Modeling nonlinearities in real exchange rates for advanced and emerging economies

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Preliminary

Abstract

Long-lasting misalignments in the real exchange rates are sometimes explained by the presence of a non-linear adjustment process toward the long-run equilibrium. However, while it is possible that for some real exchange rate evidence of nonlinearity exists, outliers and nonlinearity may easily be mistaken. This paper uses robust methods to examine and compare the behavior of Smooth Transition Autoregressive [STAR] models for the real exchange rates OF 14 countries. The results show that the evidence of nonlinearity is reduced when considering outliers. Non-linearity is also more common among developing economies, where the transition from one regime to the other is faster.

Keywords: real exchange rate, outliers, smooth transition autoregressive models, robust estimation.

JEL classification: F31, C12, C5.

1 Introduction

The Purchasing Power Parity (PPP) is a simple and popular concept of equilibrium exchange rate in the economic literature. In general terms, the PPP hypothesis suggests that the nominal exchange rate between two currencies should be equal to the ratio of aggregate price levels between two countries, so that a unit of currency of one country will have the same purchasing power in a foreign country. Since the real exchange rate (RER) is the nominal exchange rate adjusted for relative national price levels, variations in the real exchange rate represent deviations

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from PPP. In other words, when the PPP holds, the real exchange rate tends to a constant mean in the long-run.

Even though the PPP has motivated a lot of research, the question of what exactly explains the deviations from it remains unanswer. Among the most common argument to explain the recurrent deviations of the real exchange rate from a constant long-run value is the Harrod-Balassa-Samuelson (HBS) effect which, in simple words, implies that increases in productivity are associated with a real appretiation[1]. Nonetheless, whether the HBS effect has been strong is today a subject of controversy.

An alternative explanation for the persistent misalignments in the real exchange rates during the post-Bretton Woods period (sustained periods of overvaluation and undervaluation), is the presence of a nonlinear adjustment process toward the long-run equilibrium. While the most common causes for nonlinearity are the existence of transactions costs, mainly due to the cost of transportation, other arguments, such as the heterogeneity of opinion in the foreign exchange market concerning the equilibrium level of the nominal exchange rate (see Taylor and Allen (1992) and Kilian and Taylor (2003)), the speculative attacks on currencies (Flood and Marion (1998)), the presence of target zones (Krugman (1991)), noisy traders (De Long, Shleifer, Summers, and Waldmann (1990)) or the heterogeneity of the interventions of central banks (Dominguez (1998)) have also been advanced.

To capture the phase-dependent properties, most of the empirical studies on real exchange rates specify the switch between regimes as a function of past values of the real exchange rate by means of smooth transition autoregressive (STAR) models, as proposed by Terasvirta and Anderson (1992), Granger and Terasvirta (1993) and Terasvirta (1994). Based on this approach, these studies conclude that RER in industrialized countries behave more like a unit root process the closer they are to long-run equilibrium and, conversely, become more mean reverting the further they are from it[2].

However, even if STAR characterizations may be useful to explain deviations from PPP in some cases, it is possible that this apparent nonlinearity is due to outlier observations in the series[3]. Indeed, many linear economic time series are contaminated by occasional outliers. In particular, given the changes in exchange rate regimes, financial or political crisis and other sharp disturbances, real exchange rates are subject to substantial variations. These turbulent

[1]Some empirical evidence for the HBS effect in industrial countries can be found in Heston, Nuxoll, and Summers (1994) and Lane and Milessi-Ferreti (2002). Choudhri and Khan (2004) analyze the Balassa-Samuelson effect in developing countries.

[2]See Michael, Nobay, and Peel (1997), Sarantis (1999), Taylor, Peel, and Sarno (2001). They all investigate non-linear adjustment dynamics of exchange rates in major industrial countries.

[3]Although it is not a formal definition, an outlier can be described as an observation (or a subsect of observations) which appears to be inconsistent with the remaining of that set of data (Barnett and Lewis (1994)).

histories appear as structural shifts or as outliers in the real exchange rates. In the presence of some aberrant observation, nonlinearity tests may wrongly point towards nonlinear structures.

Yet, it is also possible that the non-linear properties of the series are reflected in a few observations. One may be tempted to view this nonlinear data points as aberrant observations and remove them dramatically, destroying intrinsic nonlinearity (van Dijk (1999)). That is, it may be the case that the presence of these outliers allows to detect the way and speed of adjustment towards equilibrium. Thus, eliminating outliers sometimes can lead to the omission of valuable information about the equilibrium.

In this context, the aim of this paper is to analyze whether the characteristics of the adjustment process of the real effective exchange rates for a group of 14 countries can be explain by STAR-type models and, if so, to investigate non-linearities in real effective exchange rates. Our study differs from most of the earlier literature on nonlinear exchange rate modeling at least in two important ways. First, contrary to previous investigations, we adopt a robust test for STAR type non-linearity. This test allows us to distinguish between nonlinearity and outliers without removing these observations too severely. The second advantage of our study is that we not only investigate nonlinearities for a group of industrialized countries, but also analyzed developing countries, a fact that has been largely unexplore.

Our results show that, sometimes, apparent nonlinearity is due to just some aberrant observations that once taken into account reduce considerably the presence of nonlinear features in the data. We also provide evidence that non-linearity is most common among developing countries, where also the transition from one regime to the other is faster than in advanced countries.

The rest of the paper is organized as follows. In the next section we describe the smooth transition autoregressive models and we discuss how these models can capture real exchange rate dynamics. In section 3 we consider the implications of outliers in the series. Section 4 discusses the methodology. In section 5 we give the empirical results. Finally, we conclude in section 6.

2 Nonlinear adjustment of real exchange rates

Formal tests for evidence of PPP as a long-run phenomenon have often been based on an empirical examination of the real exchange rate which, in its logarithm form, label as q_t, may be expressed as:

$$q_t = e_t - p_t + p_t^* \quad , \tag{1}$$

where e_t is the logarithm of the nominal exchange rate (domestic price of foreign currency), and p_t and p_t^* denote the logarithms of the domestic and foreign price levels, respectively. If the RER is to settle down at any level at all, including a level consistent with PPP, q_t must display mean reversion. On the contrary, if the real exchange rate is the realization of a unit-root process, then while changes in it may be to some extend predictable, the variable may still never settle down at any particular level, even in the long-run.

Hence, in the procedures conventionally applied to test for long-run PPP using equation (1), the null hypothesis is usually that the process generating the real exchange rate series has a unit root, while the alternative hypothesis is that all of the roots of the process lie outside the unit circle. Thus, the maintained hypothesis in the conventional framework assumes a linear autoregressive process for the real exchange rate, implying that adjustment is both continuous and of constant speed, regardless of the deviations from PPP (Taylor, Peel, and Sarno 2001).

However, it has been suggested that deviations from PPP may follow a nonlinear process that is mean reverting, with the speed of adjustment toward equilibrium varying directly with the extend of the deviation from PPP[4]. This implies that deviations from PPP last for a very long time although they do not follow a random walk[5].

In general terms, the idea of nonlinearities in the adjustment process implies that there exists a band for the real exchange rate within which the marginal cost of arbitrage exceeds the marginal benefit. The thresholds then not only reflect shipping costs and trade barriers *per se*, but also are the result of the sunk cost of international arbitrage and the resulting tendency for traders to wait for sufficiently large arbitrage opportunities to open up before entering the market[6].

Several models have been suggested to capture the nonlinear nature of the adjustment process of the RER. In general, while in some models the jump to mean-reverting behavior is sudden (threshold autoregressive or TAR models as proposed by Tong (1990), for example) the empirical evidence has suggested that smooth, rather than discrete adjustment may be more appropriate. To catch this feature, Granger and Terasvirta (1993) and Terasvirta (1994) suggested the smooth transition autoregressive (STAR) model. A STAR model of order p for the (log) real exchange

[4]See, for example, Benninga and Protopapadakis (1988), Dumas (1992), Sercu, Uppal, and Van Hulle (1995), Michael, Nobay, and Peel (1997) or Dufrénot, Mathieu, Mignon, and Peguin-Feissolle (2002).

