Public and Private Firms' Locations under Endogenous Timing of Choices

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Abstract

This paper studies the choices of locations in a mixed duopoly when production costs are endogenously determined. The public firm maximizes a weighted sum of social surplus and its profits. We find that the locations of the two firms are decided simultaneously when the weight of the public firm's profits in its objective function is high enough. When this is not the case we find that one firm (not only the public firm but sometimes also the private one) behaves as a leader in the choice of location. Besides, in equilibrium, the production cost of the public firm never is higher than that of the private firm.

JEL Classification: J51, L13, L33.

Key words: Endogenous costs, firms' locations, mixed duopoly.

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1 Introduction

In a mixed oligopoly publicly-owned firms compete against private firms. A mixed oligopoly is a market structure that is very common in many sectors and many countries. For example, it can be widely found in post services, passengers' transportation, cares for elderly people, health care and many other services. In these markets, as in many markets with only private firms, the locations of the firms play a central role to understand their performance. Moreover, the production costs of the firms may be, at least at some point, far from their control because of the existence of monopolist providers of some input or due to the presence of strong unions within the firms. Therefore, it is an relevant issue to analyze the location of the firms in a mixed duopoly when production costs are endogenously determined. However, the literature on public firms has dedicated little attention to investigate whether firms want to decide their locations sequential or simultaneopusly. Thus, in this paper we analyze the endogenous order of moves in a mixed duopoly where firms choose whether to set locations sequentially or simultaneously. This issue is important since a sequential order of moves may give rise to significantly different results than a simultaneous one.

The endogenous determination of the order of decisions has been studied before in the literature on mixed markets mainly considering short run decisions: whether firms take quantity and price decisions sequential or simultaneously. For example, Pal (1998) shows that when the product is homogeneous and firms decide quantities, the game will be played sequentially.[1] However, Cremer et al. (1991) point out that homogeneous goods markets are rare, especially mixed homogeneous goods markets are quite exceptional. They analyze the simultaneous choice of locations with horizontal product differentiation and study how the presence of private firms and one or several public firms in the market affects social welfare. Following this path, Matsumura and Matsushima (2003) analyze the endogenous choice of locations by the firms in a mixed duopoly and a Hotelling-type spatial model. In both papers, Cremer et al. (1991) and Matsumura and Matsushima (2003), production costs are exogenous.[2]

Regarding the issue of heterogeneous goods markets, we study whether public and private firms want to be leaders or followers in the choice of location

[1] Matsumura (2003) and Lu (2006) show that production decisions are taken sequentially assuming that the public firm competes with foreign private firms. Bárcena-Ruiz and Garzón (2010) extend the above analysis considering a firm jointly owned by the public sector and private domestic shareholders (a semipublic firm) rather than a private firm. They obtain that there is an equilibrium in which the owners of the firms take production decisions simultaneously. Other extension made by Bárcena-Ruiz (2007) assumes that firms have to decide whether to set prices sequential or simultaneously and shows that firms choose prices simultaneously.

[2] There are other papers that analyze a mixed duopoly assuming spatial competition, although they do not consider the timing of location decisions. For example, Matsushima and Matsumura (2003, 2006) investigate a mixed market considering a circular city model with quantity-setting competition and analyze the location decisions of the firms when these decisions are taken simultaneously. Inoue et al. (2009) analyze an interregional mixed duopoly employing a spatial model with price competition.

when production costs will be set later by the input supplier. In spatial competition models (see, e.g., d'Aspremont et al., 1979) firms are usually restricted to locate within city limits. In our paper we adopt the approach of Lambertini (1994; 1997) and Tabuchi and Thisse (1995). Then, we allow locations outside the city boundaries. In a private duopoly setting it is well known that if firms can locate outside the city limits they will do so. Most of the papers on firms' location in spatial competition models consider that production costs are exogenously given. One of the few exceptions, assuming privately-owned firms, is a paper by Brekke and Straume (2004), who analyze bilateral monopolies and location choice in a duopoly model in which firms can choose their locations simultaneously or sequentially, but bargain over their input prices simultaneously. Bárcena-Ruiz and Casado-Izaga (2008) extend the above paper by considering that, when locations are chosen simultaneously, not only wages but also the timing of wage negotiations is endogenously determined. Besides, Matsumura and Matsushima (2004) investigate a mixed duopoly, where a welfare-maximizing public firm competes with a profit-maximizing private firm. They consider endogenous cost differentials between a private firm and a public one in a standard Hotelling spatial framework. They show that the private firm's cost becomes lower than the public firm's because the private firm engages in excessive strategic cost-reducing activities.

