Monetary-Fiscal Policy Interactions: Interdependent Policy Rule Coefficients

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Abstract

In this paper I formulate and solve a new Keynesian dynamic stochastic general equilibrium model with monetary and fiscal policy rules whose coefficients are time-varying and contemporaneously interdependent. I implement time variation in the policy rules by specifying coefficients that are logistic functions of correlated latent factors. This specification allows for smooth transition of the coefficients and coordination between policies. I present a solution method to the model that allows for these characteristics. The paper uses Bayesian methods for nonlinear state-space models to estimate the policy rules with time-varying coefficients, endogeneity and stochastic volatility. The paper also performs policy analysis with the model solved using the proposed solution method and the estimated policy rule coefficients.

Keywords: Time-varying policy rules, Monetary and Fiscal policy interactions, Nonlinear state-space models.

JEL Classification Numbers: C11, C32, E63.

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1 Introduction

The great recession has raised awareness of and interest in the role of monetary-fiscal policy interactions in determining the behavior of economic aggregates. Fiscal stimulus policies have been accompanied by reductions in discount and funds rates, and even by unconventional quantitative easing measures. Fears of a second great depression led to implicit coordinated policies between central banks and fiscal authorities. Moreover, given the high level of projected fiscal liabilities in developed as well as in some developing countries, interactions are likely to figure more prominently in determining economic outcomes.

In a historical context, Friedman (1948) is the first to design a scheme of monetary-fiscal policy interactions to deliver economic stability in terms of cyclical fluctuations. The proposal involves the monetary system, government expenditures on goods and services, government transfer payments and the tax structure. Sargent and Wallace (1981) provide the first formal work to illustrate how monetary and fiscal policy interact to determine inflation and how, in an environment of fiscal dominance, the ability of the monetary authority to control inflation disappears. The first contributions to the literature of monetary-fiscal policy interactions in a dynamic stochastic general equilibrium (DSGE) framework were made by Leeper (1991), Sims (1994) and Woodford (1994). Canzoneri et al. (2010) offers a comprehensive review of the positive and normative aspects of monetary-fiscal policy interactions present in the literature.

This paper formulates and solves a new Keynesian DSGE model that incorporates feedback monetary and fiscal policy rules whose coefficients are time-varying and contemporaneously interdependent. Time variation and contemporaneous interdependence allow for coordinated changes between monetary and fiscal policy, introducing a direct channel of interactions.

Conventional monetary policy analysis makes use of macroeconomic DSGE models that specify a Taylor (1993) interest rate rule under which the central bank reacts to increases in inflation with increases more than proportional in the nominal interest rate. Monetary policy provides the nominal anchor to deliver price level determinacy in this scenario. This conventional setup assumes that fiscal policy will accommodate increases in the nominal interest rate with increases in (lump-sum) taxes to stabilize debt. Another strand of the literature emphasizes that fiscal policy may play a more important role than just accommodating monetary policy in achieving inflation stabilization, in particular when monetary policy is not or can not be used as the conventional models propose. The role of fiscal policy at providing a nominal anchor has been studied by Leeper (1991), Sims (1994), Woodford (1994, 1995, 1996) and Cochrane (1998, 2001, 2005). The principle is that fiscal policy, through expectations about future surpluses, can provide the nominal anchor for price level determinacy. In this paper I offer a framework for analyzing monetary as well as fiscal policy in a setup that stipulates that they can be equally important in achieving inflation stabilization.

The literature on monetary-fiscal policy interactions with feedback rules has modeled interactions by specifying policy rule coefficients that take on a finite set of states. Interactions emerge in equilibrium as a result of the combination between states of the monetary and fiscal policy rule coefficients that place the economy in different regimes, which results in different model dynamics (see Leeper, 1991; Davig and Leeper, 2006; Chung et al., 2007; Bianchi, 2012). Given that there are not driving forces behind policy rule coefficients that make them in fact interact, these equilibrium interactions are casual in some sense. In this paper I specify a direct channel of interactions between monetary and fiscal policy. Policy rule coefficients are defined as logistic functions of stationary latent factors specific to each of the policy rules. These latent factors are permitted to be correlated, introducing a direct channel of interactions in addition to the equilibrium interactions that exist in the literature. Allowing for interdependence between policy rule coefficients generalizes the current results and offers a more realistic way of incorporating fiscal policy in macroeconomic modeling.

Time variation in policy making has been documented by substantial empirical literature that argues that policy rules have not remained invariant over the course of the last six decades (see Clarida et al., 2000; Cogley and Sargent, 2002; Favero and Monacelli, 2003; Lubik and Schorfheide, 2004; Primiceri, 2005; Davig and Leeper, 2006; Fernandez-Villaverde et al., 2010; Bianchi, 2010). There is also theoretical literature arguing that, in designing policy rules, policy authorities may have asymmetric preferences with respect to deviations of variables of interest from target, or state-dependent loss functions, resulting in state-dependent policy rules (see Dolado et al., 2005; Svensson and Williams, 2007). The events of the great recession evidence how policies have shifted in some countries: monetary policy has apparently switched from following the Taylor principle to a pegged interest rate, while fiscal policy has moved from stabilizing debt to actively aiming to stimulate the economy. This paper incorporates time-varying monetary and fiscal policy rule coefficients in a macroeconomic model to be consistent with both the empirical and the theoretical literature.

As Davig and Leeper (2007) emphasize, policymaking is a complicated process that involves analyzing data, receiving advise, interpreting data, and applying judgment. I use latent factors to represent a combination of political and institutional determinants of policymaking beyond the systematic or nonsystematic components of a policy rule. During some periods policymakers may give more attention to inflation or debt stabilization, while in other periods more attention may be given to output stabilization. In this paper, policy coefficients move across regimes as functions of latent factors. This is analogous to having a random-coefficient specification, or a Markov-switching specification for policy rules. The difference is that the function considered here is bounded, as opposed to the random-coefficient setup, and continuous, as opposed to the Markov-switching setup. A function that satisfies these requirements is the logistic function. Boundedness is important because some policy rule coefficients make sense only if they are positive or, in terms of determinacy of a linear rational expectations model, if they have an upper or lower bound. Smoothness or continuity of the transition between states is also relevant since policies do not necessarily switch abruptly from one regime to another, and if they do, the logistic function still allows to have that type of behavior.

Given the logistic specification of policy rule coefficients and the existence of latent factors as additional states, the DSGE model is intrinsically nonlinear. In the paper I implement a solution method that takes into consideration these nonlinearities. This solution method incorporates agents' expectations with respect to the joint evolution of the policy rule coefficients, and makes the model suitable to consistently analyze the impacts of monetary and fiscal policies in a framework of interactions. I estimate the policy rule coefficients with Bayesian methods appropriate for nonlinear state-state models and then incorporate the estimated coefficients, along with other calibrated coefficients, in the model to carry out policy analysis.

Results from the paper can be summarized as follows: 1) The econometric estimation shows that there is important persistence in policymaking with fiscal policy being slightly more persistent than monetary policy. There is also a small degree of direct interactions between policies given by a positive estimated correlation between latent factors. Monetary policy switches more frequently than fiscal policy and the former loosens during recessions almost systematically. 2) Policy experiments reveal that taxes have effects on output and inflation as the literature on the fiscal theory of the price level suggests, but the effects are attenuated with respect to a pure fiscal regime.

2 The Model

The economy is populated by a representative household, a final goods producing firm, a continuum of intermediate goods producing firms, a monetary authority and a fiscal authority. The model extends the setup in An and Schorfheide (2007) to incorporate a fiscal policy rule and time-varying policy rule coefficients. To keep the specification simple, the model includes neither wage rigidities nor capital accumulation. Appendix A details the derivations.

2.1 Households

The representative household derives utility from consumption, C_t,[1] relative to a habit stock, A_t, that is given by the level of technology of the economy, and real money balances, M_t/P_t; and derives disutility from working hours, H_t. Hence, a representative household chooses consumption, real balances, bond holdings and working hours to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t/A_t)^{1-\sigma}}{1-\sigma} + \chi_M \log\left(M_t/P_t\right) - \chi_H \frac{H_t^{1+\varphi}}{1+\varphi} \right),$$

where $0 < \beta < 1$ is the discount factor, $\sigma > 0$ is the inverse of the elasticity of intertemporal substitution, $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply, and $\chi_M > 0$ and $\chi_H > 0$ are constants that determine the steady state level of real money balances and hours worked. The household saves in the form of nominal government bonds, B_t, that pay a gross interest rate R_t each period, and by accumulating money balances that do not pay interests. It supplies labor services to the firms taking the nominal wage ,W_t, as given; it also receives its aggregate share on the firms' nominal profits, D_t, and pays lump-sum taxes, T_t. Thus the household's budget constraint is expressed as

$$P_t C_t + M_t + B_t + P_t T_t \leq H_t W_t + D_t + M_{t-1} + R_{t-1} B_{t-1} \text{ for } t \geq 0,$$

[1] C_t is a composite consumption good given by $C_t = \left(\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$ and $\theta \geq 1$. The household chooses $C_t(j)$ to minimize expenditure on the continuum of goods indexed by $j \in [0,1]$ which yields j's good demand as $C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} C_t$, where P_t is the final good price at t and $P_t(j)$ is the price of the consumption good indexed by j.

given the initial value of nominal assets $M_{-1}+R_{-1}B_{-1}$, and where the transversality condition that rules out Ponzi schemes holds.

2.2 Firms

There are two types of producers: perfectly competitive final goods producers, and a continuum of monopolistic intermediate goods producers.

2.2.1 Aggregation Firms

Given the composite good price, P_t, and intermediate goods prices, $P_t(j)$, for $j \in [0,1]$, producers ensemble the intermediate goods, $Y_t(j)$, to obtain a composite final good, Y_t, according to a CES technology, so that

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\theta_t - 1}{\theta_t}} dj \right)^{\frac{\theta_t}{\theta_t - 1}}, \tag{1}$$

where $\theta_t \in [0,1]$ is the (time varying) price elasticity of demand for each intermediate good. Here θ_t represents a markup, or cost-push, shock in the Phillips curve relationship. This cost-push shock follows the autoregresive process

$$\log \theta_t = (1-\rho_\theta) \log \theta + \rho_\theta \log \theta_{t-1} + \varepsilon_t^\theta,$$

with $\rho_\theta \in (0,1)$, $\theta > 1$ and $\varepsilon_t^\theta \sim$ iid $\mathbb{N}(0, \sigma_\theta^2)$.

Final good producers choose the demand of intermediate goods, $Y_t(j)$, to maximize profits given by

$$P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj.$$

Optimization yields the demand function of intermediate good j,

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta_t} Y_t. \tag{2}$$

Combining (2) and (1) yields the expression of the final good price

$$P_t = \left(\int_0^1 P_t(j)^{(1-\theta_t)} dj \right)^{\frac{1}{1-\theta_t}}.$$

2.2.2 Intermediate Goods firms

Intermediate goods firms produce type j good according to the linear technology

$$Y_t(j) = A_t L_t(j), \tag{3}$$

where $L_t(j)$ are hours of work employed by the producer of intermediate good j, and A_t is an exogenous technology shock identical across producers following the stochastic process

$$A_t = \delta A_{t-1} \exp(\nu_t),$$

where δ is a trend, and ν_t is a stochastic component following the process

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu,$$

with $\rho_\nu \in (0, 1)$ and $\varepsilon_t^\nu \sim$ iid $\mathbb{N}(0, \sigma_\nu^2)$.

Intermediate good producers face an explicit cost of adjusting their price, measured in units of the finished good, and given by

$$\frac{\phi}{2}\left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1\right)^2 Y_t,$$

where $\phi \geq 0$ measures the magnitude of the price adjustment cost, and Π is the steady state gross inflation rate associated with the final good.

Producers in the intermediate goods sector take wages as given and behave as monopolistic competitors in their goods market, choosing the price for their product taking the demand in (2) as given. Hence, firm j chooses its labor input, $L_t(j)$, and its price, $P_t(j)$, to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \mathrm{MRS}_{0,t} \left[\frac{P_t(j)}{P_t} Y_t(j) - \frac{W_t}{P_t} L_t(j) - \frac{\phi}{2}\left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1\right)^2 Y_t\right], \tag{4}$$

where $\mathrm{MRS}_{0,t}$ is the household's marginal rate of substitution between periods 0 and t, which is given exogenously to the firm. Notice that firm j's nominal labor cost is given by $W_t Y_t(j)/A_t$, and its real marginal labor cost is given by $\psi_t = W_t/P_t A_t$, which is the same across firms in the intermediate goods sector.

