta = 0.8$		$\vartheta = 0.4$		$\vartheta = 0.8$	
ϕ_i	T	Z_τ	$CADFC_P$	Z_τ	$CADFC_P$	Z_τ	$CADFC_P$	Z_τ	$CADFC_P$
1	50	0.106	0.137	0.110	0.239	0.115	0.086	0.143	0.103
1	100	0.121	0.153	0.130	0.259	0.127	0.088	0.161	0.108
1	250	0.122	0.146	0.131	0.254	0.127	0.093	0.153	0.113
0.99	50	0.108	0.139	0.113	0.242	0.119	0.089	0.143	0.105
0.99	100	0.142	0.157	0.139	0.268	0.152	0.098	0.180	0.121
0.99	250	0.271	0.207	0.266	0.311	0.278	0.149	0.288	0.170
0.95	50	0.191	0.196	0.154	0.288	0.190	0.137	0.215	0.155
0.95	100	0.540	0.380	0.407	0.425	0.518	0.320	0.524	0.333
0.95	250	0.998	0.965	0.979	0.870	0.996	0.991	0.991	0.984
0.90	50	0.401	0.352	0.250	0.389	0.392	0.306	0.405	0.318
0.90	100	0.963	0.848	0.818	0.742	0.956	0.914	0.932	0.891
0.90	250	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

		SEC1				SEC2			
		$\vartheta = 0.4$		$\vartheta = 0.8$		$\vartheta = 0.4$		$\vartheta = 0.8$	
ϕ_i	T	Z_τ	$CADFC_P$	Z_τ	$CADFC_P$	Z_τ	$CADFC_P$	Z_τ	$CADFC_P$
1	50	0.055	0.047	0.055	0.052	0.058	0.051	0.065	0.057
1	100	0.062	0.047	0.062	0.047	0.063	0.048	0.073	0.058
1	250	0.053	0.050	0.057	0.050	0.052	0.050	0.059	0.059
0.99	50	0.062	0.049	0.061	0.053	0.063	0.050	0.066	0.057
0.99	100	0.089	0.058	0.086	0.055	0.088	0.056	0.101	0.068
0.99	250	0.218	0.092	0.221	0.095	0.218	0.098	0.229	0.108
0.95	50	0.201	0.087	0.196	0.085	0.197	0.090	0.201	0.105
0.95	100	0.724	0.289	0.726	0.292	0.730	0.293	0.722	0.312
0.95	250	1.000	0.999	1.000	0.999	1.000	1.000	1.000	0.999
0.90	50	0.666	0.271	0.667	0.271	0.670	0.271	0.667	0.286
0.90	100	0.999	0.958	1.000	0.957	1.000	0.956	0.999	0.947
0.90	250	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

		SEC3			
		$\vartheta = 0.4$		$\vartheta = 0.8$	
ϕ_i	T	Z_τ	$CADFC_P$	Z_τ	$CADFC_P$
1	50	0.065	0.061	0.078	0.065
1	100	0.081	0.067	0.088	0.081
1	250	0.066	0.065	0.071	0.073
0.99	50	0.071	0.063	0.077	0.067
0.99	100	0.109	0.077	0.117	0.095
0.99	250	0.238	0.117	0.247	0.123
0.95	50	0.198	0.114	0.190	0.120
0.95	100	0.711	0.325	0.692	0.333
0.95	250	1.000	0.998	1.000	0.995
0.90	50	0.651	0.292	0.538	0.298
0.90	100	0.999	0.938	0.996	0.929
0.90	250	1.000	1.000	1.000	1.000

Table 13: Empirical size and power of the panel cointegration test using PCCE estimator with spatial dependence, imposing one common factor ($r = 1$) with $N = 10$. SAR and SMA specifications

