Common Periodic Cycles and Periodic Multicointegration in Daily Airport Transit Data.

by

Niels Haldrup*, Svend Hylleberg*, Gabriel Pons**, Jaume Rosselló***, and Andreu Sansó***

Abstract

Motivated from an interesting data set of daily arrivals and departures in the Sant Joan Airport in Mallorca we present a periodic model applied to explain the complex nature of the seasonal variation in the airport transit data. Analyses of periodic

*Department of Economics, University of Aarhus, Building 350, DK-8000 Aarhus C, Denmark. E-mail: nhaldrup@econ.au.dk, shylleberg@econ.au.dk.**Institute of Economics, University of Copenhagen, E-mail: Gabriel.pons@econ.ku.dk ***Departament d'Economia Aplicada. Universitat de les Illes Balears. Gabriel Pons appreciates support from the Danish Research Council. Andreu Sanso and J.Rosselló acknowledge the financial support of the Direcció General de Recerca de la Conselleria d'Economia, Hisenda i Innovació del Govern de les Illes Balears as well as the financial support of the Ministerio de Ciencia y Tecnología SEC2002-01512.

models at a daily frequency of observations are rather limited in the literature. However, we demonstrate how a periodic set up is natural as a benchmark for testing e.g. periodic integration and cointegration as well as for and seasonal unit roots for daily observations. Based upon the multivariate representation of periodic models we suggest a new notation, *common periodic cycles*, which are common features that covary - possibly with a phase shift - across the different days of the week. Finally, the possibility of multicointegration in the periodic model is examined to explore the link existing between the flow variables (arrivals, departures, and net flow of passengers) on the one side, and the stock of passengers in residence on the other side.

Key Words: Periodic autoregression, seasonality, high frequency data, cointegration, multicointegration, common features.

1 Introduction

We consider the first-order unit root nonstationary periodic model to describe the daily dynamics of airport of Mallorca transit data. Periodically integrated processes were introduced by Osborn (1988) in an attempt to extend the periodic autoregressive process to quarterly trending economic time series. We provide strong evidence in this paper that such nonstationary periodic model describes properly the changing seasonal pattern of daily arrivals and daily departures series. We also find strong evidence on the presence of nonperiodic cointegration between arrivals and departures, and periodic multicointegration between such variables as well.

We extend the notion of polynomial common features (see Cubadda and Hecq, 2001) to the periodic framework, and motivate such common dynamic factors as the evidence on colinealities among the different autocorrelation functions of the periodic process. The presence of multiple polynomial common features implies the presence of a nested reduced rank structure (Ahn and Reinsel, 1988) which opens the door to modelling periodic daily process in the multivariate framework under complex restrictions across the autocorrelations functions, restrictions which only can be detected in the multivariate analysis by means of common periodic cycles.

The paper is organized as follows. Section 1 describes the data set, section 2 summarizes the periodic autoregressive process for univariate and bivariate daily time series. Then, section 3 discusses the notion of common periodic cycles, section 4 presents the empirical results, and section 5 concludes the paper with some remarks.

2 The data set

The data set used in this paper consists of daily arrivals and departures in the Airport of Mallorca. The data spans the period from 1. January 1994 to 28. February, 2002. This corresponds to 2981 daily observations (about 427 weeks). The Balearic Islands are one of the most important touristic destinations in the Mediterranean. The annual volume of tourists is around 10 million people of whom more than 95%

travel by plane. More than 80% of total transit are tourists visiting Mallorca, the biggest of the Balearic Islands.

Being a touristic destination it is not surprising that arrivals and departure series exhibit a high degree of seasonal variation. This is verified by inspection of figure 1 which shows the variation of the data over the entire sample period. In addition to the arrivals and departures data, the figure shows the net flow of passengers to Mallorca, (arrivals minus departures), as well as the cumulation of the net flows denoted the stock. Note that the net flow variable will indicate the contribution to the number of airline passengers staying on the islands and hence the stock variable is really a measure of the stock (level) of air-passengers (visitors) who stay at Mallorca at a given date[1].

From figure 1 the yearly variation of the transit data is most obvious. In particular, the very close co-movement of arrivals and departures is apparent and hence suggesting a strong common seasonal pattern over the year. However, the week-of-day effect is also very apparent as can be seen from figure 2 where the daily variation for the year 2001 is displayed. Whereas the arrivals and departures exhibit very strong seasonal fluctuations the net flow and the stock variable obviously have much less variation within the week. It is hard from eye-balling to say whether cointegration in

[1] The stock variable indicates the *level* of people staying on Mallorca and not the actual figure because the initial value of observations is unknown.

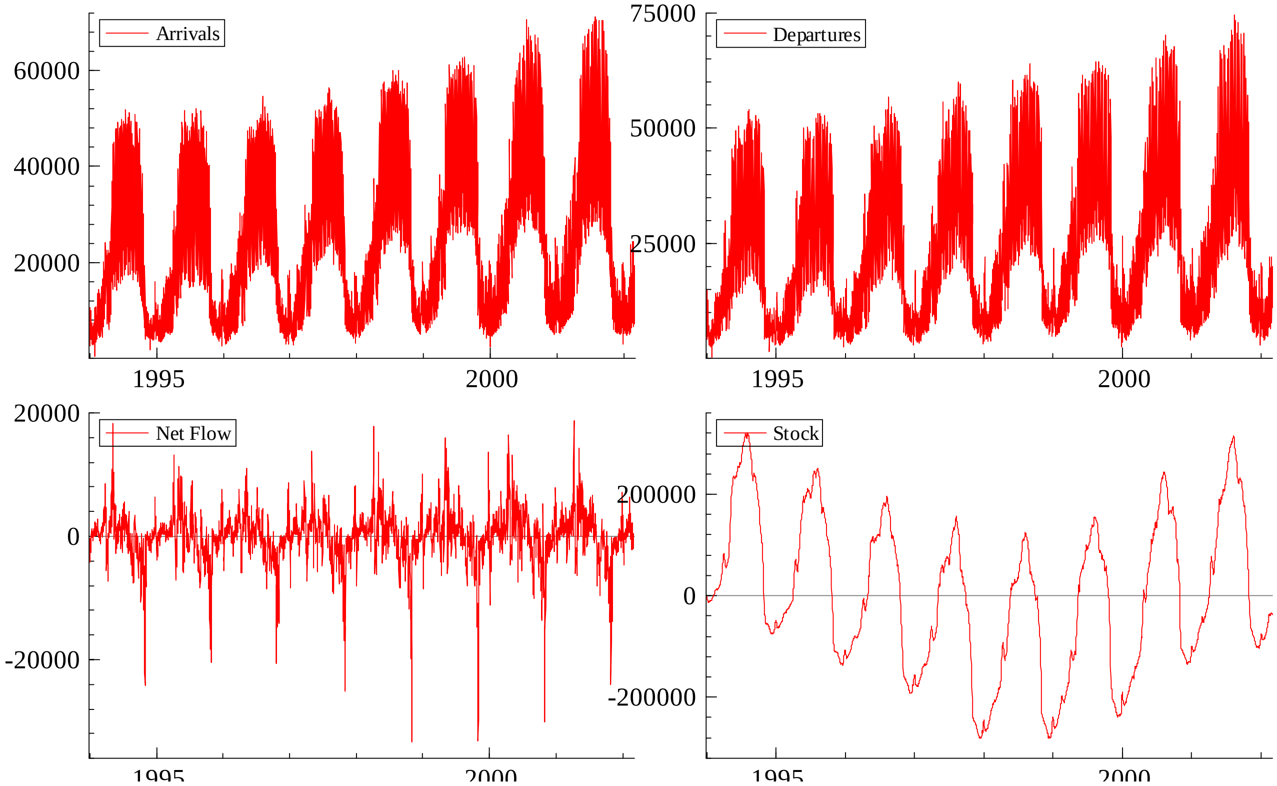

Figure 1: Arrivals, departures, net flow (i.e. arrivals minus departures), and the level of stock (i.e. the cumulated net flow) at the Son Sant Joan Airport, Mallorca, 1. January, 1994 - 28. February, 2002.

