Spatial Competition under Market Regulation: Maximum and Minimum Differentiation

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Abstract

This article considers a regulated circular space where consumers and firms are located in different sides of the circle. We consider a three stage game in which in the first stage the regulator chooses the size of the arch where firms will be located (the commercial area), in the second stage firms choose locations within the commercial zone and in the third stage they compete in prices.

We find that with this type of market configuration and with concave transportation costs, there exists a price equilibrium for every possible firms' location. Furthermore, this equilibrium is unique and characterized by maximum differentiation. We also find that the optimal size of the commercial area will depend on the objective function of the regulator and, in particular, that once a regulator is considered, maximum differentiation, minimum differentiation or intermediate cases may be obtained.

Keywords: spatial competition, circular model, concave transportation costs, regulator, sequential equilibrium.

JEL Classification: C72, D43.

1. Introduction

This paper studies optimal urban design using a circular model of spatial competition. We consider the problem of a central planner or regulating authority in charge of designing a new city located in a circular space of perimeter 1. In particular, the regulator has to divide the circle in two different regions, a commercial area where firms will locate of length v, and a residential area where consumers will live of length $(1-v)$. Our aim is to determine what is the optimal size of v, that is, how much space should be set aside for firms, which in our model is equivalent to determining the optimal size of the population in the city.

In order to study this problem we analyze a standard model of spatial competition as a three stage game, where in the first stage the regulator chooses the size of the commercial area, v, in the second stage firms choose locations within v, and in the third stage they compete in prices. That is, the regulator will choose v, taking into account that once the size of the commercial area is determined, firms will choose their locations within this area and then compete in prices.

As in all models of horizontal spatial competition *a la Hotelling*, we assume two firms selling and homogeneous product and that consumers will choose to address the firm who offers the least full price, (mill price plus transportation costs), and we assume that transportation costs are concave and equal to $d-d^2$, where d is the distance between the consumer and the chosen firm. This is a technical assumption chosen to warrantee the existence of a perfect location-price equilibrium in the last two stages of the game. The existence of equilibrium in these two stages allows us to study the optimal size of the commercial area in the first stage.

In the literature of horizontal spatial competition it is known that quadratic transportation costs functions of the form d^2 have given good results in terms of achieving perfect equilibrium, as it is shown in D'Aspremont et al, (1979) for the linear case and de Frutos et al, (1999), for the circular space. For other types of transportation costs functions the results are rather negative. For example, the linear function: ad, has been studies by D'Aspremont et al, (1979) for the linear space; and by Kats, (1995) using the circular configuration; in both cases there is no perfect price-location equilibrium using this function. The transportation costs function: bd^a, $1 < a \leq 2$, $b > 0$, was studied by Economides, (1986) in the linear model. With this function, equilibrium can only be attained for certain values of a. While the linear quadratic function $ad + bd^2$, $a > 0$, has been analyze be Gabszewicz & Thisse, (1986) for the linear model with $b > 0$; de Frutos et al, (1999) using the circular model and taking $b \in R$; and Hamoudi & Moral, (2005), for the linear space and $b<0$. In all cases a price equilibrium does not exist for every possible firms' locations. For a recent review of this literature see Brenner, (2001).

Unlike the literature cited above, when a regulated spatial configuration, of the type we propose, is assumed, the transportation costs function d^2 cannot deliver a price equilibrium for every possible firms' location as have been shown in Arguedas et al, (2006). Therefore to determine the optimal market size in the first stage of the game, the space that can be considered is the circular model and the only possible transportation costs function is $(d\text{-}d^2)$ [1]. Other type of regulated space, similar to this model, has been studied in Hamoudi & Risueño, (2006). They considers a circular spatial model where firms and consumers are located together in a segment of the circle, the market, while in the other segment no commercial activity takes place. They show that in a two stages game and with the same transportation costs function assumed in this paper, perfect equilibrium exists provided the length of the market is greater than ¾.

As mentioned above, we study a model of horizontal spatial competition, where in the first stage the regulator chooses the size of the commercial area, in the second stage the firms choose locations within the commercial zone, and in the third stage they compete in prices. The model is solved by backward induction: first we look for the price equilibrium, secondly we study optimal locations given equilibrium prices, and finally the optimal size of the commercial area is determined taking into account different welfare objectives of the central planner.

