Leveraged Buy Out and Tax saving advantage: a double-sided moral hazard model

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Abstract:

We consider a double sided moral hazard model with three agents: the entrepreneur, the LBO fund and the bank. The entrepreneur and the LBO fund have to provide efforts in order to improve the productivity of their project; efforts are not observable. When the bank's payments decrease with the revenues, the efforts depend on the project's quality: if the project is not very risky, the entrepreneur and the LBO fund provide the first best efforts and they share equally the benefit. If it is highly risky, they provide the second best efforts and the benefit's share given to each agent depends on the impact of his effort on the project's performance. When the bank's payments are non-decreasing, the agents' efforts do not depend on the project's quality. Whether the project is financed through the bank and the LBO fund or solely through the latter, the entrepreneur and the LBO fund provide the same levels of efforts.

We show that the excessive use of debt is explained by the tax saving advantage: the interests of the debt are tax-deductible which creates additional revenues but has no impacts on the agents' incentives.

Key words: LBO, tax advantage, double moral hazard, debt, capital structure.

JEL classification: G23, G24, G32.

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Introduction

Leveraged Buy Out commonly known as LBO[1] accounts for a significant part of the corporate finance and plays a major role in structuring mergers, acquisitions and transmissions.

There are three main facts about LBO finance:

First, the LBO fund (hereafter he) is an active investor and he is well connected with many industries: he is engaged in the day to day operations of the firm, he helps to recruit key personnel, he negotiates with suppliers, bank (hereafter he) and other financial partners and he advises the entrepreneur (hereafter she) on all the strategic decisions.

Second, the level of debt is significantly high. Moreover, there are many kinds of debt, such as the mezzanine debt, the subordinate debt and the convertible debt, each debt has a specific level of risk.

Finally, the use of convertible securities becomes prevalent in LBO finance. It is surprising because these securities are very rarely issued in the presence of banks or passive outside equity holders who finance more established and less risky companies.

This paper provides a theory of LBO financing based on a contractual approach. The theoretical model that we present deals with the two first facts. The model describes the relationship between the LBO fund and the entrepreneur engaged in the acquisition of a firm.

The success of these acquisitions is explained not only by the use of debt but it depends also on the market conditions, the performance of the *Op Co* and the partners' abilities. The entrepreneur is endowed with technical skills and he knows well the acquired company while the LBO fund plays a dual role: he is a financier and an adviser at the same time.

The question raised in this paper is the following: *why does the entrepreneur prefer asking for financing and for advice from the LBO fund and the bank, while the LBO fund alone is able to advise her and to finance the project?*

The LBO fund is usually not wealth-constrained, the private equity funds are well established financiers: these funds participate in the formation of large companies and they issue high level of equity. In Venture capital[2], the venture capitalist fund (hereafter VC fund) is usually the only financier in the project and he is also providing advice.

To answer the question, we consider three agents: the entrepreneur, the LBO fund[3] and the bank. The entrepreneur is the manager of the *Op Co* and she wants to acquire it. The entrepreneur

[1] LBO is the acquisition of a company, called the *Op Co*, using mostly debt and a small amount of equity. The debt is secured by the *Op Co* assets. The acquiring company, called the *holding* company or the *New Co*, uses these assets as collateral for the debt in hopes that the future cash-flows will cover the debt's payments.

[2] The venture capital is a type of the private equity capital provided by professional, outside investors to new high-potential-growth companies. The objective is to take the company to an IPO (*Initial Public Offering*).

[3] In the last section, we discuss the case where the LBO fund does not contribute financially into the project: he will be considered as a consultant.

is wealth-constrained this is why she asks first for advice and for money from the LBO fund. These two partners sign a first contract: the holding contract and they establish the holding company (hereafter the holding). They can also ask for additional funding from the bank and then they sign a second contract: the debt contract. The entrepreneur and the LBO fund have to exert non observable efforts which induce a double-sided moral hazard problem.

The optimal financial contracts have to meet three objectives: (1) each agent gets at least the cost of his initial investment (2) to determine the payments of each agent when the project succeeds and when it fails and (3) to incite the entrepreneur and the LBO fund to exert efficient efforts.

We study the impacts of financial capital structure on the efforts when there is a double-sided moral hazard problem. To our knowledge, there are no papers exploring this issue in LBO. In the opposite with LBO, a number of papers study this question on the venture capital.

Several papers focus on the relationship between capital structure of the start-ups and the incentives to efforts in the presence of a double-sided moral hazard problem. Bergemann and Hege (1998) present a dynamic agency model with learning and moral hazard problems. They show that short-term refinancing[4] is never optimal but long-term contract allowing for intertemporal risk-sharing such as the stage financing[5] is optimal. This financial regime constrains the entrepreneur to provide optimal effort so that she could obtain funding for his project. If not, the VC fund will not invest additional capital and will abandon the project.

Cornelli and Yosha (2003) show that the stage financing may induce a *window dressing* problem: the entrepreneur is tempted to announce a good short-term performance. Her aim is to reduce the probability that the project will be abandoned or liquidated. They conclude that the use of the convertible debt solves the "*window dressing*" problem: when the project looks too profitable, the VC fund will convert his debt into equity which reduces the entrepreneur's profit.

Schmidt (1999, 2003) does not study the stage financing regime but he focuses on the incentive properties of the convertible securities. He proposes a model where the profit that can be generated by the entrepreneur and the VC fund depends on three factors: the quality of the project and/or the abilities of the entrepreneur (the state of the world), their efforts, and further financial investment of the VC fund. The state of the world has an impact on the results of the project. It is unknown to both parties at date 0 and can be observed only after the initial investment has been sunk. Schmidt shows that there is no debt-equity contract that induces both parties to invest efficiently. He joins Cornelli and Yosha and he deduces that convertible securities constraint both agents to exert optimal efforts. When he considers a multi-dimensional investment[6], the convertible debt contract still implements the first best investments of both parties.

[4] The entrepreneur sign a new contract with a new VC fund, but only for one period. The next period, she is looking for another one; the VC market is supposed to be competitive.

[5] While the commitment is to fund the entire amount, in venture capital projects, the funding is contingent on the company attaining his short-term objectives. At each stage, the entrepreneur asks for additional capital from the VC fund.

[6] First, he assumed that entrepreneur's investment is one-dimensional. Then, he supposed that the entrepreneur has to choose a multi-dimensional investment vector such as to invest in R&D, to spend effort in order to organize the firm, to hire the key staff and to invest in marketing, in supplier and customer relations.

My model is closely related to those of Casamatta (2003) and Repullo and Suarez (2004). They present a double-sided moral hazard model with a pure financier.

Casamatta considers a model with three agents: an entrepreneur endowed with an innovative idea, an adviser and a pure financier. When the adviser invests money in the project he is a VC fund, and he is a consultant when he only exerts effort. The entrepreneur and the advisor have to provide substitute efforts. When these efforts are observable, whether the advisor is a consultant or a VC fund is irrelevant: there are many ways to implement the first best. This is no longer true when efforts are unobservable. The consultant's effort is less efficient than her proper effort. Thus, she prefers asking for the fund's advice.

In order to solve the double moral hazard problem, she shows that all agents must participate financially in the project. When the project is not very risky, the presence of the pure financier induces both agents to exert first best efforts. On the contrary, when it is very risky, they need high powerful incentives to make efficient efforts. Casamatta proposes to pledge the revenue to the pure financier when the project fails and to let the entrepreneur and the VC fund share the revenue in case of success: it is a "*live or die*" contract (Iness, 1990). This is still not sufficient to induce them to provide the first best efforts; the moral hazard problem induces them to provide the second best efforts.

Repullo and Suarez consider a stage financing model. The entrepreneur is wealth-constrained and she asks for advice and for money from two VC funds. One of them does not provide effort so he may be considered as a pure or passive financier. The entrepreneur and the other fund have to exert non observable efforts. They conclude that the entrepreneur must ask for advice and fund from the partner who provides both money and advice. The project gives no gains when it fails and φ otherwise, where φ is a continuous random variable. φ is non observable when they sign the contract, they will learn more about it at the end of the first period. When the project is profitable, it is continued, otherwise it is abandoned. When it is continued, the VC fund must invest additional capital.

We consider a single-period model and that the entire cost of the project is invested when the contracts are signed. In contrast with Casamatta, we suppose that the entrepreneur is the only wealth-constrained agent and that the efforts are complementary. There is a continuum of LBO funds and banks. If the debt and the holding contracts are signed, the partners cannot abandon the project.

We determine endogenously the financial contribution of each agent and their revenues' shares. We focus particularly on the presence of the bank and its impact on the agents' incentives. We find that the LBO fund must issue a significant part of equity and that the bank's payments decrease with the project's revenues. We join Casamatta (2003), Jensen (1986, 1989), Jensen and Meckling (1976) and show that the debt induces the agents to provide the highest efforts. Furthermore, the way the project's outcome is shared between the agents, the bank's payments and the provided efforts depend on the project's quality but they do not depend on the financial capital structure. Our results show that:

- When the project is not very risky, we generalize the Casamatta's model and we show that both agents exert the first best efforts. We conclude that they must get equal shares of the project's benefit.

- When the project is very risky, the entrepreneur and the LBO fund provide the second best efforts. Double moral hazard problem induces them to make less efficient investment decisions. They share the revenue in the good state of nature and pledge the entire revenue to the bank in the bad state of the nature. The sharing rule of the benefit depends on the impact of each effort on the project's performance.

If the debt's payments are non-decreasing with the results' project, the presence of the bank has no impacts on the agents' incentives: whether the project is financed through the LBO fund and the bank or only through the fund, the agents provide equal levels of efforts: these efforts do not depend on the financial capital structure.

The taxation creates additional revenues which explain why the entrepreneur asks for advice anf for money from the bank and the LBO fund while the latter is not wealth constrained. But, we show that the tax saving advantage does not influence the efforts provided by the entrepreneur.

The model and the assumptions are presented in the section 1. Section 2 solves the model and derives the properties of the optimal financial contracts when all agents can invest financially in the entrepreneur's project. In section 3, we suppose that the debt's payments are non-decreasing with the revenues of the project and study the agents' incentives to efforts. Section 4 compares these contracts with those provided when the LBO fund does not contribute financially to the acquisition. In Section 5, we introduce the tax advantage in the model and deduce the optimal financial capital. Concluding remarks are in Section 6. All the proofs are presented in the Appendix.

1. The model

We consider an entrepreneur E who wants to acquire a company where she is the manager. The cost of the project is equal to K. She invests W but she is wealth-constrained. So she asks first for advice and for money from the LBO fund A. She concedes a part of the company's benefit $1-\beta$ $(0 \leq \beta \leq 1)$ and he issues equity i. These two partners may ask for additional financing from the bank B. The latter may lend them an amount $I = K - (W + i)$ which corresponds to a debt D.

The project is risky and generates an observable random revenue θ. It depends on the quality of the project, the entrepreneur skills, the market conditions... It can take two values: θ^u when the project succeeds and θ^d when it fails where $\theta^u > \theta^d$. When the project fails, θ^d is equal to its liquidation value. The probability of success is denoted $p(e,a)$, where e and $a \in \mathbb{R}_+$ are the efforts provided respectively by the entrepreneur and the LBO funds. The entrepreneur's effort is related to technical skills. The LBO fund may exert a technical effort or a managerial effort such as the monitoring. We assume that the probability function $p(e,a)$ is increasing and concave. We add the condition $\frac{\partial^2 p(e,a)}{\partial e \partial a} > 0$ so that we ensure that efforts are complementary. Furthermore, $p(e,0) = p(0,a) = 0$: it means that both agents must provide strictly positive efforts so that the probability of success would be strictly positive. Making efforts $(e,a) = (0,0)$ is a Nash equilibrium but it means that the project fails with probability one.

There is a continuum of LBO fund and banks. All agents are risk neutral and the risk-free rate is normalized to 0. When contracts are signed, agents cannot abandon the project before the date of exit.

1.1. The sequence of events in the model

The sequence of events is summarized in the following figure:

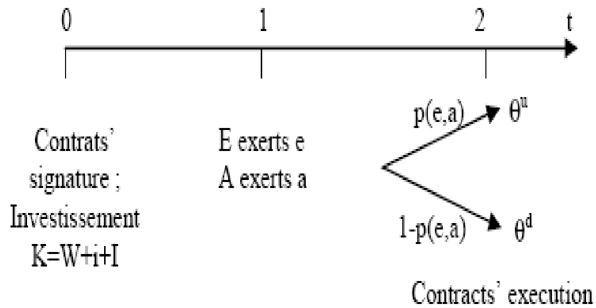

FIG. 1 - The sequence of events in the model.

- At the date 0, E and A negotiate and sign a first contract: the holding contract. If they need further financial investment, they can ask for required money from the bank. Then, the holding and the bank B sign a second contract: the debt contract.
- At the date 1, E and A have to exert respectively the non contractible efforts e and a. Let $u(e)$ and $v(a)$ denote their respective cost functions. These functions are increasing, convex and satisfy $u(0) = v(0) = u'(0) = v'(0) = 0$.
- At the date 2, the project is completed. If it succeeds, the bank gets the payment D and the two agents share the residual amount; they get respectively $\beta(\theta^u - D)$ and $(1-\beta)(\theta^u - D)$. Otherwise, the bank perceives the collateral H ($H \leq \theta^d$). If $H = \theta^d$, the entrepreneur and the LBO fund get zero payoffs. If $H < \theta^d$, they obtain respectively $\beta(\theta^d - H)$ and $(1-\beta)(\theta^d - H)$.

1.2. Financial contracts

Two financial contracts must be specified in this model:

1. The holding contract: it determines the amount of equity that must be issued by the LBO fund and his share of benefits. He participates in the project if his revenue is positive. This constraint is written:

$$EU^A = (1-\beta)[p(e,a)(\theta^u - D) + (1-p(e,a))(\theta^d - H)] - v(a) - i \geq 0 \qquad (PC_A)$$

where (PC_A) is the participation constraint of the LBO funds. The agent E asks for money and for advice from many LBO funds. He deals with the one who makes the best offer (i, β). This offer gives him a zero payoff; (PC_A) is binding which implies that $EU^A = 0$.

2. The debt contract: the bank lends I at the date 0. At the date 3, he is paid D when the project's result is success, otherwise he gets H. The bank is willing to lend money only if he will recoup what he would get if he makes a risk-free investment. It means that:

$$EU^B = p(e,a)D + (1 - p(e,a))H - I \geq 0 \qquad (PC_B)$$

where (PC_B) is the participation constraint of the bank. Because of the competition between the banks, we deduce that $EU^B = 0$.