2.3 Government

The government finances purchases of goods, G_t, with a combination of lump-sum taxes, T_t, and money creation, $M_t - M_{t-1}$, so that the implied process for nominal debt, B_t, satisfies the budget constraint:

$$B_t + M_t + P_t T_t = P_t G_t + M_{t-1} + R_{t-1} B_{t-1} \text{ for } t \geq 0, \tag{5}$$

given $M_{-1} + R_{-1} B_{-1}$. Each period, the government demand of the final good is given by

$$G_t = \zeta_t Y_t,$$

where $\zeta_t \in (0, 1)$ is an exogenous process defined by the transformation $g_t = 1/(1 - \zeta_t)$ with

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \varepsilon_t^g,$$

where $\rho_g \in (0,1)$, $g = 1/(1-\zeta)$ with ζ being the steady state ratio of government spending to output, and $\varepsilon_t^g \sim$ iid $\mathbb{N}(0, \sigma_g^2)$.

2.4 Policy Rules

Time-varying policy rule coefficients have received special attention in recent years. Dolado et al. (2005) offers a survey of the literature to support on theoretical grounds the existence of nonlinear responses of an interest rate rule with respect to inflation and/or output. In particular, one of the arguments goes along the line of asymmetric preferences by the central bank with respect to deviations of inflation and/or output with respect to target. In a similar line, Svensson and Williams (2007), in a context of model uncertainty, specify a loss function whose weights are not fixed and obtain a monetary policy rule whose coefficients take on different values across states.

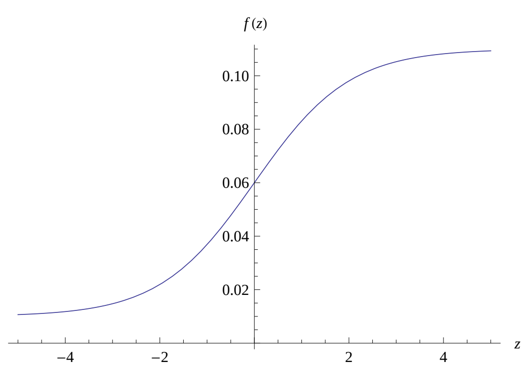
I specify policy rules with coefficients that are time varying. The time varying coefficients of a particular policy rule are specified as logistic functions of a latent state. More specifically, if ϱ_t is a time varying coefficient of a policy rule, it has the following functional form:

$$\begin{aligned} \varrho_t &\equiv \varrho(z_t) \\ &= \varrho_0 + \frac{\varrho_1}{1 + \exp\left(-\varrho_2(z_t - \varrho_3)\right)}, \end{aligned}$$

where $z_t = \rho_z z_{t-1} + \xi_t$ is a latent factor, $0 < \rho_z \leq 1$ and $\xi_t \sim \text{iid}\mathbb{N}(0,1)$.

Under this specification, ϱ_0 denotes the lower (upper) bound of ϱ_t, while $\varrho_0 + \varrho_1$ denotes its upper (lower) bound (if $\varrho_1 < 0$). $\varrho_2 > 0$ is a transition coefficient affecting with its magnitude the speed of the transition between the lower and the upper bounds, and ϱ_3 is a location parameter determining the value of z_t at which ϱ_t crosses the y-axis. A graph for $\varrho(z_t)$ with $\varrho_0 = 0.01$, $\varrho_1 = 0.1$, $\varrho_2 = 1$, and $\bar{\varrho}_3 = 0$ is reproduced in Figure 1.

Figure 1: Logistic Function



Two approaches have been proposed in the empirical literature to model time-varying policy rule coefficients: 1) A specification with two-state Markov-switching coefficients (Davig and Leeper, 2006; Eo, 2009; Davig and Doh, 2009; Bianchi, 2010), and 2) a random-coefficient specification (Kim and Nelson, 2006a; Fernandez-Villaverde et al., 2010). An advantage of the Markov-switching specification is that it implies bounded coefficients, which in terms of determinacy and relevance of the equilibrium can be important. A disadvantage of this specification is that it implies sudden changes from one policy regime to the other. With respect to the random-coefficient specification, an advantage is that it implies smooth transitions between states. A disadvantage is that it does not bound the evolution of policy rule coefficients. The logistic specification proposed in this paper allows to generalize both approaches in the literature: On one hand it allows a policy rule coefficient to switch smoothly from one regime to another, while on the other allows for a bounded evolution of the coefficient.[2]

Davig and Leeper (2007) argue that a policy rule, in particular the monetary policy rule, is a "complicated, probably non-linear, function of a large set of information about the state of the economy" (p. 607). In this paper, latent factors represent a combination of political and institutional determinants of policymaking beyond the systematic or nonsystematic components of a policy rule. I assume that the variables summarized in the latent factors are exogenous with respect to the variables in the model, and that the information that they provide is part of the information set of the agents in the model. That is, agents and policymakers share the same information set that is transmitted by the latter to the former as part of the political e institutional process of electing monetary and fiscal authorities. Notice that using latent factors as the processes driving policy rule coefficients avoids having to choose an observable macroeconomic variable to drive the transition between states, like in the threshold autoregressive (TAR) and the smooth threshold autoregressive (STAR) literatures.

2.4.1 Monetary Policy Rule

Monetary policy takes place by means of an interest rate feedback rule of the form

$$R_t = R_{t-1}^{\rho_R} \bar{R}_t^{(1-\rho_R)} \exp\left(\varepsilon_t^R\right),$$

where $\rho_R \in (0,1)$ indicates the degree of interest rate smoothing, and $\varepsilon_t^R \sim \mathrm{iid}\mathbb{N}(0, \sigma_R^2)$ is a monetary policy shock. $\bar{R}_t$ is the target short-term nominal interest rate. The central bank sets the interest rate to react to deviations of inflation from target and to the output gap according to

$$\bar{R}_t = R \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\alpha^{\pi}(z_t^m)} \left(\frac{Y_t}{Y_t^*}\right)^{\alpha^{y}(z_t^m)},$$

where $\Pi_t = P_t/P_{t-1}$, Y_t^* is output in the absence of price rigidities, and R is the steady state nominal interest rate, which is guaranteed to be state independent if the target inflation rate, $\bar{\Pi}$, is set equal to Π, the steady state inflation.[3]

The time varying monetary policy rule coefficients are denoted by $\alpha^{\pi}(z_t^m)$ for inflation deviations from steady state, and $\alpha^{y}(z_t^m)$ for the output gap. Both are logistic functions of the monetary policy latent factor, z_t^m.

[2] The Markov-switching specification is a particular case of the logistic specification, when $\varrho_2 = \infty$.

[3] The target inflation rate is constant to allow the linearization of the model around the steady state conditional on a realization of the latent factor at each period. A time-varying target inflation rate goes beyond of the scope of this work.

2.4.2 Fiscal Policy Rule

The fiscal rule is a feedback rule for the ratio of lump-sum taxes net of transfers to output, $\tau_t = T_t/Y_t$, of the form

$$\tau_t = \tau_{t-1}^{\rho_\tau} \bar{\tau}_t^{(1-\rho_\tau)} \exp(\varepsilon_t^\tau),$$

where $\rho_\tau \in (0,1)$ indicates the degree of tax rate smoothing, and $\varepsilon_t^\tau \sim \text{iid}\mathbb{N}(0, \sigma_\tau^2)$ is a fiscal policy shock. $\bar{\tau}_t$ is the target level of the ratio of taxes net of transfers to output. The fiscal authority sets lump-sum taxes to respond to debt deviations, the output gap and government spending according to

$$\bar{\tau}_t = \tau \left(\frac{b_{t-1}}{\bar{b}}\right)^{\gamma^b(z_t^f)} \left(\frac{Y_t}{Y_t^*}\right)^{\gamma^y(z_t^f)},$$

where $b_t = B_t/(P_t Y_t)$ denotes the debt-to-output ratio in period t, and $\bar{b}$ is its target level. τ denotes the steady state level of τ_t, which is guaranteed to be state independent in the steady state equilibrium if $\bar{b}$ is set equal to its steady state value, denoted by b.

The time varying fiscal policy rule coefficients are $\gamma^b(z_t^f)$ for debt deviations from steady state and $\gamma^y(z_t^f)$ for the output gap. All are logistic functions of the fiscal policy latent factor, z_t^f. This fiscal policy rule setup generalizes the specification in Chung et al. (2007) by adding the output gap and the smoothing component and Davig and Leeper (2006) by adding the smoothing component, although the latter includes feedback from government spending.

2.5 Interactions Between Monetary and Fiscal Policies

The introduction of monetary and fiscal policy interactions in the context of dynamic stochastic general equilibrium models dates back to the work of Leeper (1991) where different combinations of monetary and fiscal policy rule coefficients lead to different equilibrium outcomes and local dynamics. There, the terms "active" and "passive" monetary and fiscal policies are introduced to describe how the central bank adjusts interest rates with respect to inflation deviations from target, and how fiscal policy adjusts taxes to changes in public debt. A change more than proportional in nominal interest rates with respect to inflation deviations from target is called "active" monetary policy (AM), while a Ricardian view of fiscal policy, where taxes adjust enough to cover interest payments and to retire debt, is called "passive" fiscal policy (PF). The alternative scenario with respect to monetary and fiscal policies is called "passive" monetary policy (PM) and "active" fiscal policy (AF).[4] Leeper finds that the model delivers a bounded unique rational expectations equilibrium as long as monetary policy is active and fiscal policy is passive -Monetary (or M) Regime-, or if monetary policy is passive and fiscal policy is active -Fiscal (or F) Regime-. Also, the model delivers indeterminacy if both monetary policy and fiscal policy are passive, and no bounded solution if both are active. Other works along this line are Sims (1994) and Leith and Wren-Lewis (2000).

[4] Any positive response of taxes to debt constitutes a passive fiscal policy. If one only wants to consider equilibria with bounded real debt, then real taxes have to respond to real debt deviations with increases higher than the real interest rate.

From a normative perspective, Nordhaus (1994) carries on a game theoretical approach to understand monetary-fiscal policy coordination. He finds that a deficit-reduction package should be accompanied by a cooperative monetary policy to offset declines in aggregate demand and increases in unemployment, so that the economy ends up in a recovery with higher domestic and foreign investment. From an optimality perspective, Dixit and Lambertini (2003) find that a second-best outcome can be achieved if the monetary and the fiscal authorities both choose to be equally and optimally conservative with respect to the price level.

As for a quantitative approach to measure the interdependence of monetary and fiscal policies, Muscatelli et al. (2004) estimate a new Keynesian model with an interest rate rule and government-spending and tax rules with U.S. data. Fiscal rules are specified to act as automatic stabilizers, and do not incorporate feedback from debt or deficit. Their results show that when an output shock hits the economy, monetary and fiscal policies tend to be complements, while if an inflation shock hits the economy, policies tend to act as substitutes. This channel of interactions occurs since the monetary policy rule reacts to inflation and output, but fiscal policy rules react mainly to output.

In a time-varying coefficient setup, Davig and Leeper (2006) estimate Markov-switching models of monetary and fiscal policy rules with U.S. data. Their results show that there have been numerous switches in monetary and fiscal policy rule coefficients. Whenever the interest rate rule pays more (less) attention to inflation deviations, less (more) weight is given to output deviations. That corresponds to the AM (PM) regime. Also, when the tax rule pays more (less) attention to debt deviations, more (less) weight is given to output deviations (in line with an automatic stabilizers argument). That corresponds to the PF (AF) regime. These switches have led to four regimes of interactions: AM/PF (M Regime), PM/AF (F Regime), PM/PF, AM/AF. One of the conclusions of Davig and Leeper is that, to better understand macroeconomic policy effects, it is essential to model policy rules as governed by a stochastic process over which agents form expectations.

