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New alternative normalization and aggregation formulas for the Human Development Index

Abstract

The Human Development Index (HDI) constitutes a widely used tool of analysis to evaluate human well-being and progress across countries and over time. Since it was launched, the HDI has generated an extensive literature, which includes numerous critiques and potential improvements. In 2010 it was revised with several major changes. Many of the problems pointed out by critics were tackled with the changes introduced, although serious drawbacks still persist, particularly related to the potential trade-offs between the HDI components. In this paper we propose new alternative normalization and aggregation formulas for the HDI and assess the problem of substitutability. To this end, we implement an approach based on the double reference point methodology with data from the *Human Development Report 2011*. For each component, the value of each country is normalized by means of two reference values (aspiration and reservation values) by using an achievement scalarizing function which is piecewise linear. Aggregating the values of the components, we calculate: (1) a weak index that allows total substitutability; (2) a strong index that measures the state of the worst component and allows no substitutability; and (3) a mixed index that is a linear combination of the first two. The resulting values of these indices and country rankings are analyzed and compared with the official HDI, evidencing the problem of substitutability and how it may seriously distort the data of human well-being and their policy implications.

Keywords: Human Development Index (HDI); substitutability; multi-criteria approach; double reference point methodology; aspiration and reservation values

JEL Classification: O15, O57, C02, C44

1. Introduction

The Human Development Index (HDI) was presented in 1990 in the first global *Human Development Report* (HDR) of the United Nations Development Program (UNDP) as an alternative to gross domestic product (GDP) or gross national product (GNP) per head; since then this measure of human well-being and progress has aroused great interest among researchers, practitioners and policy makers. For more than 20 years, the HDI has been a useful tool of analysis for governments, the media and civil society in order to evaluate human development across countries and over time, helping guide policy discussions and enlightening decisions.

Since its launch, the HDI has generated in the academic field an extensive literature assessing its properties, providing numerous critiques, and proposing a number of potential improvements (McGillivray 1991; Trabold-Nubler 1991; Desai 1991; Kelley 1991; McGillivray and White 1993; Murray 1993; Srinavasan 1994; Dossel and Grounder 1994; Gormely 1995; Ravallion 1997; Doraid 1997; Noorbakhsh 1998; Palazzi and Lauri 1998; Anand and Sen 2000; Chakravarty 2003; Chatterjee 2005; Foster et al. 2005; Chowdhury and Squire 2006; Gaertner and Xu 2006; Lind 2010; Herrero et al. 2010a and 2010b; De Muro et al. 2011; Nguefack-Tsague et al. 2011; Pinar et al. 2012; Foster et al. 2013; Rende and Donduran 2013, among others)[1].

In 2010, coinciding with the twentieth anniversary of the first global HDR, the United Nations Development Program (UNDP) decided to revise the HDI and introduced several major changes. Though this is not the first time that the HDI was modified, it was the first time that major changes were simultaneously made to its components and to the functional form used. Many of the problems pointed out by critics were tackled with the changes introduced in the manner in which the new HDI (UNDP 2010) is calculated, although some authors consider that serious drawbacks still persist (see Ravallion 2010, 2011 and 2012; Klugman et al. 2011a and 2011b; Chakravarty 2011; Tofallis 2012; Herrero et al. 2012; Bilbao-Ubillos 2012; among others).

The HDI's functional form is one the methodological issues that has attracted the most interest among researchers, focusing on aspects such as the substitutability assumptions, the normalization of indicators, the asymmetric treatment of income, and the choice of weights (see, e.g., Klugman et al. 2011a, director and lead author of the 2009, 2010 and 2011 HDR). One of the major modifications introduced in the new HDI is the replacement of the arithmetic mean of country-level attainments in health, education and income for the

[1] For a survey, see Kovacevic (2011), which was part of a comprehensive review undertaken by the Human Development Report Office (HDRO) of UNDP.

geometric mean as the aggregation formula. In fact, the main reason given for introducing the new HDI was to avoid the past assumption of perfect substitutability between the HDI components[2].

The new HDI allows imperfect substitutability between its three components, as the new functional form continues to a certain extent to entail implicit trade-offs. In this context, Herrero et al. (2012) highlight that, although the choice of the geometric mean is certainly an important improvement, UNDP (2010) does not provide any theoretical justification of the new aggregation method. Among other contributions, they propose an elementary characterization of the geometric mean following the axiomatic method.

Although it is obvious that any composite index of this sort will entail potentially troubling trade-offs, as Ravallion (2010 and 2012) recognizes, he highlights that the new multiplicative form appears to generate highly problematic trade-offs from the standpoint of assessing human development. In particular, he shows that the new HDI has greatly reduced its implicit weight on longevity in poor countries, and the valuations of extra schooling as a whole seem high. Ravallion (2010 and 2012) and Chakravarty (2011) agree that the troubling trade-offs found in the new HDI could have been avoided to a large extent by using an alternative aggregation function from the literature, namely the generalized form of the old HDI proposed by Chakravarty (2003).

There exist in the literature other recent contributions proposing alternative measures of human development. For instance, Bilbao-Ubillos (2012) proposes a supplementary index, called 'Composite, Dynamic Human Development Index', in order to palliate some limitations of the HDI. This index incorporates significant additional points related to the concept of human development, and provides an interesting dynamic factor that distinguishes between countries on the basis of achievements attained. However, the author ignores the problem of substitutability.

In this paper we propose new alternative normalization and aggregation formulas for the HDI and assess the problem of substitutability between the HDI components posed by the new functional form. To this end, we implement an approach based on the double reference point methodology (aspiration and reservation) by employing data from the 2011 HDR. For each component, the value of each country is normalized in the range [-1, 2] by means of two reference values (aspiration and reservation values) using an achievement scalarizing function which is piecewise linear. This normalization entails an advantage in respect to the one used in calculating the official HDI, in so far as normalization of the range [0, 1] between maximum and minimum

[2] Let us recall that, as highlighted by authors such as Desai (1991) and Palazzi and Lauri (1998), the additive form of the HDI is problematic because it implies perfect substitution across components. It assumes that the level of priority to be given to a component is invariant to the level of attainments. In addition, if a society were to seek policies to maximize its HDI, it might emphasize one component and disregard the others (see Klugman et al. 2011a).

values means that a specific improvement in some HDI component may have a similar impact on the measurement of human development in countries with very different levels of development. Aggregating the new values (values of the achievement scalarizing functions), we calculate three indices: (1) a weak index that allows full compensation between the various components; (2) a strong index that measures the state of the worst component and allows no compensation; (3) and a mixed index that is a linear combination of the first two. Subsequently, the resulting values of these indices and country rankings are analyzed and compared with the official HDI, in particular taking into account the problem of substitutability. The remainder of the paper is as follows. Section 2 briefly describes the calculation of the HDI in the 2011 HDR. Section 3 states our methodological approach. Section 4 presents and discusses the results. The final section presents our conclusions.

2. Calculation of the HDI

The HDI is a summary measure of human development which measures a country's average achievements in the three core dimensions of human development[3]:

i. A long and healthy life, by using life expectancy at birth (LE) as an indicator. This is the only core dimension that was not changed in 2010.

ii. Access to knowledge, measured as mean years of schooling (MS) and expected years of schooling (ES), the latter defined as the years of schooling that a child can expect to receive given current enrolment rates. These indicators have replaced literacy and gross enrolment rate. Both new indicators are summarized by using the geometric mean (S).

iii. A decent standard of living, measured as the natural log of per capita gross national income (GNI) at purchasing-power parity (PPP) (Y). In this case, GNI has replaced GDP, also at PPP and logged[4].

Following UNDP (2011), there are two steps to calculate the HDI. Firstly, the dimension indexes are created. To this end, minimum and maximum values (goalposts) are set in order to transform the indicators into indices between 0 and 1. The maximums are the highest observed values in the time series (1980–2011) and the

[3] The HDI excludes other 'broader dimensions' of the concept of human development, such as empowerment, sustainability and equity. The 2010 HDR decided not to introduce any new dimensions in the HDI, stressing that the HDI can be characterized as an index of opportunities and freedoms, according to the two types of freedoms (opportunity freedoms and process freedoms) suggested by Sen (2002), that are valued by the human development approach (see, e.g., Klugman et al. 2011a).

[4] Given that the transformation function from income to capabilities is likely to be concave (Anand and Sen 2000), the natural logarithm is now used for per capita GNI, whereas before it was for per capita GDP.

minimum values can be appropriately conceived as subsistence values. In particular, in the 2011 HDR the minimum values are set at 20 years for life expectancy, at 0 years for both education variables and at $100 for per capita GNI[5].

Table 1. Goalposts for the Human Development Index in 2011 HDR

Dimension	Observed maximum	Minimum
Life expectancy	83.4 (Japan, 2011)	20.0
Mean years of schooling	13.1 (Czech Republic, 2005)	0
Expected years of schooling	18.0 (capped at)	0
Combined education index	0.978 (New Zealand, 2010)	0
GNI (PPP $)	107,721 (Qatar, 2011)	100

Source: UNDP (2011)

$$I_{LE} = \frac{LE - LE^{min}}{LE^{max} - LE^{min}} \quad (1)$$

$$I_S = \frac{S - S^{min}}{S^{max} - S^{min}} \quad (2)$$

$$I_Y = \frac{\ln Y - \ln Y^{min}}{\ln Y^{max} - \ln Y^{min}} \quad (3)$$

Having defined the minimum and maximum values and calculated the normalized dimension indices in the zero to one range, these are aggregated to produce the HDI as the geometric mean of the three dimension indices instead of the arithmetic mean considered in the old aggregation formula. In this way, in a multiplicative setting the weights are applied by raising each variable to a power. Equal weights continue to be taken[6]. Thus, the HDI is calculated through the geometric mean of normalized indices measuring achievements in each core dimension.

$$\text{HDI} = I_{LE}^{1/3} \cdot I_S^{1/3} \cdot I_Y^{1/3} \quad (4)$$

3. Methodology

3.1. Multi-criteria approach

Many real life problems involve dealing with optimization problems, in which multiple objective functions are maximized or minimized simultaneously within a feasible set of solutions or alternatives. The general form of a multi-objective optimization problem (MOP) can be represented by:

[5] UNDP (2011) reminds us that the low value for income can be justified by the considerable amount of unmeasured subsistence and nonmarket production in economies close to the minimum, not reflected in the official data.

[6] The choice of equal weights has been widely criticized, with diverse methodologies proposed to set weights (see, e.g., Kelley 1991; Chowdhury and Squire 2006; Lind 2010; Nguefack-Tsague et al. 2011; Tofallis 2012; Pinar et al. 2012; Foster et al. 2013), was also unchanged in 2010.

$$\begin{aligned} &\max \ f(\mathbf{x}) = \big(f_1(\mathbf{x}), \dots, f_k(\mathbf{x})\big) \\ &s.t.: \qquad \mathbf{x} \in X \end{aligned} \tag{5}$$

where $\boldsymbol{x} = (x_1, \dots, x_n)^T$ is an n-dimensional vector of *decision variables*, $X \subset \mathbb{R}^n$ is the *feasible region*, $Z = f(X)$ is the *feasible objective space*, and $\boldsymbol{z} = f(\boldsymbol{x})$ an *objective vector* where $\boldsymbol{z} \in Z$ if $\boldsymbol{x} \in X$ exists. The purpose is to simultaneously maximize all the k ($k \geq 2$) objective functions. All the objective functions can be considered in the same sense (all maximizing or all minimizing), since minimizing an objective function is equivalent to maximizing the opposite one.

In multi-objective optimization, which generally lacks a feasible solution to simultaneously maximize all objective functions, there appears another concept of optimal where none of the components can be improved without deteriorating at least one of the others. A decision vector $\boldsymbol{x}' \in X$ is called efficient or Pareto optimal of the problem MOP if there does not exist another $\boldsymbol{x} \in X$ such as $f_i(\boldsymbol{x}') \leq f_i(\boldsymbol{x})$ for all $i = 1, \dots, k$ and $f_j(\boldsymbol{x}') < f_j(\boldsymbol{x})$ for at least one index j. In this case, $\boldsymbol{z}' = f(\boldsymbol{x}')$ is called nondominated objective vector. The efficient set is denoted by E and $f(E)$ is the nondominated objective set. A decision vector $\boldsymbol{x}' \in X$ is called weakly efficient or weakly Pareto optimal if there does not exist another $\boldsymbol{x} \in X$ such as $f_i(\boldsymbol{x}') < f_i(\boldsymbol{x})$ for all $i = 1, \dots, k$. The corresponding objective vectors are called weakly nondominated objective vectors. Note that the set of efficient solutions is a subset of weakly efficient solutions.