In this paper we extend the analysis of Brekke and Straume (2004) by considering a mixed duopoly rather than a private duopoly. Thus, our paper is related with the model used by Matsumura and Matsushima (2003) who analyze the sequential choice of location assuming a Hotelling-type spatial framework. They consider a mixed duopoly where the private firm maximizes its profits, the public firm maximizes social surplus, and production costs are exogenously given. They find that both the private and the public firm locate inside the linear city under both sequential and simultaneous locations. Moreover, they obtain two equilibria in the location game: in one of them the public firm is the leader and the private firm the follower; in the other, both firms take location decisions simultaneously.[3] The above cited paper analyzes the endogenous order of moves by considering that public firms maximize social surplus. However, with regard to the objective function of public firms, White (2002, p. 489) argues that "while the standard, equally-weighted welfare function may be desirable for normative reasons, based on utilitarianism or fairness doctrines (as in Harsanyi, 1995), it may be restrictive for purposes of predicting the behavior of actual public firms and the resulting market outcomes". White (2002) considers a weighted welfare function to analyze how the public firm responds to different objective functions, and how this impacts on the industry as a whole. In this paper we follow this approach considering a weighted welfare function: the public firm maximizes the weighted sum of social surplus and its profit. Besides, as in Brekke and Straume (2004) and Bárcena-Ruiz and Casado-Izaga (2008) we assume that production costs are endogenously determined. We also use the

[3]When price regulation is imposed the public firm is the follower and the private firm the leader.

observable delay game of Hamilton and Slutsky (1990) in order to analyze the choice of locations by the public and the private firm.

Our paper presents two main differences regarding the paper by Matsumura and Matsushima (2003). First, we assume that the public firm maximizes the weighted sum of social surplus and its own profit while they consider that this firm maximizes social surplus. Second, we assume that production costs are not exogenously given. Specifically, we assume that the workers of the firms have the power to set the wage (see, Booth, 1995). The first assumption, like in De Fraja (1993), is necessary to obtain a solution for the wage negotiation problem between the public firm and its workers. If the public firm maximizes social surplus it is not possible to obtain an expression for the wage paid by the public firm. Besides, it can be shown that if the public firm maximizes the weighted sum of social surplus and its profit when production costs are exogenous, in equilibrium location decisions are taken simultaneously. Therefore, the sequential equilibrium obtained by Matsumura and Matsushima (2003) only arises if the public firm maximizes social surplus.

We find that the choice of location may be simultaneous or sequential depending on the weight of its profits in the objective function of the public firm. If the weight is great enough, the profit of the public firm has a greater effect than social surplus and thus location decisions are taken simultaneously.[4] If the weight has an intermediate value, the public firm is the leader and the private firm the follower. Finally, if the weight is low enough there are two sequential equilibria: the public firm is the leader in one of them and the follower in the other one.

Considering the timing of the choice of location as endogenous we can get the wages paid by the public and the private firms. We obtain that, in equilibrium, the wages paid by the public firm never exceed the wages paid by private firm. This result is in contrast with other previous results obtained by Willner (1997), De Fraja (1993) or Bárcena-Ruiz and Garzón (2009). They obtain, considering Cournot competition, that the public firm pays greater wages than the private firm due to the different objective functions that the firms have.[5] Therefore, the result obtained under Cournot competition does not hold in a spatial competition model. In fact we obtain the surprising result that when location are taken simultaneously the two firms locate simetrically and pay the same wage in spite of the two firms have different objective functions.

The rest of the paper is organized as follows: first we study the model, second the main results and, finally we draw conclusions.

2 The model

[4] Thus, we obtain a similar result than when firms are privately-owned. Brekke and Straume (2004) analyze a private duopoly in which firms take location decisions, sequential or simultaneously, and wages are endogenously determined. Although they do not analyze the timing of location decisions it is easy to see that, in their model, when wages are set by unions firms take location decisions simultaneously.

[5] It has to be noted that De Fraja (1993) shows that the wage paid by the public firm could be lower than that paid by the private firm. However, the model cannot be solved explicitly but some simulations show this result.

Consumers are distributed uniformly and with unitary density along the interval $[0, 1]$. They transport their purchase home at a cost td^2, where t is a positive constant and d is the distance travelled from the firm's location to consumer's home. Each consumer buys only one unit of the good at the lowest delivered price, considered as the mill price plus transportation cost. Each consumer derives a surplus from consumption, gross of price and transportation costs, equal to s. We assume that s is so large that every consumer buys one unit of product.

There are two firms indexed by i ($i = 0, 1$) competing in the market. Firm 0 is public and firm 1 is private; therefore, we are considering a mixed duopoly. Both firms are free to locate wherever they like. Let $x_i \in [-\infty, +\infty]$ denote the location of firm i. If x_i is negative the firm is located to the left of the $[0, 1]$ city. If $x_i > 1$, the firm is located to the right of all consumers. For the sake of simplicity, we assume that the public firm is located to the left or on the same point as the private firm: $x_0 \leq x_1$. Once both firms choose their locations they cannot be changed in the future.

Labor is the only factor required to produce. Firm i hires L_i workers and pays a uniform wage rate w_i. All workers are unionized and there is a separate, independent union in each firm. The utility of each union is its wage bill: $U_i(w_i,L_i)=L_i w_i; i = 0, 1$. Technology exhibits constant returns to scale such that aggregate output of firm i is $q_i=L_i$. As Ishida and Matsushima (2009) do we consider the monopoly-union model where the labor union has the power to impose its preferred wage while employment is set unilaterally by the firm (see Booth, 1995). The private firm seeks to maximize profits, the public firm maximizes a weighted sum of social surplus and its profits, while unions seek to maximize their own wage bill.

