On risk aversion in RBC models

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Abstract

This paper points out that changes in the parameterization of the relative risk aversion parameter alter the internal propagation mechanism of an RBC model when the presumption underlying standard RBC models about the exogeneity of growth components is relaxed. We focus on the specific contribution of the utility function curvature in characterizing observed U.S. GNP dynamics in the context of an endogenous growth model. In particular, we evaluate the model over all dimensions of the business cycle phenomenon. In constrast with the results obtained following the traditional approach in this literature, we find that the curvature of the utility function is crucial in determining the performance of the endogenous growth model.

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I. Introduction

Prescott (1986) points out that the business cycle phenomenon has three dimensions: the comovements of other variables with output, the relative volatilities of various series and the periodicity of output. Standard Real Business Cycle (RBC) models provide satisfactory results on the first and second dimensions but, as Cogley and Nason (1995) noted, they fail to reproduce two stylized facts about U.S. output dynamics: first, GNP growth is positively autocorrelated over short horizons and has a weak and possibly insignificant negative autocorrelation over longer horizons; and second, GNP appears to have an important trend-reverting component that has a hump-shaped MA representation. Furthermore, they show that even non-standard RBC models that incorporate lags or costs of adjusting labor input have weak internal propagation mechanisms and, as a consequence, exogenous sources of dynamics are needed in order to replicate both stylized facts. This result reveals the relevance of studying the internal propagation mechanism embodied in RBC models in order to obtain realistic ones.

Cogley and Nason (1995) follow the traditional approach in RBC literature, which assumes a strictly exogenous engine of growth. An alternative approach is to consider a model that encompasses both cycles and endogenous growth. This paper pursues this second line of research.

As known in the literature, endogenous growth models provide a stronger internal propagation mechanism which improves the model's ability to reproduce not only the first and second dimensions of the business cycle, but also the observed autocorrelation of output growth. On the one hand, King, Plosser and Rebelo (1988 b), Gomme (1993), Ozlu (1996) and Einarsson and Marquis (1997), among others, show that endogenous growth models perform better than the standard exogenous growth model in explaining labor market fluctuations due to the strength of their internal propagation mechanism. On the other hand, Collard (1999), Hercowitz and Sampson (1991), Perli and Sakellaris (1998), Jones, Manuelli and Siu (2000) and Matheron (2003) point out that these models generate the kind of internal propagation mechanism necessary to match the observed autocorrelation of output growth. That is why we think that this is a good platform to work on.

Another way to get a stronger propagation mechanism endogenously consists of including 'rigidities' in the labor market. Several authors, including

Andolfato (1996), Burnside and Eichenbaum (1996), Cogley and Nason (1995) and Hashimzade and Ortigueira (2005)[1], have shown that the introduction of these 'rigidities' partially improves the model's ability to reproduce the third dimension of the business cycle. In particular, this feature helps in replicating the observed autocorrelation function for output growth, but a stronger internal propagation mechanism is needed in order to match the magnitude of the observed transitory impulse response function of output.

To summarize, a stronger propagation mechanism can be obtained by including labor adjustment costs or by considering endogenous growth, but none of these features by itself is able to explain all the dimensions of the business cycle simultaneously. Barañano and Moral (2003) show that combining the endogenous character of the engine of growth with labor adjustment costs improves the model's ability to explain the third dimension of the business cycle, due to the strength of its internal propagation mechanism.

The aim of this paper is to explore a different way of enhancing the internal propagation mechanism of an RBC model and analyze its contribution in characterizing observed U.S. GNP business cycle dynamics. As mentioned above, the traditional approach in RBC literature assumes an exogenous character of the engine of growth. In this context, the intertemporal elasticity of substitution in consumption (measured by the inverse of the relative risk aversion parameter) plays a minor role in determining the performance of the model, though intertemporal substitution is crucial to understanding how shocks are propagated. In this paper, we study whether this result also works in a different context, that is, an endogenous growth model.

This paper considers an extended version of the model by Barañano and

[1]These authors consider an alternative approach to RBC models: an expectations-driven business cycle model in which fluctuations are driven by revisions in expectations. They find that such business cycle models are partially successful when frictions in the labor market are introduced into the model, since they generate the kind of internal propagation mechanism that is necessary to account for the positive autocorrelation of output growth found in US data. However, they do not examine their model's ability to match the important trend-reverting component observed. In accordance with the view expressed by Cogley and Nason (1995), they also conclude that the persistence in growth rates does not depend on whether fluctuations stem from shocks to fundamentals or from shocks to expectations, but on the propagation mechanism embodied in the model. Indeed, the relative success of their model is due to the introduction of frictions into the labor market that introduce a lag in the determination of employment, which enhances the propagation of shocks.