We have considered a first case, which we call the *socialist regulator*, where the central planner only cares about consumers' welfare, a second case in which we have

[1] De Frutos et al (1999) have proven the existence of a maximum differentiation equilibrium in the circumference model with this type of transportation costs function.

considered a *capitalist regulator* that only cares about firms profits, and finally, we have considered a *mixed economy regulator* that cares about both consumers and firms welfare. In the first case, the regulator minimizes consumers' disutility, measured as the full costs of purchasing the goods. In the second case, the regulator maximizes the sum of firms' profits and in the third case the regulator maximizes total welfare as the difference between firms' profits and consumers' disutility, which is equivalent to minimizing total transportation costs in the economy.

We find that when the regulator only cares about consumers welfare, the optimal policy for the regulator is to choose a commercial area as small as possible, since optimal $v^* = 0$. That is, consumers are better off when firms face maximum price competition and get to the Bertrand solution, although in this situation total transportation costs faces by consumers are maximized.

In the second case, when the regulator only cares about firms profits, the optimal size of the commercial area is 1/3. Unlike the typical maximum differentiation result obtained in the literature, in our model, although the regulator only cares about firms' profits she will not allow firms to locate at a greater distance than 1/3. This result is obtained because the decrease in prices due to the relative competition that firms face at this distance is compensated by the larger demand that a longer residential area implies.

Surprisingly, in the third case, when the social planner cares about both consumers and firms, the optimal size of the commercial area is ½. Firms' prices are maximized but the distance travelled by consumers and market demand is minimized.

The remainder of the paper is organized as follows. In Section 2, we present the model. Sections 3 and 4 are devoted to analyzing equilibrium existence in a regulated market in the prices and location stages respectively. In Section 5, we study the optimal size of the commercial area given the results obtained in the previous stages of the game and under different assumptions about the motivations of the social planner. We conclude in Section 6.

2. The model

We consider a circular city of length 1 which the regulator has to divide into two distinct parts, a commercial area, where firms locate, of length $0 = v = \frac{1}{2}$ and a residential area of length $1 - v$ (see *Figure 1*). There are two firms selling a homogeneous product, with zero production costs, located at x_1 and x_2, with $0 \leq x_1 \leq x_2 \leq v$.

Consumers are evenly distributed along the residential area $1\text{-}v$, and each consumer buys a single unit of this product per unit of time, irrespective of its price. Since the product is homogeneous, consumers will buy from the firm who offers the least delivered price, that is, the mill price plus transportation costs. Let p_1, p_2, denote the mill prices charged by firms located at x_1 and x_2, respectively. The distance between consumer x and firm i is given by $d_i = |x - x_i|$, $i = 1,2$. We will consider that transportation costs are concave. In particular, we will assume that $C(d_i) = (d_i - d_i^2)$ (1)

Although consumers can only be located within $1\text{-}v$, consumers can travel along the whole circle and they will always take the direction that implies the shorter distance to the chosen firm.

The model described above gives rise to a three-stage game in which in the first stage the regulator chooses the size of the commercial area, v, in the second stage firms decide simultaneously their location and in the third stage they simultaneously choose prices. The choice of the transportation costs function described in (1) is a technical assumption that avoids non-existence of equilibrium problems in the second and third stages of the game.

In order to determine the market boundaries and derive the demands faced by each firm, we will have to find the marginal consumers. A consumer is indifferent to buying from one firm or the other if and only if: $p_1 + C(d_1) = p_2 + C(d_2)$ (2)
That is, a consumer is indifferent if she faces the same full price from the two firms.