The timing of the game is as follows. In the first stage, both firms simultaneously decide whether their choice of location will be made in the first period or in the second period of stage two. In stage two there are two periods. In period one (t=1), the firm that decided to choose its location in the first period of this stage takes this decision. In period two (t=2) the firm that did not take the location decision in period one takes now this decision. In stage three, both independent unions simultaneously choose the wages imposed to the public firm and to the private firm. Finally, in stage four, firms take price and employment decisions simultaneously. It has to be noted that although location decisions can be taken in two different periods there is only one production period. In order to get subgame perfect Nash equilibria we solve the game by backward induction from the last stage of the game.

3 Results

Let p_i denote the price set by firm i ($i = 0, 1$). The location of the consumer who is indifferent between the two firms ($\overline{x}$) is such that:

$$p_0 + t(\overline{x} - x_0)^2 = p_1 + t(\overline{x} - x_1)^2. \quad (1)$$

From (1) we obtain:

$$\overline{x} = \frac{t(x_0^2 - x_1^2) + p_0 - p_1}{2t(x_0 - x_1)}. \tag{2}$$

Thus, the respective demands of firms 0 and 1, when both firms do not locate at the same point ($x_0 \neq x_1$) are given by q_0 and q_1:

$$q_0 = \begin{cases} \overline{x} & \text{if } 0 \leq \overline{x} \leq 1 \\ 1 & \text{if } \overline{x} > 1 \\ 0 & \text{if } \overline{x} < 0 \end{cases}, \quad q_1 = \begin{cases} 1 - \overline{x} & \text{if } 0 \leq 1 - \overline{x} \leq 1 \\ 1 & \text{if } 1 - \overline{x} > 1 \\ 0 & \text{if } 1 - \overline{x} < 0 \end{cases} \tag{3}$$

We solve first the forth stage of the game in order to get equilibrium prices.

3.1 Price competition

In this stage of the game both firms simultaneously choose prices and employment. Once both firms decide their prices, their output and employment levels are determined by expressions (2) and (3): $L_0 = q_0$ and $L_1 = q_1$. The objective function of the private firm, firm 1, is its profit function:

$$\pi_1(p_0, p_1) = (p_1 - w_1)q_1. \tag{4}$$

The objective function of the public firm (W) is the weighted sum of its profits, π_0, and social surplus and then we can write the objective function as:[6]

$$W(p_0, p_1) = \alpha\pi_0(p_0, p_1) + s - \int_0^{\overline{x}} t(x - x_0)^2 dx - \int_{\overline{x}}^1 t(x - x_1)^2 dx. \tag{5}$$

We define (5) as weighted welfare. As usual (see, for example, Lommerud et al., 2003; Mezzetti and Dinopoulos, 1991), social surplus comprises consumer surplus, CS, producer surplus, PS, and the rents obtained by the workers, U. Specifically, we assume the following social surplus function: $SS = CS + PS + U$, where $CS = s - p_0q_0 - p_1q_1 - \int_0^{\overline{x}} t(x - x_0)^2 dx - \int_{\overline{x}}^1 t(x - x_1)^2 dx$, $PS = \pi_0 + \pi_1 = (p_0 - w_0)q_0 + (p_1 - w_1)q_1$ and $U = w_0q_0 + w_1q_1$. Therefore: $SS = s - \int_0^{\overline{x}} t(x - x_0)^2 dx - \int_{\overline{x}}^1 t(x - x_1)^2 dx$.[7] Union rents are included as that

[6] See White (2002) for a justification of this objective function. White (2002) considers a weighted welfare function to analyze how the public firm responds to different objective functions, and how this impacts on the industry as a whole. Ogawa and Kato (2006) consider a similar objective for the public firm to analyze a mixed duopoly with homogeneous products and price competition. See also Bös (1991, pp. 135-138) for an interpretation of this function as the objective function of a partially privatized firm. De Fraja (1993) also assumes a weighted welfare function to be able to find a solution for the wage negotiation problem between the public firm and its workers.

[7] It has to be noted that the incomes of firms are a transfer from consumer to producers and that the wage bill is a transfer from firms to workers.

part of the producer surplus which is absorbed by the unions (see, for example, Brander and Spencer, 1988; Bughin and Vanini, 1995).[8]

From the first order condition for each firm we get the equilibrium prices when both firms sell the good:[9]

$$p_0 = \frac{1}{1+3\alpha}(2\alpha w_0 + (1+\alpha)w_1 + 2t(1+\alpha)(x_1 - x_0) + t(1-\alpha)(x_0^2 - x_1^2)), \quad (6)$$

$$p_1 = \frac{1}{1+3\alpha}(\alpha w_0 + (1+2\alpha)w_1 + 2t(1+2\alpha)(x_1 - x_0) + t(1+\alpha)(x_0^2 - x_1^2)). \quad (7)$$

Therefore, the market price of each firm, public or private, increases both with its own wage and with the wage paid by the rival. We can obtain now the demands of firms 0 and 1 and their employment levels:

$$q_0 = L_0 = \frac{(w_0 - w_1)\alpha + 2t\alpha(x_0 - x_1) + t(1+\alpha)(x_0^2 - x_1^2)}{2t(1+3\alpha)(x_0 - x_1)}, \quad (8)$$

$$q_1 = L_1 = \frac{(w_1 - w_0)\alpha + 2t(1+2\alpha)(x_0 - x_1) + t(1+\alpha)(x_1^2 - x_0^2)}{2t(1+3\alpha)(x_0 - x_1)}. \quad (9)$$

As usual, the output and employment levels of each firm decrease with its wage and increase with the wage paid by its rival. We now solve the third stage of the game, so we determine the wages paid by the public and the private firm.

