The role of volatility regimes on volatility transmission patterns

Abstract

This paper investigates volatility transmission patterns between US and Eurozone stock markets differentiating between low and high volatility periods which tend to be related with international crisis. Our approach let us distinguish the spillover intensities between markets in calm and crisis periods and it also tests for a potential increase of market comovements during these periods of market jitters. State-Dependent Volatility Impulse-Response Functions (SD-VIRF) are also introduced considering different responses of stock markets during the detected high and low volatility periods. The results show that both spillovers intensities and conditional correlation increase in times of market unstability.

Keywords: Regime-Switching, Market volatility, Spillover effects, State-Dependent Volatility Response Functions, Correlation

JEL codes: G01, G10, G15

1.-Introduction

With the recent process of globalization, there is an increasingly interrelation between stock markets all over the world. The relationships and transmission mechanisms among different countries is a topic that has received lots of attention in the financial literature (Rachmand and Susmel (1998), Brooks (2000), Bekaert et. al(2005) among others) and seems even a more attractive topic in the current moment of global financial crisis. Although interactions and transmission patterns among countries have been analyzed both in mean (Eun and Shim (1989) Kroner and Ng (1998)) and variance (Koutmos and Booth (1995), Beirne et al. (2009)), in this paper we focus our attention on the relationships between conditional second moments. We are interested in knowing how the volatility of one market reacts to innovations in other markets and whether or not this reaction is different during low and high volatility periods. Moreover, previous studies show evidence of nonlinearities in volatility dynamics (Frijns and Schotman, 2006) and the no consideration of them could lead to misleading conclusions about the significance and magnitude of volatility transmission mechanisms.

In the literature the studies have focused on spillovers analyzing how shocks from one market affect volatility in other markets (Hamao et al. (1990), Lee et. al (2004)). Another volatility transmission mechanism treated in the literature is financial contagion, often detected as changes in the correlations level during crisis periods (Forbes and Rigobon, 2002). One of the attractive features of our study is that it presents a time-varying shock spillover sensitivity that is driven by a regime-switching model (Baele, 2005). This approach let us observe the magnitude of the volatility transmission between markets and how it evolves during time. Moreover, as far as the markets are more interconnected during high volatility periods, a potential increase in the correlations between them may suggest a sign of higher market comovements or interdependence (Forbes and Rigobon (2002), Bekaert et. al(2005)). Therefore, this study may shed light on the topic discussing if it is worth using country diversification during periods of market jitters. Some studies focus on analyzing the role of several crises (for example Mexico, Russia, Argentina) in the volatility transmission patterns between markets (Forbes and Rigobon (2002). In this study we do not consider the effect of a specific crisis but we let the data decide which periods corresponds to high volatility states (associated with crises) and to low volatility states (associated with stable periods).

From a methodological point of view, in the vast stream of literature analyzing volatility transmission, one of the most popular approaches is the one using GARCH processes. In its univariate specification, the conditional variance of one market may be affected by additional information (such as innovations, past variance levels in other markets)[1] which

[1] This univariate GARCH models are the second phase of a 2-stage methodology. Generally, the goal of the first phase is to obtain series of unexpected innovations or volatilities of a specified market. In the second phase, the univariate GARCH models are estimated including the series from the first phase as explanatory variables (see e.g. Hamao et al. (1990)).

let us analyze the relationship between the domestic market variance and the unexpected volatility/innovations of the foreign market. Previous studies using this univariate methodology already concern about the potential influence of changes in regime in the volatility transmission patterns. Some of them defined several regimes through sudden changes in unconditional volatility. Aggarwal et. al (1999) use a model combing GARCH specification with dummy variables which represent sudden changes in variance detected with an iterative cumulative sum of squares (ICSS) algorithm. However, studies using univariate models merely propose the conditional variance of one market as explanatory variable for the conditional volatility of the other and do not consider potential interrelations between volatilities also ignoring the role of the covariances. Therefore, a multivariate GARCH model (Darbar and Deb (1997), Meneu and Torro (2003)) seems more attractive in order to analyze the volatility transmission patterns between two markets. Sudden switches in bivariate models have also been introduced in studies such as in Ewing and Malik (2005). Other studies considering potential changes in regime use the Switching ARCH (SWARCH) model of Hamilton and Susmel (1994) starting with univariate estimation of each series and then use the bivariate version of a SWARCH model. Edwards and Susmel (2001) apply this methodology to transmission volatility in emerging markets finding that high volatility states tend to be related to international crises[2].

Another line of research recently developed that treat to solve the problem showed above is based on the use of Markov-Switching models. The studies in this stream propose the analysis of volatility transmission patterns by testing several hypotheses on how changes in regime in one country may lead to changes in volatility regime in other countries. Sola et. al (2002), Gallo and Otranto (2007), Bialkwosky et. al (2006) use a methodology that allows them to test different types of volatility transmission mechanisms by proposing hypothesis on the parameters that define the transition probability matrix. These studies identify the direction of the volatility spillovers and the moments when they occur. However, although these studies identify properly the volatility transmission among markets they ignore potential differences in the magnitude of spillovers during high and volatility periods.

In our study we try to develop a model that reflects the advantages of these two approaches. We propose a multivariate GARCH framework where we allow for state-dependent parameters. With our approach we can overcome the limitation of single-regime GARCH models (Brooks and Henry (2000), Caporale et. al (2002) among others) when they consider the same volatility transmission patterns for all periods (calm and crisis). Another advantage of our model is that we do not need to establish calm and crises periods a priori but is the estimation procedure itself which decides when the markets are in periods of calm or financial turmoil[3]. Our analysis also lets us analyze the magnitude of the spillovers

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2 The studies using SWARCH models are similar than the approach we used in this paper, but our model introduces a GARCH structure in conditional variance and considers the role of covariances between markets while SWARCH models are based on univariate ARCH specifications of the conditional variance.

3 In the approach used in papers such as Ewing and Malik (2005) the sudden changes in variances are associated with regime shifts and are computed in a previous stage using an ICSS algorithm and then they are incorporated to the variance equation using dummy variables. In our approach we define the different states in the estimation process.

during time. Moreover, with time-varying volatilities we are able to propose volatility impulse response functions (VIRF) measuring how an unexpected shock in one market can affect the volatility in another market. We are able to perform this analysis in times of calm and crisis and see if a shock of the same magnitude has a similar effect under different volatility market scenarios through the state-dependent VIRF we define in the paper (SD-VIRF)[4].

In this study we detect volatility spillovers if the past volatility of one market has a significant impact on the volatility formation of the other market. We perform a similar analysis and we define shock spillovers when we discuss how the past shocks generated in one market have a significant impact on the other market volatility. We also analyze changes in correlation levels during crises periods and associate them as increases of market comovements. Although some studies (Forbes and Rigobon, 2002) associate increases in correlations during crises periods as a sign of financial contagion, as we do not know the direction or the origin of the potential contagion, we just establish these higher correlations as market comovements.

The contributions of the study to the current literature are the followings. First, we study the volatility transmission between the US and the Eurozone using a sample period including the recent global financial crisis period. Second, we also present the shock spillover intensity between these two markets which varies over time according to the regime-switching process. Third, we present a simple procedure to test differential market comovements during crises periods based on the estimated time-varying correlations. Finally, we introduce a State-Dependent Volatility Impulse-Response Function which distinguishes between calm and crises periods and show how different the conditional second moments in each market react during calm and turmoil periods.

The main results of the study can be summarized as follows. First, the intensity of the spillovers during high volatility periods is significantly higher than the observed during low volatility periods. Second, the impact of an unexpected shock in the volatility formation depends essentially on the market scenario (low or high volatility). The magnitude and decay of these shocks is higher and takes longer to disappear during high volatility periods. Third, we find a significant difference between the magnitudes of conditional correlations during periods of high and low volatility observing that markets present a higher level of correlation during market turmoil periods. Fourth, generally the Eurozone stock market seems to be more sensitive to volatility spillovers and react more against unexpected shocks occurring in any of two markets.

The rest of the paper is organized as follows. Section 2 presents the database and performs some previous statistical analysis on the data. Section 3 introduces the econometric approach. Section 4 shows some empirical results about volatility and shock spillovers and

4 To the best of our knowledge this is one of the first attempts analyzing regime-dependent volatility response functions. Some works (Ehrman et. al (2003)) propose state-dependent impulse-response functions for autoregressive processes in mean but the application on volatility is a topic yet to analyze.

analyzes the role of conditional correlation. Section 5 deals with the state-dependent volatility impulse-response functions and finally, Section 6 concludes.

2.- Data description

The data used in this study includes weekly closing prices[5] (Brooks and Henry (2000), Billio and Pellizon (2003)) for the US (S&P100 index) and the Eurozone (Eurostoxx50 index) stock markets. The time horizon includes observations since 1 January 1988 until 31 December 2010. We obtained the market stock indexes data from Thomson Datastream.

[INSERT FIGURE 1]

Figure 1 displays the evolution of the returns[6] and prices series of Eurostoxx50 and SP 100 during the sample period considered.