Moral (2003)[2], and studies the specific contribution of each model's assumption in explaining the above mentioned three dimensions of the business cycle phenomenon. Moreover, the sensitivity of the results to the model's assumptions is also analyzed. It is found that, by contrast with exogenous growth models, changes in the relative risk aversion parameter have a large impact on the performance of endogenous growth models. In particular, the higher the relative risk aversion parameter, the worse the match.

The rest of the paper is as follows: We briefly describe the endogenous growth model considered in Section II. Section III assesses the ability of the model to generate realistic output dynamics and analyzes the specific contribution of each assumption of the model in explaining all the dimensions of the business cycle phenomenon. Section IV concludes.

II. The Model

This paper considers a generalized version of the model studied in Barañano and Moral (2003). In particular, it consists of a stochastic discrete time version of Lucas'(1988) model in the absence of externalities with two modifications. On the one hand, agents derive utility not only from consumption but also from leisure. On the other hand, as suggested by Shapiro (1986) and Cogley and Nason (1995), labor adjustment costs are included. The model is explained in detail in the above mentioned work. Barañano and Moral (2003) only considered the logarithmic utility function. In this work we extend the model in order to analyze the dependence of the results on utility curvature.

Preferences are described by the following utility function[3]:

$$U(c_t, l_t) = \begin{cases} \frac{(c_t^{\lambda} l_t^{1-\lambda})^{1-\gamma} - 1}{1-\gamma}, & 0 \leq \lambda \leq 1, \quad \gamma > 0 \text{ and } \gamma \neq 1 \\ \lambda \ln c_t + (1-\lambda) \ln l_t & \text{for } \gamma = 1 \end{cases} \tag{1}$$

where c_t denotes the consumption and l_t is the fraction of time given over to leisure. Since the utility function is multiplicatively separable, it can be written as $U(c, l) = u(c)v(l)$, where $u(c)$ is homogeneous of degree $1 - \sigma$. Note that $(1-\gamma)\lambda = 1-\sigma$. The parameter value that measures the relative risk aversion,

[2]In this paper, we generalize the utility function. This allows us to analyze the effect of the curvature of the utility function on the results.

[3]Note that this function satisfies the conditions needed to ensure the existence of a balanced growth path. For more details on this issue, see King, Plosser and Rebelo (1988a, pp. 201-202).

σ, is established to be within the interval [1, 2], as suggested by Mehra and Prescott (1985). We consider two values for σ, 1 and 2, which allows us to analyze the sensitivity of the results to changes in this parameter[4]. The value for γ can be derived from the expression $(1-\gamma)\lambda = 1-\sigma$. The values for the rest of the structural parameters and some steady state variables are displayed in Table 1 and the selection criteria are explained in Barañano and Moral (2003).

III. Output dynamics

This section presents the main output dynamic properties of the model. The purpose of this paper is to analyze under what context the curvature of the utility function alters the strength of the internal propagation mechanism of a RBC model and whether the performance of the model depends on the relative risk aversion considered.

The simulation procedure followed to analyze the dynamic properties can be summarized as follows. We generate artificial time series for output by simulating various RBC models. This allows us to evaluate the specific contribution of the endogenous character of growth and the effects of changing the specification of preferences. First, we analyze the improvements provided by the model related to the first and second dimensions of the business cycle phenomenon. To that end, we perform an RBC standard exercise and as usual we compare HP filtered second moments generated by the models with those obtained from U.S. data. Next, we focus our attention on the third dimension of the business cycle. In particular, model performance is compared with U.S. quarterly data from 1955:3 to 1984:1 to test the hypothesis that the data were generated by a particular RBC model. The autocorrelation function, spectrum and impulse response functions are estimated for each artificial sample (each model was simulated 1000 times) and the corresponding empirical probability distributions are computed.

The resolution method used in this paper is Uhlig's (1999) Log-Linear Method (LLM)[5].

[4]Note that when $\sigma = 1$ the utility function is logarithmic.

[5]For more details, a computational appendix is available from the authors upon request.

(i) First and second dimensions: aggregate fluctuations

This section assesses the endogenous growth model and its exogenous growth counterpart by comparing the second order moments generated by them with those obtained from U.S. economic data. The moments to be compared are the volatility of output, productivity and hours worked, and the correlation between hours worked and productivity. We also study the impact of changes in the relative risk aversion parameter on these second moments.

When comparing these statistics we proceed as an RBC researcher would. The cycle component of output is extracted by applying the Hodrick and Prescott (1997) filter. Both the data obtained from simulating the model and the U.S. data are logged and then detrended by using this filter. Since we have quarterly data, the smoothing parameter used is 1600.

Table 2 shows the sample means of the statistics across 1000 simulations, which are 115 periods long (the number of quarters in our data set). Values in parentheses are standard deviations across simulations.