3.2 Wages decided by both unions

We assume the monopoly union model and, thus, in this stage of the game unions set the wage that maximize their wage bill.[10] Wages are simultaneously decided. From the first order condition for each union it is straightforward to show that equilibrium wages in the second stage of the game are:[11]

$$w_0 = \frac{(x_1 - x_0)(2 + 8\alpha + (1+\alpha)(x_0 + x_1))t}{3\alpha}, \quad (10)$$

[8] An alternative justification of the objective function of the public firm is the following. We could assume that firm 0 is jointly owned by the public sector and private domestic shareholders. The governent owns β% of the shares of firm 0, $0 \leq \beta \leq 1$. Therefore, the objective of this function is to maximize the weighted average of the payoff of the government and its own profit (see, for example, Matsumura, 1998; Barcena-Ruiz and Garzón, 2003): $\beta(SS) + (1-\beta)\pi_0$. To maximize the above expression is equal to maximize $\frac{\beta}{\beta}SS + \frac{1-\beta}{\beta}\pi_0 = SS + \alpha\pi_0$, where $\alpha = \frac{1-\beta}{\beta}$, $0 \leq \alpha < \infty$ (when β=1 then α=0, when $\beta \to 0$ then $\alpha \to \infty$). As a result: $\alpha \in [0, \infty)$.

[9] The second order conditions in the problems we analyze are always satisfied.

[10] Other recent paper in which the monopoly union model is used to analyze bargaining is due to Ishida and Matsushima (2009). They work with a unionized mixed duopoly model to study whether the employees in the public sector should be allowed to bargain collectively.

[11] It can be shown that, as usual, wages are strategic complements. Thus, the greater the wage of the rival the greater the own wage is.

$$w_1 = \frac{(x_1 - x_0)(4 + 10\alpha - (1+\alpha)(x_0 + x_1))t}{3\alpha}. \tag{11}$$

We can now solve equations (8) and (9):

$$q_0(x_0, x_1) = \frac{2 + 8\alpha + (1+\alpha)(x_0 + x_1)}{6 + 18\alpha}, \tag{12}$$

$$q_1(x_0, x_1) = \frac{4 + 10\alpha - (1+\alpha)(x_0 + x_1)}{6 + 18\alpha}. \tag{13}$$

3.3. Location choice

We solve now the stage in which locations are chosen. In this stage of the game we have three different possibilities: first, locations are chosen simultaneously (it does not matter if it is in the first or during the second period), second, the public firm is the leader and, finally, the case in which the private firm acts as the leader. Let us solve first the simultaneous game.

3.3.1 Simultaneous location choice

We solve now the subgame in which locations are simultaneously decided by both firms. As far as prices, wages and quantities are a function of locations x_0 and x_1, we have that the objective functions of the public and the private firm only depend on locations x_0 and x_1. So we can get equilibrium locations from the first order conditions. Manipulating the FOCs we first get the reaction functions of both the public and the private firms:

$$x_0(x_1) = \frac{x_1}{3} - \frac{2 + 8\alpha}{3(1+\alpha)}, \tag{14}$$

$$x_1(x_0) = \frac{x_0}{3} + \frac{4 + 10\alpha}{3(1+\alpha)}. \tag{15}$$

Then we can get equilibrium locations, the wages paid by both firms, outputs, prices and employment levels for both firms and their prices, and the utilities from both unions (the superscript C stands for the simultaneous choice of locations):

$$x_0^C = -\frac{1 + 7\alpha}{4(1+\alpha)},\ x_1^C = \frac{5 + 11\alpha}{4(1+\alpha)}, \tag{16}$$

$$w_0^C = w_1^C = \frac{3t(1+3\alpha)^2}{2\alpha(1+\alpha)},\ q_0^C = L_0^C = q_1^C = L_1^C = \frac{1}{2}, \tag{17}$$

$$p_0^C = p_1^C = \frac{3t(1+3\alpha)(1+4\alpha)}{2\alpha(1+\alpha)},\ U_0^C = U_1^C = \frac{3t(1+3\alpha)^2}{4\alpha(1+\alpha)}. \tag{18}$$

It is easy to see that $\forall\alpha$, $x_0^C<0$ and that $x_1^C>1$, and thus the two firms locate outside the market. Moreover, as $\frac{\partial x_0^C}{\partial \alpha} < 0$ and $\frac{\partial x_1^C}{\partial \alpha} > 0$, firms locate further from the market as the weight of the own profit in the objective function of the public firm increases. As α increases the weight of the profit of the public firm in its objective function is greater and the weight of social surplus is lower. Thus, as α increases firms move away from the market to reduce market competition. Note that, althoug firms have different objective functions, both firms locate symmetrically with regard to the center of the market:$\left|x_0^C\right| = \left|x_1^C - 1\right|$. Both firms pay the same wages, set the same prices and obtain the same market share (and thus hire the same employment) and profits. As a result, bot unions obtain the same utility.