[INSERT TABLE 1]

Several statistical tests performed over the weekly returns are presented in Table 1. Panel A shows the main summary statistics for the EU and US indexes. Certain results are noteworthy. For the returns, negative values are present in the third-order moments. There is also excess kurtosis in the returns (fat tails); this finding suggests that the variances of the series may be time varying. Finally, note that the Jarque-Bera normality test is rejected, due to the asymmetric and leptokurtic characteristics of the series. Panel B shows several tests to identify serial autocorrelation in the return series and in their squares (heterokedasticity). The first one reveals evidence for serial autocorrelation in returns levels[7]. There are also displayed two tests to detect potential ARCH effects. The first one uses the serial autocorrelation tests for the squared series. The second one uses the Engle's ARCH test. It is noteworthy that the statistics for both tests suggest evidence of conditional heteroskedasticity for EU and US series. Panel C reflects the stationarity tests performed over the returns and prices series and reveals that the prices series are I(1), so we have to work with the returns series for stationarity reasons.

5 Most of the authors using non-linear switching models in a bivariate framework (Edwards and Susmel (2001), Baele (2005) among others) use data at weekly frequency because does not reflect the noise of higher frequencies and, therefore, let identify more accurately the different regimes in the volatility processes.
6 We use logarithmic returns multiplied by 100 to facilitate the convergence process of our model.
7 Despite the presence of serial correlation in the returns series we do not consider any structure in the mean equation as the standardized residuals from the estimations are free of serial autocorrelation.

3. Methodology

In contrast to previous studies in which the dynamic relationship between the returns of two markets is characterized by linear patterns (see Soriano and Climent, 2006 for a review), the model presented by Lee and Yoder (2007) allows for regime shifts in this relationship. This type of non-linear models opens up a new line for modeling conditional volatilities in a multivariate framework. In that study, the Gray's (1996) method to solve the problem of path dependency is extended to a bivariate case.

Let $r_{EU,t,st}$ and $r_{US,t,st}$ be the state-dependent Eurostoxx and SP500 returns at t respectively; we define the state-dependent mean equations as:

$$r_{EU,t,st} = \mu_{EU,st} + e_{EU,t,st} \tag{1}$$

$$r_{US,t,st} = \mu_{US,st} + e_{US,t,st} \tag{2}$$

$$e_{t,st} \left| \Omega_{t-1} = \begin{pmatrix} e_{EU,t,st} \\ e_{US,t,st} \end{pmatrix} \right| \Omega_{t-1} \sim BN\left(0, H_{t,st}\right) \tag{3}$$

where $\mu_{EU,st}$ and $\mu_{US,st}$ for $s_t = \{1,2\}$ are parameters to be estimated.

The state-dependent innovations $e_{t,st}$ follow a bivariate normal distribution that depends on the state variable $s_t = \{1,2\}$ [8]. This state variable follows a two-state first-order Markov process with transition probabilities:

$$P = \begin{pmatrix} \Pr\left(s_t = 1 \middle| s_{t-1} = 1\right) = p & \Pr\left(s_t = 1 \middle| s_{t-1} = 2\right) = (1-q) \\ \Pr\left(s_t = 2 \middle| s_{t-1} = 1\right) = (1-p) & \Pr\left(s_t = 2 \middle| s_{t-1} = 2\right) = q \end{pmatrix} \tag{4}$$

where *p* represents the probability of continuing in state 1 at period t if it was previously in state 1 and *q* represents the probability of continuing in state 2 if it was previously in state 2.

The state-dependent conditional second moments $H_{t,st}$ follow a BEKK specification model (Baba et. al, 1990) that takes different values depending on the value of $s_t = \{1,2\}$. Thus, the variance specification in each state is defined as follows:

8 We want to thank an anonymous referee for suggesting us a more general model with two states variables (one for each market). After a careful study evaluating the costs and benefits of each specification we decided to go with the more parsimonious model. The expected benefits of the unrestricted model would be a more realistic representation of the regime-switching (it would let the markets change regimes in different periods). However, we observed that in most of the sample both markets are in the same state (fact which is reflected in the more parsimonious model). The costs of the unrestricted model would be the overparameterized optimization problem we would face (35 parameters) with bad consequences for the accuracy of the estimations. Given this evidence, the restricted variance specification is chosen. Results of this analysis are available upon request.

$$H_{t,st} = \begin{pmatrix} \sigma^2_{EU,t,st} & \sigma_{EU,t,st}\sigma_{US,t,st} \\ \sigma_{US,t,st}\sigma_{EU,t,st} & \sigma^2_{US,t,st} \end{pmatrix} = C'_{st}C_{st} + A'_{st}e_{t-1}e'_{t-1}A_{st} + B'_{st}H_{t-1}B_{st} \quad (5)$$

where $\sigma^2_{EU,t,st}$ and $\sigma^2_{US,t,st}$ are the conditional variances for the EU and US returns in period t for each state s_t and $\sigma_{EU,t,st}\sigma_{US,t,st}$ is the conditional covariances in t for each state s_t. C_{st}, A_{st} and B_{st} are matrices of parameters to be estimated.

If we develop the compact form we obtain the following specification for each state-dependent variance equation:

$$H_{t,st} = \begin{pmatrix} \sigma^2_{EU,t,st} & \sigma_{EU,t,st}\sigma_{US,t,st} \\ \sigma_{US,t,st}\sigma_{EU,t,st} & \sigma^2_{US,t,st} \end{pmatrix} = \begin{pmatrix} c_{11,st} & 0 \\ c_{12,st} & c_{22,st} \end{pmatrix}' \begin{pmatrix} c_{11,st} & 0 \\ c_{12,st} & c_{22,st} \end{pmatrix} + \quad (6)$$

$$\begin{pmatrix} a_{11,st} & a_{21,st} \\ a_{12,st} & a_{22,st} \end{pmatrix}' \begin{bmatrix} e^2_{EU,t-1} & e_{EU,t-1}e_{US,t-1} \\ e_{US,t-1}e_{EU,t-1} & e^2_{US,t-1} \end{bmatrix} \begin{pmatrix} a_{11,st} & a_{21,st} \\ a_{12,st} & a_{22,st} \end{pmatrix} + \begin{pmatrix} b_{11,st} & b_{21,st} \\ b_{12,st} & b_{22,st} \end{pmatrix}' \begin{bmatrix} \sigma^2_{EU,t-1} & \sigma_{EU,t-1}\sigma_{US,t-1} \\ \sigma_{US,t-1}\sigma_{EU,t-1} & \sigma^2_{US,t-1} \end{bmatrix} \begin{pmatrix} b_{11,st} & b_{21,st} \\ b_{12,st} & b_{22,st} \end{pmatrix}$$

The consideration of several states leads to a noteworthy rise in the number of parameters to estimate. In order to reduce this over-parameterization the difference among states is defined by three new parameters *sa*, *sb*, and *sc* that properly weight the estimations obtained in one state for the other state. Therefore, the state-dependent covariance matrices in our model[9] are:

$$H_{t,s_t=1} = \begin{pmatrix} \sigma^2_{EU,t,1} & \sigma_{EU,t,1}\sigma_{US,t,1} \\ \sigma_{US,t,1}\sigma_{EU,t,1} & \sigma^2_{US,t,1} \end{pmatrix} = C'_1C_1 + A'_1e_{t-1}e'_{t-1}A_1 + B'_1H_{t-1}B_1 \quad (6.1)$$

$$H_{t,s_t=2} = \begin{pmatrix} \sigma^2_{EU,t,2} & \sigma_{EU,t,2}\sigma_{US,t,2} \\ \sigma_{US,t,2}\sigma_{EU,t,2} & \sigma^2_{US,t,2} \end{pmatrix} = C'_2C_2 + A'_2e_{t-1}e'_{t-1}A_2 + B'_2H_{t-1}B_2 \quad (6.2)$$

where $C_2 = sc \cdot C_1$, $A_2 = sa \cdot A_1$ and $B_2 = sb \cdot B_1$, A_1 and B_1 are 2x2 matrices of parameters and C_1 is a 2x2 lower triangular matrix of constants.

Because of this state dependence, the model will become intractable as the number of observations increases. In order to solve this problem we apply the recombining method used in Gray (1996) where the path dependency problem is solved for univariate models. Lee and Yoder (2007a) extend this recombining method for the bivariate case.

9 Following the insightful suggestions from an anonymous referee, we have also considered a variance specification where all parameters are estimated freely. In order to choose between this more general model and the one presented in the paper we analyzed the costs and benefits of using each model. The expected benefits of the more general model would be a more realistic representation of dynamics for volatilities and covariances among regimes. This is expected to lead to a better representation and fit of this model to the financial data. To check this hypothesis, we run a likelihood-ratio test between both models and we observed that the more general model cannot improve the fitting of the data obtained with the restricted one. Again, the costs of the unrestricted model would be the overparameterized optimization problem we would face (28 parameters) with bad consequences for the accuracy of the estimations. Given this evidence, one state variable for both markets is chosen. Results of this analysis are available upon request.