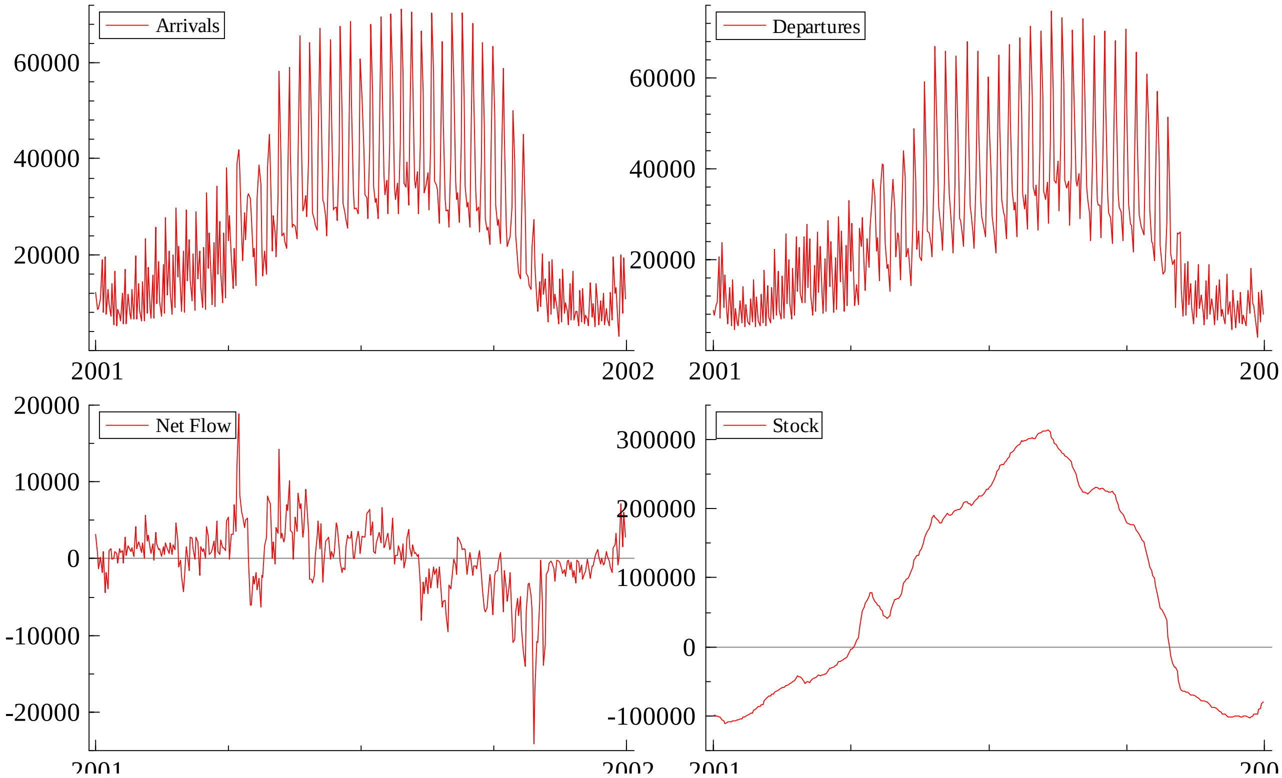

Figure 2: Arrivals, departures, net flow (i.e. arrivals minus departures), and the level of stock (i.e. the cumulated net flow) at the Son Sant Joan Airport, Mallorca, 1. January, 2001 - 31. December, 2001.

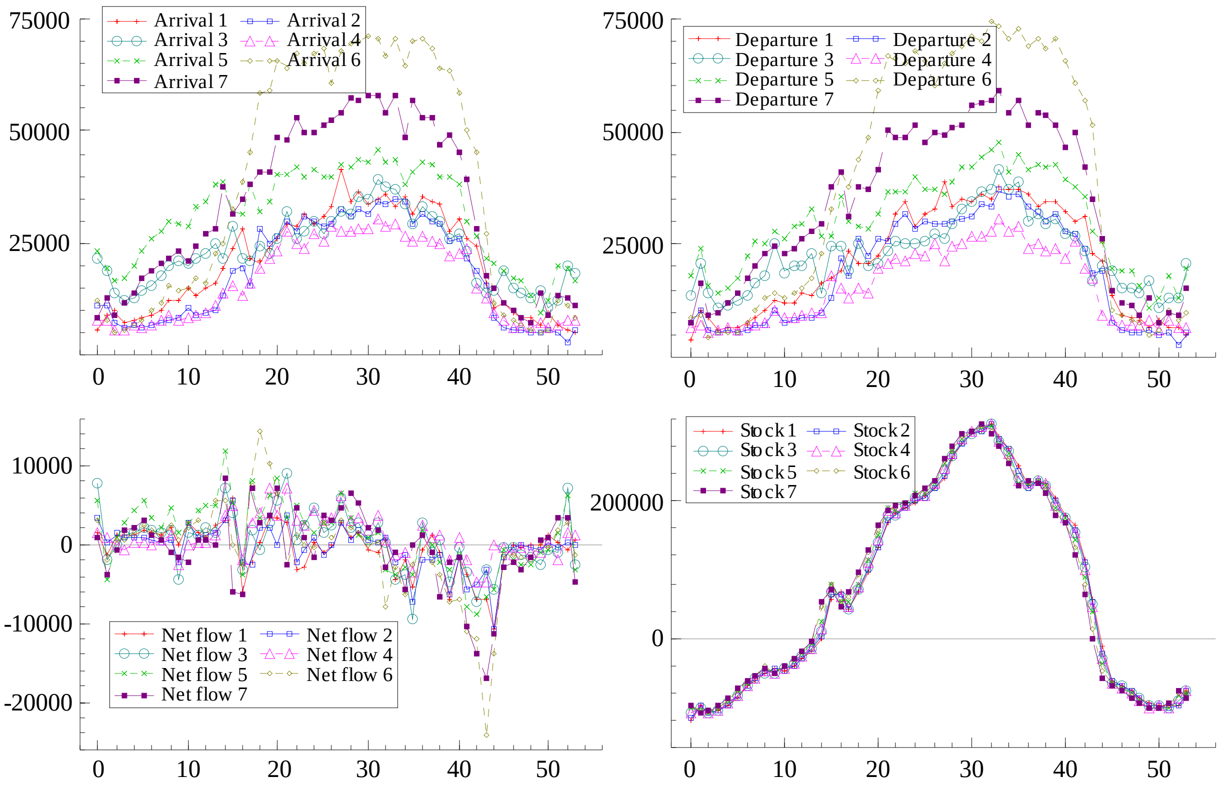

Figure 3: Arrivals, departures, net flow (i.e. arrivals minus departures), and the level of stock (i.e. the cumulated net flow) at the Son Sant Joan Airport, Mallorca, for each week-day (Monday 1, Tuesday 2,..., Sunday 7) for each week in 2001.

one form or the other characterizes the data. However, what seems obvious is that some kind of common seasonal feature exists amongst the arrivals and departures series.

To further focus on the weekly periodicity figure 3 displays the individual weekday observations for each of the series for 2001. As can be seen the arrivals and departures have strong day-of-week effects and this feature seems to change over the year. This might indicate that these series potentially can be modelled as periodic seasonal processes. For the net-flow series and the stock series no periodic seasonal variation seems to be present.

A further aspect of the present data set concerns the possibility of a multicointegration like feature amongst the series. If we assume that the arrivals and departures series are cointegrated in some sense, then it is of interest to look at the cumulated net flow series, i.e. the stock variable generated from arrivals and departures. Interestingly, it appears from figures 1 and 2 that although the stock series has much less weekly variation, the level around some trend co-varies with both the arrivals and departures series. This is an interesting phenomenon because it allows for the possibility of more than just one cointegrating relationship existing between just two series. The property is often being referred to as multicointegration. There are numerous examples of multicointegration (at the zero frequency) in the literature as indicated in the introduction. What is of particular interest in the present case is the fact that a similar property is likely to arise in the daily transit data. The challenge here is to examine simultaneously the interactions between stocks and flows as well as the strong seasonal pattern in the data.

3 A Periodic Autoregressive Model for daily observations

3.1 The representation and properties of the model

As argued in section 2 it is likely that the arrivals and departures series follow periodic processes. Periodic models have frequently been criticized because they require a lot of parameters to be estimated. However, in the present case data is not scarce and a periodic modelling framework seems feasible as well as reasonable. It further has the implications that models with fixed parameters and standard seasonal ARIMA processes, including seasonal unit root processes, are encompassed within the periodic model for certain restrictions on the parameters, and hence can be tested.

Seasonal processes with a periodic structure can be represented by periodic ARMA (henceforth PARMA) models which allow for different parameters across the seasons. In practice, the estimation of pure PAR models provide certain advantages, see McLeod (1995), over PARMA models because every season can be treated independently whereby a multivariate representation of the univariate series can be given (see Gladishev, 1961).

Let us describe the most relevant characteristics of the periodic model for a univariate time series where the periodicity is allowed for the day of the week.[2] General comprehensive surveys of periodic models and required inferential tools can be found in e.g Franses (1996) and Ghysels and Osborn (2001).