Then, we have the equilibrium values for weighted welfare and the profits of both firms in the simultaneous case:

$$W^C = s + \frac{t(-13 - 62\alpha - 49\alpha^2 + 108\alpha^3)}{48(1+\alpha)^2}, \tag{19}$$

$$\pi_0^C = \pi_1^C = \frac{3t(1+3\alpha)}{4(1+\alpha)}. \tag{20}$$

3.3.2 Sequential location choice: the public firm is the leader.

We solve now the subgame in which the public firm chooses its location first and then the private firm does it. Equation (15) is the reaction function in locations of the follower so we substitute this value of x_1 in W and we have it as a function of x_0 solely. Let the superscript L stand for the leader role and the superscript F for the follower. From the first order condition we get the solution:[12]

$$x_0^L = \frac{(1+4\alpha)(-7-(18-\alpha)\alpha)}{4(1+2\alpha)(1+\alpha)^2},\ x_1^F = \frac{3+14\alpha+27\alpha^2+28\alpha^3}{4(1+2\alpha)(1+\alpha)^2}. \tag{21}$$

The wages paid by both firms, their outputs and employment levels, their prices and the utility of both unions are :

$$w_0^L = \frac{t(1+3\alpha)^2(5+15\alpha+4\alpha^2)(1+3\alpha+8\alpha^2)}{6\alpha(1+\alpha)^3(1+2\alpha)^2}, \tag{22}$$

$$w_1^F = \frac{t(1+3\alpha)^2(5+15\alpha+4\alpha^2)^2}{6\alpha(1+\alpha)^3(1+2\alpha)^2}, \tag{23}$$

$$q_0^L = L_0^L = \frac{1+3\alpha+8\alpha^2}{6(1+\alpha)(1+2\alpha)}, q_1^F = L_1^F = \frac{5+15\alpha+4\alpha^2}{6(1+\alpha)(1+2\alpha)}, \tag{24}$$

$$p_0^L = \frac{t(1+3\alpha)(1+4\alpha)(5+15\alpha+4\alpha^2)(5+16\alpha+7\alpha^2+8\alpha^3)}{6\alpha(1+\alpha)^4(1+2\alpha)^2}, \tag{25}$$

$$p_1^F = \frac{t(1+3\alpha)(1+4\alpha)(5+15\alpha+4\alpha^2)^2}{6\alpha(1+\alpha)^3(1+2\alpha)^2}, \tag{26}$$

[12] We discard the solution that does not verify the second order condition. It can be shown that $x_0^L < 0$ if $\alpha < 9 + 2\sqrt{22}$, and $x_1^F > 1$ if $\alpha > \frac{1}{60}(-7 + (3797 - 360\sqrt{74})^{1/3} + (3797 + 360\sqrt{74})^{1/3})$. Take also into account that $x_0^L < 1/2$ and $x_1^F > 1/2$.

$$U_0^L = \frac{t(1+3\alpha)^2(5+15\alpha+4\alpha^2)(1+3\alpha+8\alpha^2)^2}{36\alpha^2(1+\alpha)^4(1+2\alpha)^3}, \tag{27}$$

$$U_1^F = \frac{3t(1+3\alpha)^2(5+15\alpha+4\alpha^2)^3}{36\alpha^2(1+\alpha)^4(1+2\alpha)^3}. \tag{28}$$

Then, we have the equilibrium values for weighted welfare and profits of the two firms in the sequential case where the leader is the public firm:

$$W^L = s + \frac{t(-11-84\alpha-234\alpha^2-204\alpha^3+429\alpha^4+1560\alpha^5+2192\alpha^6+1536\alpha^7)}{144(1+\alpha)^4(1+2\alpha)^2}, \tag{29}$$

$$\pi_0^L = \frac{t(1+3\alpha)(5+15\alpha+4\alpha^2)(1+3\alpha+8\alpha^2)(4+29\alpha+48\alpha^2-5\alpha^3+8\alpha^4)}{36\alpha(1+\alpha)^5(1+2\alpha)^3}, \tag{30}$$

$$\pi_1^F = \frac{3t(1+\alpha)(5+15\alpha+4\alpha^2)^3}{36(1+\alpha)^4(1+2\alpha)^3}. \tag{31}$$

It is easy to see that if $\alpha > \frac{3+\sqrt{13}}{2}$ then $\left|x_0^L\right| < \left|x_1^F - 1\right|$,[13] $w_0^L > w_1^F$, $q_0^L = L_0^L > q_1^F = L_1^F$, $p_0^L > p_1^F$, $U_0^L > U_1^F$ and $\pi_0^L > \pi_1^F$. Therefore, if parameter α is great enough the public firm locates closer to the market, which implies that it pays greater wages, has greater marker share and hires more workers, sets greater prices and obtains greater profits. In this case, the workers of the public firm obtain the greatest utility. If $\alpha > \frac{3+\sqrt{13}}{2}$, then the opposite result is obtained.