As is well known, the standard exogenous growth model provides a high correlation between hours and productivity, in sharp contrast with the features observed in actual data (this is the so-called *productivity puzzle*). This high correlation is usually attributed to the existence of a single technology shock that causes shifts in the labor demand curve but not in the labor supply curve. When the endogenous growth model is considered shifts in the labor supply curve are obtained by an additional shock (the human capital shock). As a consequence, a lower correlation between hours and productivity is obtained. In addition, hours fluctuate much more due both to the second sector considered and the introduction of this second shock. The former increases the intratemporal substitution between activities and the latter increases the cost of current leisure and, as a consequence, individuals' willingness to substitute leisure intertemporally increases. Therefore, encompassing cycles and endogenous growth is a good startpoint given our purposes.

In this subsection we show that these results are not robust to changes in the values of η and σ. The higher these parameters, the lower the volatility of output and hours worked and the higher the correlation between hours and productivity. On the one hand, due to the presence of labor adjustment costs, firms do not adjust labor input completely in the current quarter in response to a favorable technology shock. Their optimal response is to defer part to the subsequent quarter. This reduces the volatility of hours and output but increases the contemporaneous correlation between hours and productivity. On the other hand, regardless of the value of η, as the degree of risk aversion σ becomes higher, the willingness of individuals to smooth consumption increases which decreases not only the impact effect on output and hours but also their lagged effects. This generates a lower volatility of hours and output and a higher contemporaneous correlation between hours and productivity. Note that, in sharp contrast, these results are robust to changes in σ when the exogenous growth model is considered. This result is not surprising. After all, in an exogenous growth model technology shocks have transitory effects on income and individuals behave as expected: they smooth consumption by saving. But, in an endogenous growth model the same transitory shock has permanent effects on income and so, due to the permanent change in the level of income (and consumption), individuals do not need to smooth consumption by saving. As a result, the internal propagation mechanism of endogenous growth models does depend on the value of σ since, as stated above, a higher σ will lead to a lower effect on hours and output in all periods.

The sensitivity of labor market fluctuations to changes in σ in the endogenous growth model is consistent with sensitivity results found in Barañano (2001) with a different endogenous growth model[6].

In summary, results are sensitive to changes in the values of η and σ, since they have effects on the internal propagation mechanism of the model. In any case, the generalized endogenous growth model outperforms the standard exogenous growth RBC model on the first and second dimensions of the business cycle phenomenon.

[6]Barañano (2001) examines the contribution of a generalized version of the Uzawa-Lucas model in explaining labor market fluctuations. This paper points out that certain second moment statistics are sensitive to changes in the relative risk aversion parameter. In comparing the second moment statistics we must take into account that the models considered differ in many aspects. In particular, in Barañano (2001) physical capital is included as an input in the human capital production function, qualified leisure is assumed to have a positive effect on agent's utility, a single source of uncertainty (technology shocks) is considered and there are no labor adjustment costs.

(ii) Third dimension

As Cogley and Nason (1995) note, standard exogenous RBC models fail to reproduce two stylized facts about U.S. output dynamics: first, GNP growth is positively autocorrelated over short horizons and has a weak and possibly insignificant negative autocorrelation over longer horizons; and second, GNP appears to have an important trend-reverting component that has a hump-shaped MA representation. In this section we study whether the performance of the model in terms of the third dimension of the business cycle depends on the curvature of the utility function as well as the sensitivity of the results to changes in η and σ.

Autocorrelation Functions

We analyze whether all the above mentioned models replicate the sample autocorrelation function (ACF) for output growth. In order to do this, we compute a generalized test statistic to analyze the goodness of fit between actual and theoretical ACF's:

$$Q_{acf} = (\widehat{c} - c)' \widehat{V}_c^{-1} (\widehat{c} - c),$$

where $\widehat{c}$ stands for actual ACF and c for the model-generated one. It is estimated by averaging the ACF values across the ensemble of artificial series. The matrix $\widehat{V}_c$ denotes the sampling covariance matrix for the autocorrelation coefficients. A high value of Q_{acf} indicates a poor fit between the theoretical ACF and the actual ACF. The generalized Q_{acf} statistic is approximately $\chi^2(p)$, where p is the number of lags in c.

Following Cogley and Nason (1995), we compute the generalized statistic for the first $p = 8$ autocorrelations. Results are summarized in Table 3, Table 4 and Figure 1. The first and second columns of Table 3 report values of the Q_{acf} statistic for each model with probability values in parentheses. The first and second columns of Table 4 focus on the performance of the endogenous growth model when both the labor adjustment costs and the curvature of the utility function change.

The performance of the endogenous growth model depends on the curvature of the utility function. As σ becomes higher, the p-values decrease and the model is rejected when $\eta = \frac{0.36}{3}$ at conventional significance levels (see the first and second columns of Table 4). Figure 1 shows that the higher the relative risk aversion parameter σ, the lower the first autocorrelation coefficients are.