The daily periodic autoregressive process of order p, PAR(p), reads:

$$y_t = \phi_{s,1} y_{t-1} + \phi_{s,2} y_{t-2} + \cdots + \phi_{s,p} y_{t-p} + \mu_t + \varepsilon_t,\ s = 1, .., 7 \tag{1}$$

where all the autoregressive parameters $\phi_{s,j}$ $(j = 1, ..., p)$ are allowed to vary with the season s, i.e. the day of the week. It should hold that at least one $\phi_{s,p} \neq 0$ $(s = 1, ..., 7)$. The component μ_t collects deterministic terms, and ε_t is a Gaussian white noise error term potentially with periodic heteroskedasticity, $E\left(\varepsilon_t^2\right) = \sigma_s^2$.[3] Note that, in this model, the parameters are allowed to be different for each day of the week, and therefore the PAR process is non-stationary since the autocovariance function varies with the season. A more interesting source of nonstationarity, the presence of stochastic trends, can be discussed within a multivariate representation of the PAR process which we denote the vector of days (VD) representation.

This representation defines the 7-dimensional weekly multivariate process $x_\tau \equiv (y_{1,\tau}, ..., y_{7,\tau})'$ with $\tau \equiv [(t-1)/7] + 1$ (see Gladyshev, 1961; or Tiao and Grupe,

[2] Franses and Paap (2004) apply a daily periodic autoregressive model with 5 seasons for financial data.

[3] Note that the season s is related to the observation number t through $s \equiv t - 7\left[(t-1)/7\right]$ where $[\cdot]$ stands for the integer part of its argument.

1980), which has the following nonperiodic representation:

$$\Phi_0 x_\tau = \Phi_1 x_{\tau-1} + \cdots + \Phi_P x_{\tau-P} + \upsilon_\tau + e_\tau \tag{2}$$

where Φ_k $(k = 0, ..., P;\ P = [(p+6)/7];\ p \leq (P+1)s - 1)$ are 7×7 matrices of parameters

$$\Phi_0 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -\phi_{2,1} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ -\phi_{7,6} & \cdots & -\phi_{7,1} & 1 \end{bmatrix}, \Phi_k = \begin{bmatrix} \phi_{1,7k} & \cdots & \cdots & \phi_{1,7k-6} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \phi_{7,7k+6} & \cdots & \cdots & \phi_{7,7k} \end{bmatrix}$$

for $k = 1, ..., P$, $\upsilon_\tau = \left(\mu_{1,\tau}, ..., \mu_{7,\tau}\right)'$ and $e_\tau \equiv (\varepsilon_{1,\tau}, ..., \varepsilon_{7,\tau})'$ ~N(0,Σ), where

$$\Sigma = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_7^2 \end{bmatrix}$$

For instance, the PAR(1) model, $y_t = \phi_s y_{t-1} + \varepsilon_t$, can be written as

$$\begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 \\ -\phi_2 & 1 & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -\phi_7 & 1 \end{pmatrix} \begin{pmatrix} y_{1,\tau} \\ y_{2,\tau} \\ \vdots \\ \vdots \\ y_{7,\tau} \end{pmatrix} = \begin{pmatrix} 0 & \cdots & 0 & \phi_1 \\ 0 & \cdots & \cdots & 0 \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & 0 \end{pmatrix} \begin{pmatrix} y_{1,\tau-1} \\ y_{2,\tau-1} \\ \vdots \\ \vdots \\ y_{7,\tau-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,\tau} \\ \varepsilon_{2,\tau} \\ \vdots \\ \vdots \\ \varepsilon_{7,\tau} \end{pmatrix}. \tag{3}$$

The unit root properties of the multivariate process x_τ determine the unit root properties of the daily process y_t. Define the matrix lag polynomial

$$\Phi\left(L^7\right) = \Phi_0 - \Phi_1 L^7 - \cdots - \Phi_P L^{7P},$$

where $Ly_{s,\tau} = y_{s-1,\tau}$ (with $Ly_{1,\tau} = y_{7,\tau-1}$) and $L^7 y_{s,\tau} = y_{s,\tau-1}$. When all the roots of the characteristic equation $|\Phi\left(L^7\right)| = 0$ lie outside the unit circle, the process x_τ is second order stationary and y_t is PI(0). Following the PAR(1) example, the necessary and sufficient condition for stationarity of this model is $|\phi_1\phi_2\cdots\phi_7| < 1$. The multivariate process is integrated at the zero frequency if $|\Phi\left(L^7\right)| = 0$ has some roots equal to one and all other roots outside the unit circle. Of particular interest are those situations where every weekly processes $y_{s,\tau}$ $(s = 1, ..., 7)$ is I(1). This property is known as first order nonstationarity (see Osborn, 2002).

A convenient way to represent the different possibilities of first order nonstationary processes is the Error Correction Representation (ECM). Consider the VAR representation of (2):

$$\Pi(L^7)x_\tau = \varsigma_\tau + u_\tau,$$

where $\Pi(L^7) = I_7 - \Pi_1 L^7 - ... - \Pi_P L^{7P}$, with $\Pi_k \equiv \Phi_0^{-1}\Phi_k$, $\varsigma_\tau \equiv \Phi_0^{-1}\upsilon_\tau$ contains the deterministic components, and $u_\tau = \Phi_0^{-1}e_\tau \sim$N$(0, \Phi_0^{-1}\Sigma(\Phi_0^{-1})')$, and decompose the matrix lag polynomial as $\Pi(L^7) = \Pi L^7 + \Gamma(L^7)(1-L^7)$ where $\Pi = \Phi_0^{-1}\left(\sum_{j=1}^{P}\Phi_j\right) - I_7$,

$\Gamma_0 = I_7$ and $\Gamma_k = \Phi_0^{-1}\Sigma_{j=k+1}^{P}\Phi_j$ $(k = 1, .., P-1)$ such that we obtain the VAR model:

$$\Delta_7 x_\tau = \varsigma_\tau + \Pi x_{\tau-1} + \sum_{k=1}^{P-1} \Gamma_k \Delta_7 x_{\tau-k} + u_\tau \tag{4}$$

with $\Delta_7 = 1 - L^7$. The different cases of first order nonstationarity are associated with different properties of the impact matrix Π. In particular, when Π has rank 7 the process y_t is periodically integrated of order zero, PI(0), and when Π has rank 6, y_t is periodically integrated of order one, PI(1). In this case we may distinguish two important cases. When the 6 cointegrating relations are given by $y_{2,\tau} - y_{1,\tau}$, $y_{3,\tau} - y_{2,\tau}$, ..., $y_{7,\tau} - y_{6,\tau}$, then in fact y_t is a nonseasonally and nonperiodically integrated process, that is a standard I(1) process. When the 6 cointegrating relations read $y_{2,\tau} - \phi_2 y_{1,\tau}$, $y_{3,\tau} - \phi_3 y_{2,\tau}$, ..., $y_{7,\tau} - \phi_7 y_{6,\tau}$ with at least one $\phi_s \neq 1$ $(s = 2, ..., 7)$ then y_t is PI(1). Hence the I(1) model appears as a special case of the PI(1) model with certain restrictions on the parameters. When $y_t \sim$PI(1), the first-difference operator Δ does not remove the stochastic trend from y_t. In this case it is necessary to apply a specific difference filter for every season, the quasi-difference filter, $\delta_s(L) \equiv 1 - \phi_s L$ where ϕ_s are named the periodic integration coefficients, such that $\delta_s(L)y_t \sim$PI(0).

When Π has rank $0 \leq r < 6$, then y_t is a seasonally integrated process, see Hylleberg, Engle, Granger and Yoo (1990) and Franses (1996).

Generally, under the reduced rank of Π $(0 < r < 7)$, the impact matrix can be decomposed $\Pi = \alpha\beta'$ where α and β are $7 \times r$ matrices of full rank, that contain the adjustment vectors and the cointegrating vectors, respectively. Then, we can rewrite

(4) as the ECM:

$$\Delta_7 x_\tau = \varsigma_\tau + \alpha\beta' x_{\tau-1} + \sum_{k=1}^{P-1} \Gamma_k \Delta_7 x_{\tau-k} + u_\tau. \tag{5}$$

We denote the r-dimensional cointegrating relations process by $z_\tau \equiv \beta' x_\tau$.

