

Endogenous Timing in a Mixed Duopoly: Price Competition

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The endogenous order of moves is analyzed in a mixed duopoly for differentiated goods, where firms choose whether to set prices sequentially or simultaneously. It is shown that, in contrast to the private duopoly where firms set prices sequentially, in the mixed duopoly firms choose prices simultaneously. Moreover, the result obtained in the mixed duopoly under price competition differs from the one under quantity competition, since in the latter case decisions are taken sequentially.

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1 Introduction

Pal (1998) points out that whether firms take decisions sequentially or simultaneously is important, since a sequential order of moves may give rise to significantly different results. However, the literature on mixed oligopoly takes the order of moves as exogenously given. This paper addresses the issue of the endogenous order of moves in a mixed duopoly by considering the observable delay game of Hamilton and Slutski (1990).

It is well known that strategic effects may arise when firms take decisions sequentially. The literature on industrial organization has analyzed whether firms take decisions sequential or simultaneously (see, for example, Gal-Or, 1985; Dowrick, 1986; Hamilton and Slutsky, 1990). This literature shows that in a private duopoly, if both firms have upward-sloping reaction functions and one firm prefers to be the leader, the other firm must prefer to be the follower; in this case, firms take decisions sequentially. If both firms have downward-sloping reaction functions, each firm prefers being the leader to being a follower. As a result, firms take decisions simultaneously.

The above issue has been considered by the literature on mixed duopoly only when firms decide quantities and the product is homogeneous (in this case, firms have downward-sloping reaction functions). Private firms maximize profits and public firms maximize social welfare. Pal (1998) shows that when firms produce a homogeneous good, they decide quantities sequentially in a mixed duopoly.¹ However, in a private duopoly firms decide quantities simultaneously.

There are few papers analyzing the case in which firms decide prices. Anderson *et al.* (1997) consider a mixed oligopoly with product differentiation and price competition and study whether or not there is a welfare loss when a public firm is privatized, both in the short run and in the long run. Tasnádi (2006) considers a model in which some firms use their price while the remaining

¹ Matsumura (2003) and Lu (2006) extend this analysis to consider that the public firm competes with foreign private firms. They also obtain that decisions are taken sequentially.

firms use their quantity as a decision variable. He assumes that the choice of the decision variable can be endogenized. Ogawa and Kato (2006) consider a duopoly model in which firms have quadratic cost functions, produce homogeneous goods and compete à la Bertrand. They compare the prices set by firms when prices are set sequentially and simultaneously.

Under price competition with substitute goods, firms have upward-sloping reaction functions. Considering a mixed duopoly where the firms decide the timing of choosing their prices, it is shown that the public and private firms set prices simultaneously since it is a dominant strategy for the two firms to set prices as soon as possible.² However, in a private duopoly where both firms maximize profits, firms set prices sequentially. In this case, if one firm wants to be the leader the other firm must prefer to be the follower. Therefore, the results obtained in a mixed duopoly and in a private duopoly are different. On the other hand, it has to be noted that in a mixed duopoly the result obtained under price competition is different from that obtained under quantity competition. In this second case, firms take decisions sequentially while in the first case decisions are taken simultaneously.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes whether firms set prices sequentially or simultaneously, and conclusions are drawn in Section 4.

2 The model

² As the two firms have the same constant marginal cost of production, if the product is homogeneous the price equals the marginal cost regardless of whether the firms set their prices simultaneously or sequentially (and of whether the public firm is the leader or the follower). Therefore, firms are indifferent as to whether to set prices sequentially or simultaneously. As a result, when the two firms sell a homogeneous product, equilibrium in the Bertrand model is quite different from equilibrium in the Cournot model. To avoid the trivial solution arising under the assumption of homogeneous product I consider heterogeneous goods in this paper.

