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THE ECONOMIC PRODUCTION FUNCTION.
WAGES AND GROWTH

MARIANO ALIERTA

REAL SOCIEDAD ECONOMICA ARAGONESA

JEL O 47

Mariano Alierta
Zaragoza
m.alierta@terra.es

THE ECONOMIC PRODUCTION FUNCTION. WAGES AND GROWTH

The Economic Production Function describes the relation between the output produced, measured in terms of value, the amounts and the prices of the resources employed in its production, that is, between the value of production and the value of the resources employed. It is a relation, $Y = f(L, K, w, i)$, between the value of output and the amount of labour and capital, wages and interest. The simplest form of this relation is $Y = Lw + Ki$.

There are reasons for a relation of this type. In the long run there have been increases in the amount of labour and capital. But also there has been an increase of wages which affects also output in the long run. Wages have been growing in the long run in economies that have had technical change. Economic historians have collected data which show this process of growth of wages, which started about two hundreds of years ago in those countries that initiated technical change and afterwards in others. The statistical data in the last decades show an important growth of wages in many countries (Note 1). The price of capital tends, instead, to stay around a certain value in the long term and does not affect greatly the growth of output.

The paper develops these points: first the definition of the Economic Production Function and then describes the evolution of the resources and of their prices in the United States from 1950. It is explained, afterwards, how the economic production function complies with the data of the US economy. The paper continues with the relations deduced from the Economic Production Function that express the increase of production and the rate of increase of production. These relations allow the identification of the sources of growth and estimate their importance in the growth of the US economy.

In the second part of the paper it is briefly explained that there is a close relation between wages and growth. Growth is the result of the variation in the amount of resources, capital and labour, and also the consequence of the variation of wages through time. The growth of wages is an important part of the total growth of the economy. The

differences in the process of evolution of wages help to understand the differences in the development of the different countries

This approach does not eliminate the influence of technology in growth. As briefly stated above, technology influences the process of rising wages (in a sufficiently closed economy). Therefore technology also influences growth through the process of rising wages which it stimulates.

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I. THE ECONOMIC PRODUCTION FUNCTION

1. THE RELATION

The output produced can be measured in terms of the volume of resources employed and of the price of the resources employed. If we take labour (L) and capital (K) as the factors of production and their prices, wages (w) and interest (i), the relation is like

$$Y = Lw + Ki,$$

In this Incomes Relation, Y is the National Income, which has the known relation to the Net National Product, and therefore can be a measure of output

In this relation the amount of labour, -L-, is expressed in average annual number of workers if we take annual data and -w- is annual wages (or salaries) of employees in monetary units. It could also -L- be expressed in annual hours of labour and wages and salaries in the compensation per hour. The amount of capital, -K-, is expressed in monetary units and -i-, interest rate, is a percentage. This rate of interest, -i-, is an average of the retribution of capital which groups all the perceptions that receives the different classes of capital

All the elements of this relation are expressed in value per time. Labour (L) is a number, wages (w) are value per time, therefore -Lw- is value per time. Capital (K) is value, interest (i) is percentage per time, then -Ki- is also value per time. As -Y- is value per time all the three elements -Y-, -Lw- and -Ki- are value per time. It is a relation homogeneous in terms of units. As the unit used is value per time it can be considered an economic production function. (See Appendix I).

2. EVOLUTION OF THE RESURCES AND THEIR PRICES IN THE US ECONOMY.

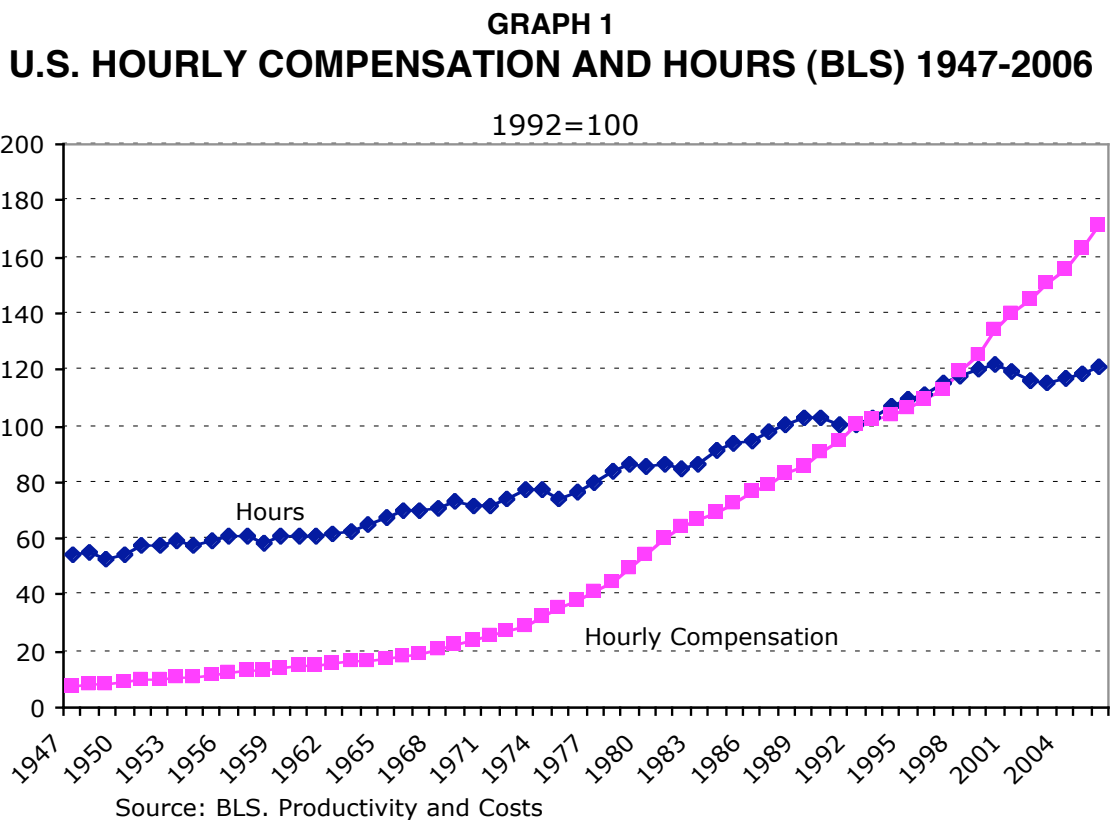
To describe the evolution of the variables used in the economic production function, it is employed data from the US in the last fifty years. In the long run labour, capital and wages have grown while

interest has remained stable. It is described the evolution of each one.

a) Labour and Wages

In the long run both nominal hourly compensation and the hours of work have increased.