The basic equations of the recombining method used to collapse the variances and covariances and the innovations, and to ensure the model is tractable are described below:

$$e_{i,t} = \Delta R_t - \left(\pi_{1,t} r_{i,t,1} + \left(1-\pi_{1,t}\right) r_{i,t,2}\right) \qquad i = \{EU, US\} \tag{7}$$

$$\sigma_{i,t}^2 = \pi_{1,t}\left(r_{i,t,1}^2 + \sigma_{i,t,1}^2\right) + \left(1-\pi_{1,t}\right)\left(r_{i,t,2}^2 + \sigma_{i,t,2}^2\right) - \left(\pi_{1,t} r_{i,t,1} + \left(1-\pi_{1,t}\right) r_{i,t,2}\right)^2 \text{ for } i = \{EU, US\} \tag{8}$$

$$\begin{aligned}\sigma_{EU,t}\sigma_{US,t} &= \pi_{1,t}\left(r_{EU,t,1} r_{US,t,1} + \sigma_{EU,t,1}\sigma_{US,t,1}\right) + \left(1-\pi_{1,t}\right)\left(r_{EU,t,2} r_{US,t,2} + \sigma_{EU,t,2}\sigma_{US,t,2}\right) \\ &-\left(\pi_{1,t} r_{EU,t,1} + \left(1-\pi_{1,t}\right) r_{EU,t,2}\right) - \left(\pi_{1,t} r_{US,t,1} + \left(1-\pi_{1,t}\right) r_{US,t,2}\right)\end{aligned} \tag{9}$$

Where ΔR_t are the observed EU and US returns, $\sigma_{i,t}^2$, $\sigma_{EUUS,t}$ are the state-independent variances and covariances aggregated by the recombining method, and $\sigma_{i,t,st}^2, \sigma_{EUUS,t,st}$ are the state-dependent variances and covariances for $i = \{EU, US\}$ and $s_t = \{1, 2\}$.

The terms $r_{i,t,st}$ represent the state-dependent mean equations and $\pi_{1,t}$ is the probability of being in state 1 at time t obtained by the expression:

$$\pi_{1,t} = p\left(\frac{g_{1,t-1}\pi_{1,t-1}}{g_{1,t-1}\pi_{1,t-1} + g_{2,t-1}\left(1-\pi_{1,t-1}\right)}\right) + (1-q)\left(\frac{g_{2,t-1}\left(1-\pi_{1,t-1}\right)}{g_{1,t-1}\pi_{1,t-1} + g_{2,t-1}\left(1-\pi_{1,t-1}\right)}\right) \tag{10}$$

Where $$g_{i,t} = f\left(r_t \middle| s_t = i, \Omega_{t-1}\right) = (2\pi)^{-1}\left|H_{t,st}\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2} e_{t,st} H_{t,st}^{-1} e_{t,st}\right\} \tag{11}$$

for $s_t = \{1, 2\}$ and *p*, *q* were described in (4).

Thus, the parameters of the model can be estimated with the following maximum likelihood function $L(\theta) = \sum_{t=1}^{T} \log f\left(r_t;\theta\right)$ (12) where

$$f\left(r_t;\theta\right) = \pi_{1,t}\left[(2\pi)^{-1}\left|H_{t,1}\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2} e_{t,1} H_{t,1}^{-1} e_{t,1}\right\}\right] + \pi_{2,t}\left[(2\pi)^{-1}\left|H_{t,2}\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2} e_{t,2} H_{t,1}^{-1} e_{t,2}\right\}\right] \tag{13}$$

The interpretation of the parameters obtained in 6 cannot be done individually. Instead, we have to interpret the non-linear functions of the parameters which form the intercept terms and the coefficients of the lagged variances, covariances and lagged error terms. So, the conditional variance for each equation can be expanded for each state-dependent bivariate GARCH as follows expanding (6):

$$\begin{aligned}\sigma_{EU,t,s_t=1}^2 &= c_{11,s_t=1}^2 + b_{11,s_t=1}^2 \sigma_{EU,t-1}^2 + 2 b_{11,s_t=1} b_{12,s_t=1} \sigma_{EU,t-1}\sigma_{US,t-1} + b_{21,s_t=1}^2 \sigma_{US,t-1}^2 \\ &+ a_{11,s_t=1}^2 e_{EU,t-1}^2 + 2 a_{11,s_t=1} a_{12,s_t=1} e_{EU,t-1}\, e_{US,t-1} + a_{21,s_t=1}^2 e_{US,t-1}^2\end{aligned} \tag{14}$$

$$\sigma^2_{EU,t,s_t=2} = c^2_{11,s_t=2} + b^2_{11,s_t=2}\sigma^2_{EU,t-1} + 2b_{11,s_t=2}b_{12,s_t=2}\sigma_{EU,t-1}\sigma_{US,t-1} + b^2_{21,s_t=2}\sigma^2_{US,t-1}$$
$$+a^2_{11,s_t=2}e^2_{EU,t-1} + 2a_{11,s_t=2}a_{12,s_t=2}e_{EU,t-1}\, e_{US,t-1} + a^2_{21,s_t=2}e^2_{US,t-1} \tag{15}$$

$$\sigma^2_{US,t,s_t=1} = c^2_{12,s_t=1} + c^2_{22,s_t=1} + b^2_{12,s_t=1}\sigma^2_{EU,t-1} + 2b_{12,s_t=1}b_{22,s_t=1}\sigma_{EU,t-1}\sigma_{US,t-1} + b^2_{22,s_t=1}\sigma^2_{US,t-1}$$
$$+a^2_{12,s_t=1}e^2_{EU,t-1} + 2a_{12,s_t=1}a_{22,s_t=1}e_{EU,t-1}\, e_{US,t-1} + a^2_{22,s_t=1}e^2_{US,t-1} \tag{16}$$

$$\sigma^2_{US,t,s_t=2} = c^2_{12,s_t=2} + c^2_{22,s_t=2} + b^2_{12,s_t=2}\sigma^2_{EU,t-1} + 2b_{12,s_t=2}b_{22,s_t=2}\sigma_{EU,t-1}\sigma_{US,t-1} + b^2_{22,s_t=2}\sigma^2_{US,t-1}$$
$$+a^2_{12,s_t=2}e^2_{EU,t-1} + 2a_{12,s_t=2}a_{22,s_t=2}e_{EU,t-1}\, e_{US,t-1} + a^2_{22,s_t=2}e^2_{US,t-1} \tag{17}$$

Equations (14) to (17) reveal how past shocks and volatilities are transmitted over time and across the EU and US stocks indexes in each volatility state. Note that these coefficient terms are nonlinear functions of the estimated elements from (6). We follow Kearney and Patton (2000) and compute the expected value and the standard error of those non-linear functions.

The expected value of a non-linear function of random variables is calculated as the function of the expected value of the variables, if the estimated variables are unbiased. In this case we just develop the compact form in (6) and compute the products between the corresponding coefficients. This leads to equations (14) to (17). In order to calculate the standard errors of the function, a first order Taylor approximation is used[10]. This approach linearizes the standard error function by using a transformation of the variance-covariance matrix of the parameters.

4.- Empirical results

Table 2 shows the estimated parameters for the model proposed. The estimations for the mean equation reveal that for state 1(associate with low volatility states) the average returns for both countries present a positive and significant value while they present a significant negative value for state 2 (high volatility state). This result is consistent with some literature (Lundblad, 2007) which discuss that realized returns, particularly in the less common high volatility states (corresponding generally with recession periods) are often associated with low or even negative markets returns while during calm periods the returns

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10 To calculate the standard errors, the function must be linearized using a first order Taylor series expansion using the covariance matrix of the parameters (which in QML estimation is approximated by a sandwich expression with the outer product of the gradient and the inverse of the hessian at the optiumum $V(\theta) = (OPG * Hess^{-1} * OPG)$ (Bollerslev-Wooldrige(1992)). Note that for the parameters in table 2, the standard errors are computed using the square root of the diagonal in this matrix $V(\theta)$.

However, for the non-linear expression in equations 14-17 we need to use a transformation of this matrix using the mentioned Taylor series expansion. For this specific case, the non-linear transformations involving just one of the original parameters in θ, (for example $a^2_{11,st=1}$) is given by this expression:

$$V(Y) \approx (\Delta Y)^2 \approx \frac{\delta Y}{\delta X}(\Delta X)^2 \approx \frac{\delta Y}{\delta X}V(Y) \text{ when } Y = f(X)$$

And for transformations involving two parameters in θ (for example $a_{11,st=1} \cdot a_{12,st=1}$), the expression to use is:

$$V(Y) \approx (\Delta Y)^2 \approx \frac{\delta Y}{\delta X}V(Y) + \frac{\delta Y}{\delta Z}V(Z) + 2\left(\frac{\delta Y}{\delta X}\right)\left(\frac{\delta Y}{\delta Z}\right)Cov(X,Z) \text{ when } Y = f(X,Z)$$

observed in the markets are usually higher. As we mentioned above, the parameters of the model in the variance equation cannot be interpreted individually, so we have to develop the compact form and focus on the non-linear combinations of parameters in the expanded model. However, we can observe that the estimated parameters for the model are generally significant and the standardized residuals obtained from the estimation do not present significant evidence of serial autocorrelation or conditional heterokedasticity.