[5]It can also be the case that deviations of the exchange rates from their equilibrium value (i.e., the PPP or an equilibrium value according to the fundamentals) are described by long-memory process. In this case it would be more natural to study the RER misalignment using fractional cointegration models (see, for example, Dufrénot, Lardic, Mathieu, Mignon, and Peguin-Feissolle (2006))

[6]That is, the profits from commodity arbitrage, which is generally thought to be the ultimate force behind maintaining PPP, do not make up for the cost involved in the necessary transactions for small deviations from the equilibrium real exchange rate. This implies the existence of a band around the equilibrium rate in which there is no tendency of the real exchange rate to revert to its equilibrium value. Outside this band, commodity arbitrage becomes profitable , which forces the real exchange rate towards the band.

rate may be written as:

$$q_t = \alpha + \sum_{j=1}^{p} \pi_j q_{t-j} + (\alpha^* + \sum_{j=1}^{p} \pi_j^* q_{t-j}) F(s_{t-d}; \gamma, c) + \varepsilon_t \quad , \tag{2}$$

where α and α^* are regime constants, q_t, is assumed to be a stationary ergodic process, ε_t is $\sim iid(0, \sigma^2)$, and F is a transition function which is bounded by zero and one. This function is governed by the parameters γ, which is the speed of transition from one regime to the other, s_{t-d} which is the transition variable (with d the delay parameter) and c, which is the threshold[7]. In these models, nonlinearities arise through conditioning on lagged real exchange rates. The adjustment takes place in every period, but the speed varies with the extend of the deviation from parity.

In (2) F can be a first-order logistic function, in which case the model is called a logistic STAR model (LSTAR):

$$F(s_{t-d}; \gamma, c) = 1 + exp[-\gamma(s_{t-d} - c)]^{-1}, \gamma > 0 \quad , \tag{3}$$

In (3) the parameter c can be interpreted as the threshold between the two regimes corresponding to $F(s_{t-d}; \gamma, c) = 0$ and $F(s_{t-d}; \gamma, c) = 1$, in the sense that the logistic function changes monotonically from 0 to 1 as q_t increases while $F(s_{t-d}; \gamma, c) = 0.5$. The parameter γ determines the smoothness of the change in the value of the logistic function and, thus, the smoothness of the transition from one regime to the other. In the LSTAR model, the two regimes are associated with small and large values of the transition variable s_{t-d} relative to the threshold c.

It has been argued that the logistic function (3) is not the most plausible for the modeling of the real exchange rate series, since it is not symmetric for positive and negative deviations from PPP[8]. The need for symmetry leads to the exponential STAR or ESTAR model which can be expressed as:

$$F(s_{t-d;\gamma,c}) = 1 - exp[-\gamma(s_{t-d} - c)^2], \gamma > 0 \quad , \tag{4}$$

The ESTAR model suggests that the two regimes have rather similar dynamics, while the transition period can have different dynamics. The transition function is U-shaped and symmetric around c in the sense that local dynamics are the same for high and for low values of the real exchange rate series.

[7]The transition variable, s_{t-d}, can be thought as lags of the real effective exchange rate in levels, its first difference, an exogenous variable, a function of lagged endogenous variables or a linear time trend, which gives rise to a model with smoothly changing parameters.

[8]See Michael, Nobay, and Peel (1997) and Taylor, Peel, and Sarno (2001)

In the LSTAR (or ESTAR) model, the lower (middle) regime corresponds to $s_{t-d} = c$ when $F = 0$ and thus the model (2) becomes a linear AR(p) model:

$$q_t = \alpha + \sum_{j=1}^{p} \pi_j q_{t-j} + \varepsilon_t \quad , \tag{5}$$

The upper or outer regime for the LSTAR or ESTAR models respectively corresponds to $s_{t-d} = \pm\infty$ when $F = 1$ and (2) becomes a different type of AR(p) model:

$$q_t = \alpha + \alpha^* + \sum_{j=1}^{p} (\pi_i \pi_j^*) q_{t-j} + \varepsilon_t \quad , \tag{6}$$

A drawback of the exponential function (4) is that for either $\gamma \to 0$ or $\gamma \to \infty$, the function collapses to a constant (equal to 0 or 1, respectively). Hence, the model becomes linear in both cases. This can be solved by using the quadratic logistic function, LSTAR$_2$:

$$F(q_{t-d}; \gamma, c) = 1 + exp[-\gamma(s_{t-d} - c_1)(s_{t-d} - c_2)]^{-1}, c_1 \leq c_2; \gamma > 0 \quad , \tag{7}$$

where now $c = (c_1, c_2)'$. In this case, if $\gamma \to 0$, the model becomes linear, whereas if $\gamma \to \infty$, the function $F(s_{t-d}; \gamma, c)$ is equal to 1 for $s_{t-d} < c_1$ and $s_{t-d} < c_2$ and equal to zero in between[9].

Finally, since the STAR models require the dependent variable to be stationary, equation (2) can be reparameterized as follows:

$$\Delta q_t = k + \lambda q_{t-1} + \sum_{j=1}^{p-1} \phi_j [\Delta q_{t-j}] + (k^* + \lambda^* q_{t-1} + \sum_{j=1}^{p-1} \phi_j^* [\Delta q_{t-j}]) F(s_{t-d}; \gamma, c) + \varepsilon_t \quad , \tag{8}$$

where k and k^* are regime constants, $\Delta q_t = q_{t-j} - q_{t-j-1}$, $\phi_j = 1 - \pi$ and $\phi_j^* = 1 - \pi^*$.

3 The presence of outliers

Recently, regime switching models, in general, and smooth transition autoregressive models as the LSTAR and ESTAR, in particular, have been applied to study possible non-linearities in the real exchange rates. Based on these non-linear models, several empirical studies conclude that the behavior of the real exchange rates depend nonlinearly on the size of the deviation from purchasing power parity[10].

Certainly, it is possible that for many series empirical evidence for nonlinearity does exists. However, most of the studies do not consider that many linear economic time series are

[9] Note that for moderate values of γ, the minimum value taken by the function (7) is not equal to zero.

[10] See, for example, Obstfeld and Taylor (1997) who investigate the nonlinear nature of the adjustment process in terms of a threshold autoregressive (TAR) model or Michael, Nobay, and Peel (1997) for an application of ESTAR models in real exchange rates.

contaminated by occasional outliers. If that was the case, tests for STAR nonlinearity would tend to reject the correct null-hypothesis of linearity too often. In other words, the apparent nonlinearity could be simply due to structural breaks or outliers in the series.

Indeed, according to Koop and Potter (2000) models that allow for dynamics which vary over the business cycle in a predictable way are the nonlinear models. Still, another possibility is that apparent departures from linearity are due to unpredictable large shocks which have only temporary effects. Models with this property are the "outliers" models. Since they have very different consequences for forecasting and understanding the real exchange rate dynamics, it is important to test whether nonlinearities or outliers are present in the data.

Two types of outliers are specially interesting in time series. First, the additive outlier (AO), which gives a one time effect on the level of the time series, as only the current observation is affected. Second, the innovative outlier (IO) implies that a shock at time t also influences future observations through the same dynamics as the linear part of the model. It has been shown (van Dijk (1999), van Dijk, Franses, and Lucas (1996) and van Dijk, Terasvirta, and Franses (2000)) that the presence of both kinds of outliers may distort the results of testing and estimation. In particular the Ordinary Least Squares estimates of the autoregressive parameters though consistent are inefficient in the presence of AOs and biased towards zero in the presence of IOs.

