Are there threshold effects in the stock price-dividend relation?: The case of the US stock market, 1871-2004*

Vicente Esteve†
Universidad de Valencia, Spain

María A. Prats
Universidad de Murcia, Spain

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Abstract

We use recent developments on threshold autoregressive (TAR) models that allow to derive endogenously threshold effects, to analyze the evolution of the US stock price-dividend relation over the period 1871-2004. More specifically, a mean-reverting dynamic behaviour of the stock price-dividend ratio should be expected once such threshold is reached. Our empirical results showed that significant adjustments would occur when, in a particular year, the stock price-dividend ratio had shown a decrease of more than 8.0% between the previous year and the fourth year before, which implies nonlinearities in the dynamic behaviour of the US stock price-dividend relation.

Keywords: Present value model; Nonliearities; Threshold effects; TAR models; Stock prices; Dividends

JEL classification: G12.

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†*Corresponding author*: Departamento de Economia Aplicada II, Universidad de Valencia, Avda. dels Tarongers, s/n, 46022 Valencia, Spain. Fax: +34-96-3828354. e-mail: vicente.esteve@uv.es.

1 Introduction

Over the last decades, the influence of the linear present value (PV) model to explain the behavior of aggregate US stock prices has been actively investigated. In particular, and according to several empirical studies, the linear PV model fails to explain the behavior of stock prices in the long-run; see, e. g., the surveys by Bohl and Siklos (2004) and Kanas (2005). This article examines whether this failure of the linear PV model can be attributed to non-linearities in the stock price-dividend relation.

As discussed in Campbell, Lo and MacKinlay (1997), when expected stock returns are time varying, the correct theoretical framework for the analysis of the PV model is nonlinear. In addition, several recent theoretical models have explicitly introduced nonlinearities in the relationship between stock prices and dividends. A possible reason for such non-linear effects has been pointed out in an important paper by Krugman (1987). More specifically, this author suggests that trigger-price sell strategies are followed by private investors participating in portfolio insurance schemes who commit themselves to buying or selling when the stock price reaches a predetermined threshold level. Cecchetti, Lam and Mark (1990) show that nonlinearity in the stock price-dividend relation may arise within an equilibrium model of asset price determination which combines a nonlinear endowment process of dividends (a Markov switching model), and investors that attempt to smooth their consumption. Finally, Froot and Obstfeld (1991) propose a standard PV model with intrinsic bubbles where speculation by rational investors would create threshold effects in the stock price-dividend relation.

On the other hand, some empirical studies that investigate the presence of nonlinearities in the stock price-dividend relation have recently appeared. For instance, Kanas (2003) provides some empirical evidence of nonlinearities in the PV model using US annual data for 1871-1999, and following the procedure for non-linear cointegration suggested by Granger and Hallman (1991). More recently, Kanas (2005) has used three nonlinear nonparametric techniques (i.e., nonlinear cointegration, locally-weighted regression, and nonlinear Granger causality tests), also obtaining evidence on the existence of nonlinearities in the stock price-dividend relation for the UK, the US, Japan, and Germany, using monthly data for the period 1978:1-2002:5.

This article tests empirically whether there have been nonlinearities in the stock price-dividend relation for the US market over the period from 1871 to 2004. We use recent developments on threshold autoregressive (TAR) models that allow to derive endogenously threshold effects in the evolution of the US stock price-dividend ratio. Nonlinearity is tested by means of the technique developed by Hansen (1996, 1997, 2000) and Caner and Hansen (2001), to test for a threshold whose location is unknown a priori. In this way, a mean-reverting dynamic behavior of the US stock price-dividend ratio should be expected once such threshold is reached.

The rest of the paper is organized as follows. The econometric methodology is outlined in Section 2, the empirical results are presented in Section 3, and the main conclusions are summarized in Section 4.

2 Econometric methodology

Here we follow the framework of Hansen (1996, 1997, 2000) and Caner and Hansen (2001) who have developed new methods for TAR models along the lines of previous work by Tong (1978, 1983, 1990), who considered the possibility of a mean reverting time series only after hitting a certain threshold. Hansen (1996, 1997, 2000) and Caner and Hansen (2001) showed how to test for threshold effects, estimate the threshold parameter, and construct asymptotic confidence intervals for the threshold parameter.

