Nonlinear adjustment in the real dollar-euro exchange rate: the role of the productivity differential as a fundamental

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Abstract

In this paper we analyze the influence of productivity differentials in the dynamics of the real dollar-euro exchange rate. Using nonlinear procedures for the estimation and testing of ESTAR models during the period 1970-2009 we find that the dollar-euro real exchange rate shows nonlinear mean reversion towards the fundamental represented by the productivity differential. In addition, we provide evidence about the ability of this variable to capture the overvaluation and undervaluation of the dollar against the euro.

JEL classification: C22, F31.

Keywords: nonlinearities, real exchange rate, productivity differential fundamental.

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1 Introduction

The purchasing power parity (PPP) theory postulates that national price levels should be equal when expressed in a common currency. Since the real exchange rate is the nominal exchange rate adjusted for relative national price levels, variations in the real exchange rate represent deviations from PPP. It has become something of a stylized fact that the PPP does not hold continuously. Thus deviations of spot exchange rates away from PPP are persistent and this is consistent with a unit root or near-unit root behavior of the real exchange rates. The persistent divergence from equilibrium causes that linear PPP-based fundamentals exchange rates models do not perform well in predicting or explaining future or past exchange rate movements (Frankel and Rose, 1995, Taylor, 1995, Sarno and Taylor, 2002). Other authors still believe that some form of PPP does in fact hold at least as a long run relationship (MacDonald, 1999, 2004). The issue of whether the real exchange rate tends to revert towards a long-run equilibrium has been a topic of considerable debate in the literature (e.g. Lothian and Taylor, 1996, Lothian and Taylor, 1997, and Taylor and Taylor 2004, Lothian and Taylor, 2008, among others). Panel unit root and long-run studies have reported favourable evidence to parity reversion (see Taylor, 1995, for a survey), however, as pointed out by Rogoff (1996), it is impossible to reconcile the high short-term volatility of real exchange rates with the slow rate at which shocks in the real exchange rate appear to die out in those studies. This conclusion, known as the PPP-Puzzle, constitutes one of the most controvertial issues related to real exchange rates.

The relatively recent literature on nonlinearities and exchange rates can be considered a possible solution to such puzzles. Taylor, Peel and Sarno (2001) and Kilian and Taylor (2003) argue that allowing for nonlinearities in real exchange rate adjustment are key both to detect mean reversion in the real exchange rate and to solve the PPP-puzzle. Following their argument, the further away the real exchange rate is from its long-run equilibrium, the stronger will be the forces driving it back towards equilibrium. Another way to reconsider the linear PPP-based models is to integrate in this basic model the impact of shocks coming from real variables. Thus, persistent shocks might be supply-related and incorporate, for example, the Harrod-Balassa-Samuelson (HBS) effect which postulates that productivity shocks affect the equilibrium real exchange rate. From the empirical evidence, it seems that the productivity differential plays a very important role in explaining some real exchange rate movements. Alquist and Chinn (2002) find supporting evidence for the productivity differential as the most important fundamental that explains the behavior of the real dollar-euro exchange rate since the mid 1980s. Furthermore, they argue that the magnitude of the correlation between the two variables is much larger than what would predict the HBS effect. Camarero, Ordóñez and Tamarit (2002) have also estimated a long-run model for the euro-dollar exchange rate, finding that the main factor explaining the dynamic adjustment in the error correction model is again the productivity differential. In contrast, Schnatz et al. (2004) find that although the productivity differential is an important determinant of the real dollar-euro exchange rate, its ability to explain the real depreciation of the euro in the late nineties is very limited. Lothian and Taylor (2008) investigate the influence of productivity differentials on the equilibrium level of the pound-dollar and pound-franc real exchange rates. Although these authors found statistically significant evidence of the HBS effect for the pound-dollar real exchange rate, they failed to find any significant evidence of the HBS effect for the pound-franc real exchange rate.

In this paper we focus on testing for and estimating some form of nonlinear adjustment in the real dollar-euro exchange rate towards the productivity differential. The purpose of the analysis is to check the ability of this fundamental to capture the dollar-euro exchange rate behavior.

The remainder of this paper is organized as follows. Section 2 briefly describes the methodology used in the empirical analysis. In Section 3 we present the data, as well as the estimated nonlinear model. We also analyze the adjustment of the real exchange rate towards the productivity differential. The last section concludes.