SAR

		$\vartheta = 0.4$			$\vartheta = 0.8$		
ϕ_i	T	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$
1	50	0.135	0.060	0.129	0.127	0.065	0.125
1	100	0.143	0.050	0.134	0.135	0.058	0.132
1	250	0.131	0.055	0.121	0.125	0.057	0.120
0.99	50	0.138	0.061	0.128	0.128	0.068	0.127
0.99	100	0.169	0.052	0.141	0.148	0.062	0.139
0.99	250	0.240	0.060	0.158	0.232	0.064	0.162
0.95	50	0.215	0.069	0.163	0.200	0.079	0.166
0.95	100	0.453	0.086	0.289	0.440	0.098	0.281
0.95	250	0.961	0.296	0.835	0.955	0.308	0.842
0.90	50	0.415	0.101	0.261	0.400	0.113	0.266
0.90	100	0.869	0.213	0.658	0.870	0.236	0.655
0.90	250	1.000	0.851	1.000	1.000	0.880	1.000

SMA

		$\vartheta = 0.4$			$\vartheta = 0.8$		
ϕ_i	T	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$
1	50	0.105	0.060	0.093	0.128	0.061	0.117
1	100	0.108	0.060	0.098	0.132	0.061	0.123
1	250	0.098	0.054	0.092	0.114	0.054	0.119
0.99	50	0.106	0.061	0.093	0.136	0.061	0.119
0.99	100	0.129	0.062	0.102	0.152	0.061	0.126
0.99	250	0.201	0.065	0.131	0.215	0.067	0.157
0.95	50	0.184	0.077	0.131	0.203	0.072	0.151
0.95	100	0.454	0.102	0.246	0.446	0.102	0.263
0.95	250	0.979	0.318	0.874	0.969	0.310	0.850
0.90	50	0.403	0.117	0.234	0.404	0.115	0.252
0.90	100	0.903	0.243	0.687	0.880	0.238	0.672
0.90	250	1.000	0.881	1.000	1.000	0.865	1.000

Table 14: Empirical size and power of the panel cointegration test using PCCE estimator with spatial dependence, imposing one common factor ($r = 1$) with $N = 10$. SEC specifications

SEC1

		$\vartheta = 0.4$			$\vartheta = 0.8$		
ϕ_i	T	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$
1	50	0.065	0.055	0.053	0.064	0.060	0.052
1	100	0.063	0.056	0.048	0.067	0.058	0.048
1	250	0.056	0.056	0.045	0.060	0.054	0.044
0.99	50	0.069	0.056	0.051	0.070	0.061	0.054
0.99	100	0.079	0.062	0.053	0.082	0.058	0.053
0.99	250	0.155	0.068	0.073	0.156	0.063	0.070
0.95	50	0.135	0.082	0.076	0.134	0.077	0.078
0.95	100	0.432	0.112	0.186	0.426	0.109	0.189
0.95	250	0.994	0.366	0.939	0.995	0.357	0.938
0.90	50	0.375	0.127	0.173	0.371	0.125	0.178
0.90	100	0.952	0.292	0.735	0.951	0.288	0.731
0.90	250	1.000	0.941	1.000	1.000	0.937	1.000

SEC2

		$\vartheta = 0.4$			$\vartheta = 0.8$		
ϕ_i	T	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$
1	50	0.069	0.061	0.055	0.075	0.060	0.061
1	100	0.062	0.054	0.050	0.073	0.056	0.059
1	250	0.063	0.051	0.044	0.070	0.052	0.054
0.99	50	0.075	0.061	0.057	0.076	0.058	0.063
0.99	100	0.083	0.056	0.056	0.087	0.064	0.066
0.99	250	0.158	0.062	0.075	0.159	0.062	0.083
0.95	50	0.135	0.077	0.084	0.136	0.079	0.092
0.95	100	0.429	0.103	0.192	0.429	0.109	0.203
0.95	250	0.995	0.349	0.937	0.994	0.323	0.927
0.90	50	0.375	0.131	0.183	0.374	0.124	0.191
0.90	100	0.952	0.286	0.730	0.948	0.259	0.727
0.90	250	1.000	0.927	1.000	1.000	0.904	1.000