Since the set of non-dominated objective vectors contains more than one vector, it is useful to know the bounds for the objective vectors in the non-dominated set. Upper bounds are given by the ideal values $\mathbf{z}^* = (z_1^*, \dots, z_k^*)$, easily obtained by maximizing each objective function separately $z_i^* = \max_{\mathbf{x} \in E} f_i(\mathbf{x}) = \max_{\mathbf{x} \in X} f_i(\mathbf{x})$ for all $i = 1, \dots, k$. However, nadir vector $\mathbf{z}^{nad} = \big(z_1^{nad}, \dots, z_k^{nad}\big)$, where $z_i^{nad} = \min_{\mathbf{x} \in E} f_i(\mathbf{x})$ for all i = 1, … , k, is usually difficult to obtain (see Miettinen (1999) and references therein).

A very common way to express preferences about the efficient solutions is given by the so-called *reference point* $\mathbf{q} = (q_1, \dots, q_k)^T$, which consists of reference values for the objective functions. The multi-objective MOP problem and the reference point are combined in an achievement scalarizing function (ASF), which is optimized to generate (weakly) efficient solutions.

One of the most commonly used ASFs was proposed by Wierzbicki (1980):

$$s(\mathbf{q}, f(\mathbf{x}), \mu) = \min_{i=1,\dots,k} \{\mu_i (f_i(\mathbf{x}) - q_i)\} + \rho \sum_{i=1}^{k} (f_i(\mathbf{x}) - q_i) \tag{6}$$

which must be maximized in the feasible region:

$$\begin{aligned} &\max \ \ s(\mathbf{q}, f(\mathbf{x}), \mu) \\ &s.t.: \qquad \mathbf{x} \in X \end{aligned} \tag{7}$$

The parameter $\rho > 0$ is the so-called augmentation term, which must be a small value, and which assures the efficiency of the solutions generated. If the second term is not used, then only the weak efficiency of the solution is assured. The vector $\mu = (\mu_1, \dots, \mu_k)^T$ with $\mu_i > 0$ for all $\mathrm{i} = 1, \dots, \mathrm{k}$ is formed by the weights assigned to reach the reference values, which can range from a purely normalizing coefficient to a preferential parameter (Ruiz et al., 2009, Luque et al. 2009). Along the same line, the ASF proposed in Luque et al. (2012):

$$\begin{aligned} \bar{s}(\mathbf{q}, f(\mathbf{x}), \mu) &= \bar{s}_0(\mathbf{q}, f(\mathbf{x}), \mu) + \rho \textstyle\sum_{i=1}^{k} (f_i(\mathbf{x}) - q_i) \\ \bar{s}_0(\mathbf{q}, f(\mathbf{x}), \mu) &= \min_{i=1,\dots,k} \{\min \ \ \{\mu_i^U (f_i(\mathbf{x}) - q_i), 0\} + \max \ \ \{\mu_i^A (f_i(\mathbf{x}) - q_i), 0\}\} \end{aligned} \tag{8}$$

allows considering different weights depending on the reference point.

Another achievement scalarizing function (Wierzbicki et al., 2000), used in both continuous and discrete programming, normalizes the objective functions (or indicators in our case) in a very appropriate way, taking into account two types of values of reference for each objective function. This type of ASF, called the double reference point (aspiration and reservation values) scheme, is based on considering an aspiration value z_i^a for each objective function f_i (it being desirable to reach that value) and a reservation value z_i^r (level under which the objective function is not considered acceptable). Concretely, let us consider the achievement scalarizing function:

$$s(f(\mathbf{x}),\, \mathbf{z}^a,\, \mathbf{z}^r) = \min_{i=1,\dots,k} \{s_i(f_i(\mathbf{x}),\, z_i^a,\, z_i^r)\} + \rho \textstyle\sum_{i=1}^{k} s_i(f_i(\mathbf{x}),\, z_i^a,\, z_i^r) \tag{9}$$

where s_i for all $i = 1, \dots, k$ are the individual achievement scalarizing functions:

$$s_i(f_i(\mathbf{x}),\, z_i^a,\, z_i^r) = \begin{cases} 1 + \frac{f_i(\mathbf{x}) - z_i^a}{z_i^{max} - z_i^a} & \text{if } z_i^a \le f_i(\mathbf{x}) \le z_i^{max} \\ \frac{f_i(\mathbf{x}) - z_i^r}{z_i^a - z_i^r} & \text{if } z_i^r \le f_i(\mathbf{x}) \le z_i^a \\ \frac{f_i(\mathbf{x}) - z_i^r}{z_i^r - z_i^{min}} & \text{if } z_i^{min} \le f_i(\mathbf{x}) \le z_i^r \end{cases} \tag{10}$$

The values z_i^{max} and z_i^{min} are upper and lower bounds for each objective function f_i in the feasible region or even in the efficient set, if possible. $z_i^{max} = z_i^*$ and, $z_i^{min} = z_i^{nad}$ can be considered if available. Two parameters of the original formulation have been considered equal to 1. For more details about this ASF, see Wierzbicki et al. (2000).

This kind of ASF allows scaling all indicators in the interval [-1, 2], so that different interpretations are given on the basis of the aspiration and reservation values. Although in the continuous case the ASF function must be maximized in the feasible region, as mentioned previously, in the discrete case (our case) it allows us to establish a ranking for the different alternatives. For our purposes, the values of the objective functions $f_i(\mathbf{x})$ are substituted by the values of the indicators in the different alternatives (countries).

3.2. Application to measure the Human Development Index (HDI)

Let us consider a total of N_I indicators and N_C the number of alternatives (countries). In our case, $N_I = 3$ (*Life expectancy at birth, Combined Education Index, Gross National Income (GNI) per capita)* and N_C is the number of countries considered (N_C = 187). Let us denote by y_{ij} ($i = 1, \dots, N_C$ and $j = 1, \dots, N_I$) the value of the country *i* and the indicator *j*. For each indicator it is necessary to determine whether it is of the type "more is better" (equivalent to maximizing in the continuous case) or "less is better" (equivalent to minimizing in the continuous case); in our case, the three are of type "more is better".

For each indicator *j*, we have to calculate the maximum and minimum values:

$$y_j^{max} = \max_{i=1,\dots,N_C} y_{ij} \tag{11}$$

$$y_j^{min} = \min_{i=1,\dots,N_C} y_{ij} \tag{12}$$

However, these values can be modified by other values considered more appropriate.

The values of the aspiration and reservation levels, denoted by y_j^a and y_j^r respectively, are key to interpreting and analyzing the results. In the next section, we will explain which values are considered in our study.

Taking into account all the previous values calculated for each indicator *j*, let us consider the value given by the individual achievement scalarizing function in each alternative *i* (country):

$$s_j\left(y_{ij},\, y_j^a,\, y_j^r\right) = \begin{cases} 1 + \frac{y_{ij} - y_j^a}{y_j^{max} - y_j^a} & \text{if } y_j^a \le y_{ij} \le y_j^{max} \\ \frac{y_{ij} - y_j^r}{y_j^a - y_j^r} & \text{if } y_j^r \le y_{ij} \le y_j^a \\ \frac{y_{ij} - y_j^r}{y_j^r - y_j^{min}} & \text{if } y_j^{min} \le y_{ij} \le y_j^r \end{cases} \tag{13}$$

Given a country *i* and an indicator *j*, if y_{ij} is between -1 and 0, it means that the value of the indicator for this country is under the reservation value; between 0 and 1, that it is between reservation and aspiration values; and between 1 and 2, that it is over the aspiration value.

For each country *i*, let us define the *weak index* (W_i) as the arithmetic mean of the N_I values of the indicators and the *strong index* (S_i) as the minimum of all, that is, the worst one:

$$W_i = \frac{1}{N_I} \sum_{j=1}^{N_I} y_{ij} \tag{14}$$

$$S_i = \min_{j=1,\dots,N_I} y_{ij} \tag{15}$$

While the weak index allows compensation among different indicators (substitutability), the strong index does not allow any compensation since it represents the worst value. In case we want to assign different weights to the indicators, let ω_j with $j = 1, \dots, N_I$ be the weight values, which have to be strictly positive ($\omega_j > 0 \; \forall \; j = 1, \dots, N_I$). The weak index is calculated directly:

$$W_i = \sum_{j=1}^{N_I} \bar{\omega}_j \; y_{ij} \tag{16}$$

where $\bar{\omega}_j$ is the normalized weight ($\bar{\omega}_j = \frac{\varpi_j}{\sum_{k=1}^{N_I} \varpi_k} \; \forall \; j = 1, \dots, N_I$). However, for the strong index, it is necessary to make some changes to avoid unwanted effects. Specifically, let us consider the following weights normalized by its maximum value:

$$\bar{\omega}_j = \frac{\varpi_j}{\max_{k=1,\dots,N_I} \varpi_k} \;\; \forall \; j = 1, \dots, N_I \tag{17}$$

and for each country *i*, we define the following values:

$$\bar{y}_{ij} = y_{ij} - C_i \;\; \forall j = 1, \dots, N_I \;\; \text{with } C_i = \left[\min_{j=1,\dots,N_I} y_{ij}\right] + 1 \tag{18}$$

where [ ] is the integer part of a real number. Then, the strong index is given by:

$$S_i = C_i + \min_{j=1,\dots,N_I} \bar{\omega}_j \; \bar{y}_{ij} \tag{19}$$

The strong index indicates that if its value is below 0, at least one indicator is under 0 (at least one indicator does not reach its corresponding reservation value). If the strong index is above 1, it means that all the indicators improve their corresponding aspiration values.

As a combination of both we propose a *mixed indicator* (MI_i), which is a linear combination of the previous ones:

$$MI_i = \lambda W_i + (1-\lambda)S_i \text{ with } 0 < \lambda < 1 \quad (20)$$

and reflects an intermediate state between total substitutability (weak index) and no substitutability (strong index).

4. Results

4.1. Calculation of aspiration and reservation values

As mentioned in the previous section, in order to apply the proposed normalization, an aspiration level and reservation level have to be defined for each component. Let us recall that a component's level of aspiration is the desirable level to be achieved by said component, whereas the level of reservation is the value below which all values are considered unacceptable.

In our methodological proposal, these levels are essential for normalization. These values can be defined exogenously in an absolute manner, although in the literature such universally accepted values do not exist. Thus, in this paper they have been calculated in a relative manner, taking into account for each component the situation of some countries in respect to others[7].

This type of normalization has not yet been used to calculate the HDI, but could open up a new line of analysis in which the human development of countries could be assessed depending on the aspiration and reservation values considered.

In this paper, we are going to use two different criteria which we consider reasonable to calculate these values:

i. *Criterion I: Weighted mean of first and third group countries*. The UNDP (2011) classifies countries as *Very High Human Development*, *High Human Development*, *Medium Human Development* and *Low Human Development*. To do so, it divides the countries listed according to their HDI level into 4 equal parts. Similarly, we ranked countries according to the values of the respective components, taking as level of aspiration the corresponding mean values weighted by population of the countries with *Very High* levels for the component in question. On the other hand, as level of reservation we used the mean weighted values of the group of countries with *Medium* levels for the respective components. The figures for each component are shown Table 2.

ii. *Criterion II: First and second quartile*. The second criterion takes as level of aspiration the first quartile (value below which 75 per cent of the countries -127 countries- appear for each component), according to the order of the list of countries mentioned above for each component. We consider the third quartile as the

[7] Another option could have been to apply levels of reference defined by a panel of experts, which could lead to establishing, by consensus, absolute aspiration and reservation levels for each component.

reservation value; in other words, the value below which 25 per cent of the countries -42 countries- appear for each component. The figures are shown in Table 2.

Table 2. Reference Values

	Life expectancy	**Ln GNI per capita**	**Combined Education Index**
Aspiration Values I	80.000	10.416	13.381
Reservation Values I	66.825	8.173	9.111
Aspiration Values II	76.128	9.729	11.852
Reservation Values II	64.228	7.728	7.131

4.2. Calculation of normalized components

For the normalization of the components, we need, in addition to the aspiration and reservation values, a maximum and a minimum value for each indicator, which do not have to coincide with the values of the study. As mentioned in Section 2, UNDP (2011) specifies the maximum and minimum goalposts for each indicator used to calculate the official HDI (see Table 1). We thus use them as a reference for our calculations. The respective maximum and mínimum values for each component are shown in Table 3.