2 Methodology

A number of authors have reported evidence of nonlinear adjustment in the real exchange rate[1]. Such nonlinearities can be modelled using a smooth transition autoregressive (STAR) process, proposed by Granger and Teräsvirta (1993). In this model, the adjustment takes place in every period at a speed that varies with the extent of deviation from equilibrium. A STAR model can be formulated as follows:

$$y_t = (\alpha + \sum_{i=1}^{p} \phi_i y_{t-i})[1 - G(\gamma, y_{t-d} - c)] + (\tilde{\alpha} + \sum_{i=1}^{p} \tilde{\phi}_i y_{t-i}) G(\gamma, y_{t-d} - c) + \varepsilon_t \quad (1)$$

where α, $\tilde{\alpha}$, γ and c are constant terms; ε_t is an i.i.d. error term with zero mean and constant variance σ^2. The transition function $G(y_{t-d}; \gamma, c)$ is con-

[1]See for example Taylor (2006) for a recent overview of the real exchange rate and purchasing power parity debate.

tinuous and bounded between 0 and 1.

The STAR models can be considered regime-switching models that allow for two regimes associated with the extreme values of the transition function $G(y_{t-d}; \gamma, c) = 1$ and $G(y_{t-d}; \gamma, c) = 0$, where the transition between these two regimes is smooth.

A very popular transition function used to model real exchange rates is the exponential function suggested by Granger and Teräsvirta (1993):

$$G(y_{t-d}; \gamma, c) = 1 - \exp\{-\gamma(y_{t-d} - c)^2\}, \ \ \gamma > 0 \tag{2}$$

where c is the equilibrium level of y_t and γ the transition parameter, which determines the speed of transition between the two extreme regimes, with higher values of γ implying faster transition.

Combining (1) and (2) we obtain an exponential STAR or ESTAR model. The exponential function is symmetric and U-shaped around zero. The ESTAR model collapses to a linear AR(p) model for either $\gamma \to 0$ or $\gamma \to \infty$, and it is therefore useful to capture symmetric adjustment of the endogenous variable above and below the equilibrium level. Symmetric adjustment is a condition frequently assumed to model real exchange rates. The reason for this assumption is the difficulty to justify any economic reasons for different speeds of adjustment depending on whether one currency is overvalued or undervalued.

In our research, we will use the procedure suggested by Granger and Teräsvirta (1993) and Teräsvirta (1994) for the specification and estimation of parametric STAR models. Their technique consists of the "specific-to-general" strategy for building nonlinear time series models suggested by Granger (1993) and, as indicated by van Dijk (2002), it comprises the following steps: (a) specify a linear AR model of order p for the time series under investigation; (b) test for the null hypothesis of linearity against the alternative of STAR nonlinearity; (c) if linearity is rejected, select the appropriate transition variable and the form of the transition function; (d) estimate and evaluate the model; (e) use the model for descriptive or forecasting purposes.

Testing for linearity against a STAR is a complex matter because, under the null of linearity, the parameters in the STAR model are not identified. Luukkonen et al. (1988) and Teräsvirta (1994) suggest a sequence of tests to evaluate the null of an AR model against the alternative of a STAR model. These tests are conducted by estimating the following auxiliar regression for a chosen set of values of the delay parameter d, with $1 < d < q$:[2]

$$y_t = \beta_0 + \sum_{i=1}^{p} \beta_{1i} y_{t-i} + \sum_{i=1}^{p} \beta_{2i} y_{t-i} y_{t-d} + \sum_{i=1}^{p} \beta_{3i} y_{t-i} y_{t-d}^2 + \sum_{i=1}^{p} \beta_{4i} y_{t-i} y_{t-d}^3 + \epsilon_t. \tag{3}$$

The null of linearity against a STAR model corresponds to: $H_0 : \beta_{2i} = \beta_{3i} = \beta_{4i} = 0$ for $i = 1, 2, ..., p$. The corresponding LM test has an asymptotic χ^2 distribution with $3(p+1)$ degrees of freedom under the null of linearity. If linearity is rejected for more than one value of d, the value of d corresponding to the lowest p-value of the joint test is chosen. In small samples, it is advisable to use F-versions of the LM test statistics because these have better size properties than the χ^2 variants (the latter may be heavily oversized in small samples). Under the null hypothesis, the F version of the test is approximately F distributed with $3(p+1)$ and $T-4(p+1)$ degrees of freedom. Escribano and Jordà (2001) propose an extension of the Teräsvirta (1994) linearity test by adding a fourth order regressor.[3] Below we use both tests.