This paper explicitly introduces interactions between the coefficients of the monetary and fiscal policy rules. This setup generalizes the Markov-switching setup in Davig and Leeper (2006) and Davig and Leeper (2011) by introducing correlation in the evolution of policy rule coefficients. With this addition, the model allows for direct interactions or coordinated changes in policies that deliver a long-run scenario for agents to form expectations accordingly, not only in terms of the individual future evolution of policy rule coefficients, but as a framework of joint future policy making. To incorporate direct interactions between policies I specify the latent factors driving the evolution of the monetary and fiscal policy rule coefficients as follows:

$$z_t^m = \rho_{z^m} z_{t-1}^m + \xi_t^m \tag{6}$$

$$z_t^f = \rho_{z^f} z_{t-1}^\tau + \xi_t^f, \tag{7}$$

where ξ_t^m and ξ_t^f are normally distributed with zero mean, unit variance and $\text{corr}(\xi_t^m, \xi_t^f) = \kappa$. Notice that under this specification, if κ is different from zero, there exist explicit interactions or coordinated changes between monetary and fiscal policy.

In the present context, policies become active or passive depending on the values of the policy rule coefficients and the evolution of the latent factors z_t^m and z_t^f, and there will be combinations of policies depending on the relationship between these factors. The full specification of the monetary and fiscal policy rule coefficients is as follows:

$$\begin{aligned}
\alpha^\pi(z_t^m) &= \alpha_0^\pi + \frac{\alpha_1^\pi}{1+\exp(-\alpha_2^\pi(z_t^m - \alpha_3^\pi))} \\
\alpha^y(z_t^m) &= \alpha_0^y + \frac{\alpha_1^y}{1+\exp(-\alpha_2^y(z_t^m - \alpha_3^y))} \\
\gamma^b(z_t^f) &= \gamma_0^b + \frac{\gamma_1^b}{1+\exp(-\gamma_2^b(z_t^f - \gamma_3^b))} \\
\gamma^y(z_t^f) &= \gamma_0^y + \frac{\gamma_1^y}{1+\exp(-\gamma_2^y(z_t^f - \gamma_3^y))}.
\end{aligned}$$

If the values of the parameters defining each of the policy rule coefficients and the correlation between the latent factors are chosen in a way that whenever the monetary authority reacts strongly enough to inflation deviations, the fiscal authority reacts strongly enough to debt deviations (M Regime), or viceversa (F Regime), then a unique bounded long-run equilibrium will be delivered with short-run deviations from this equilibrium. These interactions with long-run determinacy of the equilibrium (as long as the coefficients satisfy the determinacy conditions) are only well defined if $\kappa > 0$: a high value of $\alpha^\pi(\cdot)$ is likely to be associated with a high value of $\gamma^b(\cdot)$, and viceversa. $\kappa = 0$ is the case present in the existent literature, where interactions only emerge in equilibrium, depending on the values of the policy rule coefficients. As can be seen, the specification proposed here allows a rich possibility of combinations, and explicitly introduces interactions between monetary and fiscal policies.

Even though the latent factors are potentially correlated, I assume that they are independent of the structural shocks and part of the information set of agents, as mentioned above. This assumption is necessary to obtain a solution to the model that is based on the minimum state variable solution approach as will be shown.

2.6 Symmetric Equilibrium

In a symmetric equilibrium, all the intermediate goods producing firms make identical decisions, the money supply equals the money demand, the labor supply equals the labor demand, and the net supply of government bonds is zero. Hence, the equilibrium conditions for $t \geq 0$ are given by

$$Y_t = C_t + G_t + \frac{\phi}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2 Y_t, \tag{8}$$

$$\begin{aligned}
0 = {} & 1 - \theta_t + \theta_t\psi_t - \phi\left(\frac{\Pi_t}{\Pi} - 1\right)\frac{\Pi_t}{\Pi} + \\
& + \beta\phi\mathbb{E}_t\left(\frac{C_t/A_t}{C_{t+1}/A_{t+1}}\right)^\sigma \frac{Y_{t+1}/A_{t+1}}{Y_t/A_t}\left(\frac{\Pi_{t+1}}{\Pi} - 1\right)\frac{\Pi_{t+1}}{\Pi}
\end{aligned} \tag{9}$$

$$1 = \beta R_t \mathbb{E}_t\left(\frac{C_t/A_t}{C_{t+1}/A_{t+1}}\right)^\sigma \frac{A_t}{A_{t+1}}\frac{1}{\Pi_{t+1}}, \tag{10}$$

$$\frac{W_t}{P_t} = \chi_H L_t^{\varphi} A_t \left(\frac{C_t}{A_t}\right)^{\sigma}, \tag{11}$$

$$\frac{M_t}{P_t} = \chi_M A_t \left(\frac{C_t}{A_t}\right)^{\sigma} \left(\frac{R_t}{R_t - 1}\right), \tag{12}$$

with $B_{-1}, R_{-1} > 0, A_{-1} > 0$ and $M_{-1} > 0$ given. The symmetric equilibrium is complemented with the monetary and fiscal policy rules, and the exogenous processes for G_t, θ_t and A_t.

2.7 Frictionless Equilibrium

The frictionless equilibrium is given by the above equilibrium with no frictions ($\phi = 0$). Aggregate output in the frictionless equilibrium is given by

$$Y_t^* = A_t \left(\frac{\frac{\theta_t - 1}{\theta_t}}{\chi_H}\right)^{1/(\sigma+\varphi)} g_t^{\sigma/(\sigma+\varphi)}. \tag{13}$$

The above is the potential output over which the output gap in the monetary and fiscal policy rules is defined.

2.8 Steady State Equilibrium

Since technology, A_t, is a non-stationary process, it introduces a stochastic trend in output, consumption, real money balances, and the real wage. We define the stationary variables as: $y_t = Y_t/A_t, c_t = C_t/A_t, w_t = W_t/(A_t P_t)$ and $v_t = Y_t/(M_t/P_t)$. The steady state equilibrium is the stationary equilibrium in the absence of shocks, and is defined by the following equations:

$$\Pi = \frac{R\beta}{\delta}, \tag{14}$$

$$y = \left(\frac{\frac{\theta-1}{\theta}}{\chi_H}\right)^{1/(\sigma+\varphi)} g^{\sigma/(\sigma+\varphi)} = y^*, \tag{15}$$

$$c = \left(\frac{\frac{\theta-1}{\theta}}{\chi_H}\right)^{1/(\sigma+\varphi)} g^{\varphi/(\sigma+\varphi)}, \tag{16}$$

$$\frac{1}{v} = \chi_M y^{(\sigma-1)} g^{-\sigma} \left(\frac{R}{R-1}\right), \tag{17}$$

$$b = \left(\frac{\beta}{\beta - 1}\right) \left(1 - \frac{1}{g} - \tau - \frac{1}{v}\left(1 - \frac{1}{\Pi}\frac{1}{\delta}\right)\right), \tag{18}$$

where b is the steady state level of the debt-to-output ratio, $B_t/(P_t Y_t)$.

2.9 Log-linearized Model and Solution Method

Since the coefficients of the policy rules are time varying, and the time variation depends on the latent factors, the log-linearization is performed conditioning on the latent factors being at their current values each period. That is the essence of the quasi-linearity of the model. Similar approaches have been taken by Liu et al. (2009) and Davig and Doh (2009) in a context of Markov-switching policies, where the linearization is performed conditioning on a given regime. I present here the model in log-deviations from the non-stochastic steady state, and show a way to solve it using a method in line with the minimum state variable (MSV) solution approach (McCallum, 1983).

Conditioning on a value of the latent factors at period t, z_t^m and z_t^f, the log-linearized equations characterizing the economy in equilibrium are ($\hat{x}_t = \ln(x_t/x)$ denotes the log-deviation of variable x_t with respect to its non-stochastic steady state, x):

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} - \frac{1}{\sigma}(\hat{R}_t - \mathbb{E}_t\hat{\Pi}_{t+1}) + (1-\rho_g)\hat{g}_t + \frac{\rho_\nu}{\sigma}\hat{\nu}_t \tag{19}$$

$$\hat{\Pi}_t = \beta\mathbb{E}_t\hat{\Pi}_{t+1} + \frac{(\theta-1)(\varphi+\sigma)}{\phi}\left(\hat{y}_t - \hat{y}_t^*\right) \tag{20}$$

$$\hat{\upsilon}_t = (1-\sigma)\hat{y}_t + \sigma\hat{g}_t + \left(\frac{1}{R-1}\right)\hat{R}_t \tag{21}$$

$$\hat{b}_t = \frac{1}{bg}\hat{g}_t - \frac{\tau}{b}\hat{\tau}_t + \frac{1}{b\upsilon}\hat{\upsilon}_t - \frac{1}{b\upsilon\Pi\delta}\hat{\upsilon}_{t-1} - \left(\frac{1}{b\upsilon\Pi\delta} + \frac{1}{\beta}\right)(\hat{\Pi}_t + \Delta\hat{Y}_t) + \frac{1}{\beta}(\hat{R}_{t-1} + \hat{b}_{t-1}) \tag{22}$$

$$\hat{y}_t^* = \frac{1}{(\varphi+\sigma)(\theta-1)}\hat{\theta}_t + \frac{\sigma}{\varphi+\sigma}\hat{g}_t \tag{23}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left(\alpha^\pi(z_t^m)\hat{\Pi}_t + \alpha^y(z_t^m)\left(\hat{y}_t - \hat{y}_t^*\right)\right) + \varepsilon_t^R \tag{24}$$

$$\hat{\tau}_t = \rho_\tau\hat{\tau}_{t-1} + (1-\rho_\tau)\left(\gamma^b(z_t^f)\hat{b}_{t-1} + \gamma^y(z_t^f)\left(\hat{y}_t - \hat{y}_t^*\right)\right) + \varepsilon_t^\tau, \tag{25}$$

where $\Delta\hat{Y}_t = \hat{y}_t - \hat{y}_{t-1} + \hat{\nu}_t$. The exogenous shocks that complete the equilibrium are the government spending shock, the cost-push shock, and the technology shock, given by

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + \varepsilon_t^g \tag{26}$$

$$\hat{\theta}_t = \rho_\theta\hat{\theta}_{t-1} + \varepsilon_t^\theta \tag{27}$$

$$\hat{\nu}_t = \rho_\nu\hat{\nu}_{t-1} + \varepsilon_t^\nu. \tag{28}$$

To solve the model, let $\boldsymbol{\omega}_t = [\hat{y}_t, \hat{\pi}_t]'$, $\mathbf{k}_t = [\hat{\upsilon}_t, \hat{b}_t, \hat{R}_t, \hat{\tau}_t, \Delta\hat{Y}_t, y_{t-1}, \hat{y}_t^*]'$, $\mathbf{u}_t = [\hat{g}_t, \hat{\theta}_t, \hat{\nu}_t, \varepsilon_t^R, \varepsilon_t^\tau]'$, $\boldsymbol{\varepsilon}_t = [\varepsilon_t^g, \varepsilon_t^\theta, \varepsilon_t^\nu, \varepsilon_t^R, \varepsilon_t^\tau]'$, $\mathbf{z}_t = [z_t^m, z_t^f]'$ and rewrite (19)-(28) as

$$0 = \mathbf{A}(\mathbf{z}_t)\mathbf{k}_t + \mathbf{B}(z_t^f)\mathbf{k}_{t-1} + \mathbf{C}(\mathbf{z}_t)\boldsymbol{\omega}_t + \mathbf{D}\mathbf{u}_t \tag{29}$$

$$0 = \mathbf{G}\mathbf{k}_t + \mathbf{J}\mathbb{E}_t\boldsymbol{\omega}_{t+1} + \mathbf{K}\boldsymbol{\omega}_t + \mathbf{M}\mathbf{u}_t \tag{30}$$

$$\mathbf{u}_{t+1} = \mathbf{N}\mathbf{u}_t + \boldsymbol{\varepsilon}_{t+1}, \tag{31}$$

where $\mathbf{A}, \mathbf{B}(z_t^f), \mathbf{C}(\mathbf{z}_t), \mathbf{D}, \mathbf{G}, \mathbf{J}, \mathbf{K}, \mathbf{M}$ and $\mathbf{N}$ are appropriate coefficient matrices shown in Appendix B.