More specifically, we consider a two-regime TAR (k) model with an autoregressive unit root that takes the form:

$$\Delta y_t = \theta_1' x_{t-1} 1_{\{Z_{t-1}<\lambda\}} + \theta_2' x_{t-1} 1_{\{Z_{t-1}\geq\lambda\}} + e_t \tag{1}$$

for $t = 1, ..., T$, where $x_{t-1} = (y_{t-1} r_t' \Delta y_{t-1} ... \Delta y_{t-k})'$, $1(.)$ is the indicator function, e_t is an iid error, $Z_t = y_t - y_{t-m}$ for some $m \geq 1$ is the threshold variable, and r_t is a vector of deterministic components including an intercept, μ, and possibly a linear time trend, t. The autoregressive order is $k \geq 1$. The threshold parameter λ is unknown and represents the level of the variable y_t that triggers a "regime change".

We find convenient to discuss separately the components of the two regimes $\theta_1(\rho_1, \beta_1, \alpha_1)'$ and $\theta_2(\rho_2, \beta_2, \alpha_2)'$, so that equation (1) can be written with $k = 1$, a linear trend, and $Z_{t-1} = \Delta y_{t-1}$, as:

$$\begin{aligned} \Delta y_t &= (\rho_1 y_{t-1} + \beta_1 t + \mu_1 + \alpha_1 \Delta y_{t-1}) 1_{\{\Delta y_{t-1}<\lambda\}} \\ &\quad + (\rho_2 y_{t-1} + \beta_2 t + \mu_2 + \alpha_2 \Delta y_{t-1}) 1_{\{\Delta y_{t-1}\geq\lambda\}} + e_t \end{aligned} \tag{2}$$

where (ρ_1, ρ_2) are the slope coefficients on the lagged levels, (β_1, β_2) the slopes on the deterministic components, and (α_1, α_2) the slope coefficients on the lagged differences for the two regimes (θ_1, θ_2).

Since either (1) or (2) are both regression equations (albeit nonlinear in parameters), an appropriate estimation method for the TAR (k) model is least squares (LS). Caner and Hansen (2001) show how under the auxiliary assumption that e_t is iid $N(0, \sigma^2)$, LS is equivalent to the maximum likelihood estimation. In this case, the estimates can be used to conduct inference concerning the parameters of (1) using standard Wald statistics. The LS point estimate of the threshold λ and of the corresponding vectors $\theta_1$and θ_2, are those that minimize the residual sum of squares.

The main question in model (1) is whether or not there is a threshold effect. In order to test the null hypothesis of linearity (i.e., the threshold effect disappears and $\theta_1 = \theta_2$), against the alternative of a threshold effect (i.e., the process is nonlinear), Hansen (1997) and Caner and Hansen (2001) propose a standard heteroskedastic-consistent Wald or Langrage multiplier test, $\sup W_T(\lambda)$, where the estimated threshold point estimate, $\hat{\lambda}$, and the corresponding vectors θ_1 and θ_2 are estimated by LS. They show that the asymtotic distribution of $W_T(\lambda)$ has a nonstandard asymptotic null distribution. This is partially due to the presence of a parameter that is not identified under the null, and partially due to the assumption of a nonstationary autoregression, so that critical values cannot be tabulated. Caner and Hansen (2001) suggest calculating the bootstrap critical values and p-values both ways, and base inference on the more conservative (larger) p-values. Furthermore, the authors suggest that setting the bounds of the trimming regions to $\pi_1 = 0.15$ and $\pi_2 = 0.85$ provides a reasonable trade-off between power and size properties of the test for threshold effects; see Caner and Hansen (2001) for details.

3 Empirical results

In this section, we analyze the possible presence of nonlinearities in the US stock price-dividend ratio, pdr_t, over the period 1871-2004, using the methodology presented in the previous section. The series on real stock price and dividends are taken from Robert Shiller's website http://www.econ.yale.edu/~shiller/data/. The stock price index is the January values of the Standard & Poor's 500 Composite Stock Price index; the evolution of the real stock price-dividend ratio is shown in Figure 1.

As a first step of the analysis, we have tested for the order of integration of the stock price-dividend ratio. To this end, we have used a modified version of the Dickey-Fuller and Phillips-Perron tests proposed by Ng and Perron (2001), which try to solve the main problems present in these conventional tests for unit roots.