If linearity is rejected, we need to test for LSTAR against ESTAR nonlinearity. For this purpose, Granger and Teräsvirta (1993) and Teräsvirta (1994) propose the following sequence of tests within the auxiliar regression (3):

$$\begin{aligned} &H_{03} : \beta_{4i} = 0 \;\; i = 1, 2, ..., q \\ &H_{02} : \beta_{3i} = 0 | \beta_{4i} = 0 \;\; i = 1, 2, ..., q \\ &H_{01} : \beta_{2i} = 0 | \beta_{3i} = \beta_{4i} = 0 \;\; i = 1, 2, ..., q. \end{aligned}$$

An ESTAR model is selected if H_{02} has the smallest p-value, otherwise the selected model is the LSTAR.

Since linearity type tests assume stationarity we first need to check whether y_t is a stationary variable. Kapetanios, Shin and Snell (2003) propose a frameworh test for nonstationarity against nonlinear but globally stationary exponential smooth transition autoregressive processes. Consider a univariate smooth transition autoregressive of order 1, STAR(1) model:

$$y_t = \phi y_{t-1} + \tilde{\phi} y_{t-1}(1 - exp\{-\gamma y_{t-d}^2\}) + \varepsilon_t \tag{4}$$

As suggest by Kapetanios, et al. (2003), KSS hereafter, equation (4) can be conveniently reparameterised as:

[2]Equation (3) is obtained by replacing the transition function in the STAR model (1) by a suitable Taylor series approximation (see Granger and Teräsvirta, 1993).

[3]They claim that this provides better results when the data are mainly in one of the regimes and when there is uncertainty about the lag length of the autoregressive part. The corresponding LM test statistic has an asymptotic χ^2 distribution with $4(p+1)$ degrees of freedom under the null of linearity.

$$\Delta y_t = \beta y_{t-1} + \tilde{\phi} y_{t-1}(1 - exp\{-\gamma y_{t-d}^2\}) + \varepsilon_t \tag{5}$$

where $\beta = \phi - 1$. Imposing $\beta = 0$ (that is, the variable is a nonstationary process in the central regime) and $d = 1$, our specific ESTAR model is:

$$\Delta y_t = \tilde{\phi} y_{t-1}(1 - exp\{-\gamma y_{t-1}^2\}) + \varepsilon_t \tag{6}$$

where $\varepsilon_t \sim iid(0, \sigma^2)$. In order to test the null hypothesis of a unit root $H_0 : \gamma = 0$ against $H_1 : \gamma > 0$ outside of the threshold[4], Kapetanios et al. (2003) propose a Taylor approximation of the ESTAR model since, in practice, the coefficient γ cannot be identified under H_0. Thus, under the null, the model becomes

$$\Delta y_t = \delta y_{t-1}^3 + \eta_t \tag{7}$$

where η_t is an error term. Now, it is possible to apply a t-test to analyze whether y_t is a nonstationary process, $H_0 : \delta = 0$, or whether it is a nonlinear stationary process, such that $H_1 : \delta < 0$.

Equation (6) can be extended to include a constant and a trend as well as the more general case where the errors are serially correlated so that equation (7) becomes:

$$\Delta y_t = \sum_{i=1}^{p} \rho_i \Delta y_{t-i} + \delta y_{t-1}^3 + \eta_t \tag{8}$$

Once nonlinearities are proved to be significant, the adequacy of the estimated STAR model can be evaluated using the tests suggested by Eitrheim and Teräsvirta (1996). They proposed three LM tests for the hypotheses of no error autocorrelation, no remaining nonlinearity and parameter constancy.

3 Empirical results

3.1 Data

The data is quarterly and covers the period 1970:Q1 to 2009:Q2. We use the series from Camarero, Ordóñez and Tamarit (2005) for the nominal (synthetic) dollar-euro exchange rate data for the period 1970:Q1 to 1997:Q4 and from the European Central Bank Monthly Bulletin for the rest of the sample. The real dollar-euro exchange rate is computed using consumer price indices. CPI-data are obtained from the OECD Main Economic Indicators

[4]The process is globally stationary provided that $-2 < \tilde{\phi} < 0$.

database for the US and from the European Central Bank Monthly Bulletin for the EMU. The productivity differential is proxied by labor productivity differential, computed as GDP per employed person. The data of employment and GDP are taken from the OECD Main Economic Indicators with the exception of the German labor data for the period 2001:Q1 to 2006:Q4 which have been obtained from the German Statistisches Bundesamt. European productivity is a weighted average based on the relative GDP of the four largest euro-area economies, with fixed weights and base year 2005.[5] The productivity differential is plotted in Figure 1. All the variables are in natural logarithms.