Graph 1 shows the nominal hourly compensations of employees and the number of hours of work in the US, 1947-2006, provided by, Bureau of Labour Statistics, Department of Labour, USA, Productivity and Costs Series. From 1947 the nominal hourly compensation has grown in the US much quickly than the number of hours of works. All the curves are indexes (1992 = 100)

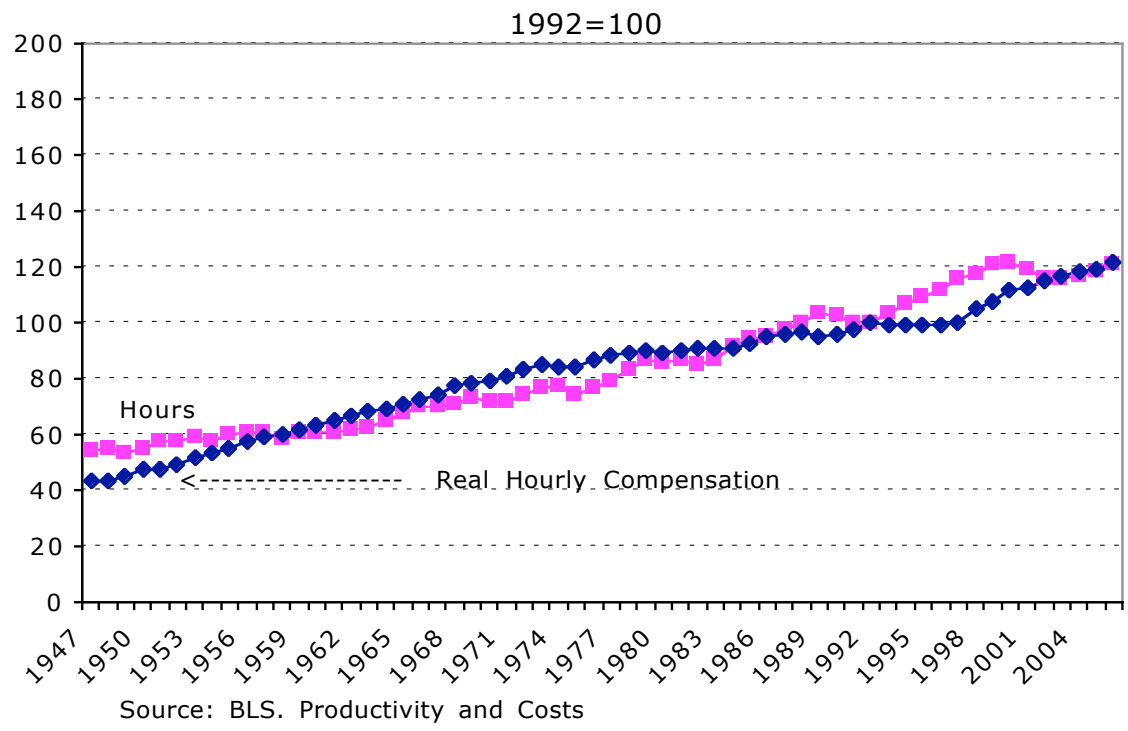


Although the short run it is not considered in this paper it can be of interest to say that the amounts of labour, capital and level of wages usually grow although in some periods they can stay stable and in

some cases employment, and less frequently wages, can become smaller. Their movements are more volatile than in the long run.

In real terms the hourly compensation has also grown in the US from 1947. The Graph 2 shows the real hourly compensation in 1947 - 2006 (BLS, Productivity and Costs). The growth of real hourly compensation has been in the US bigger than the growth of hours.

GRAPH 2
U.S. REAL HOURLY COMPENSATION AND HOURS (BLS) 1947-2006



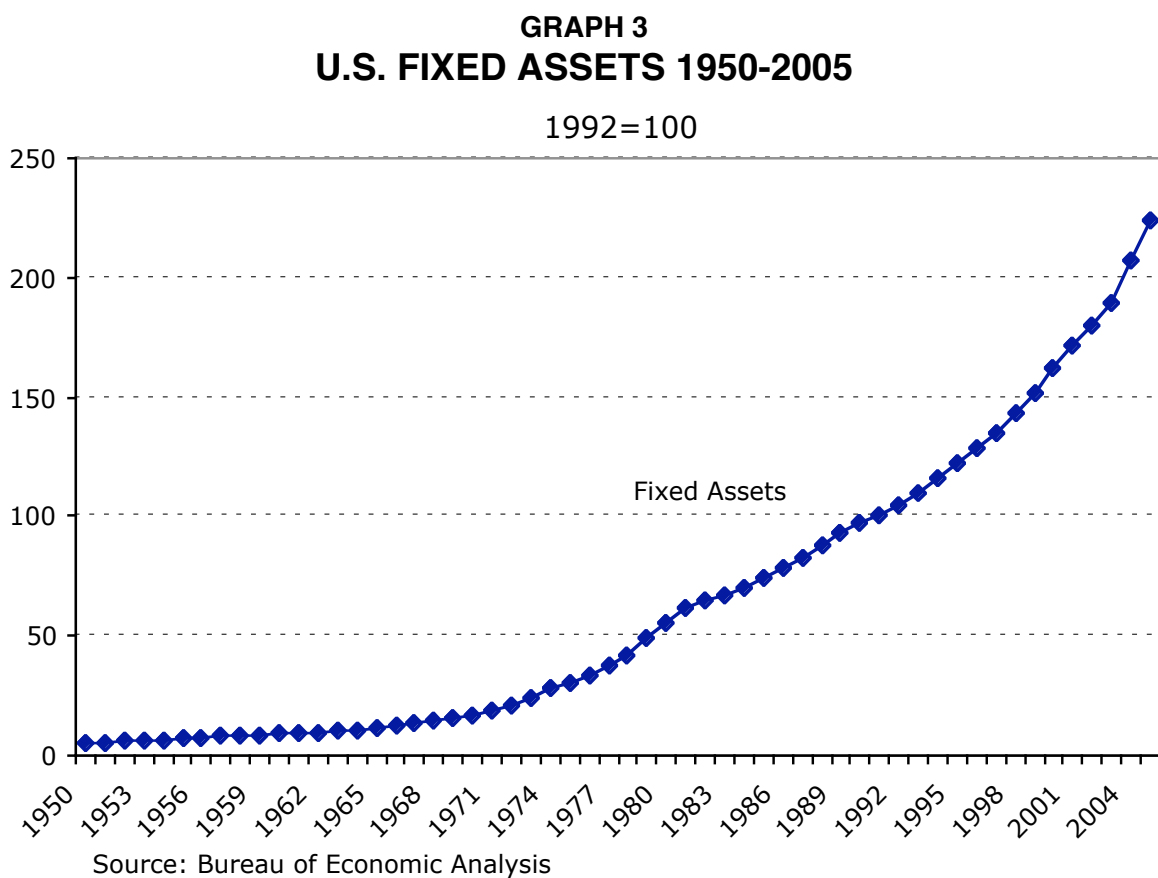
Normally the growth of the employment (labour) has been considered. But as it is possible to see from the previous two graphs the growth of wages has been even more important even in real terms than the growth of employment. The growth of wages has a long tradition in economies with technical change.