Also, in terms of a simple regression model, $y_t = x_t\beta + \varepsilon_t$, outliers can also be classified into three groups. The first one, the vertical outliers, are characterized because the x_t values of these observations fall inside the range of the bulk of the observations. However, the observations depart markedly from the linear relationship. The second set of outliers, the "good" leverage points satisfy the linear relationship, but have x_t values outside the usual range. Finally, the "bad" leverage points have aberrant values for x_t and do not fit the (linear) pattern set out by the bulk of the data (Lucas (1996)).

A significant literature now exists suggesting different outlier detection and correction procedures. Here, we follow a robust procedure as suggested by Martin (1981), van Dijk (1999) and van Dijk, Terasvirta, and Franses (2000) among others. This methodology has the advantage that it is not based on a estimation-detection-correction-estimation procedure and therefore it is not subjective and time consuming.

4 Methodology

Model selection and estimation is based on the procedure suggested by Terasvirta (1994) for single transition models. The first step is to specify a linear AR model. Second, based on this model, we test the null hypothesis of linearity against the alternative of nonlinearity. Third, if linearity is rejected, an appropriate transition variable and the form of the transition function are selected. Fourth, the parameters in the selected transition function are estimated. Finally, the model must be evaluated and modified if necessary.

In what follows, we briefly explain these steps. Yet, since we are mainly concern with linearity tests in the presence of outliers, we dedicate more time to discuss this point.

4.1 The linear AR model specification

A linear AR(p) model is selected, with the lag length p chosen (to a maximum value of 12) according to the Schwarz Information Criterium (BIC). In order to anticipate the structure of the STAR model, we specified an AR(p) model in first differences, including a constant and the first lag of the variable in levels. Thus, the estimated linear models can be characterized as follows:

$$\Delta q_t = \beta_0 + \lambda q_{t-1} + \sum_{j=1}^{p-1} \phi_j [\Delta q_{t-j}] \ , \tag{9}$$

It must be kept in mind that if linearity is rejected, the lag order used in the AR model is not necessarily the appropriate lag order in the alternative STAR model, although usually it gives a good first guess.

4.2 Testing linearity

4.2.1 Standard linearity tests

A crucial step in specifying a STAR model is, of course, to test linearity of the model against the STAR specification. If the null hypothesis of linearity is accepted, the conclusion is that the real exchange rate can be adequately described by a linear AR model. On the contrary, if the null is rejected, we can suggest that the real exchange rate follows a non-linear adjustment process towards equilibrium.

For this purpose, the null hypothesis of linearity can be expressed as equality of the autoregressive parameters in the two regimes of the STAR models in (8). That is, under the null hypothesis, $H_0 : \phi_j = \phi_j^*$ whereas the alternative hypothesis is $H_1 : \phi_j \neq \phi_j^*$ for at least one $j \in \{0, ..., p\}$.

However, the STAR specifications share with many other nonlinear models the property that it is not identified under the alternative hypothesis of non-linearity. Indeed, the STAR model contains parameters which are not restricted by the null hypothesis, but which nevertheless are no longer present in the model when the null hypothesis holds true. For example, the null hypothesis $H_0 : \phi_j = \phi_j^*$ does not restrict the parameters γ and c in the transition function, but when $\phi_j = \phi_j^*$ the function, $F(s_{t-d}; \gamma, c)$ and therefore γ and c drop out of the equation (van

Dijk (1999)).

Also, the null hypothesis of linearity can be formulated in different approaches. For example, one way to achieve an AR model is if $\gamma = 0$ in (8). Alternatively, the null hypothesis can be set as the equality of the AR parameters in the two regimes as asserted before.

The consequence of the presence of unidentified nuisance parameters is that the conventional statistical theory is not available for obtaining the asymptotic null distribution of the test statistics. Instead, the test statistics tend to have non-standard distribution and, therefore, the critical values have to be determined by simulation.

To deal with this problem, Lukkonen, Saikkonen, and Terasvirta (1988) suggested to replace the transition function $F(s_{t-d};\gamma,c)$ in equation (2) with a suitable Taylor approximation. The re-parameterized model is no longer associated with an identification problem, and linearity testing proceeds by using regular Lagrange multiplier (LM) tests with a standard asymptotic χ^2 distribution under the null hypothesis.

Then, the procedure for testing linearity against LSTAR is as follows. Rewriting (2) as:

$$q_t = \phi' x_t + (\phi * - \phi)' x_t F(\gamma; s_{t-d}, c*) + \varepsilon_t \quad , \tag{10}$$

were $x_t = (1, \tilde{x_t}')'$, $\tilde{x_t} = (q_{t-1}, ..., q_{t-p}')'$, and $\phi_j = (\phi_{j,0}, \phi_{j,1} ... \phi_{j,p})'$, the logistic function can be approximated with a first-order Taylor approximation around $\gamma = 0$. This results in the auxiliary equation:

$$q_t = \beta_0' x_t + (\beta_1)' x_t s_{t-d} + e_t \quad , \tag{11}$$

where $\beta_j = (\beta_{j,0}, \beta_{j,1}, ...,)'$, $j = 1, 0$, and $e_t = \varepsilon_t + (\phi_2 - \phi_1)' x_t R_1(s_{t-d}; \gamma, c)$, with R_1 the remainder term from the Taylor expansion. Notice that under the null hypothesis, $R_1(s_{t-d}; \gamma, c) \equiv 0$ and $e_t = \varepsilon_t$. Therefore, the remainder term of the Taylor approximation does not affect the properties of the test statistics. The parameters $\beta_j = (\beta_{j,0}, \beta_{j,1}, ...,)'$, $j = 1, 0$ in the auxiliary regression (11) are functions of the parameters in the STAR model (10) such that the restriction $\gamma = 0$ implies $\beta_{0,j} \neq 0$ and $\beta_{1,j} = 0$ for $j = 0, ..., p$. Therefore, testing the null hypothesis of linearity (i.e. $H_0' : \gamma = 0$ or, similarly, $H_0 : \phi_1 = \phi_2$) in equation (2) is alternative to testing for the null hypothesis $H_0'' : \beta_1 = 0$ in (11). This non-linear test, called the LM_1, test may be conducted by using either an asymptotic χ^2 test, with $p + 1$ degrees of freedom under the null hypothesis of linearity, or an appropriate F version of the test.

However, as Lukkonen, Saikkonen, and Terasvirta (1988) noticed, the LM_1 test has low power in cases where only the intercept varies across regimes. A test that apparently does have power in this situation can be obtained by replacing $F(s_{t-d}; \gamma, c)$ in (2) for a third-order Taylor series approximation. This gives the next auxiliary regression:

$$q_t = \beta_0' x_t + \beta_1' x_t s_{t-d} + \beta_2' x_t s_{t-d}^2 + \beta_3' x_t s_{t-d}^3 + e_t \quad , \tag{12}$$

where $e_t = \varepsilon_t + (\phi_2 - \phi_1)' x_t R_3(\gamma; s_{t-d}, c)$ and $\beta_j = (\beta_{j,0}, \beta_{j,1}, ...,)'$, $j = 1, 0$ are also functions of the parameters ϕ_1, ϕ_2, γ and c. Now a test of linearity involves testing $H_{03} : \beta_1 = \beta_2 = \beta_3 = 0$ against the alternative that H_{03} is not true by a standard LM-type test. This test, denoted the LM_3, may be conducted by using either an asymptotic χ^2 test with $3(p+1)$ degrees of freedom or its F-test counterpart. A parsimonious version of the LM_3 test can be obtained by including only s_{t-d}^2 and s_{t-d}^3 as additional regressors in the auxiliary model (11). In this case, the artificial regression is:

$$q_t = \beta_0' x_t + \beta_1' x_t s_{t-d} + \beta_{2,0} s_{t-d}^2 + \beta_{3,0} s_{t-d}^3 + e_t \quad , \tag{13}$$

A test of the null hypothesis $H_{03}^e : \beta_1 = 0$ and $\beta_{2,0} = \beta_{3,0} = 0$ yields the LM_3^e test which has an asymptotic χ^2 distribution with $p + 3$ degrees of freedom. Again, an F version of the test can be used.