Table 3: Maximum and Minimum

	Life expectancy	**Ln GNI per capita**	**Combined Education Index**
Max	83.394	11.587	15.709
Min	20	4.605	0

After calculating the necessary parameters, we obtained the normalized components by applying equation (13). Since we are working with two criteria to calculate aspiration and reservation values, we obviously obtained two different results. In Table 4 we show the components normalized for a selection of countries[8] according to reference values calculated by means of criterion I and in Table 5 the components normalized according to criterion II.

[8] For the presentation of the results, we have selected the 10 most populated countries, which represent about 60 per cent of world population. These countries, listed on the basis of their HDI, are distributed amongst the 4 groups of countries as defined in the 2011 HDR: *Very High Human Development* (United States, Japan), *High Human Development (*Russian Federation, Brazil), *Medium Human Development* (China, Indonesia, India, Pakistan) and *Low Human Development* (Bangladesh, Nigeria).

Table 4: Normalized Components (I)

	Life expectancy	Life expectancy (Normalized)	Ln GNI per capita	Ln GNI per capita (Normalized)	Combined Education Index	Combined Education Index (Normalized)
Min	20.000		4.605		0.000	
Reservation	66.825		8.173		9.111	
Aspiration	80.000		10.416		13.381	
Max	83.394		11.587		15.709	
United States	78.531	0.889	10.669	1.216	14.095	1.307
Japan	83.394	2.000	10.383	0.985	13.248	0.969
Russian Federation	68.823	0.152	9.586	0.630	11.769	0.623
Brazil	73.488	0.506	9.226	0.469	9.944	0.195
China	73.456	0.503	8.920	0.333	9.344	0.054
Indonesia	69.366	0.193	8.220	0.021	8.760	-0.039
India	65.438	-0.030	8.151	-0.006	6.748	-0.259
Pakistan	65.437	-0.030	7.844	-0.092	5.789	-0.365
Bangladesh	68.944	0.161	7.333	-0.236	6.232	-0.316
Nigeria	51.879	-0.319	7.635	-0.151	6.631	-0.272

Table 5. Normalized Components (II)

	Life expectancy	Life expectancy (Normalized)	Ln GNI per capita	Ln GNI per capita (Normalized)	Combined Education Index	Combined Education Index (Normalized)
Min	20.000		4.605		0.000	
Reservation	64.228		7.728		7.131	
Aspiration	76.128		9.729		11.852	
Max	83.394		11.587		15.709	
United States	78.531	1.331	10.669	1.506	14.095	1.581
Japan	83.394	2.000	10.383	1.352	13.248	1.362
Russian Federation	68.823	0.386	9.586	0.929	11.769	0.982
Brazil	73.488	0.778	9.226	0.749	9.944	0.596
China	73.456	0.775	8.920	0.595	9.344	0.469
Indonesia	69.366	0.432	8.220	0.246	8.760	0.345
India	65.438	0.102	8.151	0.212	6.748	-0.054
Pakistan	65.437	0.102	7.844	0.058	5.789	-0.188
Bangladesh	68.944	0.396	7.333	-0.127	6.232	-0.126
Nigeria	51.879	-0.279	7.635	-0.030	6.631	-0.070

Tables 4 and 5 show, for example, that Japan's normalized life expectancy value is 2. This means that its value for this component coincides with the maximum considered. Indeed, let us recall that in this case the maximum considered for this component was taken from the value registered in Japan in 2011 (see Table 1). In respect to the other results, normalized values above 1 mean that countries are above the aspiration level for that

component, whereas values below 0 indicate that they are below the reservation value. If we look at Table 4, we can see, for example, that the United States is above the level of aspiration in attainments in education and income, although in respect to life expectancy it is below said level, indicating a very high, though slightly imbalanced, level of human development. Nigeria, on the other hand, shows values below the level of reservation for all the components in both tables, thus indicating low but relatively balanced levels of human development in the three core dimensions.

4.3. Calculation of strong, weak and mixed indices

Let us calculate the weak and strong indices using the equations [16-19]. Although our methodology allows establishing different weights for each component, in this paper we have considered the same weights for all components, in line with the position followed by the UNDP in calculating the official HDI. To calculate the mixed index with the equation (20), we use a value of $\lambda = 0.5$.

We show the values of these indices for the group of countries selected using values of reference I (Table 6) and values of reference II (Table 7), denoted as DRP-WI (Double Reference Point - Weak Index), DRP-SI (Double Reference Point - Strong Index) and DRP-MI (Double Reference Point - Mixed Index).

Table 6: DRP-WI, DRP-SI and DRP-MI (I)

Country	DRP-WI	DRP-SI	DRP-MI
United States	1.137	0.889	1.013
Japan	1.318	0.969	1.143
Russian Federation	0.468	0.152	0.310
Brazil	0.390	0.195	0.293
China	0.297	0.054	0.176
Indonesia	0.058	-0.039	0.010
India	-0.098	-0.259	-0.179
Pakistan	-0.162	-0.365	-0.263
Bangladesh	-0.130	-0.316	-0.223
Nigeria	-0.247	-0.319	-0.283

Table 7: DRP-WI, DRP-SI and DRP-MI (II)

Country	DRP-WI	DRP-SI	DRP-MI
United States	1.473	1.331	1.402
Japan	1.571	1.352	1.461
Russian Federation	0.766	0.386	0.576
Brazil	0.708	0.596	0.652

China	0.613	0.469	0.541
Indonesia	0.341	0.246	0.293
India	0.086	-0.054	0.016
Pakistan	-0.010	-0.188	-0.099
Bangladesh	0.048	-0.127	-0.039
Nigeria	-0.126	-0.279	-0.203

Tables 6 and 7 show that the DRP-SI -which measures the state of the worst component and allows no compensation between components- is much stricter than the DRP-WI -which allows compensation among components-, and is lower in all cases. The DRP-MI, in turn, always presents values between the other two indices.

4.4. Rankings and differences in respect to HDI rank

If we take into account country rankings for each new index calculated (DRP-WI, DRP-SI, DRP-MI), we should analyze the consistency of those rankings with HDI country rankings, and the differences in the number of positions between said rankings for each country. The results for the group of countries selected obtained using criteria I and II appear in Tables 8 and 9, respectively (the position in the corresponding ranking appears in parenthesis).

Table 8: Rankings and Differences (I)

Country	**HDI**		**DRP-WI**		**DRP-SI**		**DRP-MI**		**HDI rank - DRP-WI rank**	**HDI rank - DRP-SI rank**	**HDI rank - DRP-MI rank**
United States	0.910	(4)	1.137	(14)	0.889	(17)	1.013	(16)	-10	-13	-12
Japan	0.901	(12)	1.318	(3)	0.969	(7)	1.143	(3)	9	5	9
Russian Federation	0.755	(66)	0.468	(71)	0.152	(86)	0.310	(80)	-5	-20	-14
Brazil	0.718	(84)	0.390	(86)	0.195	(79)	0.293	(83)	-2	5	1
China	0.687	(101)	0.297	(100)	0.054	(98)	0.176	(100)	1	3	1
Indonesia	0.617	(124)	0.058	(131)	-0.039	(108)	0.010	(121)	-7	16	3
India	0.547	(134)	-0.098	(136)	-0.259	(134)	-0.179	(138)	-2	0	-4
Pakistan	0.504	(145)	-0.162	(144)	-0.365	(152)	-0.263	(147)	1	-7	-2
Bangladesh	0.500	(146)	-0.130	(141)	-0.316	(143)	-0.223	(143)	5	3	3
Nigeria	0.459	(156)	-0.247	(157)	-0.319	(144)	-0.283	(152)	-1	12	4

Table 9: Rankings and Differences (II)

Country	**HDI**		**DRP-WI**		**DRP-SI**		**DRP-MI**		**HDI rank - DRP-WI rank**	**HDI rank - DRP-SI rank**	**HDI rank - DRP-MI rank**
United States	0.910	(4)	1.473	(16)	1.331	(11)	1.402	(11)	-12	-7	-7
Japan	0.901	(12)	1.571	(3)	1.352	(9)	1.461	(6)	9	3	6
Russian Federation	0.755	(66)	0.766	(73)	0.386	(95)	0.576	(89)	-7	-29	-23
Brazil	0.718	(84)	0.708	(85)	0.596	(73)	0.652	(78)	-1	11	6
China	0.687	(101)	0.613	(98)	0.469	(88)	0.541	(91)	3	13	10
Indonesia	0.617	(124)	0.341	(126)	0.246	(107)	0.293	(115)	-2	17	9
India	0.547	(134)	0.086	(137)	-0.054	(129)	0.016	(134)	-3	5	0
Pakistan	0.504	(145)	-0.010	(146)	-0.188	(143)	-0.099	(145)	-1	2	0

Bangladesh	0.500	(146)	0.048	(141)	-0.127	(138)	-0.039	(139)	5	8	7
Nigeria	0.459	(156)	-0.126	(157)	-0.279	(155)	-0.203	(158)	-1	1	-2

In general, the correlation between HDI's ranking of countries and the rankings according to DRP-WI, DRP-SI and DRP-MI is obviously quite high, in particular if we bear in mind that we are only working with three components and are analyzing a relatively large simple (N=187). Specifically, using criterion I for the reference values, Spearman's correlation coefficients *rho* (ρ) are 0.997, 0.976 and 0.994, respectively, and using criterion II, they are 0.997, 0.974 and 0.992. On the other hand, Kendall's correlation coefficients *tau* (τ) are 0.958, 0.874 and 0.939 (criterion I) and 0.958, 0.867 and 0.93 (criterium II).

For example, Table 8 shows how the United States descends 13 positions in the DRP-SI country ranking as compared to the HDI rank, influenced by its worse relative situation in terms of life expectancy; whereas Nigeria, with less inequality between components than other countries with similar levels of human development, rises 12 positions. Likewise, the cases of the Russian Federation and Indonesia are interesting. In Tables 8 and 9, the former loses 20 and 29 positions in the DRP-SI ranking as compared to the HDI ranking, also affected by its relatively low attainments in health, whereas the latter rises 16 and 17 positions.

Appendices 1a and 1b show the results of Tables 8 and 9 for all countries. In order to facilitate analysis of the results, the differences between the HDI and DRP-WI and DRP-WI country ranks have been represented graphically (Figures 1-4).

Figure 1. Differences between HDI rank and DRP-WI rank (I)

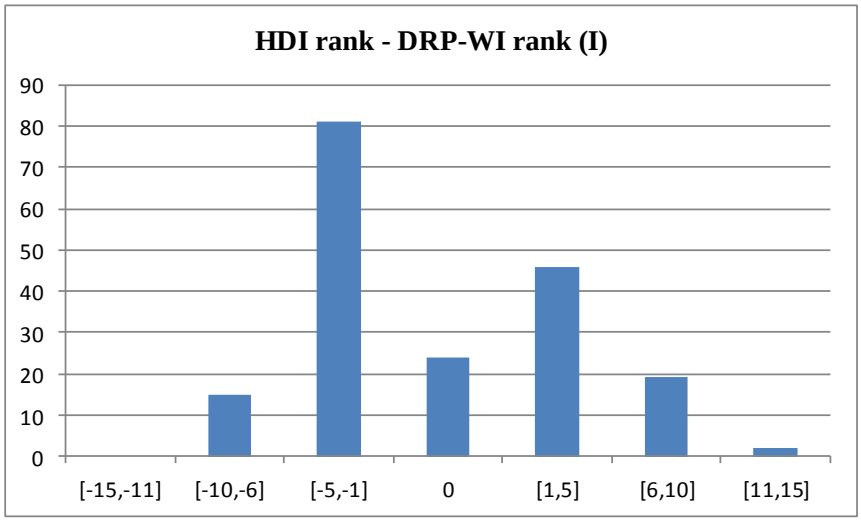


Figure 2. Differences between HDI rank and DRP-WI rank (II)

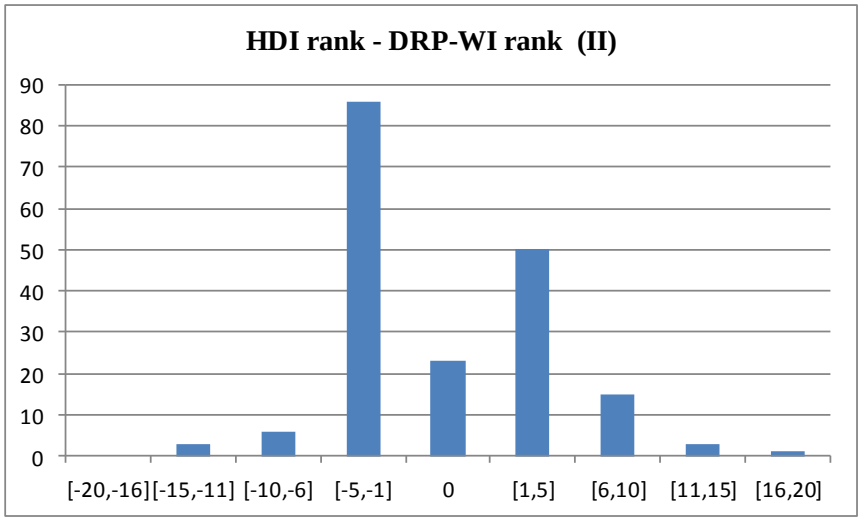


Figure 3. Differences between HDI rank and DRP-SI rank (I)