I consider an economy made up of a monopolistic sector and a competitive numeraire one. The monopolistic sector comprises one public firm, 0, and one private firm, 1. On the consumption side, there is a continuum of consumers of the same type whose utility function is linear and separable in the numeraire good. The representative consumer maximizes $U(q_0, q_1) - p_0q_0 - p_1q_1$, where $q_i \geq 0$ is the amount of the good i and p_i is its price ($i = 0, 1$). The function $U(q_0, q_1)$ is assumed to be quadratic, strictly concave and symmetric in q_0 and q_1 :

$$U(q_0, q_1) = a(q_0 + q_1) - \frac{1}{2}((q_0)^2 + 2bq_0q_1 + (q_1)^2), \quad 1 > b > 0,^3$$

where parameter b measures the degree to which goods are substitutes. Then, demand functions are given by:

$$q_i = \frac{a(1-b) - p_i + bp_j}{1-b^2}, \quad i \neq j; \quad i, j = 0, 1. \quad (1)$$

The marginal cost of production of both firms is c . The profit function of firm i is given by:

$$\pi_i = (p_i - c) q_i, \quad i \neq j; \quad i, j = 0, 1, \quad (2)$$

where q_i is given by (1). The private firm chooses the price, p_1 , that maximizes its profit. The public firm chooses the price, p_0 , that maximizes social welfare. I measure social welfare as the sum of consumer surplus (denoted by CS) and producer surplus (denoted by PS). Therefore, social welfare is given by:

$$W = CS + PS, \quad (3)$$

³ I consider a simplified version of the model used by Vives (1984). Following Vives, I assume that $b < 1$ to ensure that the function $U(q_0, q_1)$ is strictly concave.

where $PS = p_0 + p_1$ and consumer surplus is given by:

$$CS = U(q_0, q_1) - p_0 q_0 - p_1 q_1 = \frac{2a^2(1-b) + (p_0)^2 - 2bp_0p_1 + (p_1)^2 - 2a(1-b)(p_0 + p_1)}{2(1-b^2)}. \quad (4)$$

I propose a two stage game with the following timing. In the first stage, the private and public firms decide whether to set prices at $t=1$ or at $t=2$. Therefore, I consider the observable delay game of Hamilton and Slutsky (1990) in the context of a price setting mixed duopoly where the firms choose when to set their prices. If both firms choose their prices in the same period, firms take decisions simultaneously. If one firm sets its price at $t=1$ and the other at $t=2$, the game is sequential. In the second stage, the firms decide their prices either sequentially or simultaneously. To obtain a subgame perfect equilibrium, the game is solved backwards.

3 Results

As a benchmark case, I first solve the case in which both firms are private (see Hamilton and Slutsky, 1990).

3.1 Private duopoly

I denote the case in which firms set prices simultaneously by superscript S . When firms decide prices sequentially, superscript L (F) denotes the firm leader (follower) in prices.

I first solve the case in which prices are set simultaneously. Each firm chooses the value of p_i that maximizes (2).⁴ Solving this problem, I obtain that the price set by each firm and its profit are:

⁴ It is easy to see that reaction functions in prices are upward-sloping.

$$p^S = \frac{a(1-b) + c}{2-b}, \mathbf{p}^S = \frac{(1-b)(a-c)^2}{(1+b)(2-b)^2}.$$

Next I consider the case in which the firms decide their prices sequentially and firm i decides its price first. The follower j chooses the value of p_j that maximizes its profit. Solving this problem, I get:

$$p_j = \frac{1}{2}(a(1-b) + c + bp_i), \quad i \neq j; i, j = 0, 1. \quad (5)$$

The leader i chooses the value of p_i that maximizes its profit taking (5) into account. Solving this problem, I get:

$$p^L = \frac{a(1-b)(2+b) + c(1+b)(2-b)}{2(2-b^2)}, \quad p^F = \frac{a(1-b)(4 + (2-b)b) + c(4 + b(2-b-b^2))}{4(2-b^2)},$$

$$\mathbf{p}^L = \frac{(1-b)(2+b)^2(a-c)^2}{8(1+b)(2-b^2)}, \quad \mathbf{p}^F = \frac{(1-b)(4 + 2b - b^2)^2(a-c)^2}{16(1+b)(2-b^2)^2}.$$

A comparison of the results obtained in the simultaneous and sequential cases reveals that $p^L > p^F > p^S$ and $\mathbf{p}^F > \mathbf{p}^L > \mathbf{p}^S$. Given that prices are strategic complements (i. e. reaction functions are upward-sloping) the leader firm sets a higher price than the follower and both firms set a higher price than in the simultaneous case. As a result, market competition in the sequential setting is lower than in the simultaneous game. The follower firm obtains greater profits than the leader and both firms obtain greater profits than in the simultaneous game and thus firms will choose prices sequentially.