Therefore, on the one hand, LM_1, LM_3 and LM_3^e statistics are used to test against LSTAR with the delay parameter d assumed unknown. On the other hand, Escribano and Jordá (1999) suggested that linearity might be tested against an ESTAR alternative replacing the exponential transiton function in 4 with a second-order Taylor approximation around $\gamma = 0$, which gives rise to the auxiliary model:

$$y_t = \beta_0' x_t + \beta_1' x_t s_{t-d} + \beta_2 x_t s_{t-d}^2 + \beta_3 x_t s_{t-d}^3 + \beta_4 x_t s_{t-d}^4 + e_t \quad , \tag{14}$$

where $e_t = \varepsilon + (\phi_2 - \phi_1)' x_t R_3(s_{t-d}; \gamma, c)$. The null hypothesis to be tested now is $H_0' : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$. The resulting LM-type statistic, denoted LM_4, has an asymptotic χ^2 distribution with $4(p+1)$ degrees of freedom under the null hypothesis.

Finally, in small samples, van Dijk (1999) recommends to use F-versions of the LM-tests statistics because these have better size and power properties than the χ^2 variants.

4.2.2 Outliers robust tests

van Dijk (1999) and van Dijk, Terasvirta, and Franses (2000) suggest that the LM tests discussed above are sensitive to several kinds of misspecification of the model under the null hypothesis. In particular, they show that in the presence of additive outliers and given the properties of OLS, these tests tend to reject the correct null hypothesis of linearity too often, even asymptotically. As a solution, they suggested to used outlier-robust estimation techniques.

Therefore, to avoid the deficiencies of the OLS estimator in the presence of outliers, the autoregressive parameter can be estimated robustly using maximum likelihood (M) type or generalized M (GM) estimators. The class of GM estimators are design to obtain better estimates in the presence of contamination by giving less weight to influential observations such as outliers.

That is, the OLS estimate of the autoregressive parameter ϕ_1 in the $AR(p)$ model $y_t = \phi' x_t + \varepsilon_t$ minimizes the sum of square residuals, which can be characterized by the following first order condition:

$$\sum_{t=1}^{T} x_t(y_t - \phi' x_t) = 0 \quad , \tag{15}$$

Instead, a robust estimator for the $AR(p)$ model gives less weight to influential observations such as outliers. Thus, these estimators solve the alternative first order condition:

$$\sum_{t=1}^{T} \omega_r(r_t) x_t(y_t - \phi' x_t) = 0 \quad , \tag{16}$$

where r_t are the standardized residuals, $r_t \equiv (y_t - \phi' x_t)/(\sigma_\varepsilon \omega_x(x_t))$, with σ_ε a measure of scale of the residuals $\varepsilon_t \equiv y_t - \phi' x_t$ and $\omega_x(\cdot)$ and $\omega_r(\cdot)$ are weight functions that are bounded between 0 and 1. This estimator is a type of weighted least squares estimator, with the weight of the t-th observation given by the value of $\omega_r(\cdot)$. The functions $\omega_x(\cdot)$ and $\omega_r(\cdot)$ should be chosen such that the t-th observation receives a relatively small weight if either the regressor x_t or the standardized residual becomes unusually large. In this way, outliers and influential observations automatically receive less weight. For the OLS estimator, $\omega_x(\cdot) \equiv 1$ and $\omega_r(\cdot) \equiv 1$, such that all observations receive the same weight. Several forms of bounded functions for $\omega_x(\cdot)$ and $\omega_r(\cdot)$ are suggested in the literature. Usually, the weight function $\omega_r(\cdot)$ is specified in terms of a function $\psi(r_t)$ as $\omega_r = \psi(r_t)/r_t =$ for $r_t \neq 0$ and $\omega_x(0) = 1$.

This outlier robust estimation procedure allows to modify the LM tests discussed before. In particular a robust test can be obtained by using a robust estimator to obtain the model under the null hypothesis. Under conventional assumptions, the test statistics retain their standard limiting χ^2 distributions. Therefore, as shown by van Dijk (1999), these robust tests have good size properties in small samples, also in the presence of outliers.

4.3 The choice of the transition variable and transition function

If linearity is rejected we proceeded to select the appropriate transition variable, s_t. For this purpose, even though the LM_3 statistic was developed as a test against LSTAR it has power against ESTAR alternatives as well. Therefore, computing the LM_3 for the candidate variables, $s_{1t-d}, ..., s_{mt-d}$ and selecting the one with the smallest p-value gives a good starting point.

To select the transition function $F(s_{t-d};\gamma,c)$ we limit our choice to that between the first-order logistic function (LSTAR) or the second-order logistic function (LSTAR$_2$). Therefore, when choosing between these two models for the real effective exchange rate series where linearity is rejected, we used the following sequence of nested tests:

$$\begin{aligned} &H_{03} : \beta_3 = 0, \\ &H_{02} : \beta_2 = 0 \mid \beta_3 = 0, \\ &H_{01} : \beta_1 = 0 \mid \beta_3 = \beta_2 = 0 \end{aligned} \tag{17}$$

Note that given the properties of the auxiliary parameters in (14), if the p-value of the test corresponding to H_{02} is the smallest, an ESTAR model should be selected (or, alternatively, an LSTAR$_2$ model), while in all the other cases, an LSTAR model is prefered.

Alternatively, Escribano and Jordá (1999) proposed an alternative transition function selection which makes use of LM_4 as a test for general STAR nonlinearity. They suggest to test the hypothesis:

$$\begin{aligned} &H_{0E} : \beta_2 = \beta_4, \\ &H_{0L} : \beta_1 = \beta_3 \end{aligned} \tag{18}$$

in (14) and to select an LSTAR (LSTAR$_2$) model if the minimum p-value is obtained for $H_{0L}(H_{0E})$

4.4 The estimation

Once the correct transition function has been selected, the LSTAR or ESTAR specifications are estimated by non-linear least squares, which gives consistent and asymptotically normal estimates. For this, finding good starting values for the algorithm is important to ensure that a global minimum is achieve. One way of obtaining them is by a two-dimensional grid search over γ and c. When constructing the grid, note that γ is not a scale free parameter. Therefore, the exponent of the transition function is standardized by dividing it by the k-th sample standard deviation of the transition variable s_t, named $\hat{\sigma}_s$, where K=1 for the LSTAR and K=2 for the LSTAR$_2$. The transition function becomes:

$$F(\gamma; s_t, c) = \left(1 + exp\{(-\gamma/\hat{\sigma}_s^k)\sum_{k=1}^{K}(s_t - c_k)\}\right)^{-1}, \gamma > 0 \ , \tag{19}$$

This makes the parameter γ in (19) scale-free, which in turn facilitates the construction of an effective grid. A significant set of grid values for the location parameter c may be defined as sample percentiles of the transition variable s_t. For each value of c and γ the residual sum of squares is computed and the values that correspond to the minimum of that sum are taken as starting values. Once good starting points have been found, the unknown parameters c, γ, ϕ, θ can be estimated by using a form of the Newton-Raphson algorithm to maximize the conditional maximum likelihood function [11].

It is important to notice that a specific numerical problem exists in the estimation of STAR models when γ in (19) is large and the model is consequently close to a switching regression model (see Terasvirta (1994)). To obtain an accurate estimate of γ, one needs many observations in the immediate neighborhood of c, because even large values in γ only have a small effect on the shape of the transition function. It is unlikely that such clusters can be found in small samples. The estimate of γ may therefore be rather imprecise and often appears to be insignificant when judged by its t-statistic. Yet, this does not suggest redundancy of the non-linear component.