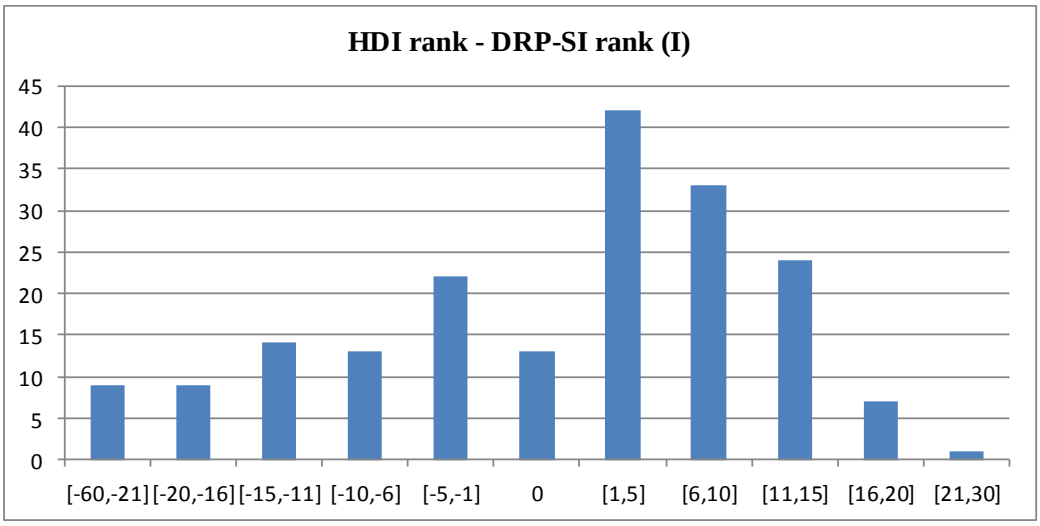
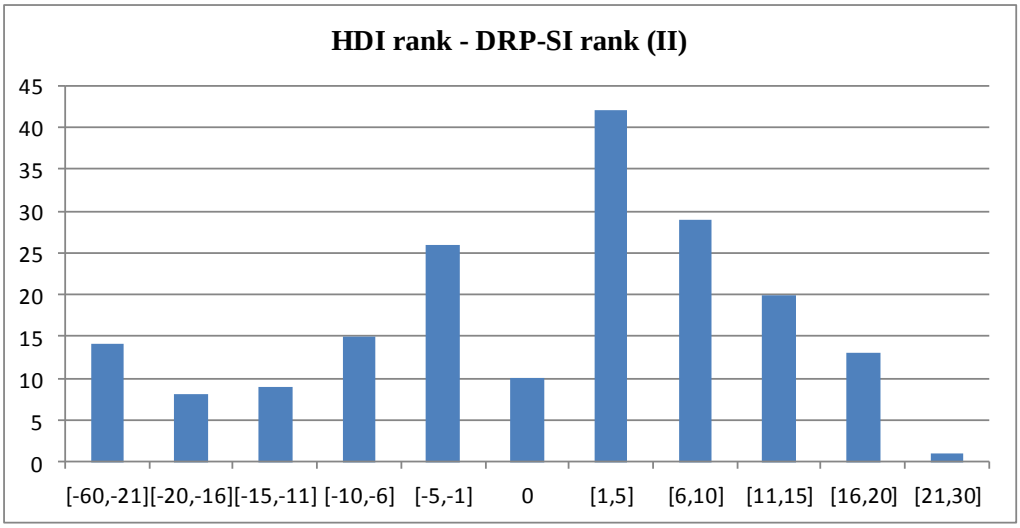


Figure 4. Differences between HDI rank and DRP-SI rank (II)



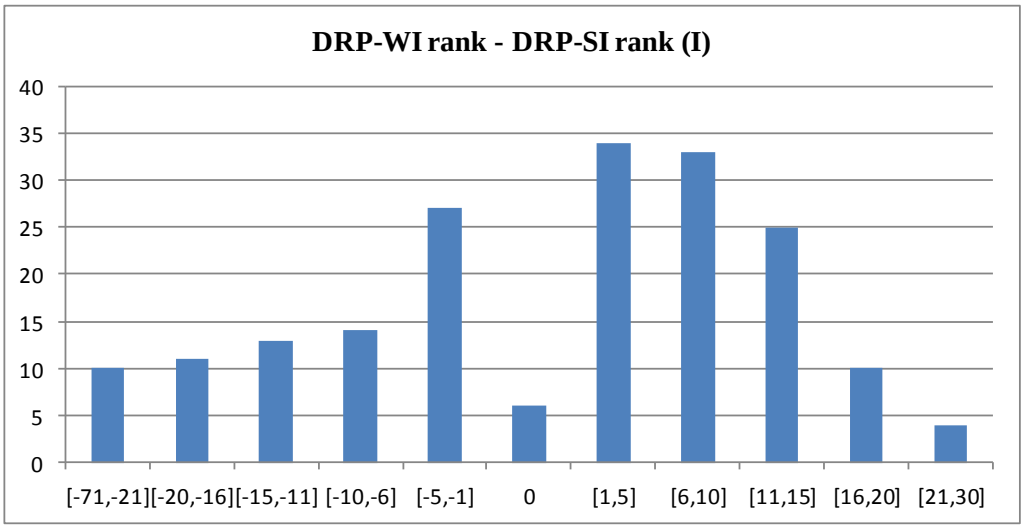
As can be seen, the distribution in Figures 1 and 2, on one hand, and Figures 3 and 4, on the other, is similar, without any significant differences in respect to country rankings between the two criteria used to determine the values of reference[9]. In this sense, in order to avoid reiterations we are going to focus our analysis on the results obtained with the values of reference calculated using criterion I.

In respect to the differences in country positions between the HDI and DRP-WI rankings, it should be noted that the maximum variation of positions is (+13), whereas it reaches (-60) if we compare HDI and DRP-SI rankings. Furthermore, in the first case 80% of countries change positions in the range of [-5, 5], whereas in the second case only 40% of countries are in that range. This clearly shows that the difference of positions of the HDI ranking in respect to DRP-SI's are greater than in respect to DRP-WI's, which seems to confirm that the valuations of the level of human development stemming from the official HDI are more similar to the values of our weak index than to those of the our strong index.

4.5. Analyzing differences between DRP-WI rank and DRP-SI rank

In order to compare a composite index with total substitutability and one that does not allow any substitutability between components, the country ranks resulting from DRP-WI and DRP-SI should also be compared. Figure 5 shows the differences in country rank positions between the alternative indicators using reference values I.

Figure 5. Differences between DRP-WI rank and DRP-SI rank (I)



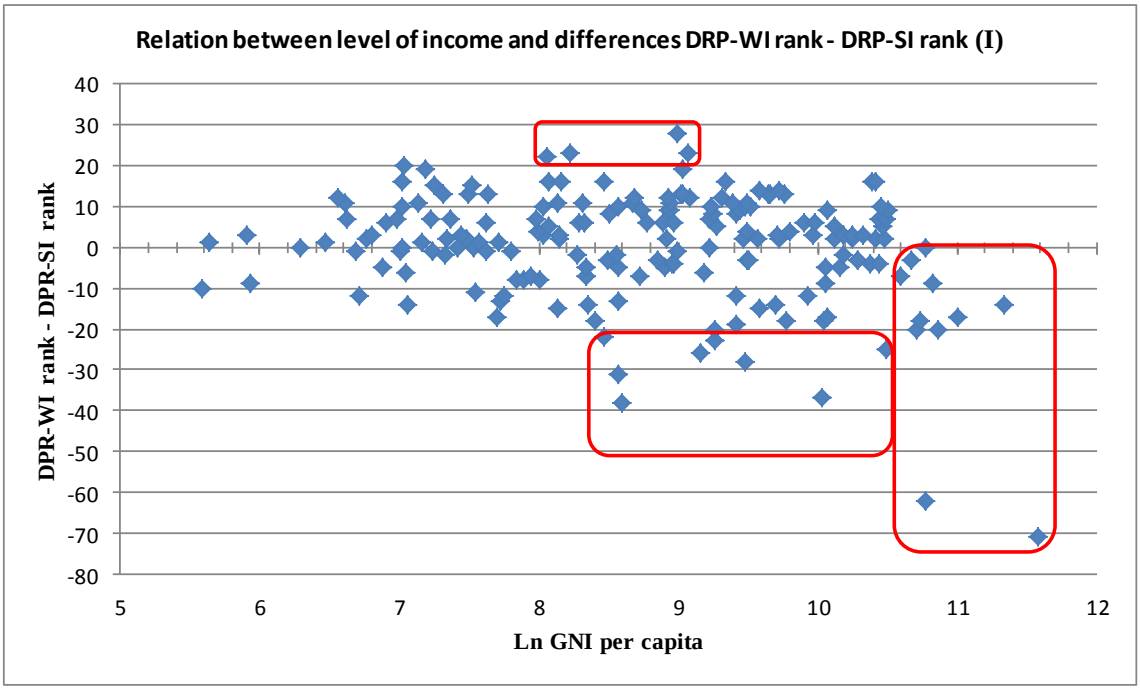
The graph shows, first of all, that 6 countries maintain their position in both rankings: Eritrea, Iran, Lesotho, Mali, Mauritania and Norway (with first place in the HDI, it maintains that position in all the indices calculated). Likewise, note should be taken of the countries which lose more than 20 positions with DRP-SI as

[9] Specifically, Spearman's correlation coefficient *rho* (ρ) amongst the ranks of countries listed according to the DRP-WI using criteria I and II is 0.999, and 0.992 amongst the DRP-SI ranks using both criteria.

compared to DRP-WI, and in some cases up to 71 positions (left side of Figure 5). These countries are Qatar, Kuwait, Cuba, Oman, Bhutan, Botswana, South Africa, Andorra, Kazakhstan and Georgia, all having very imbalanced levels in respect to health, education and income achievements. On the other hand, the maximum number of positions a country has gained with the DRP-SI is 28, the following four having improved more than 20 positions: Saint Vincent and the Grenadines, Indonesia, Azerbaijan and Kiribati (right side of Figure 5). For these countries, the three components show highly similar levels, meaning that their human development is very balanced.

In order to assess the degree of imbalance amongst components of countries on the basis of their level of income, Figure 6 relates the difference of positions in the DRP-WI and DRP-SI rankings to the level of the countries' GNI per capita. In general, differences of rank increase as country income level rises, both in positive and negative terms, this difference tending to be solely negative in countries with a high level of income. In this sense, 3 areas of the graph are noteworthy: the upper area, where we find positive differences of more than 20 positions which begin at a level that corresponds to a GNI per capita of $3,140 (constant 2005 PPP$), and which includes the countries of Kiribati, Indonesia, Azerbaijan and Saint Vincent and the Grenadines; a lower middle area where we find the first negative differences of more than 20 positions, which occur starting at a GNI per capita of $4,780 (constant 2005 PPP$), with Georgia, Bhutan, Cuba, South Africa, Kazakhstan, Botswana, Oman and Andorra; and lastly, a striking lower area to the right where starting at a level of GNI per capita of $39,924 (constant 2005 PPP$) all the countries (except Norway, with a difference of 0) show negative rank differences, namely Switzerland, United States, Hong Kong, China (SAR), Brunei Darussalam, Norway, Kuwait, Luxembourg, Singapore, United Arab Emirates, Liechtenstein and Qatar.

Figure 6. Relation between level of income and DRP-WI rank - DRP-SI rank differences (I)



To underscore the imbalances existing between the different core dimensions in countries with negative rank differences of more than 20 positions between DRP-WI and DRP-SI, Table 10 shows the normalized components, highlighting in each case the one which is in a worse situation. Countries are ordered according to their rank differences, also specifying the income group to which they belong pursuant to the World Bank classification (2013), which distinguishes 4 categories: low-income, lower-middle-income, upper-middle-income, and high-income.

