KEY FACTORS IN THE TERM STRUCTURE OF VOLATILITY OF INTEREST RATES

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Abstract. This paper tackles with the estimation of the Term Structure of Volatility (TSV) of interest rates. Analogously to the Term Structure of Interest Rates (TSIR), the TSV illustrates the relationship of zero-coupon rate volatility to time to maturity, and shows the existence of strong co-movements in the volatility of interest rates. Exploiting this feature, we firstly apply the orthogonal GARCH model introduced by Alexander and Chibumba (1997) and Alexander (2000) to obtain the set of unobservable factors that summarize interest rate volatility in the Spanish Public Debt Market during the period 1996-2006. In a second stage, we identify the observable factors, given interest rates at specific maturities, which best approximate the volatility of the unobservable factors and still provide an efficient representation of interest rate volatility. This two-stage approach establishes a link between the research aimed to identify the key rates of the TSIR using econometric techniques and those studies that use principal component analysis or factor analysis to offer a parsimonious representation of interest rates dynamics. This link allow us to avoid the use of large multivariate GARCH models and also overcome the main drawbacks of the two methods, given as the lack of economic interpretation of the unobservable factors and the difficulties found in econometric studies to identify the set of key rates due to the high number of interest rates to be tested and their significant correlations.

JEL classification: E43, G10; G11.

Key words: Interest rate volatility, term structure of volatility, term structure of interest rates, factor models, principal component analysis

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1. INTRODUCTION.

Analogously to the Term Structure of Interest Rates (hereafter TSIR), the Term Structure of Volatility of interest rates (hereafter TSV) illustrates the relationship of zero-coupon rate volatility to time to maturity. The TSV has important practical implications from different points of view. On one hand, it is a critical issue in the context of risk management in order to accurately measure the risk in fixed-income portfolios using Value at Risk (VaR) or other measures. On the other hand, in the framework of monetary policy it is crucial to be able to know the transmission of uncertainty from the short-term interest rates to the long-term ones and vice versa in order to assess the effectiveness of the monetary policy measures adopted on firms' investments and on household's consumption decisions. Furthermore, the correct modelling and forecasting of interest rate volatility plays also a prominent role in other areas of finance such as pricing of interest rate derivatives, asset allocation, and portfolio selection.

However, the estimation of the TSV faces a major difficulty, given as the large dimension of the set of interest rates that constitutes the TSIR. For this reason, and also the benefits that derive for the different applications of the TSV, the representation of the TSV through a small number of factors becomes an issue of special relevance. The existence of strong co-movements in the volatility of interest rates makes this reduction of dimensionality feasible, but raises the challenge to identify the reduced number of factors that best describe interest rate movements.

The literature offers two main approaches to achieve this goal. The first one uses econometric techniques to identify a few number of observable factors, given as some interest rates corresponding to different maturities, the so-called key rates, that best describe interest rate movements (see, for instance, Elton et al., 1990 for the U.S. market; and Navarro and Nave, 1997 and 2001, for the Spanish market). The second approach assumes that interest rate dynamics are driven by a number of common unobservable factors that are usually defined using factor analysis or principal components analysis (PCA, hereafter). Examples of the use of PCA for the description of interest rate movements are found in Litterman and Scheinkman (1991), Barber and Cooper (1996) and Bliss (1997) for the U.S. market and Abad and Benito (2006) for the Spanish market, and extensions to GARCH-type models can be found in Alexander and Chibumba (1997), Alexander (2000), and Benito and Novales (2007).

The two approaches show different drawbacks. On the one hand, the key-rate approach has to deal with a large number of combinations of interest rates to find the one that best describes interest rate dynamics. Also, the correlation between interest rates should be taken into account in order to not to distort the econometric results. On the other hand, principal components are usually criticized because of the lack of economic interpretation and its instability along time (see Perignon and Villa, 2006).

In this paper we try to reconcile both approaches and overcome these difficulties by proposing an efficient way of representing interest rate changes and generating volatility term structures of interest rates that only requires principal component analysis and the estimation of univariate GARCH models and simple regression models.

The starting point is the orthogonal GARCH model proposed by Alexander and Chibumba (1997), and Alexander (2000). This model relies on principal component analysis to obtain a small number of orthogonal factors that can each be modelled as univariate GARCH processes. The model can be thought as an extension of the factor GARCH model of Engle et al. (1990) to a multifactor model with orthogonal factors and has been proven to work particularly well in highly correlated systems.

The orthogonal GARCH model allows us to estimate the TSV of interest rates of the Spanish public debt market using historical time series data of interest rates for the period 1996-2006 and draw some conclusions about the shape and dynamics of interet rate volatility during this period.

Finally, the conditional volatility of the principal components are compared to the volatility of each interest rate obtained from univariate GARCH models in order to identify the set of key rates that best describe the TSV. Our results indicate that a set of four interest rates is able to reproduce the shape and dynamics of the TSV effectively.