It is worth to point that the hourly compensation and the number of hours reflected in these graphs are only a part of the total number of hours of US economy. It is, however, an important and representative sample.

In real terms, wages also grow but sometimes remains constant. Rarely they decreases (See Appendix 2)

b). Capital

There is also growth in the amount of capital. The **Graph 3** shows that the increase in fixed assets in the US, 1950-2005 (BEA, D. of C., US). The graph is also an index, 1992=100



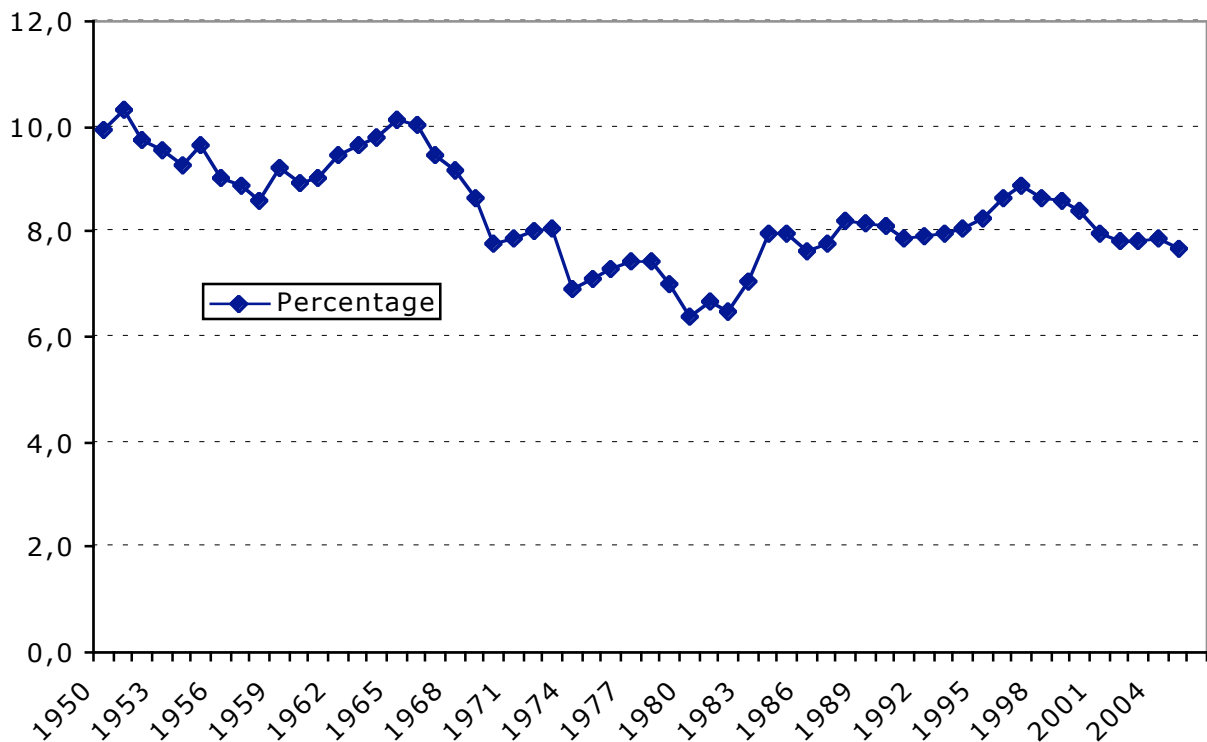
The graph shows that the US Fixed Assets grow in that period.

In the short run the amount of capital usually grows although in some periods they can grow at a smaller rate being their movements more volatile than in the long run.

c) Interest.

In the Graph 4 it is shown the relation between the non-labor income (NIPA, BEA) and the US Fixed Assets. This graph can represent the rate of profit in the long run. Interest through time is shown as a percentage. As the graph shows it remains more or less stable, perhaps somewhat decreasing (there is a low interval from 1980 to 1983).

GRAPH 4
INDEX OF RATE OF PROFIT 1950-2005



Source: Bureau of Economic Analysis

In the short run the behaviour of interest is very different. It has a great degree of variability and this variation has great importance. It could be of interest to distinguish between the incomes of the different classes of capital (profits, interest, real state rents, etc.) as long as the data could be available.

The rate of interest seems to be the variable that plays the principal adjusting role to the equilibrium of the whole in the short run, being profits the part which takes most of the adjustment.