In general, the majority of the conventional unit root tests suffer from three problems. First, many tests have low power when the root of the autoregressive polynomial is close to, but less than, unit (Dejong *et al.,* 1992). Second, the majority of the tests suffer from severe size distortions when the moving-average polynomial of the first-differenced series has a large negative autoregressive root (Schwert, 1989; Perron and Ng, 1996). Third, the implementation of unit root tests often needs the selection of an autoregressive truncation lag, k; however, as discussed in Ng and Perron (1995) there is a strong association between k and the severity of size distortions and/or the extend of power loss.

Recently, Ng and Perron (2001) have proposed a methodology that solves these three problems. This method consists of a class of modified tests, called $\bar{M}^{GLS}_{MAIC}$, originally developed in Stock (1999) as M tests, with GLS detrending of the data as proposed in Elliot *et al.* (1996), and using the Modified Akaike Information Criteria ($MAIC$).[1] Also, Ng and Perron (2001) have proposed a similar procedure to correct for the problems of the standard Augmented Dickey-Fuller (ADF) test, ADF^{GLS}_{MAIC}.[2]

In Table 1 we report the results of the $\bar{M}^{GLS}_{MAIC}$ tests and the ADF^{GLS}_{MAIC} test. In all these tests the null hypothesis is that a series is I(1) against the alternative that it is I(0).[3] The null hypothesis of nonstationarity for the series in levels cannot be rejected, independently of the test, whereas the existence of two unit roots is clearly rejected at the usual significance levels. Therefore, according to the results of these tests, the US stock price-dividend ratio would be I(1), so we work with the variable in first differences to ensure stationarity.

In the TAR(k) model, the threshold variable is $Z_{t-1} = y_{t-1} - y_{t-m}$ for some integer $m \in [1, M]$ called the delay lag, which is unknown a priori so it has to be estimated. For the threshold variable, we use a long difference for some $m \leq 8$, as suggested by Hansen (1997). Following Franses and Van Dijk (2000), we use both the minimization of Akaike's (1973) information criteria (AIC) and Hannan and Quinn (1979) (HQ) statistics to choose the appropriate lag order k. Both criteria lead to $k = 8$.

Table 2 reports the sum of squared errors (SSE) from the various TAR models from $m = 1$ to $m = 8$, and the bootstrap-calculated asymptotic p-value (using 5,000 replications) for the Wald test statistic, $\sup W_T(\lambda)$, on the null of linearity against a particular threshold model. Hansen (1997, 2000) suggests to select the delay lag through the minimization of the SSE, so in this case $m = 5$. The model is highly statistically significant.

Next, Table 3 presents the parameter estimates for the TAR model selected. Setting $m = 5$, the LS estimate of the threshold parameter (or trigger point) would be -8.0. The estimate $\hat{\lambda} = -8.0$ means that the estimated TAR model splits the regression into two regimes, depending on whether, in a particular year, the US stock price-dividend ratio, has shown an decrease of more than 8.0% between the previous year and the fourth year before. Of the 134 observations in the fitted sample, 34 observations lie in regime 1 where $y_{t-1}-y_{t-5} \leq -8.0$, and 86 observations lie in regime 2 where $y_{t-1}-y_{t-5} > -8.0$.

Figure 2 depicts the threshold variable, $Z_{t-1} = y_{t-1} - y_{t-5}$, together with the estimated threshold parameter, -8.0, for the case of the US stock price-dividend ratio over the period 1876-2004. As can be seen, four trigger points, according to Krugman's (1987) model, would appear: 1884, 1904, 1951 and 1978, which can be mainly related to recessions and/or wartime.

[1] These tests are the $\bar{M}Z^{GLS}_{\alpha}$, $\bar{M}SB^{GLS}$ and $\bar{M}Z^{GLS}_{t}$.

[2] See Ng and Perron (2001) and Perron and Ng (1996) for a detailed description of these tests.

[3] Note that for the $\bar{M}SB^{GLS}$ test, the null hypothesis is rejected in favour of stationarity when the estimated value is smaller than the critical value.

The two shifts in 1884 and 1904 were caused by two short-lived recessions. A major shift in the evolution of the stock price-dividend ratio occurs in 1951, coinciding with the Korean War. Finally, the figure shows another major shift in 1978, in the aftermath of the 1973 oil crisis, which plunged the US economy into a deep recession.