1.3. The first best efforts

The social value of the project $V(e,a)$ is given by:

$$V(e,a) = p(e,a)\Delta\theta + \theta^d - u(e) - v(a) - K$$

where $\Delta\theta = \theta^u - \theta^d$. The first best efforts e^{FB} and a^{FB} are deduced from the first order conditions of $V(e,a)$. They are given respectively by:

$$\left.\frac{\partial V(e,\,a)}{\partial e}\right|_{e^{FB},\,a^{FB}} = p_e(e^{FB}, a^{FB})\Delta\theta - u'(e^{FB}) = 0 \qquad (1)$$

$$\left.\frac{\partial V(e,\,a)}{\partial a}\right|_{e^{FB},\,a^{FB}} = p_a(e^{FB}, a^{FB})\Delta\theta - v'(a^{FB}) = 0 \qquad (2)$$

The equations (1) and (2) can be written:

$$\Delta\theta = \frac{u'(e^{FB})}{p_e(e^{FB}, a^{FB})} = \frac{v'(a^{FB})}{p_a(e^{FB}, a^{FB})} \qquad (3)$$

The ratios of the marginal cost to the marginal probability of efforts are equal to the difference between the revenues of the bad and the good states of nature. The probability of success when the agents provide first best effort is denoted $p^{FB} = p(e^{FB},\, a^{FB})$. Then, the optimal social value of the project is given by:

$$V^{FB} = p^{FB}\Delta\theta + \theta^d - u(e^{FB}) - v(a^{FB}) - K$$

V^{FB} is assumed to be strictly positive.

The first best solution can be implemented in a number of ways: the entrepreneur and the LBO fund may ask money from the bank or the LBO fund may invest $i = K - W$. Without transaction costs, there are many ways to implement the first best solution: the entrepreneur can ask for advice and for money from the LBO fund (they sign one contract: the holding contract) or she can rely on the LBO fund and the bank (they sign two contracts: the holding and the debt contracts). She may ask also for advice from the consultant and for money from the bank. In this case, the entrepreneur is indifferent towards the identity of the agent providing money and advice.

2. The optimal financial contracts with a double sided moral hazard problem

2.1. The reaction functions

We suppose now that efforts are unobservable. The entrepreneur (respectively the LBO fund) exerts the level of effort e (respectively a) that maximizes her (respectively his) expected profit given the efforts' costs and the established contracts: hence, each effort is an increasing function of the other one.

The reaction functions of E and A are given by their incentive constraints. They are respectively written:

$$e(a) \in \arg\max_{e \in \mathbb{R}_+} \beta[p(e,a)(\theta^u - D) + (1-p(e,a))(\theta^d - H)] - u(e) - W \qquad (IC_E)$$

$$a(e) \in \arg\max_{a \in \mathbb{R}_+} (1-\beta)[p(e,a)(\theta^u - D) + (1-p(e,a))(\theta^d - H)] - v(a) - i \qquad (IC_A)$$

These functions $e(a)$ and $a(e)$ are respectively the solutions of the following equations:

$$\beta(\Delta\theta - D + H) = \frac{u'(e)}{p_e(e,a)} \qquad (4)$$

$$(1-\beta)(\Delta\theta - D + H) = \frac{v'(a)}{p_a(e,a)} \qquad (5)$$

By differentiating the equations (4) and (5), we obtain:

$$e'(a) = \frac{u'(e)p_{ea}(e,a)}{u''(e)p_e(e,a) - u'(e)p_{ee}(e,a)} \geq 0$$

$$a'(e) = \frac{v'(a)p_{ea}(e,a)}{v''(a)p_a(e,a) - v'(a)p_{aa}(e,a)} \geq 0$$

The effort of the entrepreneur (respectively the LBO fund) is an increasing function of the effort of the LBO fund (respectively of the entrepreneur): the efforts are strategic complements.

2.2. The optimal financial contracts when the efforts are unobservable

Competition among the LBO funds and the banks induces A and B to propose respectively holding and debt contracts that maximize the expected profit of the entrepreneur. The latter must maximize her expected gain under their participation constraints and the incentive constraints. The program to be solved is therefore given by:

$$(\beta^*, i^*, I^*, D^*, H^*) \in \arg\max_{\beta,\, i,\, I,\, D,\, H} EU^E = \beta[p(e,a)(\Delta\theta - D + H) + \theta^d - H] - u(e) - K + i + I$$

$$\text{s.t} \qquad (PC_A),\ (PC_B),\ (IC_A) \text{ and } (IC_E)$$

with the additional conditions:

$$0 \leq D \leq \theta^u,\ 0 \leq H \leq \theta^d \text{ and } 0 \leq \beta \leq 1 \qquad (6)$$

The participation constraints (PC_A) and (PC_B) are binding, so the amounts of equity and debt issued respectively by the LBO fund and the bank are given by:

$$i = (1-\beta)[p(e,a)\,(\Delta\theta - D + H) + \theta^d - H] - v(a) \quad (7)$$

$$I = p(e,a)\,(D-H) + H \quad (8)$$

We replace (7) and (8) in the objective function. The optimal financial contracts induce the entrepreneur to maximize the social value of the project under the incentive constraints (IC_E) and (IC_A). Hence, the optimal financial contracts induce the entrepreneur to maximize the social value of the project under the incentive constraints:

$$\max_{\beta,\ D,H} V(e,a) = \quad p(e,a)\Delta\theta + \theta^d - u(e) - v(a) - K$$
$$\text{s.t} \qquad (4) \text{ and } (5)$$

with the conditions (6).

2.2.1. The project is not very risky

The social value of the project is optimal if the agents exert first best efforts, in other words, the efforts e^{FB} and a^{FB} must satisfy the incentive constraints (4) and (5).

Given the equations (3), we replace in (4) and (5). We obtain:

$$\Delta\theta \;= \beta(\Delta\theta - D + H) = (1-\beta)(\Delta\theta - D + H) \quad (9)$$

It means that the benefit's share of the entrepreneur is equal to $\beta = \frac{1}{2}$. The revenues of the project are fixed, if the optimal contracts give powerful incentives to one agent, it will reduce the incentives of the other agent. The optimal financial contracts must boost the incentives of both agents such that they provide the first best efforts; so they must get equal profit shares. We notice that this sharing rule depends neither on the capital structure, nor on the efforts' efficiency.

When we substitute $\beta = \frac{1}{2}$ in (9), we get:

$$H = \Delta\theta + D$$

This means that $\Delta\theta + D < \theta^d \iff D < 2\theta^d - \theta^u$, but $D \geq 0$, in other words, the project must be not very risky in the sense $\theta^u \leq 2\theta^d$. We conclude that the bank payments are decreasing with the outcome of the project: the collateral is larger than his payment when the project succeeds.

The following proposition presents the properties of the optimal financial contracts:

Proposition 1 *If the project is not very risky* $\left(\theta^u \leq 2\theta^d\right)$, *under the conditions* $\theta^d \in [l + max\{v(a^{FB}) - p^{FB}\Delta\theta, 0\},\ 2l - p^{FB}\Delta\theta]$ *and* $\theta^u \in [\ l,\ 2l\ ]$, *the optimal financial contracts are given by:*

$$H^{*} = 2[K - W + v(a^{FB})] - \theta^d$$
$$D^{*} = 2[K - W + v(a^{FB})] - \theta^u$$
$$I^{*} = 2[K - W + v(a^{FB})] - p^{FB}\Delta\theta - \theta^d$$

and

$$i^{*}= V^{FB}+W + u(e^{FB}) - v(a^{FB})$$
$$\beta^{*}=\frac{1}{2}$$

where $l = K - W + v(a^{FB})$.

The proof of this proposition is presented in the appendix A.

When the project is not very risky, the LBO fund issues a high amount of equity. Moreover the LBO fund and the entrepreneur provide the first best efforts and they get equal shares of the benefit. Although i is larger than W, their payments are equal. They get $\theta^u + W - K - v(a^{FB})$, if the project succeeds and $\theta^d + W - K - v(a^{FB})$ when it fails. The difference between these payments is equal to the difference between the revenues of the project $\Delta\theta < \theta^d$.

The intuition of the proposition 1 is the following: in order to induce the agents to provide optimal efforts, the bank payments must be decreasing with the outcome of the project. Indeed, despite the fact that $\Delta\theta$ is not very large, they are constrained to provide first best efforts in order to perceive the highest payments; the success payments.

We conclude that when the project is not very risky in the sense defined in proposition 1, the presence of the bank constrains the agents to exert optimal efforts. In addition, the entrepreneur captures the whole social value of the project while the LBO fund and the bank recoup the costs of their investments.

2.2.2. The project is very risky

When the project is very risky ($\theta^u > 2\theta^d$), the general model does not enable us to deduce the properties of the optimal financial contracts. This is why, we rely in the following specification:

$$p(e, a) = e^{1-\alpha}a^{\alpha}$$

where $\alpha \in]0, 1[$ measures the impact of the effort of the LBO fund on the success' probability. In other words, it is the LBO's effort elasticity of the success probability. The functions of cost are given by:

$$u(e) = \frac{e^2}{2\lambda} \text{ and } v(a) = \frac{a^2}{2\mu}$$

where e and a take values on $[0\ ,\ 1]$ and $\lambda > \mu > 0$: the LBO's effort is more costly than the entrepreneur's effort. We substitute $e^{1-\alpha}a^{\alpha}$, $\frac{e^2}{2\lambda}$ and $\frac{a^2}{2\mu}$ in the expected gain of the entrepreneur and the LBO fund.

The optimal solution

The social value of the project is given by:

$$V\,(e\ ,\ a) = e^{1-\alpha}a^{\alpha}\Delta\theta - \frac{e^2}{2\lambda} - \frac{a^2}{2\mu} + \theta^d - K$$

The first best efforts are given by the first order conditions of $V(e\ ,\ a)$:

$$e^{FB} = \phi(\alpha)\Delta\theta \tag{10}$$

$$a^{FB} = \left[\frac{\mu\alpha}{\lambda(1-\alpha)}\right]^{\frac{1}{2}} \phi(\alpha)\Delta\theta \tag{11}$$

where $\phi(\alpha) = [\lambda(1-\alpha)]^{1-\alpha/2}[\mu\alpha]^{\alpha/2}$. These efforts are increasing with the difference between the revenues of the project. When the project is very risky, the levels of the first best efforts become very large. Furthermore, when the impact of the effort of the LBO fund on the success probability is high such that $\alpha \geq \frac{\lambda}{\mu+\lambda}$, his first best effort is larger than the effort provided by the entrepreneur. Otherwise, the entrepreneur provides the highest effort.

We denote by $p^{FB} = \frac{\phi^2(\alpha)}{\lambda(1-\alpha)}\Delta\theta$ the probability of success of the project when agents provide the first best efforts. To ensure that this probability is inferior to 1, we assume that:

$$\Delta\theta \leq \frac{\lambda(1-\alpha)}{\phi^2(\alpha)}$$

The optimal social value of the project is given by:

$$V^{FB} = \frac{[\phi(\alpha)\Delta\theta]^2}{2\lambda(1-\alpha)} + \theta^d - K$$

When α is not high enough such that $\alpha \in]0, \frac{\lambda}{\mu+\lambda}[$, this value decreases with α and it increases when α becomes high $\left(\alpha \in [\frac{\lambda}{\mu+\lambda}, 1[\right)$.

The solution when the agents' efforts are unobservable

When there is asymmetric information, the reaction functions (4) are (5) give the following efforts in equilibrium:

$$e^* = [\lambda\beta(1-\alpha)\rho(\beta)]^{1/2}(\Delta\theta - D + H) \tag{12}$$

$$a^* = [\mu\alpha(1-\beta)\rho(\beta)]^{1/2}(\Delta\theta - D + H) \tag{13}$$

where $\rho(\beta) = [\lambda\beta(1-\alpha)]^{1-\alpha}[\mu\alpha(1-\beta)]^{\alpha}$. It is easy to check that the difference between the bank's payments in cases of success and failure must be inferior to the difference between the project's revenues θ^d and θ^u, i.e.

$$D - H \leq \Delta\theta \tag{14}$$

Otherwise, the efforts (12) and (13) are negative. Besides, the agents' efforts are increasing with the difference between the project's revenues. The more high the bank's payment in case of success, the less the entrepreneur and the LBO fund will be induced to provide efforts. At the opposite, the increase of H increases the efforts e and a.

Then, the project succeeds with the following probability:

$$p^* = \rho(\beta)(\Delta\theta - D + H).$$

We add the following assumption:

$$(\Delta\theta - D + H) \leq \rho^{-1}(\beta)$$

to ensure that this probability is inferior to 1.

If the financial contracts attribute all the gain to the entrepreneur or to the LBO fund, the project will fail with probability equal to 1 ($\rho(0) = \rho(1) = 0$). This enables us to conclude that: $0 < \beta < 1$.

The expected gain of the entrepreneur is given by:

$$EU^E = \frac{1}{2}(1+\alpha)\beta\rho(\beta)(\Delta\theta - D + H)^2 + \beta\left(\theta^d - H\right) - W \tag{15}$$

Because of the competition among the LBO funds and the bank, the participation constraints (CP_A) and (CP_B) are binding. We deduce that:

$$i = (1 - \frac{1}{2}\alpha)(1-\beta)\rho(\beta)(\Delta\theta - D + H)^2 + (1-\beta)\left(\theta^d - H\right) \tag{16}$$

$$I = \rho(\beta)(\Delta\theta - D + H)(D - H) + H \tag{17}$$

We substitute that (12), (13), (16) and (17) in (15). Consequently, the entrepreneur is induced to maximize the social value of the project:

$$\begin{aligned}\max_{\beta,\ D,\ H} V(.) = \ & \rho(\beta)\Delta\theta(\Delta\theta - D + H) + \theta^d - K \\ & -\tfrac{1}{2}(\alpha + \beta - 2\alpha\beta)\rho(\beta)(\Delta\theta - D + H)^2\end{aligned}$$

with the conditions:

$$0 < \beta < 1,\ 0 \leq D \leq \theta^u \text{ and } 0 \leq H \leq \theta^d \tag{18}$$

The following proposition summarizes the properties of the optimal financial contracts when the project is very risky:

Proposition 2 *If the project is very risky* $\left(\theta^u > 2\theta^d\right)$, *the optimal financial contracts are given by:*

$$\hat{\beta}(\alpha,k) = \begin{cases} \frac{(1-\alpha)(2-\alpha)-k-\sqrt{\Delta}}{2(1-2\alpha)} & \textit{if } \alpha \in \left]0\ ,\ 1\right[/\{\frac{1}{2}\} \textit{ and } k \in \left[0\ ,\ \frac{1}{2}\right[\\ \frac{1}{2} & \textit{if } \alpha = \frac{1}{2} \end{cases}$$

$$\hat{\imath} = (1 - \frac{1}{2}\alpha)(1-\hat{\beta})\rho(\hat{\beta})(\theta^u)^2$$

and

$$\hat{D} = 0\ ,\quad \hat{H} = k\theta^u \quad \textit{and} \quad \hat{I} = k\theta^u(1 - \theta^u\rho(\hat{\beta}))$$

where $k = \frac{\theta^d}{\theta^u}$ *and* $\Delta = \alpha(1-\alpha)\theta^u\left[(1+\alpha)(2-\alpha)\theta^u - 6\theta^d\right] + (\theta^d)^2$.