The STAR models are evaluated using diagnosis tests that include tests of no error autocorrelation, normality in the errors, no additive nonlinearity and parameter constancy. Finally, the models are modified if necessary.

4.5 Dynamic properties of the STAR models

According to Terasvirta and Anderson (1992) it is usually difficult to interpret the individual coefficients of STAR models, but the roots of the characteristic polynomial associated with these models provide information which is useful for understanding their dynamic properties. This can be done by computing the roots of the STAR(p) model by solving:

$$z^p - \sum_{j=1}^{p} (\hat{\theta}_j + \hat{\theta^*}_j F) z^{p-j} = 0 \quad , \tag{20}$$

It is possible to compute the roots of the characteristic polynomial of the models at each point in the sample period and for several values of the transition function. However, the roots of the respective extreme regimes describe the local dynamics of appreciation and depreciation. In particular, we considered two regimes: first, F=0, which corresponds to the lower (rising q) regime in the LSTAR model and the middle regime in the ESTAR model. Second, F=1, which corresponds to the upper (falling q) regime in the LSTAR model, and the outer regime in the STAR model.

[11] We should keep in mind that one problem with NLS is that the algorithm may give local maxima or minima instead of a global solution.

5 Empirical Analysis

5.1 Data Sources and Construction of the Variables

For the empirical analysis we considered the real effective exchange rate (REER) of the following countries: Argentina, Australia, Brazil, Canada, Euro-zone, India, Indonesia, Japan, Korea, Mexico, Norway, Turkey, UK and the USA. The data are monthly and covers the period January 1980 until December 2005. The REER, based on consumer prices, of the country i was constructed as follows:

$$REER_{it} = \frac{P_{it}E_{it}}{\prod_{j \neq i}^{N}(P_{jt}E_{jt})^{\omega_{ij}}} \quad , \tag{21}$$

where j is an index of country i's trade partners; N is the number of countries, ω_{ij} is the competitiveness weight put by country i on country j, P_i and P_j are Consumer Price Indices (CPI) in countries i and j; and E_i and E_j represent the nominal exchange rates of countries i and j's currencies in US dollars, all of them obtained from the the IMF's *International Financial Statistics*[12]. We work with the logarithm of the real effective exchange rate and its first difference, denoting this variable as q_t and Δq_t[13].

6 Empirical results

As stated before, we analyze possible non-linearities in the real effective exchange rates for a group of advanced and developing countries. A preliminary evaluation of the data shows that real exchange rates are non-stationary in levels. Therefore, in order to anticipate the structure of the STAR/ESTAR models, the real exchange rates are made approximately stationary by taking first differences of logarithmic transformed data (see figure (1)).

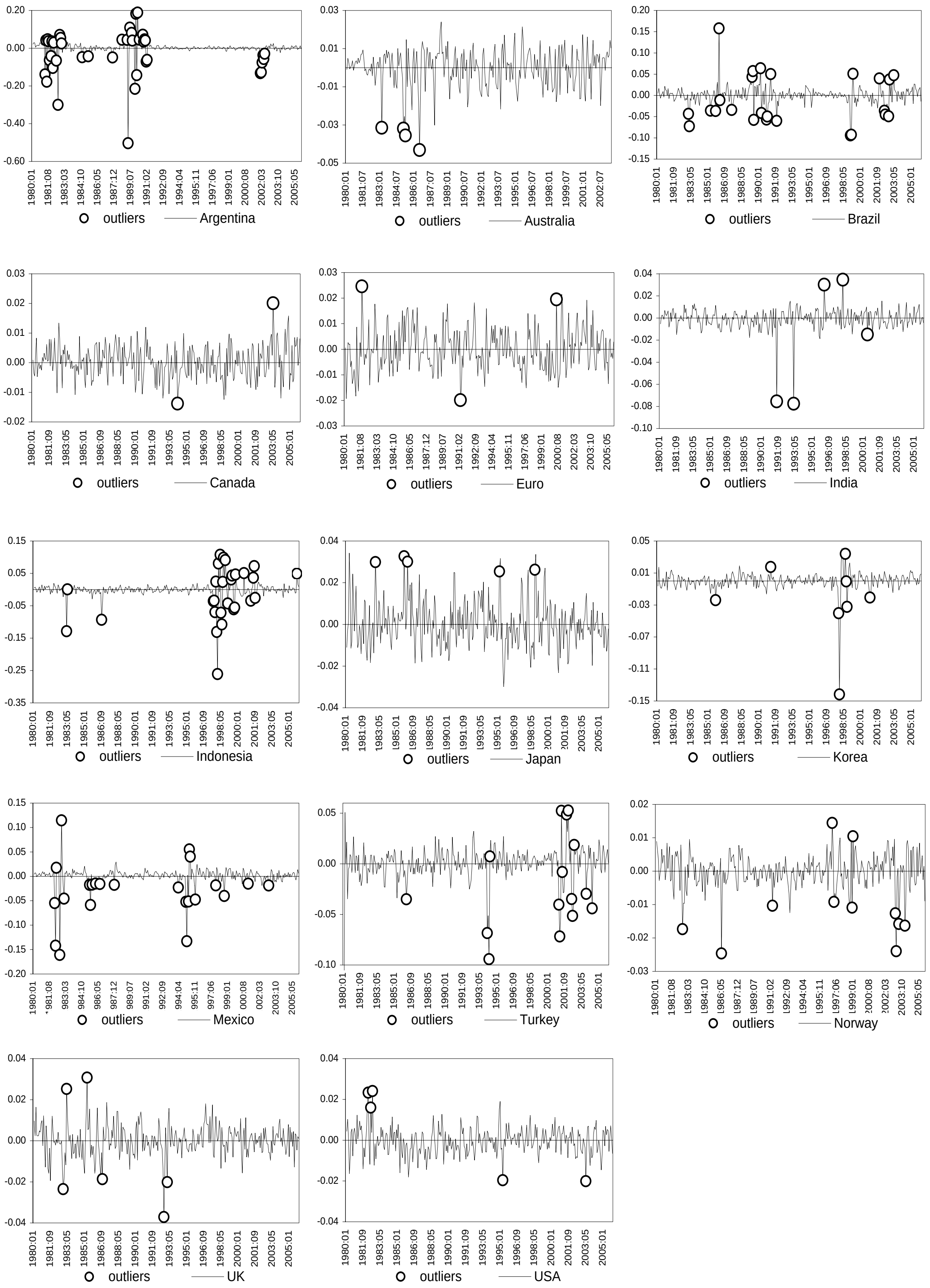
6.1 Standard linearity tests

Once we specified the linear AR(p) models, to test for real exchange rate linearity against LSTAR/ESTAR specification, we used the auxiliary regressions (11), (12), (13) and (14). In carrying out these tests we have considered values for the delay parameter, d, over the range $1 \leq d \leq 12$, and calculated the p-values for the LM_1, LM_3, LM_3^e and LM_4 standard and robust linearity tests in each case. We also considered the first lag of the level term and the time trend as possible transition variables. We varied the delay parameter in order to provide the strongest probability of non-linearity. If linearity was rejected for more than one value of d, then the

[12] In (21) i's N partners are exclusively the members of our group of countries, which had a competitiveness weight in i's trade equal or bigger than 2 percent, normalizing these weights to sum to one. For the weights we used data from Durand, *et al.* (1998) and correspond to the 1995 matrix.

[13] An increase (decrease) in a country's index indicates an appreciation (depreciation).

Figure 1: Monthly growth rates of the real effective exchange rates and outliers according to robust estimation



estimate of d was chosen by the lowest p-value of all (or most) of the LM-type tests. Thus, we chose the delay length for which non-linearity is strongest according to the tests.