4 Conclusions

In this paper we contribute to the debate on the ability of the PV model to explain the behaviour of stock prices. Specifically, we examine whether this failure of the linear PV model can be attributed to non-linearities in the stock price-dividend relation. To this end, we use recent developments on TAR models that have allowed us to derive endogenously threshold effects in the evolution of the US stock price-dividend relation, which could explain the changes in the trigger stock prices selling strategies followed by private investors participating in portfolio insurance schemes. More specifically, we should expect a mean-reverting dynamic behavior of the US stock price-dividend ratio once such threshold is reached, according to the theoretical model of Krugman (1987).

Our empirical results showed that significant adjustments would occur when, in a particular year, the stock price-dividend ratio had shown a decrease of more than 8.0% between the previous year and the fourth year before, which implies nonlinearities in the dynamic behaviour of the US stock price-dividend relation. There is also evidence of four trigger points, according to the theoretical model of Krugman (1987), which can be mainly related to recessions and/or wartime. The first and second one would occur at 1884 and 1904, following two short-lives recessions. In turn, a third trigger point would emerge at 1951, coinciding with the Korean War. Finally, a major shit appears in the aftermath of the 1973 oil crisis.

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Table 1
Ng and Perron[a,b] tests for a unit root

I(2) vs. I(1)	Case: $p = 0, \bar{c} = -7.0$			
	$\bar{M}^{GLS}_{MAIC}$ tests			
Variable	MZ^{GLS}_{α}	MZ^{GLS}_{t}	MSB^{GLS}	ADF^{GLS}
Δpdr_t	-63.69***	-5.64***	0.088	-13.83***
I(1) vs. I(0)	Case: $p = 1, \bar{c} = -13.5$			
	$\bar{M}^{GLS}_{MAIC}$ tests			
Variable	$\bar{M}Z^{GLS}_{\alpha}$	$\bar{M}Z^{GLS}_{t}$	$\bar{M}SB^{GLS}$	ADF^{GLS}
pdr_t	-10.14	-2.06	0.203***	-2.15

Notes:
[a] *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.
[b] The $MAIC$ information criteria is used to select the autoregressive truncation lag, k, as proposed in Perron and Ng (1996). The critical values are taken from Ng and Perron (2001), Table 1.

Critical values:	Case: $p = 0, \bar{c} = -7.0$			Case: $p = 1, \bar{c} = -13.5$		
	10%	5%	1%	10%	5%	1%
$\bar{M}Z^{GLS}_{\alpha}$	-5.7	-8.1	-13.8	-14.2	-17.3	-23.8
$\bar{M}SB^{GLS}$	0.275	0.233	0.174	0.185	0.168	0.143
$\bar{M}Z^{GLS}_{t}, ADF^{GLS}$	-1.62	-1.98	-2.58	-2.62	-2.91	-3.42

Table 2
TAR models for the US stock price-dividend ratio

$Z_{t-1} = y_{t-1} - y_{t-m}$								
m	1	2	3	4	5	6	7	8
SSE	2.43	2.33	2.55	2.38	**2.30**	2.36	2.54	2.48
$p - value^a$	0.31	0.98	0.35	0.98	**0.01**	0.92	0.73	0.68

Note:
[a] From the Wald or Lagrange multiplier test, $\sup W_T(\lambda)$, that tests the null hypothesis of linearity (i.e., the threshold effect disappears and $\theta_1 = \theta_2$) against the alternative of a threshold effect (i.e., the process is nonlinear), as proposed in Hansen (1997) and Caner and Hansen (2001).

Table 3
TAR estimates for the US stock price-dividend ratio

Regime 1: $y_{t-1}-y_{t-5} \leq -8.0$									
Variable	Intercept	Δy_{t-1}	Δy_{t-2}	Δy_{t-3}	Δy_{t-4}	Δy_{t-5}	Δy_{t-6}	Δy_{t-7}	Δy_{t-8}
$\hat{\theta}_1$	0.04	0.07	-0.23	0.19	-0.03	0.25	0.10	0.52	0.23
S.E.	(0.03)	(0.17)	(0.14)	(0.18)	(0.11)	(0.14)	(0.11)	(0.10)	(0.13)