For instance, the general PAR(1) process can be represented as

$$\Delta_7 x_\tau = \begin{pmatrix} -1 & 0 & \cdots & 0 & \phi_1 \\ 0 & \ddots & \ddots & 0 & \phi_1\phi_2 \\ \vdots & \ddots & \ddots & 0 & \vdots \\ \vdots & \cdots & \ddots & -1 & \phi_1\phi_2\phi_3\phi_4\phi_5\phi_6 \\ 0 & \cdots & \cdots & 0 & \phi_1\phi_2\phi_3\phi_4\phi_5\phi_6\phi_7 - 1 \end{pmatrix} x_{\tau-1} + u_\tau.$$

For this latter process, if $\phi_1\phi_2\cdots\phi_7 = 1$ but not all $\phi_s = 1$, then $y_t \sim$PI(1). In the case of PI(1), the cointegrating matrix β will contain six of the coefficients ϕ_s, for example ϕ_s $(s = 2, ..., 7)$:

$$\beta' = \begin{pmatrix} -\phi_2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\phi_3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\phi_4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi_5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\phi_6 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\phi_7 & 1 \end{pmatrix}, \tag{6}$$

and the remaining coefficient is $\phi_1 = (\phi_2\phi_3\phi_4\phi_5\phi_6\phi_7)^{-1}$.

The weekly multivariate representation can be used to select among the different first-order non-stationary possibilities, by means of a multivariate cointegration analysis (see Johansen, 1991). This procedure is proposed by Franses (1994) for quarterly PAR models. The same method can be used to test for PI(1) at the daily arrivals, departures, net flow, and stock series.

3.2 Bivariate Periodic Models

Consider the daily bivariate PAR(p) process $\mathbf{y}_t = (y_t^1, y_t^2)'$

$$\mathbf{y}_t = \boldsymbol{\phi}_{s,1}\mathbf{y}_{t-1} + \boldsymbol{\phi}_{s,2}\mathbf{y}_{t-2} + \cdots + \boldsymbol{\phi}_{s,p}\mathbf{y}_{t-p} + \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t,\ s = 1, .., 7,$$

where now $\boldsymbol{\phi}_{s,j}$ $(j = 1, .., p)$ are 2-dimensional matrices of coefficients $\boldsymbol{\phi}_{s,j}^{i,h}$ $(i, h = 1, 2)$ which may vary with the day of the week. Under cointegration we can represent the PAR(p) as

$$D_s\mathbf{y}_t = \boldsymbol{\alpha}_s\mathbf{k}_s'\mathbf{y}_{t-1} + \Sigma_{j=1}^{p-1}\gamma_{sj}D_{s-j}\mathbf{y}_{t-j} + \boldsymbol{\varepsilon}_t,$$

where $D_s = diag(\delta_s^1(L), \delta_s^2(L))$ (with $D_{1-1} = D_7$) is the quasi-difference operator turning the bivariate PI(1) process into a bivariate PI(0) process (see Ghysels and Osborn, 2001).

3.2.1 Cointegration between Flow Variables

In this section we consider the multivariate representation of the daily flow variables, (e.g. arrivals which we denote y_t^1 and departures denoted y_t^2). Then, paralleling the univariate model consider the weekly representation of the daily bivariate process $\mathbf{y}_t = (y_t^1, y_t^2)'$, where now we define the 14 dimensional VD process $\mathbf{x}_\tau \equiv (y_{1,\tau}^1, ..., y_{7,\tau}^1, y_{1,\tau}^2, ..., y_{7,\tau}^2)'$, and assume that it can be represented by a VAR(P)

$$\mathbf{\Pi}(L^7)\mathbf{x}_\tau = \boldsymbol{\varsigma}_\tau + \mathbf{u}_\tau,$$

where $\mathbf{\Pi}(L^7) = I_7 - \mathbf{\Pi}_1 L^7 - ... - \mathbf{\Pi}_P L^{7P}$ with $P = [(p+6)/7]$, $\boldsymbol{\varsigma}_\tau$ contains the deterministic components, and $\mathbf{u}_\tau$ is a 14-dimensional white noise. The ECM representation of the VAR model reads:

$$\Delta_7 \mathbf{x}_\tau = \boldsymbol{\varsigma}_\tau + \mathbf{\Pi}\mathbf{x}_{\tau-1} + \sum_{k=1}^{P-1} \mathbf{\Gamma}_k \Delta_7 \mathbf{x}_{\tau-k} + \mathbf{u}_\tau. \tag{7}$$

Under the presence of (periodic) cointegration between the daily series the 14 weekly series have a common stochastic trend and the impact matrix $\mathbf{\Pi}$ can be written $\mathbf{\Pi} = \mathbf{AB}$, where $\mathbf{A}$ and $\mathbf{B}'$ are full column $14 \times 13-$matrices

$$\mathbf{B} = \begin{bmatrix} \mathbf{I}_7 & \mathbf{K} \\ \mathbf{0} & \mathbf{\Phi}^2 \end{bmatrix}.$$

$\mathbf{I}_7$ is the 7-dimensional identity matrix, $\mathbf{0}$ is the 6×7-dimensional null matrix, $\mathbf{K}$ is a 7-dimensional matrix containing on the diagonal the cointegrating coefficients:

$$\mathbf{K} = \begin{bmatrix} -k_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -k_7 \end{bmatrix},$$

such that $y^1_{s,\tau} - k_s y^2_{s,\tau}$ ~PI(0) (s=1,...,7) and $\mathbf{\Phi}^2$ is the 6 × 7-dimensional matrix containing the periodic integration coefficients associated with y^2_t:

$$\mathbf{\Phi}^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -\phi^2_1 \\ -\phi^2_2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\phi^2_3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\phi^2_4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi^2_5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\phi^2_6 & 1 & 0 \end{bmatrix}.$$

It should be noted that the periodic integration coefficients for the arrivals can be obtained from $\phi^1_s = \phi^2_s k_s / k_{s-1}$ ($k_{1-1} = k_7$) where ϕ^1_s ($s = 1, 2, ..., 7$) are the periodic integration coefficients for the arrivals series y^1_t.

Osborn (2002) discusses how periodically and non-periodically integrated processes can potentially cointegrate in various cases. When both daily variables y_t^i $(i = 1, 2)$ are I(1), then $k_s = k$ $(s = 1, ..., 7)$, and $\phi_s^2 = 1$ $(s = 1, ..., 6)$, that is, in this case the daily variables are nonperiodically cointegrated. When both daily variables y_t^i $(i = 1, 2)$ are PI(1), then the cointegrating vectors may be different across the different days of the week, i.e. such that the series are fully periodically cointegrated $k_s \neq k$ (at least for some s) and $k_s \neq 0$ $(s = 1, ..., 7)$. However, the series could also be non-periodically cointegrated such that $k_s = k$ $(s = 1, ..., 7)$. When, one daily variable is I(1) and the other one is PI(1), then the variables may only be fully periodically cointegrated.

Under the absence of (periodic)cointegration between y_t^1 and y_t^2 the 14 weekly series have two common stochastic trends. In particular, the impact matrix reads $\mathbf{\Pi} = \mathbf{AB}$, where $\mathbf{A}$ and $\mathbf{B}'$ are full column $14 \times 12-$matrices

$$\mathbf{B} = \begin{bmatrix} \mathbf{\Phi}^1 & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}^2 \end{bmatrix},$$

where $\mathbf{0}$ is the 6×7-dimensional null matrix, $\mathbf{\Phi}^i$ is the 6×7-dimensional matrix containing the periodic integration coefficients associated with the variable y_t^i:

$$\mathbf{\Phi}^i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -\phi_1^i \\ -\phi_2^i & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\phi_3^i & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\phi_4^i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi_5^i & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\phi_6^i & 1 & 0 \end{bmatrix}. \tag{8}$$

The multivariate representation of the flow variables (7) is the basis to test for periodic cointegration between daily arrivals and daily departures. First, one should test for cointegration with Johansen (1991). Then, one may test for non periodic cointegration through the hypothesis $k_s = k$ $(s = 1, ..., 7)$. Given, the relation $\phi_s^1 = \phi_s^2 k_s / k_{s-1}$, then the test for non periodic cointegration can be also interpreted as a test of equivalent periodic integration coefficients $\phi_s^1 = \phi_s^2$ $(s = 1, ..., 7)$, such that under non periodic cointegration the estimated periodic integration coefficients for the departures are also the estimated periodic integration coefficients of the arrivals.