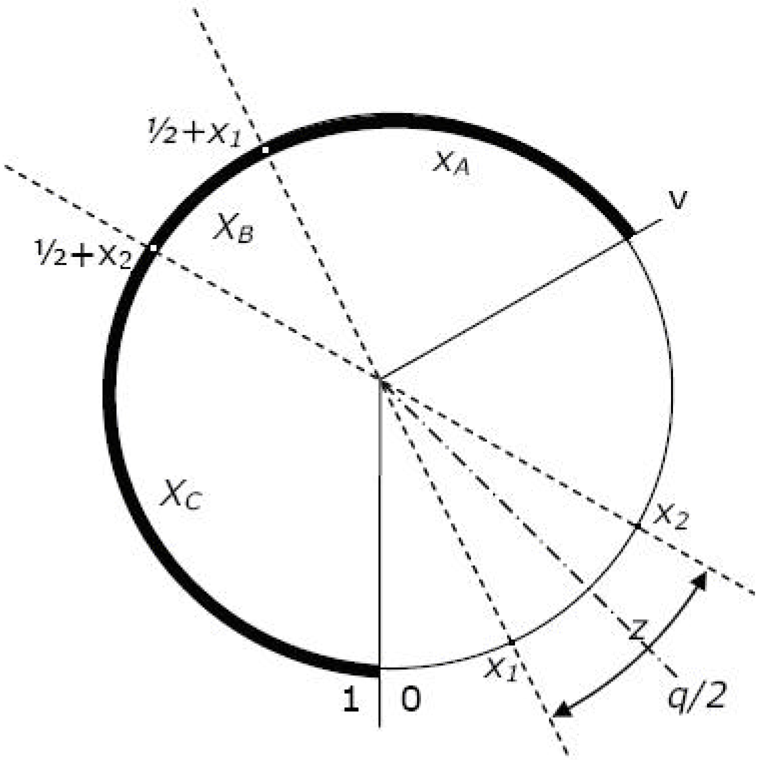

Figure 1: The circular city

Since consumers will travel to firms taking the direction that minimizes the distance to the chosen firm, there are three possible indifferent consumers in the residential area each one belonging to a different segment of the circumference given by $x_A \in [v, x_1+½]$, $x_B \in [x_1+½, x_2+½]$, and $x_C \in [x_2+½, 1]$, respectively (see *Figure 1*). In order to calculate the location of the indifferent consumer let $q = x_1 + x_2$ and $z = x_2 - x_1$ substituting (1) into (2) and computing the expression for the marginal consumers, it turns out that

$$a = x_A = x_B = x_C = \frac{p_1 - p_2}{2z} + \frac{q}{2} + \frac{1}{2}, \text{ for } v \le a \le 1 \qquad (3)$$

Every consumer $x \le a$ will buy from firm 2.

Condition (3) is equivalent to: $-z(1-2v+q) \le p_1 - p_2 \le z(1-q)$ (4)

Consequently, the demand function of firm 1 can be expressed as follows:[2]

$$D_1(p_1, p_2, x_1, x_2, v) = \begin{cases} 1-v & p_1 - p_2 \le -z(1-2v+q) \\ 1-a & -z(1-2v+q) \le p_1 - p_2 \le z(1-q) \\ 0 & z(1-q) \le p_1 - p_2 \end{cases}$$

Note that in this model total market demand as well as firms individual demands depend on v. This means that the total number of consumers in the city is a variable to be determined within the model.

In order to analyze the model we will start by seeking for equilibrium in the prices stage, we will then turn to look for equilibrium locations given equilibrium prices found in the previous stage, and finally we will analyze the optimal market size under three different perspectives, that we denote as the *socialist regulator,* the *capitalist regulator* and the *mix-economy regulator*.

3. Price Equilibrium

We define a non-cooperative price equilibrium as:

For a given v (the commercial area), and firms' locations, a Nash-price equilibrium is a pair $(p_1^*(x_1, x_2, v), p_2^*(x_1, x_2, v))$ such that each firm selects the price which maximizes profits, considering the other firm's price as given. Therefore, no firm can raise its profits by changing its price unilaterally.

That is $(p_1^*(x_1, x_2, v), p_2^*(x_1, x_2, v))$ satisfies: $B_i(p_i^*, p_j^*) = B_i(p_i, p_j^*)$, for all i and j, $i \neq j$.

Since production costs are zero, firms' profits are:

$B_i(p_1, p_2, x_1, x_2, v) = D_i(p_1, p_2, x_1, x_2, v)\, p_i$, substituting in the demand functions the expression for the indifferent consumer, we obtain the following profit functions:

[2] The corresponding demand of firm 2 is simply $D_2 = 1 - v - D_1$

$$B_1(p_1,p_2,x_1,x_2,v)=\begin{cases} p_1(1-v) & p_1-p_2\leq -z(1-2v+q) \\ p_1\left[\dfrac{p_2-p_1}{2z}-\dfrac{q}{2}+\dfrac{1}{2}\right] & -z(1-2v+q)\leq p_1-p_2\leq z(1-q) \\ 0 & z(1-q)\leq p_1-p_2 \end{cases}$$

$$B_2(p_1,p_2,x_1,x_2,v)=\begin{cases} 0 & p_1-p_2\leq -z(1-2v+q) \\ p_2\left[\dfrac{p_1-p_2}{2z}+\dfrac{q}{2}+\dfrac{1-2v}{2}\right] & -z(1-2v+q)\leq p_1-p_2\leq z(1-q) \\ p_2(1-v) & z(1-q)\leq p_1-p_2 \end{cases}$$

Given that the profit functions are quasi-concave in prices, the existence of a price equilibrium is assured whatever the size of the market, v, and locations of firms', x_1, x_2, may be.