3.2. Mixed duopoly

Given that firms may decide prices sequentially or simultaneously, we have three cases. First, the public firm and the private one decide prices simultaneously. Second, the public firm decides its

price before the private firm does. Finally, the private firm decides its price before the public firm does.

I first solve the case in which the prices are set simultaneously. The private firm chooses the value of p_1 that maximizes (2), for $i=1$ and $j=0$. The public firm chooses the value of p_0 that maximizes (3). Solving these two problems simultaneously, I obtain the reaction functions in prices of the two firms:

$$p_1 = \frac{1}{2}(a(1-b) + c + bp_0). \quad (6)$$

$$p_0 = c(1-b) + bp_1. \quad (7)$$

From (6) and (7) the following is obtained:

$$p_0^S = \frac{ab(1-b) + c(2-b)}{2-b^2}, \quad p_1^S = \frac{a(1-b) + c(1+b-b^2)}{2-b^2}, \quad \mathbf{p}_0^S = \frac{(1-b)b(a-c)^2}{(1+b)(2-b^2)},$$

$$\mathbf{p}_1^S = \frac{(1-b)(a-c)^2}{(1+b)(2-b^2)^2}, \quad CS^S = \frac{(5-b-3b^2+b^3)(a-c)^2}{2(1+b)(2-b^2)^2},$$

$$PS^S = \frac{(1-2b^2+b^3)(a-c)^2}{(2-b^2)^2}, \quad W^S = \frac{(7+b-7b^2-b^3+2b^4)(a-c)^2}{2(1+b)(2-b^2)^2}.$$

Next I consider the case in which the firms decide their prices sequentially and the public firm decides its price first. The private firm chooses the value of p_1 that maximizes (2) for $i=1$ and $j=0$. Solving this problem, I obtain equation (6). The public firm chooses the value of p_0 that maximizes (3) taking into account (6). Solving this problem, I get:

$$p_0^L = \frac{ab(1-b) + c(4-b-2b^2)}{4-3b^2}, \quad p_1^F = \frac{a(1-b)(2-b^2) + c(2+2b-2b^2-b^3)}{4-3b^2},$$

$$\mathbf{p}_0^L = \frac{(4-3b-4b^2+2b^3+b^4)b(a-c)^2}{(1+b)(4-3b^2)^2}, \quad \mathbf{p}_1^F = \frac{(1-b)(2-b^2)^2(a-c)^2}{(1+b)(4-3b^2)^2},$$

$$CS^L = \frac{(5+b-3b^2-b^3)(a-c)^2}{2(1+b)(4-3b^2)}, \quad PS^L = \frac{(1-b)(a-c)^2}{4-3b^2},$$

$$W^L = \frac{(7+b-5b^2-b^3)(a-c)^2}{2(1+b)(4-3b^2)}.$$

Finally, I consider the case in which firms decide prices sequentially and the private firm decides its price first. The public firm chooses the value of p_0 that maximizes (3). Solving this, I obtain equation (7). The private firm chooses the value of p_1 that maximizes (2) for $i=1$ and $j=0$, taking into account (7). Solving this, I get:

$$p_0^F = \frac{ab+c(2+b)}{2(1+b)}, \quad p_1^L = \frac{a+c(1+2b)}{2(1+b)}, \quad p_0^F = \frac{(a-c)^2 b}{2(1+b)^2}, \quad p_1^L = \frac{(a-c)^2}{4(1+b)^2},$$

$$CS^F = \frac{(5+4b)(a-c)^2}{8(1+b)^2}, \quad PS^F = \frac{(1+2b)(a-c)^2}{4(1+b)^2}, \quad W^F = \frac{(7+8b)(a-c)^2}{8(1+b)^2}.$$

From the results obtained in the three cases, the following results are obtained.