3.2.2 Cointegration between Stock and Flow Variables

Assuming that the net flow $y_t^3 = y_t^1 - y_t^2$ is PI(0), and therefore that there exists fully nonperiodic cointegration between the daily arrivals y_t^1 and the daily departures y_t^2, then the stock variable $y_t^4 = y_0^4 + \sum_{j=1}^t y_t^3$ is I(1) by definition. Then, it is feasible that the daily stock y_t^4 may cointegrate with the daily arrivals or with the daily departures. This phenomenon is known as multicointegration (see Granger and Lee, 1989, 1990), and it implies that cointegration may occur not only between the flow variables but also between the flow and the stock variable which itself is generated from the flows. The presence of multicointegration can be analyzed in the same way as the cointegration between the arrivals and departures by substituting for example the seven weekly arrivals series by the seven stock series in $\mathbf{x}_\tau$.[4]

Potentially, the presence of multicointegration can be analyzed in a smaller system when the stock variable is non-periodically integrated. Hence, if the flow variable is PI(1) and the stock variable is non-periodic I(1), then if these variables are cointegrated, then multicointegration can only be periodic (see Osborn, 2002)

$$y_{s,\tau}^4 - k_s y_{s,\tau}^2 \text{ with } k_s \neq k \text{ for some } k_s. \tag{9}$$

Therefore, the periodicity of multicointegration, which can only be tested at the 14-dimensional system (7), does not need to be checked due to the results obtained at the cointegration analysis of the flows or at the univariate analysis. A possibility is thus to include only one of the departure series in the multivariate system such that $\mathbf{x}_\tau^* \equiv (y_{1,\tau}^2, ..., y_{7,\tau}^2, y_{i,\tau}^4)'$ is analyzed. Under the presence of periodic cointegration

[4] The daily arrivals y_t^1 could be used in place of the daily departures.

between the seven departure series and the stock variable,

$$\Delta_7\mathbf{x}_\tau^* = \boldsymbol{\varsigma}_\tau^* + \mathbf{\Pi}^*\mathbf{x}_{\tau-1}^* + \sum_{k=1}^{P-1}\mathbf{\Gamma}_k^*\Delta_7\mathbf{x}_{\tau-k}^* + \mathbf{u}_\tau^*,$$

where $\mathbf{\Pi}^* = \mathbf{A}^*\mathbf{B}^*$, with $\mathbf{A}^*$ and $\mathbf{B}^{*\prime}$ being full column $8 \times 7-$matrices and the cointegration matrix can be written

$$\mathbf{B}^* = \begin{bmatrix} \mathbf{I}_7 & -\mathbf{k} \end{bmatrix},$$

where $\mathbf{k} \equiv (k_1, k_2, k_3, k_4, k_5, k_6, k_7)'$ is a 7×1 column vector containing the multicointegrating coefficients such that $y_{s,\tau}^2 - k_s y_{s,\tau}^4$ is PI(0).

When y_t^4 and y_t^2 are not cointegrated, then $\mathbf{\Pi}^* = \mathbf{A}^*\mathbf{B}^*$, where $\mathbf{A}^*$ and $\mathbf{B}^{*\prime}$ are full column $8 \times 6-$matrices and the cointegration matrix can be written

$$\mathbf{B}^* = \begin{bmatrix} \mathbf{\Phi}^2 & \mathbf{0} \end{bmatrix},$$

where $\mathbf{0}$ is 6×1 column vector of zeros and $\mathbf{\Phi}^2$ is given by (8).

4 Common Periodic Correlation

4.1 Contemporaneous Common Periodic Correlation

When building PAR models it is recommended (see Ghysels and Osborn, 2001) to introduce restrictions on the periodic components of the model to increase the degrees of freedom. One way to do this is by imposing (after appropriately tested) common feature restrictions on the model like common business cycles, common stationary annual seasonality, or common deterministic annual seasonality[5]. It is a very plausible assumption that the daily series $y_{s,\tau}$, in addition to the trend, will share common cyclical features across the days of the week. To our knowledge, these kind of restrictions to describe the common periodicity of the cycles to further restrict the seasonal features of a univariate time series model have not yet been proposed in the literature.

Engle and Kozicki (1993) introduced the notion of serial correlation common features to represent common cycles among different economic time series. A multivariate time series x_τ is said to pose a serial correlation common features if there exists a $n \times q$ matrix $\widetilde{\beta}$ that satisfies, see (4):

$$\begin{aligned} &\text{(i)} \quad \widetilde{\beta}'\Gamma_k &=& \ 0 \qquad (k = 1, ..., P-1), \\ &\text{(ii)} \quad \widetilde{\beta}'\Pi &=& \ -\widetilde{\beta}'\alpha\beta' = 0. \end{aligned}$$

Under these two conditions the cofeature matrix $\widetilde{\beta}$ turns the differenced variables into a s-dimensional white noise process $\widetilde{\beta}'\Delta_7 x_\tau = \widetilde{\beta}' u_\tau$, and the short-run dynamics of the n series are driven by $n-q$ dynamic factors In our periodic setting, conditions i) and ii) imply the restriction $\Phi_0 = \Phi_1$. However, in this case the number of common features $n-q$ is bounded by the cointegration rank r, $r \leq n-q \leq n$, and then when

[5] In the sequel we refer the notion 'periodic' to the daily variation of the data whereas the remaining seasonality (e.g. within the year) is referred to simply as 'seasonality'.

the daily series is PI(1)/I(1), then the number of common features may be 6 or 7 ($6 \leq 7-q \leq 7$; or $0 \leq q \leq 1$). Thus, the serial correlation common features allows a very minor restriction of the periodicity of the process.

Hecq *et al.* (2004) consider a less restrictive form of common cycles and introduce the idea of a Weak Form (WF) Reduced Rank Structure, which implies that only the condition (i) should hold. Under i) but not ii) different common factors generate the long-run and the short-run dynamics of the variables, see Hecq *et al.*, (2004) for details. In the present case, under the WF structure there exists a $n \times q$ dimensional matrix $\widetilde{\beta}$, the cofeature matrix, that turns the differenced variables adjusted for long-run effects into a q-dimensional white noise process $\widetilde{\beta}'\left(\Delta_7 x_\tau - \alpha z_{\tau-1}\right) = \widetilde{\beta}'\varsigma_\tau + \widetilde{\beta}' u_\tau$, such that the cointegrated system can be expressed as

$$\Delta_7 x_\tau = \varsigma_\tau + \alpha z_{\tau-1} + \widetilde{\beta}_\perp \mathbf{w}_{\tau-1} + u_\tau,$$

where $\mathbf{w}_\tau = \boldsymbol{\Upsilon}\boldsymbol{\omega}_\tau$ are the common features, $\boldsymbol{\omega}_\tau = (\Delta_7 x'_\tau,...,\Delta_7 x'_{\tau-P+2},)'$, $\boldsymbol{\Upsilon} \equiv [\Upsilon_1,...,\Upsilon_{P-1}]$ is a $(7-q) \times (7(P-1))$ matrix, $\widetilde{\beta}_\perp$ is a $n \times (n-q)$ full column rank matrix satisfying $\widetilde{\beta}'\widetilde{\beta}_\perp = 0$ (and $\widetilde{\beta}'\widetilde{\beta}_\perp \boldsymbol{\Upsilon}\boldsymbol{\omega}_\tau = 0$). Condition i) applied to our periodic model implies the following restrictions

$$\begin{aligned}
\Phi_1 &= \Phi_0(\alpha\beta' - \widetilde{\beta}_\perp \Upsilon_1) + I_n, \\
\Phi_k &= \Phi_0 \widetilde{\beta}_\perp (\Upsilon_{k-1} - \Upsilon_k) \; k = 2, ..., P-1, \\
\Phi_P &= \Phi_0 \widetilde{\beta}_\perp \Upsilon_{P-1}.
\end{aligned}$$

The notion of WF Reduced Rank Structure is much more flexible than the serial correlation common features, since now the number of common features $(n-q)$ is not bounded by the cointegrating rank. Concretely, the number of common features may take any value

$$1 \leq n - q \leq n$$

For example, in the case of the multivariate representation of one of the daily series, then $1 \leq 7 - q \leq 7$. When $7 - q = 1$ or $q = 6$, then the short-run dynamics of the seven weekly series associated to the daily series are driven by the same dynamic factor. On the other extreme when $7 - q = 7$ or $q = 0$, then the short-run dynamics of the seven weekly series is generated by different factors. In our framework when $0 < q < 7$, then we name such common features as common periodic correlation (CPC), and $n - q$ will denote the number of CPCs.