3.3.3 Sequential location choice: the private firm acts as a leader.

We solve now the subgame in which the private firm chooses its location first and then the public firm does it. In this case equation (14) is the reaction function in locations of the follower so we substitute this value of x_0 in π_1 and the profits of the private firm are merely a function of x_1. From the first order condition we get the solution:

$$x_0^F = -\frac{1+5\alpha}{2(1+\alpha)},\ x_1^L = \frac{1}{2}. \tag{32}$$

The wages paid by both firms, their outputs, their employment levels, their prices and the utilities of both unions are:

[13] When the two firms are privately-owned and assuming that unions set the wage (a result that can be analyzed from Brekke and Straume, 2004) the leader locates in the middle of the market and the follower outside the market. When α is great enough, the weight of the own profit in the objective function of the public firm is great, and thus we obtain a similar result, except that the public firm never locates exactly in the middle of the market as it takes nor profits but weighted welfare into account.

$$w_0^F = \frac{2t(1+3\alpha)^2}{3\alpha(1+\alpha)}, w_1^L = \frac{4t(1+3\alpha)^2}{3\alpha(1+\alpha)}, q_0^F = L_0^F = \frac{1}{3}, q_1^L = L_1^L = \frac{2}{3}, \tag{33}$$

$$p_0^F = \frac{2t(1+3\alpha)(2+9\alpha+4\alpha^2)}{3\alpha(1+\alpha)^2}, p_1^L = \frac{t(1+3\alpha)(1+4\alpha)}{3\alpha(1+\alpha)}, \tag{34}$$

$$U_0^F = \frac{2t(1+3\alpha)^2}{9\alpha(1+\alpha)}, U_1^L = \frac{8t(1+3\alpha)^2}{9\alpha(1+\alpha)}. \tag{35}$$

Then, we have the equilibrium values for weighted welfare and profits of the two firms in the sequential case where the leader is the private firm:

$$W^F = s + \frac{t(1+17\alpha+24\alpha^2)}{36(1+\alpha)}, \tag{36}$$

$$\pi_0^F = \frac{2t(1+3\alpha)(1+5\alpha+\alpha^2)}{9\alpha(1+\alpha)^2}, \pi_1^L = \frac{8t(1+3\alpha)}{9(1+\alpha)}. \tag{37}$$

The above results show that the private firm locates in the middle of the market ($x_1^L = \frac{1}{2}$) while the public firm locates outside the market ($x_0^F < 0$).[14] Besides, it can be shown that the public firm locates further from the market as parameter α becomes greater. As the public firm is located further from the market pays lower wages ($w_0^F < w_1^L$), sets a lower price ($p_0^F < p_1^L$) and obtains lower market share and hires less workers ($q_0^F = L_0^F < q_1^L = L_1^L$). As a result, the workers of the public firm obtain lower utility ($U_0^F < U_1^L$). Besides, $\pi_0^L > \pi_1^F$ if and only if $\alpha < \frac{1+\sqrt{13}}{6}$.

Now that we have got the firms' locations in the three different settings we can compare our results with those obtained by Brekke and Straume (2004) with two private firms. They use the Nash bargaining solution in their paper so these comparisons are made considering that the bargaining power of both unions in Brekke and Straume's model is set at its highest level. They show that when production costs are endogenously determined and wage negotiation is simultaneous, firms locate further from the market than when costs are exogenously given. In our model, as one firm is publicly-owned and takes into account not only profits but also social surplus, the distance between the two firms is lower in the case of simultaneous choice of location and also in the case in which the private firm acts as a leader. The distance between the two firms may be higher than in the case of a private duopoly when the leader in the choice of location is the public firm and this firm is very concerned about its profits.

With regard to the results obtained by Matsumura and Matsushima (2003) when no regulation exists, we have that when the private firm is the leader

[14] We obtain a similar result than when the two firms are privately-owned (see, Brekke and Straume, 2004). In that case, when the unions set the wages they obtain that the leader firm locates in the middle of the market and gets a market share of 2/3.

it locates in the middle of the market in their model and in ours. But in our model the distance between the two firms is higher as the public firm is partially concerned by its own profits. When the public firm is the leader they get that firms adopt the first best locations, but in our model the distance between the two firms is higher and firms never adopt the locations that maximize social welfare.

3.4 Choice of the timing of the location decision

In the first stage of the game both firms decide the period of time (t =1 or t =2) of the second stage of the game in which they are going to choose their location. We can gather together the information in the following matrix for the first period (see table 1):

		Private firm	
		location at $t = 1$	location at $t = 2$
Public firm	location at $t = 1$	W^C, π_1^C	W^L, π_1^F
	location at $t = 2$	W^F, π_1^L	W^C, π_1^C

Table 1. First stage of the game: Choice of the period to decide locations.

The equilibrium in the first stage of the game is the following.

Proposition 1. In equilibrium:
(i) Both firms choose their locations at $t = 1$ if $\alpha \geqslant \frac{3+\sqrt{13}}{2}$.
(ii) When $\frac{3+\sqrt{13}}{2} > \alpha \geqslant \frac{1}{152}(129 + \sqrt{29713})$ the public firm is the leader and the private one acts as a follower.
(iii) When $\alpha < \frac{1}{152}(129 + \sqrt{29713})$ there are two equilibria: one firm is the leader and the rival is the follower in the choice of location.