[INSERT TABLE 2]

Table 3 represents the expected value and the standard errors of these non-linear functions in each volatility regime which are expressed as a function of the lagged variances and innovations in both markets and their cross products.

[INSERT TABLE 3]

Regarding other studies using multivariate GARCH (Chulia et. al, 2009, Ewing and Malik, 2005) our model let us distinguish the volatility formation in each market during periods of low and high volatility. During calm market situations $\left(s_t = 1\right)$, the Eurostoxx50 volatility is directly affected by its own past volatility $\left(\sigma^2_{EU,t-1}\right)$and by the past volatility of SP500 at 1% of significance $\left(\sigma^2_{US,t-1}\right)$. Our findings also suggest that the past Eurostoxx shocks $\left(e^2_{EU,t-1}\right)$do not have a significant impact on the Eurozone contemporaneous volatility. However, the shocks occurred in the US $\left(e^2_{US,t-1}\right)$ have a significant effect on Eurozone volatility at 10% of significance. In these low volatility periods, the SP100 volatility is affected by its own past volatility $\left(\sigma^2_{US,t-1}\right)$ and by the Eurostoxx past volatility $\left(\sigma^2_{EU,t-1}\right)$ at 1% of significance. However, in this case only the shocks occurred in the US market $\left(e^2_{US,t-1}\right)$ have a significant impact on the volatility formation at 10% of significance but not those ones occurring in the Eurozone$\left(e^2_{EU,t-1}\right)$.

The situation during high volatility periods$\left(s_t = 2\right)$ varies from the observed in low volatility periods. First of all the impact of both past variance and innovations is higher during this turmoil periods. All the coefficients measuring the sensitivities of the own, foreign and cross-product variances and innovations increase regarding the observed in low volatility states. The significance is similar than the observed in low volatility states obtaining that Eurostoxx50 volatility is affected at 1% level by its own $\left(\sigma^2_{EU,t-1}\right)$ and the SP100 past volatility $\left(\sigma^2_{US,t-1}\right)$ but only by the shocks occurring in US affect $\left(e^2_{US,t-1}\right)$at 10% of significance the Eurozone volatility. The volatility formation in the SP100 index during turmoil periods is affected at 1% of significance by its own $\left(\sigma^2_{US,t-1}\right)$and the Eurozone volatility $\left(\sigma^2_{EU,t-1}\right)$ but only the shocks occurring in the domestic US market $\left(e^2_{US,t-1}\right)$ affect the volatility of the SP100 at 5% of significance.

Besides this, the estimation of our model also reflects other interesting results. We can compute the average duration of low and high volatility states according to the transition probability estimates *p* and *q* in equation (4). These parameters present a value of *p=0.9502* and *q=0.9112*; this means that once in state 1, the probability of remaining in that state is 95,02%, while the probability of remaining in state 2 if previously the process was in state 2 is 91.12%. Therefore, the average duration of being in state 1 when the volatility process is governed by this state will be approximately 20 weeks *(1/(1-0.9502))* and it is significantly less longer in high volatility regimes *(1/(1-0.9112))* being the average duration of this state 11 weeks. This indicates that the regime switches present a smooth evolution staying the process in each state during relatively long periods.

[INSERT FIGURE 2]

We show in figure 2 the smooth probability[11] of being in the low volatility state for the sample period used. The figure is governed essentially by this state[12] which corresponds with calm periods of financial markets. When the state governing the process is the state 2, this corresponds to periods of international market jitters. The most relevant periods where the high volatility states are the dominant states correspond to the dot-com bubble (2002-2003) and the last financial crisis (2008).

Using the figure 2 and the plots in figure 3 which represent the estimated conditional volatilities we can associate states 1 and 2 to calm (low) and crises (high volatility) periods respectively. Moreover, the estimated conditional volatilities tend to be higher in the Eurostoxx50 than in the SP100.

[INSERT FIGURE 3]

Finally, to obtain an understanding of the magnitude and evolution of shock spillover intensity through time and among countries we plot in figure 4 the weighted foreign shock spillover in both markets.

[INSERT FIGURE 4]

The magnitude of spillovers is clearly distinct during different periods of the sample. In those periods corresponding to high volatility states the magnitude of foreign shock spillovers is higher both in the US and the Eurozone markets. Furthermore, this magnitude is also higher in the direction of the impact of US shock to Eurozone volatility than in the direction from Eurozone to US. If we consider the average spillovers, we obtain that during low volatility periods the shock spillover is more than 6 times higher during crises periods

11 The smooth probability is computed following the Kim and Nelson (1999) algorithm.

12 Assuming that 0.5 is the threshold between low volatility states (probabilities higher than 0.5) and high volatility states (probabilities lower than 0.5), during 816 periods the dominant volatility process is the low volatility state against only 384 periods where both markets are in high volatility states.

than during calm periods; for example, in average, the sensitivity of shock spillovers is 2.82% and 0.72% during stable periods for the Eurozone and the US market respectively but these percentages increase until 18.09% and 4.61% during times of financial turmoil.

4.1.- Obtaining time-varying correlations

In the previous subsection we analyzed how the past variance and shock spillovers are transmitted among US and EU stock markets. We focused essentially on the patterns followed by conditional volatilities and the factors that may affect them. Using those conditional volatilities it is easy to obtain conditional correlations just by using the following expression:

$$\rho_{EU/US,t} = \frac{\sigma_{EUUS,t}}{\sqrt{\sigma^2_{EU,t}\sigma^2_{US,t}}} \tag{18}$$

[INSERT FIGURE 5]

Figure 5 plots the conditional correlations according to expression 18. It is observed that conditional correlations tend to be higher during those periods governed by high volatility states. For example, in the periods from 1998 to 2003 and during 2008-2009 is observed an increase in the correlation levels between the two markets. It is noteworthy that these periods coincide with situations of market jitters in EU and US markets.

To contrast this potential fact that during high volatility (crisis) periods it is observed a rise in the correlation level we propose the following simple regression:

$$\rho_{EU/US,t} = c + \gamma D_t + \varepsilon_t \tag{19}$$

where D_t is a dummy variable taking the value of 1 during detected high volatility periods[13]. So the coefficient γ represents the increase of correlation levels during periods of market turmoil.

[INSERT TABLE 4]

Panel A of Table 4 shows the value of the estimated coefficients in this regression. The coefficient γ takes a significant value of 0.1067 which means that there is an increase of 0.1067 points in the correlation levels during high volatility periods regarding those of low volatility. Panel B reflects a battery of simple mean equality tests between the two samples of conditional correlations (corresponding to periods of low and high volatility) and we conclude that we cannot accept the null hypothesis of equal average correlation during low and high volatility states. So, these results suggest that correlation dynamics dependence increases during periods of high volatility indicating a higher degree of co-movement between the two markets during these periods.

13 We define high volatility periods as those observations in which the smooth probability for state 1 is lower than 0.5.

Finally we also regress the conditional volatilities of EU and US on the computed conditional correlations to detect the contribution of the observed variance level in each country to the obtained conditional correlation. The results are summarized in table 5.

[INSERT TABLE 5]

It is observed that during high volatility periods is the volatility generated in the US market which has a greater impact on the conditional correlation. However, during low volatility states is the volatility generated in the Eurozone which seems more important. The results for the whole period also reflect a higher influence of the US volatility on the observed correlation between EU and US. Therefore, although increases in volatilities in both markets lead to higher correlations, it seems that it is the volatility generated in the US market which generates a greater commovemet between markets. This can be viewed as more evidence for our previous results. It suggests that increases in US market volatility are transmitted to the Eurozone market making the volatility of the Eurozone increase as well and this fact is reflected in the estimated correlations of how similar these two markets behave. The transmission in the other direction (effect of increases in EU and then transmitted to US) is less strong and the effect on the correlations (how similar the two markets evolve) is lower.

5.- State-dependent volatility impulse-response functions (SD-VIRF)

Volatility Impulse-Response Functions (VIRF) (Lin(1997), Hafner and Herwartz(2006)) are useful tools to analyze the second moment interaction between related markets since they measure the impact of an unexpected shock on the predicted volatility. The regime-dependent impulse response functions we develop in this paper are slightly different form the traditional VIRF since they describe the interaction between volatility markets within each Markov-Switching regime. Regime dependent impulse response functions are conditional on a given regime prevailing at the time of the disturbance and thorough the time of response. The validity of regime conditioning depends on the time horizon of the impulse response and the time and the expected duration of the regime. As long as the time horizon is not excessive long and the transition matrix predicts regimes which are highly persistent then the conditioning is valid and regime dependent impulse response functions are a useful tool (Ehrmann et al., 2003).