To get some intuition on these results, let us analyze how the internal propagation mechanism is affected by changes in σ.

When a favorable technology shock (Z_t) takes place, the higher the degree of aversion, the less resources are devoted to producing goods. This, in the end, will lead to a smaller reduction of the growth rate (look at g_t in Figure 4). Due to the existence of labor adjustment costs, output not only rises at impact but also in the subsequent period. As shown in this figure, not only the impact effect on output of Z_t but also the lagged effects become smaller the higher σ is. This generates a smaller serial correlation in output growth. In addition to this effect, we must take into account that which results from considering human capital shocks (θ_t). In that case, the higher the degree of aversion, the less resources are devoted to human capital accumulation, which will lead to a smaller increase of the growth rate (see Figure 5). Output falls during some periods and subsequently rises back toward its initial trend, but the higher σ is, the smaller the response. Hence, this second effect also generates a smaller correlation in output growth[7].

The analysis in the frequency domain complements the time domain analysis. The spectrum of a time series captures the portion of the sample variance that can be attributed to cycles of different frequencies. A peak in the spectrum indicates that the corresponding frequency band contributes a greater portion of the variance. Figure 2 compares the estimated spectrum for output growth with the spectrum generated by the endogenous growth model. We analyze the performance of this model when the curvature of the utility function changes. The spectrum of output growth is estimated by smoothing the periodogram using a Barlett window and the model-generated spectrum is computed by smoothing the averaged periodogram obtained with 1000 artificial series using a Barlett window. The 95% confidence bands are computed using the approximation given by Brillinger (1981).

The estimate of the spectrum of GNP growth shows a broad peak that ranges approximately from 3 to 6 years. Results are sensitive to changes in the value of the relative risk aversion parameter. Figure 2 shows that only when σ equals 1 does the sample spectrum lie between the 95% confidence bands at business cycle frequencies.

[7]Note that this second effect is only observed when the human capital production sector is considered.

Impulse Response Functions

We also analyze whether these models replicate observed impulse response functions (IRF's). The IRF's are obtained by using the structural VAR model with long-run restrictions developed by Blanchard and Quah (1989). To implement this technique, a third-order VAR for per-capita output growth and ln(hours) is estimated[8]. We analyze whether the theoretical IRF is close to the actual IRF by using the following test statistic:

$$Q_{irf} = (\widehat{r} - r)' \widehat{V}_r^{-1} (\widehat{r} - r),$$

where $\widehat{r}$ is the actual IRF and r is the model-generated one. It is estimated by averaging across the ensemble of artificial series. The matrix $\widehat{V_r}$ denotes the covariance matrix which is estimated as from the Monte-Carlo samples.

A high value of Q_{irf} indicates that the theoretical model does not perform consistently with actual data. We compute this statistic for endogenous growth models with coefficients up to lag 8. Exogenous growth models are driven by a single shock, so their bivariate VAR models have stochastic singularities. The last four columns on the right of Table 3 and Table 4 report values of the Q_{irf} statistics with Monte Carlo probability values in parentheses. Figure 3 shows the transitory impulse response functions for Lucas' (1988) model with different values of σ.

Let us consider the logarithmic utility function[9]. In this case, results show that Lucas' (1988) model has some success in matching the transitory IRF, but not the permanent IRF. In contrast with well-known results on standard RBC models, our setting is not rejected at conventional significance levels even when $\eta = 0$ (see the fifth column of Table 3). As shown in Table 3, transitory IRF results are not sensitive to changes in the value of η. Moreover, the higher the labor adjustment cost parameter, the higher the hump displayed. Hence, Lucas' (1988) human capital investment model not only generates the right qualitative response to transitory shocks but is also able to match the magnitude of the transitory impulse response. Note however that the introduction of labor adjustment costs in Lucas' (1988) endogenous growth model is needed in order to

[8]We select the order of the VAR model following the model selection criteria given by Lütkepohl (1991).

[9]This is the case studied in Barañano and Moral (2003). In comparing the results obtained in that work with those obtained in this paper, the reader should notice some differences due to the order of the VAR selected.

match the magnitude of the permanent IRF. Since the generalized endogenous growth model is not rejected even when $\eta = \frac{0.36}{4}$ at conventional significance levels, the permanent IRF results are also successful (see the third column of Table 4).

Regardless of the value of η, results are very sensitive to changes in the value of the relative risk aversion parameter σ. Note that when $\eta = 0.36$ and $\sigma = 2$ the model does not pass the tests at conventional significance levels. Hence, the curvature of the utility function also plays an important role in generating an important trend-reverting component in output. As shown in Figure 3, increasing relative risk aversion leads to decreasing transitory IRF. The reason is that, as stated above, when a favorable technology shock (Z_t) takes place, the higher σ is, the less resources are devoted to producing goods and, as a consequence, the hump displayed by output is smaller and this is reflected in transitory IRF in Figure 3.