The proof of this proposition is presented in the appendix B.

This proposition shows that the optimal sharing rule $\hat{\beta}$ is a function of α. When $\alpha > \frac{1}{2}$, the fund's share of benefit is larger than the entrepreneur's share. Despite the fact that the effort of LBO fund is more costly than the entrepreneur's effort, it has the most important impact on the probability of success. It is more efficient to give him powerful incentives to induce him to provide effort; for example attributing him the highest share of the project's outcome. When the impact of the fund's effort on the success' probability is low, it is efficient to give powerful incentives to the agent who provides the less costly effort.

When the project is highly risky, the agents make the efforts:

$$\hat{e} = [\lambda\hat{\beta}(1-\alpha)\rho(\hat{\beta})]^{\frac{1}{2}}\theta^u \quad \text{and} \quad \hat{a} = [\mu\alpha(1-\hat{\beta})\rho(\hat{\beta})]^{\frac{1}{2}}\theta^u \tag{19}$$

If we substitute $\rho(\hat{\beta})$ for its expression in (19), we can write:

$$\hat{e} = \hat{\beta}^{1-\frac{\alpha}{2}}(1-\hat{\beta})^{\frac{\alpha}{2}}\phi(\alpha)\theta^u \quad \text{and} \quad \hat{a} = \hat{\beta}^{\frac{1-\alpha}{2}}(1-\hat{\beta})^{\frac{1+\alpha}{2}}\left[\frac{\mu\alpha}{\lambda(1-\alpha)}\right]^{\frac{1}{2}}\phi(\alpha)\theta^u \tag{20}$$

These efforts depend on α.

We focus on the particular case where $\alpha = \frac{1}{2}$. Hence $\hat{e}$ and $\hat{a}$ are written:

$$\hat{e} = \frac{1}{4}\lambda^{\frac{3}{4}}\mu^{\frac{1}{4}}\theta^u = \frac{1}{2}\frac{\theta^u}{\Delta\theta}e^{FB} \quad \text{and} \quad \hat{a} = \frac{1}{4}\lambda^{\frac{1}{4}}\mu^{\frac{3}{4}}\theta^u = \frac{1}{2}\frac{\theta^u}{\Delta\theta}a^{FB} \tag{21}$$

The double moral hazard induces the agents to make less efficient efforts: the levels of first best efforts are high. We need powerful incentive mechanisms to constrain the entrepreneur and the LBO to exert the optimal efforts. The revenue should be shared between them in the case of success, and the entire revenue should be paid to the bank when the project fails. However it is still not sufficient to induce them to spend the first best efforts.

Notice that the LBO fund issues a significantly high amount of equity $i = \frac{3}{32}(\mu\lambda)^{\frac{1}{2}}(\theta^u)^2$. Under the condition $\theta^u < 4(\mu\lambda)^{-\frac{1}{2}}$, the amount of debt is given by $\hat{I} = \theta^d\left[1-\frac{1}{4}(\mu\lambda)^{\frac{1}{2}}\theta^u\right]$. The entrepreneur cannot get the whole optimal social value, her expected gain is given by:

$$EU^E = \left(2\alpha\theta^u - \theta^d\right)\frac{\phi^2(\alpha)}{2\lambda(1-\alpha)}\theta^u + \theta^d - K < V^{FB}$$

It is straightforward to see that the failure payment of the bank is superior to his success payment. To get the highest revenues, the entrepreneur and the LBO fund may be tempted to announce a success whether the project is succeeded or failed. If the project's result is a failure, they sell the assets and the equipment, pay D to the bank and share the remaining amount of money.

To avoid such behavior, we consider the hypothesis where the bank's payments are non-decreasing with the projects' revenues.

3. The optimal financial contracts when the debt's payments are non-decreasing with the project's revenues

Hereafter, we assume that the bank's payment is the highest in case of success. In other words, we add the following condition:

$$D \geq H. \tag{22}$$

to the program described in the previous section.

The objective of the entrepreneur is to maximize the social value of the project:

$$\begin{aligned} \max_{\beta,\ D,\ H} V(.) = \quad & \rho(\beta)\Delta\theta(\Delta\theta - D + H) + \theta^d - K \\ & -\tfrac{1}{2}\left(\alpha + \beta - 2\alpha\beta\right)\rho(\beta)(\Delta\theta - D + H)^2 \end{aligned}$$

with the following conditions:

$$(18) \text{ and } (22)\,.$$

The optimal financial contracts are characterized in the following proposition:

Proposition 3 *When the debt's payments are non-decreasing with the project's revenues, it is optimal to set* $H \in \left[0\ ,\ \theta^d\right]$*, the optimal financial contracts are given by*

$$\breve{\beta}(\alpha) = \begin{cases} \frac{(2-\alpha)(1-\alpha)-\sqrt{\alpha(1-\alpha^2)(2-\alpha)}}{2(1-2\alpha)} & \text{if } \alpha \in \left]0,\ 1\right[/ \left\{\frac{1}{2}\right\} \\ \frac{1}{2} & \text{if } \alpha = \frac{1}{2} \end{cases} \tag{23}$$

$$\breve{\imath} = (1-\frac{1}{2}\alpha)(1-\breve{\beta})\rho(\breve{\beta})(\Delta\theta)^2.$$

and

$$\breve{D} = \breve{H} = \breve{I} = K - W - (1-\frac{1}{2}\alpha)(1-\breve{\beta})\rho(\breve{\beta})(\Delta\theta)^2$$

The proof of this proposition is presented in the appendix C.

In contrast with the previous propositions, this proposition states that the bank may not contribute financially to the entrepreneur's project. In this case, it is financed solely with equity; the LBO fund issues all the needed capital $i = K - W$.

Whether the project is very risky or not, the non-decreasing revenue constraint imposes to give to the bank at least as much in the good state as in the bad state. He perceives a fixed payment in cases of success and failure. This payment is equal to the amount of the issued debt which implies that the interest rate of the bank is null. Consequently, the amount of issued debt is equal to his payment. Lending money to the entrepreneur and the LBO fund is not risky investment to the bank. He will get his money back whatever the results' project.

Moreover, there are an infinity number of financial contracts $n \in \left[0\ ,\ \theta^d\right]$ that enable to implement this solution.

The optimal sharing rule $\breve{\beta}(\alpha)$ depends neither on the financial capital structure nor on the project's revenues. But, it depends on the impact of each effort on the project's performance:

If the LBO's effort has a low impact on the success probability, in the sense $\alpha \in]0, \frac{1}{2}[$, the LBO fund perceives the lowest benefit's share. At the opposite, the effort a is more efficient than e, this is why the optimal financial contracts must induce him to provide the highest effort. So, he gets the highest benefit's share.

If $\alpha = \frac{1}{2}$, both efforts have equal impacts on the success probability: given that e and a are complementary efforts and the revenues are fixed, the optimal financial contracts must boost simultaneously the agents' incentives: the entrepreneur and the LBO fund have equal payments.

Let' us now compare the sharing rules $\breve{\beta}(\alpha)$ and $\hat{\beta}(\alpha, k)$ given by the proposition 2: when k converges to 0, whatever the value of α, we find $\breve{\beta} = \hat{\beta}$. However, when k and α converge respectively to $\frac{1}{2}$ and 1, we get $\breve{\beta}(\alpha) < \hat{\beta}(\alpha, k)$. In the opposite, if k and α converge respectively to $\frac{1}{2}$ and 0, $\breve{\beta}(\alpha) > \hat{\beta}(\alpha, k)$.

The entrepreneur and the LBO fund provide the following efforts:

$$\breve{e} = [\lambda\breve{\beta}(\alpha)(1-\alpha)\rho(\breve{\beta}(\alpha))]^{\frac{1}{2}}\Delta\theta \quad \text{and} \quad \breve{a} = [\mu\alpha(1-\breve{\beta}(\alpha))\rho(\breve{\beta}(\alpha))]^{\frac{1}{2}}\Delta\theta \tag{24}$$

These efforts are also written:

$$\breve{e} = \left[\breve{\beta}(\alpha)\right]^{1-\frac{\alpha}{2}}[(1-\breve{\beta}(\alpha)]^{\frac{\alpha}{2}}e^{FB} \quad \text{and} \quad \breve{a} = \left[\breve{\beta}(\alpha)\right]^{\frac{1-\alpha}{2}}[(1-\breve{\beta}(\alpha)]^{\frac{1+\alpha}{2}}a^{FB} \tag{25}$$

They induce the success of the entrepreneur's project with the following probability:

$$\breve{p} = \rho(\breve{\beta}(\alpha))\Delta\theta \tag{26}$$

Financing the acquisition with a mixture of debt and equity or just with equity, the provided efforts do not depend on the financial capital structure: the entrepreneur and the LBO fund provide the same levels of efforts.

According to these results, when we consider that the bank's payments are non-decreasing with the project's revenues, the debt has no impacts on the agents' incentives: the levels of efforts (25) are lower than the first best levels and even than the efforts provided by the entrepreneur and the LBO when the project is very risky and the debt's payments are decreasing with the project's payoff (see proposition 2).

When $\alpha = \frac{1}{2}$, the entrepreneur and the LBO fund provide the following efforts:

$$e^c = \frac{1}{4}\lambda^{\frac{3}{4}}\mu^{\frac{1}{4}}\Delta\theta < e^{FB} \quad \text{and} \quad a^c = \frac{1}{4}\lambda^{\frac{1}{4}}\mu^{\frac{3}{4}}\Delta\theta < a^{FB} \tag{27}$$

In this case, the expected gain of the entrepreneur is written:

$$EU^E = \frac{\alpha[\phi(\alpha)\Delta\theta]^2}{2\lambda(1-\alpha)} + \theta^d - K < V^{FB} \tag{28}$$

4. Optimal financial contracts when the LBO fund does not issue equity: the case of a consultant

We suppose now that the LBO fund does not contribute financially to the project but he still spend the effort a. The LBO fund can be considered as a simple consultant. The two partners will ask for the remaining amount $K - W$ from the bank.

The participation constraints of the consultant and the bank (CP_A) and (CP_B) are written:

$$EU^A = (1-\beta)[p(e,a)\,(\Delta\theta - D + H) + \theta^d - H] - v(a) \geq 0 \tag{29}$$
$$EU^B = p(e,a)\,(D-H) + H - (K-W) \geq 0 \tag{30}$$

The efforts which maximize the expected gain of the entrepreneur and the LBO fund are given by (4) and (5).

The optimal financial contracts maximize the expected gain of the entrepreneur under the participation constraints of the bank and the consultant and the incentive constraints. Consequently, the program to be solved is given by:

$$\max_{\beta,\, e,\, a,\, D,\, H} EU^E = \beta[p(e,a)\,(\Delta\theta - D + H) + \theta^d - H] - u(e) - W$$
$$\text{s.t} \qquad (4)\,,\ (5)\,,\ (29) \text{ and } (30)$$

with the conditions (6).

The participation constraints of the LBO fund and the bank give

$$(1-\beta)\,[p(e,a)\,(\Delta\theta - D + H) + \theta^d - H] - v(a) \geq 0 \tag{31}$$
$$W = K - H - p(e,a)\,(D-H) \tag{32}$$

In contrast with the participation constraint of the bank, the participation constraint of the consultant may be not binding.

When we substitute (32) in the objective function of the entrepreneur, we get the following program:

$$\max_{\beta,\, e,\, a,\, D,\, H} EU^E = \beta[p(e,a)\,(\Delta\theta - D + H) + \theta^d - H] - u(e) - K$$
$$+H + p(e,a)\,(D-H)$$

$$\text{s.t} \qquad (4)\,,\ (5) \text{ and } (31)$$

with the conditions (6).

It is easy to check that the social value of the project is optimal if the agents provide the first best efforts, in other words, when $\beta = \frac{1}{2}$ and the bank's payments are decreasing with the revenues' project, i.e. $H - D = \Delta\theta$.

The following proposition summarizes the properties of the optimal financial contracts.

Proposition 4 *When the project is not very risky ($\theta^u \leq 2\theta^d$) and $I = K - W$, under the condition $\Delta\theta \leq K - W + p^{FB}\Delta\theta \leq \theta^d$, the optimal financial contracts are given by:*

$$\tilde{\beta} = \frac{1}{2}$$
$$\tilde{D} = K - W + p^{FB}\Delta\theta - \Delta\theta$$
$$\tilde{H} = K - W + p^{FB}\Delta\theta$$

The proof of this proposition is presented in the appendix D.

We get the properties of the optimal financial discussed in the proposition 1: when the project is not very risky, the entrepreneur and the consultant provide the first best efforts and share equally the revenues of the project. This results show that the identity of the agent providing advice is irrelevant for the entrepreneur. In other words, it is the debt which constraints both agents to provide optimal efforts.

When the project is very risky, as shown previously we get $H = \theta^d$. We come back to the specified model.

Proposition 5 *If $\theta^u > 2\theta^d$ and $I = K - W$, under the condition $\theta^u < \frac{\alpha+2}{\alpha}\theta^d$, the optimal financial contracts are given:*

$$\tilde{\beta} = \frac{2}{2+\alpha} \tag{33}$$
$$\tilde{D} = \left(\tfrac{1}{2}\alpha + 1\right)\theta^d - \tfrac{1}{2}\alpha\theta^u \quad \text{and} \quad \tilde{H} = \theta^d \tag{34}$$

The proof is presented in the appendix E.

The proposition 5 states that the bank's payments are decreasing with the revenues of the project. In contrast with the proposition 2, the debt contract is not a "*live or die*" contract. But, the bank gets the entire revenue in case of failure. Notice that the bank's payment in case of success depends on the impact of the LBO's effort on the success probability α.

Besides, the optimal sharing benefit is decreasing with α. In contrast with the previous propositions, if $\alpha = \frac{1}{2}$, the entrepreneur has the highest share of benefit. Furthermore, if α converges to 1, despite the fact that the consultant's effort is the most efficient, the entrepreneur has the highest benefit's share (β converges to $\frac{2}{3}$). Notice that $\beta \in \left]\frac{2}{3}\ ,\ 1\right[$.