The state-dependent VIRF is based on the paper of Hafner and Herwartz (2006) which define the VIRF as follows:

$$V_h\left(\varepsilon_t,\Omega_{t-1}\right)=E\left[vech\left(H_{t+h}\right)\middle|\varepsilon_t,\Omega_{t-1}\right]-E\left[vech\left(H_{t+h}\right)\middle|\Omega_{t-1}\right] \quad (20)$$

where ε_t is a specific shock hitting the system at date t and Ω_{t-1} is the observed history up to t-1. The index h represents the forecast horizon and $V_h\left(\varepsilon_t\right)$ is the *(N(N+1)/2)* vector of the shock impact on the h-ahead conditional covariance matrix components. The VIRF is therefore the difference between the h-ahead expected conditional covariance matrix given an unexpected shock and the history up this date and the expectation given just the history.[14] The operator vech is used to eliminate the variables of the conditional covariance matrix which appear twice.

So, we have to transform our state-dependent BEKK models into its vech specifications. Notice that the BEKK (1,1) model we use in our state-dependent equations (6.1) and (6.2) is a particular case of the more general multivariate GARCH(p,q) model written as follows:

$$vech\left(H_t\right) = c + \sum_{i=1}^{p} F_i vech\left(e_{t-1}e_{t-1}^{'}\right) + \sum_{j=1}^{q} G_j vech\left(H_{t-1}\right) \tag{21}$$

Where H_t stands for the conditional covariance matrix at time t, vech $(\cdot)$ is the operator that stacks the lower fraction of an N x N matrix into an $N^* = N(N + 1)/2$ dimensional vector. F_i and G_j are parameters matrices each containing $\left(N^*\right)^2$ parameters and c is a N^* vector.

The relation between the matrices of parameters of the multivariate GARCH(1,1) and the BEKK(1,1) models[15] is:

$$\begin{gathered} F = L_N\left(A^{'} \otimes A^{'}\right)D_N \\ G = L_N\left(B^{'} \otimes B^{'}\right)D_N \,. \end{gathered} \tag{22}$$

The VIRF yields an analytical expression of the impulse response function when is applied to the previous class of MGARCH models. Computing the impact of shocks on volatility is therefore less time-consuming compared to a simulation-based estimation[16] (Le Pen and Sevi (2010)). Applied to a MGARCH(1,1) model, the one-step ahead VIRF is:

$$V_1\left(\varepsilon_t, \Omega_{t-1}\right) = FD_N^{+}\left(H_t^{1/2} \otimes H_t^{1/2}\right)D_N vech\left(\varepsilon_t \varepsilon_t^{'} - I_N\right) \tag{23}$$

Where I_N is the identity matrix, D_N is the duplication matrix previously defined and D_N^{+} its Moore-Penrose inverse. For h>1, the VIRF is:

$$V_h\left(\varepsilon_t, \Omega_{t-1}\right) = \left(F + G\right)V_{h-1}\left(\varepsilon_t\right) \tag{24}$$

For the Regime-Switching Multivariate GARCH we develop a similar approach. The regime-dependent impulse response functions are developed in (25). It shows the expected changes in conditional volatility at time t+h to a one standard deviation shock occurring in one market at time t, conditional on regime i.

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14 It is important to note that in equations 14 to 17 the analysis is performed using the information set up to t-1 (Ω_{t-1}) and in the VIRF we use a different information set because we include an unexpected shock in t (so, we use Ω_{t-1} plus ε_t). Although we showed that the European lagged shocks $e_{EU,t-1}$ have no effect in either market (table 3), using equations 14-17 we do not know the effect of unexpected contemporaneous shocks in the volatility transmission mechanisms. This is the main objective of the VIRF we present.

15 The vec operator stacks the column of a (NxN) matrix into a N2 column vector but does not eliminate redundant parameters. L_N is the elimination matrix such that $vech(A) = L_N vec(A)$ and D_N is the duplication matrix such that $vec(A) = D_N vech(A)$.

16 Koop et. al (1996) presents a unified approach to impulse-response functions using simulation-based algortihms for both linear and non-linear models. Since in their approach the impulse-response functions are derived from Monte-Carlo experiments (instead of an analytical expression) they require a higher computanional effort.

$$V_h\left(\varepsilon_t,\Omega_{t-1}\right)\Big|_{s_t=\ldots=s_{t+h}=i}=E\left[vech\left(H_{t+h}\right)\middle|\varepsilon_t,\Omega_{t-1},s_t=\ldots=s_{t+h}=i\right]-E\left[vech\left(H_{t+h}\right)\middle|\Omega_{t-1},s_t=\ldots=s_{t+h}=i\right] \quad (25)$$

The state-response vectors can be obtained similarly than those for the linear GARCH case, conditioning the horizon forecast to stay in the same regime and using the state-dependent variance parameters:

$$\begin{aligned} V_{1,s_t}\left(\varepsilon_t,\Omega_{t-1}\right)\Big|_{s_t=\ldots=s_{t+h}=i} &= F_{s_t}D_N^{+}\left(H_{t,s_t}^{1/2}\otimes H_{t,s_t}^{1/2}\right)D_N vech\left(\varepsilon_t\varepsilon_t^{'}-I_N\right) \\ V_{h,s_t}\left(\varepsilon_t,\Omega_{t-1}\right)\Big|_{s_t=\ldots=s_{t+h}=i} &= \left(F_{s_t}+G_{s_t}\right)V_{h-1}\left(\varepsilon_t\right) \qquad \textit{for} \quad s_t=1,2 \end{aligned} \quad (26)$$

Figure 6 plots the SD-VIRF for an unexpected shock in Europe and in the US. The top of figure 6 plots the results for an unexpected shock occurring in the Eurozone. The impact of a shock of a certain magnitude (one standard deviation) during low volatility states has a greater impact in the Eurozone (0,1%) than in the US market (merely 0,02%). However, when this shock occurs during a situation of market turmoil the effect of the shock on the market volatility raises the Eurozone market volatility levels in 11% and the level of US volatility in 5,5 % approximately. Another interesting result from these plots is the persistence of the unexpected shocks along time. For an unexpected shock of EU during low volatility periods the effect only is latent during 4-5 observations. However, the effect of a shock of the same magnitude still have effects on the market volatility after 12-14 observations if it is produced during periods of financial turmoil.

The results for an unexpected shock occurring in the US market have a similar interpretation. Again, the Eurozone market seems to be more sensitive against the unexpected shocks. Against shocks in the US market occurring during low volatility periods the volatility in the Eurozone and the SP100 increases 0.15% and 0.07% respectively. If the same shock is introduced during high volatility periods the volatility response arrives until 27-28% in Eurozone and 12-13% in US. Again, the impact of the shocks during time is more persistent during high volatility periods remaining their effect on volatility during 3-4 weeks and 12-15 weeks depending if they are introduced during low or high volatility scenarios.

So, summing up the SD-VIRF analysis, the most interesting results we find are: (1) Conditional variances are more sensitive to shocks occurring during high volatility states; (2) the Eurozone market is generally more sensitive to both EU and US shocks than the US market; and (3) the persistence of shocks is similar in both markets having an effect on volatility of approximately 4-5 weeks during low volatility states and 12-14 (even longer) when they occur in times of financial turmoil.

6. Conclusions

The main objective of this study is to analyze potential differences in the volatility transmission patterns during periods of low and high volatility often associated with boom and crises periods. To do this, weekly data for the US (SP100 index) and the Eurozone (Eurostoxx50) stock markets is used in a Regime-Switching Multivariate GARCH framework. Our approach presents the main advantage that it is the data evolution itself which decides the observations corresponding to low and high volatility periods.

We focus on the magnitude and direction of spillovers distinguishing those caused by foreign innovations from those by foreign past volatilities and we are able to show how these transmission patterns vary depending on the dominant market volatility regime. We also perform a study on the obtained conditional correlation searching for potential rises on the observed correlation levels in both markets in those periods corresponding to financial crises. Finally, we introduce a complementary analysis through a State-Dependent Volatility Response Function (SD-VIRF) which allows us to know how reacts the volatility of one market against an unexpected shock in any of the two markets considered distinguishing if the shock occurs during periods of stability or in periods of turmoil.

The results suggest that although the transmission of past volatility is bi-directional between US and EU, only the past shocks occurring in US have a significant impact on the volatility formation of both markets. We also observe that volatility transmission patterns are intensified with periods of financial instability. The spillover intensities during these periods are around 6 times higher than during calm periods. Moreover, we detect significant changes in the correlation levels during periods of market jitters. Some authors associate this fact with financial contagion. However, as we do not know the direction of this potential contagion we just point out that these two markets tend to be more correlated in times of instability. The response of volatility against an unexpected shock of the same magnitude also presents different patterns in low and high volatility periods. It seems that this shock has a lower impact and decay quicker in times of financial calm but the same shock presents a higher effect and their impact takes longer to disappear in times of markets jitters.

So, what it seems clear is the important role of non-linearities and regimes switching when we analyze volatility transmission patterns. The results reported in this paper regarding the magnitude of the spillover, the correlation levels and the response of volatility against an unexpected shock clearly differ if we are performing the analysis under a situation of financial stability or we are in time of market uncertainty and we would like to highlight this matter for future research.