2. THE ECONOMIC PRODUCTION FUNCTION AND THE US DATA

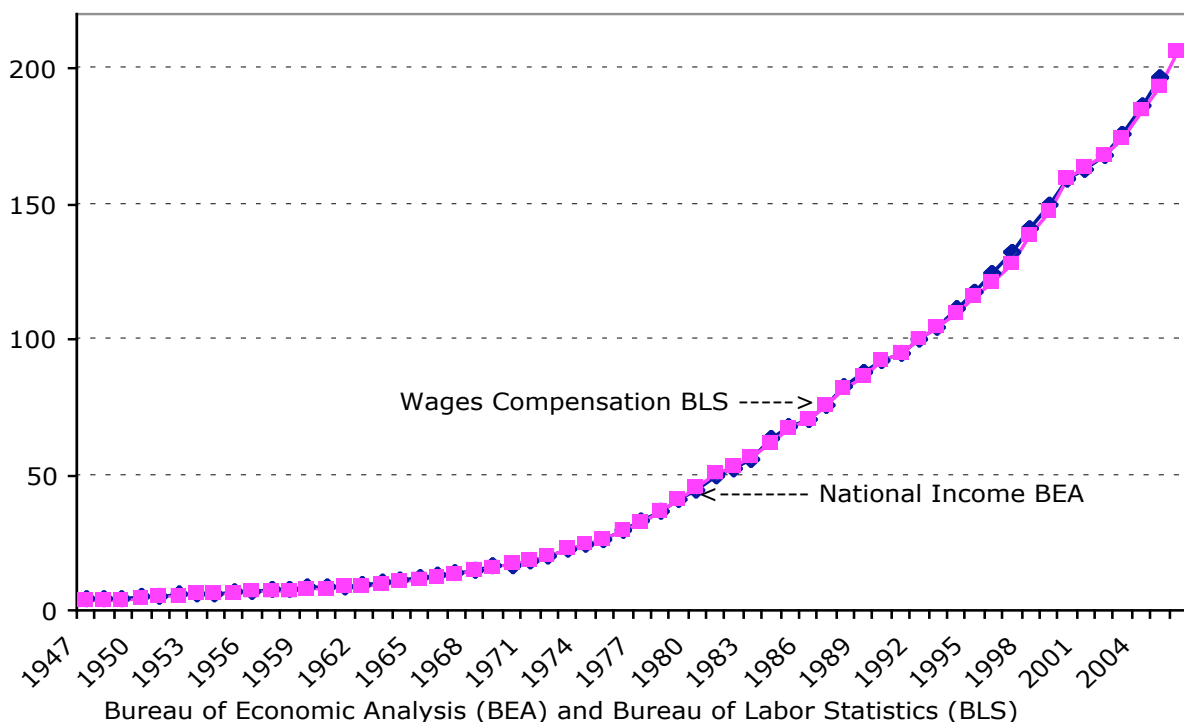
The economic production function explains the growth of the US economy and complies with the disposable economic data of the US economy. There is a comparison in terms on the evolution on the indexes of Wages Compensation (Hours and Wages) and Fixed Assets. There is also the comparison using the data of Compensation of Employees, Fixed Assets and the National Income from the BEA

A). Evolution on Indexes.

The Indexes are Wages Compensation (Hours and Wages) and Fixed Assets and the sum of both.

i). **Wages Compensation** is the product of nominal hourly compensations and numbers of hours of work which have been shown in Graph 1. The Graph 5 shows the total Wages Compensation. In the graph 5 is also reflected the National Income from 1948. Both are indexes 1992=100.

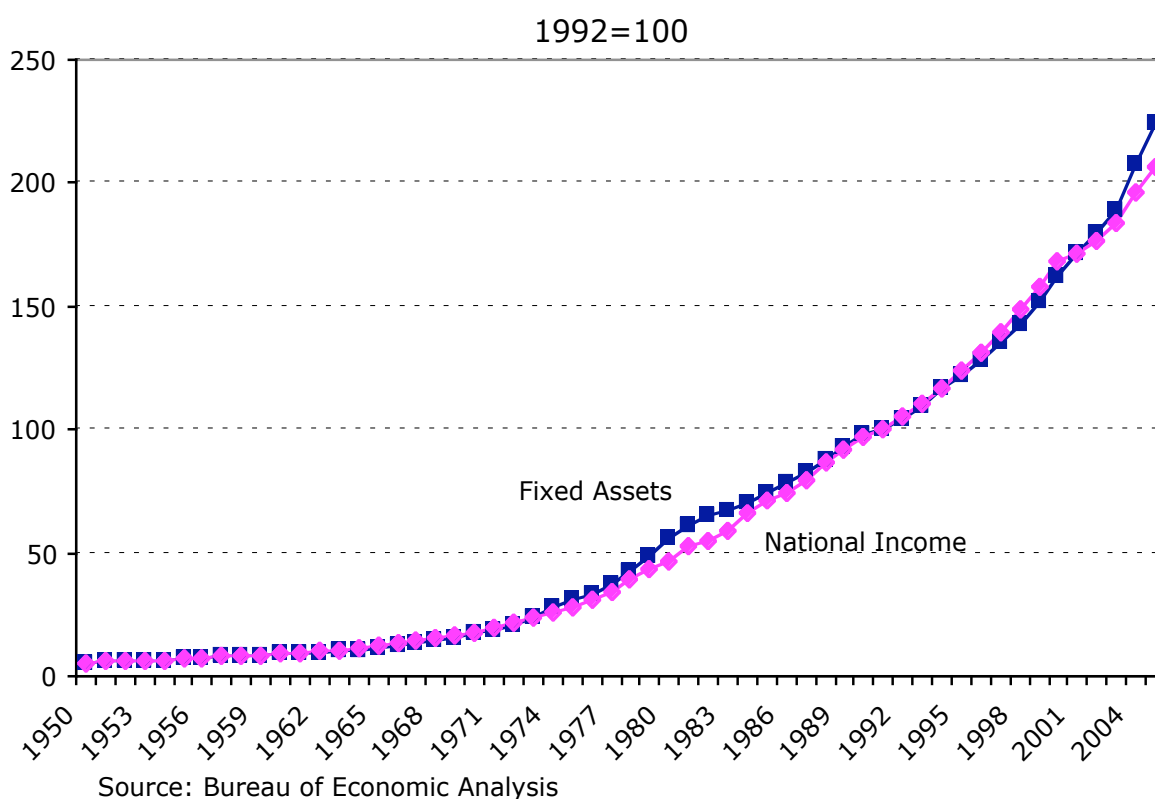
GRAPH 5
WAGES COMPENSATION (BLS) AND NATIONAL INCOME (BEA)
1947-2006



As the **Graph 5** there is a strait parallelism between the total Wages Compensations and the whole National Income

ii). **Fixed Assets** (Capital). the increase in fixed assets in the US, 1950-2005 (BEA,) is shown in **Graph 6**. It also shows the National Income. Both are indexes 1992=100.