Regime 2: $y_{t-1}-y_{t-5} > -8.0$									
Variable	Intercept	Δy_{t-1}	Δy_{t-2}	Δy_{t-3}	Δy_{t-4}	Δy_{t-5}	Δy_{t-6}	Δy_{t-7}	Δy_{t-8}
$\hat{\theta}_2$	0.01	-0.09	-0.05	-0.002	-0.01	-0.27	-0.10	-0.05	-0.30
S.E.	(0.02)	(0.18)	(0.13)	(0.12)	(0.11)	(0.12)	(0.10)	(0.09)	(0.11)

Figure 1: Annual Data on US Stock Market, 1871-2004

Real stock price-dividend ratio

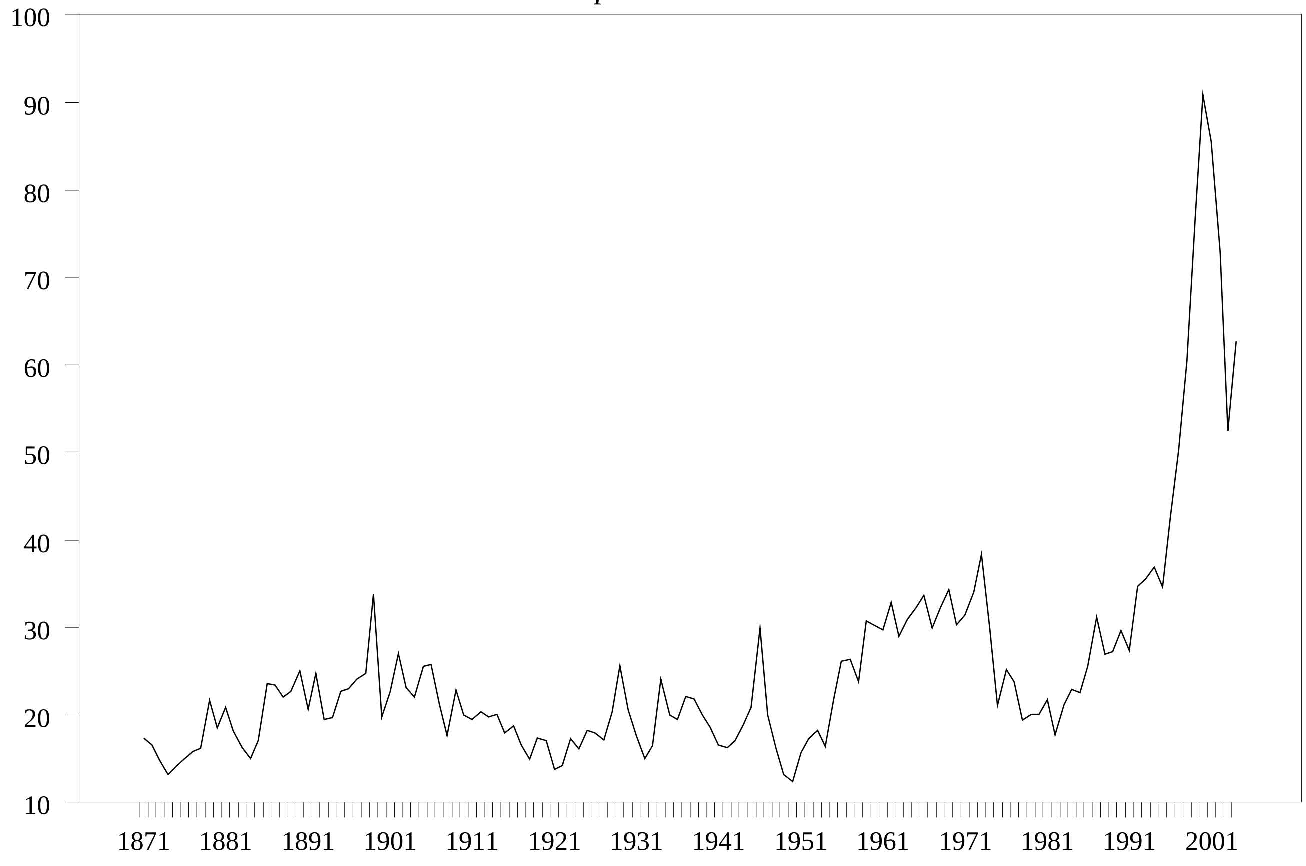

FIGURE 2

Threshold regimes for the US stock price-dividend ratio, 1876-2004