To sum up, this subsection stresses that the curvature of the utility function measured by the relative risk aversion parameter is crucial in determining the performance of this generalized endogenous growth model, since its internal propagation mechanism depends also on the parameterization considered for the curvature of the utility function.

IV. Conclusions

Cogley and Nason (1995), Rotemberg and Woodford (1996) and Watson (1993), among others, have pointed out that standard Real Business Cycle (RBC) models must rely on external sources of dynamics to replicate the three dimensions of business cycles due to the weakness of their internal propagation mechanisms. As a result, they suggest that RBC theorists ought to devote further attention to understanding how shocks are magnified and propagated over time.

We can include different features in a RBC model to get a stronger internal propagation mechanism. As is well known in the literature, the introduction of labor adjustment costs into an otherwise standard model enhances the ability of the model to reproduce the ACF for output growth. But this model will fail not only to replicate the observed hump in the transitory IRF but also to explain the first dimension of business cycles. An alternative approach is to consider an endogenous engine of growth. In this case, it is possible not only to solve the so called *productivity puzzle* but also to match the large transitory IRF found in the data. However, endogenous growth models by themselves do not internally generate the right pattern of autocorrelation in output growth. Barañano and Moral (2003) illustrates that combining labor adjustment costs and endogenous growth improves the model's ability to reproduce the third dimension of the business cycle.

This paper attemps to improve upon the standard RBC model over all dimensions of business cycle phenomenon simultaneously. In particular, this article analyzes under what context the curvature of the utility function alters the strength of the internal propagation mechanism of an RBC model and whether the performance of the model depends on the relative risk aversion considered. To that end, we consider a generalized version of the model studied in Barañano and Moral (2003) which allows us to analyze the effects of different parameterizations of the utility function curvature on the quantitative properties of the model. In contrast to well-known results on standard exogenous growth RBC models, the curvature of the utility function measured by the relative risk aversion parameter plays an important role in determining the performance of the endogenous growth model. In particular, the higher the relative risk aversion parameter, the worse the match.

Appendix A

The first-order conditions for this problem are:

$$\begin{aligned}
U_2(c_t,l_t) &= U_1(c_t,l_t)\left[\tfrac{1-\alpha}{n_t} - \eta\tfrac{\Delta(n_t h_t)h_t}{(n_{t-1}h_{t-1})^2}\right] y_t \\
&+ \beta E_t\left\{U_1(c_{t+1},l_{t+1})\,\eta\tfrac{\Delta(n_{t+1}h_{t+1})n_{t+1}h_{t+1}h_t}{(n_t h_t)^3}\,y_{t+1}\right\}, \qquad (2)\\
U_1(c_t,l_t) &= \beta E_t\left\{U_1(c_{t+1},l_{t+1})\left[\frac{\alpha}{k_{t+1}}\,y_{t+1} + 1 - \delta_k\right]\right\}, \qquad (3)\\
\frac{U_2(c_t,l_t)}{A_h\theta_t h_t} &= \beta E_t\left\{\frac{U_2(c_{t+1},l_{t+1})}{A_h\theta_{t+1}h_{t+1}}\left[A_h\theta_{t+1}(1-l_{t+1}) + 1 - \delta_h\right]\right\}, \qquad (4)\\
h_{t+1} &= A_h\theta_t(1-l_t-n_t)h_t + (1-\delta_h)h_t,\\
y_t &= \frac{A_m Z_t k_t^{\alpha}(n_t h_t)^{1-\alpha}}{\exp\left\{\frac{\eta}{2}\left[\frac{\Delta(n_t h_t)}{n_{t-1}h_{t-1}}\right]^2\right\}},\\
k_{t+1} + c_t &= y_t + (1-\delta_k)k_t,\\
\lim_{t\to\infty} E_t\beta^t U_1 k_{t+1} &= 0,\\
\lim_{t\to\infty} E_t\beta^t \frac{U_2}{A_h\theta_t h_t} h_{t+1} &= 0,
\end{aligned}$$

where E_t is an operator whose expectations are conditional on the information available up to period t and:

- c_t denotes consumption,
- l_t is the fraction of time given over to leisure,
- $n_t h_t$ represents qualified labor units,
- h_t is the stock of human capital,
- y_t denotes output,
- k_t is the stock of physical capitals,
- θ_t is a shock that affects the stock of human capital and finally
- Z_t is a technology shock. Both shocks follow (log-linear) $AR(1)$ processes.

Equation (2) shows the optimal way of determining the fraction of time devoted to the production of goods. The marginal utility from an additional labor unit has to be equal to its marginal disutility. Labor adjustment costs affect not only current marginal utility but also expected utility via future output. Hence, due to the presence of labor adjustment costs, firms do not adjust labor input completely in the current quarter. Their optimal response is to defer part to the subsequent quarter.