Table 10. Differences DRP-WI rank - DRP-SI rank and normalized components

HDI rank	Country	Income Group	DRP-WI rank	DRP-SI rank	DRP-WI rank - DRP-SI rank	Life expectancy (Normalized)	Ln GNI per capita (Normalized)	Combined Education Index (Normalized)
37	Qatar	High income	26	97	**-71**	0.876	2.000	**0.056**
63	Kuwait	High income	50	112	**-62**	0.588	1.308	**-0.049**
51	Cuba	Upper middle income	42	80	**-38**	0.935	**0.189**	0.943
89	Oman	High income	85	122	**-37**	0.467	0.831	**-0.112**
141	Bhutan	Upper middle income	135	166	**-31**	0.027	0.179	**-0.447**
118	Botswana	High income	111	139	**-28**	**-0.291**	0.581	0.303
123	South Africa	Upper middle income	116	142	**-26**	**-0.300**	0.438	0.343
32	Andorra	High income	30	55	**-25**	1.276	1.066	**0.421**

68	Kazakhstan	Upper middle income	78	101	**-23**	**0.015**	0.488	0.797
75	Georgia	Upper middle income	67	89	**-22**	0.525	**0.133**	0.816

If we observe the situation of the different components in the countries with the greatest imbalance amongst core dimensions, note should be taken, first of all, of countries such as Qatar and Kuwait, which are well ranked by DRP-WI basically due to their high value of GNI per capita, descending 71 and 62 positions, respectively, in the DRP-SI ranking. In both cases, the other components do not reach the aspiration value, with Qatar very close to the reservation value in education and Kuwait even below said value. Other countries with significant imbalances where education is the component at the lowest level are Oman, Bhutan and Andorra. On the other hand, countries whose health component is in relatively more unfavorable situation include Botswana, South Africa and Kazakhstan. Note should also be taken of countries such as Cuba and Georgia, in which the results for the income component are clearly the worst.

5. Conclusions

Since 1990 the HDI has been widely used as measure of progress closely related to the idea of human capabilities proposed by Sen (1985), and in broader terms than exclusively income-based progress. Despite its simplicity and notable limitations, the HDI has permitted evaluating the well-being of citizens from the perspective of human development across countries and over time, and has helped recommend policies which might lead to improve the lives of people and to enhance their choices and capabilities throughout the world, and particularly in developing countries.

One of major critiques of the original HDI was the perfect substitutability between different dimensions of well-being. In fact, the problem of trade-offs between components is present to a greater or lesser extent in the majority of composite indices. The new HDI has replaced the arithmetic mean with the geometric mean as the aggregation formula of country-level attainments in health, education and income, reaching a compromise between the extremes of perfect substitutability and no substitutability. However, as discussed above, some leading authors such as Ravallion (2010 and 2012) have already shown the existence of considerable troubling trade-offs that might involve inappropriate implications in terms of development policy.

By adopting a multi-criteria approach, this paper proposes new alternative normalization and aggregation formulas for the HDI. In particular, we implement an approach based on the double reference point methodology (aspiration and reservation). We use the same indicators as the new HDI and data from the 2011 HDR.

Nevertheless, we take advantage of a different normalization: for each indicator the value of each country is normalized by means of two reference values (aspiration and reservation values) by using an achievement scalarizing function which is piecewise linear. In this work we use two alternative reference values for each indicator, taking into account that the reservation value can be interpreted as a minimal level of acceptable achievement for an indicator, and the aspiration value as a minimal level of desirable achievement.

As in the official HDI, each core dimension has been equally weighted, although our methodology would enable adopting any alternative weighting scheme. Aggregating the values of the achievement scalarizing functions, we calculate three synthetic indicators: (1) a weak index that allows total substitutability; (2) a strong index that measures the state of the worst component and allows no substitutability; (3) and a mixed index that is a linear combination of the first two and would allow different degrees of substitutability. In contrast with the range [0, 1] between maximum and minimum values of the HDI, these indices range between -1 and 2, so that if their values are between -1 and 0, it means that the value of the indicator for this country is under the reservation value; between 0 and 1, that they are between reservation and aspiration values; and between 1 and 2, that they are over the aspiration value.

As an application of this methodological approach, we have calculated these indices for 2011 and analyzed and compared the resulting country ranks with the official HDI. We observe that the results of the HDI are closer to the results of the weak index than to those of the strong index. On comparing the country ranks of the strong index with those of HDI and the weak index, we note that when a country moves down in the strong index ranking with respect to the HDI ranking or weak index ranking, it is due to the imbalance of the core dimensions; that is, at least one component is significantly worse than the others. The countries that have the biggest imbalances in core dimensions include Qatar, Kuwait, Cuba, Oman, Bhutan, Botswana, South Africa, Andorra, Kazakhstan and Georgia, and they are all placed in the upper-middle- and high-income categories. On the contrary, when a country moves up in the strong index ranking, it is because its normalized values are very similar; that is, its human development is very balanced. This category includes, amongst others, Saint Vincent and the Grenadines, Indonesia, Azerbaijan and Kiribati, in addition to Norway, which maintains the first position with all the indices analyzed.

In 1990 the first global HDR already addressed the importance of seeking a balance in priorities across dimensions of human development (UNDP 1990). If every core dimension has the same significance in terms of human development, it should be desirable to achieve balanced development across dimensions. Along this line, there are even proposals of an alternative composite index to measure human development introducing penalties

for countries with 'imbalanced' indicators (see, e.g., De Muro et al. 2011). In fact, in order to achieve balanced and harmonious human development, from a development policy viewpoint it seems rational that the worse the deprivation in a particular core dimension, the more urgent should be the efforts to improve achievements in that dimension. Therefore, in addition to the progress of human development as a whole, it is worth considering the state and evolution of the weakest dimension for each country. In this context, the information provided by the official HDI appears insufficient. These new alternative normalization and aggregation formulas, and the corresponding composite indices, entail a significant complementary contribution to the measurement of human well-being, permitting a new analytical perspective of the problem of sustainability and opening the door to whole new possibilities for future research.

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Appendix 1a. Rankings and differences (I)

Country	HDI		DRP-WI		DRP-SI		DRP-MI		HDI rank - DRP-WI rank	HDI rank - DRP-SI rank	HDI rank - DRP-MI rank
Norway	0.943	(1)	1.410	(1)	1.302	(1)	1.356	(1)	0	0	0
Australia	0.929	(2)	1.388	(2)	1.026	(6)	1.207	(2)	0	-4	0
Netherlands	0.910	(3)	1.181	(11)	1.073	(2)	1.127	(4)	-8	1	-1
United States	0.910	(4)	1.137	(14)	0.889	(17)	1.013	(16)	-10	-13	-12
New Zealand	0.908	(5)	1.246	(6)	0.848	(23)	1.047	(12)	-1	-18	-7
Canada	0.908	(6)	1.189	(9)	1.044	(4)	1.117	(6)	-3	2	0
Ireland	0.908	(7)	1.190	(8)	0.942	(11)	1.066	(10)	-1	-4	-3
Liechtenstein	0.905	(8)	1.166	(12)	0.742	(26)	0.954	(23)	-4	-18	-15
Germany	0.905	(9)	1.131	(15)	1.036	(5)	1.083	(8)	-6	4	1
Sweden	0.904	(10)	1.188	(10)	1.060	(3)	1.124	(5)	0	7	5
Switzerland	0.903	(11)	1.257	(5)	0.929	(12)	1.093	(7)	6	-1	4
Japan	0.901	(12)	1.318	(3)	0.969	(7)	1.143	(3)	9	5	9
Hong Kong, China (SAR)	0.898	(13)	1.290	(4)	0.807	(24)	1.049	(11)	9	-11	2
Iceland	0.898	(14)	1.202	(7)	0.942	(10)	1.072	(9)	7	4	5
Korea (Republic of)	0.897	(15)	1.129	(16)	0.925	(13)	1.027	(13)	-1	2	2
Denmark	0.895	(16)	1.047	(22)	0.911	(15)	0.979	(19)	-6	1	-3
Israel	0.888	(17)	1.153	(13)	0.886	(18)	1.019	(14)	4	-1	3
Belgium	0.886	(18)	0.989	(24)	0.965	(8)	0.977	(20)	-6	10	-2
Austria	0.885	(19)	1.064	(21)	0.882	(19)	0.973	(21)	-2	0	-2
France	0.884	(20)	1.112	(17)	0.923	(14)	1.017	(15)	3	6	5
Slovenia	0.884	(21)	1.029	(23)	0.869	(21)	0.949	(24)	-2	0	-3
Finland	0.882	(22)	0.978	(25)	0.947	(9)	0.962	(22)	-3	13	0
Spain	0.878	(23)	1.083	(19)	0.897	(16)	0.990	(17)	4	7	6
Italy	0.874	(24)	1.106	(18)	0.874	(20)	0.990	(18)	6	4	6
Luxembourg	0.867	(25)	0.976	(27)	0.577	(36)	0.777	(29)	-2	-11	-4
Singapore	0.866	(26)	1.075	(20)	0.507	(40)	0.791	(28)	6	-14	-2
Czech Republic	0.865	(27)	0.946	(28)	0.802	(25)	0.874	(26)	-1	2	1
United Kingdom	0.863	(28)	0.926	(29)	0.730	(27)	0.828	(27)	-1	1	1
Greece	0.861	(29)	0.912	(31)	0.848	(22)	0.880	(25)	-2	7	4
United Arab Emirates	0.846	(30)	0.902	(32)	0.469	(49)	0.686	(35)	-2	-19	-5
Cyprus	0.840	(31)	0.836	(34)	0.671	(29)	0.753	(30)	-3	2	1
Andorra	0.838	(32)	0.921	(30)	0.421	(55)	0.671	(36)	2	-23	-4
Brunei Darussalam	0.838	(33)	0.853	(33)	0.442	(51)	0.647	(40)	0	-18	-7
Estonia	0.835	(34)	0.818	(35)	0.607	(33)	0.713	(34)	-1	1	0
Slovakia	0.834	(35)	0.789	(37)	0.654	(31)	0.722	(32)	-2	4	3
Malta	0.832	(36)	0.814	(36)	0.668	(30)	0.741	(31)	0	6	5
Qatar	0.831	(37)	0.977	(26)	0.056	(97)	0.517	(54)	11	-60	-17
Hungary	0.816	(38)	0.724	(40)	0.576	(37)	0.650	(39)	-2	1	-1
Poland	0.813	(39)	0.724	(41)	0.706	(28)	0.715	(33)	-2	11	6
Lithuania	0.810	(40)	0.686	(43)	0.410	(57)	0.548	(48)	-3	-17	-8
Portugal	0.809	(41)	0.736	(38)	0.462	(50)	0.599	(42)	3	-9	-1
Bahrain	0.806	(42)	0.681	(45)	0.493	(43)	0.587	(45)	-3	-1	-3
Latvia	0.805	(43)	0.684	(44)	0.494	(42)	0.589	(44)	-1	1	-1
Chile	0.805	(44)	0.731	(39)	0.590	(35)	0.661	(37)	5	9	7
Argentina	0.797	(45)	0.672	(46)	0.629	(32)	0.650	(38)	-1	13	7
Croatia	0.796	(46)	0.670	(47)	0.600	(34)	0.635	(41)	-1	12	5
Barbados	0.793	(47)	0.658	(48)	0.491	(44)	0.575	(46)	-1	3	1
Uruguay	0.783	(48)	0.636	(49)	0.548	(38)	0.592	(43)	-1	10	5
Palau	0.782	(49)	0.609	(52)	0.380	(58)	0.494	(56)	-3	-9	-7
Romania	0.781	(50)	0.612	(51)	0.507	(39)	0.559	(47)	-1	11	3
Cuba	0.776	(51)	0.689	(42)	0.189	(80)	0.439	(60)	9	-29	-9
Seychelles	0.773	(52)	0.565	(59)	0.490	(45)	0.528	(52)	-7	7	0