4.2 Non-contemporaneous Common Periodic Correlation

When considering the presence of common periodic cyclical features between different variables, say the arrivals and the departures, it is likely that such variables will exhibit non-contemporaneous cyclical co-movements, a property that is not captured by the CPC described in the preceding section. Cubadda and Hecq introduce the notion of a polynomial serial correlation common feature, which as suggested in Hecq *et al.* (2004) can be also considered in its WF form, which we name generically polynomial CPC. The notion of polynomial common periodic cycles applies to our periodic setting, better than to macroeconomic variables, typically quarterly series. Then, the PI(1) process x_τ has polynomial CPC of order $m+1$, denoted CPC($m+1$), if there exists a $7 \times q_m$ polynomial matrix $\widetilde{\beta}_m(L) = \sum_{j=0}^{m+1} \widetilde{\beta}_{mj} L^j$ such that $\widetilde{\beta}_0$ is full column rank and

$$\widetilde{\beta}'_{m0}\Gamma_k = \begin{cases} -\widetilde{\beta}'_{mk} & \text{if } k = 1, ..., \ m, \\ 0 & \text{if } k > m. \end{cases}.$$

Under CPC(m), the cofeature matrix reduces the order of the VECM model from $P-1$ to m, $\widetilde{\beta}'_{m0}\left(\Delta_7 x_\tau - \alpha z_{\tau-1}\right) = \widetilde{\beta}'_{m0}\varsigma_\tau - \widetilde{\beta}'_{m1}\Delta_7 x_{\tau-1} - ... - \widetilde{\beta}'_{mm}\Delta_7 x_{\tau-m} + \widetilde{\beta}'_{m0}u_\tau$, such that under CPC($m+1$) the cointegrated system can be written as

$$\Delta_7 x_\tau = \varsigma_\tau + \alpha z_{\tau-1} + \sum_{k=1}^{m} \Gamma_k \Delta_7 x_{\tau-k} + \widetilde{\beta}_{m0\perp} \sum_{k=m+1}^{P-1} \Upsilon_j \Delta_7 x_{\tau-j} + u_n,$$

where Υ_j are $g_m(\equiv n - q_m) \times 7$ matrices, $\widetilde{\beta}_{m0\perp}$ is $7 \times g_m$ full column rank matrix satisfying $\widetilde{\beta}'_{m0}\widetilde{\beta}_{m0\perp} = 0$, and $f_{m,n} = \sum_{k=m+1}^{P-1} \Upsilon_j \Delta_7 x_{\tau-j}$ are the g_m common features of the short-run component or the CPC(m) factor. The presence of the CPC(m) implies restrictions on the periodic coefficients in a similar way than CPC(0) but now involving only more distant lags, and therefore implies complex restrictions among the the ACFs.

Notice that CPC(m) of different orders may cohabit in the VECM model, potentially imposing a rich structure on the underlying PeACF. For the presence of different CPC(0),CPC(1),...,CPC(m^*), then there must exists the cofeature matrix polynomials $\widetilde{\beta}_0(L) = \widetilde{\beta}_0$, $\widetilde{\beta}_1(L)$,...,$\widetilde{\beta}_{m^*}(L)$ such that $0 < q_0 < q_1 < ... < q_{m^*} < 6$ or in terms of number of common factors $1 < g_{m^*} ... < g_1 < g_0 < 7$. The presence of multiple CPCs implies then that the short-run matrices of the VECM are of reduced rank, and in addittion the left null space is nested (see Ahn and Reinsel, 1988). Under such property the short-run component can be estimated with a much more parsimonious representation.

To illustrate this let us consider the case $S = 7$ and $P = 3$, and assume that the process has 2 CPC(0) and 1 CPC(1), such that the associated cofeature matrices $\widetilde{\beta}_0$ and $\widetilde{\beta}_1$ are of dimension 1×7 and 2×7, and therefore the orthogonal matrices $\widetilde{\beta}_{0\perp}$ and $\widetilde{\beta}_{1\perp}$ are of dimension 2×7 and 1×7, respectively. Then, we may write the restricted VECM model

$$\Delta_7 x_\tau = \alpha z_{\tau-1} + \widetilde{\beta}_{0\perp} \Upsilon_1 \Delta_7 x_{\tau-1} + \widetilde{\beta}_{1\perp} \Upsilon_2 \Delta_7 x_{\tau-2} + u_n, \tag{10}$$

where Υ_1 and Υ_2 are of conformable dimension. The nested reduced rank implies that $\widetilde{\beta}_{1\perp} = \widetilde{\beta}_{0\perp}(\mathbf{1}, \mathbf{0})'$, and therefore we may write the model

$$\Delta_7 x_\tau = \alpha z_{\tau-1} + \widetilde{\beta}_{0\perp} \Upsilon_1 \Delta_7 x_{\tau-1} + \widetilde{\beta}_{0\perp} \Xi_2 \Delta_7 x_{\tau-2} + u_n,$$

where $\Xi_2 = (\mathbf{1}, \mathbf{0})' \Upsilon_2$.

5 Empirical Application

To perform both the univariate and the multivariate analyses of the airport transit data described in section 2, we specify the following model

$$\Delta_7 \mathbf{x}_\tau = \boldsymbol{\mu} + \boldsymbol{\Psi} \mathbf{d}_\tau + \boldsymbol{\Theta} \mathbf{c}_\tau + \boldsymbol{\Pi} \mathbf{x}_{\tau-1} + \sum_{k=1}^{P-1} \boldsymbol{\Gamma}_k \Delta_7 \mathbf{x}_{\tau-k} + \mathbf{u}_\tau, \tag{11}$$

where $\boldsymbol{\mu}$ is a $(n \times 1)$ vector of unrestricted intercepts (and linear trends when $\mathbf{x}_\tau$ includes some or all the stock series), Ψ is a $n \times 12$ matrix of unrestricted parameters, $\boldsymbol{\Theta}$ is a $n \times 5$ matrix of unrestricted parameters, $\boldsymbol{\Pi}$ and $\boldsymbol{\Gamma}_k$ are $n \times n$ matrices possibly of reduced rank, $\mathbf{d}_\tau$ is a matrix of 12 trigonometric variables $\cos(j\pi/26 \times \tau)$ and $\sin(j\pi/26 \times \tau)$ $(j = 1, ..., 6)$ to account for the deterministic seasonality, $\mathbf{c}_\tau$ is a matrix of dummy variables to account for different calendar effects.[6] The $\mathbf{x}_t$ vector consists of various combinations of the transit series, i.e. arrivals y_t^1, departures y_t^2, net flow $y_t^3 = y_t^2 - y_t^1$, and the stock of visitors variable $y_t^4 = y_0^4 + \sum_{j=1}^{t} y_t^3$. For the periodic integration and CPC(0) analyses $n = 7$ and $\mathbf{x}_\tau = (y_{1,\tau}^i, ..., y_{7,\tau}^i)'$ $(i = 1, 2, 3, 4)$; whereas for the periodic cointegration analyses and multiple CPC(m) analysis $n = 14$ and $\mathbf{x}_\tau = (y_{1,\tau}^1, ..., y_{7,\tau}^1, y_{1,\tau}^2, ..., y_{7,\tau}^2)'$. Based upon the univariate empirical evidence it shows useful for the periodic multicointegration analysis to consider $n = 8$ and

[6] Concretely, we introduce five calendar type dummy variables: $EAST_\tau$ that takes value one the week of Easter and zero otherwise, XMA_τ takes value one each Christmas, $ENDY_\tau$ accounts similarly for the end-of-the-year, $MAY1_\tau$ deals with the May-the-first and $ALLS_\tau$ captures the All-Saints week.

$\mathbf{x}_\tau = (y^2_{1,\tau}, ..., y^2_{7,\tau}, y^4_{7,\tau})'$.

The daily series are filtered from additive outliers to prevent the potential distortionary effect of such outliers on the cointegration analysis (see Haldrup *et al.*, 2004).[7] All these outliers are capturing effects not collected by the calendar effect variables.

The order P of the VAR has been chosen according to the AIC criterion which performs reasonably well within high dimensional systems (see Gonzalo and Pitarakis, 2002) such that for the univariate analysis $P = 8$ is selected for arrivals and departures y^1_t and y^2_t and $P = 3$ for the net flow and stock variables y^3_t and y^4_t. For the multivariate analysis $P = 8$ is selected for both the periodic cointegration and the periodic multicointegration analyses.[8]

[7] Concretely, 11 outliers are detected at the arrivals series at dates 31/03/1994, 13/04/1995, 25/04/1995, 04/04/1996, 27/03/1997, 9/04/1998, 30/04/1998, 01/04/1999, 20/04/2000, 07/04/2001, and 12/04/2001; 12 outliers are found at the departures series at dates 04/04/1994, 17/04/1995, 08/04/1996, 31/03/1997, 31/10/1997, 13/04/98, 31/10/1998, 05/04/1999, 28/10/1999, 30/10/1999, 28/10/2000, 16/04/2001; 12 outliers are estimated at the flow series at time points 31/10/1997, 30/10/1998, 31/10/1998, 26/10/1999, 28/10/1999, 29/10/1999, 30/10/1999, 27/04/2000, 20/10/2000, 28/10/2000, 30/10/2000, and 07/04/2001; and one outlier at 24/10/1999 is selected at the stock series.