Proof: It is straightforward to check that $W^L > W^C$ if $\alpha \neq \frac{3+\sqrt{13}}{2}$ and $W^L = W^C$ if $\alpha = \frac{3+\sqrt{13}}{2}$. Moreover, $\pi_1^L > \pi_1^C$ so both firms are better as leaders of the game rather than playing a simultaneous game. We only need to compare W^C and W^F and π_1^C and π_1^F. From the public firm point of view $W^C \gtreqless W^F$ if and only if $\alpha \gtreqless \frac{1}{152}(129 + \sqrt{29713})$. From the private firm point of view we have that $\pi_1^C \gtreqless \pi_1^F$ if and only if $\alpha \gtreqless \frac{3+\sqrt{13}}{2}$. This proves the results.

The explanation of the results is the following. When α is low enough we have two equilibria: one firm is the leader and the rival acts as a follower. From the public firm point of view to be the follower is better than playing a simultaneous game as far as the private firm is located in the middle of the market and the location of the public firm is closer to the market because the main importance is that of the transportation costs. Thus, given that the consumers who buy to the private firm have low transportation costs this is better than to play the simultaneous game which is not interesting from transportation costs point of view as far as firms are both far apart from the market.

As α increases the profits of the public firm are more important and they initially decrease in the sequential game (the public firm is located furthest from the market) but increase in the simultaneous game as both firms are further from the market. For this reason as α increases the public firm prefers to play the simultaneous game. From the private firm point of view something similar occurs: to be the follower is right when the public firm is not located close to the market but when α is very high as its value increases the public firm will locate closer to the middle of the market and then the private firm will be located further from it. As a result when α is high enough is better to play the simultaneous game than to be the follower. In fact it can be cheeked than to play a simultaneous game choosing the locations in the first period would be the result of the game when both firms only value their own profits. This is the result obtained when α tends to ∞. It can be shown that if wages are exogenously given, the two firms take location decisions simultaneously at period t=1 since it is a dominant strategy for both firms to decide locations at t=1.[15]

Other interesting findings in this paper have to do with the wages bargained in both firms. The following proposition shows that the public firm never pays a higher wage than the private one when the timing of the choice of location is endogenously determined.

Proposition 2. In equilibrium the wage paid by the public firm never exceed the wage paid by the private firm. Besides, in equilibrium, the employment level of the public firm and the utility of its workers are no greater than those of the private firm.

Proof: When both firms choose their locations simultaneously both pay the same wage and hire the same number of workers; thus, the workers of the two firms obtain the same utility.. When the public firm is the leader, comparing w_0^L and w_1^F it is straightforward to check that $w_1^F > w_0^L$ if and only if $\alpha < \frac{3+\sqrt{13}}{2}$. Then, in the cases in which the public firm acts as a leader this firm pays the lowest wage. Finally, when the private firm is the leader and the public one acts as a follower the wage of the private firm is twice the wage of the public firm. Although in equilibrium the public firm pays lower wages under sequential locations, it hires less workers and the utility of its workers is lower. This proves the results.

The wage paid by the public firm should be higher than the wage paid by the private one only in the case in which the firms are not able to chose the timing of the choice of location. When we consider that the public firm is the leader in the choice of location the result is that this firm should pay higher wages if $\alpha > \frac{3+\sqrt{13}}{2}$. But in our model, when this is the case, both firms chose their locations simultaneously.

Our findings are in contrast to previous results obtained by Willner (1997), de Fraja (1993) or Bárcena-Ruiz and Garzón (2009). Willner obtains the results that unit costs are normally higher in the public firm. In his model there is

[15] The proof of this assertion is available from the authors on request.

quantity competition and Nash bargaining.[16] In the model analyzed by de Fraja (1993) wages paid by the public firm are usually highest, but there are some extreme cases in which wages are highest in the private firm. In our model we do not find this sort of ambiguity.[17] In other related paper, Bárcena-Ruiz and Garzón (2009) obtain that a semipublic firm pays a higher wage than a private one in a setting in which negotiation on wages takes place on the basis of the monopoly union model.

4. Conclusions

The timing of the choice of locations by a public and a private firm depends on the concern that the public firm has on its profits. When the public firm is very concerned on its profits the locations will be decided simultaneously. But when the public firm is not very concerned about its profits we find that locations will be chosen sequentially. When the weight of profits in the public firm's objective function is very low both firms could adopt the role of the leader in the location game. Thus, the objective funtion of the public firm drives the results on the choice of locations.

In this paper we get the result that in the three different equilibrium configurations for the choice of locations the public firm never pays higher wages than the private one. Thus, the idea that the public firm could be more weak negotiating wages and should pay the highest wage does not fit under endogenous choice of locations even though this result could be obtained when some scenario, i.e. the public firm is the leader, is exogenously imposed.

References

Bárcena-Ruiz, J.C. (2007) "Endogenous Timing in a Mixed Duopoly: Price Competition", Journal of Economics, 91, 263-272.