SEC3

		$\vartheta = 0.4$			$\vartheta = 0.8$		
ϕ_i	T	Z_τ	$t_{\tilde{F}}$	$CADFC_P$	Z_τ	$t_{\tilde{F}}$	$CADFC_P$
1	50	0.075	0.058	0.066	0.086	0.061	0.076
1	100	0.082	0.054	0.070	0.093	0.053	0.077
1	250	0.078	0.047	0.061	0.089	0.048	0.074
0.99	50	0.080	0.061	0.067	0.089	0.061	0.079
0.99	100	0.098	0.057	0.075	0.110	0.057	0.084
0.99	250	0.175	0.060	0.096	0.185	0.058	0.105
0.95	50	0.140	0.074	0.098	0.150	0.077	0.107
0.95	100	0.430	0.103	0.218	0.436	0.103	0.228
0.95	250	0.992	0.316	0.916	0.989	0.318	0.906
0.90	50	0.379	0.122	0.203	0.381	0.118	0.213
0.90	100	0.940	0.255	0.719	0.929	0.251	0.711
0.90	250	1.000	0.898	1.000	1.000	0.896	1.000

Table 15: Empirical size and power of the panel cointegration test using PCCE estimator with spatial dependence, with estimated number of common factors using panel BIC information criterion ($r_{max} = 6$) with $N = 10$

		SAR				SMA			
		$\vartheta = 0.4$		$\vartheta = 0.8$		$\vartheta = 0.4$		$\vartheta = 0.8$	
ϕ_i	T	Z_τ	$CADFC_P$	Z_τ	$CADFC_P$	Z_τ	$CADFC_P$	Z_τ	$CADFC_P$
1	50	0.128	0.129	0.135	0.125	0.156	0.093	0.135	0.117
1	100	0.139	0.134	0.148	0.132	0.166	0.098	0.137	0.123
1	250	0.130	0.121	0.138	0.120	0.170	0.092	0.146	0.119
0.99	50	0.134	0.128	0.140	0.127	0.155	0.093	0.136	0.119
0.99	100	0.151	0.141	0.160	0.139	0.182	0.102	0.148	0.126
0.99	250	0.219	0.158	0.224	0.162	0.260	0.131	0.229	0.157
0.95	50	0.168	0.163	0.184	0.166	0.199	0.131	0.175	0.151
0.95	100	0.364	0.289	0.356	0.281	0.401	0.246	0.362	0.263
0.95	250	0.926	0.835	0.911	0.842	0.901	0.874	0.911	0.850
0.90	50	0.269	0.261	0.275	0.266	0.301	0.234	0.272	0.252
0.90	100	0.731	0.658	0.700	0.655	0.739	0.687	0.712	0.672
0.90	250	1.000	1.000	0.998	1.000	0.997	1.000	0.999	1.000

		SEC1				SEC2			
		$\vartheta = 0.4$		$\vartheta = 0.8$		$\vartheta = 0.4$		$\vartheta = 0.8$	
ϕ_i	T	Z_τ	$CADFC_P$	Z_τ	$CADFC_P$	Z_τ	$CADFC_P$	Z_τ	$CADFC_P$
1	50	0.097	0.053	0.101	0.052	0.116	0.055	0.135	0.061
1	100	0.054	0.048	0.059	0.048	0.066	0.050	0.138	0.059
1	250	0.049	0.045	0.050	0.044	0.056	0.044	0.136	0.054
0.99	50	0.101	0.051	0.103	0.054	0.118	0.057	0.144	0.063
0.99	100	0.075	0.053	0.076	0.053	0.083	0.056	0.150	0.066
0.99	250	0.145	0.073	0.148	0.070	0.158	0.075	0.221	0.083
0.95	50	0.157	0.076	0.157	0.078	0.163	0.084	0.176	0.092
0.95	100	0.476	0.186	0.472	0.189	0.466	0.192	0.383	0.203
0.95	250	0.996	0.939	0.996	0.938	0.996	0.937	0.958	0.927
0.90	50	0.333	0.173	0.327	0.178	0.311	0.183	0.294	0.191
0.90	100	0.969	0.735	0.970	0.731	0.961	0.730	0.803	0.727
0.90	250	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