[8] These lags imply a maximum lag for the daily models of $p = 62$ and $p = 27$ days.

5.1 Testing for periodic integration and cointegration amongst stocks and flows

5.1.1 The univariate series

The rank of the matrix $\mathbf{\Pi}$ has been determined according to the Johansen procedure (see, for instance, Johansen 1991). Table 1 reports the trace statistic (LR_r) of the Johansen procedure and the estimated coefficients ϕ_s of the quasi-difference operators $\delta_s(L) \equiv 1 - \phi_s L$ associated with the cointegrating vectors of the four series. The cointegration analysis of the VD series corresponding to the daily arrivals and departures provides strong evidence favouring the PI(1)/I(1) characteristic of such series by detecting 6 cointegrating relations.[9] The fact that 6 cointegrating relations are present means that the 7 daily series exhibit the same stochastic trend and hence this implies that the series cannot be seasonally integrated, (Hylleberg *et al.* (1990) and Franses (1994)). The cointegration analysis of the net flow series does not detect any cointegrating relationships, while the cointegration analysis of the stock series again detects a cointegration rank of 6 and hence suggests the series to be PI(1) or I(1) depending upon further restrictions. PI(1) against I(1) of the arrivals, departures

[9] New critical values for the LR_r test have been computed to account for the specific characteristics of the fitted weekly multivariate model. Specifically, we tabulated critical values for a 7-dimensional random walk (independent) processes with 425 observations and including the same calendar efects and (trygonometric) deterministic seasonality specified in model (11).

and stocks can be tested by restricting the value of the cointegrating vectors. In particular, we reject the nonperiodicity of the cointegrating vectors and that this are (1,$-$1) for the arrivals and departures, while we do not reject such property for the stocks. Therefore, from the cointegration analysis we conclude that both the arrivals and departures are PI(1), and hence potentially the series are nonperiodically cointegrated with vector (1,-1), given that the net flow series is stationary, and the stock series is nonperiodically integrated I(1).

5.1.2 Bivariate analyses and multicointegration

The univariate properties detected in the preceding section have several implications for the multivariate analysis. First, the daily arrivals y_t^1 and daily departures y_t^2 are PI(1), and the net flow are I(0) which indicates that the flow variables are potentially nonperiodically cointegrated with cointegrating vector $(1, -1)'$, $y_{s,\tau}^1 - y_{s,\tau}^2$ $(s = 1, ..., 7)$. Secondly, due to the I(1)-ness of the stock variable y_t^4 and the PI(1)-ness of the flow variables, then possible cointegration amongst the stocks and flows (i.e. multicointegration) will be periodic whereby $y_{s,\tau}^4 - k_s y_{s,\tau}^2$ is I(0) $(s = 1, ..., 7$, and at least one $k_s \neq k)$.

Let us consider the multivariate analysis of the weekly flow series $\mathbf{x}_\tau \equiv (y_{1,\tau}^1, ..., y_{7,\tau}^1, y_{1,\tau}^2, ..., y_{7,\tau}^2)'$. For the extended system we follow the same sequence as we did for the univariate analysis, that is we first test for cointegration rank, and next test hypothesis regarding

Table 1: Periodic Integration Analysis.

	LR_0	LR_1	LR_2	LR_3	LR_4	LR_5	LR_6
y_t^1	168.51^{***}	114.49^{***}	76.70^{***}	50.11^{***}	28.54^{***}	11.41^{**}	1.28
y_t^2	181.64^{***}	115.55^{***}	80.39^{***}	52.99^{***}	32.09^{***}	14.65^{***}	2.31
y_t^3	985.82^{***}	733.18^{***}	545.96^{***}	397.70^{***}	272.08^{***}	168.67^{***}	70.55^{***}
y_t^4	850.82^{***}	634.46^{***}	438.52^{***}	271.06^{***}	167.71^{***}	73.15^{***}	2.91

Periodic Integration Coefficients

	$\widehat{\phi}_1^i$	$\widehat{\phi}_2^i$	$\widehat{\phi}_3^i$	$\widehat{\phi}_4^i$	$\widehat{\phi}_5^i$	$\widehat{\phi}_6^i$	$\widehat{\phi}_7^i$
y_t^1	0.910	0.561	3.039	0.683	1.100	1.390	0.617
y_t^2	0.889	0.551	3.087	0.602	1.044	1.583	0.665
y_t^3	0.000	0.000	0.000	0.000	0.000	0.000	0.000
y_t^4	0.999	0.999	1.000	0.998	0.997	1.001	1.006

Note: LR_r stands for the trace statistic of Johansen. *(**,***) indicates rejection of the null hypothesis at 10% (5%,1%).

the cointegrating space.

Table 2: Cointegration Analysis.

	Trace Test						
	LR_0	LR_1	LR_2	LR_3	LR_4	LR_5	LR_6
	769.77***	594.74***	475.16***	376.21***	300.01***	232.61***	175.08***
	LR_7	LR_8	LR_9	LR_{10}	LR_{11}	LR_{12}	LR_{13}
	124.88***	90.72***	61.25***	40.81***	22.20***	9.40	1.35
	Cointegrating Relations						
	$\widehat{k}_1$	$\widehat{k}_2$	$\widehat{k}_3$	$\widehat{k}_4$	$\widehat{k}_5$	$\widehat{k}_6$	$\widehat{k}_7$
	1.003	1.043	1.013	1.105	1.237	1.030	0.977
	$\widehat{\phi}_1^i$	$\widehat{\phi}_2^i$	$\widehat{\phi}_3^i$	$\widehat{\phi}_4^i$	$\widehat{\phi}_5^i$	$\widehat{\phi}_6^i$	$\widehat{\phi}_7^i$
y_t^1	0.895	0.592	3.125	0.646	1.156	1.331	0.609
y_t^2	0.872	0.569	3.219	0.592	1.032	1.598	0.642

Note: See note of table 1.

The trace test does not reject the null hypothesis for $r = 12, 13$, but the likelihood ratio test ($LR_{12} = 9.40$) is rather close to the 10% critical value 9.67, which leads us to conclude that the rank equals 13 and hence the daily arrivals and departures are cointegrated. The analysis of PI(1)-ness of the flow variables can also be done from the multivariate model of the flows $\mathbf{x}_\tau$ for $r = 13$. We test for non PI(1)ness through the linear hypothesis H$_0$: $\phi_1 = ... = \phi_6 = 1$, and obtain LR=22.38 which rejects at

1%, and therefore we reinforce the evidence about PI(1)-ness of the daily departures.

The next step is to test for nonperiodic cointegration between the arrivals and departures H$_0$: $k_s = 1$, but in this case, and contrary to the univariate findings we reject the null at 1% level with LR=21.21. From the estimated k_ss we recognize that the k_5 coefficient, that is the cointegrating coefficient associated with Friday arrivals and Friday departures, is significatively different from 1. A test for equal cointegrating vectors for all days with the exception of Fridays H$_0$: $k_1 = k_2 = k_3 = k_4 = k_6 = 1$ cannot be rejected (LR=7.64). This slight departure from unity of the k_5 coefficient may be explained by the different periodic integration coefficients of Saturday arrivals and Saturday departures, which are respectively $\widehat{\phi}_6^1 = 1.331$, $\widehat{\phi}_6^2 = 1.598$. This result is also found in the univariate analysis yielding the estimates $\widehat{\phi}_6^1 = 1.390$ and $\widehat{\phi}_6^2 = 1.583$ (see panel B of table 1).

Finally, we test for periodic multicointegration by considering the cointegration analysis of the 8-dimensional process $\mathbf{x}_\tau^* \equiv (y_{1,\tau}^4, ..., y_{7,\tau}^4, y_{7,\tau}^2)'$. This system is motivated by the fact that the stock series is non-periodic I(1) and hence any day of the week series will represent the stochastic trend of the stock series. The results of this analysis is presented in table 3.

As seen the cointegration rank is 7 and hence suggesting the stock to cointegrate with the departure series. Because the stock variable is defined from the net flows the series exhibit multicointegration, i.e. the weekly trend of the arrivals and departures

Table 3: Multicointegration Analysis.