The rest of the paper is organised as follows. The next section describes the data; section 3 shows the results of the orthogonal GARCH model; section 4 deals with the identification and performance of the key rates, and section 5 concludes.

2. DATA

Term structure of interest rates (TSIR) has been estimated using data from the Spanish Public Debt market by using a correction on the Nelson and Siegel (1987) method in order to avoid errors in the estimation of short-term spot interest rates. [1] [2]

The sample period extends from January 1996 to December 2006, spanning a time interval in which Spanish interest rates varied substantially within an overall downward trend. Since we use daily estimations of TSIR, a number of 2782 data of spot interest rates for eleven different maturities have been employed in our analysis. Specifically, we have used the usual spot interest rates considered by RiskMetrics for the Spanish market: one, three and six month interest rates, and one, two, three, four, five, seven, nine, and ten year interest rates. Notice that in order to do an adequate risk management, the key variable in the analysis is the volatility of interest rate *changes*, so first order differences of the spot interest rates have been calculated and those changes have been taken as our dependent variable. Additionally, this permits us to work with stationary variables, since interest rate levels are variables integrated of first order.

3. THE ORTHOGONAL GARCH MODEL

Since their introduction by Engle (1982) and Bollerslev (1986), the GARCH (generalized autoregressive conditionally heteroskedastic) models have been used extensively both in academia and by practitioners to estimate the volatility of financial variables. The success of GARCH models can be largely attributed to their ability to capture several stylised facts of financial data, such as time-varying volatility, persistence and clustering of volatility, and asymmetric reactions to positive and negative shocks of equal magnitude.

Given the empirical success of GARCH processes in the modelling of univariate volatility and since it is now widely accepted that financial volatilities move together over time across assets and markets, a natural extension has been the use of multivariate GARCH models to measure the dynamic volatilities and correlations of large dimension

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[1] The authors would like to thank our colleagues A. Diaz and E. Navarro from the University of Castilla-La Mancha (UCLM) their acceptance to use their TSIR estimated from the Spanish market for the period 1996-2006 as a database for this paper.

[2] A detailed explanation of this correction on the Nelson and Siegel method can be found in A. Diaz's website (http://www.uclm.es/area/aef/Etti.asp).

systems. However, the multivariate GARCH models present many difficulties with estimation, mainly caused by the exponential increase in the number of parameters with the dimension of the system and the restrictions required by the positive definiteness of the covariance matrix. As one solution to these problems, several factor GARCH models based on the assumption that there are few common factors whose time-varying variances drive the whole covariance matrix of the system have been introduced in the literature (see, for example, Engle et al., 1990; or Diebold and Nerlove, 1989)[3].

In this context, the orthogonal GARCH model proposed by Alexander and Chibumba (1997) and Alexander (2000) combines the GARCH methodology and the principal component analysis. Its central idea is to use principal component analysis to generate a number of orthogonal factors that can each be treated in a univariate GARCH framework. The orthogonal GARCH model reduces the dimensionality of estimating the conditional covariance matrix by generating *kxk* GARCH covariances from *m* univariate GARCH models, where *k* is the number of variables in the system and *m* is the number of principal components actually used. This allows us to capture the *k*-dimension of the system by estimating *m* univariate GARCH models of the principal components of the larger original system. The gain in efficiency clearly depends on the extent to which $m<k$.

As Alexander (2000) points out, the orthogonal GARCH method has several advantages over direct multivariate GARCH. On the one hand, it permits to reduce the dimension problems that arise when estimating covariance matrices in large systems of assets, particularly when the assets are highly correlated. On the other, it gives us the possibility of cutting out any noise in the data that would otherwise make correlation estimates unstable. Finally, this method allows us to generate estimates for volatilities and correlations of variables in the system even when data are sparse and unreliable, for example in illiquid markets.

The application of the orthogonal GARCH model to estimate the TSV of interest rates from the TSIR original data involves splitting the empirical analysis in the following three stages. First of all, a principal component analysis has been carried out on the time series of changes in spot interest rates corresponding to *k* different maturities. This

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[3] As Christiansen (1999) points out, the different factor ARCH models differ in the assumptions about the common factors. For example, Engle et al. (1990) present a factor ARCH model related with Arbitrage Pricing Theory of Ross (1976) where the excess returns of financial assets are driven by a number of common unobservable factors whose conditional variances are described by a GARCH-M specification.

analysis permits us to keep *m* principal components (where $m<k$) containing most of the information about the spot rates changes' variability. These *m* principal components constitute the key market factors that represent the most important sources of information, and all the rest of the variation is ascribed to noise.