The results of the tests suggest that in some cases, particularly for advanced economies, evidence against linearity in favor of STAR-form nonlinearity can be found just for a few number of the transition variables considered[14]. On the contrary, the evidence of non-linearity using the standard LM-type tests increases for developing countries. Second, the chosen transition variable also varies considerably among countries. Third, only for Canada, we could not find evidence of non-linearity.

6.2 Robust linearity tests

Next, we computed the observations that had large residuals in the OLS estimation of the restricted linear model (9). These observations, the outliers, receive a weight equal to zero in the robust estimation and are represented as circles in figures (1) and (2). As can be seen, outliers were found to exist in all the series with the number and timing varying across countries. Furthermore, while in developing countries outliers show up more or less in groups, indicating that there is a significant correlation between them, in advanced countries the isolated outliers are the main characteristic of the series.

Based on the outlier robust estimation procedure, we modified the previous standard LM-type tests. The outcome from these tests are particularly interesting. Certainly, once controlling for outliers, the evidence of nonlinearity is reduced considerably, particularly for advanced countries. Indeed, we found that for some countries results coincide in the sense that standard and robust based procedures both indicate the presence of nonlinearity for the same transition variables. Yet, in several cases the p-values for the robust tests are considerably larger than those from the OLS-based tests, indicating that the evidence of nonlinearity might be due to only the presence of a few outlying observations in several countries. Also, even when linearity can be rejected, care must be taken when choosing the transition variable.

6.3 Choice of the non-linear specification

Once we have tested for linearity, we followed the sequence of tests to discriminate between LSTAR and $LSTAR_2$ following the methodology by Terasvirta (1994) and Escribano and Jordá (1999). A summary of this procedure is presented in table (1) below. Contrary to conventional wisdom, the real exchange rates for all the advanced countries which rejected linearity (Australia, Euro and Norway) and for Mexico and Turkey, are better specified as LSTAR models. This suggests that the expansion and contraction phases of the exchange rates in these countries may

[14]These results, together with the robust tests are not presented here but are available upon request.

Figure 2: Outliers and standard transition functions

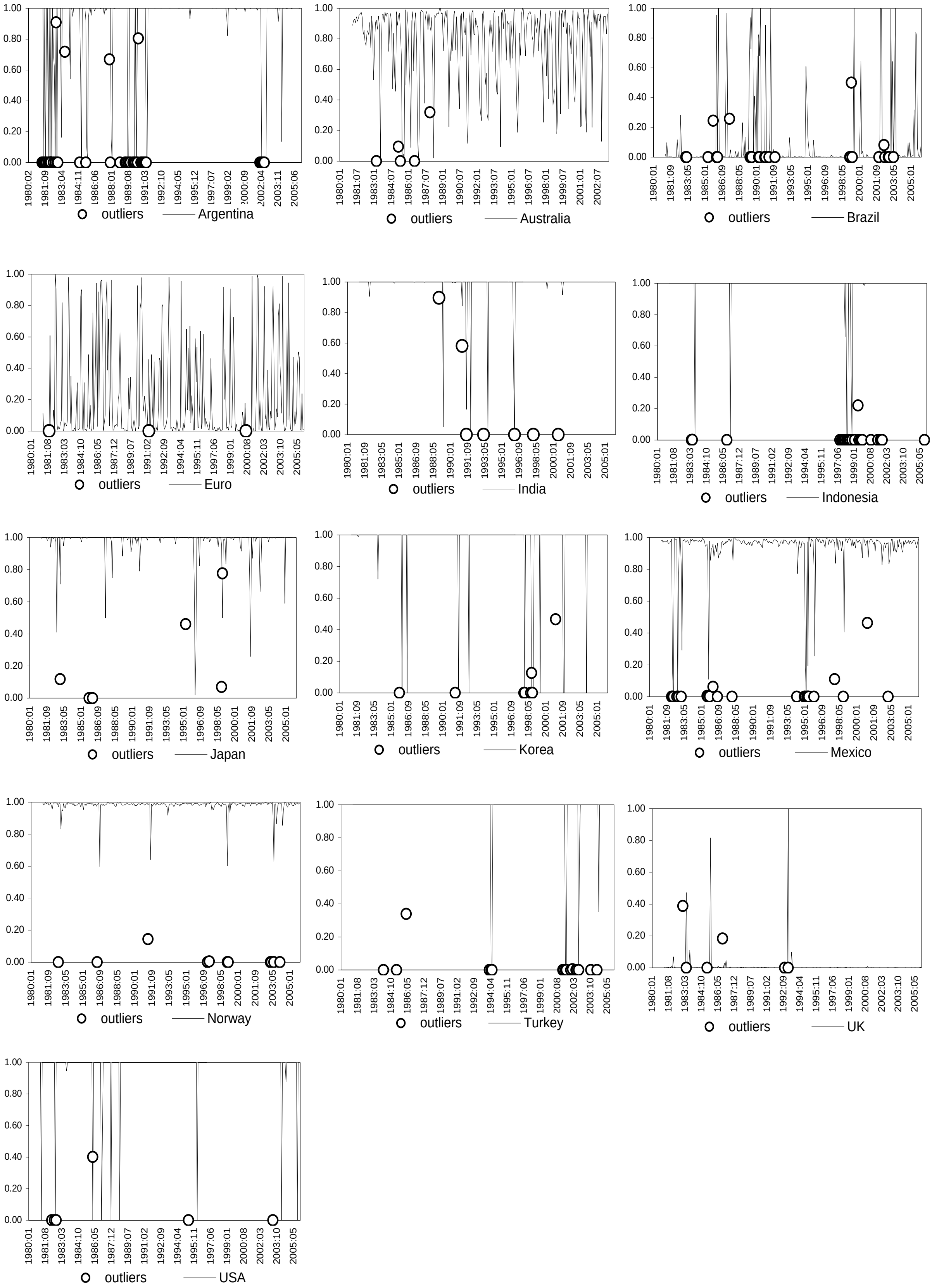


Table 1: **Model selection by standard and robust tests**

Country	p	Standard tests s_{t-d}	T	EJ	Outlier robuts tests s_{t-d}	T	EJ
Advanced countries:							
Australia	2	Δq_{t-4}	LSTAR	LSTAR	Δq_{t-4}	LSTAR	LSTAR
Canada	1		Linear	Linear		Linear	Linear
Euro	1	Δq_{t-7}	LSTAR	LSTAR	Δq_{t-7}	LSTAR	LSTAR
Japan	1	Δq_{t-6}	LSTAR	LSTAR		Linear	Linear
Norway	2	Δq_{t-3}	LSTAR	LSTAR	Trend	LSTAR	LSTAR
UK	2	Δq_{t-4}	LSTAR	ESTAR		Linear	Linear
US	2	Δq_{t-3}	ESTAR	LSTAR		Linear	Linear
Developing countries:							
Argentina	1	Δq_{t-2}	LSTAR	ESTAR	Δq_{t-11}	LSTAR	ESTAR
Brazil	2	Δq_{t-1}	LSTAR	LSTAR	Δq_{t-6}	ESTAR	ESTAR
India	1	Δq_{t-5}	ESTAR	LSTAR		Linear	Linear
Indonesia	1	Δq_{t-4}	ESTAR	LSTAR	Δq_{t-11}	LSTAR	ESTAR
Korea	3	Δq_{t-9}	ESTAR	ESTAR	Δq_{t-1}	LSTAR	ESTAR
Mexico	2	Δq_{t-1}	LSTAR	ESTAR	Δq_{t-4}	LSTAR	LSTAR
Turkey	1	Δq_{t-2}	LSTAR	LSTAR	Δq_{t-3}	LSTAR	LSTAR

Notes: (1) p is the number of autoregressive terms; (2) s_{t-d} is the transition variable, with d the delay parameter; (3) T are the sequence of tests suggested by Terasvirta (1994); (4) EJ are the tests by Escribano and Jordá (1999); (5) Δq_{t-j} is the first difference of the (log) real effective exchange rate.

have different dynamics. In contrast, results from the tests in the exchange rates of the rest of the countries are less conclusive. Based on the decision rule of the procedure of Terasvirta (1994), the outlier tests suggests that LSTAR models are appropriate for the candidate transition variables. Nevertheless, the results from the statistics used in the Escribano and Jordá (1999) procedure contradict this suggestion. Only for Brazil, there is no doubt that the real exchange rate can be better specify by a LSTAR$_2$ model, implying that exchange rate move from high or low levels towards the middle ground (or normal level) in a similar fashion.