Country	HDI		DRP-WI		DRP-SI		DRP-MI		HDI rank - DRP-WI rank	HDI rank - DRP-SI rank	HDI rank - DRP-MI rank
Bahamas	0.771	(53)	0.576	(56)	0.226	(74)	0.401	(65)	-3	-21	-12
Montenegro	0.771	(54)	0.584	(54)	0.478	(46)	0.531	(50)	0	8	4
Bulgaria	0.771	(55)	0.567	(57)	0.497	(41)	0.532	(49)	-2	14	6
Saudi Arabia	0.770	(56)	0.554	(60)	0.287	(69)	0.420	(62)	-4	-13	-6
Mexico	0.770	(57)	0.591	(53)	0.418	(56)	0.505	(55)	4	1	2
Panama	0.768	(58)	0.580	(55)	0.477	(47)	0.529	(51)	3	11	7
Serbia	0.766	(59)	0.566	(58)	0.473	(48)	0.519	(53)	1	11	6
Antigua and Barbuda	0.764	(60)	0.527	(65)	0.441	(52)	0.484	(58)	-5	8	2
Malaysia	0.761	(61)	0.532	(64)	0.432	(54)	0.482	(59)	-3	7	2
Trinidad and Tobago	0.760	(62)	0.486	(68)	0.250	(73)	0.368	(73)	-6	-11	-11
Kuwait	0.760	(63)	0.616	(50)	-0.049	(112)	0.283	(86)	13	-49	-23
Libyan Arab Jamahiriya	0.760	(64)	0.536	(62)	0.437	(53)	0.486	(57)	2	11	7
Belarus	0.756	(65)	0.485	(69)	0.267	(72)	0.376	(70)	-4	-7	-5
Russian Federation	0.755	(66)	0.468	(71)	0.152	(86)	0.310	(80)	-5	-20	-14
Grenada	0.748	(67)	0.533	(63)	0.302	(66)	0.417	(63)	4	1	4
Kazakhstan	0.745	(68)	0.433	(78)	0.015	(101)	0.224	(95)	-10	-33	-27
Costa Rica	0.744	(69)	0.538	(61)	0.184	(81)	0.361	(75)	8	-12	-6
Albania	0.739	(70)	0.506	(66)	0.352	(60)	0.429	(61)	4	10	9
Lebanon	0.739	(71)	0.444	(74)	0.310	(64)	0.377	(67)	-3	7	4
Saint Kitts and Nevis	0.735	(72)	0.440	(76)	0.302	(65)	0.371	(72)	-4	7	0
Venezuela (Bolivarian Republic of)	0.735	(73)	0.455	(73)	0.299	(68)	0.377	(69)	0	5	4
Bosnia and Herzegovina	0.733	(74)	0.473	(70)	0.344	(61)	0.409	(64)	4	13	10
Georgia	0.733	(75)	0.491	(67)	0.133	(89)	0.312	(79)	8	-14	-4
Ukraine	0.729	(76)	0.419	(83)	0.127	(90)	0.273	(87)	-7	-14	-11
Mauritius	0.728	(77)	0.418	(84)	0.181	(82)	0.300	(81)	-7	-5	-4
The former Yugoslav Republic of Macedonia	0.728	(78)	0.442	(75)	0.312	(63)	0.377	(68)	3	15	10
Jamaica	0.727	(79)	0.438	(77)	0.269	(71)	0.354	(76)	2	8	3
Peru	0.725	(80)	0.423	(81)	0.342	(62)	0.383	(66)	-1	18	14
Dominica	0.724	(81)	0.462	(72)	0.221	(76)	0.341	(78)	9	5	3
Saint Lucia	0.723	(82)	0.424	(80)	0.302	(67)	0.363	(74)	2	15	8
Ecuador	0.720	(83)	0.429	(79)	0.278	(70)	0.353	(77)	4	13	6
Brazil	0.718	(84)	0.390	(86)	0.195	(79)	0.293	(83)	-2	5	1
Saint Vincent and the Grenadines	0.717	(85)	0.382	(87)	0.364	(59)	0.373	(71)	-2	26	14
Armenia	0.716	(86)	0.423	(82)	0.170	(84)	0.296	(82)	4	2	4
Colombia	0.710	(87)	0.371	(90)	0.210	(77)	0.290	(84)	-3	10	3
Iran (Islamic Republic of)	0.707	(88)	0.351	(92)	0.116	(92)	0.233	(93)	-4	-4	-5
Oman	0.705	(89)	0.395	(85)	-0.112	(122)	0.142	(104)	4	-33	-15
Tonga	0.704	(90)	0.378	(88)	0.074	(95)	0.226	(94)	2	-5	-4
Azerbaijan	0.700	(91)	0.307	(98)	0.225	(75)	0.266	(88)	-7	16	3
Turkey	0.699	(92)	0.352	(91)	-0.040	(110)	0.156	(103)	1	-18	-11
Belize	0.699	(93)	0.373	(89)	0.197	(78)	0.285	(85)	4	15	8
Tunisia	0.698	(94)	0.346	(94)	0.134	(88)	0.240	(92)	0	6	2
Jordan	0.698	(95)	0.346	(93)	0.179	(83)	0.263	(89)	2	12	6
Algeria	0.698	(96)	0.326	(96)	0.157	(85)	0.242	(91)	0	11	5
Sri Lanka	0.691	(97)	0.340	(95)	0.148	(87)	0.244	(90)	2	10	7

Country	HDI		DRP-WI		DRP-SI		DRP-MI		HDI rank - DRP-WI rank	HDI rank - DRP-SI rank	HDI rank - DRP-MI rank
Dominican Republic	0.689	(98)	0.299	(99)	0.032	(100)	0.166	(102)	-1	-2	-4
Samoa	0.688	(99)	0.325	(97)	0.046	(99)	0.185	(96)	2	0	3
Fiji	0.688	(100)	0.294	(102)	0.070	(96)	0.182	(97)	-2	4	3
China	0.687	(101)	0.297	(100)	0.054	(98)	0.176	(100)	1	3	1
Turkmenistan	0.686	(102)	0.249	(104)	-0.039	(109)	0.105	(107)	-2	-7	-5
Thailand	0.682	(103)	0.295	(101)	-0.016	(105)	0.139	(105)	2	-2	-2
Suriname	0.680	(104)	0.241	(106)	0.102	(94)	0.171	(101)	-2	10	3
El Salvador	0.674	(105)	0.247	(105)	0.105	(93)	0.176	(98)	0	12	7
Gabon	0.674	(106)	0.217	(108)	-0.088	(120)	0.065	(112)	-2	-14	-6
Paraguay	0.665	(107)	0.227	(107)	0.124	(91)	0.176	(99)	0	16	8
Bolivia (Plurinational State of)	0.663	(108)	0.184	(113)	-0.004	(102)	0.090	(108)	-5	6	0
Maldives	0.661	(109)	0.291	(103)	-0.064	(116)	0.114	(106)	6	-7	3
Mongolia	0.653	(110)	0.172	(115)	-0.012	(104)	0.080	(109)	-5	6	1
Moldova (Republic of)	0.649	(111)	0.177	(114)	-0.041	(111)	0.068	(110)	-3	0	1
Philippines	0.644	(112)	0.137	(119)	-0.005	(103)	0.066	(111)	-7	9	1
Egypt	0.644	(113)	0.195	(112)	-0.077	(117)	0.059	(114)	1	-4	-1
Occupied Palestinian Territory	0.641	(114)	0.204	(110)	-0.081	(118)	0.061	(113)	4	-4	1
Uzbekistan	0.641	(115)	0.142	(117)	-0.050	(113)	0.046	(116)	-2	2	-1
Micronesia (Federated States of)	0.636	(116)	0.132	(121)	-0.053	(114)	0.040	(119)	-5	2	-3
Guyana	0.633	(117)	0.119	(122)	-0.029	(106)	0.045	(117)	-5	11	0
Botswana	0.633	(118)	0.198	(111)	-0.291	(139)	-0.047	(127)	7	-21	-9
Syrian Arab Republic	0.632	(119)	0.215	(109)	-0.121	(123)	0.047	(115)	10	-4	4
Namibia	0.625	(120)	0.063	(130)	-0.093	(121)	-0.015	(123)	-10	-1	-3
Honduras	0.625	(121)	0.139	(118)	-0.054	(115)	0.042	(118)	3	6	3
Kiribati	0.624	(122)	0.068	(129)	-0.034	(107)	0.017	(120)	-7	15	2
South Africa	0.619	(123)	0.160	(116)	-0.300	(142)	-0.070	(129)	7	-19	-6
Indonesia	0.617	(124)	0.058	(131)	-0.039	(108)	0.010	(121)	-7	16	3
Vanuatu	0.617	(125)	0.093	(125)	-0.087	(119)	0.003	(122)	0	6	3
Kyrgyzstan	0.615	(126)	0.099	(124)	-0.155	(125)	-0.028	(126)	2	1	0
Tajikistan	0.607	(127)	0.075	(127)	-0.169	(126)	-0.047	(128)	0	1	-1
Viet Nam	0.593	(128)	0.132	(120)	-0.172	(127)	-0.020	(125)	8	1	3
Nicaragua	0.589	(129)	0.102	(123)	-0.135	(124)	-0.016	(124)	6	5	5
Morocco	0.582	(130)	0.072	(128)	-0.264	(135)	-0.096	(130)	2	-5	0
Guatemala	0.574	(131)	0.041	(132)	-0.278	(137)	-0.119	(133)	-1	-6	-2
Iraq	0.573	(132)	-0.018	(134)	-0.192	(129)	-0.105	(131)	-2	3	1
Cape Verde	0.568	(133)	0.082	(126)	-0.299	(141)	-0.109	(132)	7	-8	1
India	0.547	(134)	-0.098	(136)	-0.259	(134)	-0.179	(138)	-2	0	-4
Ghana	0.541	(135)	-0.112	(138)	-0.226	(131)	-0.169	(137)	-3	4	-2
Equatorial Guinea	0.537	(136)	0.027	(133)	-0.336	(151)	-0.154	(134)	3	-15	2
Congo	0.533	(137)	-0.127	(140)	-0.202	(130)	-0.165	(136)	-3	7	1
Lao People's Democratic Republic	0.524	(138)	-0.122	(139)	-0.289	(138)	-0.205	(140)	-1	0	-2
Cambodia	0.523	(139)	-0.145	(143)	-0.183	(128)	-0.164	(135)	-4	11	4
Swaziland	0.522	(140)	-0.110	(137)	-0.387	(155)	-0.248	(144)	3	-15	-4
Bhutan	0.522	(141)	-0.080	(135)	-0.447	(166)	-0.264	(148)	6	-25	-7
Solomon Islands	0.510	(142)	-0.137	(142)	-0.298	(140)	-0.218	(142)	0	2	0
Kenya	0.509	(143)	-0.164	(145)	-0.243	(132)	-0.203	(139)	-2	11	4