According to equation (1), linear combinations of these *m* principal components does permit generate most of the information regarding the *k* original variables.

$$\Delta ir_{jt} = \theta_{1j} \cdot Z_{1t} + \theta_{2j} \cdot Z_{2t} + \cdots + \theta_{mj} \cdot Z_{mt} + \gamma_j \qquad (1)$$

where:
Δir_{jt} indicates the change at time *t* in the spot interest rate corresponding to a maturity *j*;
Z_i represents the i^{th}-principal component of the system of interest rate changes considered;
γ_j is a constant term, specific for every Δir_j, indicating the error assumed in cutting out the *k*-*m* remaining principal components.[4]

Second, the time-varying conditional variances of the first *m* principal components are obtained using an univariate standard GARCH(1,1) process. The GARCH(1,1) model defines the conditional mean and variance of the i^{th}-principal component at time *t* respectively as:

$$Z_{it} = \mu + \delta \cdot Z_{it-1} + \varepsilon_{it} \qquad (2)$$

$$\text{var}(Z_{it}) = \omega + \alpha \cdot \varepsilon_{it-1}^2 + \beta \cdot \text{var}(Z_{it-1}) \qquad (3)$$

where $\text{var}(Z_{it})$ denotes the conditional variance of the component principal Z_i obtained from the estimated GARCH(1,1) where $\omega > 0$, $\alpha, \beta \geq 0$, and $\alpha + \beta < 1$.

This simple GARCH model effectively captures volatility clustering and provides convergent forecasts to the long-term average level of volatility. The coefficient α measures the intensity of reaction of volatility to yesterday's unexpected return ε_{t-1}^2, and the coefficient β measures the persistence in volatility.

Third, the conditional variances of the changes in spot interest rates can be obtained as a linear combination of the conditional variances of the first *m* principal components.

[4] Provided that *k* is the number of variables considered in the analysis, every one of them can be fully described by a linear combination of the *k* principal components. However, reducing the dimension of the problem by keeping only the *m* principal components which have more information necessarily implies to lose part of that information.

Since the principal components are, by definition, orthogonal to each other, we no longer need to measure the covariances, substantially reducing the number of parameters to be estimated. Therefore, the conditional variance of the interest rate variations can be calculated following equation (4):

$$\mathrm{var}(\Delta ir_{jt}) = \theta_{1j}^2 \cdot \mathrm{var}(Z_{1t}) + \theta_{2j}^2 \cdot \mathrm{var}\ (Z_{2t}) + \cdots + \theta_{mj}^2 \cdot \mathrm{var}\ (Z_{mt}) \tag{4}$$

Analogously, and assuming that errors γ_j and γ_h are not correlated, the conditional covariance between any two changes in interest rates of different maturity can be calculated from (5) since the *m* principal components obtained are orthogonal:

$$\begin{aligned}\mathrm{cov}(\Delta ir_{jt}, \Delta ir_{ht}) = \\ &= \mathrm{cov}\left(\theta_{1j} \cdot Z_{1t} + \theta_{2j} \cdot Z_{2t} + \cdots + \theta_{mj} \cdot Z_{mt} + \gamma_{jt}\ , \theta_{1h} \cdot Z_{1t} + \theta_{2h} \cdot Z_{2t} + \cdots + \cdot\theta_{mh} \cdot Z_{mt} + \gamma_{ht}\ \right) \\ &= \theta_{1j} \cdot \theta_{1h} \cdot \mathrm{var}(Z_{1t}) + \theta_{2j} \cdot \theta_{2h} \cdot \mathrm{var}(Z_{2t}) + \cdots + \theta_{mj} \cdot \theta_{mh} \cdot \mathrm{var}(Z_{mt})\end{aligned} \tag{5}$$

Application of the above methodology to the term structure of interest rates in Spain reveal that the first three principal components are sufficient to explain more than 93 per cent of total variation of the system of interest rate changes (Table 1). In particular, the first principal component (Z_1) helps to explain more than 47% of the total variation over the period of study. The addition of a second principal component (Z_2) contributes to increase that percentage up to almost 75% and the sum of the third principal component (Z_3) does permit to explain more than 93% of the variance of the system.[5]

Table 2 presents the factor loadings of the first three principal components. The first principal component is highly and positively correlated with all interest rate changes and can be interpreted as a parallel shift of the term structure, which means that all interest rates move in the same direction and by a similar amount, so inducing a roughly parallel shift in the TSIR specially in the medium term maturities, where the coefficients are more similar. The second principal component represents the tilt of the TSIR: the factor loadings have positive values for short term interest rate changes and

[5] Even though these cumulative percentages could seem low at first sight, they are in fact totally in line with those obtained by Alexander (2000) for the UK zero-coupon yields for the sample period January 1992–March 1995. Additionally, notice that we are dealing not with the interest yield or zero coupon rates, but with their first order differences. When the variables considered are the zero-coupon rates in levels, the cumulative percentages of total variation explained by the three principal components are 96.76%, 99.19%, and 99.96%, respectively, in line with the results obtained for the Spanish market by other authors for former sample periods.

negative values for medium and long term interest rate changes. Thus, an upward movement in this second component induces a change in the slope of the most part of the TSIR, since short maturities move up whereas long maturities move down. Finally, the factor loadings on the third component are positive for very short rates, but decreasing and becoming negative for the medium-term rates, and then increasing and becoming positive again for the longer maturities. According to this, the third principal component can be interpreted as a curvature or convexity factor.