We now characterize the equilibrium in the following proposition.

Proposition 1. *For 0 = v = ½, there exist a unique price equilibrium given by:*

$p_1^* = (1/3)\, z(3 - q - 2v)$ $\qquad$ $p_2^* = (1/3) z(3 + q - 4v)$

Proof:

Using the first order conditions, $\frac{\partial B_1}{\partial p_1}=0, \frac{\partial B_2}{\partial p_2}=0$, simple calculations lead to the solutions: $p_1^* = (1/3)\, z(3 - q - 2v)$, $p_2^* = (1/3) z(3 + q - 4v)$.

We must now verify that these solutions satisfy the price interval for which demand exists given in (4):

The left hand side of the equation requires: $\frac{1}{3}z(3-2q-2v)>0$; while the right hand side of the equation is equivalent to: $\frac{1}{3}z(3+q-4v)>0$; both conditions are always satisfied for any possible firms' locations. Therefore, we can conclude that there exists a unique price equilibrium.

Using the equilibrium prices obtained above and substituting z and q by its expressions ($q = x_1 + x_2$ and $z = x_2 - x_1$) equilibrium profits can be written as:

$$B_1^*(x_1,x_2,v)=\frac{1}{18}(x_2-x_1)(3-x_1-x_2-2v)^2,\ B_2^*(x_1,x_2,v)=\frac{1}{18}(x_2-x_1)(3+x_1+x_2-4v)^2.$$

4. Location Equilibrium

We turn now to the second stage of the game. We will compute equilibrium locations, given, the size of the commercial area, v, and taking into account the equilibrium prices p_1*, p_2*, obtained above. A price-location equilibrium is defined as a location pair $\left[x_1^*(v),x_2^*(v)\right]$, and a price pair $\left[p_1^*(x_1^*,x_2^*,v),\quad p_2^*(x_1^*,x_2^*,v)\right]$ such that:

$$B_i\left[v,x_i^*,x_j^*,p_i^*(v,x_i^*,x_j^*),p_j^*(v,x_i^*,x_j^*)\right]\geq B_i\left[v,x_i,x_j^*,p_i^*(v,x_i^*,x_j^*),p_j^*(v,x_i^*,x_j^*)\right]$$

For $\forall i,j=1,2;\ i\neq j$ and $\forall x_i\in[0,v]$

Proposition 2:

For 0 = v = ½, there exist a unique location equilibrium given by: $x_1^*=0;\quad x_2^*=v$

Proof:

Using the first order conditions, $\frac{\partial B_1^*}{\partial x_1}=0,\frac{\partial B_2^*}{\partial x_2}=0$, simple calculation shows that $\frac{\partial B_1^*}{\partial x_1}=0$, is equivalent to $(3-x_1-x_2-2v)(3-3x_1+x_2-2v)=0$, while $\frac{\partial B_2^*}{\partial x_2}=0$, is equivalent to $(3+x_1+x_2-4v)(3-x_1+3x_2-4v)=0$. To find the solution we must solve the equations system given by:

$$\begin{cases}(3-x_1-x_2-2v)(3-3x_1+x_2-2v)=0\\(3+x_1+x_2-4v)(3-x_1+3x_2-4v)=0\end{cases}$$

Solving the system, different solutions are obtained none of which satisfies the conditions upon: x_1, x_2 and v. We find that $\frac{\partial B_1^*}{\partial x_1}$<0 and $\frac{\partial B_2^*}{\partial x_2}$ >0. It follows that $B_1^*(x_1,x_2,v)$ reaches a maximum at $x_1^*(v)=0$, while $B_2^*(x_1,x_2,v)$ reaches its maximum at $x_2^*(v)=v$, Therefore, there exist a unique location equilibrium.

The result is the well-known principle of maximum differentiation obtained in the literature when quadratic transportation costs functions are assumed. Firms choose the two extremes of the commercial area to keep price competition as low as possible. This result is relevant since, as mentioned above, when regulated markets are considered, other costs functions that give good results in terms of equilibrium existence in the standard model have been proven to exhibit equilibrium-existence-problems when regulation is introduced.

Given that the optimal locations are $x_1^*(v) = 0$, $x_2^*(v) = v$; substituting in the previous functions we obtain that: $B_1^{**}(x_1^*, x_2^*) = B_2^{**}(x_1^*, x_2^*) = \frac{1}{2} v(1-v)^2$.