GRAPH 6
U.S. FIXED ASSETS AND NATIONAL INCOME (BEA) 1950-2005



The **Graph 6** shows also a parallelism from 1950 between the Produced Assets, and the National Income 1950-2005 (BEA) (the greatest difference is also between 1980 and 1983)

The similar evolution of Fixed Assets and National Income points to the correction of the observation that the rate of interest shows certain stability in the long run. If the variations that the rate of interest has in Graph 4 were considered the coincidence will be greater.

iii). Economic Production Function and the US Data

The Economic Production Function is the sum of Wages Compensations, Graph 5, and the Capital Remuneration, Graph 6. Both graphs show that the income of labour (L_w) and the income of capital (K_i) are practically coincident with National Income. Being of a similar form the sum of the income of labour and the income of capital will be also similar and also coincident with the National Income. These curves are indexes. The sum will depend of the weight which is applied to each element (which more or less remains stable in the long run). Therefore if an Economic Production Function is applied to the US data there will a coincidence.

B). National Income and Products Accounts Data (BEA)

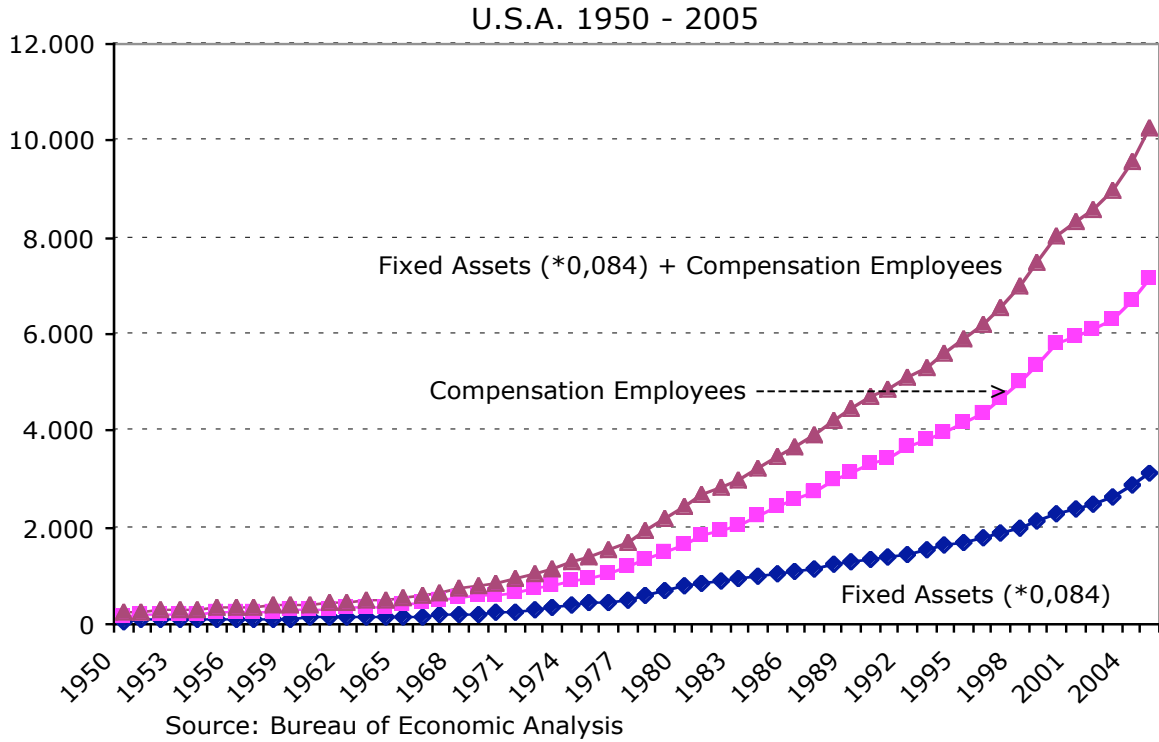
It has been pointed out that the compensation and the number of number of hours are very representative of the total data of the US economy although it does not represent the whole.

It is posible to take the data from the National Income. although this method leads to a rather tautological conclusión. But it seems to confirmate the theory that the National Income is the result of an Economic Production Function (which in fact is an Incomes Relation)

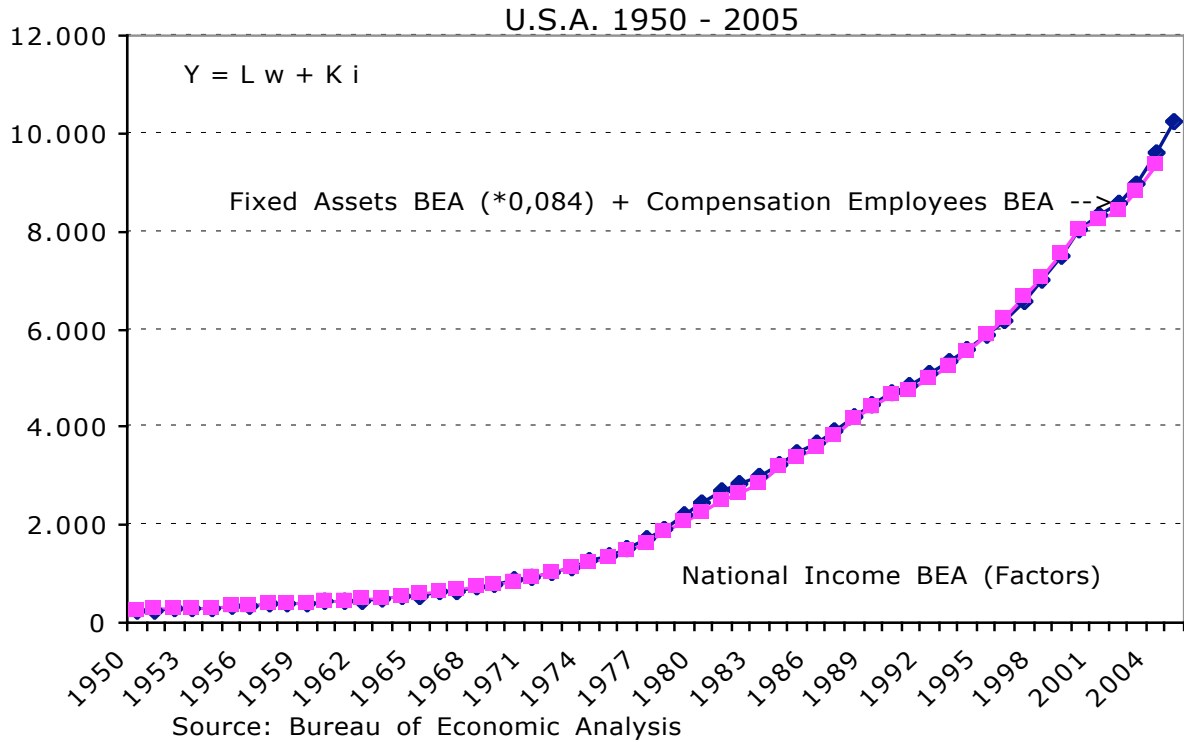
If the data for the compensation of employees (L_w) is taken from the National Income and Products Accounts from the Bureau of Economic Analysis, Department of Commerce, USA. it is posible to see the sum of the labour and non labour incomes.