Therefore, the results on the number of retained principal components and their interpretation are widely consistent with those of several empirical studies for the US and the European interest rate markets. See, for example, Litterman and Scheinkman (1991) for the US, Alexander (2000) for UK, and Benito and Novales (2007) for Spain.

Table 1. Principal Component Analysis

Proportion of variance explained by the first three principal components

Component	Eigenvalue	Cumulative Proportion
Z_1	5.218	47.342%
Z_2	2.935	74.118%
Z_3	2.092	93.136%

Table 2: Factor loadings of the principal component analysis

Maturity	Z_1	Z_2	Z_3
1 month	.214	.779	.474
3 months	.309	.892	.285
6 months	.434	.854	.048
1 year	.564	.595	-.368
2 years	.758	.130	-.610
3 years	.847	-.113	-.500
4 years	.897	-.231	-.281
5 years	.917	-.292	-.038
7 years	.858	-.328	.383
9 years	.707	-.305	.607
10 years	.630	-.286	.656

Once principal component analysis has been performed and the time series of the first three principal components have been generated from the matrix of factor score coefficients,[6] the next step has been to estimate univariate GARCH(1,1) models for each of the principal components. The results obtained are shown in Table 3.

Table 3: GARCH (1,1) models of the first three principal components

	Mean equation		Variance equation		
	μ	δ	ϖ	α	β
Z_1	0.0136 (0.0170)	0.1716*** (0.0205)	0.0073 (0.0074)	0.0488** (0.0245)	0.9441*** (0.0309)
Z_2	0.04266*** (0.0117)	-0.2826*** (0.0254)	0.0138** (0.0058)	0.2023*** (0.0486)	0.7976*** (0.0486)
Z_3	-0.0431* (0.0249)	-0.2979*** (0.0845)	0.0464** (0.0200)	0.2977*** (0.0636)	0.7022*** (0.0636)

Standard error values of estimates are in brackets. *, **, and *** indicate significance at the 10, 5, and 1 percent levels, respectively.

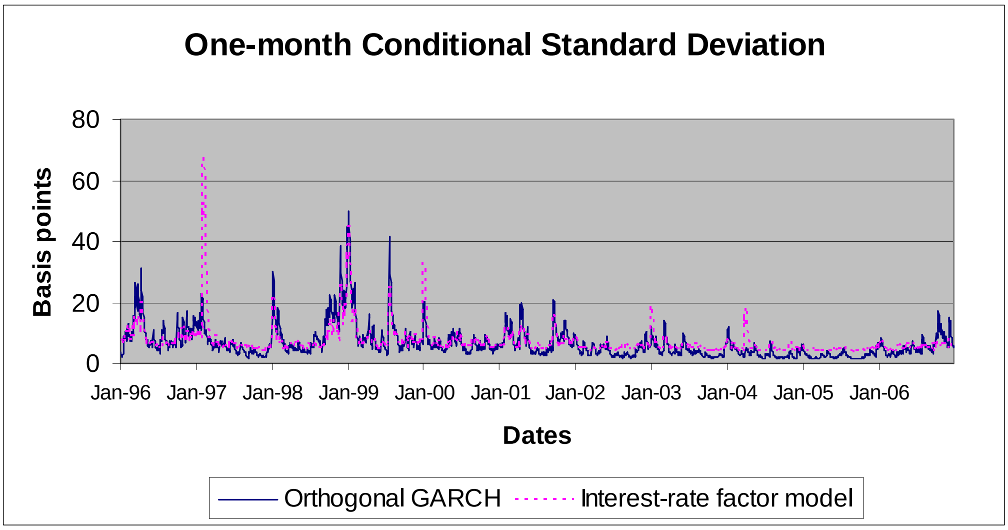
As it can be seen, coefficient α, which measures the intensity of reaction of volatility to the previous period unexpected market return ε_{it-1}^2, and coefficient β, which measures the persistence in volatility are both clearly significant at usual levels of significance for the three principal components. Note, however, that the first principal component has a lower market reaction but higher persistence than the other two principal components.

Finally, the full covariance matrix for interest rate changes of different maturities can be constructed from the principal component loading matrix using equations (4) and (5).