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Table 1.- Table statistics for the Eurostoxx50 and SP100 returns

Panel A.- Summary statistics		
Region / Statistic	***Eurozone***	***United States***
Mean	*0.1298*	*0.1221*
Standard Deviation	*2.3731*	*2.7864*
Minimum	*-19.8641*	*-25.1308*
Maximum	*10.4644*	*13.5923*
Skewness	*-0.6964*	*-0.8386*
Kurtosis	*9.2497*	*10.4309*
JB test	*2048.31****	*2090.53****
Panel B.- Test for serial autocorrelation		
Region / Statistic	***Eurozone***	***United States***
LB-Q (7)	*49.3817*	*54.1454****
ARCH test	*67.9206****	*21.1097****
LB-Q2 (7)	*279.317****	*342.021****
Region / Statistic	***Eurozone***	***United States***
Dickey-Fuller	*-38.3380*	*-35.7643*
Phillips-Perron	*-38.3380*	*-35.7643*

This table presents the descriptive statistics for the returns series for the Eurozone and US stock markets. The JB-test is the Jarque-Bera (1980) test for normality. LB-Q (7) is the Ljung-Box (1978) test for serial autocorrelation for the series in levels and squares and ARCH-test is the Engle's test for 7th order ARCH.

Table 2.- Estimation results

Panel A.-Variance equation estimations

$$r_{EU,t,st} = \mu_{EU,st} + e_{EU,t,st}$$

$$r_{US,t,st} = \mu_{US,st} + e_{US,t,st}$$

$$H_{t,st} = \begin{pmatrix} \sigma^2_{EU,t,st} & \sigma_{EU,t,st}\sigma_{US,t,st} \\ \sigma_{US,t,st}\sigma_{EU,t,st} & \sigma^2_{US,t,st} \end{pmatrix} = C'_{st}C_{st} + A'_{st}\varepsilon_{t-1}\varepsilon'_{t-1}A_{st} + B'_{st}\ H_{t-1}B_{st} \quad for \quad s_t = 1,2$$

$$where\ C_2 = scC_1 \quad A_2 = saA_1 \quad B_2 = sbB_1$$

$$P = \begin{pmatrix} \Pr(s_t = 1|s_{t-1} = 1) = p & \Pr(s_t = 1|s_{t-1} = 2) = (1-q) \\ \Pr(s_t = 2|s_{t-1} = 1) = (1-p) & \Pr(s_t = 2|s_{t-1} = 2) = q \end{pmatrix}$$

Parameters	State 1	State 2
μ_{EU}	0.4287*** (0.0638)	-0.6574*** (-0.2132)
μ_{US}	0.3469** (0.0515)	-0.3873** (-0.1704)
c_{11}	0.7650*** (0.1422)	
c_{12}	0.1777 (0.2325)	
c_{22}	1.1071*** (0.2646)	
a_{11}	0.0007 (0.0128)	
a_{12}	-0.0271** (0.0122)	
a_{21}	0.0538*** (0.0142)	
a_{22}	0.0591*** (0.0136)	
b_{11}	0.9424*** (0.0503)	
b_{12}	0.5978*** (0.0506)	
b_{21}	-0.7833*** (0.0803)	
b_{22}	-0.2608** (0.1122)	
Sc	1.5631*** (0.1093)	
Sa	8.7942*** (1.7405)	
Sb	1.61215*** (0.0875)	
p	0.9502*** (0.0312)	
q	0.9112*** (0.0638)	

Panel B.- Serial correlation test on standardized residuals

	$e_{11}/\sqrt{\sigma^2_{11}}$	(p-value)	$e_{22}/\sqrt{\sigma^2_{22}}$	(p-value)
Q(7)	11.5624	0.1159	11.3252	0.1250
Q^2 (7)	8.7837	0.2686	8.7510	0.2710
ARCH (7)	0.8315	0.3618	2.5463	0.1105

*Panel A shows the estimation results MRS-BEKK (robust standard errors in parenthesis). ***,** and * represents significance at 1%, 5% and 10% level. Panel B perform several tests for serial correlation on the standardized residuals (Q(7) and Q(7) represents the Ljung-Box test for series in levels and squares and ARCH is the Engle's test for ARCH effects.*

Table 3.- Results for the linearized state-dependent variance equation

Low volatility states
Eurostoxx50 conditional variance
$\sigma^2_{EU,t,s_t=1} = \underset{(0.2176)^{***}}{0.5852} + \underset{(0.0950)^{***}}{0.8881}\sigma^2_{EU,t-1} + \underset{(0.0816)^{***}}{0.5634}\sigma_{EUUS,t-1} + \underset{(0.1258)^{***}}{0.6135}\sigma^2_{US,t-1} + \underset{(1.766E-05)}{4.722E-07}e^2_{EU,t-1} - \underset{(0.0031)}{1.866E-05}e_{EU,t-1}e_{US,t-1} + \underset{(0.0015)^{*}}{0.0029}e^2_{US,t-1}$
SP100 conditional variance
$\sigma^2_{US,t,s_t=1} = \underset{(0.0827)}{0.0316} + \underset{(0.3434)^{***}}{1.2257} + \underset{(0.0605)^{***}}{0.3574}\sigma^2_{EU,t-1} - \underset{(0.1789)}{0.1559}\sigma_{EUUS,t-1} + \underset{(0.0293^{**})}{0.0680}\sigma^2_{US,t-1} + \underset{(0.0007)}{0.0007}e^2_{EU,t-1} - \underset{(0.0072)}{0.0016}e_{EU,t-1}e_{US,t-1} + \underset{(0.0016)^{**}}{0.0035}e^2_{US,t-1}$
High volatility states
Eurostoxx50 conditional variance
$\sigma^2_{EU,t,s_t=2} = \underset{(0.9514)}{1.4299} + \underset{(0.2454)^{***}}{2.3065}\sigma^2_{EU,t-1} + \underset{(0.6816)^{***}}{1.4632}\sigma_{EUUS,t-1} + \underset{(0.3182)^{***}}{1.5934}\sigma^2_{US,t-1} + \underset{(0.0014)}{3.652E-05}e^2_{EU,t-1} - \underset{(0.0803)}{0.0014}e_{EU,t-1}e_{US,t-1} + \underset{(0.1317^{*})}{0.2238}e^2_{US,t-1}$
SP100 conditional variance
$\sigma^2_{US,t,s_t=2} = \underset{(0.5850)}{0.0772} + \underset{(1.5534)^{*}}{2.9950} + \underset{(0.2616)^{***}}{0.9282}\sigma^2_{EU,t-1} - \underset{(0.4510)}{0.4050}\sigma_{EUUS,t-1} + \underset{(0.0812^{**})}{0.1767}\sigma^2_{US,t-1} + \underset{(0.0500)}{0.0570}e^2_{EU,t-1} - \underset{(0.1893)}{0.1240}e_{EU,t-1}e_{US,t-1} + \underset{(0.1389)^{*}}{0.2698}e^2_{US,t-1}$

Parameters values for the linearized state-dependent volatilities. Robust standard errors in parenthesis (computed following the Kearney and Patton (2000 transformation). ***, ** *and* * *represent significance at 1%, 5% and 10% levels.*

Table 4.- Equality of conditional correlation during low and high volatility periods

Panel A.- Dummy regression $\rho_{EU/US,t} = c + \gamma D_t + \varepsilon_t$		
	c	γ
Parameter *(standard error)*	0.6111*** (0.0026)	0.1067** (0.0047)

	t-test	Satterwith-Welch t-test	ANOVA F-test	Welch F-test
Statistic *(p-value)*	**22.8795** *(0.00)*	**21.2951** *(0.00)*	**523.475** *(0.00)*	**453.483** *(0.00)*

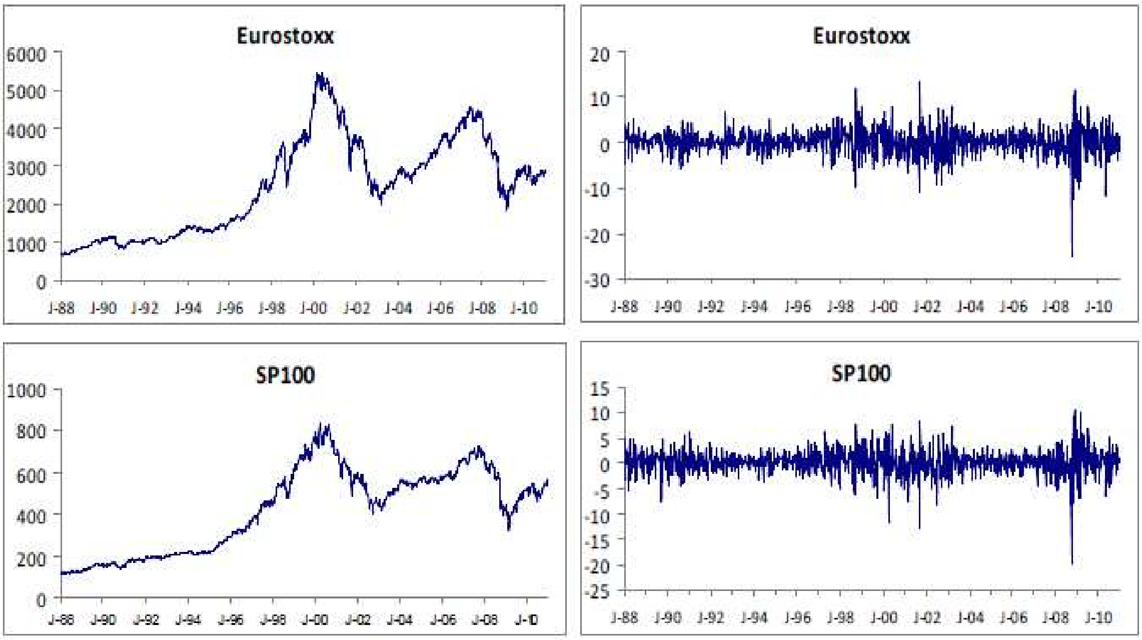
This table represents several tests analyzing differences on conditional correlation during low and high volatility periods.