Graph 7 shows the compensation of employees in the United Sates from 1950 (BEA). The graph also shows the Fixed Assets in the USA, (BEA), taken as a measure of capital and multiplied by a factor 0,084 that can be taken as the income of capital. In the graph is also represented the sum of these incomes of wages and capital.

GRAPH 7
LABOR AND NON LABOR INCOME (BEA)



GRAPH 8
LABOR AND NON LABOR INCOME AND NATIONAL INCOME (FACTORS)



Graph 8 that shows the sum of labour and non labour incomes and the National Income in the USA from 1950. Both curves are practically coincident.

When the evolution of wages is taken into account the evolution of the US economy is correctly accounted, at least for the last fifty years.

The new model allows to explain the facts in the accountancy of growth and this explanation also helps to confirm that the new model is justified. In fact the National Income takes into consideration the growth of wages through its incorporation to the compensation of employees. Therefore is evident that if the growth of wages is taken into account the National Account data is explained. Net National Product reflects the growth of resources and the growth of wages (Note 2).

4. THE INCREASE OF PRODUCTION AND THE SOURCES OF GROWTH.

From the Incomes Relation it is possible to analyse the factors that contribute to the increase of production

This increase in production in a certain period is

$$\Delta Y = w \Delta L + i \Delta K + (L + \Delta L) \Delta w + (K + \Delta K) \Delta i$$

The first two terms are due to the increase of the (physical) amounts of resources: number of workers, ΔL , and capital ΔK .

Increase of employees $w \Delta L$,

Increase of capital $i \Delta K$

The last two terms are due to the increase in the value of the resources: increase in wages, $-\Delta w$ -, and variation of the rate of interest, $-\Delta i$ -.

Increase de wages	$(L + \Delta L)$	Δw
Increase of interest	$(K + \Delta K)$	Δi

The relation shows that the increase in production can be due to the increase in the (physical) amounts of labour (ΔL) and capital (ΔK) and also of the evolutions of the prices of labour (Δw , wages) and capital (Δi , interest).

In the long run the increase in production

$$\Delta Y = w \Delta L + i \Delta K + (L + \Delta L) \Delta w + (K + \Delta K) \Delta i$$

is due to the increase in the amounts labour (ΔL), the increase in the amounts of capital (ΔK), and also the increase of the wages of workers or employees (Δw). In the long run labour, capital and wages have grown while interest has remained stable.

5. THE RATE OF INCREASE OF PRODUCTION

The rate of increase of total production is:

$$\frac{\Delta Y}{Y} = \frac{Lw}{Y} \frac{\Delta L}{L} + \frac{Ki}{Y} \frac{\Delta K}{K} + \frac{Lw}{Y} \frac{L + \Delta L}{L} \frac{\Delta w}{w} + \frac{Ki}{Y} \frac{K + \Delta K}{K} \frac{\Delta i}{i}$$

The rate of increase of the product depends of the four factors:

Two factors that reflect the increase of resources:

Rate of growth of wages	$\frac{Lw}{Y} \frac{L + \Delta L}{L}$	$\frac{\Delta w}{w}$
Rate of growth of capital	$\frac{Ki}{Y}$	$\frac{\Delta K}{K}$

Two factors that reflect the increase in the retributions of the resources are:

Rate of growth of hours	$\frac{Lw}{Y}$	$\frac{\Delta L}{L}$
Rate of change of interest	$\frac{Ki}{Y} \frac{K + \Delta K}{K}$	$\frac{\Delta i}{i}$

The rates of increase in the number of workers and in wages are multiplied by the share of labour in national income and the rates of increase in capital and in the rate of interest are multiplied by the share of capital in national income.

6. THE IMPORTANCE OF THE SOURCES OF GROWTH IN THE LONG RUN.

The importance of each factor in the long run is seen after the study of the influence that each element have in the rate of increase of the production.

The importance of each of these sources of growth can be estimated for USA with the data for the last fifty years.

It can be estimated, in the period 1951–2000, the annual rates of increase of production in USA, and the annual rates of increase of the four components, which reflect the influences of rates increase of labour, capital, wages and interest.

The average values of these rates in the period 1951-2000 are:

		Nominal Terms %	Real Terms %
Rate of growth of output			
	$\frac{\Delta Y}{Y}$	7,07	3,65
Growth due to growth of:			
wages	$\frac{Lw}{Y} \frac{L + \Delta L}{L} \frac{\Delta w}{w}$	3,90	1,50
capital	$\frac{Ki}{Y} \frac{\Delta K}{K}$	2,13	1,09
hours	$\frac{Lw}{Y} \frac{\Delta L}{L}$	1,02	1.02
interest	$\frac{Ki}{Y} \frac{K + \Delta K}{K} \frac{\Delta i}{i}$	0,03	0.03

The data of nominal output (Y), labour (L) and wages (w) are from BLS, Productivity and Costs (L expressed in hours and w nominal compensation per hour) The values of capital (K) is from the BEA. The values of the terms are expressed in monetary terms.

The fourth term is calculated from the difference between the value of Y and the sum of other three terms. It is considered that the labour and capital shares are:

$$\frac{Lw}{Y} = 0,7 \qquad \frac{Ki}{Y} = 0,3:$$

The increase in wages is very important and has a big influence in the increase of production, $(L + \Delta L) \Delta w$. This factor explains the evolution of production that the production function usually used, $Y = f(L, K)$, cannot explain. The rate of interest, which shows certain stability, does not influence production in the long run.

II. WAGES AND GROWTH

The incomes relation shows that the growth of wages plays an important role in the growth of the economy. It complies with the disposable economic data as it is shown in the paper and permits to explain, when the growth of wages is taken into account, the evolution of the US economy for the last fifty years at least. The model besides takes into account the historical facts and the data of the last decades about the growth of wages.