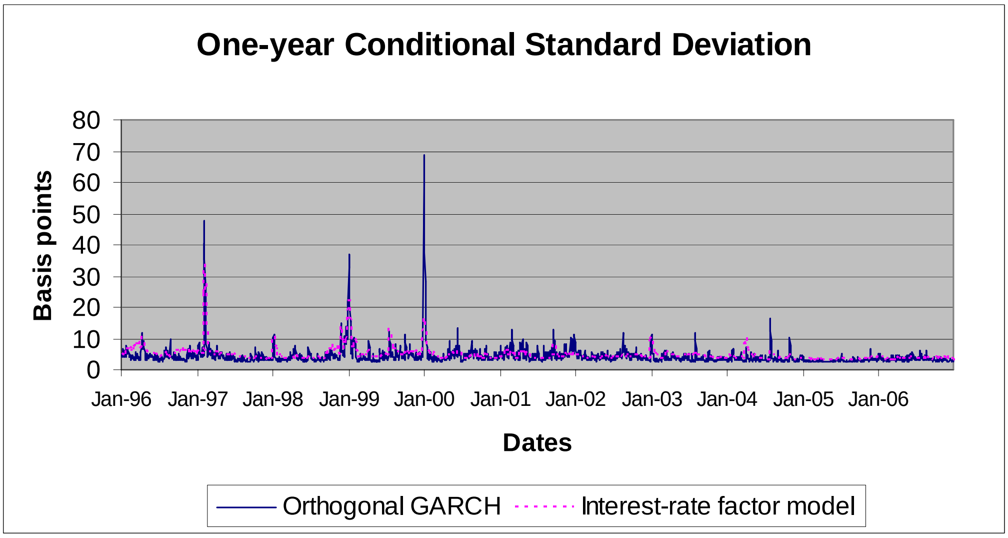
Specifically, conditional standard deviation for the interest rate changes for the whole sample period using this method has been calculated using equation (4). Graphs 1 to 3 show the comparison of this conditional standard deviation with the one obtained directly from the estimation of an univariate GARCH(1,1) model for the changes in interest rates with maturity of one-month, one-year, and ten-years.[7]

[6] The transpose of this matrix is the inverse of the component loading matrix when keeping all the principal components.
[7] Only three out of eleven graphs, corresponding to the key interest rate maturities, are plotted for space reasons.