In this paper we want to assess to what extent the productivity differential governs the real dollar-euro exchange rate behavior and whether deviation of the exchange rate from its productivity differential may followed a nonlinear process. Thus, we focus on $y_t = rer_t - difpro_t$ where rer_t and $difpro_t$ denote respectively the real dollar-euro exchange rate and the productivity differential between the Euro Area and the US. Furthermore, this choice of the variable of interest will allow us to gauge the degree of overvaluation of the US dollar relative to the Euro through the time path of the transition function as demonstrated in the following section.

Previous to the STAR modelling we test whether rer_t, $difpro_t$ and y_t are stationary processes. For this purpose we use the Kapetanios, et al (2003) test for a unit root in the nonlinear STAR framework. Table (1) presents the results for the KSS test applied to rer_t, $difpro_t$ and y_t allowing for a constant and a constant plus a trend. The lag length for i in equation (8) can be chosen using an information criteria (AIC, BIC, HQ and MAIC in Table 1). Critical values have been obtained by Monte Carlo simulation for a sample of 150 observations and 50,000 replications and are shown at the bottom of Table 1. According to the results, $Difpor_t$ is nonstationary when allowing for a trend, and rer_t is stationary only at 10% significance level. The variable y_t is, however, clearly stationary in levels.

3.2 Nonlinear estimation results

Once we have checked whether the variable of interest, y_t, is stationary, we can test for linearity, since the linearity tests are only valid under this assumption. Table 2 reports values of the test statistics H_0, H_{01}, H_{02} and H_{03}. Given the quarterly frequency of the data employed, we consider d=1,...,

[5]The choice of the countries used for aggregation is mainly due to problems of data availability. However, even if we consider only four countries, Germany, France, Italy and Spain account for over 80% of the euro-area GDP.

8 as plausible values for the delay parameter[6]. From Table 2 we conclude that the hypothesis of linerity is rejected at 5% level of significance when d=5 and 6. Furthermore, according to the sequence of test statistics H_{01}, H_{02} and H_{03} the ESTR representation of the data is preferred to the LSTAR, i.e. H_{02} presents the smallest p-value. Our results suggest that there is a significant evidence of nonlinearity in the exchange rate adjustment to its productivity fundamental which appears to be reasonably approximated by an ESTAR model with a delay of five or six.

Table 3 presents the estimated ESTAR model for d=6 [7] as well as a series of misspecification tests suggested by Eitrheim and Teräsvirta (1996). Following Teräsvirta (1994), γ has been standarized to make easier to compare speeds of adjustment when γ is divided by the standard deviation of y_t. Concerning the identification of this model we could not reject the four restrictions $c = 0$, $\alpha = \tilde{\alpha} = 0$, $\phi_1 = -\tilde{\phi_1}$ and $\phi_2 = -\tilde{\phi_2}$ with a likelihood ratio test p-value of 0.37. These restrictions imply an equilibrium level for y_t in the neighborhood of which y_t is close to a second-order unit root process, becoming increasingly mean reverting with the absolute size of the deviation from equilibrium. Thus, our model reconciles two apparently contradictory results commonly found in the literature: the real exchange rates behavies as a random walk in the neighborhood of the equilibrium fundamental in a model which is globally stationary implying that real exchange rates are mean-reverting. As pointed out by Taylor and Peel (2000), the "t-ratios" for the transition parameter γ should be interpreted with caution, since under the null hypothesis $H_0 : \gamma = 0$, y_t follows a unit root process. We have therefore computed the empirical marginal significance level of γ using Monte Carlo methods. The empirical marginal significance level appears in square brackets under the estimated transition parameter in Table 3.

The adequacy of the model is proved through the evaluation tests proposed by Eithreim and Teräsvirta (1996). The results of the misspecification tests suggest that the model is well specified since there is no evidence of autocorrelation, all possible nonconstancies in the parameters have been properly captured by our model and there are no STAR-type nonlinearities in the data that have not been captured by the model.