But the importance of the growth of wages in the growth of the economy implies a modification of the conception of the way that growth happens. The growth of an economy relies in an important way in the growth of wages. The US economy, for instance, has grown because the wages and salaries of its workers and employees have grown. Wages and salaries have to rise for the economy to grow.

This role of wages permits also understands the differences between the developments of the different countries. In the developed countries there is usually a process of growing wages. In the not developed countries frequently there is not such a process. It points, in an increasingly global world, where are the real differences between countries and why sometimes the differences diminish and why in other cases they increase.

From a social point of view the evolution of wages is probably the most important aspect in the evolution of the economies. They affect the welfare of the principal part of the population of each country. They have a great social significance. The tendencies of wages is probably the main basis to justify (or not to justify) the social role of capital markets economies.

But it not possible to forget that at the root of processes of rising wages and, therefore, of growth of the economies are the introduction of new capital goods and new methods of production. Behind the economic and the social progress of societies are the process of innovations.

It has been said that growth is the consequence of growth of resources, labour and capital, and technical change. The modification is that technical change leads to rising wages that growth consequence of growth of resources, labour and capital and the growth of wages.

NOTES

Note 1. This paper does not enter into the explanation of the theoretical reasons that make wages grow in economies with technical progress. The paper “Technical Change and Rising Wages in a Closed Economy” presented at the International J.A Schumpeter Society Conference analyses the influence of technical change in the compensation of employees.

A conclusion of that paper is that in a closed economy, after the introduction of technical change in the consumer industries there must be an increase in the real compensation of employees. This increase is usually due in part to the increase of employment and also to the growth of real wages (See Appendix 2).

Note 2. The production function (a kind of physical relation), $Y=f(L, K)$, does not help to explain adequately total growth because it does not take account of the growth of wages that is an important source of total growth.

APPENDIX I. The Economic Production Function. Variations

To explain evolution of production is better a relation in economic terms that collects the variation of the (physical) amounts of resources and also the variation of the value of those resources.

The Economic Production Function takes account of these factors of growth

$$Y = f(L, K, w, i)$$

And is a kind of economic production function. All its elements are value per time. Labour (L) is a number, wages (w) are value per time, capital (K) is value, interest (i) is percentage per time. Therefore all the three elements -Y-, -Lw- and -Ki- are value per time because wages and interest are economic magnitudes (Note 2).

The traditional production function used:

$$Y = f(L, K)$$

Is a kind of physical relation. Y, L and K must be physical quantities. But output is bigger that what would result using this relation. There is a residual. The relation has to be changed and the new form is usually like:

$$Y = A(t) f(L, K)$$

It is usually said that the growth of output is due to bigger employment, capital accumulation and technical change.

This statement has to be exposed more extensive and precisely. As said briefly in introduction, technical change leads to rising wages and this growth of wages is an important source of total growth. When the growth of wages is considered as an additional factor of growth, the evolution of the US economy is correctly accounted by the economic production function at least for the last fifty years as it is shown in this paper. There is no residual to explain.

The production function has seen some developments which permits some rapprochement between both functions. If in the

Economic Production Function it is considered that wages grow through time,

$$w = w (t)$$

and the rate of interest is considered constant, the Economic Production Function is like this:

$$Y = w(t) f(L, K)$$

Which is similar to the modified production function,

$$Y = A (t) f(L, K).$$

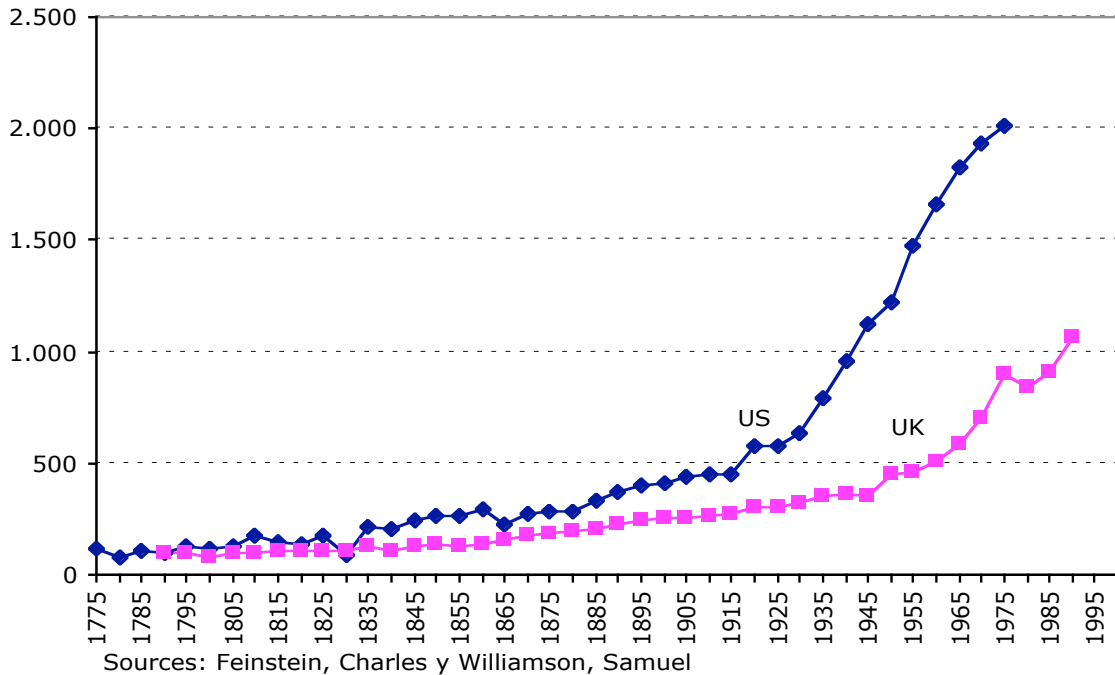
If the element $A (t)$ in the modified production function, $Y = A (t) f(L, K)$, is considered to represent the education of the population and the evolution of education is considered similar to the evolution of wages, it is possible to get good results with education as the basis of $A(t)$. But the relations in the economy are different. Education per se does not lead necessarily to bigger wages. Technical change leads to rising wages which influences output. Of course education is very important and in order to be able to have rising wages it is completely necessary to have improving education.

APPENDIX II. Wages in Economic History and in Recent Decades.