Based on the previous results, we proceeded to estimate the LSTAR or LSTAR$_2$ models for the first difference of the (log) real effective exchange rate series. For this, we used non-linear least squares, which, as said before, provides estimators that are consistent and asymptotically normal[15]. Following the recommendation by Granger and Terasvirta (1993) we standardized the transition parameter (γ) by the sample variance or sample standard deviation of the transition variable and we used a grid-search that creates a linear grid in c and a log-linear grid in γ[16]. In order to obtain more parsimonious models, it is possible to reduce them by imposing certain restrictions. For example, when the intercept in the two regimes is similar in value, we can imposed $k = -k^*$ in (8). Also, if certain coefficients are not significant in the linear or non-linear part, they can be eliminated. We did this whenever it was possible, and chose the best fitted model according to information criterium.

[15]Tong (1990) shows that this condition hold for STAR models if series are stationary, ergodic and the error terms are independently and identically distributed.

[16]Note that the grid search is made for c_1 and c_2 in case LSTAR$_2$ was selected and only for c_1 in case of a LSTAR model. The grid search is performed for values of $0.5 \leq \gamma \leq 10$.

The first thing worth mentioning is that the STAR models offer an improvement over the linear specification in terms of the R^2 and the Schwarz Information Criterium (BIC)[17]. Also, the residual variances of the nonlinear models are just slightly smaller than those of the AR models. Both, the skewness and excess kurtosis are reduced in the STAR models, although normality of the errors is still rejected in most of the countries, probably due to the presence of outliers and residuals corresponding to currency crisis.

Second, the key parameters in the STAR models are γ and c. At this respect it is interesting to notice that though the transition parameter varies across countries it is, in general, higher in developing countries than in advanced countries (see table 2). This suggests that the transition from one regime to the other is rather slow for the latter group and much faster for developing countries. However, it would be a mistake to judge the significance of this variable by means of its t-value. In fact, as noticed by Terasvirta (1994), the t-statistic of γ does not have its customary asymptotic t-distribution under the hypothesis that $\gamma = 0$, due to identification problems[18]. Therefore, we could not conclude, for example, that there is evidence of nonlinearities based on finding a transition parameter, γ, negative and significantly different from zero at standard levels.

On the other side, the estimated parameter c indicates the half way point between appreciation and depreciation phases of the real exchange rates. As shown, not all the estimated parameters are in the neighborhood of the sample mean, implying that the observations are not always distributed equally between the left-hand and the right-hand tails of their respective functions.

The previous statements can be seen in figure (3) which shows the transition function $F(s_{t-d}, \lambda, c)$ against the transition variable in the analyzed period. The estimate of the parameters (γ) and c are such that the change from the functions from 0 to 1 takes places for very different values and ranges of the transition variable. For instance, in Mexico the function changes from 0 to 1 for values of the transition variable between 0.007 to 0.016, while for the Euro the change is smoother, between -0.005 to 0.0192. Notice also that in Argentina, Euro, Mexico and Norway, most of the observations are in the lower or middle regime (i.e. the linear part). On the contrary, the maintenance of this function in its maximum value during most of the period is significant for Brazil, Korea and Turkey, indicating that the RER in these cases are usually far from its equilibrium level, with a tendency to mean revert[19].

[17] The models are available upon request.

[18] Under the null hypothesis, each of the real exchange rates follows a unit root process. Hence, the presence of a unit root under the null hypothesis complicates the testing procedure.

[19] Of course, this reversion can be more or less abrupt depending on the country.