Figure 2 plots the estimated transition function (on the vertical axis) against lagged values of the real dollar-euro exchange rate deviations from its productivity fundamental (y_{t-6}). It seems to be a reasonable number

[6] Table 2 reports the linearity test only for $d = 2$, since for this lag length the obtained p-values are the lowest. Linearity test for different values of $d = 2$ are available upon request.

[7] The model with d=5 has been also estimated however it delivers slightly poorer forecast when compared with d=6, so that the last is preferred.

of observations above and below the equilibrium, although those above the equilibrium are higher in number, so that we can be reasonably confident with our selection of the ESTAR specification. Furthermore, even if we had assumed symmetric adjustment to some degree, although not to the degree implied by LSTAR model, the ESTAR specification could be a good approximation of the real exchange rate adjustment. These deviations are clustered around a value of plus or minus 20%, with a very low speed of mean reversion.

The transition function provides some idea of the degree of nonlinear mean reversion exhibited by the real exchange rate deviation from its fundamental. However, to gain full insight into the mean-reverting properties of the estimated nonlinear model, we have carried out a dynamic stochastic simulation. Figure 3 plots the impulse response function obtained by Monte Carlo simulation of shocks to y_t of sizes 25, 15, 10, 5 and 1%. The nonlinear nature of the process is clearly shown since the speed of mean reversion dramatically increases with the size of the shock.

To conclude with the results of the estimation procedure, we test whether the estimated ESTAR model can beat the AR(2) linear model in terms of out-of-sample forecasting. The relative forecast performance can also be used as a model selection criterion and thus, as a way to evaluate the estimated models. We use the data from 2004:Q2 up to 2009:Q2 to evaluate the forecasting performance of the estimated AR and ESTAR models. The results of the forecasting procedure are shown in Table 4. We compute 1 to 4 steps ahead forecasts from the estimated linear AR model and the ESTAR model. The forecast evaluation criteria shown in Table 4 are based upon the entire forecast period, that is, they are not conditioned to the value of the transition function. The forecasting accuracy is evaluated using the mean prediction error (MPE) and the mean squared prediction error (MSPE). The results of both criteria show that the ESTAR model offers better forecast performance than the AR model and thus, our estimated ESTAR model is preferred to the linear one for the estimation of the real dollar-euro exchange rate.

Finally, using the estimated transition function it is possible to obtain the degree of over- or undervaluation of the dollar relative to the euro according to the productivity fundamental. Taylor and Peel (2000) propose a series of transformations to the transition function, which allow to assess whether the dollar is overvalued or undervalued.[8] Panel (a) in figure 4 displays the time series plot of the transformed transition function. Values above the

[8] They argue that the transition function itself cannot be used as an indicator of either overvaluation or undervaluation, as it is only a measure of the importance of the deviation from equilibrium regardless of the sign.

horizontal axis indicate dollar overvaluation and those below it show dollar undervaluation. From our results, it appears that the dollar has been most of the time undervalued against the euro. However, following the path described by the logarithm of the real dollar-euro exchange rate in panel (c) there are two periods of dollar appreciation, where our estimated model also indicates dollar overvaluation. The first one starts in 1981 and reverts in 1986 with the Plaza Agreement (1985) and the Louvre Accord (1987). The other one runs from 1999 to 2002, coinciding with the first three years after the introduction of the euro and giving rise to an interesting debate in the empirical literature of exchange rates (De Grauwe (2000), Meredith (2001), Alquist and Chinn (2002), Schnatz et al. (2004)). The fact that our model captures well these two important episodes, which characterized the real dollar-euro exchange rate during our sample period, highlights the importance of the nonlinear models against the linear ones, as well as the robustness of our estimated model. Panel (b) in figure 4 plots the the sum of the coefficients in the instantaneous AR(2) process y_t. As suggested by Taylor and Peel (2000) this sum can be viewed as a measure of the degree of mean reversion of the real exchange rate at a particular point in time, reversion is strong in mid-seventies, mid-eighties and from 2007 onwards.

4 Conclusion

In this paper we estimate a nonlinear model for the real dollar-euro exchange rate determination based on the productivity differential. As shown by the empirical literature on exchange rates, nonlinear models offer satisfactory results in dealing with some of the real exchange rate puzzles, so that they are able to explain the persistent behavior of the real dollar-euro exchange rate.