Table 5.- Impact of country volatility on conditional correlation

$\rho_{EU/US,t} = c + \gamma\sigma_{EU,t} + \beta\sigma_{US,t} + \varepsilon_t$			
Panel A.- State-independent volatility			
	c	γ	β
Parameter *(standard error)*	0.42745^{***} (0.00622)	0.03491^{***} (0.01403)	0.05947^{***} (0.01824)
Panel B.- Low volatility periods			
	c	γ	β
Parameter *(standard error)*	0.3783^{***} (0.0044)	0.0832^{***} (0.0106)	0.0304^{***} (0.0138)
Panel C.- High volatility periods			
	c	γ	β
Parameter *(standard error)*	0.4869^{***} (0.0079)	0.0202^{***} (0.0190)	0.0566^{***} (0.0246)

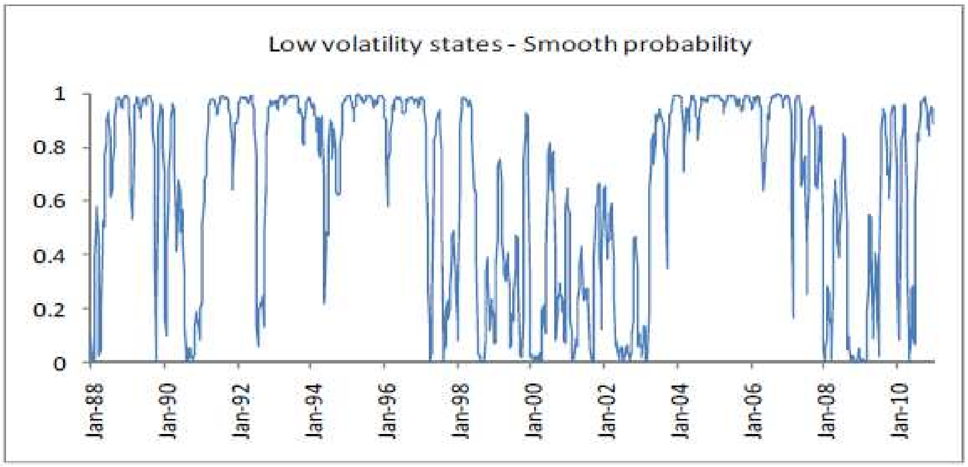
This table represents the estimation for equation ( ) distinguishing for the entire sample and during low and high volatility periods.

Figure 1.- Price indexes and returns for Eurostoxx50 and SP100



This figure shows the evolution for the Eurostoxx50 and the SP100 indexes from January 1988 to December 2010.

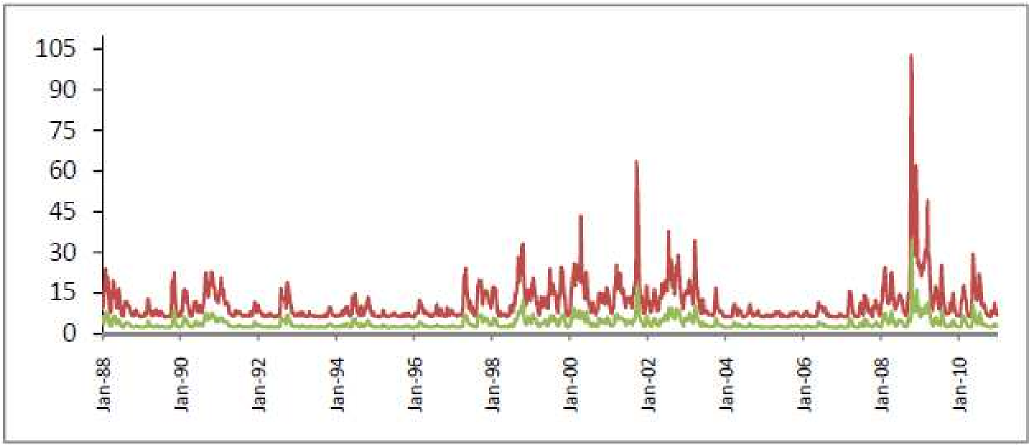
Figure 2.- Smooth probability of being in low volatility states



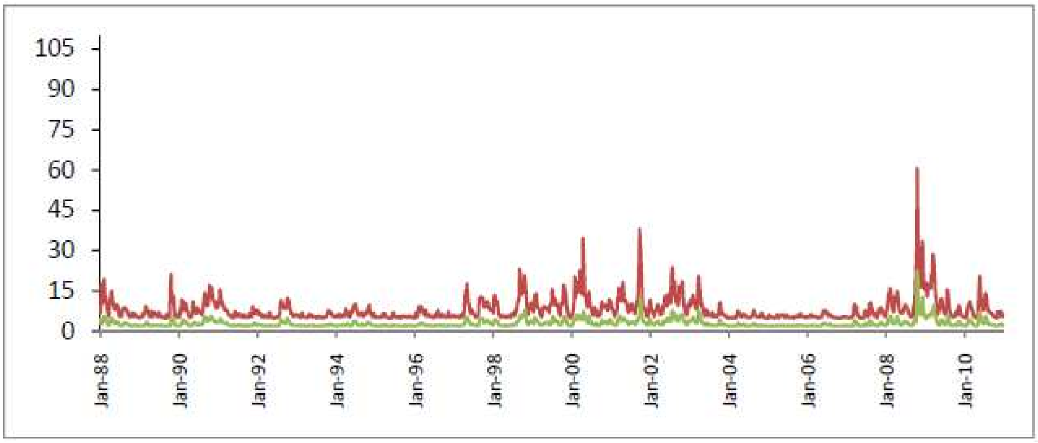
This figure displays the smooth probability of being in a low volatility state during the sample period considered.