Equation (3) governs the accumulation of physical capital and establishes that, at the margin, the expected return to acquiring an additional unit of physical capital must equal the cost it causes in utility terms today.

Equation (4) governs the accumulation of human capital. Given that $1-l_t$ denotes the fraction of time not allocated to leisure, this equation establishes that, at the margin, the expected return in current period utility from an additional unit of human capital must equal its cost.

In the steady state, the variables k_t, y_t and c_t grow at a constant rate, which is equal to the human capital growth rate, while n_t and l_t remain constant. Therefore, non-stationary time series are obtained from the first order conditions characterizing the social planner problem. For the sake of simplicity in computations, the first-order conditions can be rewritten as:

$$\begin{aligned}
U_1(\hat{c}_t, l_t) &= \beta \left(\frac{h_{t+1}}{h_t}\right)^{-\sigma} E_t \left\{ U_1(\hat{c}_{t+1}, l_{t+1}) \left[\frac{\alpha \hat{y}_{t+1}}{\hat{k}_{t+1}} + 1 - \delta_k \right] \right\}, \\
U_2(\hat{c}_t, l_t) &= U_1(\hat{c}_t, l_t) \left[\frac{1-\alpha}{n_t} - \eta \frac{n_t - n_{t-1}\frac{h_{t-1}}{h_t}}{n_{t-1}\frac{h_{t-1}}{h_t}} \frac{h_t}{n_{t-1}h_{t-1}} \right] \hat{y}_t + \\
&\quad \beta \left(\frac{h_{t+1}}{h_t}\right)^{1-\sigma} E_t \left\{ U_1(\hat{c}_{t+1}, l_{t+1})\, \eta \frac{n_{t+1} - n_t \frac{h_t}{h_{t+1}}}{n_t \frac{h_t}{h_{t+1}}} \frac{n_{t+1} h_{t+1} \hat{y}_{t+1}}{{n_t}^2 h_t} \right\}, \\
\frac{U_2(\hat{c}_t, l_t)}{A_h \theta_t} &= \beta \left(\frac{h_{t+1}}{h_t}\right)^{-\sigma} E_t \left\{ \frac{U_2(\hat{c}_{t+1}, l_{t+1})}{A_h \theta_{t+1}} \left[A_h \theta_{t+1}(1 - l_{t+1}) + 1 - \delta_h \right] \right\}, \\
\frac{h_{t+1}}{h_t} &= A_h \theta_t (1 - l_t - n_t) + 1 - \delta_h, \\
\hat{c}_t + \hat{k}_{t+1} \frac{h_{t+1}}{h_t} &= \frac{A_m Z_t \hat{k}_t^{\alpha} n_t^{1-\alpha}}{\exp\left\{ \frac{\eta}{2} \left[\frac{n_t - n_{t-1}\frac{h_{t-1}}{h_t}}{n_{t-1}\frac{h_{t-1}}{h_t}} \right]^2 \right\}} + (1 - \delta_k)\hat{k}_t,
\end{aligned}$$

where $\hat{c}_t = \dfrac{c_t}{h_t}$ and $\hat{k}_t = \dfrac{k_t}{h_t}$.

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Table 1

Parameter and steady state values [a]

Parameter	Value[b]	Interpretation
β	0.9936/0.9972	Subjective discount factor
γ	1/3.6529	Risk aversion parameter
α	0.36	Share of physical capital in the final good technology
δ_k	0.025	Depreciation rate of physical capital
δ_h	0.005	Depreciation rate of human capital
A_m	1	Scale parameter in the final good technology
A_h	0.0266666	Scale parameter in the human capital production function
λ	0.3769	Consumption weight in utility function
η	0.36	Size of labor adjustment costs
σ_Z	0.007	Standard deviation of ε_t
σ_θ	0.004	Standard deviation ofϵ_t
ρ_2	0.95	Persistence of θ_t
ρ_1	0.95	Persistence of Z_t
v	0.0036	Growth rate
$\bar{n}$	0.24	Hours worked
$\bar{r}$	0.01	Real interest rate

[a] For parameters with a time dimension, the unit of time is a quarter of a year.

[b] Following RBC tradition, when changing σ we recalibrate other parameters such as β and γ. The first value corresponds to $\sigma = 1$ and the second to $\sigma = 2$.