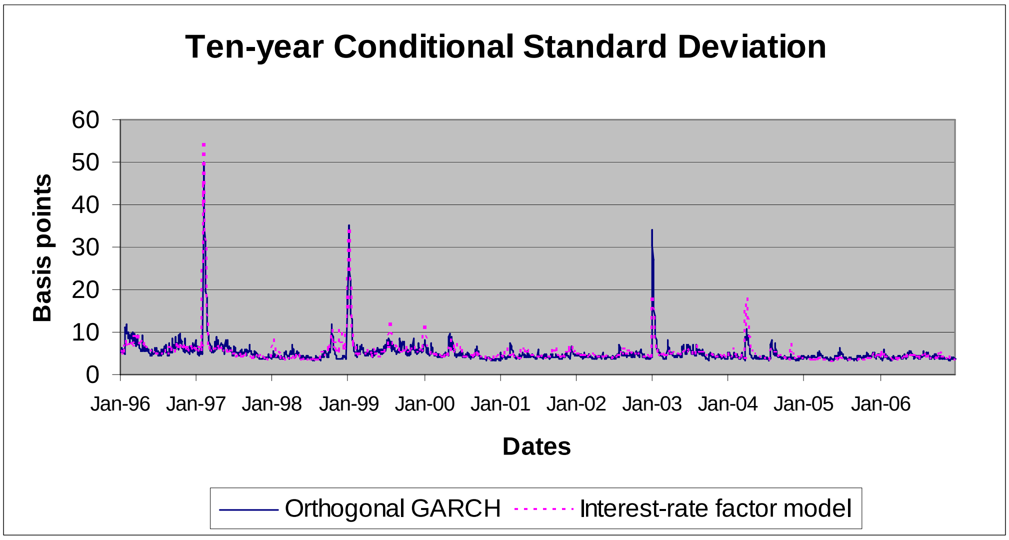
Graph 1



Graph 2



Graph 3

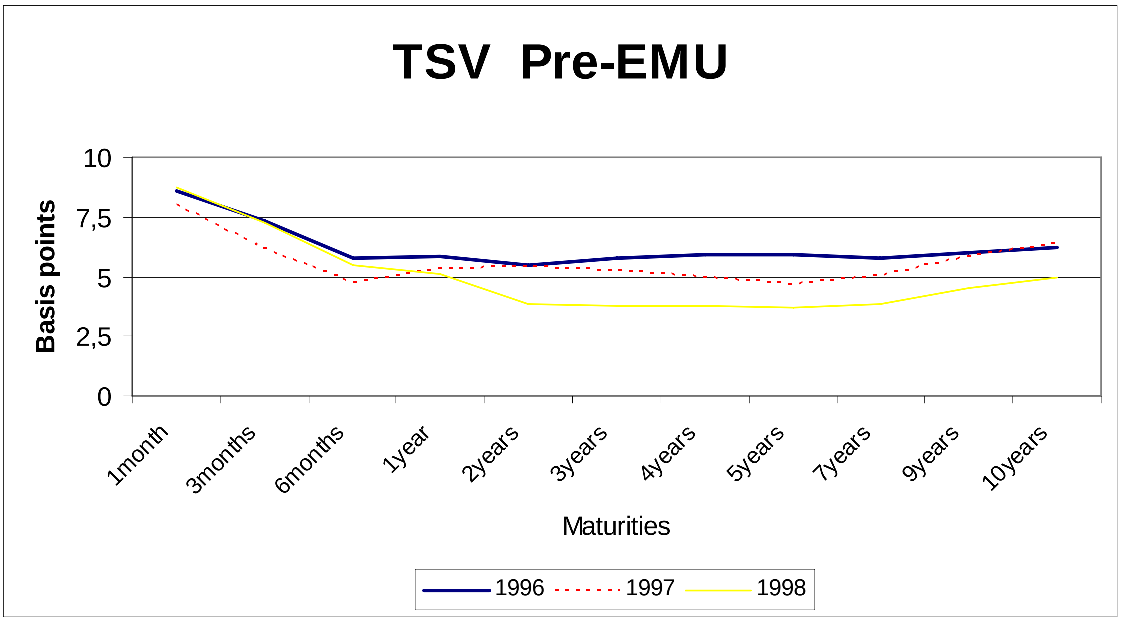


As it can be seen, in all cases the behaviour of the conditional standard deviation is very similar in both models but the results obtained working with the orthogonal GARCH model are smoother for short and long maturities.

Alternatively, and more interestingly for our purposes, 2781 daily TSV can be obtained drawing for each day the conditional standard deviation of spot rates changes as a function of their maturity. As the graphical representation of 2781 curves is not at all practical to extract conclusions, to summarize we have obtained the annual averages of these TSV. Graphs 4 to 6 show these 11 curves obtained as annual averages of TSV corresponding to the periods 1996-1998 (Pre-European Monetary Union period), 1999-2001 (European Monetary Union period), and 2002-2006 (Post-European Monetary Union period), respectively.