Country	HDI		DRP-WI		DRP-SI		DRP-MI		HDI rank - DRP-WI rank	HDI rank - DRP-SI rank	HDI rank - DRP-MI rank
Sao Tome and Principe	0.509	(144)	-0.165	(146)	-0.256	(133)	-0.210	(141)	-2	11	3
Pakistan	0.504	(145)	-0.162	(144)	-0.365	(152)	-0.263	(147)	1	-7	-2
Bangladesh	0.500	(146)	-0.130	(141)	-0.316	(143)	-0.223	(143)	5	3	3
Timor-Leste	0.495	(147)	-0.176	(148)	-0.389	(156)	-0.282	(151)	-1	-9	-4
Angola	0.486	(148)	-0.167	(147)	-0.336	(150)	-0.251	(145)	1	-2	3
Myanmar	0.483	(149)	-0.202	(151)	-0.335	(149)	-0.268	(150)	-2	0	-1
Cameroon	0.482	(150)	-0.208	(152)	-0.325	(146)	-0.267	(149)	-2	4	1
Madagascar	0.480	(151)	-0.198	(150)	-0.409	(162)	-0.303	(156)	1	-11	-5
Tanzania (United Republic of)	0.466	(152)	-0.237	(155)	-0.275	(136)	-0.256	(146)	-3	16	6
Papua New Guinea	0.466	(153)	-0.220	(154)	-0.449	(167)	-0.335	(161)	-1	-14	-8
Yemen	0.462	(154)	-0.217	(153)	-0.490	(170)	-0.354	(166)	1	-16	-12
Senegal	0.459	(155)	-0.244	(156)	-0.366	(153)	-0.305	(157)	-1	2	-2
Nigeria	0.459	(156)	-0.247	(157)	-0.319	(144)	-0.283	(152)	-1	12	4
Nepal	0.458	(157)	-0.192	(149)	-0.413	(163)	-0.303	(155)	8	-6	2
Haiti	0.454	(158)	-0.251	(158)	-0.331	(148)	-0.291	(153)	0	10	5
Mauritania	0.453	(159)	-0.252	(159)	-0.398	(159)	-0.325	(159)	0	0	0
Lesotho	0.450	(160)	-0.258	(160)	-0.398	(160)	-0.328	(160)	0	0	0
Uganda	0.446	(161)	-0.270	(161)	-0.322	(145)	-0.296	(154)	0	16	7
Togo	0.435	(162)	-0.282	(163)	-0.418	(164)	-0.350	(165)	-1	-2	-3
Comoros	0.433	(163)	-0.284	(164)	-0.395	(157)	-0.339	(163)	-1	6	0
Zambia	0.430	(164)	-0.294	(165)	-0.380	(154)	-0.337	(162)	-1	10	2
Djibouti	0.430	(165)	-0.274	(162)	-0.515	(174)	-0.395	(170)	3	-9	-5
Rwanda	0.429	(166)	-0.298	(167)	-0.330	(147)	-0.314	(158)	-1	19	8
Benin	0.427	(167)	-0.299	(168)	-0.399	(161)	-0.349	(164)	-1	6	3
Gambia	0.420	(168)	-0.304	(169)	-0.450	(168)	-0.377	(168)	-1	0	0
Sudan	0.408	(169)	-0.295	(166)	-0.594	(177)	-0.444	(172)	3	-8	-3
Côte d'Ivoire	0.400	(170)	-0.335	(170)	-0.499	(171)	-0.417	(171)	0	-1	-1
Malawi	0.400	(171)	-0.343	(172)	-0.434	(165)	-0.389	(169)	-1	6	2
Afghanistan	0.398	(172)	-0.347	(173)	-0.396	(158)	-0.371	(167)	-1	14	5
Zimbabwe	0.376	(173)	-0.342	(171)	-0.629	(180)	-0.485	(178)	2	-7	-5
Ethiopia	0.363	(174)	-0.378	(174)	-0.610	(179)	-0.494	(179)	0	-5	-5
Mali	0.359	(175)	-0.402	(176)	-0.555	(176)	-0.479	(177)	-1	-1	-2
Guinea-Bissau	0.353	(176)	-0.419	(179)	-0.502	(173)	-0.461	(174)	-3	3	2
Eritrea	0.349	(177)	-0.399	(175)	-0.554	(175)	-0.476	(176)	2	2	1
Guinea	0.344	(178)	-0.421	(180)	-0.595	(178)	-0.508	(180)	-2	0	-2
Central African Republic	0.343	(179)	-0.439	(181)	-0.472	(169)	-0.455	(173)	-2	10	6
Sierra Leone	0.336	(180)	-0.449	(183)	-0.500	(172)	-0.474	(175)	-3	8	5
Burkina Faso	0.331	(181)	-0.418	(178)	-0.692	(184)	-0.555	(184)	3	-3	-3
Liberia	0.329	(182)	-0.406	(177)	-0.727	(187)	-0.567	(185)	5	-5	-3
Chad	0.328	(183)	-0.445	(182)	-0.639	(183)	-0.542	(181)	1	0	2
Mozambique	0.322	(184)	-0.458	(184)	-0.635	(181)	-0.546	(182)	0	3	2
Burundi	0.316	(185)	-0.468	(185)	-0.635	(182)	-0.552	(183)	0	3	2
Niger	0.295	(186)	-0.482	(186)	-0.708	(185)	-0.595	(186)	0	1	0
Congo (Democratic Republic of the)	0.286	(187)	-0.506	(187)	-0.712	(186)	-0.609	(187)	0	1	0

Appendix 1b. Rankings and differences (II)

Country	HDI		DRP-WI		DRP-SI		DRP-MI		HDI rank - DRP-WI rank	HDI rank - DRP-SI rank	HDI rank - DRP-MI rank
Norway	0.943	(1)	1.668	(1)	1.560	(1)	1.614	(1)	0	0	0
Australia	0.929	(2)	1.642	(2)	1.386	(6)	1.514	(2)	0	-4	0
Netherlands	0.910	(3)	1.533	(8)	1.416	(2)	1.475	(3)	-5	1	0
United States	0.910	(4)	1.473	(16)	1.331	(11)	1.402	(11)	-12	-7	-7
New Zealand	0.908	(5)	1.542	(5)	1.186	(23)	1.364	(17)	0	-18	-12
Canada	0.908	(6)	1.534	(7)	1.398	(4)	1.466	(5)	-1	2	1
Ireland	0.908	(7)	1.528	(9)	1.300	(15)	1.414	(9)	-2	-8	-2
Liechtenstein	0.905	(8)	1.486	(14)	1.110	(26)	1.298	(24)	-6	-18	-16
Germany	0.905	(9)	1.507	(12)	1.393	(5)	1.450	(7)	-3	4	2
Sweden	0.904	(10)	1.528	(10)	1.408	(3)	1.468	(4)	0	7	6
Switzerland	0.903	(11)	1.546	(4)	1.318	(12)	1.432	(8)	7	-1	3
Japan	0.901	(12)	1.571	(3)	1.352	(9)	1.461	(6)	9	3	6
Hong Kong, China (SAR)	0.898	(13)	1.541	(6)	1.183	(24)	1.362	(18)	7	-11	-5
Iceland	0.898	(14)	1.519	(11)	1.300	(14)	1.410	(10)	3	0	4
Korea (Republic of)	0.897	(15)	1.487	(13)	1.279	(16)	1.383	(15)	2	-1	0
Denmark	0.895	(16)	1.425	(21)	1.371	(7)	1.398	(12)	-5	9	4
Israel	0.888	(17)	1.481	(15)	1.232	(20)	1.356	(19)	2	-3	-2
Belgium	0.886	(18)	1.420	(22)	1.358	(8)	1.389	(13)	-4	10	5
Austria	0.885	(19)	1.441	(18)	1.266	(17)	1.353	(20)	1	2	-1
France	0.884	(20)	1.459	(17)	1.311	(13)	1.385	(14)	3	7	6
Slovenia	0.884	(21)	1.404	(24)	1.212	(21)	1.308	(23)	-3	0	-2
Finland	0.882	(22)	1.407	(23)	1.338	(10)	1.373	(16)	-1	12	6
Spain	0.878	(23)	1.433	(19)	1.245	(18)	1.339	(21)	4	5	2
Italy	0.874	(24)	1.430	(20)	1.245	(19)	1.337	(22)	4	5	2
Luxembourg	0.867	(25)	1.354	(26)	0.941	(33)	1.148	(28)	-1	-8	-3
Singapore	0.866	(26)	1.393	(25)	0.878	(38)	1.135	(30)	1	-12	-4
Czech Republic	0.865	(27)	1.290	(30)	1.130	(25)	1.210	(27)	-3	2	0
United Kingdom	0.863	(28)	1.341	(27)	1.097	(27)	1.219	(26)	1	1	2
Greece	0.861	(29)	1.329	(28)	1.186	(22)	1.257	(25)	1	7	4
United Arab Emirates	0.846	(30)	1.195	(35)	0.844	(43)	1.020	(35)	-5	-13	-5
Cyprus	0.840	(31)	1.240	(32)	1.032	(28)	1.136	(29)	-1	3	2
Andorra	0.838	(32)	1.291	(29)	0.800	(48)	1.046	(32)	3	-16	0
Brunei Darussalam	0.838	(33)	1.205	(34)	0.819	(45)	1.012	(36)	-1	-12	-3
Estonia	0.835	(34)	1.127	(37)	0.891	(35)	1.009	(37)	-3	-1	-3
Slovakia	0.834	(35)	1.123	(38)	0.943	(32)	1.033	(33)	-3	3	2
Malta	0.832	(36)	1.214	(33)	1.029	(29)	1.122	(31)	3	7	5
Qatar	0.831	(37)	1.260	(31)	0.470	(86)	0.865	(48)	6	-49	-11
Hungary	0.816	(38)	1.048	(42)	0.856	(41)	0.952	(42)	-4	-3	-4
Poland	0.813	(39)	1.048	(41)	1.000	(30)	1.024	(34)	-2	9	5
Lithuania	0.810	(40)	1.006	(44)	0.673	(62)	0.839	(51)	-4	-22	-11
Portugal	0.809	(41)	1.137	(36)	0.837	(44)	0.987	(39)	5	-3	2
Bahrain	0.806	(42)	1.018	(43)	0.865	(39)	0.941	(43)	-1	3	-1
Latvia	0.805	(43)	1.003	(45)	0.766	(53)	0.884	(46)	-2	-10	-3
Chile	0.805	(44)	1.109	(39)	0.884	(36)	0.996	(38)	5	8	6
Argentina	0.797	(45)	0.990	(48)	0.927	(34)	0.959	(41)	-3	11	4
Croatia	0.796	(46)	1.000	(46)	0.962	(31)	0.981	(40)	0	15	6
Barbados	0.793	(47)	0.999	(47)	0.864	(40)	0.931	(44)	0	7	3
Uruguay	0.783	(48)	0.972	(49)	0.881	(37)	0.927	(45)	-1	11	3
Palau	0.782	(49)	0.919	(54)	0.638	(65)	0.779	(59)	-5	-16	-10
Romania	0.781	(50)	0.923	(52)	0.790	(50)	0.857	(50)	-2	0	0
Cuba	0.776	(51)	1.061	(40)	0.434	(90)	0.748	(63)	11	-39	-12
Seychelles	0.773	(52)	0.882	(59)	0.786	(51)	0.834	(52)	-7	1	0

Country	HDI		DRP-WI		DRP-SI		DRP-MI		HDI rank - DRP-WI rank	HDI rank - DRP-SI rank	HDI rank - DRP-MI rank
Bahamas	0.771	(53)	0.917	(55)	0.624	(67)	0.770	(61)	-2	-14	-8
Montenegro	0.771	(54)	0.892	(57)	0.759	(54)	0.825	(55)	-3	0	-1
Bulgaria	0.771	(55)	0.874	(60)	0.768	(52)	0.821	(56)	-5	3	-1
Saudi Arabia	0.770	(56)	0.889	(58)	0.679	(60)	0.784	(58)	-2	-4	-2
Mexico	0.770	(57)	0.931	(50)	0.798	(49)	0.864	(49)	7	8	8
Panama	0.768	(58)	0.899	(56)	0.846	(42)	0.872	(47)	2	16	11
Serbia	0.766	(59)	0.872	(61)	0.752	(55)	0.812	(57)	-2	4	2
Antigua and Barbuda	0.764	(60)	0.840	(64)	0.706	(56)	0.773	(60)	-4	4	0
Malaysia	0.761	(61)	0.849	(63)	0.810	(47)	0.829	(54)	-2	14	7
Trinidad and Tobago	0.760	(62)	0.809	(68)	0.495	(84)	0.652	(77)	-6	-22	-15
Kuwait	0.760	(63)	0.920	(53)	0.325	(100)	0.622	(82)	10	-37	-19
Libyan Arab Jamahiriya	0.760	(64)	0.853	(62)	0.815	(46)	0.834	(53)	2	18	11
Belarus	0.756	(65)	0.787	(70)	0.514	(82)	0.651	(79)	-5	-17	-14
Russian Federation	0.755	(66)	0.766	(73)	0.386	(95)	0.576	(89)	-7	-29	-23
Grenada	0.748	(67)	0.837	(65)	0.561	(77)	0.699	(70)	2	-10	-3
Kazakhstan	0.745	(68)	0.725	(82)	0.234	(108)	0.480	(101)	-14	-40	-33
Costa Rica	0.744	(69)	0.930	(51)	0.586	(74)	0.758	(62)	18	-5	7
Albania	0.739	(70)	0.835	(66)	0.617	(69)	0.726	(66)	4	1	4
Lebanon	0.739	(71)	0.761	(74)	0.700	(57)	0.730	(65)	-3	14	6
Saint Kitts and Nevis	0.735	(72)	0.756	(76)	0.693	(58)	0.725	(67)	-4	14	5
Venezuela (Bolivarian Republic of)	0.735	(73)	0.772	(72)	0.690	(59)	0.731	(64)	1	14	9
Bosnia and Herzegovina	0.733	(74)	0.785	(71)	0.608	(71)	0.696	(71)	3	3	3
Georgia	0.733	(75)	0.788	(69)	0.372	(96)	0.580	(87)	6	-21	-12
Ukraine	0.729	(76)	0.708	(84)	0.358	(98)	0.533	(93)	-8	-22	-17
Mauritius	0.728	(77)	0.740	(78)	0.583	(75)	0.662	(75)	-1	2	2
The former Yugoslav Republic of Macedonia	0.728	(78)	0.757	(75)	0.677	(61)	0.717	(68)	3	17	10
Jamaica	0.727	(79)	0.735	(80)	0.524	(80)	0.630	(81)	-1	-1	-2
Peru	0.725	(80)	0.734	(81)	0.653	(63)	0.694	(72)	-1	17	8
Dominica	0.724	(81)	0.809	(67)	0.619	(68)	0.714	(69)	14	13	12
Saint Lucia	0.723	(82)	0.738	(79)	0.646	(64)	0.692	(73)	3	18	9
Ecuador	0.720	(83)	0.744	(77)	0.603	(72)	0.674	(74)	6	11	9
Brazil	0.718	(84)	0.708	(85)	0.596	(73)	0.652	(78)	-1	11	6
Saint Vincent and the Grenadines	0.717	(85)	0.687	(88)	0.630	(66)	0.658	(76)	-3	19	9
Armenia	0.716	(86)	0.720	(83)	0.413	(92)	0.566	(90)	3	-6	-4
Colombia	0.710	(87)	0.685	(89)	0.609	(70)	0.647	(80)	-2	17	7
Iran (Islamic Republic of)	0.707	(88)	0.669	(90)	0.524	(81)	0.597	(85)	-2	7	3
Oman	0.705	(89)	0.701	(86)	0.204	(111)	0.453	(102)	3	-22	-13
Tonga	0.704	(90)	0.662	(93)	0.306	(101)	0.484	(100)	-3	-11	-10
Azerbaijan	0.700	(91)	0.613	(99)	0.547	(78)	0.580	(86)	-8	13	5
Turkey	0.699	(92)	0.668	(91)	0.342	(99)	0.505	(98)	1	-7	-6
Belize	0.699	(93)	0.687	(87)	0.470	(87)	0.578	(88)	6	6	5
Tunisia	0.698	(94)	0.662	(92)	0.540	(79)	0.601	(83)	2	15	11
Jordan	0.698	(95)	0.647	(95)	0.423	(91)	0.535	(92)	0	4	3
Algeria	0.698	(96)	0.639	(96)	0.562	(76)	0.600	(84)	0	20	12
Sri Lanka	0.691	(97)	0.647	(94)	0.389	(93)	0.518	(96)	3	4	1