The entrepreneur and the consultant provide the following efforts:

$$\tilde{e} = \left(\frac{\alpha}{2}\right)^{\frac{\alpha}{2}} \phi(\alpha)\Delta\theta < e^{FB} \tag{35}$$
$$\tilde{a} = \left(\frac{\alpha}{2}\right)^{\frac{1+\alpha}{2}} \left[\frac{\mu\alpha}{\lambda(1-\alpha)}\right]^{\frac{1}{2}} \phi(\alpha)\Delta\theta < a^{FB} \tag{36}$$

It is straightforward to see that these efforts are lower than those provided in the presence of the LBO fund. If he does not contribute financially to the acquisition, the entrepreneur must give him a share of his revenue to induce him to provide effort. This affects the entrepreneur's incentives in three ways:

- The first way is a direct and has negative effect: the entrepreneur's revenue decreases when she pays the consultant, which decreases her effort's incentives.
- The second way is not direct but has a negative effect: given that the efforts e and a are complementaries, the increase of one effort leads to an increase in the other effort. Recall that a is more costly than e $(\mu < \lambda)$. If α is large, to boost the LBO's incentives, the entrepreneur has to pay more money. The optimal sharing rule depends on α, this means that the entrepreneur must increase her effort ; a costly effort (otherwise inducing the LBO fund to provide high effort has no impacts on the project's performance).
- The third way is positive effect: the more high the levels of efforts e and a, the more the success probability will be high.

The positive effect is not high enough to compensate the other negative effects. Fix $\beta = \frac{1}{2}$ boosts both agents' incentives. In the previous sections, when the LBO fund contributes financially to the acquisition, he pays a part of the cost of his effort and his incentives.

In the previous sections, when the LBO fund contributes financially to the acquisition, he fund him self a part of the cost of his effort and his incentive. Accordingly, we conclude that the entrepreneur prefers asking for advice from the LBO fund rather than the consultant.

These results are in line with those of Casamatta (2003) who shows that when the efforts are perfect substitutes and the entrepreneur is not wealth constrained, she does not hire the consultant if he does not contribute financially into the acquisition.

4.1. What happens if the bank's payments are non-decreasing with the project's revenues?

The participation constraint of the LBO fund (the consultant) is binding (see appendix D and E). Then, we can write

$$\beta\left\{e^{1-\alpha}a^{\alpha}\left(\Delta\theta - D + H\right) + \theta^{d} - H\right\} = \left\{e^{1-\alpha}a^{\alpha}\left(\Delta\theta - D + H\right) + \theta^{d} - H\right\} - \frac{a^{2}}{2\mu}$$

Substituting the first term of this equation and the efforts (12) and (13) in the expected gain of the entrepreneur gives:

$$\begin{aligned} \max_{\beta,\, D,\, H} EU^{E} = \quad & \rho\left(\beta\right)\left(\Delta\theta - D + H\right)\Delta\theta - \tfrac{1}{2}\left[\alpha + \left(1-2\alpha\right)\beta\right]\rho\left(\beta\right)\left(\Delta\theta - D + H\right)^{2} \\ & +\theta^{d} - K \\ \text{s.t} \qquad & D \geq H \text{ and } (6) \end{aligned}$$

We deduce the following lemma:

Lemma 1 *When the bank's payments are non-decreasing with the project's revenues and the LBO fund does not contribute financially to the project, the optimal financial contracts are given by:*

$$\beta' = \begin{cases} \frac{(2-\alpha)(1-\alpha)-\sqrt{\alpha(1-\alpha^2)(2-\alpha)}}{2(1-2\alpha)} & \text{if } \alpha \in]0,\, 1[\, / \left\{\frac{1}{2}\right\} \\ \frac{1}{2} & \text{if } \alpha = \frac{1}{2} \end{cases}$$

$$D' = H' = I = K - W$$

The proof is presented in the appendix F.

In contrast with the proposition 3, the debt contract is unique. The bank's payments do not depend on the project's revenues: whether the project succeeds or fails, he gets the same amount of money. This implies that the bank interest is null. In this case, the entrepreneur and the consultant provide the efforts (24).

When the debt's payments are non-decreasing with the revenues, the identity of the agents providing the effort a is irrelevant to the entrepreneur: the consultant and the LBO fund provide the same levels of efforts whatever the amount of the debt.

5. LBO debt and Tax saving advantage

In this section, we consider that the debt's interests are tax-deductible and study the impact of the tax saving advantage on the agents' incentives and on the financial capital structure.

Let τ denotes the corporate income tax, $0 \leq \tau \leq 1$.

Hereafter the success payment of the bank D is equal to $(1+r)\, I$ where r is the interest rate of the bank.

If the debt's interests are deduced from the taxes of the holding and the *Op Co* companies, the success revenue after taxation is given by:

$$(1-\tau)\left[\theta^u - (1+r)\, I\right] + (1+r)\, I = (1-\tau)\, \theta^u + \tau\, (1+r)\, I$$

But in case of failure, the revenue after taxation is given by:

$$(1-\tau)\left(\theta^d - H\right) + H = (1-\tau)\, \theta^d + \tau H$$

If the project is liquidated $\left(H = \theta^d\right)$, the entrepreneur and the fonds LBO have no payments.

5.1. The optimal financial contracts when the efforts are observable

Hereafter, we study the impact of taxation on the agents' incentives first when the efforts are observable. The expected gain of the entrepreneur after taxation is written:

$$\begin{aligned} EU^E = \;& \beta\,(1-\tau)\left\{p(e,a)\theta^u + [1-p(e,a)]\,\theta^d - p(e,a)\,(1+r)\, I \right. \\ & \left. - [1-p(e,a)]\, H\right\} - u(e) - W \end{aligned} \tag{37}$$

Because of the competition among the LBO funds and the banks, after taxation the participation constraints of the agents A and B enable us to write:

$$\begin{aligned} i = \quad & (1-\beta)(1-\tau)\left\{p(e,a)\theta^u + [1-p(e,a)]\,\theta^d \right. \\ & \left. -p(e,a)(1+r)I - [1-p(e,a)]\,H\right\} - v(a) \end{aligned} \tag{38}$$

$$I = p(e,a)\left[(1+r)I - H\right] + H \tag{39}$$

If we substitute the second term of the equation (39) in (37) and (38), the expected gain of the entrepreneur and the amount of the equity issued by the LBO can be written respectively:

$$\begin{aligned} EU^E = \quad & \beta(1-\tau)\left\{p(e,a)\theta^u + [1-p(e,a)]\,\theta^d - I\right\} \\ & -u(e) - W \end{aligned} \tag{40}$$

$$\begin{aligned} i = \quad & (1-\beta)(1-\tau)\left\{p(e,a)\theta^u + [1-p(e,a)]\,\theta^d - I\right\} \\ & -v(a) \end{aligned} \tag{41}$$

Given the financial constraint $K = W + i + I$, if we substitute (41) in (40), the expected gain of the entrepreneur becomes:

$$\begin{aligned} EU^E = \quad & (1-\tau)\left[p(e,a)\Delta\theta + \theta^d\right] - u(e) - v(a) \\ & -K + \tau I \end{aligned} \tag{42}$$

When the efforts are observable, the levels of efforts e^{PI} and a^{PI} which maximize the expected gain of the entrepreneur are given by:

$$(1-\tau)\Delta\theta = \frac{u'(e)}{p_e(e,a)} = \frac{v'(a)}{p_a(e,a)} \tag{43}$$

These conditions (43) imply that the agents cannot provide the first best efforts. Besides, the two agents are better off if the amount of the debt is very large. The expected gain of the entrepreneur is a strictly increasing function of I. Given the financial constraint, the LBO fund does not issue equity ($i = 0$). Then, the amount of the debt is equal to $K - W$. This result means that the identity of the agent providing the effort a is irrelevant: whether her partner is the LBO fund or the consultant, the entrepreneur prefers asking for money from the bank.

If W is an endogenous variable, the entrepreneur does not issue equity and the acquisition is financed solely through the debt: $I = K$ and $W = i = 0$. Notice that the entrepreneur may perceive an amount of money if $I > K$.

If we consider the specified model, the efforts e^{PI} and a^{PI} are written:

$$e^{PI} = \phi(\alpha)(1-\tau)\Delta\theta \tag{44}$$

$$a^{PI} = \left[\frac{\mu\alpha}{\lambda(1-\alpha)}\right]^{\frac{1}{2}} \phi(\alpha)(1-\tau)\Delta\theta \tag{45}$$

These efforts are increasing with the corporate income tax τ: if τ converges to 0, (44) and (45) converge to the first best efforts. At the opposite, when τ converges to 1, the revenues after taxation are very low, consequently, the agents are not induced to provide efforts.

The efforts (44) and (45) induce the success with the following probability:

$$p^{PI} = \frac{\phi^2(\alpha)}{\lambda(1-\alpha)}\,(1-\tau)\,\Delta\theta \tag{46}$$

The bank's payment of success is equal to $D^{PI} = (1+r)\,(K-W)$.

The bank's payment of failure is given by the participation constraint (39), it is written:

$$H^{PI} = \frac{1-(1+r)\,p^{PI}}{1-p^{PI}}\,(K-W) \tag{47}$$

H^{PI} exists under the following condition:

$$\Delta\theta < \frac{\lambda(1-\alpha)}{(1+r)\,(1-\tau)\,\phi^2(\alpha)}$$

This condition ensures also that the success probability p^{PI} is inferior to 1.

Notice that the debt's payments are non-decreasing with the project revenues. We substitute (44), (45), (46) and $i=0$ in the participation constraint of the LBO fund. The optimal sharing rule is therefore given by :

$$\beta^{PI} = 1 - \frac{\frac{\alpha}{2\lambda(1-\alpha)}\left[\phi(\alpha)\,(1-\tau)\,\Delta\theta\right]^2}{\frac{1}{\lambda(1-\alpha)}\left[\phi(\alpha)\,(1-\tau)\,\Delta\theta\right]^2 + (1-\tau)\left(\theta^d - K + W\right)}$$

5.2. The optimal financial contracts when the efforts are unobservable

5.2.1. The project is financed solely through equity

The expected gain of the entrepreneur and the LBO fund are written:

$$EU^E = (1-\tau)\,\beta\left[p(e,a)\Delta\theta + \theta^d\right] - u(e) - W \geq 0$$
$$EU^A = (1-\tau)\,(1-\beta)\left[p(e,a)\Delta\theta + \theta^d\right] - v(a) - i = 0$$

The efforts in equilibrium are given by the first order conditions of EU^E and EU^A. They are written:

$$e_T^c = \left[\lambda\beta(1-\alpha)\rho(\beta)\right]^{\frac{1}{2}}\,(1-\tau)\,\Delta\theta \tag{48}$$
$$a_T^c = \left[\mu\alpha(1-\beta)\rho(\beta)\right]^{\frac{1}{2}}\,(1-\tau)\,\Delta\theta \tag{49}$$

Given $0 < \beta < 1$, these efforts are strictly lower than the first best efforts. The success probability is therefore given by $p_T^c = (1-\tau)\,\rho(\beta)\Delta\theta$.

The optimal financial contracts induce the entrepreneur to maximize her expected gain:

$$\max_{\beta} \; EU^E = \tfrac{1}{2}\left[2-\alpha-(1-2\alpha)\,\beta\right]\rho(\beta)\,(1-\tau)^2\,\Delta\theta^2+(1-\tau)\,\theta^d-K$$
$$\text{s.t} \quad 0<\beta<1$$

The first order condition of EU^E enables us to deduce that the optimal sharing rule is given by (23).

If the project is financed only through the equity issued by the LBO fund, the agents provide the following efforts:

$$e_T^c = \left[\lambda\beta^c\,(\alpha)\,(1-\alpha)\rho(\beta^c\,(\alpha))\right]^{\frac{1}{2}}\,(1-\tau)\,\Delta\theta \tag{50}$$
$$a_T^c = \left[\mu\alpha(1-\beta^c\,(\alpha))\rho(\beta^c\,(\alpha))\right]^{\frac{1}{2}}\,(1-\tau)\,\Delta\theta \tag{51}$$

It is straightforward to see that the efforts (50) and (51) are inferior to (24). There is no advantage tax when the entrepreneur and the LBO fund issue only equity to fund the acquisition. Accordingly, their net revenues are reduced due to taxation. This is why the agents' incentives are decreasing with the corporate income tax.

Consider the particular case $\alpha=\frac{1}{2}$, the entrepreneur and the LBO fund provide the following efforts:

$$e_T^c = \frac{1}{4}\lambda^{\frac{3}{4}}\mu^{\frac{1}{4}}\,(1-\tau)\,\Delta\theta \tag{52}$$
$$a_T^c = \frac{1}{4}\lambda^{\frac{1}{4}}\mu^{\frac{3}{4}}\,(1-\tau)\,\Delta\theta \tag{53}$$

5.2.2. The project is financed through the debt and the equity

The efforts of equilibrium e_T^* and a_T^* are given by the first order conditions of EU^E and EU^A given by(40) and (41):

$$\beta\,(1-\tau)\,\Delta\theta = \frac{u'\,(e)}{p_e\,(e\;,\;a)} \tag{54}$$
$$(1-\beta)\,(1-\tau)\,\Delta\theta = \frac{v'\,(a)}{p_a\,(e\;,\;a)} \tag{55}$$

When the efforts are unobservable, the entrepreneur and the LBO fund do not provide the first best efforts. If we rely on the specified model, the conditions (54) and (55) enable us to deduce the following efforts:

$$e_T^* = \left[\lambda\beta(1-\alpha)\rho(\beta)\right]^{1/2}(1-\tau)\,\Delta\theta \tag{56}$$
$$a_T^* = \left[\mu\alpha(1-\beta)\rho(\beta)\right]^{1/2}(1-\tau)\,\Delta\theta \tag{57}$$

Then, the project succeeds with the probability:

$$p_T^* = \rho(\beta)\,(1-\tau)\,\Delta\theta \tag{58}$$

To ensure that this probability is inferior to 1, we add the following condition:

$$\Delta\theta \leq \frac{1}{\rho(\beta)\,(1-\tau)} \tag{59}$$

The optimal financial contracts induce the entrepreneur to maximize her expected gain given by:

$$\underset{\beta,\, I}{\arg\max} EU^E = \tfrac{1}{2}\left[2-\alpha-(1-2\alpha)\beta\right]\rho(\beta)\left[(1-\tau)\,\Delta\theta\right]^2 + (1-\tau)\,\theta^d - K + \tau I$$

$$\text{s.t} \qquad 0 < \beta < 1 \text{ and } 0 \leq I \leq K - W$$

The first order conditions of EU^E give the results presented in the lemma 2:

Lemma 2 *In the presence of taxation, the optimal financial contracts are given by:*

$$\beta_T^* = \begin{cases} \frac{(2-\alpha)(1-\alpha)-\sqrt{\alpha(1-\alpha^2)(2-\alpha)}}{2(1-2\alpha)} & \textit{if } \alpha \in \left]0,\, 1\right[/ \left\{\frac{1}{2}\right\} \\ \frac{1}{2} & \textit{if } \alpha = \frac{1}{2} \end{cases}$$

$$i_T^* = 0$$

and

$$I_T^* = K - W$$

$$H_T^* = \frac{\left[1-\rho\left(\beta_T^*\right)(1-\tau)\,(1+r)\,\Delta\theta\right](K-W)}{1-\rho\left(\beta_T^*\right)(1-\tau)\,\Delta\theta}$$

This lemma states that under the condition (59), the bank's payments are non-decreasing with the project's revenues. Furthermore, the LBO fund does not contribute financially into the acquisition which means that the entrepreneur is indifferent whether her partner is the LBO fund or the consultant. Because of the tax saving advantage of the debt, the acquisition is financed through the equity of the entrepreneur and the debt.