And the corresponding demand functions and prices are:

$$D_1^{**}(x_1^*, x_2^*) = D_2^{**}(x_1^*, x_2^*) = \frac{1}{2}(1-v), \quad p_1^{**}(x_1^*, x_2^*) = p_2^{**}(x_1^*, x_2^*) = v(1-v)$$

The indifferent consumer is then $a^{**} = \frac{1+v}{2}$

5. Optimal size of the commercial area

To end the analysis of our model, we will focus our interest in searching for the optimal size of the commercial area, v^*, (or equivalently, the optimal size of market demand) given the results obtained above. More precisely we will denote by v^* the optimal strategy of the regulator such that $v^* = \underset{v}{ArgMax}\, W(v) \quad s.t. \quad 0 \le v \le \frac{1}{2}$, where *W(v)* is the regulator's objective function.

We will consider three possible specifications for *W(v)*:

$$W_1(v) = -[p(1-v) + C_T(v)], \; W_2(v) = B_1^* + B_2^* = p(1-v), \; W_3(v) = W_1(v) + W_2(v)$$

Where $p = p_1^{**}(x_1^*, x_2^*) = p_2^{**}(x_1^*, x_2^*) = v(1-v)$ and $C_T(v)$ is the total transportations cost endured by all consumers, given the optimal locations obtained above, and defined as:

$$C_T(v) = \int_{\frac{1+v}{2}}^{1} \left[(1-x)-(1-x)^2\right]dx + \int_{v}^{\frac{1+v}{2}} \left[(x-v)-(x-v)^2\right]dx$$

where the first integral corresponds to the transportations costs paid by all consumers that address firm 1 and the second integral to the transportation costs paid by all consumers that address firm 2.

It is easy to check that $C_T(v) = \frac{1}{6} - \frac{v}{12}(3-v^2)$.

We are going to analyze the optimal size of the commercial area depending on the objective function of the regulator:

Case 1. The socialist regulator: The regulator is only concern about consumers' welfare and maximizes $W_1(v)$, which is equivalent to minimizing consumers full costs of purchasing the goods, that is, mill prices plus transportation costs, i.e.

$$Max\, W_1(v) \Leftrightarrow Min\left[-W_1(v)\right] = Min\; p(1-v) + C_T(v) \quad s.t. \quad 0 \le v \le \frac{1}{2}$$

We obtain that the solution is given by $v^*=0$.

And the corresponding equilibrium locations and prices are given by: $x_1^* = x_2^* = 0$ and $p_1^*(x_1^*(v^*), x_2^*(v^*), v^*) = p_2^*(x_1^*(v^*), x_2^*(v^*), v^*) = 0$. Therefore, equilibrium profits are zero and the corresponding demands are given by: $D_1^* = D_2^* = \frac{1}{2}$.

This result implies that consumers' welfare is maximized when firms are forced to compete *a la Bertrand* and prices are driven to the perfectly competitive outcome even if this means that the distance that consumers have to travel to purchase the goods is maximized. An alternative interpretation in terms of product differentiation is that product variety cannot compensate consumers for the increases in prices that customized products imply. Note that with this welfare function the size of market demand is maximized and equal to 1. The result obtained is in the tradition of the minimum differentiation principle as stated by Hotelling (1929), but as opposed to this article and others[3], minimum differentiation is obtained as the result of welfare maximization instead of as the outcome of a non-cooperative Nash game.

Case 2. The capitalist regulator: The regulator is only concern about firms' profits and maximizes $W_2(v)$.The solution to this problem is given by: $v^* = \frac{1}{3}$. And the corresponding equilibrium locations and prices are given by: $x_1^* = 0;\ x_2^* = \frac{1}{3}$ and $p_1^*(x_1^*(v^*), x_2^*(v^*), v^*) = p_2^*(x_1^*(v^*), x_2^*(v^*), v^*) = \frac{2}{9} = 0.22$.

The corresponding demand and profit functions are given by: $D_1^{**}(0, v^*) = D_2^{**}(0, v^*) = \frac{1}{3} = 0.33$ and $B_1^{**}(0, v^*) = B_2^{**}(0, v^*) = \frac{2}{27} = 0.074$

The solutions obtained under this welfare function may seem rather surprising since, although firms choose locations at opposite extremes of the commercial area, total profits are maximized when they locate at a distance of 1/3 from each other. This result differs from the standard solution with quadratic transportation costs, when a non-regulated circular market is considered, of maximum product differentiation and firms locating at a distance of ½ from each other. The outcome is due to the fact that in our model, when the regulator chooses the size of the commercial area, what is being determined is the optimal size of market demand, that is, the optimal number of dwellers of our model city. As firms move apart from each other equilibrium prices increase but market demand, and therefore the demand faced by each firm, decreases. However, at a distance of 1/3 firms are relatively protected from competition, prices are positive and demand is large enough to maximize profits.