The proposed solution is given by

$$\mathbf{k}_t = \mathbf{P}(\mathbf{z}_t)\mathbf{k}_{t-1} + \mathbf{Q}(\mathbf{z}_t)\mathbf{u}_t, \tag{32}$$
$$\boldsymbol{\omega}_t = \mathbf{R}(\mathbf{z}_t)\mathbf{k}_{t-1} + \mathbf{S}(\mathbf{z}_t)\mathbf{u}_t, \tag{33}$$

where, for $\mathbf{F}(\mathbf{z}_t) = \mathbf{P}(\mathbf{z}_t), \mathbf{Q}(\mathbf{z}_t), \mathbf{R}(\mathbf{z}_t), \mathbf{S}(\mathbf{z}_t)$, the $i, j - th$ entry is given by

$$F(\mathbf{z}_t) = \frac{\left(F_{0m} + \frac{F_{1m}}{1+\exp\left(-F_{2m}(z_t^m - F_{3m})\right)}\right)\left(1 + \frac{F_{1f}}{1+\exp\left(-F_{2f}\left(z_t^f - F_{3f}\right)\right)}\right)}{1 - F_4 \frac{\exp\left(-F_{2m}(z_t^m - F_{3m})\right)}{1+\exp\left(-F_{2m}(z_t^m - F_{3m})\right)} \frac{\exp\left(-F_{2f}\left(z_t^f - F_{3f}\right)\right)}{1+\exp\left(-F_{2f}\left(z_t^f - F_{3f}\right)\right)}}, \tag{34}$$

with $F_4 \in [0, 1]$. This functional form is known as a bivariate logistic function and was introduced by Ali et al. (1978).[5]

Appendix C illustrates the procedure to obtain the parameters of the bivariate logistic functions. Appendix D shows that the coefficients of the solution indeed follow a bivariate logistic function.

The model is solved using an undetermined coefficients method approach where not only the linear solution has to be guessed and verified, but also the functional form of the coefficients of the solution has to be guessed and verified. Within the logistic specification of policy rules, the bivariate logistic function (34) satisfies this requirement.

2.10 On Existence, Stability and Uniqueness of the Solution

Since the method used to obtain the solution is based on the undetermined coefficients method, existence is guaranteed. Time-varying coefficients pose a potential difficulty at guaranteeing stability and/or uniqueness of the solution, in particular if one thinks of stability and/or uniqueness holding at each period of time. The method presented here finds a solution that is based on the values of the time-varying policy rule coefficients at their limits or long-run bounds. These limiting coefficient values are chosen to deliver stability and uniqueness of the solution in a constant-coefficient version of the model, offering well defined bounds between which the economy evolves and between which agents form expectations. Davig and Leeper (2007) and Farmer et al. (2008) emphasize that stability and uniqueness of Markov-switching rational expectations models have to be discussed in a framework of how agents form expectations about the evolution of policy rule coefficients, and that is partially the approach taken at deriving the solution of the model in this paper, with the difference that here the conditional expectations are taken to logistic functions.

The debate on uniqueness of the solution of nonlinear models has attracted attention of the DSGE modeling and estimation literatures in recent years, and it is still an open field to future research (see Davig and Leeper, 2006, 2007; Chung et al., 2007; Fernandez-Villaverde et al., 2010; Farmer et al., 2011; Bianchi, 2012).

[5]For identification of the latent factors and the coefficients of the policy rules in a estimation setting it is necessary to impose that $F_{2m}^{ij} > 0$, $F_{2f}^{ij} > 0$, $F_{2mf}^{ij} > 0$, and $F_{2fm}^{ij} > 0$.

3 Estimation Strategy

This section presents the estimation of the policy rules with time-varying coefficients driven by latent factors as specified for the new Keynesian model presented in Section 2. The estimation employs Bayesian methods that allow obtaining the set of parameters characterizing the policy rules, denoted by Θ_y, the set of parameters of the latent factors, denoted by Θ_z, and the latent factors themselves, using the approach proposed by Geweke and Tanizaki (2001).

3.1 Time-varying Coefficients State-Space Model, Stochastic Volatility and Endogeneity

Let INT_t denote the demeaned nominal federal funds rate in period t, TAX_t the demeaned ratio of federal receipts net of transfers to output in period t, INF_t the demeaned annual inflation rate in period t, GAP_t the output gap in period t and DBT_t the demeaned average debt to output ratio over the last four quarter. The state-space model is composed of the observation equations

$$\text{INT}_t = \rho_R \text{INT}_{t-1} + (1-\rho_R)\left(\alpha^\pi(z_t^m)\text{INF}_t + \alpha^y(z_t^m)\text{GAP}_t\right) + \upsilon_t^R \tag{35}$$

$$\text{TAX}_t = \rho_\tau \text{TAX}_{t-1} + (1-\rho_\tau)\left(\gamma^b(z_t^f)\text{DBT}_t + \gamma^y(z_t^f)\text{GAP}_t\right) + \upsilon_t^\tau, \tag{36}$$

and the transition equations

$$z_t^m = \rho_{z^m} z_{t-1}^m + \xi_t^m \tag{37}$$

$$z_t^f = \rho_{z^f} z_{t-1}^\tau + \xi_t^f. \tag{38}$$

The assumptions about the distributions of υ_t^R and υ_t^τ are made explicit in the following section.

3.1.1 Stochastic Volatility

The existence of stochastic volatility in the shocks of policy rules with time-varying coefficients has been examined by Davig and Leeper (2006), Fernandez-Villaverde et al. (2010), Bianchi (2010) and Fernandez-Villaverde et al. (2011b), who find that not only switches in policy rule coefficients are detectable from estimations, but also a fair amount of stochastic volatility.[6] Hence, the distribution of the error terms in the policy rules is specified as $\upsilon_t^R \sim \mathbb{N}(0, \sigma_{R,t}^2)$ and $\upsilon_t^\tau \sim \mathbb{N}(0, \sigma_{\tau,t}^2)$, where

$$\ln \sigma_{R,t} = (1-\rho_{\sigma_R})\ln \sigma_R + \rho_{\sigma_R}\ln \sigma_{R,t-1} + \eta_R \xi_t^R \tag{39}$$

$$\ln \sigma_{\tau,t} = (1-\rho_{\sigma_\tau})\ln \sigma_\tau + \rho_{\sigma_\tau}\ln \sigma_{\tau,t-1} + \eta_\tau \xi_t^\tau \tag{40}$$

with $\xi_t^R \sim \text{iid}\mathbb{N}(0,1)$ and $\xi_t^\tau \sim \text{iid}\mathbb{N}(0,1)$.

[6] Sims and Zha (2006), on the other hand, argue that only changes in volatility can be detected in estimations, and not changes in coefficients.

In what follows, let $\mathbf{h}_t = [\ln \sigma_{R,t}, \ln \sigma_{\tau,t}]$ and let the set of parameters of the stochastic volatility processes be denoted Θ_h.

Equations (39) and (40) are added to the state-space model (35)-(38) to introduce stochastic volatility to the specification of the policy rules with time-varying coefficients.

3.1.2 Endogeneity

Since the work of Clarida et al. (2000), the estimation of monetary policy rules with constant coefficients, in particular the Taylor rule, has taken into account the endogeneity that exists between the shocks of the policy rule and inflation and output. The instrument set used in their GMM estimation contains four lags of: inflation, the output gap, the Federal funds rate, the short-long spread, and commodity price inflation.

With respect to fiscal policy rules with constant coefficients, Li (2009) illustrates the endogeneity/simultaneity problem that arises when estimating a fiscal policy rule like the one presented in this work. In estimating a fiscal policy rule that reacts to contemporary debt and the output gap, Claeys (2008) uses a set of instrumental variables in their GMM estimation that contains lags of: the output gap, debt, unit labor costs, growth in labor productivity, NAIRU, a broad money aggregate, a synthetic interest rate of the EURO area, oil price index, and the SEK/DEM exchange rate.

In terms of estimating linear equations with time-varying coefficients, either in the conventional random coefficients or Markov-switching setups, Kim (2006) and Kim (2009) establish a Heckman-type two-stage maximum likelihood estimation technique to deal with the endogeneity problem to yield consistent estimates of the hyper-parameters, as well as to provide correct inferences on the time-varying coefficients. Kim and Nelson (2006b) estimate a random coefficients monetary policy rule for the United States using as the set of instruments four lags of: the Federal funds rate, output gap, inflation, commodity price inflation, and M2 growth. On a related work, Bae et al. (2011) estimate a Markov-switching coefficients monetary policy rule for the United States using as the set of instruments three lags of: the Federal funds rate, GDP gap, inflation, commodity price changes, and spread between the long-term bond rate, and the three-month Treasury Bill rate.

In the present work, the set of instruments for both, the monetary and the fiscal policy rules, is given by four lags of: inflation, the output gap, government spending as proportion of GDP, M2 growth, and commodity price inflation. In a constant coefficients version of the policy rules, the GMM estimation obtains the following results with respect to the instruments set: 1) The J test statistic of overidentification restrictions for both of the rules does not reject the null hypothesis that the instrument set is appropriate at the 5% level of significance. 2) The exogeneity C test statistic implies that different subsets of instruments are exogenous at the 5% level of significance. 3) The Cragg-Donald test statistic rejects the null hypothesis of weak instruments at the 5% level of significance for both policy rules.

Let GSP_t the demeaned government spending to output ratio in period t, M2G_t the annual rate of growth of M2 in period t, and CMP_t the annual commodity price inflation. In order to account for the existence of endogeneity, the observation equations (35)-(36) of the state-space model have to be modified by introducing a system of simultaneous equations.

To that end, let

$$
\begin{aligned}
y_{1,t} &= \mathrm{INT}_t, \\
y_{2,t} &= \mathrm{TAX}_t, \\
\mathbf{x}_{1,t} &= [\mathrm{INF}_t, \mathrm{GAP}_t], \\
\mathbf{x}_{2,t} &= [\mathrm{DBT}_{t-1}, \mathrm{GAP}_t], \\
\boldsymbol{\alpha}(z_t^m) &= [\alpha^\pi(z_t^m), \alpha^y(z_t^m)]', \\
\boldsymbol{\gamma}(z_t^f) &= [\gamma^b(z_t^f), \gamma^y(z_t^f)]', \\
\mathbf{w}_{1,t} &= [\{\mathrm{INF}_{t-s}\}_{s=1}^4, \{\mathrm{GAP}_{t-s}\}_{s=1}^4, \{\mathrm{GSP}_{t-s}\}_{s=1}^4, \{\mathrm{M2G}_{t-s}\}_{s=1}^4, \{\mathrm{CMP}_{t-s}\}_{s=1}^4], \\
\mathbf{w}_{2,t} &= [\mathrm{DBT}_{t-1}, \{\mathrm{INF}_{t-s}\}_{s=1}^4, \{\mathrm{GAP}_{t-s}\}_{s=1}^4, \{\mathrm{GSP}_{t-s}\}_{s=1}^4, \{\mathrm{M2G}_{t-s}\}_{s=1}^4, \{\mathrm{CMP}_{t-s}\}_{s=1}^4].
\end{aligned}
$$

Hence, conditional on $\mathbf{z}_t$, the state-space model (35)-(40) can be divided into two models: one for the interest rate equation, and another for the tax rate equation. The observation equations of each of the models are:

$$
\begin{aligned}
y_{1,t} &= \rho_R y_{1,t-1} + (1-\rho_R)\mathbf{x}_{1,t}\boldsymbol{\alpha}(z_t^m) + \upsilon_t^R && (41) \\
\mathbf{x}_{1,t} &= \mathbf{w}_{1,t}\boldsymbol{\Pi}_1 + \mathbf{v}_{1,t}, && (42)
\end{aligned}
$$

and

$$
\begin{aligned}
y_{2,t} &= \rho_\tau y_{2,t-1} + (1-\rho_\tau)\mathbf{x}_{2,t}\boldsymbol{\gamma}(z_t^f) + \upsilon_t^\tau && (43) \\
\mathbf{x}_{2,t} &= \mathbf{w}_{2,t}\boldsymbol{\Pi}_2 + \mathbf{v}_{2,t}. && (44)
\end{aligned}
$$

Here, $\boldsymbol{\Pi}_1$ and $\boldsymbol{\Pi}_2$ are conformable parameter matrices, and $\mathbf{v}_{i,t} \sim \text{iid } \mathbb{N}(0, \boldsymbol{\Psi}_i)$ for $i = 1, 2$. We introduce endogeneity in (41)-(42) and (43)-(44) by specifying

$$
\begin{aligned}
\upsilon_t^{R*} &= \mathbf{v}_{1,t}\delta_1 + e_t^R \\
\upsilon_t^{\tau*} &= \mathbf{v}_{2,t}\delta_2 + e_t^\tau,
\end{aligned}
$$

where $\upsilon_t^j = \sigma_{j,t}\upsilon_t^{j*}$ for $j = R, \tau$, and

$$
\begin{aligned}
e_t^R | y_{1,t-1}, \mathbf{v}_{1,t} &\sim \text{iid } \mathbb{N}\left(0, 1 - \delta_1'\boldsymbol{\Psi}_1\delta_1\right) \\
e_t^\tau | y_{2,t-1}, \mathbf{v}_{2,t} &\sim \text{iid } \mathbb{N}\left(0, 1 - \delta_2'\boldsymbol{\Psi}_2\delta_2\right).
\end{aligned}
$$

Let $\mathbf{y}_t = [y_{1,t}, y_{2,t}]'$ and $\mathbf{Y}_t = \{\mathbf{y}_s\}_{s=1}^t$. Appendix F shows how to obtain the conditional likelihood function of $\mathbf{Y}_T$. Details on the implementation of the Bayesian estimation of Θ_y, Θ_z, Θ_h, $\{\mathbf{z}_s\}_{s=0}^T$ and $\{\mathbf{h}_s\}_{s=0}^T$ appear in Appendix G.