Figure 3: Transition function versus transition variable

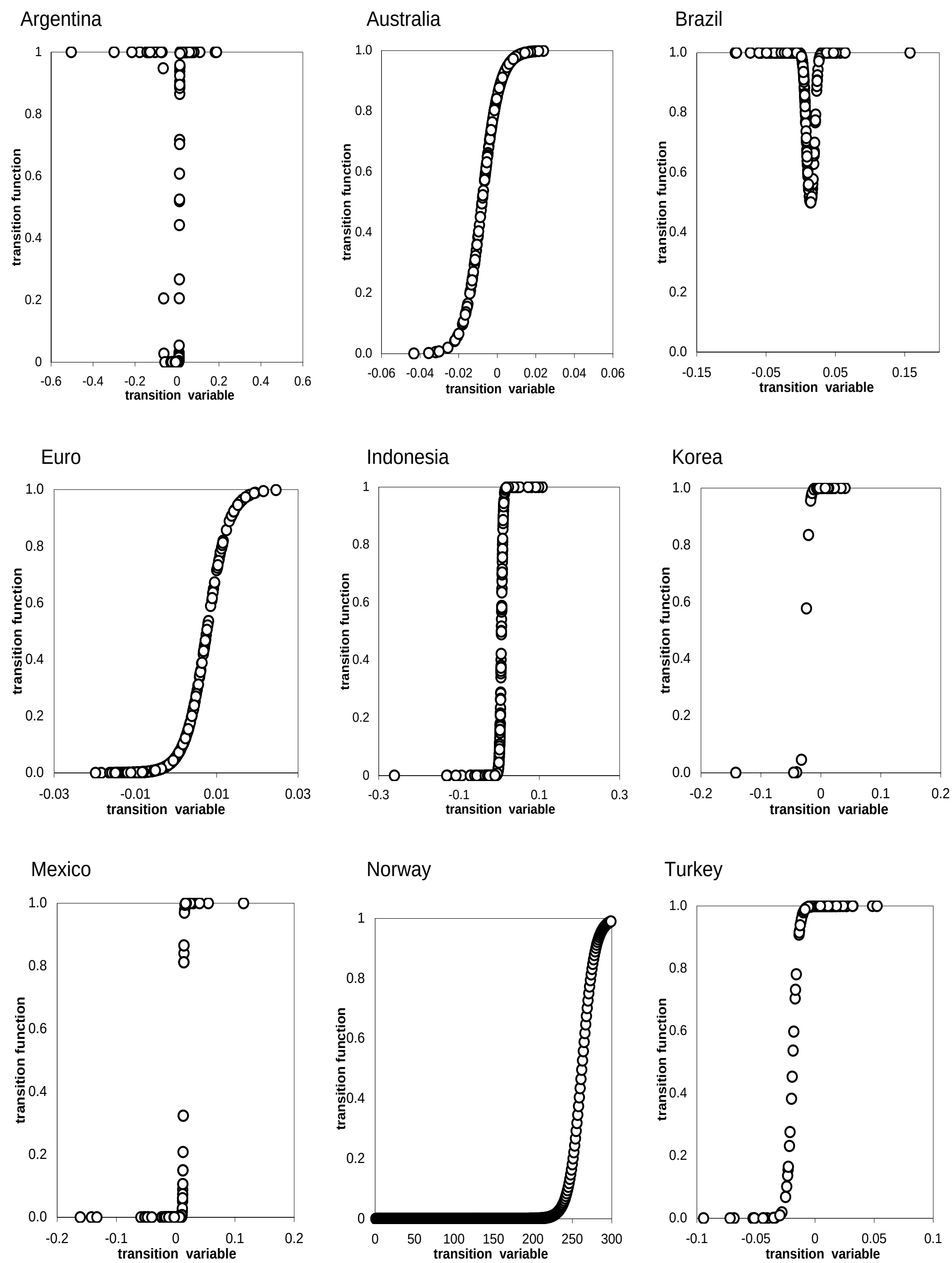


Table 2: Selected models and estimated key parameters

	Argentina		Australia		Brazil	
	Standard	Robust	Standard	Robust	Standard	Robust
Model	LSTAR	ESTAR	LSTAR	LSTAR	LSTAR	ESTAR
$\hat{\lambda}$	-0.196	0.001	0.035	0.035	-0.015	-0.102
$\hat{\lambda^*}$	0.168	-0.061	-0.055	-0.055	-0.028	0.094
$\hat{\gamma}$	-26.19	-54.99	2.090	2.090	14.93	11.14
$\hat{c}$	-0.019	-0.064;	-0.007	-0.007	0.050	0.015
		0.012				0.014
	Euro-zone		Indonesia		Korea	
	Standard	Robust	Standard	Robust	Standard	Robust
Model	LSTAR	LSTAR	ESTAR	ESTAR	ESTAR	LSTAR
$\hat{\lambda}$	-0.008	-0.008	-0.130	0.182	0.001	-2.325
$\hat{\lambda^*}$	-0.030	-0.030	0.117	-0.227	-1.208	2.315
$\hat{\gamma}$	2.976	2.976	-251.549	2.976	23.39	4.747
$\hat{c}$	0.008	0.008	-0.048	0.023	-0.009	-0.024
			-0.021	0.023	-0.013	
	Mexico		Norway		Turkey	
	Standard	Robust	Standard	Robust	Standard	Robust
Model	LSTAR	LSTAR	LSTAR	LSTAR	LSTAR	LSTAR
$\hat{\lambda}$	-0.662	-0.011	-1.060	-0.013	-0.046	-0.097
$\hat{\lambda^*}$	0.662	0.045	1.060	-0.120	0.026	0.071
$\hat{\gamma}$	30.204	39.84	0.838	10.738	46.845	6.863
$\hat{c}$	-0.023	0.013	-0.025	262.123	0.002	-0.019

Notes: (1) Standard and robust correspond to the estimated parameters according to the standard and robust linearity tests; (2) $\hat{\lambda}$ and $\hat{\lambda^*}$ are the estimated coefficients of the lagged level in the linear and nonlinear parts, respectively; (3) $\hat{\gamma}$ is the estimated transition parameter; (4) $\hat{c}$ is the estimated half-way point between the two regimes

6.4 Dynamic properties

Once we estimated the STAR models, we can deduce their dynamic properties through the roots of the characteristic polynomial. Table (3) shows the most prominent roots for each regime. Broadly speaking, most of the countries are characterized by complex roots in the upper (or outer) or in the lower (middle) regime, which implies that real exchange rates in these cases display cyclical movements during contraction (depreciation) and expansion (appreciation) phases.

In addition, the models for several countries are stable in both regimes. Yet, in Brazil, Indonesia and Korea, the characteristic polynomials of the the lower (or middle) regime contain a large explosive root, whereas the upper (outer) regimes are stationary. Quite the opposite, the largest modulus for the upper regime in Mexico is also not far from the unit circle. This suggests that the real effective exchange rates for these countries move very aggressively from a low level into a higher growth rate whereas once they are in the expansionary phase they will tend to remain there.

In contrast, the rest of the countries are completely characterized by stable roots in the outer and middle regime. This implies that there is nothing in the dynamics of the regimes to suggest

Table 3: **Roots of the characteristic polynomial in each regime**

Country	Regime	Root(s)	Modulus
Argentina	O	-0.3095	0.3095
	M	0.1213	0.1213
Australia	U	0.3217 ± 0.4604i	0.5619
	L	-0.2731± 0.2524i	0.3718
	L	-0.2532	0.2532
Brazil	O	0.1862± 0.2831i	0.3388
	M	-2.1820	2.1820
	M	0.6594	0.6594
Euro	U	0.5348	0.5348
	L	0.2103	0.2103
Indonesia	O	-0.0512 ± 0.2062i	0.2125
	M	1.4417	1.4417
Korea	U	0.595 ± 0.4561i	0.5808
	U	-0.1378 ± 0.1065i	0.1742
	L	1.7104	1.7104
	L	1.6666	1.6666
Mexico	U	-0.0457 ± 0.9216i	0.9227
	L	0.1984 ± 0.1682i	0.2601
Norway	U	0.3443 ± 0.5614i	0.6586
	U	-0.3078	0.3078
	L	0.2128 ± 0.1706i	0.2727
Turkey	U	0.5316	0.5316
	L	-0.0446 ± 0.3088i	0.312

(1) only dominant roots are shown; (2) U: upper regime in LSTAR (F=1), L: lower regime in LSTAR (F=0), O: outer regime ESTAR (F=1), M: middle regime ESTAR (F=0)

a rapid change from one to the other. Therefore, only large shocks could cause these changes.

7 Conclusions

The emerging literature on real exchange rate determination and purchasing power parity suggests that persistent misalignments in the real exchange rates can be explained by the presence of a nonlinear adjustment process toward the long-run equilibrium. In simple words, this means that the real exchange rate can have different characteristics according to the size of the deviations from PPP.

While it is true that for a number of countries the real exchange rates are well characterized by a process which adjust nonlinearly toward its long-run equilibrium, this is not always the case. Indeed, we showed evidence that, in some cases, aberrant observations in the series render the standard nonlinear tests to point towards non-linear structures when there are none. Even when the real exchange rate can be characterized by a nonlinear model, the presence of these outliers can lead to wrong decisions, particularly in terms of the transition variable. Therefore, care must be taken when modeling nonlinearities.

According to this, we performed robust linearity tests, finding partial evidence of nonlinearity for 9 out of 14 countries. Particularly, we were not able to reject linearity in most of the advanced countries of our sample. For the real effective exchange rates that had some evidence of nonlinearity, we estimated STAR type-models for the period 1980:01 to 2005:12.

Most of the exchange rates were found to be better classified by logistic STAR models, implying that the expansion and contraction phases of the exchange rates may have different dynamics. Although the estimated STAR models perform better than the linear autoregressive models, in several cases we failed to obtain gaussian residuals, probably due to the outlier observations.

The estimates of the transition parameter indicate that the speed of transition from one exchange rate regime to the other is slower for advanced countries than for the developing ones. This may imply that sizeable over or under valuations adjust more brusque in developing countries. The abrupt devaluations and financial crisis experienced in Asian and Latin American countries can be examples of a situation when changes from one regime to the other occur in a more aggressive way. Instead, adjustment in industrial countries is smooth.

Regarding the dynamics, the exchange rates are characterized by cyclical movements mainly in the upper or outer regime and less often in the lower or middle regime. The fact that in most of the countries the upper regime (i.e., when the real exchange rate is far from its equilibrium value) is stable, suggests that once the exchange rate is in this phase, it will tend to stay there for long periods of time. That is, when the real exchange rate is far from equilibrium, convergence to long-run PPP is rather slow. There is nothing in the dynamics of the upper regime to suggest a rapid fall into a contraction. Only sufficiently large shocks could cause this.

We also found that in Brazil, Indonesia and Korea, exhibit explosive roots in the lower regime, entailing that the real exchange rate in these countries moves abruptly from low levels to high rates of appreciation. It is interesting to notice that countries that have been affected by financial crisis exhibit explosive behavior in the lower (or middle) regime.

It must be added that, while non-linear models represent an improvement over autoregressive linear models in some cases, questions remain unanswer. Particularly, investigating the properties of STAR models in the presence of outliers may be a major area of future research. In connection with this, it would also be interesting to study the properties of the linearity tests due to different types of outliers (additive, innovative, correlated outliers, etc.).

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