Country	HDI		DRP-WI		DRP-SI		DRP-MI		HDI rank - DRP-WI rank	HDI rank - DRP-SI rank	HDI rank - DRP-MI rank
Dominican Republic	0.689	(98)	0.618	(97)	0.448	(89)	0.533	(94)	1	9	4
Samoa	0.688	(99)	0.613	(100)	0.274	(104)	0.443	(105)	-1	-5	-6
Fiji	0.688	(100)	0.570	(103)	0.301	(102)	0.435	(106)	-3	-2	-6
China	0.687	(101)	0.613	(98)	0.469	(88)	0.541	(91)	3	13	10
Turkmenistan	0.686	(102)	0.496	(107)	0.064	(122)	0.280	(117)	-5	-20	-15
Thailand	0.682	(103)	0.610	(101)	0.388	(94)	0.499	(99)	2	9	4
Suriname	0.680	(104)	0.548	(105)	0.512	(83)	0.530	(95)	-1	21	9
El Salvador	0.674	(105)	0.554	(104)	0.479	(85)	0.517	(97)	1	20	8
Gabon	0.674	(106)	0.465	(111)	-0.034	(125)	0.215	(123)	-5	-19	-17
Paraguay	0.665	(107)	0.530	(106)	0.366	(97)	0.448	(104)	1	10	3
Bolivia (Plurinational State of)	0.663	(108)	0.453	(112)	0.201	(112)	0.327	(109)	-4	-4	-1
Maldives	0.661	(109)	0.605	(102)	0.296	(103)	0.451	(103)	7	6	6
Mongolia	0.653	(110)	0.448	(113)	0.200	(113)	0.324	(110)	-3	-3	0
Moldova (Republic of)	0.649	(111)	0.448	(114)	0.149	(117)	0.298	(114)	-3	-6	-3
Philippines	0.644	(112)	0.419	(117)	0.213	(109)	0.316	(111)	-5	3	1
Egypt	0.644	(113)	0.483	(109)	0.270	(105)	0.376	(107)	4	8	6
Occupied Palestinian Territory	0.641	(114)	0.478	(110)	0.078	(121)	0.278	(118)	4	-7	-4
Uzbekistan	0.641	(115)	0.408	(118)	0.134	(118)	0.271	(119)	-3	-3	-4
Micronesia (Federated States of)	0.636	(116)	0.402	(119)	0.128	(119)	0.265	(120)	-3	-3	-4
Guyana	0.633	(117)	0.401	(120)	0.170	(115)	0.286	(116)	-3	2	1
Botswana	0.633	(118)	0.439	(115)	-0.250	(152)	0.095	(132)	3	-34	-14
Syrian Arab Republic	0.632	(119)	0.492	(108)	0.186	(114)	0.339	(108)	11	5	11
Namibia	0.625	(120)	0.304	(131)	-0.040	(127)	0.132	(127)	-11	-7	-7
Honduras	0.625	(121)	0.423	(116)	0.208	(110)	0.316	(112)	5	11	9
Kiribati	0.624	(122)	0.345	(124)	0.162	(116)	0.253	(121)	-2	6	1
South Africa	0.619	(123)	0.395	(121)	-0.258	(154)	0.068	(133)	2	-31	-10
Indonesia	0.617	(124)	0.341	(126)	0.246	(107)	0.293	(115)	-2	17	9
Vanuatu	0.617	(125)	0.366	(123)	0.252	(106)	0.309	(113)	2	19	12
Kyrgyzstan	0.615	(126)	0.342	(125)	-0.035	(126)	0.153	(125)	1	0	1
Tajikistan	0.607	(127)	0.318	(128)	-0.051	(128)	0.133	(126)	-1	-1	1
Viet Nam	0.593	(128)	0.371	(122)	0.088	(120)	0.230	(122)	6	8	6
Nicaragua	0.589	(129)	0.339	(127)	0.034	(124)	0.187	(124)	2	5	5
Morocco	0.582	(130)	0.304	(130)	-0.059	(130)	0.123	(129)	0	0	1
Guatemala	0.574	(131)	0.270	(132)	-0.078	(133)	0.096	(131)	-1	-2	0
Iraq	0.573	(132)	0.207	(134)	0.049	(123)	0.128	(128)	-2	9	4
Cape Verde	0.568	(133)	0.310	(129)	-0.105	(136)	0.103	(130)	4	-3	3
India	0.547	(134)	0.086	(137)	-0.054	(129)	0.016	(134)	-3	5	0
Ghana	0.541	(135)	0.067	(138)	-0.115	(137)	-0.024	(136)	-3	-2	-1
Equatorial Guinea	0.537	(136)	0.209	(133)	-0.297	(160)	-0.044	(140)	3	-24	-4
Congo	0.533	(137)	0.049	(140)	-0.155	(140)	-0.053	(142)	-3	-3	-5
Lao People's Democratic Republic	0.524	(138)	0.059	(139)	-0.091	(134)	-0.016	(135)	-1	4	3
Cambodia	0.523	(139)	-0.002	(144)	-0.066	(131)	-0.034	(138)	-5	8	1
Swaziland	0.522	(140)	0.105	(136)	-0.351	(166)	-0.123	(147)	4	-26	-7
Bhutan	0.522	(141)	0.126	(135)	-0.293	(157)	-0.084	(144)	6	-16	-3
Solomon Islands	0.510	(142)	0.041	(142)	-0.103	(135)	-0.031	(137)	0	7	5
Kenya	0.509	(143)	0.014	(143)	-0.160	(141)	-0.073	(143)	0	2	0

Country	HDI		DRP-WI		DRP-SI		DRP-MI		HDI rank - DRP-WI rank	HDI rank - DRP-SI rank	HDI rank - DRP-MI rank
Sao Tome and Principe	0.509	(144)	-0.030	(149)	-0.076	(132)	-0.053	(141)	-5	12	3
Pakistan	0.504	(145)	-0.010	(146)	-0.188	(143)	-0.099	(145)	-1	2	0
Bangladesh	0.500	(146)	0.048	(141)	-0.127	(138)	-0.039	(139)	5	8	7
Timor-Leste	0.495	(147)	-0.040	(150)	-0.219	(145)	-0.129	(148)	-3	2	-1
Angola	0.486	(148)	-0.009	(145)	-0.297	(159)	-0.153	(151)	3	-11	-3
Myanmar	0.483	(149)	-0.065	(152)	-0.151	(139)	-0.108	(146)	-3	10	3
Cameroon	0.482	(150)	-0.060	(151)	-0.285	(156)	-0.172	(154)	-1	-6	-4
Madagascar	0.480	(151)	-0.016	(147)	-0.324	(162)	-0.170	(153)	4	-11	-2
Tanzania (United Republic of)	0.466	(152)	-0.118	(155)	-0.172	(142)	-0.145	(150)	-3	10	2
Papua New Guinea	0.466	(153)	-0.110	(154)	-0.296	(158)	-0.203	(159)	-1	-5	-6
Yemen	0.462	(154)	-0.084	(153)	-0.349	(165)	-0.216	(163)	1	-11	-9
Senegal	0.459	(155)	-0.131	(158)	-0.191	(144)	-0.161	(152)	-3	11	3
Nigeria	0.459	(156)	-0.126	(157)	-0.279	(155)	-0.203	(158)	-1	1	-2
Nepal	0.458	(157)	-0.027	(148)	-0.250	(153)	-0.139	(149)	9	4	8
Haiti	0.454	(158)	-0.139	(159)	-0.225	(147)	-0.182	(155)	-1	11	3
Mauritania	0.453	(159)	-0.141	(160)	-0.230	(149)	-0.186	(156)	-1	10	3
Lesotho	0.450	(160)	-0.120	(156)	-0.362	(170)	-0.241	(164)	4	-10	-4
Uganda	0.446	(161)	-0.151	(161)	-0.229	(148)	-0.190	(157)	0	13	4
Togo	0.435	(162)	-0.167	(162)	-0.335	(163)	-0.251	(166)	0	-1	-4
Comoros	0.433	(163)	-0.179	(165)	-0.238	(151)	-0.209	(161)	-2	12	2
Zambia	0.430	(164)	-0.173	(164)	-0.344	(164)	-0.258	(167)	0	0	-3
Djibouti	0.430	(165)	-0.170	(163)	-0.381	(174)	-0.275	(168)	2	-9	-3
Rwanda	0.429	(166)	-0.189	(166)	-0.223	(146)	-0.206	(160)	0	20	6
Benin	0.427	(167)	-0.193	(168)	-0.232	(150)	-0.212	(162)	-1	17	5
Gambia	0.420	(168)	-0.203	(170)	-0.297	(161)	-0.250	(165)	-2	7	3
Sudan	0.408	(169)	-0.201	(169)	-0.481	(177)	-0.341	(172)	0	-8	-3
Côte d'Ivoire	0.400	(170)	-0.239	(171)	-0.359	(169)	-0.299	(171)	-1	1	-1
Malawi	0.400	(171)	-0.239	(172)	-0.354	(168)	-0.297	(169)	-1	3	2
Afghanistan	0.398	(172)	-0.244	(173)	-0.352	(167)	-0.298	(170)	-1	5	2
Zimbabwe	0.376	(173)	-0.193	(167)	-0.576	(182)	-0.384	(177)	6	-9	-4
Ethiopia	0.363	(174)	-0.295	(174)	-0.502	(179)	-0.399	(179)	0	-5	-5
Mali	0.359	(175)	-0.315	(176)	-0.432	(175)	-0.374	(176)	-1	0	-1
Guinea-Bissau	0.353	(176)	-0.331	(178)	-0.364	(171)	-0.347	(173)	-2	5	3
Eritrea	0.349	(177)	-0.318	(177)	-0.463	(176)	-0.390	(178)	0	1	-1
Guinea	0.344	(178)	-0.340	(179)	-0.482	(178)	-0.411	(180)	-1	0	-2
Central African Republic	0.343	(179)	-0.352	(181)	-0.374	(173)	-0.363	(174)	-2	6	5
Sierra Leone	0.336	(180)	-0.364	(182)	-0.372	(172)	-0.368	(175)	-2	8	5
Burkina Faso	0.331	(181)	-0.342	(180)	-0.607	(184)	-0.475	(183)	1	-3	-2
Liberia	0.329	(182)	-0.311	(175)	-0.688	(187)	-0.499	(185)	7	-5	-3
Chad	0.328	(183)	-0.367	(183)	-0.539	(181)	-0.453	(181)	0	2	2
Mozambique	0.322	(184)	-0.382	(184)	-0.534	(180)	-0.458	(182)	0	4	2
Burundi	0.316	(185)	-0.384	(185)	-0.583	(183)	-0.484	(184)	0	2	1
Niger	0.295	(186)	-0.416	(186)	-0.627	(185)	-0.521	(186)	0	1	0
Congo (Democratic Republic of the)	0.286	(187)	-0.426	(187)	-0.671	(186)	-0.549	(187)	0	1	0