Table 2

HP Filtered Autocorrelation Coefficients of U.S. and Model Generated Output[a]

Variable	σ_y	σ_w	σ_n	$corr_{nw}$
GNP	1.977	1.242	1.903	-0.265
Exogenous growth model with $\eta = 0.36$				
$\sigma = 1$	1.860	0.953	0.942	0.925
	(0.254)	(0.119)	(0.139)	(0.016)
$\sigma = 2$	1.665	1.037	0.639	0.971
	(0.221)	(0.130)	(0.092)	(0.005)
Endogenous growth model with $\eta = 0.36$				
$\sigma = 1$	1.606	0.648	1.305	0.263
	(0.226)	(0.079)	(0.202)	(0.163)
$\sigma = 2$	1.446	0.661	1.010	0.472
	(0.199)	(0.080)	(0.155)	(0.131)
Endogenous growth model with $\eta = 0$				
$\sigma = 1$	1.834	0.627	1.668	0.087
	(0.232)	(0.087)	(0.206)	(0.158)
$\sigma = 2$	1.627	0.623	1.302	0.344
	(0.205)	(0.088)	(0.161)	(0.134)

[a] σ_x denotes the volatility of variable x. Volatilities are expressed in percentage terms.
Variable y denotes GNP, variable w productivity and variable n hours.
Standard deviations are in parentheses.

Table 3

Test statistics for the autocorrelation and impulse response functions

Model	Q_{acf}		$Q_{irf}(Y_P)$		$Q_{irf}(Y_T)$	
Exogenous growth	$\sigma = 1$	$\sigma = 2$	$\sigma = 1$	$\sigma = 2$	$\sigma = 1$	$\sigma = 2$
$\eta = 0$	31.82	29.17	–	–	–	–
	(0.0001)	(0.0003)				
$\eta = 0.36$	16.60	20.69	–	–	–	–
	(0.034)	(0.008)				
Endogenous growth	$\sigma = 1$	$\sigma = 2$	$\sigma = 1$	$\sigma = 2$	$\sigma = 1$	$\sigma = 2$
$\eta = 0$	28.66	32.45	23.71	51.01	17.23	38.08
	(0.0003)	(0.00008)	(0.048)	(0.012)	(0.108)	(0.028)
$\eta = 0.36$	10.17	12.63	22.91	46.57	12.02	29.43
	(0.25)	(0.126)	(0.052)	(0.013)	(0.20)	(0.044)

Table 4

Sensitivity analysis

	Endogenous growth model with adjustment costs					
	Q_{acf}		$Q_{irf}(Y_P)$		$Q_{irf}(Y_T)$	
Adjustment costs	$\sigma = 1$	$\sigma = 2$	$\sigma = 1$	$\sigma = 2$	$\sigma = 1$	$\sigma = 2$
$\eta = 0.36$	10.17	12.63	22.91	46.57	12.02	29.43
	(0.25)	(0.126)	(0.052)	(0.013)	(0.20)	(0.044)
$\eta = \frac{0.36}{2}$	12.54	15.46	22.78	47.75	11.97	29.85
	(0.128)	(0.051)	(0.06)	(0.014)	(0.198)	(0.043)
$\eta = \frac{0.36}{3}$	14.49	17.67	22.80	48.27	12.54	30.94
	(0.07)	(0.024)	(0.059)	(0.014)	(0.18)	(0.044)
$\eta = \frac{0.36}{4}$	16.03	19.38	22.87	48.64	13.07	31.85
	(0.041)	(0.013)	(0.058)	(0.014)	(0.168)	(0.041)

Figure 1:

ACF for output growth. Labor adjusment costs $\eta = 0.36$

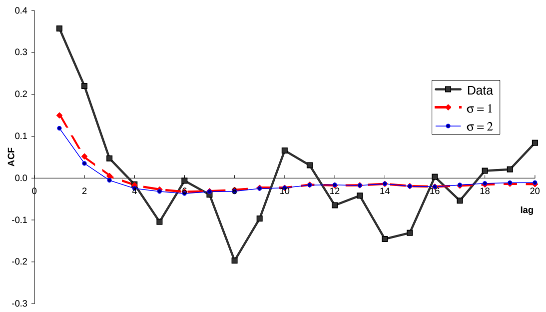

Figure 2:

Spectrum for output growth

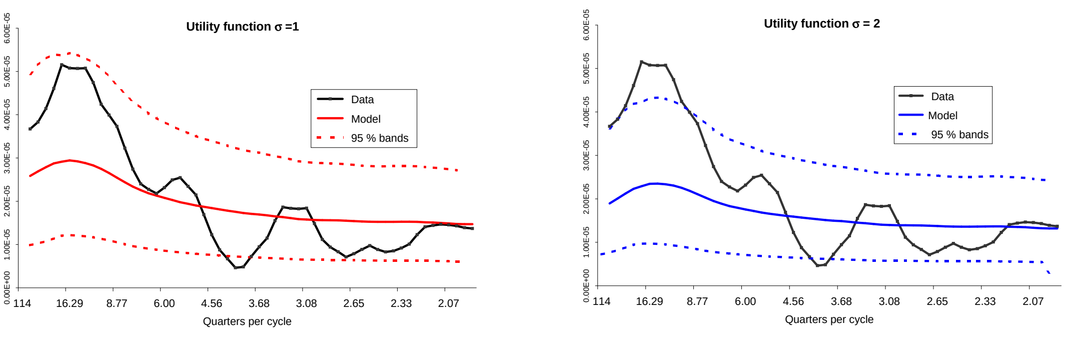

Figure 3:

Transitory IRF function for $\ln(GNP)$ *from the Blanchard-Quah technique. Labor adjusment costs* $\eta = 0.36$

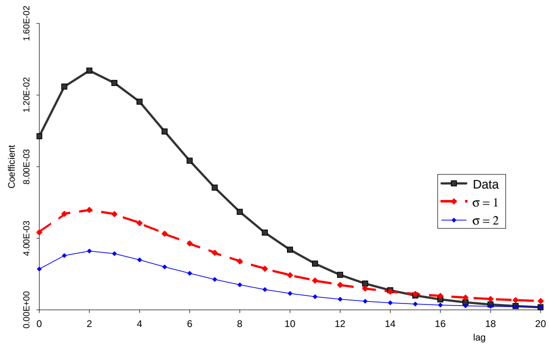

FIGURE 4:

Response functions to a 1% technology shock

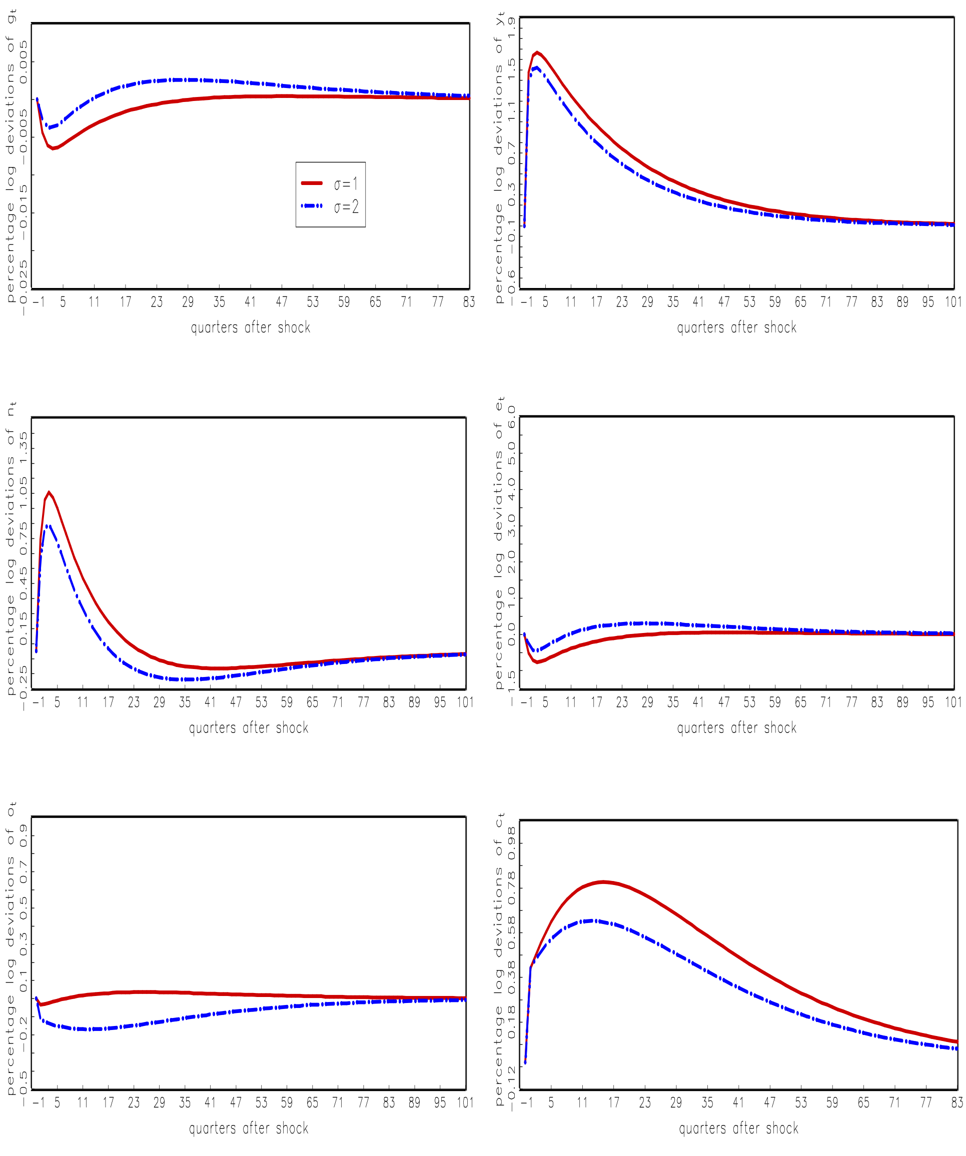

Figure 5:

Response functions to a 1% human capital shock