3.2 Data

I use quarterly data from 1960:1 to 2008:3. The sample is not extended beyond 2008:3 to avoid having to deal with the binding of the zero lower bound (ZLB) of interest rates. It is still possible to estimate the interdependence between monetary and fiscal policy under the ZLB, but that is object of future research. Inflation is the percentage change over the last four quarters of the price level given by the GDP price deflator. The nominal interest rate is the quarterly average of the monthly rate of the 3-month T-Bill in the secondary market. The output gap is the log difference between real GDP and the Congressional Budget Office's measure of potential real GDP. M2 growth is the percentage change over the last four quarters of seasonally adjusted M2. Commodity price inflation is the percentage change over the last four quarters of the commodity price index. Government spending is the federal consumption expenditures and gross investment. These variables are obtained from FRED. Lagged debt is the average debt-output ratio over the previous four quarters, where debt is the TreasuryDirect par value of gross marketable federal debt held by the public. Tax net of transfers corresponds to the seasonally adjusted quarterly current receipts of the federal government from which the current transfer payments have been deducted. This variable is obtained from the NIPA Table 3.2.

4 Estimation Results

The choice of prior distributions, hyper parameters, means of $5,000$ draws from the posterior distribution after trimming the first $1,000,000$ out of $2,000,000$ draws and thinning every 200th draw[7], along with 90% confidence sets appear in Table 1. In order to keep the estimation relatively simple, we impose two restrictions that do not change the results quantitatively: 1) We assume that the output gap coefficients of both policy rules are not time varying, which allows us to focus on capturing the interdependence between monetary and fiscal policy making in terms of the co-movement of the Taylor rule coefficient of inflation and the tax rule coefficient of lagged debt, and 2) We assume that the location coefficients of the logistic policy rule coefficients, α_3^π and γ_3^b, are zero.

Figure 2 shows that the estimated model does acceptably well at explaining the observed time series of interest and tax rates. Figures 8 and 9 in Appendix H are indicative that the Monte Carlo Markov Chain converged. The Geweke (1991) test shows convergence of the Gibbs sampler at the 1% level of significance for all the estimated parameters.

4.1 Choice of Prior Distributions

To allow the Taylor rule coefficient on inflation to evolve between the active and passive monetary policy regimes, the lower bound of the logistic function that characterizes this coefficient must be roughly less than unity, while the upper bound must be roughly greater than unity. On the other hand, to allow the tax rule coefficient to evolve between the active and passive fiscal policy regimes, the lower bound of the logistic function that characterizes this coefficient must be roughly lower than the real interest rate charged on debt, and the upper bound greater than this threshold. There exist results in the literature about the values of these coefficients take in different regimes. Clarida et al. (2000) estimate that the inflation coefficient is 0.83 in the pre-Volcker era and 2.15 in the Volcker-Greenspan era. Lubik and Schorfheide (2004) find that the coefficient on inflation is estimated at 0.77 or

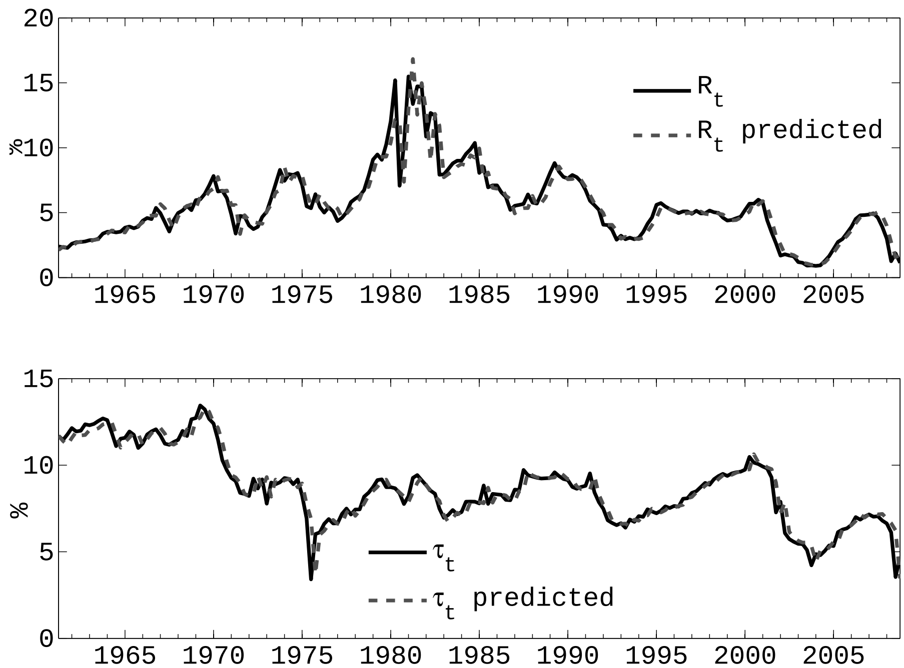
[7] The Raftery and Lewis (1992) diagnostic test determines that 676,113 draws from the posterior distribution should be taken to estimate the 50th percentile within 0.1 with 95% confidence level. It also determines that thinning to achieve an independent chain should occur every 100th draw.

Table 1: Results from the Bayesian Estimation

	Prior			Posterior	
Parameters	Density	Mean	SD	Mean	90% Conf. Set
α_0^π	Gamma	0.8	0.2	0.53	[0.36, 0.71]
α_1^π	Gamma	1.2	0.3	1.28	[0.68, 1.83]
α_2^π	Gamma	10	8	7.11	[0.75, 18.18]
α^y	Gamma	0.5	0.4	0.17	[0.07, 0.27]
ρ_R	Beta	0.9	0.05	0.91	[0.87, 0.96]
γ_0^b	Normal	-0.025	0.01	-0.024	[-0.04, -0.01]
γ_1^b	Gamma	0.125	0.03	0.13	[0.08, 0.18]
γ_2^b	Gamma	10	8	4.42	[0.05, 14.71]
γ^y	Gamma	0.5	0.4	0.79	[0.44, 1.22]
ρ_τ	Beta	0.9	0.05	0.94	[0.93, 0.96]
ρ_{z^m}	Beta	0.8	0.1	0.74	[0.48, 0.91]
ρ_{z^f}	Beta	0.8	0.1	0.80	[0.61, 0.93]
κ	TransformedBeta	0.5	0.25	0.18	[-0.13, 0.48]
z_0^m	Normal	0	1	-0.83	[-2.96, 1.31]
z_0^f	Normal	0	1	0.07	[-2.24, 2.39]
$\ln \sigma_R$				-2.17	[-2.56, -1.80]
ρ_{σ_R}				0.78	[0.65, 0.89]
η_R				0.58	[0.42, 0.76]
$\ln \sigma_{R,0}$				-3.18	[-5.05, -1.22]
$\ln \sigma_\tau$				-1.17	[-1.49, -0.91]
ρ_{σ_τ}				0.46	[0.24, 0.62]
η_τ				0.92	[0.68, 1.14]
$\ln \sigma_{\tau,0}$				-6.31	[-13.39, 0.62]

The parameters of the stochastic volatility processes have flat priors.

Figure 2: Observed and Predicted Series



0.89, depending on the prior used, in the pre-Volcker era, and 2.19 in the post-1982 era. Bianchi (2012) finds that the Taylor rule coefficient on inflation is estimated to be 0.94, 1.25, or 1.6 in a three-regime Markov-switching specification, while the tax rule coefficient on debt is estimated to be 0.0006, -0.0007, or -0.0036 in the same regime specification. Davig and Leeper (2011) estimate the Taylor rule coefficient on inflation to be 0.53 or 1.29 in a two-regime Markov-switching specification, while the tax rule coefficient on debt is -0.025 or 0.071. The specification in this document sets the parameter that fixes the lower bound of the logistic function of the inflation coefficient with a mean of 0.8, while the upper bound is set with a parameter whose mean is 2 (the sum of 0.8 and 1.2). With respect to the parameter that sets the lower bound of the logistic function of the debt coefficient, its mean is specified to be -0.025, and the parameter that sets the upper bound of the logistic function is specified to have a mean of 0.1 (the sum of -0.025 and 0.125).

The prior distribution specification of the transition coefficients of both logistic policy rule coefficients is set to have a mean of 10 with a wide range. Recall that the larger the coefficient, the more rapid the transition between states. Values of the coefficient greater than 20 imply a specification that mimics closely a Markov-switching transition. The choice of this prior distribution allows the coefficient to take low or high values.

The smoothing coefficients of the policy rules have prior distributions that are standard in terms of the persistence that they represent for interest and tax rates in the data.

A coefficient that deserves attention in terms of its prior distribution is the correlation coefficient between the shocks of the latent factors, that ultimately determines the degree of interdependence between monetary and fiscal policy making. The distribution chosen has its domain on [-1,1] and a mean of 0.5. Recall that a positive value of this parameter implies that monetary and fiscal policy are coordinated in such a way that they tend to deliver outcomes in the M or the F regime more frequently than outcomes in the indeterminacy or no solution regimes. The distribution chosen has enough dispersion to allow the likelihood function to play its role more prominently.

Finally, following Fernandez-Villaverde et al. (2011a) we do not specify prior distributions for the parameters of the stochastic volatility processes.

4.2 Parameter Estimates

The estimated parameters of the monetary and fiscal policy rule coefficients have expected magnitudes and signs. In particular, for the monetary policy rule, the inflation coefficient takes on values in (0.53, 1.81), while the output gap coefficient has a posterior mean of 0.17, below the values obtained by Clarida et al. (2000), around 0.27 and 0.93 for the two eras that they analyze, and below the values that Bianchi (2012) finds, around 0.58, 0.23, or 0.58 for the three regimes that he assumes. With respect to the fiscal policy rule, the debt coefficient takes on values in (-0.024, 0.106), while the output gap coefficient has a posterior mean of 0.79. Here the results with respect to the debt coefficient coincide closely with the results in Davig and Leeper (2011). The posterior mean of the tax rule coefficient on output is found to be larger than the results in the literature, where the values range between 0.11 and 0.5. The speed of transition of the policy rule coefficients is estimated to be higher for the inflation coefficient of the monetary policy rule than for the debt coefficient of the tax rule. This implies that the fiscal authority has a slower switching of taxes with respect to deviations of debt than the monetary authority does of the interest rate with respect to inflation. This result is consistent with the legislative and implementation lags of fiscal policymaking and the argument that these lags imply that policy takes time to influence macroeconomic aggregates.