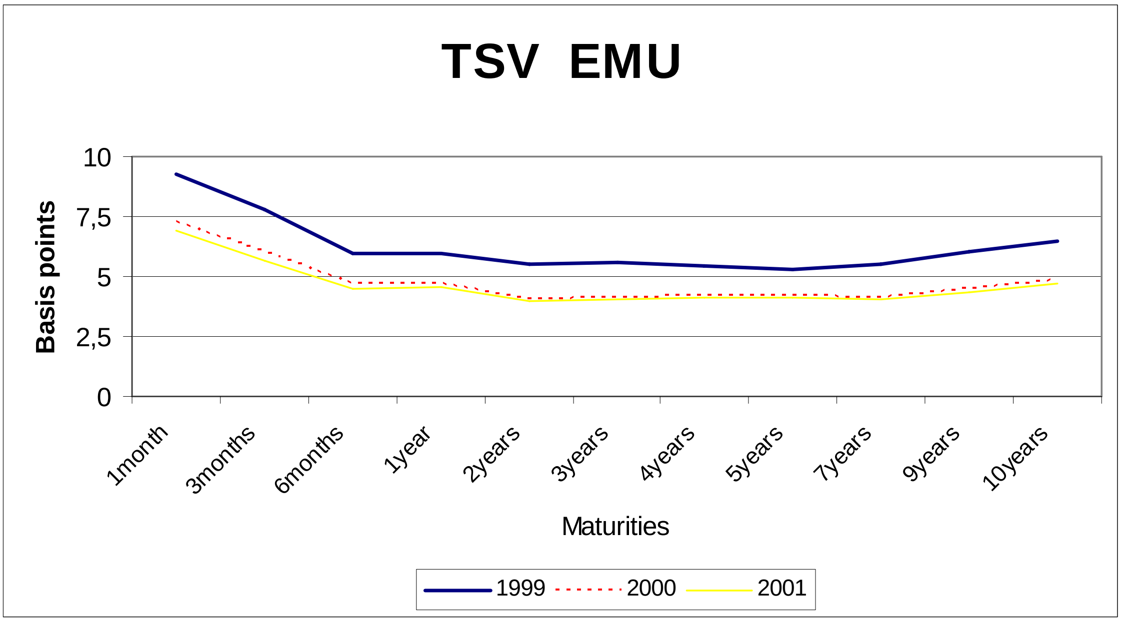
Graph 4

Annual averages of TSV for the period 1996-1998



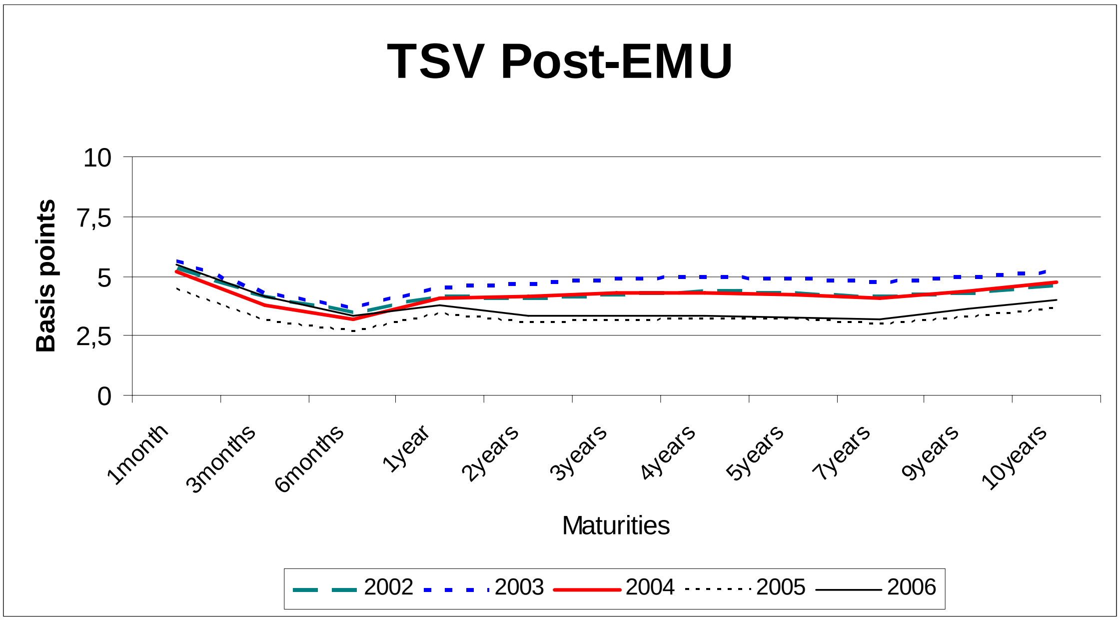
Graph 5

Annual averages of TSV for the period 1999-2001



Graph 6

Annual averages of TSV for the period 2002-2006



Several notable features emerge from these graphs. First, it is apparent that the estimated term structures of volatility of interest rates of the Spanish Public Debt market exhibit a quite similar pattern over the period of study, since most of the curves of volatility are roughly parallel. Specifically, as expected the volatility of changes in interest rates with shorter maturities is larger than the volatility of interest rates with longer maturities, although the changes in medium-term rates are the ones which show in general lower volatility.

Second, it can also be observed that the term structures of volatility of interest rates vary over time, showing a decreasing trend along the sample period. This decline in volatility of interest rates over the last few years may be closely related with some important changes and improvements in the conduct of monetary policy. In particular, the increasing gradualism in policy action (more frequent policy moves of smaller size), the greater transparency of central banks through improved communication about policy intentions, together with the historically low levels of interest rates, given the positive correlation between level and variability of interest rates usually observed, may have played a key role in this context.

4. KEY RATES IDENTIFICATION AND PERFORMANCE

Despite principal components might be seen as extremely useful for describing the TSV, they show two main drawbacks. First, they are synthetic variables that summarize information in correlated systems, as the term structure of interest rates, but they lack any economical interpretation; and second, and related to this, the principal components might be very unstable because the changes in the covariance matrix of interest rates might be transferred to the factor loadings (see, for example, Bliss, 1997, Soto, 2004 or Perignon and Villa, 2006). For these reasons, principal components are usually found in pure empirical research, and not in theoretical or structural models for interest rate dynamics. By the contrary, the state variables in these models are usually defined as interest rates of different maturities.

The orthogonal factor GARCH model, however, can shed light on the identification of the interest rates that best describe the TSV more than other methods for three reasons. A first and major reason refers to the problem of dependence observed among interest rates, which might distort the results of the econometric analysis of interest rate data (for example, using our data, the coefficients of correlation between the changes in the 5-year interest rate and the 4-year and 7-year rates are 97% and 89%, respectively). Principal components, however, are independent by construction, and then it might be expected that the interest rates that contribute to each component to a larger extent represents different dynamics. Secondly, the principal components are ranked according to their explanatory power and hence, the identification of the key rates can derive from the first few components. Finally, principal components can be rotated so that each new component shows the highest correlation with a single interest rate while the correlations with the rest of interest rates are the lowest[8]. This feature facilitates the identification of the key rates.