Proposition 1: In equilibrium,

- i) $p_1^L > p_0^F$, $p_1^F > p_0^L$, $p_1^S > p_0^S$, $p_1^L > p_1^S > p_1^F$ and $p_0^F > p_0^S > p_0^L$.
- ii) $PS^F > PS^S > PS^L$ and $CS^L > CS^S > CS^F$.
- iii) $W^L > W^S > W^F$, $p_1^L > p_1^S > p_1^F$ and $p_0^F > p_0^S > p_0^L$.

This proposition shows that the private firm sets higher prices than the public firm in all cases ($p_1^L > p_0^F$, $p_1^F > p_0^L$, $p_1^S > p_0^S$). This is due to the fact that the public firm maximizes social welfare, and thus takes the consumer surplus into account. By setting a lower price it produces more than the private firm. In the private duopoly, when prices are set sequentially both the leader and the follower set higher prices than in the simultaneous setting and the leader sets a higher price than the follower. This is due to the fact that the reaction functions are upward sloping and private firms want to reduce market competition. The private firm in the mixed duopoly has the

same incentives as in the private duopoly. However, as the public firm cares about the consumer surplus, it seeks to increase market competition. When the private firm is the leader in prices, it raises its price (in comparison to the simultaneous case) since it knows that the follower (the public firm) will raise its price too, because prices are strategic complements. When the public firm is the leader in prices, it lowers its price (in comparison to the simultaneous case) since it knows that the follower (the private firm) will lower its price too because prices are strategic complements. As a result, when the private firm is the leader it sets a higher price than when decisions are set simultaneously and, in this last case, its price is higher than when it is the follower ($p_1^L > p_1^S > p_1^F$). When the public firm is the follower it sets a higher price than when decisions are set simultaneously and, in this last case, its price is higher than when it is the leader ($p_0^F > p_0^S > p_0^L$).

Market competition is greater when the public firm is the leader in prices than otherwise. In this case, the two firms set lower prices than otherwise and thus the producer surplus is lower and the consumer surplus higher. This means that when the public firm is the leader in prices, both firms obtain lower profits and social welfare is greater than when it is the follower or prices are set simultaneously.

Market competition is lower when the public firm is the follower in prices than otherwise. In this case, both firms set higher prices than otherwise and thus the producer surplus is higher and the consumer surplus is lower. This means that both firms obtain higher profits and social welfare is lower when the public firm is the follower than when it is the leader or prices are set simultaneously.

Taking into account proposition 1, the following result is obtained.

Proposition 2: In equilibrium, firms decide prices simultaneously at $t=1$.

From Proposition 1 it is obtained that it is a dominant strategy for each firm to set prices at $t=1$. Thus, both firms want to be leaders in prices: the greatest profit for the private firm and the greatest social welfare are obtained in this case. By contrast, neither of the two firms wants to be the followers in prices: the lowest profit for the private firm and the lowest social welfare are obtained in this case. As a result, the two firms decide prices simultaneously at $t=1$. Therefore, a different result is obtained from when the two firms are private, where firms decide prices sequentially.

4 Conclusions

This paper studies a setting where firms decide about prices rather than quantities and goods are substitutes. Considering a mixed duopoly where the firms choose whether to set prices sequentially or simultaneously, it is shown that they set prices simultaneously. However, in a private duopoly firms set prices sequentially. On the other hand, in a mixed duopoly, the result obtained under price competition is different from that obtained under quantity competition. In this second case, firms take decisions sequentially while in the first case decisions are taken simultaneously. In this paper it is assumed that public and private firms have the same marginal costs of production. It can be shown that the results obtained here hold if the private firm is more efficient than the public firm.

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