The growth of wages is a theoretical conclusion in the new model. As it is a rather novel statement it can be of interest to know what has happened to wages in economic history. Some of the available information about the evolution of wages is reflected in the following graphs and data:

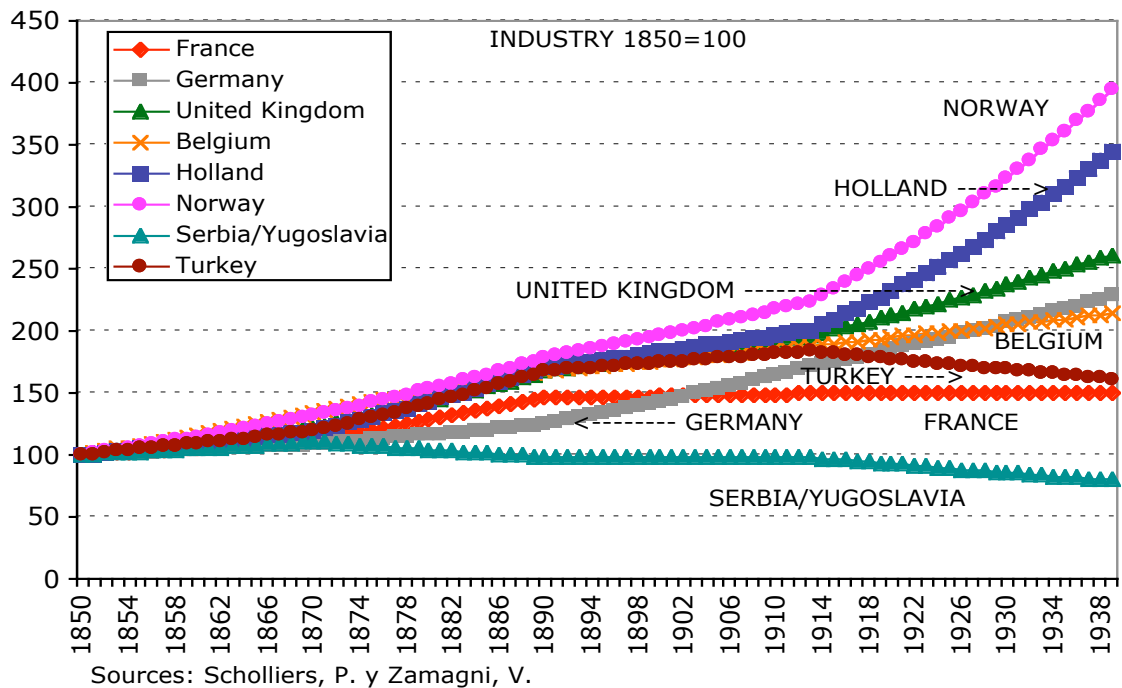
Graph 1 shows the evolution of real earnings in the UK from 1790. Charles Feinstein (1995) and the real earnings in the US from 1775 to 1975, Samuel H. Williamson (2003).

GRAPH 1
REAL WAGES UK AND US (1790=100)



Graph 2 shows the evolution of real earnings in several European countries from 1850 to 1939,

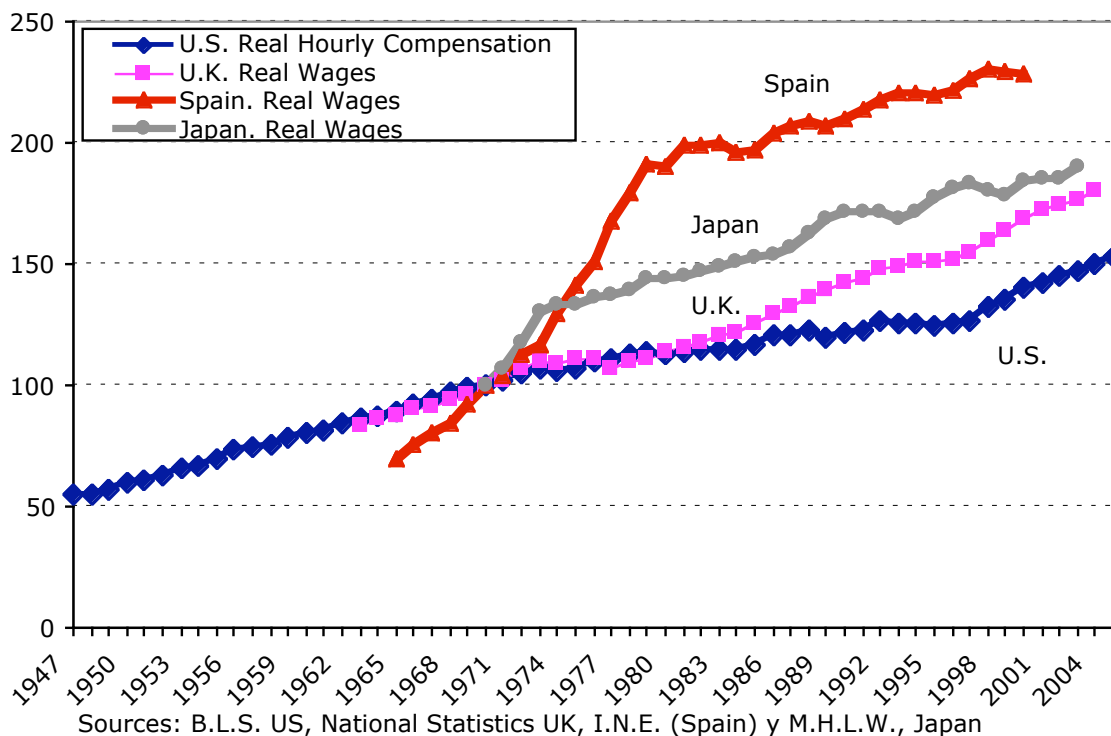
GRAPH 2
EUROPEAN COUNTRIES. REAL WAGES 1850-1938



Economic historians have shown in their work that the growth of wages has been occurring for a long time in countries that have incorporated capital, technical and organizational progress to production. In those countries the growth of wages has been occurring for a long time.

The increase of wages has been more extended in the last sixty years. There are also data about wages in official publications which show that wages have been growing. **Graph 3** reflects evolution of real wages in the last decades in the US., UK., Japan and Spain. Data from other countries can also be included to show that the process of growth of (real) wages that has been happening in them.

GRAPH 3
REAL WAGES AFTER 1945
1970=100



Many factors influence the process of history and this also applies to the evolution of wages but the graphs which are shown reflect the growth of wages in economies with capital accumulation and innovation.

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