		SEC3			
		$\vartheta = 0.4$		$\vartheta = 0.8$	
ϕ_i	T	Z_τ	$CADFC_P$	Z_τ	$CADFC_P$
1	50	0.134	0.066	0.126	0.076
1	100	0.139	0.070	0.141	0.077
1	250	0.136	0.061	0.133	0.074
0.99	50	0.136	0.067	0.132	0.079
0.99	100	0.149	0.075	0.158	0.084
0.99	250	0.230	0.096	0.222	0.105
0.95	50	0.184	0.098	0.194	0.107
0.95	100	0.388	0.218	0.392	0.228
0.95	250	0.957	0.916	0.955	0.906
0.90	50	0.312	0.203	0.324	0.213
0.90	100	0.820	0.719	0.809	0.711
0.90	250	1.000	1.000	1.000	1.000

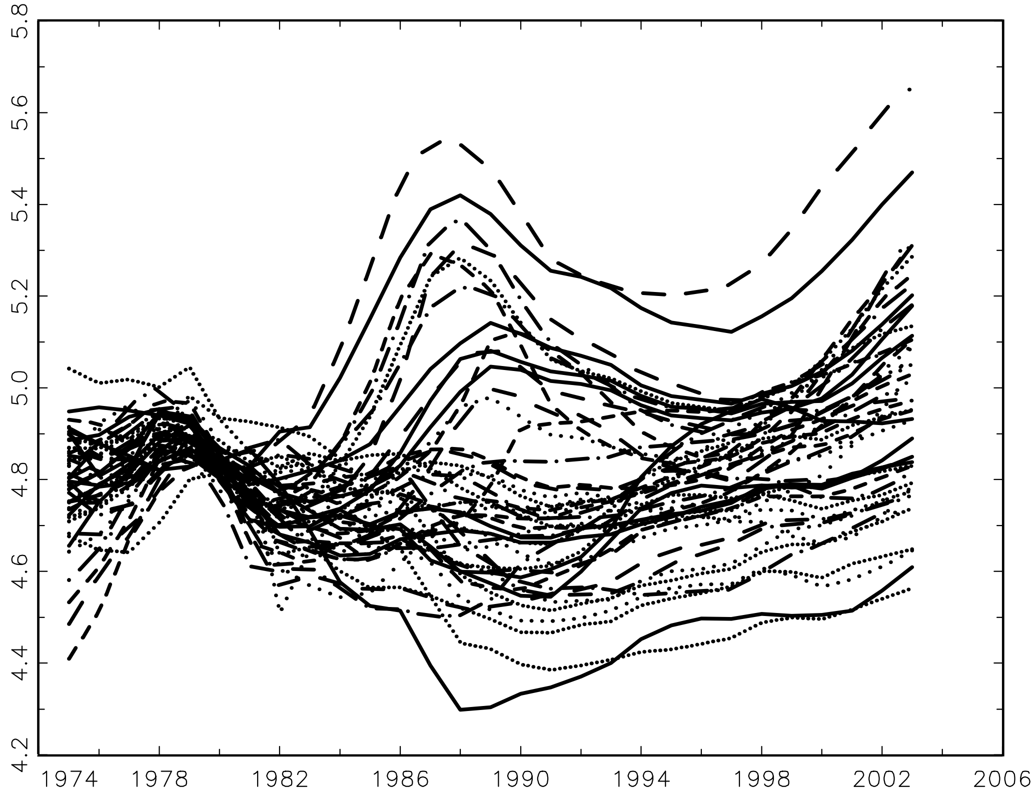

Figure 1: US State real house price

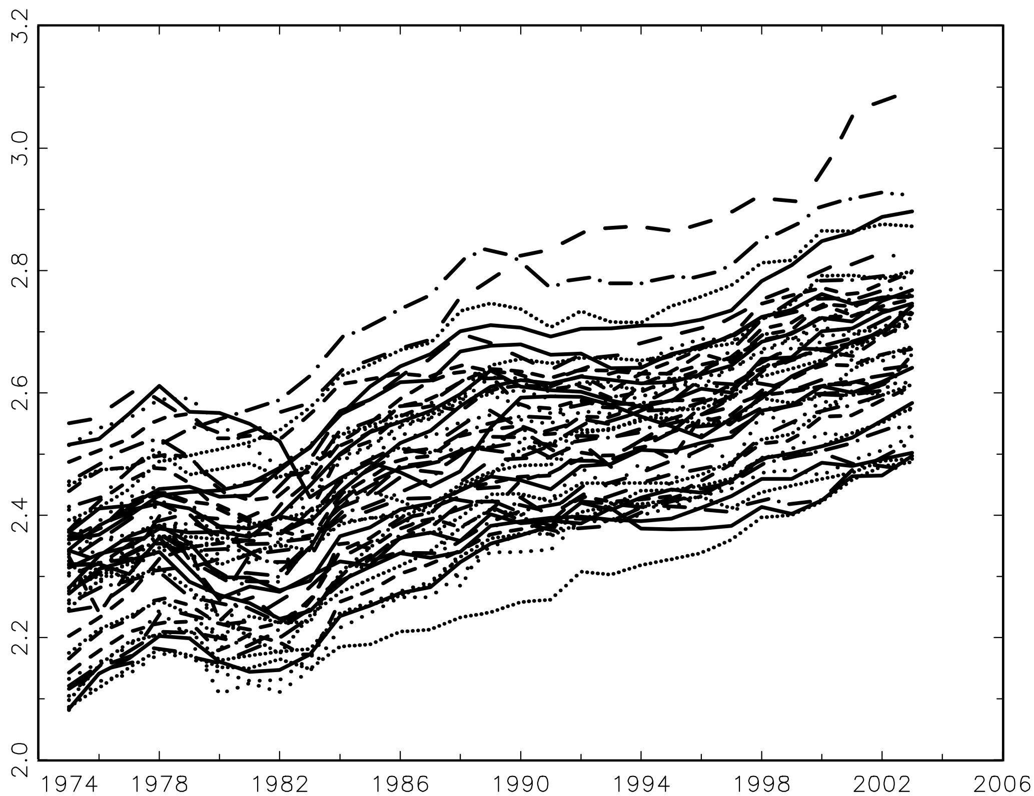

Figure 2: US State real per capita disposable income

Table 16: US Housing price and disposable income relationship. Individual cointegration statistics