Bárcena-Ruiz, J.C. and Casado-Izaga, FJ. (2008): "Timing of Endogenous Bargaining over Costs and Firms' Locations", Journal of Economics, 95, 149-166.

Barcena-Ruiz, J.C. and M.B. Garzón (2003). "Mixed duopoly, merger and multiproduct firms", Journal of Economics, 80, 27-42.

Bárcena-Ruiz, J.C. and M.B. Garzón, (2009). "Relocation and public ownership of firms", Journal of the Japanese and International Economies, 23, 71-85.

Bárcena-Ruiz J.C. and M.B. Garzón (2010). "Endogenous timing in a mixed oligopoly with semipublic firms", Portuguese Economic Journal, forthcoming, DOI 10.1007/s10258-010-0054-8.

Booth, A., (1995): The Economics of the Trade Unions. Cambridge: Cambridge University Press.

Bös, D., (1991): Privatization: A Theoretical Treatment, Oxford, Clarendon Press.

[16] In his model the public firm maximizes total surplus with the restriction that the price should not fall bellow a floor.

[17] In the paper by de Fraja (1993) he considers quantity competition and the objective function of the public firm includes a different weight for the incomes of both unions.

Brander, J. A. and B. J. Spencer (1988): “Unionized oligopoly and international trade policy”, Journal of International Economics, 24, 217-234.

Brekke, KR., and Straume, OR. (2004): "Bilateral Monopolies and Location Choice", Regional Science and Urban Economics, 34, 275-288.

Bughin, J. and S. Vannini (1995): “Strategic direct investment under unionized oligopoly”, International Journal of Industrial Organization, 13, 127-145.

Cremer,H., Marchand, M. and Thisse, J.-F. (1991): "Mixed Oligopoly with Differentiated Products", International Journal of Industrial Organization, 9, 43-53.

d’Aspremont, C., and Gabszewicz, JJ., Thisse, J-F. (1979): "On Hotelling’s Stability in Competition", Econometrica, 47, 1145-1150.

de Fraja, G. (1993): "Unions and wages in public and private firms: A game-theoretic analysis", Oxford Economic Papers, 45, 457-469.

Hamilton, J.H. and Slutsky S.M. (1990): “Endogenous Timing in Duopoly Games: Stackelberg or Cournot Equilibria”, Games and Economic Behavior 2: 29-46.

Harsanyi, J.C. (1995): "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility", Journal of Political Economy, 63, 309-321.

Inoue, T., Kamijo, Y and Y. Tomaru (2009). "Interregional mixed duopoly", Regional Science and Urban Economics, 39, 233-242.

Ishida, J. and N. Matsushima (2009). “Should civil servants be restricted in wage bargaining? A mixed duopoly approach”, Journal of Public Economics, 93, 634-646.

Lambertini, L. (1994): "Equilibrium Locations in the Unconstrained Hotelling Game", Economic Notes, 24, 438-446.

Lambertini, L. (1997): "Unicity of the Equilibrium in the Unconstrained Hotelling Model", Regional Science and Urban Economics, 27, 785-798.

Lommerud, K., Meland, F., and L. Sogard (2003) “Unionised oligopoly, trade liberalisation and location choice”, The Economic Journal, 113, 782-800.

Lu, Y. (2006): "Endogenous timing in a mixed oligopoly with foreign competitors: The linear demand case", Journal of Economics, 88, 49-68.

Mezzetti, C. and E. Dinopoulos (1991): “Domestic unionization and import competition”, Journal of International Economics, 31, 79-100.

Matsumura, T. (1998): “Partial Privatization in Mixed Duopoly”, Journal of Public Economics, 70, 473-483.

Matsumura, T. (2003): "Stackelberg mixed duopoly with a foreign competitor", Bulletin of Economic Research, 55, 275-87.

Matsumura, T., and N. Matsushima, (2003): "Mixed Duopoly with Product Differentiation: Sequential Choice of Location", Australian Economic Papers 1, 18-34.

Matsumura, T., and N. Matsushima, (2004): "Endogenous cost differentials between public and private enterprises: A Mixed Duopoly Approach", Economica 71, 671-688.

Matsushima, N. and T. Matsumura, (2003): "Mixed oligopoly and spatial agglomeration", Canadian Journal of Economics, 36, 62-87.

Matsushima, N. and T. Matsumura, (2006): "Mixed oligopoly, foreign firms, and location choice", Regional Science and Urban Economics, 36, 753-772.

Ogawa, A. and Kato, K. (2006): "Price Competition in a Mixed Duopoly", Economics Bulletin, 4, 1-5.

Pal, D. (1998): “Endogenous timing in a mixed oligopoly”, Economics Letters 61: 181-185.

Tabuchi, T., and Thisse, J-F. (1995): "Asymmetric Equilibria in Spatial Competition", International Journal of Industrial Organization 13, 213-227.

White, M.D. (2002): "Political Manipulation of a Public Firm’s Objective Function", Journal of Economic Behavior and Organization, 49, 487-499.

Willner, J. (1999): "Policy objectives and performance in a mixed with bargaining", International Journal of Industrial Organization 17, 137-145.