Case 3. The mix-economy regulator: The regulator maximizes social welfare taking into account both consumers and firms welfare. In this case the regulator objective function is: $W_3(v) = B_1^* + B_2^* - p(1-v) - C_T(v)$. Substituting B_1^*, B_2^* for their expressions we obtain $W_3(v) = -C_T(v)$. That is, in this case, the objective of the regulator would be to minimize the total costs function, $C_T(v)$. The solution is given by $v^* = \frac{1}{2}$. Equilibrium

[3] Although the results on Hotelling's article were proven to be wrong (D'Aspremont et al, 1979) other articles have obtained the minimum differentiation solution.

locations are: $x_1^*(v^*)=0; x_2^*(v^*)=\frac{1}{2}$ and equilibrium prices are $p_1^*=p_2^*=\frac{1}{4}=0.25$. The profit functions are: $B_1^*=B_2^*=\frac{1}{16}=0.0625$ and the demands are: $D_1^*=D_2^*=\frac{1}{4}=0.25$.

When the welfare of both firms and consumers is considered the principle of maximum differentiation is restored. The reason is that in our model total revenue paid by consumers to firms from purchasing the goods is equal to total firms' profits. When the regulator maximizes social welfare it is equivalent to minimizing total transportation costs across the economy. In this case, firm prices are maximized but transportation costs and demands are minimized, that is, the optimal size of the population of our model city under this assumption is only ½ of the circumference. Note, however, that this result differs in interpretation from the one obtained in the traditional two stage game. In the standard model, the maximum differentiation principle is the outcome of a non-cooperative Nash equilibrium and as such, the solution is not necessarily a social optimum. However, in our case, the maximum differentiation principle stems from a problem of welfare maximization, where the interests of all parties involved has been considered and the social optimum is achieved through a demand adjustment. The intuition behind being that transportation costs are a sort of dead weight loss to the economy that do not accrue to consumers or firms. The result implies that maximum product differentiation is optimal provided that the distance that consumers have to travel from their preferred variety to the offered one is not large.

6. Concluding comments

In this paper, we have considered a regulated circular space where firms and consumers are restricted to locate in different sides of the circle. We study, under concave transportation costs, a three stages game in which in the first stage the regulator chooses the size of the commercial area, in the second stage firms choose locations within the commercial zone, and in the third stage they compete in prices. We find that with this type of market regulation, unlike when convex transportation costs are considered, there exists a unique price-location equilibrium. This result is relevant since under location restrictions, the choice of other type of costs functions leads to equilibrium existence problems as have been shown in Arguedas et all (2006) and Hamoudi & Risueño (2006). Furthermore, we find that when welfare considerations are made, the optimal size of the commercial area will depend on the objective function of the regulator with optimality being achieved through adjustments in the size of market demand.

We find that depending on the social welfare function considered, maximum differentiation, minimum differentiation or intermediate cases may be obtained. When the regulator is only concerned about consumers welfare, the social optimum is achieved when firms are forced to locate side by side and prices are driven to the competitive solution even if that means that no product variety will be offered, so that a result in the tradition of the minimum differentiation principle is obtained. However, in our model, minimum differentiation is achieved as the result of welfare maximization instead of the outcome of a non-cooperative game. On the other hand, when the regulator is assumed to be only concerned about firms' profits, social welfare is maximized when firms locate at a distance equal to 1/3. This result differs from the standard solution in the literature of maximum product differentiation because as firms move apart from each other equilibrium prices increase but market demand, and therefore the demand faced by each firm, decreases; that is, there is trade off between demand size and equilibrium prices level. Finally, when the regulator takes into account both consumers and firms, the principle of maximum differentiation is restored since the optimal size of the commercial area is half of the circle and firms locate at a distance of ½ from each other. In this case, maximizing social welfare is equivalent to minimizing total transportation costs across the economy, and the size of market demand is minimized. The result can be interpreted as an argument for the optimality of maximum product differentiation when consumers' tastes are not very far apart from the varieties offered in the market.

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