The entrepreneur and the LBO fund provide the following efforts :

$$e_T^* = \left[\lambda\beta_T^*(1-\alpha)\rho(\beta_T^*)\right]^{1/2}(1-\tau)\,\Delta\theta \tag{60}$$

$$a_T^* = \left[\mu\alpha(1-\beta_T^*)\rho(\beta_T^*)\right]^{1/2}(1-\tau)\,\Delta\theta \tag{61}$$

Given $0 < \beta < 1$, the efforts (60) and (61) are lower than e^{PI} and a^{PI} but equal to the efforts provided when the project is financed through equity. This result means that the tax davantage of the debt does not influence the agents' incentives but it induces them to get high level of debt.

- Discussion

These results are in line with those of Modigliani and Miller (1963) who correct that the tax advantage of debt financing is greater than what they originally suggested in a first paper in 1958. They show that under some restrictive conditions, the value of the levered company is equal to the amount of the tax-deductible interests and the value of the unlevered company. The relationship between the financial capital structure and the investment decision is therefore established when there is tax.

In contrast with Modigliani and Miller (1963), Miller (1977) distinguish between the corporate and the personal income taxes and find that for individual, the effects of the two types of taxes end up cancelling each other. He suggests that there is a tax saving advantage only in macroeconomic level.

In LBO, leveraged also potentially increases firm value through the tax-deductibility of interests. Lowenstein (1985) mentions incentive effects of debt, but argues tax effects play a major role in explaining the value increase of the firm's value.

Kaplan (1989a) consider a sample of 76 LMBO[7] projects exited between 1980 and 1986, their results show that there is a tax saving advantage in LBO acquisitions: the median value of tax benefits estimated at the time of going private via LBO has a lower bound of 21% and an upper bound of 143% of the premium paid to the pre-buyout shareholders. Notice that their estimated value depend on the rate of the debt of the acquired company paid and on the tax rate applied to the interest deductions. In a second paper (Kaplan, 1989b), he estimates that interest tax shield can explain from 4% to 40% of the firm's value.

However, Kaplan and Strömberg (2008) explain that the value of this tax shield is notoriously difficult to calculate because it requires restrictive assumptions of the tax advantage of debt (net of personal taxes), the expected permanence of the debt, and the riskiness of the tax shield.

"*It is safe to say, therefore, that taxes create some value, but difficult to say exactly how much*".

Jensen (1989) notices that usually the other tax revenues are ignored in LBO acquisitions. He counts five sources of additional tax revenues generated in these projects:

1. Capital gains taxes paid by pre-buyout shareholders;
2. Capital gains taxes paid on post-buyout asset sales;
3. Tax payments on the large increases in operating earnings generated by efficiency gains;
4. Tax payments by creditors who receive interest payments on the LBO debt;
5. Taxes generated by more efficient use of the company's total capital.

He explains that these taxes compensate the tax saving advantage of the LBO debt (as an example, see the RJR Nabisco, in the same paper).

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[7] LMBO (*Leveraged Management Buy Out)* is a LBO project where the entrepreneur(s) is/are the manager(s) and/or the employees of the acquired company.

6. Conclusion

This paper has studied the properties of the optimal financial contracts in LBO. We have focused on the incentive effects of debt in the presence of a double-sided moral hazard problem. The model explains why the entrepreneur prefers asking for money and for advice from the bank and the LBO fund, despite that the latter can advise her and is not wealth-constrained.

We show that all agents have to participate financially in the project and that the payments of the bank are decreasing with the outcome of the project. The agents' efforts depend on the quality's project. If the project is not very risky, the entrepreneur and the LBO fund provide the first best efforts and they get equal shares of the outcome of the project. This is no longer true when the acquisition is very risky; the moral hazard problem induces them to provide the second best efforts. They must share the revenue in case of success and pledge the entire revenue to the bank when the project fails.

At the opposite, the debt has no impacts on the agents' incentives when the bank's payments are non-decreasing with the project's revenues. Whether the project is financed through the bank and the LBO fund or solely through the LBO fund, the efforts do not depend on the financial capital structure.

To explain the excessive use of debt in LBO acquisitions, we show that the tax saving advantage creates extra revenues which explains the high level of debt in LBO acquisitions. However, we show that tax advantage does not influence the agents' incentives.

Exploring in full details how these optimal contracts are implemented in LBO is an important topic for future research. In such research, it would probably be reasonable to study the use of convertible securities in the presence of a passive financier as the bank. Another topic for further research is how the cash-flow and control rights are allocated when the profitability of the project varies. To our knowledge, research in this direction is still pending.

Moreover, in a dynamic model, when the LBO fund has convertible securities, the share of benefit attributed to the entrepreneur fund may be considered as a quality's signal of the project if there is asymmetric information on the market. When this share is significantly high, it means that the project is profitable: the LBO fund cannot exert his conversion option. He will be a minority shareholder and he will look for a quick exit of the project (Signal theory, Leland and Pyle, 1977 and Ross, 1977).

Appendix

A. Proof of proposition 1

Given the participation constraints of the LBO fund and the bank (PC_A) and (PC_B), the amounts of i and I are written respectively:

$$i = \frac{1}{2}[2p^{FB}(e,a)\Delta\theta + \theta^d - H] - v(a^{FB}) \tag{A1}$$

$$I = -p^{FB}(e,a)\Delta\theta + H \tag{A2}$$

Moreover, given $K - W = i + I$, we conclude that:

$$\begin{aligned} H^* &= 2[K - W + v(a^{FB})] - \theta^d \\ D^* &= 2[K - W + v(a^{FB})] - \theta^u \end{aligned}$$

We replace H^* and D^* in the participation constraints (PC_A) and (PC_B), we get the optimal amounts of equity and debt which must be issued respectively by the LBO fund and the bank:

$$\begin{aligned} i^* &= p^{FB}\Delta\theta + \theta^d + W - K - 2v(a^{FB}) \\ &= V^{FB} + u(e^{FB}) - v(a^{FB}) + W \\ I^* &= 2[K - W + v(a^{FB})] - p^{FB}\Delta\theta - \theta^d \end{aligned}$$

These optimal solutions exist under the following conditions:

$$H^* \geq 0 \Leftrightarrow \theta^d \leq 2[K - W + v(a^{FB})] \tag{A3}$$

$$D^* \geq 0 \Leftrightarrow \theta^u \leq 2[K - W + v(a^{FB})] \tag{A4}$$

$$i^* \geq 0 \Leftrightarrow \theta^d \geq K - W + 2v(a^{FB}) - p^{FB}\Delta\theta \tag{A5}$$

$$I^* \geq 0 \Leftrightarrow \theta^d \leq 2[K - W + v(a^{FB})] - p^{FB}\Delta\theta \tag{A6}$$

$$H^* \leq \theta^d \Leftrightarrow \theta^d \geq K - W + v(a^{FB}) \tag{A7}$$

$$D^* \leq \theta^u \Leftrightarrow \theta^u \geq K - W + v(a^{FB}) \tag{A8}$$

The equation ($A3$) (respectively ($A8$)) is satisfied if ($A4$) (respectively ($A7$)) is satisfied.

On one hand, the conditions ($A4$) and ($A7$) imply that:

$$l \leq \theta^d < \theta^u \leq 2l \tag{A9}$$

where $l = K - W + v(a^{FB})$.

On the other hand, the conditions ($A5$) and ($A6$) give:

$$l + v(a^{FB}) - p^{FB}\Delta\theta \leq \theta^d \leq 2l - p^{FB}\Delta\theta \tag{A10}$$

but $l-v(a^{FB}) = K-W > 0$ which means that $(A10)$ is always satisfied. Given that $2l-p^{FB}\Delta\theta < 2l$, $(A9)$ and $(A10)$, we deduce that:

$$l \leq \theta^u \leq 2l \tag{A11}$$

$$l + \min\{v(a^{FB}) - p^{FB}\Delta\theta\ ,\ 0\} \leq \theta^d \leq 2l - p^{FB}\Delta\theta \tag{A12}$$

which completes the proof of the proposition 1.

B. Proof of proposition 2

The Lagrangian is given by:

$$\begin{aligned}\mathcal{L}: &= \mathcal{L}(\beta,\, D,\, H,\, \delta_j,\, j=1..6) \\ &= \rho(\beta)\Delta\theta(\Delta\theta - D + H) + \theta^d - K - \tfrac{1}{2}\left(\alpha + \beta - 2\alpha\beta\right)\rho(\beta)(\Delta\theta - D + H)^2 \\ &\quad + \delta_1 D + \delta_2 H + \delta_3\left(\theta^u - D\right) + \delta_4\left(\theta^d - H\right) + \delta_5\beta + \delta_6\left(1-\beta\right).\end{aligned}$$

where δ_j, $j = 1..6$ are the Kuhn-Tucker multipliers.

The Kuhn-Tucker conditions are given by:

$$\begin{aligned}\tfrac{\partial\mathcal{L}}{\partial\beta} = &\ \rho'(\beta)\Delta\theta(\Delta\theta - D + H) - \tfrac{1}{2}\left(1-2\alpha\right)\rho(\beta)(\Delta\theta - D + H)^2 \\ &-\tfrac{1}{2}\left(\alpha + \beta - 2\alpha\beta\right)\rho'(\beta)(\Delta\theta - D + H)^2 + \delta_5 - \delta_6 \leq 0\end{aligned} \tag{B1}$$

$$\beta \geq 0 \tag{B2}$$

$$\beta\frac{\partial\mathcal{L}}{\partial\beta} = 0 \tag{B3}$$

$$\begin{aligned}\tfrac{\partial\mathcal{L}}{\partial D} = &\ -\rho(\beta)\Delta\theta + \left(\alpha + \beta - 2\alpha\beta\right)\rho(\beta)(\Delta\theta - D + H) \\ &+\delta_1 - \delta_3 \leq 0\end{aligned} \tag{B4}$$

$$D \geq 0 \tag{B5}$$

$$D\frac{\partial\mathcal{L}}{\partial D} = 0 \tag{B6}$$

$$\begin{aligned}\tfrac{\partial\mathcal{L}}{\partial H} = &\ \rho(\beta)\Delta\theta - \left(\alpha + \beta - 2\alpha\beta\right)\rho(\beta)(\Delta\theta - D + H) \\ &+\delta_2 - \delta_4 \leq 0\end{aligned} \tag{B7}$$

$$H \geq 0 \tag{B8}$$

$$H\frac{\partial\mathcal{L}}{\partial H} = 0 \tag{B9}$$

The complementary slackness conditions are written:

$$\tfrac{\partial\mathcal{L}}{\partial\delta_1} = D \geq 0\ ,\quad \delta_1 \geq 0\ ,\quad \delta_1 D = 0 \tag{B10}$$

$$\tfrac{\partial\mathcal{L}}{\partial\delta_2} = H \geq 0\ ,\quad \delta_2 \geq 0\ ,\quad \delta_2 H = 0 \tag{B11}$$

$$\tfrac{\partial\mathcal{L}}{\partial\delta_3} = \theta^u - D \geq 0\ ,\quad \delta_3 \geq 0\ ,\quad \delta_3\left(\theta^u - D\right) = 0 \tag{B12}$$

$$\tfrac{\partial\mathcal{L}}{\partial\delta_4} = \theta^d - H \geq 0\ ,\quad \delta_4 \geq 0\ ,\quad \delta_4\left(\theta^d - H\right) = 0 \tag{B13}$$

$$\tfrac{\partial\mathcal{L}}{\partial\delta_5} = \beta \geq 0\ ,\quad \delta_5 \geq 0\ ,\quad \delta_5\beta = 0 \tag{B14}$$

$$\tfrac{\partial\mathcal{L}}{\partial\delta_6} = 1-\beta \geq 0\ ,\quad \delta_6 \geq 0\ ,\quad \delta_6\left(1-\beta\right) = 0 \tag{B15}$$

1. $0 < D < \theta^u$, $0 < H < \theta^d$ and $0 < \beta < 1$

   According to the conditions $(B10)$, $(B11)$, $(B12)$, $(B13)$, $(B14)$ and $(B15)$, all the multipliers are null: $\delta_j = 0$, $j = 1..6$. We substitute them in the equations $(B1)$, $(B4)$ and $(B7)$. We get the following system:

$$\begin{aligned} &2\tfrac{1-\alpha-\beta}{\beta(1-\beta)}\rho(\beta)\Delta\theta - (1-2\alpha)\,\rho(\beta)(\Delta\theta - D + H) \\ &-(\alpha+\beta-2\alpha\beta)\,\tfrac{1-\alpha-\beta}{\beta(1-\beta)}\rho(\beta)(\Delta\theta - D + H) = 0 \end{aligned} \tag{B16}$$

$$-\rho(\beta)\Delta\theta + (\alpha+\beta-2\alpha\beta)\,\rho(\beta)(\Delta\theta - D + H) = 0 \tag{B17}$$

$$\rho(\beta)\Delta\theta - (\alpha+\beta-2\alpha\beta)\,\rho(\beta)(\Delta\theta - D + H) = 0 \tag{B18}$$

   Notice that the equations $(B17)$ and $(B18)$ can be written

$$\rho(\beta)\Delta\theta = (\alpha+\beta-2\alpha\beta)\,\rho(\beta)(\Delta\theta - D + H) \tag{B19}$$

   We substitute $\rho(\beta)\Delta\theta$ for its expression in $(B16)$, we get $\beta = \frac{1}{2}$. Substituting $\beta = \frac{1}{2}$ in $(B19)$, given our starting hypothesis, we deduce that the bank's payments are decreasing with the project's revenues: $D - H = -\Delta\theta < 0$. This is true only when the project is not very risky.

2. $0 < D < \theta^u$, $H = \theta^d$ and $0 < \beta < 1$

   According to $(B10)$, $(B11)$, $(B12)$, $(B14)$ and $(B15)$, $\delta_1 = \delta_2 = \delta_3 = \delta_5 = \delta_6 = 0$. Substituting them in the first order conditions $(B1)$, $(B4)$ and $(B7)$ gives:

$$\begin{aligned} &2\rho'(\beta)\Delta\theta - (1-2\alpha)\,\rho(\beta)(\theta^u - D) \\ &-(\alpha+\beta-2\alpha\beta)\,\rho'(\beta)(\theta^u - D) = 0 \end{aligned} \tag{B20}$$

$$\rho(\beta)\Delta\theta = (\alpha+\beta-2\alpha\beta)\,\rho(\beta)(\theta^u - D) \tag{B21}$$

$$\delta_4 \geq \rho(\beta)\Delta\theta - (\alpha+\beta-2\alpha\beta)\,\rho(\beta)(\theta^u - D) \tag{B22}$$

   If we substitute $(B21)$ in $(B20)$ and $(B22)$, we deduce that $\beta = \frac{1}{2}$ and the Kuhn-Tucker multiplier $\delta_4 \geq 0$. To deduce the bank's payments, we substitute $\beta = \frac{1}{2}$ in $(B21)$ which leads to a contradiction $D = \theta^u - 2\Delta\theta < 0$.