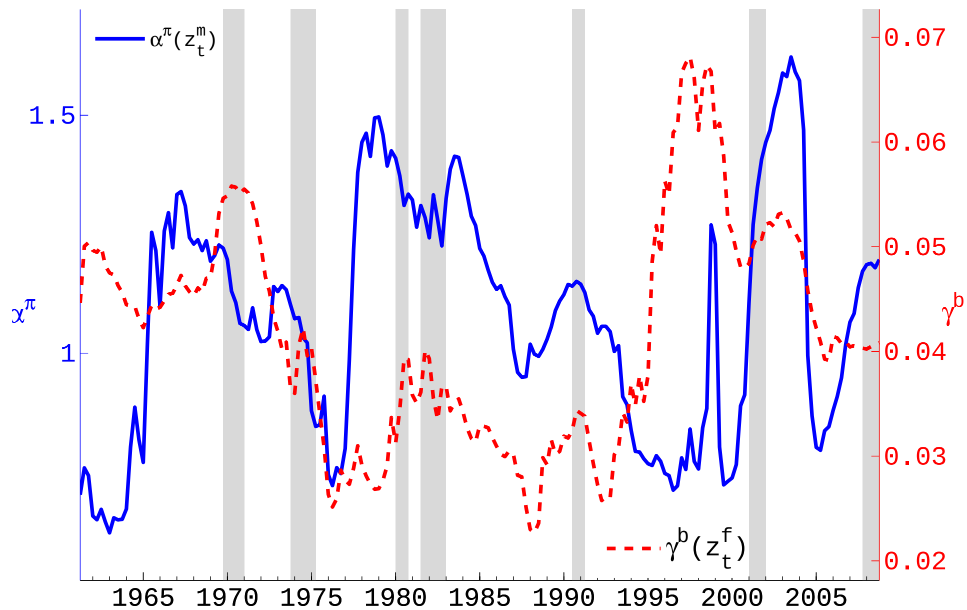
The latent factors show relatively high persistence, with the fiscal policy latent factor being more persistent than the monetary policy one. This result shows that fiscal policy adjustments to stabilize debt are likely to last longer than monetary policy adjustments to inflation deviations from target. This result finds an intuitive explanation if one realizes that monetary policy changes do not require as many institutional constraints as changes in fiscal policy do.

Finally, the correlation between the two latent factors has a estimated posterior mean of 0.18, and a 90% confidence set given by (-0.13, 0.48). This result implies that there is a degree of direct interactions between policies. Monetary tightenings to stabilize inflation tend to be accompanied by fiscal policy that stabilizes debt, while fiscal policy that deviates from debt stabilization tend to be accompanied by a loose monetary policy to keep debt stable.

4.3 Evolution of Policy Rule Coefficients

Figure 3 shows the posterior mean evolution of the monetary policy rule coefficient for inflation on the left axis, and of the fiscal policy rule coefficient for debt on the right axis. Three situations are apparent from this figure: 1) The fiscal policy rule coefficient does not show sudden changes as frequently as the monetary policy rule coefficient does, 2) The fiscal policy rule coefficient on debt was lower during most of the 1970s, the 1980s, and the first

Figure 3: Evolution of Policy Rule Coefficients and NBER Recession Periods



half of the 1990s, 3) Monetary policy was loose for a good portion of the 1990s,. I will start with a discussion about the evolution of each of the policy rule coefficients to match the econometric results with the narrative about policy making.

The evolution of the monetary policy rule coefficient reveals that the Federal Reserve Board reduced its reaction to inflation during at least a few quarters of every NBER recession, except the recession of 2001. The graph shows that during the 1960s, monetary policy was tight during the second half of the decade as Davig and Leeper (2011), Bianchi (2010), Eo (2009) and Fernandez-Villaverde et al. (2010) find. During the 1970s monetary policy did not actively fight inflation, with the exception of a few periods in the first half and at the end of the decade. With the exception of Boivin (2006), who finds that monetary policy was tight during the first half of the 1970s, all the studies in the literature find a passive monetary policy during this decade. At the end of the 1970s, and after the appointment of Fed Chairman Paul Volcker, monetary policy switched rapidly to fight inflation. This abrupt switch is also found by Davig and Doh (2009), Eo (2009) and Bianchi (2010) who find, based on estimations of Markov-switching policy rule coefficients, that the active monetary policy periods started around, or a little earlier than, the mid 1980s. The graph shows that monetary policy had a "hawkish" regime during most of the 1980s. The 1990s show monetary policy that was actively fighting inflation for a few periods at the beginning and at the end of the decade. The only study that supports a finding of this nature is Fernandez-Villaverde et al. (2010).[8] The 2000s show that monetary policy increased its reaction to inflation until 2005, when it sharply decreases it. The empirical evidence is divided with respect to this result: On one hand Eo (2009), Davig and Doh (2009) and Bianchi (2010) find that monetary policy was

[8]Kim and Nelson (2006a) find that their confidence interval starts including the passive monetary policy region at the beginning of the 1990s.

actively fighting inflation, while on another Fernandez-Villaverde et al. (2010) and Davig and Leeper (2011) find the opposite. It has to be noted that during the first half of the 2000s the Federal Reserve Board was aggressively lowering interest rates while inflation did not necessarily showed significant increases.

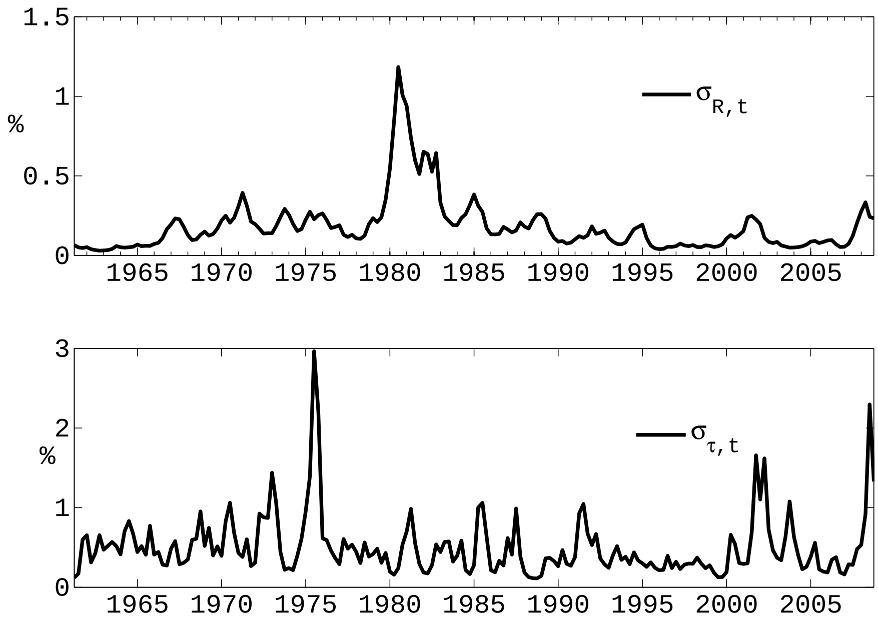
The evolution of the fiscal policy rule coefficient shows less variations than the evolution of the monetary policy rule coefficient. Compared to the reaction of monetary policy to recessions, the reaction of fiscal policy is not as strongly related to recessions, with the exception of the first two decades of the sample where fiscal policy tried to loosen its reaction to debt whenever a recession hit the economy. The tax incentives of President Kennedy lowered the fiscal reaction to debt in the early 1960s, with an increase in the second half of the decade due to President Johnson's policies to face the Vietnam war. The 1970s saw a loose fiscal policy due to the fiscal incentives to face the recessions of the beginning and the middle of the decade. This loose fiscal policy stayed in place during the last years of President Carter in the early 1980s to face the recession of those years. President Reagan continued with an even looser fiscal policy during the first years of his mandate. President Bush, in early 1990s, basically continued with his predecessor's policies with no substantial changes in the reaction of fiscal policy with respect to debt. President Clinton starts a new regime where taxes start to respond strongly to debt in order to decrease it, taking debt from nearly 44% of GDP in 1995 to nearly 27% in 2001. President Bush's cuts imply a reduction in the debt coefficient of the tax policy rule as expected.

It is important to point out that the sample was not extended beyond the third quarter of 2008 because the zero lower bound constraint starts to bind after that quarter. Different econometric techniques and models are required to deal with a situation of this nature, in particular in a scenario of policy interdependence, and that constitutes an extension of current research of this paper.

4.4 Evolution of Stochastic Volatilities

Figure 4 shows the evolution of the estimated stochastic volatility for the interest and tax rate rules. The results show that the volatility of interest rates was significantly higher during the beginning of the 1980s due to the important change in conducting monetary policy during those years. Also, volatility is significantly lower after the second half of the 1980s and stays lower until the end of the sample period. This result has also been found by Justiniano and Primiceri (2008), Fernandez-Villaverde et al. (2010), Davig and Leeper (2006), Davig and Doh (2009) and Bianchi (2010). On the other hand, fiscal policy volatility shows spikes in the mid 1970s, the first half of the 2000s, and the beginning of the great recession. All these events are triggered for some kind of fiscal stimulus. There is not significant increase in volatility during the years of President Clinton. It is worth noticing that the volatility of monetary policymaking (ρ_{σ_R}) is significantly more persistent than the volatility of fiscal policymaking (ρ_{σ_τ}).

Figure 4: Evolution of Stochastic Volatilities



4.5 Nonlinear Impulse-Response Analysis for the New Keynesian Model

With the estimated parameters of the policy rule coefficients and the parameterization in Section 3.2, this section performs policy experiments with the new Keynesian model with time-varying policy rule coefficients solved with the proposed solution method. The new Keynesian model requires the specification of aditional parameters to perform the impulse-response analysis. In particular, it requires the specification of the markup in steady state and the price adjustment cost parameter. Keen and Wang (2007) show that, given a steady state markup and a fraction of firms that re-optimize each period, there is a corresponding value for the adjustment cost parameter. Here, we set the markup to 20% and the fraction of re-optimizing firms to 25% each period (a firm re-optimizes every 12 months). These values correspond approximately to $\theta = 6$ and $\phi = 60$. Additional parameters that need to be specified are: 1) δ, the steady state gross quarterly rate of output growth, which is set to 1.0081, the average over the sample period that implies a steady state annual growth rate of approximately 3.25%, which is the average over the sample period; 2) Π, the steady state gross quarterly rate of price inflation, which is set to 1.0084, the average over the sample period that implies a steady state annual inflation of approximately 3.4%; 3) b, the steady state level of debt to output, which is set to 0.3354, the average over the sample period; 4) ζ, the ratio of government spending to output is set to 0.081, also the average over the sample period; 5) $1/vb$, the ratio of outstanding money balances to debt, is set to 0.2 following Kim (2003); β is set to 0.995.

Figure 5 shows the response of the variables of the model to a 1% increase in the interest rate and to a 1% increase in the tax rate. Responses are calculated under three scenarios:

1) Starting at the AM/PF regime and staying there forever, 2) Starting at the PM/AF regime and staying there forever, and 3) Starting at the unconditional mean of the policy rule coefficients and averaging out responses across future random draws of coefficients.

4.5.1 A Monetary Contraction

The effects of a monetary contraction in a new Keynesian model under the AM/PF regime are well known: An open market operation that sells debt to households, and that is expected to be corrected in the future via higher taxes, does not change their wealth and only increases the nominal interest rate. With sticky prices, the real interest rate increases reducing consumption and subsequently output, which reduces inflation. As the nominal interest rate decreases as well as the real interest rate, output returns to steady state as well as inflation. Taxes decrease due to the automatic stabilizers effect and then start to increase due to their Ricardian response to lagged debt, which is now higher. Given the persistence in taxes, the present value of expected future surpluses increases and slowly returns to steady state, while the present value of expected future seignorage decreases at impact and then does not experiment major variations due to the short and not pronounced changes in inflation.

In the PM/AF regime, the effects are as follows: An open market operation that sells debt to households, and that is not expected to be corrected in the future via higher taxes, increases agents' wealth. With sticky prices, the increase in the nominal interest rate still increases the real interest rate, making output and inflation decrease at impact. Then, higher wealth and decreasing real rates increase output above its steady state level until the wealth effect is corrected via higher prices. The "price puzzle" is present in this policy experiment as it was in the Fisherian model.[9] Taxes only react to the output gap, hence they decrease during the first quarters and then increase above steady state until slowly returning to it. This makes the present value of expected future surpluses increase at impact, while higher and persistent inflation increases the present value of expected future seignorage after a small decrease at impact due to the decrease of the demand of real money balances.

With the economy starting at the mean of the stationary distribution of policy rule coefficients, the effects of a monetary contraction are between the two previously described regimes. Agents' expectations incorporate the behavior of the economy under the AM/PF and PM/AF regimes so that: 1) There are still lower output and inflation at impact, and 2) There is still a wealth effect of higher debt in their hands that is not expected to be completely corrected in the future via higher taxes. As long as the economy does not start at one of the limiting regimes and stays there forever, the solution method allows to always have some wealth effect derived from the PM/AF regime that will influence the economy's response to a monetary contraction. As in the Fisherian model, a monetary contraction decreases inflation at impact, but the long-run effect is an increase of inflation above steady state, or the "stepping on a rake" phenomenon.