Using quarterly data on the dollar-euro exchange rate and the associated productivity differential for the period 1970:Q1 to 2009:Q3, we find evidence of nonlinearities in the dynamics of the exchange rate. These nonlinearities, which are of the form of an exponential smooth transition model, allow real dollar-euro exchange rate deviations from equilibrium to be consistent with the productivity fundamental, despite the apparent persistent behavior of the series. Our results also indicate that the nonlinear model offers better forecasting performance than the linear one, and thus, it must be preferred to the linear model in order to estimate the real exchange rate.

In addition, the transformed transition function is able to capture the well-established dollar overvaluation in the mid 1980s and the weakness of the euro after its introduction in 1999. This fact reinforces the idea that the productivity differential is an adequate explanation of the behavior of the real dollar-euro exchange rate and that volatility in exchange rates should not be directly associated with disconnection from fundamentals.

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Figure 1: Euro Area-US productivity differential

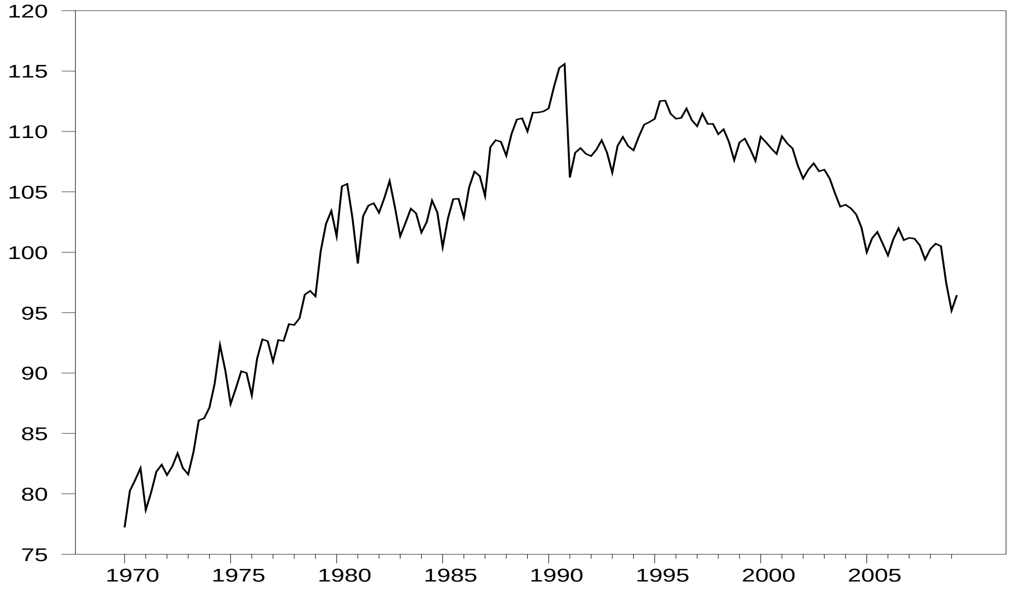


Figure 2: Estimated transition function

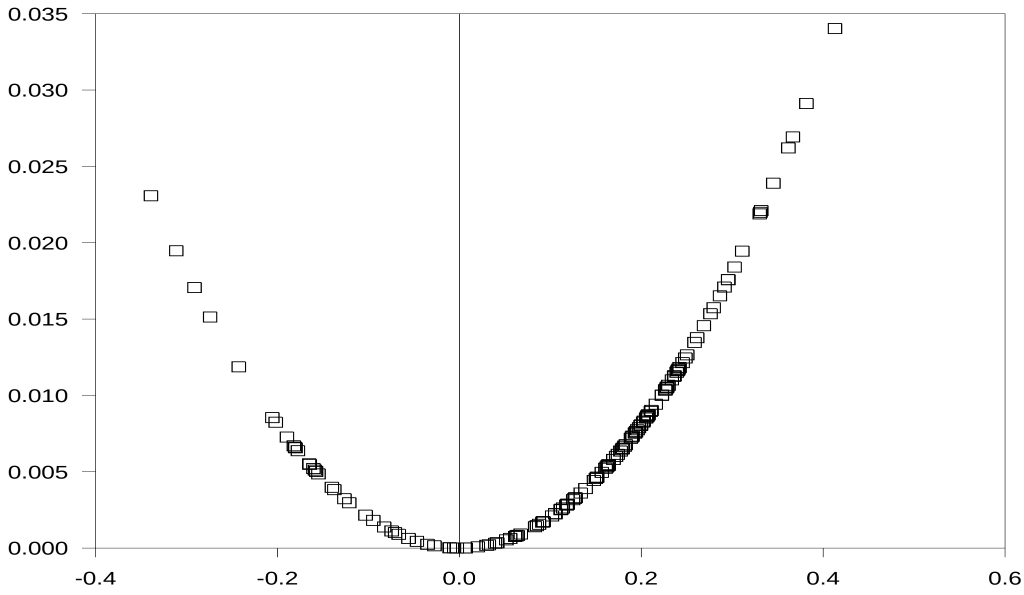


Figure 3: Impulse response functions

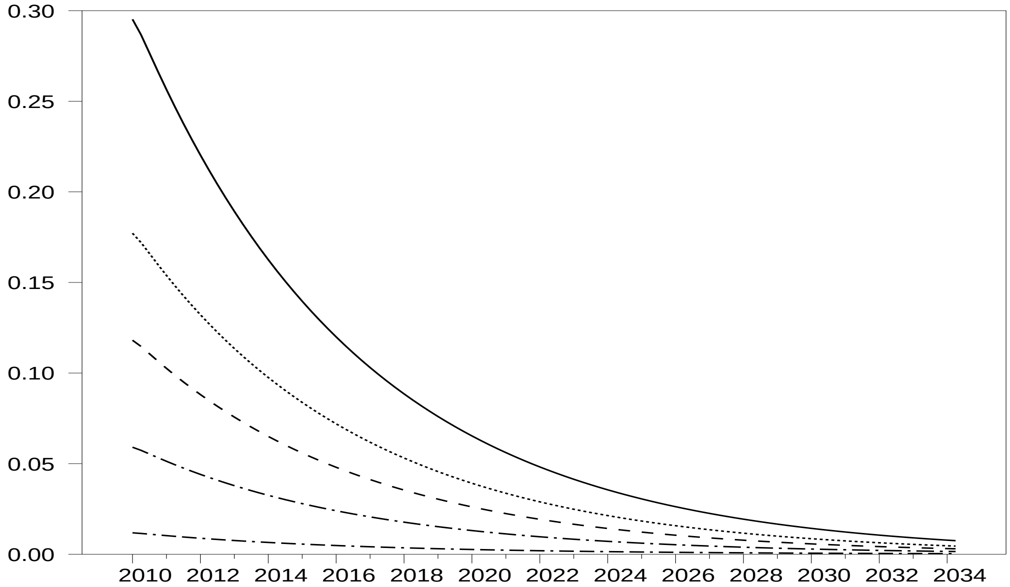
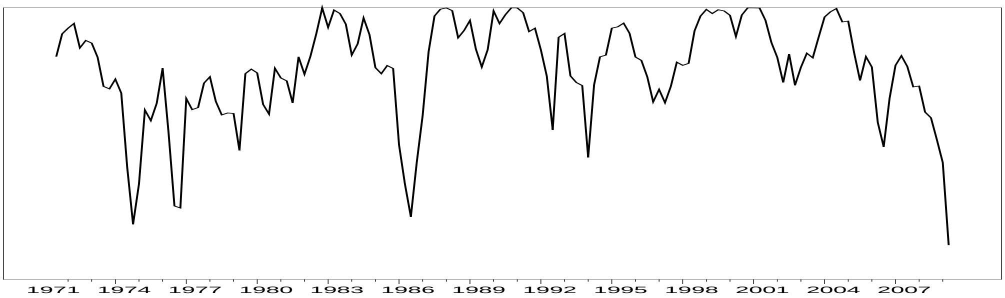
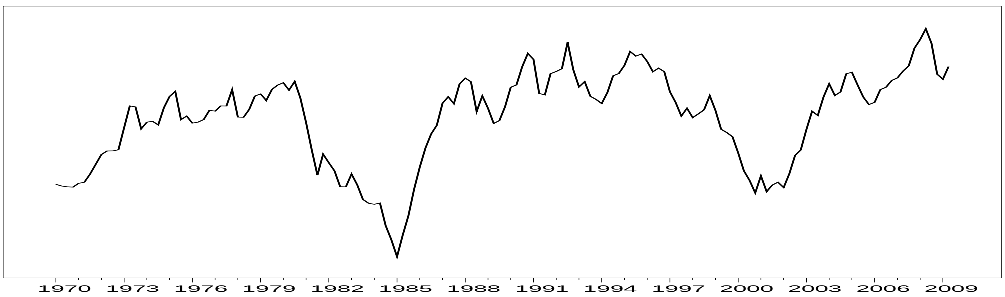