3. $0 < D < \theta^u$, $H = 0$ and $0 < \beta < 1$

   According to $(B10)$, $(B12)$, $(B13)$, $(B14)$ and $(B15)$, the multipliers $\delta_1 = \delta_3 = \delta_4 = \delta_5 = \delta_6 = 0$. We substitute $H = 0$ and $\delta_1 = \delta_3 = \delta_4 = \delta_5 = \delta_6 = 0$ in the conditions $(B1)$, $(B4)$ and $(B7)$. We get a contradiction:

$$\beta = \frac{1}{2},\ \delta_2 \leq 0 \text{ and } D = -\Delta\theta < 0.$$

4. $D = \theta^u$, $H = 0$ and $0 < \beta < 1$

   This case does not satisfy the condition (14), otherwise the entrepreneur and the LBO fund provide negative efforts.

5. $D = 0$, $H = \theta^d$ and $0 < \beta < 1$

Given the equations $(B11)$, $(B12)$, $(B14)$ and $(B15)$, the multipliers $\delta_2 = \delta_3 = \delta_5 = \delta_6 = 0$. We substitute $D = 0$, $H = \theta^d$ and $\delta_2 = \delta_3 = \delta_5 = \delta_6 = 0$ in the first order conditions $(B1)$, $(B4)$ and $(B7)$. We get the following system:

$$2\,(1-2\alpha)\,\beta^2 + 2\left[k - 2 + 3\alpha - \alpha^2\right]\beta + (1-\alpha)\,(2-2k-\alpha) = 0 \qquad \text{(B23)}$$

$$\delta_1 \leq \rho(\beta)\,(1-k)\,\theta^u - (\alpha+\beta-2\alpha\beta)\,\rho(\beta)\theta^u \qquad \text{(B24)}$$

$$\delta_4 = \rho(\beta)\,(1-k)\,\theta^u - (\alpha+\beta-2\alpha\beta)\,\rho(\beta)\theta^u \qquad \text{(B25)}$$

where $k = \frac{\theta^d}{\theta^u} < \frac{1}{2}$.

- If $\alpha = \frac{1}{2}$, the equation $(B23)$ has a single solution $\beta = \frac{1}{2}$.
  if $\alpha \in \left]0\ ,\ 1\right[/ \left\{\frac{1}{2}\right\}$, it has two solutions:

  $$\hat{\beta}\,(\alpha) \;=\; \frac{(1-\alpha)(2-\alpha) - k - \sqrt{\Delta}}{2(1-2\alpha)} \;\in \left]0\ ,\ 1\right[$$

  $$\check{\beta}\,(\alpha) \;=\; \frac{(1-\alpha)(2-\alpha) - k + \sqrt{\Delta}}{2(1-2\alpha)} \;< 0$$

  where $\Delta = \alpha\left(1-\alpha^2\right)(2-\alpha) + k\left(6\alpha^2 - 6\alpha + k\right)$.

  We conclude that the optimal sharing rule is given by:

  $$\hat{\beta}\,(\alpha\ ,\ k) = \begin{cases} \frac{(1-\alpha)(2-\alpha)-k-\sqrt{\Delta}}{2(1-2\alpha)} & \text{if } \alpha \in \left]0\ ,\ 1\right[/ \left\{\frac{1}{2}\right\} \text{ and } k \in \left[0\ ,\ \frac{1}{2}\right[\\ \frac{1}{2} & \text{if } \alpha = \frac{1}{2} \end{cases}$$

  Notice that in this case the multipliers δ_1 and δ_4 are positive only if $\hat{\beta}\,(\alpha\ ,\ k) \leq \frac{1-k-\alpha}{(1-2\alpha)}$ which is always satisfied.

We substitute $D = 0$, $H = \theta^d$ and $\hat{\beta}\,(\alpha\ ,\ k)$ in the expressions of efforts in equilibrium. Accordingly and given the participation constraint (CP_B), we deduce the amount of the issued debt I which is substituted in the financial constraint $i = K - W - I$ to deduce the amount of the equity issued by the LBO fund.

C. Proof of the proposition 3

The Lagrangian is given by:

$$\begin{aligned} \mathcal{L} :=&\ \mathcal{L}(\beta,\, D,\, H,\, \delta_i,\, i = 1..7) \\ =&\ \rho\,(\beta)\,\Delta\theta(\Delta\theta - D + H) + \theta^d - K - \tfrac{1}{2}\,(\alpha+\beta-2\alpha\beta)\,\rho\,(\beta)\,(\Delta\theta - D + H)^2 \\ &+\delta_1\,(D-H) + \delta_2 D + \delta_3\,(\theta^u - D) + \delta_4 H + \delta_5\left(\theta^d - H\right) + \delta_6\beta + \delta_7\,(1-\beta) \end{aligned}$$

where δ_j, $j = 1..7$ are the Kuhn-Tucker multipliers.

The Kuhn-Tucker conditions are given by:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \beta} = \;& \rho'(\beta)\,\Delta\theta(\Delta\theta - D + H) - \tfrac{1}{2}(1-2\alpha)\,\rho(\beta)\,(\Delta\theta - D + H)^2 \\ & -\tfrac{1}{2}(\alpha+\beta-2\alpha\beta)\,\rho'(\beta)\,(\Delta\theta - D + H)^2 + \delta_6 - \delta_7 \le 0\end{aligned} \tag{C1}$$

$$\beta \ge 0 \tag{C2}$$

$$\beta \frac{\partial \mathcal{L}}{\partial \beta} = 0 \tag{C3}$$

$$\frac{\partial \mathcal{L}}{\partial D} = -\rho(\beta)\,\Delta\theta + (\alpha+\beta-2\alpha\beta)\,\rho(\beta)\,(\Delta\theta - D + H) + \delta_1 + \delta_2 - \delta_3 \le 0 \tag{C4}$$

$$D \ge 0 \tag{C5}$$

$$D \frac{\partial \mathcal{L}}{\partial D} = 0 \tag{C6}$$

$$\frac{\partial \mathcal{L}}{\partial H} = \rho(\beta)\,\Delta\theta - (\alpha+\beta-2\alpha\beta)\,\rho(\beta)\,(\Delta\theta - D + H) - \delta_1 + \delta_4 - \delta_5 \le 0 \tag{C7}$$

$$H \ge 0 \tag{C8}$$

$$H \frac{\partial \mathcal{L}}{\partial H} = 0 \tag{C9}$$

The complementary slackness conditions are written:

$$\tfrac{\partial \mathcal{L}}{\partial \delta_1} = D - H \ge 0 \;,\;\; \delta_1 \ge 0 \;,\;\; \delta_1 (D - H) = 0 \tag{C10}$$

$$\tfrac{\partial \mathcal{L}}{\partial \delta_2} = D \ge 0 \;,\;\; \delta_2 \ge 0 \;,\;\; \delta_2 D = 0 \tag{C11}$$

$$\tfrac{\partial \mathcal{L}}{\partial \delta_3} = \theta^u - D \ge 0 \;,\;\; \delta_3 \ge 0 \;,\;\; \delta_3 (\theta^u - D) = 0 \tag{C12}$$

$$\tfrac{\partial \mathcal{L}}{\partial \delta_4} = H \ge 0 \;,\;\; \delta_4 \ge 0 \;,\;\; \delta_4 H = 0 \tag{C13}$$

$$\tfrac{\partial \mathcal{L}}{\partial \delta_5} = \theta^d - H \ge 0 \;,\;\; \delta_5 \ge 0 \;,\;\; \delta_5 \left(\theta^d - H\right) = 0 \tag{C14}$$

$$\tfrac{\partial \mathcal{L}}{\partial \delta_6} = \beta \ge 0 \;,\;\; \delta_6 \ge 0 \;,\;\; \delta_6 \beta = 0 \tag{C15}$$

$$\tfrac{\partial \mathcal{L}}{\partial \delta_7} = 1 - \beta \ge 0 \;,\;\; \delta_7 \ge 0 \;,\;\; \delta_7 (1 - \beta) = 0 \tag{C16}$$

1. <u>$0 < D < \theta^u$, $0 < H < \theta^d$ and $0 < \beta < 1$</u>

   According to the conditions $(C11)$, $(C12)$, $(C13)$, $(C14)$, $(C15)$ and $(C16)$, the Kuhn-Tucker multipliers $\delta_j = 0$, $j = 2, ..7$. Substituting $\delta_j = 0$, $j = 2, ..7$ in the equations $(C1)$, $(C4)$ and $(C7)$ gives the following system:

$$\begin{aligned}& \rho'(\beta)\,\Delta\theta(\Delta\theta - D + H) - \tfrac{1}{2}(1-2\alpha)\,\rho(\beta)\,(\Delta\theta - D + H)^2 \\ & \quad -\tfrac{1}{2}(\alpha+\beta-2\alpha\beta)\,\rho'(\beta)\,(\Delta\theta - D + H)^2 = 0\end{aligned} \tag{C17}$$

$$-\rho(\beta)\,\Delta\theta + (\alpha+\beta-2\alpha\beta)\,\rho(\beta)\,(\Delta\theta - D + H) + \delta_1 = 0 \tag{C18}$$

$$\rho(\beta)\,\Delta\theta - (\alpha+\beta-2\alpha\beta)\,\rho(\beta)\,(\Delta\theta - D + H) - \delta_1 = 0 \tag{C19}$$

These equations are also written:

$$\begin{array}{c}\rho(\beta)\Delta\theta = \frac{(1-2\alpha)\beta(1-\beta)}{2(1-\alpha-\beta)}\rho(\beta)(\Delta\theta - D + H) \\ +\frac{1}{2}(\alpha+\beta-2\alpha\beta)\rho(\beta)(\Delta\theta - D + H)\end{array} \tag{C20}$$

$$\delta_1 = \rho(\beta)\Delta\theta - (\alpha+\beta-2\alpha\beta)\rho(\beta)(\Delta\theta - D + H) \tag{C21}$$

Substituting $\rho(\beta)\Delta\theta$ for its expression $(C20)$ in $(C21)$ gives

$$\delta_1 = -\frac{\alpha(1-\alpha)(1-2\beta)}{2(1-\alpha-\beta)}\rho(\beta)(\Delta\theta - D + H) \tag{C22}$$

If $\delta_1 = 0$, we deduce that $\beta = \frac{1}{2}$ and/or $D - H = \Delta\theta$. Substituting $\beta = \frac{1}{2}$ in $(C20)$ does not satisfy $(C10)$ because $\frac{\partial L}{\partial \delta_1} = D - H = -\Delta\theta < 0$. Let us suppose $D - H = \Delta\theta$, the equation $(C20)$ leads to a contradiction because $\rho(\beta)\Delta\theta = 0$ only if $\beta \in \{0\ ,\ 1\}$.

If $\delta_1 < 0$, the condition $(C10)$ is not satisfied.

⇒ We conclude that our program has no solutions when $\delta_1 \leq 0$.

If $\delta_1 > 0$, the condition $(C10)$ enables us to deduce that $D = H$. Substituting the latter equation in $(C20)$ gives

$$2(1-2\alpha)\beta^2 - 2(1-\alpha)(2-\alpha)\beta + (1-\alpha)(2-\alpha) = 0 \tag{C23}$$

This equation has two solutions but one of them is a feasible solution and varies between 0 and 1. It is given by:

$$\beta(\alpha) = \begin{cases} \frac{(2-\alpha)(1-\alpha)-\sqrt{\alpha(1-\alpha^2)(2-\alpha)}}{2(1-2\alpha)} & \text{if } \alpha \in]0,\ 1[\,/\left\{\frac{1}{2}\right\} \\ \frac{1}{2} & \text{if } \alpha = \frac{1}{2}\end{cases} \tag{C24}$$

2. $0 < D < \theta^u$, $H = 0$ and $0 < \beta < 1$.

   The equations $(C10)$, $(C11)$, $(C14)$, $(C15)$ and $(C16)$ enable us to deduce $\delta_1 = \delta_2 = \delta_3 = \delta_5 = \delta_6 = \delta_7 = 0$. We substitute $H = 0$ and $\delta_1 = \delta_2 = \delta_3 = \delta_5 = \delta_6 = \delta_7 = 0$ in the equations $(C1)$, $(C4)$ and $(C7)$ such that we get the following system:

$$\rho(\beta)\Delta\theta = \frac{1}{2}\rho(\beta)(\Delta\theta - D)\left[\frac{\beta(1-\beta)(1-2\alpha)}{(1-\alpha-\beta)} + (\alpha+\beta-2\alpha\beta)\right] \tag{C25}$$

$$\rho(\beta)\Delta\theta = (\alpha+\beta-2\alpha\beta)\rho(\beta)(\Delta\theta - D) \tag{C26}$$

$$\delta_4 \leq -\rho(\beta)\Delta\theta + (\alpha+\beta-2\alpha\beta)\rho(\beta)(\Delta\theta - D) \tag{C27}$$

   The equations $(C25)$ and $(C26)$ give $\beta = \frac{1}{2}$ which is substituted in $(C26)$. As a result, we get the following contradiction $D = -\Delta\theta < 0$.

3. $D = \theta^u$, $H = 0$ and $0 < \beta < 1$.

   According to the equations $(C10)$, $(C11)$, $(C14)$, $(C15)$ and $(C16)$, the Kuhn-Tucker multipliers $\delta_j = 0$, $j = 1, 2, 5, 6, 7$. Substituting $D = \theta^u$, $H = 0$ and $\delta_j = 0$, $j = 1, 2, 5, 6, 7$ in the equation $(C4)$ gives:

$$\delta_3 = -\rho(\beta)\Delta\theta - (\alpha+\beta-2\alpha\beta)\rho(\beta)\theta^d < 0.$$

   But δ_3 must satisfy the condition $(C12)$.

4. $0 < D < \theta^u$, $H = \theta^d$ and $0 < \beta < 1$.