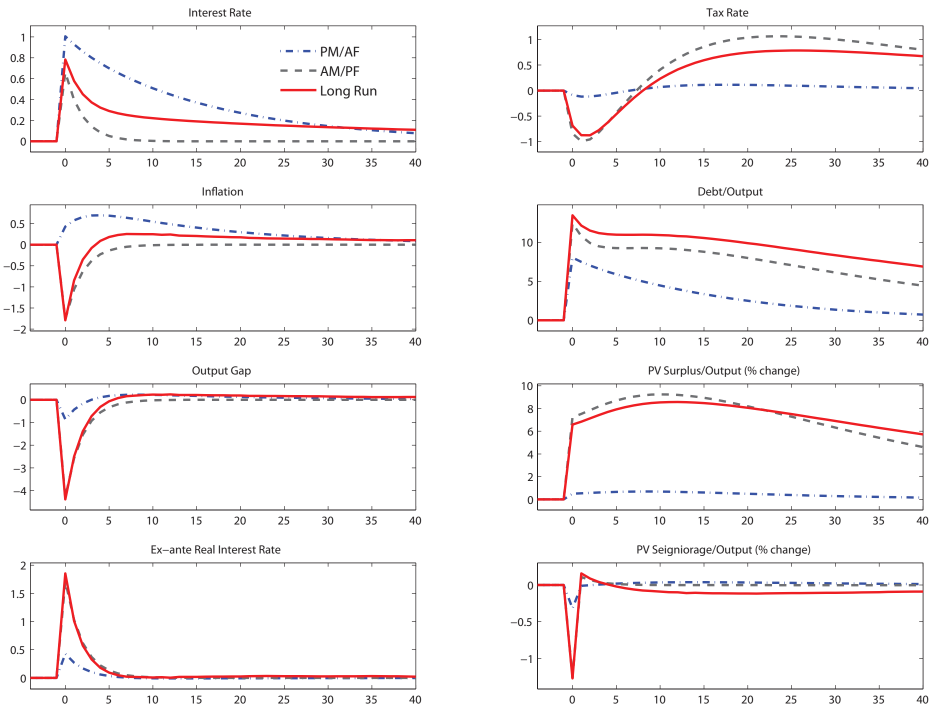
[9] To determine the initial response of inflation under the PM/AF regime, an expression like equation (**??**) can be used. In this case, the expression has to incorporate the effect of output growth:

$$R_{-1}b_{-1}\frac{1}{\Pi_0}\frac{1}{\Delta Y_0} = \mathbb{E}_0 \sum_{t=0}^{\infty} \mathrm{MRS}_{0,t}\left(s_t + m_t\right),$$

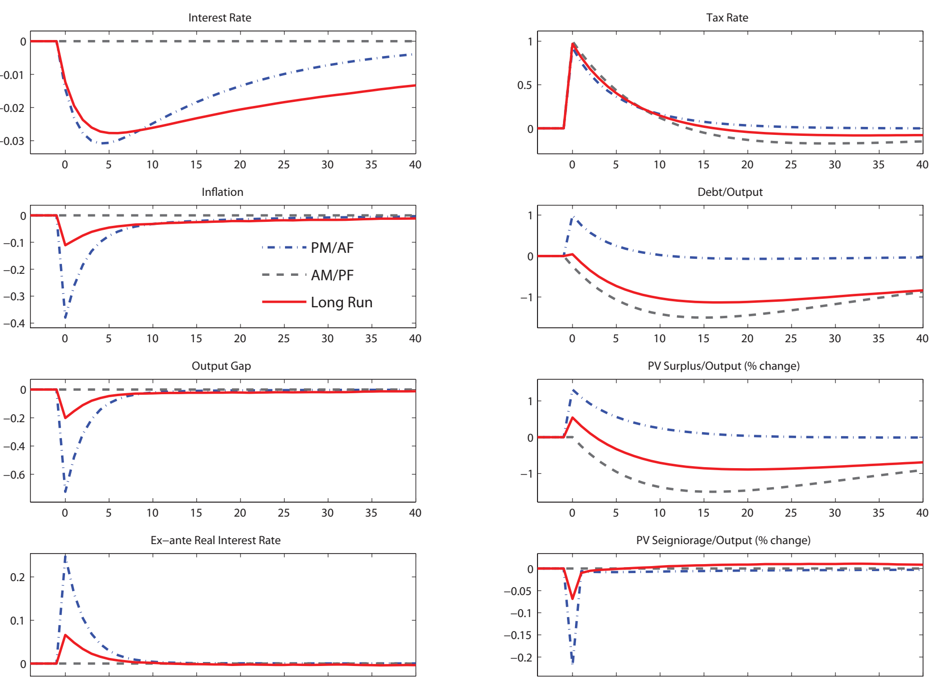
where s_t and m_t are in output terms.

Figure 5: Impulse-Response Functions

(a) Response to a 1% Increase in the Interest Rate



(b) Response to a 1% Increase in the Tax Rate



4.5.2 A Fiscal Contraction - Tax increase

In the AM/PF regime, a tax increase that retires debt does not have any effect on output or inflation due to the Ricardian nature of the equilibrium: higher taxes today are expected to be fully compensated in the future, with no wealth effect on households. The absence of change in wealth can be seen from the absence of change in the present value of expected future surpluses and seignorage at impact. Taxes decrease after impact following the debt reduction and, given their persistence and the further reduction of debt, locate below steady state at long horizons. Debt reduces anticipating the decrease in the present value of expected future surpluses. There are not changes in seignorage due to unchanged inflation.

A tax increase in the PM/AF regime leaves agents with reduced wealth since the increase is not expected to be corrected in the future. The decrease in wealth reduces demand for goods and inflation. Since the nominal interest rate does not respond strongly to the change in inflation, the real interest rate increases decreasing consumption and output. The present value of expected future surpluses increases due to the higher and persistent taxes that will not be corrected in the future. The present value of expected future seignorage reduces at impact due to lower inflation. Overall, the change in the expected present value of government net receipts implies that debt increases its value at impact and then returns to steady state.

When the economy starts at the mean of the stationary distribution of policy rule coefficients, again the effects are between the two previously described regimes. That is, a change in taxes has real effects due to the only partial correction that is implied by the active fiscal regime. Also, as long as the economy does not start at one of the limiting regimes and stays there forever, the dynamics of the economy will reflect the wealth effect of higher or lower taxes.

Appendix

A Model Setup

The representative household solves the following problem:

$$\max_{\{C_t, Mt/P_t, B_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t/A_t)^{1-\sigma}}{1-\sigma} + \chi_M \log \left(M_t^d / P_t \right) - \chi_H \frac{H_t^{1+\varphi}}{1+\varphi} \right)$$

subject to

$$\begin{aligned}
& C_t + \frac{M_t^d}{P_t} + \frac{B_t}{P_t} + \frac{T_t}{P_t} \leq H_t \frac{W_t}{P_t} + \frac{D_t}{P_t} + \frac{M_{t-1}^d}{P_t} + R_{t-1} \frac{B_{t-1}}{P_t} \text{ for } t \geq 0, \\
& \frac{M_{-1} + R_{-1} B_{-1}}{P_{-1}} \text{ given,} \\
& \lim_{t \to \infty} MRS_{0,t} \frac{M_t + B_t}{P_t} = 0,
\end{aligned}$$

where $MRS_{0,t}$ denotes the marginal rate of substitution between period 0 and period t. The necessary first order conditions are:

$$C_t \quad : \quad \frac{1}{A_t} \left(\frac{C_t}{A_t} \right)^{-\sigma} - \lambda_t = 0 \tag{45}$$

$$H_t \quad : \quad -\chi_H H_t^{\varphi} + \lambda_t \frac{W_t}{P_t} = 0 \tag{46}$$

$$\frac{M_t^d}{P_t} \quad : \quad \chi_M \left(\frac{M_t^d}{P_t} \right)^{-1} - \lambda_t + \beta \mathbb{E}_t \lambda_{t+1} \frac{P_t}{P_{t+1}} = 0 \tag{47}$$

$$B_t \quad : \quad -\frac{\lambda_t}{P_t} + \beta R_t \mathbb{E}_t \frac{\lambda_{t+1}}{P_{t+1}} = 0, \tag{48}$$

$$\lambda_t \quad : \quad C_t + \frac{M_t^d}{P_t} + \frac{B_t}{P_t} + \frac{T_t}{P_t} - H_t \frac{W_t}{P_t} - \frac{D_t}{P_t} - \frac{M_{t-1}^d}{P_t} - R_{t-1} \frac{B_{t-1}}{P_t} = 0 \tag{49}$$

where λ_t is the Lagrange multiplier associated to the budget constraint at time t.

From (45) and (48),

$$1 = \beta R_t \mathbb{E}_t \left(\frac{C_t/A_t}{C_{t+1}/A_{t+1}} \right)^{\sigma} \frac{A_t}{A_{t+1}} \frac{P_t}{P_{t+1}}. \tag{50}$$

From (45) and (46)

$$\chi_H H_t^{\varphi} A_t \left(\frac{C_t}{A_t} \right)^{\sigma} = \frac{W_t}{P_t}. \tag{51}$$

From (45), (47) and (48),

$$\frac{M_t}{P_t} = \chi_M A_t \left(\frac{C_t}{A_t}\right)^{\sigma} \left(\frac{R_t}{R_t - 1}\right). \tag{52}$$

Profits of intermediate firm j are given by

$$\frac{D_t(j)}{P_t} = \frac{P_t(j)}{P_t} Y_t(j) - \frac{W_t}{P_t} L_t(j) - \frac{\phi}{2}\left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1\right)^2 Y_t. \tag{53}$$

Substituting (2) and (3) in (53) yields

$$\frac{D_t(j)}{P_t} = \left[\left(\frac{P_t(j)}{P_t}\right)^{1-\theta_t} - \psi_t \left(\frac{P_t(j)}{P_t}\right)^{-\theta_t} - \frac{\phi}{2}\left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1\right)^2\right] Y_t.$$

Then, intermediate firm j chooses $P_t(j)$ to maximize

$$\mathbb{E}_t \sum_{k=0}^{\infty} \mathrm{MRS}_{t,t+k} \left[\left(\frac{P_t(j)}{P_t}\right)^{1-\theta_t} - \psi_t \left(\frac{P_t(j)}{P_t}\right)^{-\theta_t} - \frac{\phi}{2}\left(\frac{P_t(j)}{\Pi P_{t-1}(j)} - 1\right)^2\right] Y_t.$$

The first order condition to this maximization problem is

$$\begin{aligned} 0 \;=\; & \lambda_t Y_t \left[(1-\theta_t)\left(\frac{P_t(j)}{P_t}\right)^{-\theta_t} \frac{1}{P_t} + \theta_t \frac{\psi_t}{P_t}\left(\frac{P_t(j)}{P_t}\right)^{-\theta_t - 1} - \phi\left(\frac{P_t(j)}{\Pi P_{t-1}} - 1\right)\frac{1}{\Pi P_{t-1}(j)}\right] + \\ & + \beta \lambda_{t+1} Y_{t+1} \phi \left(\frac{P_{t+1}(j)}{\Pi P_t} - 1\right) \frac{P_{t+1}(j)}{\Pi P_t(j)^2}. \end{aligned} \tag{54}$$

In a symmetric equilibrium, $P_t(j) = P_t$, $L_t(j) = L_t$ and $Y_t(j) = Y_t$, hence

$$\frac{D_t(j)}{P_t} = \frac{D_t}{P_t} = \left[1 - \psi_t - \frac{\phi}{2}\left(\frac{P_t}{\Pi P_{t-1}} - 1\right)^2\right] Y_t. \tag{55}$$

In equilibrium, $M_t^d = M_t^s = M_t$, $B_t = 0$ and $H_t = L_t$. Then, (5), (55) and (49) imply (8). In the symmetric equilibrium, substituting (45) into (54) yields (9). Finally, the symmetric equilibrium yields (11), (52) is (12), and (50) is (10).

Before proceeding to the log-linearization of the model, it is convenient to write (5) in terms of nominal output. The resulting expression is

$$b_t = 1 - \frac{1}{g_t} - \tau_t - \frac{1}{v_t} + \frac{1}{v_{t-1}} \frac{1}{\Pi_t} \frac{1}{\Delta Y_t} + R_{t-1} b_{t-1} \frac{1}{\Pi_t} \frac{1}{\Delta Y_t}, \tag{56}$$

where $\Delta Y_t = Y_t / Y_{t-1}$.