Figure 4: Estimated dollar overvaluation against euro

(a) dollar overvaluation relative to productivity fundamental



(b) mean reversion



(c) logarithm of the dollar-euro exchange rate

Table 1: KSS nonlinear unit root test

	Test with trend			Test with constant		
	Rer_t	$Difprod_t$	y_t	Rer_t	$Difprod_t$	y_t
AIC	-2.631*	-0.783	-3.203***	-2.497*	-2.769**	-3.190***
BIC	-2.631*	-0.962	-2.809**	-2.497*	-2.769**	-2.765**
HQ	-2.631*	-0.783	-3.203***	-2.497*	-2.769**	-3.190***
MAIC	-2.058	-0.300	-2.263	-1.973	-2.582**	-2.431*

Table shows the t-statistics of the null of unit root against nonlinear stationarity for different information criteria.*, ** and *** denote rejection of the null at 10%, 5% and 1% respectively. Critical values have been tabulated by stochastic simulation with T =150 and 50,000 replications allowing for a constant (Case 1) and for a trend (Case 2) under the alternative.

Asymptotic critical values

Fractile (%)	Case 1	Case 2
1	-3.16	-3.18
5	-2.50	-2.74
10	-2.11	-2.31

Table 2: P-values for the linearity test

Transition variable	H_0	H_{01}	H_{02}	H_{03}
y_{t-1}	0.50	0.24	0.77	0.34
y_{t-2}	0.07	0.02	0.35	0.33
y_{t-3}	0.06	0.08	0.04	0.77
y_{t-4}	0.07	0.21	0.02	0.92
y_{t-5}	0.01	0.14	0.00	0.66
y_{t-6}	0.01	0.42	0.00	0.82
y_{t-7}	0.11	0.42	0.01	0.70
y_{t-8}	0.92	0.62	0.62	0.94

Note: p-values of F variants of the LM-type tests for STAR nonlinearity of the quarterly deviation or the real dollar-euro exchange rate and the productivity differential between the Euro Area and the US euro-zone for the period 1970:Q1 to 2009:Q2. For a brief description of the test statistics see Section 2.

Table 3: Estimated ESTAR model

Estimated model:

$$y_t = (\underset{(0.000)}{1.215}\, y_{t-1} - \underset{(0.014)}{0.241}\, y_{t-2})\,[\,exp\,(\underset{[0.002]}{-0.450}\, y_{t-6}^2)]$$

Sample: 1970:Q1–2009:Q2

Diagnostic tests:

Autocorrelation 1-4:	1.896 [0.085]
ARCH 1-4:	6.726 [0.151]
Test for constancy of parameters:	0.601 [0.838]
Test for non remaining nonlinearity:	1.661 [0.134]

Note: Marginal significance levels for the "t-ratio" of the estimated transition parameter was calculated by Monte Carlo methods and are given in square brackets. Figures in parentheses below coefficient estimates denote the ratio of the estimated coefficient to the estimated standard error of the coefficient estimate. Autocorrelation 1-4 stands for the autocorrelation tests for the residuals up to 4 lags; ARCH 1-4 stand for autoregressive conditional heteroskedasticity tests (ARCH) up to order 4. Misspecification tests are constructed as Eitrheim and Teräsvirta (1996).

Table 4: Forecast evaluation of the estimated AR and ESTAR models

MPE			
h	AR	ESTAR	Better
1	0.00069	0.00038	ESTAR
2	-0.00432	-0.00387	ESTAR
3	-0.00772	-0.00617	ESTAR
4	-0.00931	-0.00722	ESTAR
MSPE			
h	AR	ESTAR	Better
1	0.00016	0.00016	ESTAR
2	0.00063	0.00062	ESTAR
3	0.00115	0.00113	ESTAR
4	0.00142	0.00139	ESTAR

Note: The forecast period runs from 2004:Q2 to 2009:Q2. The evaluation criteria are unconditional upon any values of the transition function. MPE is the mean prediction error and MSPE is the mean squared prediction error. h is for the steps ahead for which the forecasts are computed. The column *Better* indicates which model offers a best prediction.