We have $\delta_2 = \delta_3 = \delta_4 = \delta_6 = \delta_7 = 0$. Substituting $H = \theta^d$ and $\delta_2 = \delta_3 = \delta_4 = \delta_6 = \delta_7 = 0$ in the Kuhn-Tucker conditions gives the following system:

$$\begin{aligned} 2(1-\alpha-\beta)\rho(\beta)\Delta\theta - (1-2\alpha)\beta(1-\beta)\rho(\beta)(\theta^u - D) \\ -(\alpha+\beta-2\alpha\beta)(1-\alpha-\beta)\rho(\beta)(\theta^u - D) = 0 \end{aligned} \tag{C28}$$

$$\delta_1 = \rho(\beta)\Delta\theta - (\alpha+\beta-2\alpha\beta)\rho(\beta)(\theta^u - D) \tag{C29}$$

$$\delta_5 = \rho(\beta)\Delta\theta - (\alpha+\beta-2\alpha\beta)\rho(\beta)(\theta^u - D) - \delta_1 \tag{C30}$$

The equations $(C29)$ and $(C30)$ give $\delta_5 = 0$. The equation $(C28)$ is therefore simplified such that

$$\rho(\beta)\Delta\theta = \frac{1}{2}\left[\frac{(1-2\alpha)\beta(1-\beta)}{(1-\alpha-\beta)} + (\alpha+\beta-2\alpha\beta)\right]\rho(\beta)(\theta^u - D). \tag{C31}$$

According to $(C29)$, the multiplier δ_1 satisfy

$$\delta_1 = -\frac{\alpha(1-\alpha)(1-2\beta)}{2(1-\alpha-\beta)}\rho(\beta)(\theta^u - D). \tag{C32}$$

If $\delta_1 = 0$, $(C32)$ is satisfied if $\beta = \frac{1}{2}$ and/or $D = \theta^u$ (in the latter case, we have a contradiction). Substituting $\beta = \frac{1}{2}$ in $(C31)$, we get:

$$D = \theta^u - 2\Delta\theta \tag{C33}$$

but this solution exists only if the project is not very risky.

If $\delta_1 < 0$, we have no solutions.

If $\delta_1 > 0$, according to $(C10)$, $D = H = \theta^d$ which is substituted in $(C31)$. We obtain the equation $(C23)$ which is solved if $(C24)$.

5. $D = H = 0$ and $0 < \beta < 1$

In this case, the bank does not contribute financially to the acquisition. The Kuhn-Tucker multipliers δ_3, δ_5, δ_6 and δ_7 are null and the conditions $(C1)$, $(C4)$ and $(C7)$ become

$$2(1-\alpha-\beta) - (1-2\alpha)\beta(1-\beta) - (\alpha+\beta-2\alpha\beta)(1-\alpha-\beta) = 0 \tag{C34}$$

$$-[1-\alpha-(1-2\alpha)\beta]\rho(\beta)\Delta\theta + \delta_1 + \delta_2 \leq 0 \tag{C35}$$

$$[1-\alpha-(1-2\alpha)\beta]\rho(\beta)\Delta\theta - \delta_1 + \delta_4 \leq 0 \tag{C36}$$

Notice that the equation $(C34)$ is satisfied if $\beta = \beta(\alpha)$ given by $(C24)$. Moreover, the conditions $(C35)$ and $(C36)$ imply

$$\delta_1 + \delta_2 \leq [1-\alpha-(1-2\alpha)\beta]\rho(\beta)\Delta\theta \leq \delta_1 - \delta_4 \tag{C37}$$

Given the fact that the Kuhn-Tucker multipliers are positive, we conclude that $(C37)$ is satisfied if $\delta_2 = \delta_4 = 0$ and $\delta_1 = [1-\alpha-(1-2\alpha)\beta]\rho(\beta)\Delta\theta > 0$.

D. Proof of the proposition 4

The Lagrangian is written:

$$\begin{aligned}\mathcal{L} = \;& \beta[p(e,a)\,(\Delta\theta - D + H) + \theta^d - H] - u(e) - K + H + p(e,a)\,(D-H) \\ & +\delta_1\left\{(1-\beta)\,[p(e,a)\,(\Delta\theta - D + H) + \theta^d - H] - v(a)\right\} + \delta_2 D + \delta_3 H \\ & +\delta_4\,(\theta^u - D) + \delta_5\left(\theta^d - H\right) + \delta_6\beta + \delta_7\,(1-\beta)\end{aligned}$$

where δ_j, $j = 1..7$ are the Kuhn-Tucker multipliers.

The Kuhn-Tucker conditions are given by:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\beta} = \;& (1-\delta_1)\,[p(e,a)\,(\Delta\theta - D + H) + \theta^d - H] \\ & +\delta_6 - \delta_7 \leq 0\end{aligned} \quad \text{(D1)}$$

$$\beta \geq 0 \quad \text{(D2)}$$

$$\beta\frac{\partial\mathcal{L}}{\partial\beta} = 0 \quad \text{(D3)}$$

$$\frac{\partial\mathcal{L}}{\partial D} = (1-\delta_1)\,(1-\beta)\,p(e,a) + \delta_2 - \delta_4 \leq 0 \quad \text{(D4)}$$

$$D \geq 0 \quad \text{(D5)}$$

$$D\frac{\partial\mathcal{L}}{\partial D} = 0 \quad \text{(D6)}$$

$$\frac{\partial\mathcal{L}}{\partial H} = (1-\delta_1)\,(1-\beta)\,[1 - p(e,a)] + \delta_3 - \delta_5 \leq 0 \quad \text{(D7)}$$

$$H \geq 0 \quad \text{(D8)}$$

$$H\frac{\partial\mathcal{L}}{\partial H} = 0 \quad \text{(D9)}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial e} = \;& \beta p_e(e,a)\,(\Delta\theta - D + H) - u'(e) + p_e(e,a)\,(D-H) \\ & +\delta_1\,(1-\beta)\,p_e(e,a)\,(\Delta\theta - D + H) = 0\end{aligned} \quad \text{(D10)}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial a} = \;& \beta p_a(e,a)\,(\Delta\theta - D + H) + p_a(e,a)\,(D-H) \\ & +\delta_1\,\{(1-\beta)\,p_a(e,a)\,(\Delta\theta - D + H) - v'(a)\} = 0\end{aligned} \quad \text{(D11)}$$

and the complementary slackness conditions are written:

$$\begin{aligned}& \frac{\partial\mathcal{L}}{\partial\delta_1} = (1-\beta)\,[p(e,a)\,(\Delta\theta - D + H) + \theta^d - H] - v(a) \geq 0\ , \\ & \delta_1 \geq 0\ ,\quad \delta_1\frac{\partial\mathcal{L}}{\partial\delta_1} = 0\end{aligned} \quad \text{(D12)}$$

$$\frac{\partial\mathcal{L}}{\partial\delta_2} = D \geq 0\ ,\quad \delta_2 \geq 0\ ,\quad \delta_2 D = 0 \quad \text{(D13)}$$

$$\frac{\partial\mathcal{L}}{\partial\delta_3} = H \geq 0\ ,\quad \delta_3 \geq 0\ ,\quad \delta_3 H = 0 \quad \text{(D14)}$$

$$\frac{\partial\mathcal{L}}{\partial\delta_4} = \theta^u - D \geq 0\ ,\quad \delta_4 \geq 0\ ,\quad \delta_4\,(\theta^u - D) = 0 \quad \text{(D15)}$$

$$\frac{\partial\mathcal{L}}{\partial\delta_5} = \theta^d - H \geq 0\ ,\quad \delta_5 \geq 0\ ,\quad \delta_5\left(\theta^d - H\right) = 0 \quad \text{(D16)}$$

$$\frac{\partial\mathcal{L}}{\partial\delta_6} = \beta \geq 0\ ,\quad \delta_6 \geq 0\ ,\quad \delta_6\beta = 0 \quad \text{(D17)}$$

$$\frac{\partial\mathcal{L}}{\partial\delta_7} = 1 - \beta \geq 0\ ,\quad \delta_7 \geq 0\ ,\quad \delta_7\,(1-\beta) = 0 \quad \text{(D18)}$$

Consider that $0 < D < \theta^u$, $0 < H < \theta^d$ and $0 < \beta < 1$. Given the complementary slackness conditions, we deduce that $\delta_j = 0$, $j = 2..7$. The first order conditions $(D1)$, $(D4)$ and $(D7)$ give

$$(1-\delta_1)\left[p(e,a)\left(\Delta\theta - D + H\right) + \theta^d - H\right] = 0$$
$$(1-\delta_1)\left(1-\beta\right)p(e,a) = 0$$
$$(1-\delta_1)\left(1-\beta\right)\left[1 - p(e,a)\right] = 0$$

This system is satisfied if and only if $\delta_1 = 1 > 0$. The condition $(D12)$ implies that the participation constraint of the consultant is binding.

Besides, the conditions $(D10)$ and $(D11)$ become

$$p_e(e,a)\Delta\theta - u'(e) = 0$$
$$p_a(e,a)\Delta\theta - v'(a) = 0$$

We conclude therefore that $\beta = \frac{1}{2}$ and the bank's payments are decreasing with the project's revenues such that $H - D = \Delta\theta$.

E. Proof of the proposition 5

We substitute $p(e,a)$ and $v(a)$ for $e^{1-\alpha}a^{\alpha}$ and $\frac{a^2}{2\mu}$ in the participation constraints of the consultant and the bank (29) and (30) which gives:

$$EU^A = (1-\beta)\left\{e^{1-\alpha}a^{\alpha}\left(\Delta\theta - D + H\right) + \theta^d - H\right\} - \frac{a^2}{2\mu} \geq 0 \tag{E1}$$

$$EU^B = e^{1-\alpha}a^{\alpha}\left(D - H\right) + H - \left(K - W\right) \geq 0 \tag{E2}$$

The expected gain of the entrepreneur is therefore written:

$$EU^E = \beta\left\{e^{1-\alpha}a^{\alpha}\left(\Delta\theta - D + H\right) + \theta^d - H\right\} - \frac{e^2}{2\lambda} - W \tag{E3}$$

The efforts of the entrepreneur and the LBO fund in the equilibrium are given respectively by (12) and (13).

Because of the competition among the banks, the participation constraint of the bank (CP_B) is binding which implies that:

$$W = K - H - e^{1-\alpha}a^{\alpha}\left(D - H\right) \tag{E4}$$

At the opposite, the participation constraint of the consultant $(E1)$ may not be binding.

We substitute (12), (13) and $(E4)$ in the objective function $(E3)$, the financial contracts induce the entrepreneur to solve the following program:

$$\max_{\beta,\, D,\, H} EU^E = \beta\left\{\tfrac{1}{2}\left(1+\alpha\right)\rho\left(\beta\right)\left(\Delta\theta - D + H\right)^2 + \theta^d - H\right\}$$
$$+H - K + \rho(\beta)(\Delta\theta - D + H)\left(D - H\right)$$

$$\text{s.t} \quad (1-\beta)\left[\left(1 - \tfrac{1}{2}\alpha\right)\rho\left(\beta\right)\left(\Delta\theta - D + H\right)^2 + \theta^d - H\right] \geq 0$$

with the conditions (6).

The Lagrangian is therefore given by:

$$
\begin{aligned}
\mathcal{L}: \; & = \mathcal{L}(\beta,\, D,\, H,\, \delta_j,\, j = 1..6) \\
& = \beta \left\{ \tfrac{1}{2} (1+\alpha)\, \rho(\beta)\, (\Delta\theta - D + H)^2 + \theta^d - H \right\} + H - K \\
& + \rho(\beta)(\Delta\theta - D + H)\,(D - H) \\
& + \delta_1 \left\{ \left(1 - \tfrac{1}{2}\alpha\right)(1-\beta)\rho(\beta)\,(\Delta\theta - D + H)^2 + (1-\beta)\left(\theta^d - H\right) \right\} \\
& + \delta_2 D + \delta_3 H + \delta_4 (\theta^u - D) + \delta_5 \left(\theta^d - H\right) + \delta_6 \beta + \delta_7 (1-\beta)
\end{aligned}
$$

where $\delta_j,\, j = 1..7$ are the Kuhn-Tucker multipliers.

The Kuhn-Tucker conditions are written:

$$
\begin{aligned}
\tfrac{\partial \mathcal{L}}{\partial \beta} = \; & \tfrac{1}{2}(1+\alpha)\, \tfrac{2(1-\beta)-\alpha}{(1-\beta)} \rho(\beta)\,(\Delta\theta - D + H)^2 + \theta^d - H \\
& + \tfrac{1-\alpha-\beta}{\beta(1-\beta)} \rho(\beta)\,(\Delta\theta - D + H)\,(D - H) + \delta_5 - \delta_6 \\
& + \delta_1 \left\{ \left(1 - \tfrac{1}{2}\alpha\right) \tfrac{1-\alpha-2\beta}{\beta} \rho(\beta)\,(\Delta\theta - D + H)^2 - \theta^d + H \right\} \leq 0
\end{aligned}
\tag{E5}
$$

$$\beta \geq 0 \tag{E6}$$

$$\beta \frac{\partial \mathcal{L}}{\partial \beta} = 0 \tag{E7}$$

$$
\begin{aligned}
\tfrac{\partial \mathcal{L}}{\partial D} = \; & \rho(\beta)\,[\Delta\theta - 2(D - H)] - \beta(1+\alpha)\,\rho(\beta)\,(\Delta\theta - D + H) \\
& - \delta_1 (2-\alpha)(1-\beta)\,\rho(\beta)\,(\Delta\theta - D + H) \\
& + \delta_2 - \delta_4 \leq 0
\end{aligned}
\tag{E8}
$$

$$D \geq 0 \tag{E9}$$

$$D \frac{\partial \mathcal{L}}{\partial D} = 0 \tag{E10}$$

$$
\begin{aligned}
\tfrac{\partial \mathcal{L}}{\partial H} = \; & \beta\,[(1+\alpha)\,\rho(\beta)\,(\Delta\theta - D + H) - 1] \\
& + 1 - \rho(\beta)\,[\Delta\theta - 2(D - H)] + \delta_3 - \delta_5 \\
& + \delta_1 (1-\beta)\,[(2-\alpha)\,\rho(\beta)\,(\Delta\theta - D + H) - 1] \leq 0
\end{aligned}
\tag{E11}
$$

$$H \geq 0 \tag{E12}$$

$$H \frac{\partial \mathcal{L}}{\partial H} = 0 \tag{E13}$$

and the complementary slackness conditions are written:

$$
\begin{aligned}
\tfrac{\partial \mathcal{L}}{\partial \delta_1} = \; & (1-\beta) \left[\left(1 - \tfrac{1}{2}\alpha\right) \rho(\beta)\,(\Delta\theta - D + H)^2 + \theta^d - H \right] \geq 0 \\
& \delta_1 \geq 0,\; \delta_1 \tfrac{\partial \mathcal{L}}{\partial \delta_1} = 0
\end{aligned}
\tag{E14}
$$

$$\tfrac{\partial \mathcal{L}}{\partial \delta_2} = D \geq 0 \;,\quad \delta_2 \geq 0 \;,\quad \delta_2 D = 0 \tag{E15}$$

$$\tfrac{\partial \mathcal{L}}{\partial \delta_3} = H \geq 0 \;,\quad \delta_3 \geq 0 \;,\quad \delta_3 H = 0 \tag{E16}$$

$$\tfrac{\partial \mathcal{L}}{\partial \delta_4} = \theta^u - D \geq 0 \;,\quad \delta_4 \geq 0 \;,\quad \delta_4 (\theta^u - D) = 0 \tag{E17}$$

$$\frac{\partial \mathcal{L}}{\partial \delta_5} = \theta^d - H \geq 0 \ , \quad \delta_5 \geq 0 \ , \quad \delta_5 \left(\theta^d - H\right) = 0 \tag{E18}$$
$$\frac{\partial \mathcal{L}}{\partial \delta_6} = \beta \geq 0 \ , \quad \delta_6 \geq 0 \ , \quad \delta_6 \beta = 0 \tag{E19}$$
$$\frac{\partial \mathcal{L}}{\partial \delta_7} = 1 - \beta \geq 0 \ , \quad \delta_7 \geq 0 \ , \quad \delta_7 (1 - \beta) = 0 \tag{E20}$$

Given the fact that the efforts e and a are complementaries, both agents have to provide strictly positive efforts otherwise the projects fails with probability equal to 1. This is satisfied only if $0 < \beta < 1$ which implies that $\delta_6 = \delta_7 = 0$. Furthermore, the following cases are not considered:

- $D = \theta^u$ and $H = \theta^d$: in this case, the agents' efforts are null and the project fails with probability equal to 1.
- $D = \theta^u$ and $0 \leq H < \theta^d$ because the efforts are strictly negative.
- $D = H = 0$, the amount of the debt is null. If the consultant does not contribute financially to the acquisition and the entrepreneur is wealth constrained ($K > W$), the entrepreneur and the LBO fund cannot acquire the company.