Given that our interest is on the TSV, the identification of the key rates in our empirical analysis is based on the coefficient of correlation between the conditional variance of each principal component and each interest rate obtained from the univariate GARCH models defined in (2) and (3)[9]. Although our principal components have not been rotated for consistency with the previous section, Table 4 clearly reveals the set of key rates that arise from our analysis. The 5-year rate is chosen as a proxy of the first principal component; the 6-months rate is the proxy for the second component and, finally, the combination of the 2-year and 10-year rates is the proxy for the third component, once the 9-year rate is discarded because its closeness to the 10-year rate. It is worth pointing out that these results are coherent with the results obtained in the previous section about the factor loadings of the principal components (see Table 2).

[8] This rotation, known as varimax rotation, is an orthogonal rotation and thus, keeps the independence of the new components. For a description of these techniques, see Jackson (2003).
[9] For brievity, we do not report the results of the estimation of the GARCH model for the individual interest rates. They are available from the authors upon request.

Table 4: Correlation between the conditional variance of principal components and interest rates

Maturity	Z_1	Z_2	Z_3
1 month	0,227	0,737	0,299
3 months	0,254	0,832	0,237
6 months	0,215	**0,892**	0,581
1 year	0,105	0,649	0,485
2 years	0,222	0,692	**0,896**
3 years	0,515	0,511	0,618
4 years	0,783	0,316	0,245
5 years	**0,913**	0,261	0,096
7 years	0,502	0,622	0,733
9 years	0,196	0,657	0,934
10 years	0,176	0,668	**0,936**

In order to corroborate the effectiveness of the four key rates for explaining the TSV in the Spanish Public Debt Market, we have carried out a simple regression analysis of the conditional variances of each interest rate and the set of key rates. Table 5 shows the results of this analysis in the third column, labelled as "key rates"; the first column, labelled as Z, shows the percentage of variance explained by the first three principal components according to the analysis in the previous section, and the second column, "all rates", shows the proportion of variance explained by the set of all interest rates excluding the one that is being analyzed.

A comparison of the data firstly reveals an obvious result, which is that, in general, the extensive parametrization based on the full set of interest rates is able to explain the highest proportion of variance, 95% on average. Despite the significant reduction of dimensionality, the percentage reduces only slightly for three principal components to 93%. Finally, the set of key rates yields a more than reasonable 86%. Evidently, this last approach leads to the highest discrepancies between interest rates, ranging from 100% for the key rates to 72% for the 3-month rate. The weird result for the 1-year rate (27%), together with the fact that this rate also shows relative low percentages for the other two models, can indicate the existence of outliers in the original series that might require an intervention analysis.

Table 5: Proportion of variance explained by the principal components, the full set of interest rates and the key rates

Maturity	Z	All rates	Key rates
1 month	87,731	91,000	88,000
3 months	97,237	93,000	72,000
6 months	91,998	93,000	100,000
1 year	80,755	77,000	27,000
2 years	96,356	96,000	100,000
3 years	98,018	99,500	87,000
4 years	93,693	99,700	91,000
5 years	92,760	99,400	100,000
7 years	99,044	98,000	84,000
9 years	96,132	99,700	99,500
10 years	90,903	99,700	100,000
Average	93,148	95,091	86,227

5. CONCLUSIONS

Accurate information about the term structure of volatility (TSV) of interest rates is particularly valuable in many practical applications, such as risk measurement using Value at Risk methods, formulation and implementation of hedging strategies, interest rate derivatives pricing, asset allocation, or monetary policy.

In this paper, the TSV of interest rates of the Spanish Public Debt market during the period 1996-2006 has been estimated using the orthogonal GARCH model introduced by Alexander and Chibumba (1997) and Alexander (2000). The orthogonal GARCH model is a method especially appropriate to be applied on interest rate term structures and other highly correlated systems. It allows for large covariance matrices to be generated from just univariate GARCH models applied on the orthogonal principal components of the system under consideration and consequently avoids the usual estimation problems arising in large dimension systems.

The results of the empirical analysis show some interesting features of the TSV of interest rates of the Spanish market during the period 1996-2006. First, interest rate volatility decreases with time to maturity, as observed in most analysis of the term structure of interest rates. Second, it can be seen that interest rate volatility has declined over the period to such an extent of reaching historically low levels. Recent improvements in monetary policymaking (greater gradualism and transparency of central banks) together with the very low level of interest rates during last years may have played a major role in the reduction of volatility.

In a second stage, we propose a method that relies in principal component analysis as a way to extract the factors, given as interest rates at specific maturities, which also offers a parsimonious representation of the TSV. The method establishes a link between the research aimed to identify the key rates of the TSIR using econometric techniques and those studies that use principal component analysis or factor analysis to obtain the set of unobservable factors that summarize interest rate movements or volatility. It also overcomes the difficulties found in econometric studies about the large number of combinations of interest rates to test and the criticism of principal component analysis about the lack of economic interpretation of the factors. Our results reveal that for the market and the period analyzed, the set of four interest rates comprised by the 6-month, 2- year, 5-year and 10-year rates is able to reproduce the dynamics of the TSV effectively.

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