	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$
Alabama	-0.827	-1.259	-3.575	-3.585	-1.858
Arkansas	-1.121	-2.557	-3.668	-2.563	-1.772
Arizona	-1.972	-2.157	-2.708	-3.491	-3.633
California	-1.418	-3.526	-3.797	-3.563	-3.555
Colorado	-1.110	-1.963	-1.867	-2.370	-1.633
Connecticut	-0.953	-2.196	-2.905	-5.117	-3.560
District of Columbia	-2.273	-2.606	-3.370*	-1.964	-1.901
Delaware	-2.346	-2.212	-3.025	-3.151*	-2.442
Florida	-2.016	-1.807	-2.664	-1.480	-1.314
Georgia	-3.758**	-3.991	-3.127*	-1.752	-4.663
Iowa	-1.409	-2.559	-2.956	-3.052	-4.393
Idaho	-1.588	-2.302	-3.826	-8.305	-4.972
Illinois	-1.669	-1.775	-1.531	-2.875	-1.736
Indiana	-1.626	-1.980	-1.839	-2.106	-2.472
Kansas	-1.343	-1.662	-1.989	-2.134	-1.758
Kentucky	-3.414*	-1.610	-1.151	-2.955	-1.420
Louisiana	-1.817	-2.932	-4.087**	-6.247**	-3.523**
Massachusetts	-1.210	-2.031	-3.119*	-4.465**	-5.050**
Maryland	-1.348	-3.061	-3.640**	-4.336**	-2.906
Maine	-0.935	-3.192*	-3.108*	-3.334*	-3.553**
Michigan	-1.521	-3.145*	-2.767	-2.450	-2.294
Minnesota	-0.379	-0.416	-2.033	-1.223	-1.121
Missouri	-1.575	-2.957	-3.456*	-1.456	-1.416
Mississippi	-3.266	-2.602	-2.341	-3.788**	-2.325
Montana	-1.578	-1.975	-2.198	-1.949	-1.316
North Carolina	-2.145	-1.814	-1.404	-3.812**	-4.361**
North Dakota	-2.636	-4.622**	-3.935**	-3.433*	-2.408
Nebraska	-1.092	-3.506**	-3.782**	-4.605**	-4.307**
New Hampshire	-0.911	-3.236*	-3.040	-3.716**	-3.744**
New Jersey	-1.798	-2.895	-2.622	-2.651	-2.565
New Mexico	-1.368	-2.165	-3.879**	-3.624**	-3.020
Nevada	-2.136	-1.832	-1.153	-2.097	-1.482
New York	-4.822**	-4.808**	-1.824	-0.230	-1.793
Ohio	-1.625	-1.792	-2.228	-2.707	-3.752**
Oklahoma	-2.114	-4.011**	-4.511**	-2.935	-5.321**
Oregon	-1.243	-1.758	-2.242	-3.802**	-3.739**
Pennsylvania	-1.769	-3.008	-2.425	-3.324*	-2.340
Rhode Island	-1.344	-3.139*	-3.317*	-5.344**	-5.342**
South Carolina	-3.112*	-5.697**	-1.794	-1.547	-1.235
South Dakota	-3.854**	-2.213	-2.229	-2.110	-3.181
Tennessee	-1.230	-1.918	-2.588	-2.714	-1.616
Texas	-2.348	-3.654**	-4.175**	-3.320*	-4.181**
Utah	-1.156	-2.919	-2.828	-3.335*	-2.552
Virginia	-1.730	-1.875	-2.767	-4.731**	-1.550
Vermont	-2.189	-2.460	-3.557	-4.176**	-2.844
Washington	-2.011	-3.409*	-2.503	-3.179*	-3.768**
Wisconsin	-1.939	-0.792	-1.184	-2.969	-2.634
West Virginia	-1.776	-1.442	-1.729	-2.155	-2.913
Wyoming	-1.927	-2.112	-3.677**	-4.694**	-3.326*

Notes: Columns 2 to 6 report the results for different lags. ** and * denote rejection of the null hypothesis of no cointegration at the 5 and 10% levels of significance, respectively.

Table 17: Production function. Individual CCE cointegration statistics

	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$
AUS	-0.881	1.071	0.149	-0.069	0.483
AUT	-3.189*	-4.439**	-6.153**	-4.667**	-2.858
BEL	1.351	1.942	0.406	0.703	0.111
CAN	-2.005	-2.694	-1.928	-1.136	-1.422
CHE	-1.236	-1.126	-1.220	-1.897	-1.977
DNK	-2.804	-2.430	-2.210	-2.124	-3.386**
ESP	-5.354**	-3.206*	-3.755**	-3.017*	-3.233*
FIN	-0.808	-2.137	-1.577	-1.482	-1.535
FRA	-2.401	-2.110	-1.451	-0.655	-0.927
GBR	-3.490**	-2.405	-2.674	-2.709	-3.031*
GRC	-2.387	-3.177*	-2.190	-2.226	-2.056
IRL	-0.366	-1.251	-1.202	-1.558	-1.955
ITA	-1.431	-1.139	-0.583	-0.418	0.304
LUX	-1.502	-1.904	-1.819	-1.761	-2.283
NLD	-2.439	-2.521	-2.317	-3.136*	-2.671
NOR	-1.613	-1.824	-1.510	-1.231	-1.087
PRT	-0.747	-0.593	-0.035	0.247	0.390
SWE	-0.225	-1.507	-0.945	-0.668	-1.298
USA	-0.367	-0.950	-0.673	-0.283	-0.451

Table 18: Production function. Banerjee and Carrion-i-Silvestre (BC) panel cointegration test

	$r=0$	$r=1$	$r=2$	$r=3$	$r=4$	$r=5$	$r=6$
BC	-3.56	-3.16	-3.09	-2.84	-2.75	-2.46	-2.53
$\hat{r}_1$ (non-parametric MQ test)	-	1	1	3	4	5	6
$\hat{r}_1$ (parametric MQ test)	-	1	2	3	4	5	6