LR_0	LR_1	LR_2	LR_3	LR_4	LR_5	LR_6	LR_7
384.81***	288.87***	214.19***	147.03***	92.61***	49.42***	18.60***	0.55
Multicointegrating Relations							
$\widehat{k}_1$	$\widehat{k}_2$	$\widehat{k}_3$	$\widehat{k}_4$	$\widehat{k}_5$	$\widehat{k}_6$	$\widehat{k}_7$	
54.797	96.082	29.852	50.332	48.654	30.493	47.775	

Note: See note of table 1.

share a common stochastic trend. It can also be seen from table 3 that the estimated multicointegrating vectors are different across the days of the week, which implies that arrivals and departures are periodically multicointegrated.

5.2 Testing for common periodic cyclical features

5.2.1 The univariate series

Given the evidence of the cointegration analysis in the previous section, we test for the presence of WF common cycles of y_t^1, y_t^2, and y_t^4 with a likelihood ratio test ($\xi_{m=0}(q)$) given by Hecq *et al.* (2004) for the case of contemporaneous cycles and with a likelihood ratio test given by Cubadda and Hecq (2001) for the case of polynomial cycles ($\xi_{m>0}(q)$). Because the estimated cointegrating rank $r = 6$ for all the cases, we can safely concentrate out the cointegrating vectors without effecting the limiting distribution.[10]

Table 4: Common Periodic Cycles Analysis of the Weekly Arrivals.

q	$\xi_{m=0}(q)$	$\xi_{m=1}(q)$	$\xi_{m=2}(q)$	$\xi_{m=3}(q)$	$\xi_{m=4}(q)$	$\xi_{m=5}(q)$	$\xi_{m=6}(q)$
1	50.39	33.38	28.34	15.34	10.71	4.37	0.02
2	107.22*	77.47	65.00	35.78	27.82	14.28	0.54
3	196.99***	139.77*	113.24*	62.61	47.59	26.05	1.82
4	306.10***	210.00***	167.96**	107.96	78.75	43.57	10.00
5	432.96***	303.89***	228.06***	154.76*	122.33**	64.56	23.61
6	581.74***	412.46***	307.06***	228.13***	185.20***	108.84**	44.82
7	862.96***	637.36***	503.77***	337.59***	277.53***	168.61***	75.92***

Note: $LR(q)_m$ stands for the likelihood ratio test. *(**,***) indicates rejection of the null hypothesis at 10% (5%,1%).

From the results of the LR test we have that, at the 1% level, $q_1 = 1$, $q_2 = q_3 = 2$, $q_4 = q_5 = 4$, $q_6 = 5$, and $q_7 = 6$, such that the weekly arrivals have 1 CPC(0), 2 CPC(1), 4 CPC(4), 5 CPC(5) and 6 CPC(6) features, which implies that the ranks of the short-run matrices $r_j = rank(\Gamma_j)$, are $r_1 = 6$, $r_2 = r_3 = 5$, $r_4 = r_5 = 3$, $r_6 = 2$, and $r_7 = 1$. Under the common features restrictions the short-run dynamics of the

[10]See the limiting result of Paruolo (2002); and the finite sample results of Hecq *et al.* (2004)).

arrivals are captured by 175 parameters, while without imposing such restrictions 343 parameters need to be estimated in the standard VECM model. To estimate the short-run parameters we should apply Ahn and Reinsel (1988) nested reduced rank regression approach.

The CPC structure for the departures is slightly different than the one for the arrivals. From table 5, the weekly arrivals have 1 CPC(0), 2 CPC(1), 3 CPC(2), 4 CPC(5), and 5 CPC(6) features.

Table 5: Common Periodic Cycles Analysis of the Weekly Departures.

q	$\xi_{m=0}(q)$	$\xi_{m=1}(q)$	$\xi_{m=2}(q)$	$\xi_{m=3}(q)$	$\xi_{m=4}(q)$	$\xi_{m=5}(q)$	$\xi_{m=6}(q)$
1	49.12	36.53	24.49	11.65	9.31	2.56	0.01
2	120.19**	81.42	58.56	40.49	30.80	13.84	1.14
3	211.65***	137.26*	107.46	81.31	58.17	31.55	4.67
4	307.36***	212.90***	173.78***	141.49***	102.82***	50.98	15.22
5	441.22***	306.19***	254.94***	209.20***	160.48***	88.58***	29.24
6	581.41***	422.86***	345.65***	292.86***	229.93***	138.92***	58.77***

See table 4.

Finally, table 6 shows that the weekly stock have 6 CPC(0), which implies that only one common factor is driving the short-run dynamics of the 7 stocks. This finding confirms the nonperiodic behavior of the stock of visitors, which allows to treat such variable as nonperiodic in the multivariate analysis, not only to capture the multicointegration property but also to capture the short-run dynamics of the flow and stock system.

Table 6: Common Periodic Cycles Analysis of the Weekly Stock

q	$\xi_{m=0}(q)$	$\xi_{m=1}(q)$	$\xi_{m=2}(q)$
1	4.90	2.10	0.04
2	17.61	8.54	0.49
3	37.09	18.73	1.54
4	59.75	32.11	3.48
5	92.30	48.72	11.32
6	135.96	72.61	31.02

See table 4.

6 Conclusion

We have proposed the nonstationary periodic autoregressive process with colineal ACFs as a very useful model to capture the complex dynamics of daily economic time series. We propose the notion of common polynomial cycles as a useful class of restrictions to capture the colinealities among the ACF of the daily periodic process.

We have illustrated the potential usefulness of our statistical model by testing for PIness, periodic cointegration, periodic multicointegration and common periodic cycles at the daily transit of Mallorca airport (1994-2002). The presence of multiple polynomial common periodic cycles is a quite promising avenue for future research since it implies a nested reduced rank structure for the multivariate weekly model which allow a more efficient estimate of a highly parameterized model.

References

Ahn, S.K. and G.C. Reinsel (1988) Nested Reduced-Rank Autoregressive Models for Multiple Time Series. *Journal of the American Statistical Association* 403, 849-856.

Cubadda, G. and A. Hecq (2001) On Non-Contemporaneous Short-Run Co-Movements. *Economics Letters* 73, 389-397.

Engle, R.F. and S. Kozicki (1993) Testing for Common Features. *Journal of Business & Economic Statistics* 11, 369-380.

Franses, P.H. (1994) A multivariate approach to modeling univariate seasonal time series. *Journal of Econometrics* 63, 133-151.

Franses, P.H. (1996) *Periodicity and Stochastic Trends in Economic Time Series.* Oxford University Press, Oxford.

Franses, P.H. and R. Paap (2004) *Periodic Time Series Models.* Oxford University Press.

Ghysels, E. and D.R, Osborn (2001) *The Econometric Analysis of Seasonal Time Series.* Cambridge University Press.

Gladyshev, E.G. (1961) Periodically correlated random sequences. *Sov. Math.* 2, 385-388.

Gonzalo, J. and J-Y. Pitarakis (2002) Lag length estimation in large dimensional systems. *Journal of Time Series Analysis* 23. 401-423.

Granger, C.W.J. and T.H. Lee (1989) Investigation of production, sales and inventory relations using multicointegration and non-symmetric error correction models. *Journal of Applied Econometrics* 4, S145-S159.

Granger, C.W.J. and T.H. Lee (1990) Multicointegration. In *Long-Run Economic Relationships. Reading in Cointegration, Advanced Texts in Econometrics*, Engle, R.F., C.W.J. Granger (eds). Oxford University Press: Oxford.

Haldrup, N., A. Montanés, and A. Sansó, 2004, Measurement Errors and Outliers in Seasonal Unit Root Testing. Forthcoming *Journal of Econometrics*

Hecq A., F.C. Palm and J-P. Urbain (2004) Common cyclical features analysis in VAR models with cointegration. *Forthcoming in Journal of Econometrics.*

Hylleberg, S., R.F. Engle, C.W.J. Granger, and S.Yoo (1990) Seasonal Integration and Cointegration. J*ournal of Econometrics* 44, 215-238.

Johansen, S. (1991) Estimation and Hypothesis Testing of Cointegrating Vectors in Gaussian Vector Autoregressive Models. *Econometrica* 59, 1551-80.

Osborn, D. (2002) Cointegration for Seasonal Time Series Processes. *Mimeo.* University of Manchester.

Paruolo, P. (2002) Common features and common I(2) trends in VAR systems. *Mimeo.* University of Insubria.

Tiao, G.C. and M.R. Grupe (1980) Hidden Periodic Autoregressive-Moving Average Models in Time Series Data. Biometrika 67, 365-373.