Figure 3.- Estimated state-dependent conditional volatilities

State 1 vs. State 2 in EU stock market

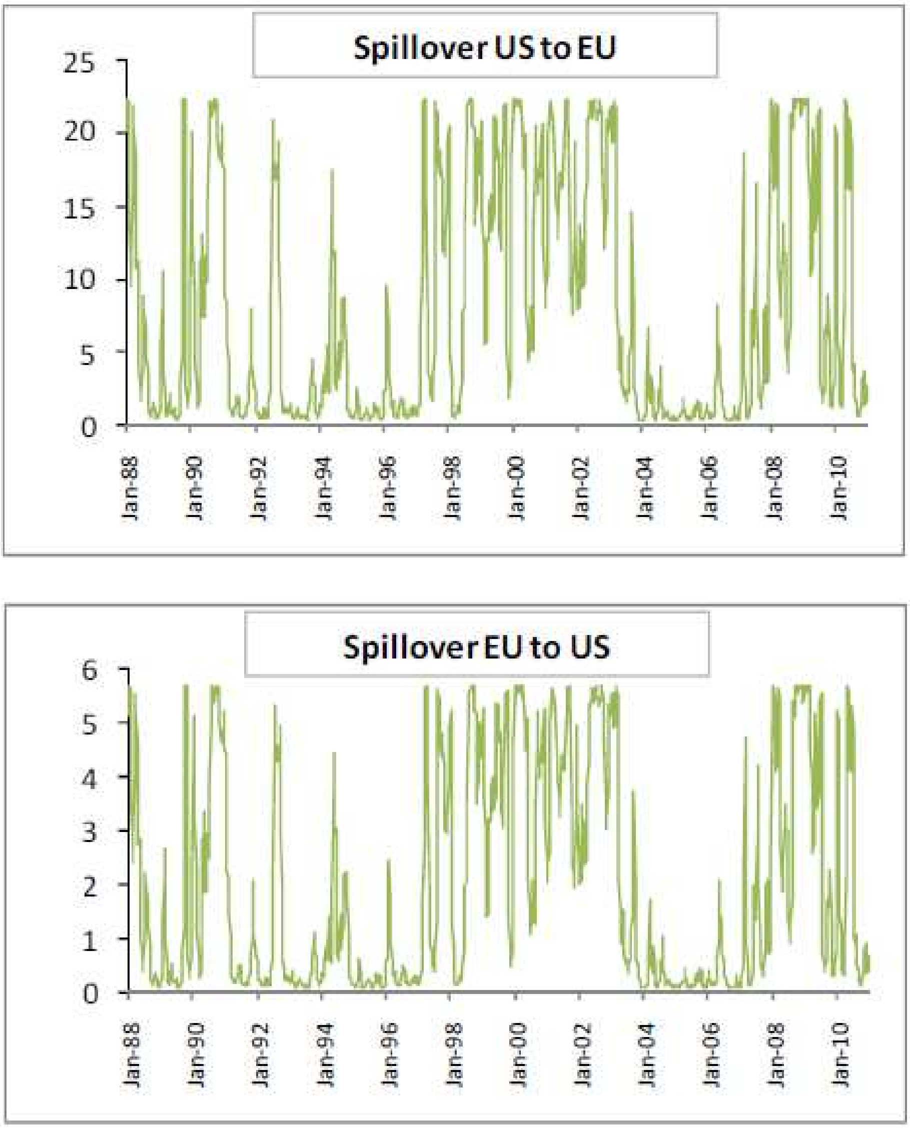


State 1 vs. State 2 in US stock market



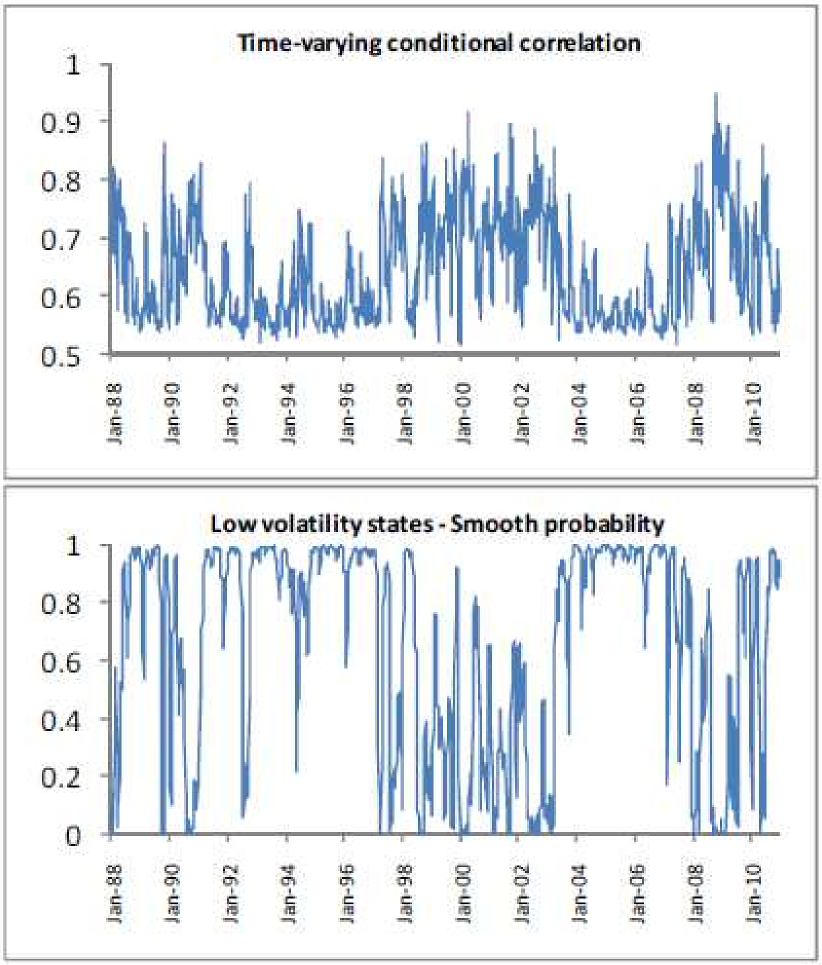
These two figures represent the estimated state-dependent volatilities in US and EU during low (green line) and (red line) volatility periods.

Figure 4.- Spillovers intensities over time



This figure reports the time-varying intensities by which shocks are transmitted from the US market to the EU market and vice-versa. The vertical axes represent the magnitude of the spillover (in % of volatility) and the horizontal axes represent the sample period

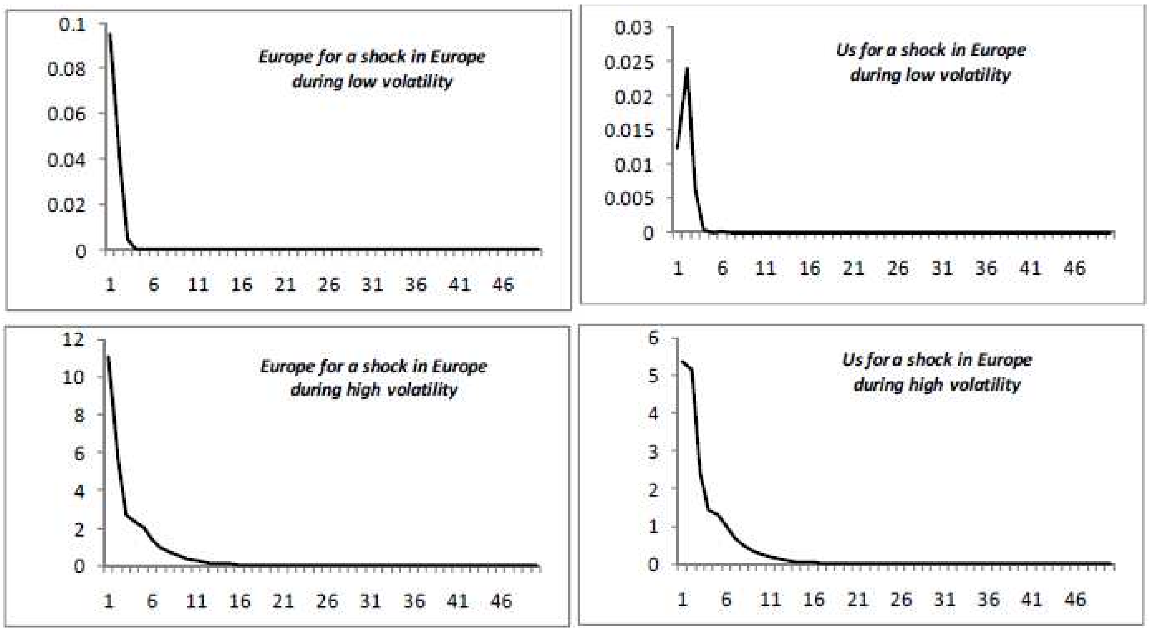
Figure 5.- Time-varying correlation against smooth probabilities



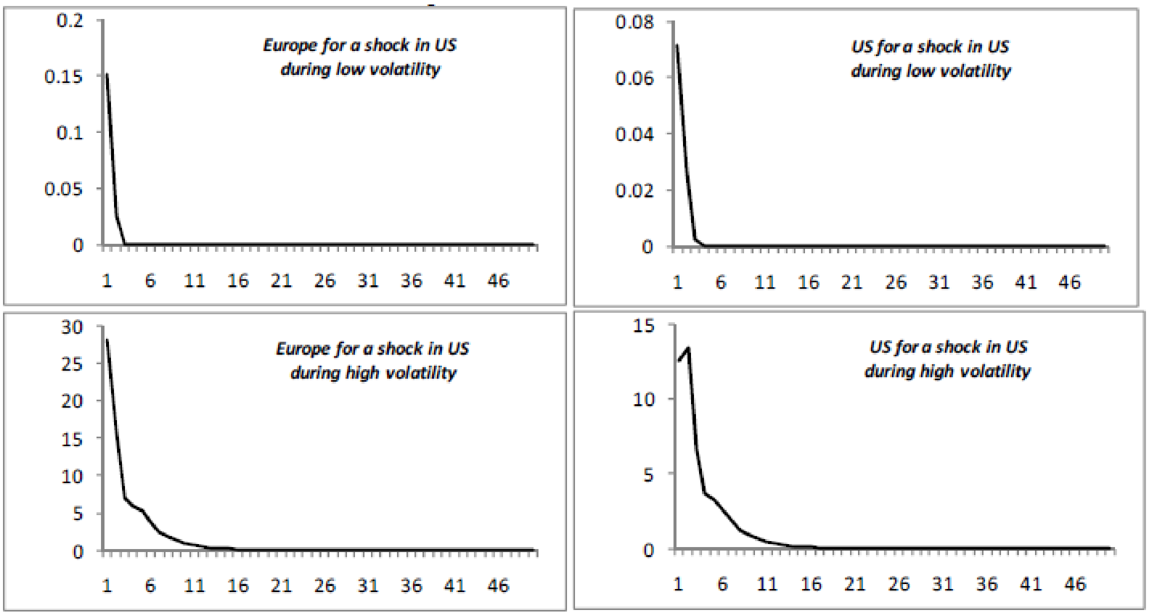
This figure compares the evolution through the sample period between the time-varying correlations and the smoothed probabilities.

Figure 6.- Volatility impulse response functions

State-dependent VIRF for a shock in Europe



State-dependent VIRF for a shock in USA



These figures represent the impact of an unexpected shock originated in a certain market on the market volatility of EU/US during the next 25 observations distinguishing between low and high volatility periods. The vertical axes represent the magnitude of the shock (in % of volatility) and the horizontal axes represents the period of decay (in weeks).