1. Consider the case $0 < D < \theta^u$ and $0 < H < \theta^d$. According to the conditions ($E15$), ($E16$), ($E17$) and ($E18$), the Kuhn-Tucker multipliers δ_2, δ_3, δ_4 and δ_5 are null.

   - Consider first that $\delta_1 > 0$, the condition ($E14$) is satisfied $\left(\frac{\partial \mathcal{L}}{\partial \delta_1} = 0\right)$ which implies that:

   $$\frac{1}{2} (2 - \alpha) \rho(\beta) (\Delta\theta - D + H)^2 + \theta^d - H = 0 \tag{E21}$$

   Notice that if $0 < \beta < 1$, $\rho(\beta) > 0$. The condition ($E21$) is satisfied only if $\Delta\theta - D + H = 0$ and $H = \theta^d$ which gives $D = \theta^u$ and $H = \theta^d$ and leads to a contradiction.
   - Consider now that $\delta_1 = 0$, the conditions ($E5$), ($E8$) and ($E11$) give the following system:

   $$\begin{array}{c} \frac{1}{2} (1 + \alpha) \frac{2(1-\beta)-\alpha}{(1-\beta)} \rho(\beta) (\Delta\theta - D + H)^2 + \theta^d - H \\ + \frac{1-\alpha-\beta}{(1-\beta)} \rho(\beta) (\Delta\theta - D + H) (D - H) + \theta^d - H = 0 \end{array} \tag{E22}$$
   $$[\Delta\theta - 2(D - H)] = \beta (1 + \alpha) (\Delta\theta - D + H) \tag{E23}$$
   $$\begin{array}{c} \beta [(1 + \alpha) \rho(\beta) (\Delta\theta - D + H) - 1] \\ +1 - \rho(\beta) [\Delta\theta - 2(D - H)] = 0 \end{array} \tag{E24}$$

   If we substitute ($E23$) in ($E24$), we obtain therefore the following contradiction:

   $$\beta = 1$$

2. Consider now the case $D = 0$ and $H = \theta^d$, the multipliers $\delta_3 = \delta_4 = 0$.

   - If $\delta_1 > 0$, the condition ($E14$) is satisfied: $\frac{\partial \mathcal{L}}{\partial \delta_1} = 0$ which gives:

   $$(1 - \beta) \left(1 - \frac{1}{2}\alpha\right) \rho(\beta) (\theta^u)^2 = 0 \tag{E25}$$

   But given $0 < \beta < 1$, the latter condition is never satisfied.

- If $\delta_1 = 0$, the equations $(E5)$, $(E8)$ and $(E11)$ are written:

$$\frac{1}{2}(1+\alpha)\frac{2(1-\beta)-\alpha}{(1-\beta)}\theta^u - \frac{1-\alpha-\beta}{\beta(1-\beta)}\theta^d = 0 \tag{E26}$$

$$\delta_2 \leq \rho(\beta)\left\{[\beta(1+\alpha)-1]\theta^u - \theta^d\right\} \tag{E27}$$

$$\delta_5 = 1-\beta-\rho(\beta)\theta^d - [1-\beta(1+\alpha)]\rho(\beta)\theta^u \tag{E28}$$

This system gives

$$\beta = \frac{1}{2}\frac{(1+\alpha)(2-\alpha)+2k-\sqrt{(1+\alpha)^2(2-\alpha)^2+4k^2+4k(1+\alpha)(3\alpha-2)}}{2(1+\alpha)} \tag{E29}$$

where $k = \frac{\theta^d}{\theta^u} < \frac{1}{2}$. If we substitute $(E29)$ in $(E27)$, we have $\delta_2 \leq 0$.

3. Consider the latter case $0 < D < \theta^u$ and $H = \theta^d$. Given the conditions $(E15)$, $(E16)$ and $(E17)$, we obtain $\delta_2 = \delta_3 = \delta_4 = 0$.

- If $\delta_1 > 0$, $\frac{\partial \mathcal{L}}{\partial \delta_1} = 0$. If we substitute $H = \theta^d$ in $(E14)$, we have a contradiction:

$$(1-\beta)\left(1-\frac{1}{2}\alpha\right)\rho(\beta)(\theta^u - D)^2 = 0$$

because this condition is satisfied only if $D = \theta^u$.

- If $\delta_1 = 0$, the first order conditions $(E5)$, $(E8)$ and $(E11)$ are written:

$$\beta(1+\alpha)(2-2\beta-\alpha)(\theta^u - D) + 2(1-\alpha-\beta)\left(D-\theta^d\right) = 0 \tag{E30}$$

$$\theta^u + \theta^d - 2D - \beta(1+\alpha)(\theta^u - D) = 0 \tag{E31}$$

$$\delta_5 = \beta[(1+\alpha)\rho(\beta)(\theta^u - D) - 1] + 1 - \rho(\beta)\left(\theta^u + \theta^d - 2D\right) \tag{E32}$$

The equations $(E30)$ and $(E31)$ enable us to deduce:

$$\beta = \frac{2}{2+\alpha}$$
$$D = \left(\frac{1}{2}\alpha + 1\right)\theta^d - \frac{1}{2}\alpha\theta^u$$

$D > 0$ if $\theta^u < \frac{\alpha+2}{\alpha}\theta^d$. If we substitute β and D in $(E32)$, it is easy to check that the multiplier

$$\delta_5 = \frac{\alpha}{\alpha+2}$$

is strictly positive.

F. Proof of the lemma 1

We add the following condition to the program to be solved:

$$D \geq H$$

The Lagrangian is therefore written:

$$\begin{aligned}\mathcal{L}: \ &= \mathcal{L}(\beta,\, D,\, H,\, \delta_j,\, j = 1..6) \\ &= \beta\left\{\tfrac{1}{2}\,(1+\alpha)\,\rho\,(\beta)\,(\Delta\theta - D + H)^2 + \theta^d - H\right\} + H - K \\ &+\rho(\beta)(\Delta\theta - D + H)\,(D - H) \\ &+\delta_1\left\{\left(1 - \tfrac{1}{2}\alpha\right)(1-\beta)\rho\,(\beta)\,(\Delta\theta - D + H)^2 + (1-\beta)\left(\theta^d - H\right)\right\} \\ &+\delta_2 D + \delta_3 H + \delta_4\,(\theta^u - D) + \delta_5\left(\theta^d - H\right) + \delta_6\beta + \delta_7\,(1-\beta) + \delta_8\,(D - H)\end{aligned}$$

δ_j, $j = 1..8$ are the Kuhn-Tucker multipliers.

The Kuhn-Tucker conditions are given by:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\beta} = \ &\tfrac{1}{2}\,(1+\alpha)\,\tfrac{2(1-\beta)-\alpha}{(1-\beta)}\rho\,(\beta)\,(\Delta\theta - D + H)^2 + \theta^d - H \\ &+\tfrac{1-\alpha-\beta}{\beta(1-\beta)}\rho\,(\beta)\,(\Delta\theta - D + H)\,(D - H) + \delta_5 - \delta_6 \\ &+\delta_1\left\{\left(1 - \tfrac{1}{2}\alpha\right)\tfrac{1-\alpha-2\beta}{\beta}\rho\,(\beta)\,(\Delta\theta - D + H)^2 - \theta^d + H\right\} \leq 0\end{aligned} \quad \text{(F1)}$$

$$\beta \geq 0 \quad \text{(F2)}$$

$$\beta\frac{\partial\mathcal{L}}{\partial\beta} = 0 \quad \text{(F3)}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial D} = \ &\rho\,(\beta)\,[\Delta\theta - 2\,(D - H)] - \beta\,(1+\alpha)\,\rho\,(\beta)\,(\Delta\theta - D + H) \\ &-\delta_1\,(2-\alpha)\,(1-\beta)\,\rho\,(\beta)\,(\Delta\theta - D + H) \\ &+\delta_2 - \delta_4 + \delta_8 \leq 0\end{aligned} \quad \text{(F4)}$$

$$D \geq 0 \quad \text{(F5)}$$

$$D\frac{\partial\mathcal{L}}{\partial D} = 0 \quad \text{(F6)}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial H} = \ &\beta\,[(1+\alpha)\,\rho\,(\beta)\,(\Delta\theta - D + H) - 1] \\ &+1 - \rho\,(\beta)\,[\Delta\theta - 2\,(D - H)] + \delta_3 - \delta_5 - \delta_8 \\ &+\delta_1(1-\beta)\,[(2-\alpha)\,\rho\,(\beta)\,(\Delta\theta - D + H) - 1] \leq 0\end{aligned} \quad \text{(F7)}$$

$$H \geq 0 \quad \text{(F8)}$$

$$H\frac{\partial\mathcal{L}}{\partial H} = 0 \quad \text{(F9)}$$

and the complementary slackness conditions are written:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\delta_1} = \ &(1-\beta)\left[\left(1 - \tfrac{1}{2}\alpha\right)\rho\,(\beta)\,(\Delta\theta - D + H)^2 + \theta^d - H\right] \geq 0 \\ &\delta_1 \geq 0,\ \delta_1\tfrac{\partial\mathcal{L}}{\partial\delta_1} = 0\end{aligned} \quad \text{(F10)}$$

$$\frac{\partial \mathcal{L}}{\partial \delta_2} = D \geq 0 \ , \quad \delta_2 \geq 0 \ , \quad \delta_2 D = 0 \tag{F11}$$
$$\frac{\partial \mathcal{L}}{\partial \delta_3} = H \geq 0 \ , \quad \delta_3 \geq 0 \ , \quad \delta_3 H = 0 \tag{F12}$$
$$\frac{\partial \mathcal{L}}{\partial \delta_4} = \theta^u - D \geq 0 \ , \quad \delta_4 \geq 0 \ , \quad \delta_4 \left(\theta^u - D\right) = 0 \tag{F13}$$

$$\frac{\partial \mathcal{L}}{\partial \delta_5} = \theta^d - H \geq 0 \ , \quad \delta_5 \geq 0 \ , \quad \delta_5 \left(\theta^d - H\right) = 0 \tag{F14}$$
$$\frac{\partial \mathcal{L}}{\partial \delta_6} = \beta \geq 0 \ , \quad \delta_6 \geq 0 \ , \quad \delta_6 \beta = 0 \tag{F15}$$
$$\frac{\partial \mathcal{L}}{\partial \delta_7} = 1 - \beta \geq 0 \ , \quad \delta_7 \geq 0 \ , \quad \delta_7 \left(1 - \beta\right) = 0 \tag{F16}$$
$$\frac{\partial \mathcal{L}}{\partial \delta_8} = D - H \geq 0 \ , \quad \delta_8 \geq 0 \ , \quad \delta_8 \left(D - H\right) = 0 \tag{F17}$$

Given $0 < \beta < 1$, the conditions $(F15)$ and $(F16)$ give $\delta_6 = \delta_7 = 0$. We will not analyze the following cases: $D = \theta^u$ and $H = \theta^d$, $D = \theta^u$ and $0 \leq H < \theta^d$ and $D = H = 0$ because of the reasons explained in the appendix E.

1. If we consider $0 < D < \theta^u$, $0 < H < \theta^d$ and $D > H$, whether $\delta_1 = 0$ or $\delta_1 > 0$, we get contradictions (see the appendix E).

2. If we consider $0 < D < \theta^u$, $0 < H < \theta^d$ and $D = H$. The Kuhn-Tucker conditions $(F4)$ and $(F7)$ give

$$\begin{aligned}
\delta_8 &= -\rho\left(\beta\right)\Delta\theta + \beta\left(1+\alpha\right)\rho\left(\beta\right)\Delta\theta + \delta_1\left(2-\alpha\right)\left(1-\beta\right)\rho\left(\beta\right)\Delta\theta \\
&= \beta\left[\left(1+\alpha\right)\rho\left(\beta\right)\Delta\theta - 1\right] + 1 - \rho\left(\beta\right)\Delta\theta + \delta_1(1-\beta)\left[\left(2-\alpha\right)\rho\left(\beta\right)\Delta\theta - 1\right]
\end{aligned}$$

The latter conditions are satisfied only if:

$$\delta_1 = 1 > 0$$
$$\delta_8 = \left[1 - \alpha - \left(1 - 2\alpha\right)\beta\right]\rho\left(\beta\right)\Delta\theta$$

According to $(F1)$, the optimal sharing rule is the solution of the following equation:

$$\frac{1}{2}\left(1+\alpha\right)\frac{2\left(1-\beta\right)-\alpha}{\left(1-\beta\right)} + \left(1 - \frac{1}{2}\alpha\right)\frac{1-\alpha-2\beta}{\beta} = 0$$

It is given by:

$$\beta\left(\alpha\right) = \begin{cases} \frac{(2-\alpha)(1-\alpha)-\sqrt{\alpha(1-\alpha^2)(2-\alpha)}}{2(1-2\alpha)} & \text{if } \alpha \in \left]0,\, 1\right[/ \left\{\frac{1}{2}\right\} \\ \frac{1}{2} & \text{if } \alpha = \frac{1}{2} \end{cases}$$

Substituting $D = H$ in the participation constraint of the bank enables us to deduce:

$$I = K - W = H$$

If we analyze the other cases of the appendix E, we get contradictions. Moreover, the hessian is negative semidefinitive when $D = H = K - W$ and $\beta = \beta\left(\alpha\right)$ which means that this solution is a local maxima.

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