To obtain (13), start with (11) and recall the definition of $\psi_T = W_t/P_tA_t$ to get

$$\psi_t = \chi_H L_t^{\varphi} A_t \left(\frac{C_t}{A_t}\right)^{\sigma}.$$

In the symmetric equilibrium, from (3), $Y_t = A_tL_t$, and without price rigidities, $\psi_t = \frac{\theta_t-1}{\theta_t}$ and $C_t = Y_t/g_t$, hence

$$\frac{\theta_t - 1}{\theta_t} = \chi_H \left(\frac{Y_t^*}{A_t}\right)^{\varphi+\sigma} g_t^{-\sigma}.$$

Once the variables have been de-trended by dividing them by A_t, the absence of shocks yields (14) from (10), (17) from (12), (15) from combining (8) and (13), (16) from (15) and $c_t = y_t/g_t$, and (18) from (56) with $\Delta y = \delta$.

B Matrices for Solving the Model

The matrices in system (29)-(31) are given by

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & -1/\sigma & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(\theta-1)(\varphi+\sigma)/\phi \end{bmatrix}, \qquad \mathbf{J} = \begin{bmatrix} 1 & 1/\sigma \\ 0 & \beta \end{bmatrix},$$

$$\mathbf{K} = \begin{bmatrix} -1 & 0 \\ (\theta-1)(\varphi+\sigma)/\phi & -1 \end{bmatrix}, \qquad \mathbf{M} = \begin{bmatrix} 1-\rho_g & 0 & \rho_\nu/\sigma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{A}(z_t) = \begin{bmatrix} -1 & 0 & 1/(R-1) & 0 & 0 & 0 & 0 \\ 1/vb & -1 & 0 & -\tau/b & -(1/bv\Pi\delta + 1/\beta) & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -(1-\rho_R)\alpha^y(z_t^m) \\ 0 & 0 & 0 & -1 & 0 & 0 & -(1-\rho_\tau)\gamma^y(z_t^f) \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

$$\mathbf{B}(z_t^f) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1/v\Pi\delta b & 1/\beta & 1/\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_R & 0 & 0 & 0 & 0 \\ 0 & (1-\rho_\tau)\gamma^b(z_t^f) & 0 & \rho_\tau & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{C}(z_t) = \begin{bmatrix} 1-\sigma & 0 \\ 0 & -(1/bv\Pi\delta + 1/\beta) \\ (1-\rho_R)\alpha^y(z_t^m) & (1-\rho_R)\alpha^\pi(z_t^m) \\ (1-\rho_\tau)\gamma^y(z_t^f) & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} \sigma & 0 & 0 & 0 & 0 \\ 1/bg & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \sigma/(\varphi+\sigma) + 1/(\varphi+\sigma) & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{N} = \begin{bmatrix} \rho_g & 0 & 0 & 0 & 0 \\ 0 & \rho_\theta & 0 & 0 & 0 \\ 0 & 0 & \rho_\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

C Obtaining the Coefficients of the Logistic Functions

Each of the elements in the matrices $\mathbf{P}(z_t), \mathbf{Q}(z_t), \mathbf{R}(z_t), \mathbf{S}(z_t)$ have the bivariate logistic functional form (34), which is reproduced here for convenience:

$$F(\mathbf{z}_t) = \frac{\left(F_{0m} + \frac{F_{1m}}{1+\exp\left(-F_{2m}(z_t^m - F_{3m})\right)}\right)\left(1 + \frac{F_{1f}}{1+\exp\left(-F_{2f}\left(z_t^f - F_{3f}\right)\right)}\right)}{1 - F_4 \frac{\exp\left(-F_{2m}(z_t^m - F_{3m})\right)}{1+\exp\left(-F_{2m}(z_t^m - F_{3m})\right)} \frac{\exp\left(-F_{2f}\left(z_t^f - F_{3f}\right)\right)}{1+\exp\left(-F_{2f}\left(z_t^f - F_{3f}\right)\right)}}.$$

The shape of a bivariate logistic function is shown in Figure 6. The bivariate logistic function yields univariate logistic functions (either of z_t^m or z_t^f) in the margins, i.e. when $z_t^m \to l$ or when $z_t^f \to l$, for $l = +\infty, -\infty$. This feature of the bivariate logistic function gives high flexibility to encompass a wide variety of linear and nonlinear combinations involving this function.

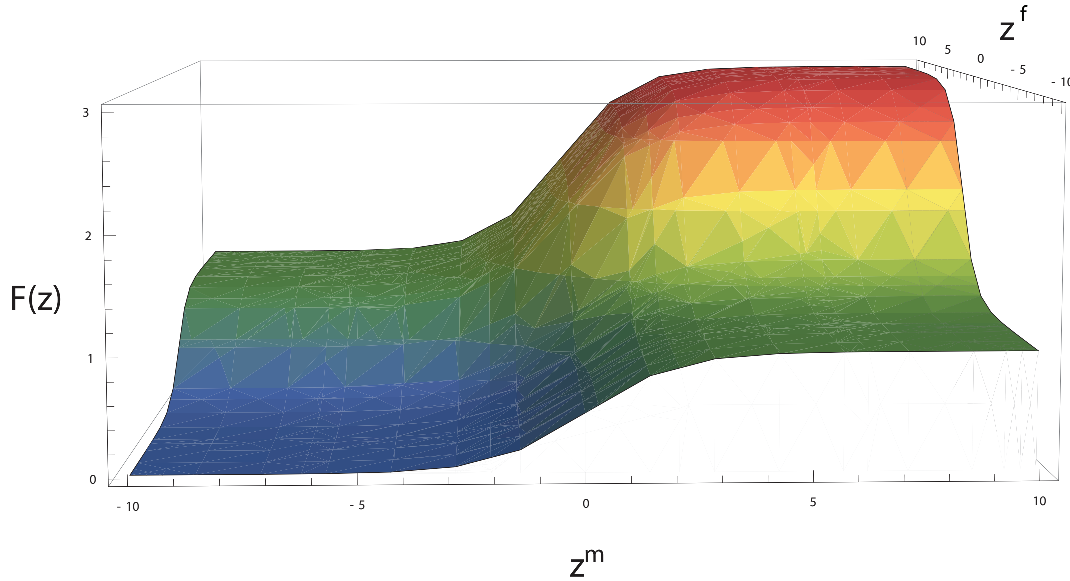
To avoid computational costs, we find the solutions to the coefficients in $\mathbf{R}(z_t)$ and $\mathbf{S}(z_t)$ only, solving for inflation and output, leaving the the full solution of the model to be accounted for the structural equations of the state variables (21)-(28). Given the structure of the vector of shocks, $\mathbf{S}(z_t)$ is a matrix with 10 distinct elements. Given the structure of the vector of state variables, $\mathbf{R}(z_t)$ is a matrix with 10 distinct elements as well, although it has 14 entries. Therefore, in total there are $20 \times 8 = 160$ coefficients of the logistic functions to be found to obtain a solution.

C.1 Finding F_{0m}, F_{1m}, F_{0f} and F_{1f} given F_4

Notice that the time-varying policy rule coefficients have the following limiting combinations (bounds):

- $\lim_{z_t^m \to -\infty} \alpha^\pi(z_t^m) = \alpha_0^\pi$, $\lim_{z_t^m \to -\infty} \alpha^y(z_t^m) = \alpha_0^y$ and $\lim_{z_t^f \to -\infty} \gamma^b(z_t^f) = \gamma_0^b$, $\lim_{z_t^f \to -\infty} \gamma^y(z_t^f) = \gamma_0^y$,
- $\lim_{z_t^m \to \infty} \alpha^\pi(z_t^m) = \alpha_0^\pi + \alpha_1^\pi$, $\lim_{z_t^m \to \infty} \alpha^y(z_t^m) = \alpha_0^y + \alpha_1^y$ and $\lim_{z_t^f \to \infty} \gamma^b(z_t^f) = \gamma_0^b + \gamma_1^b$, $\lim_{z_t^f \to \infty} \gamma^y(z_t^f) = \gamma_0^y + \gamma_1^y$,

Figure 6: Bivariate Logistic Function



- $\lim\limits_{z_t^m \to \infty} \alpha^\pi(z_t^m) = \alpha_0^\pi + \alpha_1^\pi$, $\lim\limits_{z_t^m \to \infty} \alpha^y(z_t^m) = \alpha_0^y + \alpha_1^y$ and $\lim\limits_{z_t^f \to -\infty} \gamma^b(z_t^f) = \gamma_0^b$, $\lim\limits_{z_t^f \to -\infty} \gamma^y(z_t^f) = \gamma_0^y$, and
- $\lim\limits_{z_t^m \to -\infty} \alpha^\pi(z_t^m) = \alpha_0^\pi$, $\lim\limits_{z_t^m \to -\infty} \alpha^y(z_t^m) = \alpha_0^y$ and $\lim\limits_{z_t^f \to \infty} \gamma^b(z_t^f) = \gamma_0^b + \gamma_1^b$, $\lim\limits_{z_t^f \to \infty} \gamma^y(z_t^f) = \gamma_0^y + \gamma_1^y$.

Also, notice that $F(z_t)$ has the following limiting expressions:

- $\lim\limits_{z_t^m, z_t^f \to -\infty} F(z_t) = \dfrac{F_{0m}}{1 - F_4}$,
- $\lim\limits_{z_t^m, z_t^f \to \infty} F(z_t) = (F_{0m} + F_{1m})(1 + F_{1f})$,
- $\lim\limits_{z_t^m \to \infty, z_t^f \to -\infty} F(z_t) = F_{0m} + F_{1m}$,
- $\lim\limits_{z_t^m \to -\infty, z_t^f \to \infty} F(z_t) = F_{0m}(1 + F_{1f})$.

Hence, given F_4, to obtain F_{0m}, F_{1m} and F_{1f}, we solve the constant-coefficient versions of the model four times using Uhlig (1998), at the four different limiting combinations of the latent factors z_t^m and z_t^f. However, two of the solutions for $\mathbf{R}(z_t)$ and $\mathbf{S}(z_t)$ are equal in the limits mentioned above, leaving three equations to find three coefficients. The reason is that under active monetary policy, when $z_t^m \to \infty$, the only difference that fiscal policy makes has to do with the dynamics of debt (there is not bounded solution under active fiscal policy, when $z_t^m \to \infty$) and not with the dynamics of inflation or output.

C.2 Finding $F_{2m}, F_{2f}, F_{2mf}, F_{2fm}$, $F_{3m}, F_{3f}, F_{3mf}, F_{3fm}$

Substitute (32)-(33) in (29)-(31) to obtain

$$\left[\mathbf{A}(z_t)\mathbf{P}(z_t) + \mathbf{C}(z_t)\mathbf{R}(z_t) + \mathbf{B}(z_t^f)\right] k_{t-1} + [\mathbf{A}(z_t)\mathbf{Q}(z_t) + \mathbf{C}(z_t)\mathbf{S}(z_t) + \mathbf{D}]\, u_t = 0,$$
$$\left\{\left[\mathbf{J}\bar{\mathbf{R}}(z_t) + \mathbf{G}\right]\mathbf{P}(z_t) + \mathbf{K}\mathbf{R}(z_t)\right\} k_{t-1} + \left\{\left[\mathbf{J}\bar{\mathbf{R}}(z_t) + \mathbf{G}\right]\mathbf{Q}(z_t) + \mathbf{J}\bar{\mathbf{S}}(z_t)\mathbf{N} + \mathbf{K}\mathbf{S}(z_t) + \mathbf{M}\right\} u_t = 0,$$

where $\bar{\mathbf{R}}(z_t) \equiv \mathbb{E}_t\mathbf{R}(z_{t+1})$ and $\bar{\mathbf{S}}(z_t) \equiv \mathbb{E}_t\mathbf{S}(z_{t+1})$. By the undetermined coefficients method, we have

$$\begin{aligned}
\mathbf{A}(z_t)\mathbf{P}(z_t) + \mathbf{C}(z_t)\mathbf{R}(z_t) + \mathbf{B}(z_t^f) &= 0\\
\mathbf{A}(z_t)\mathbf{Q}(z_t) + \mathbf{C}(z_t)\mathbf{S}(z_t) + \mathbf{D} &= 0\\
\left[\mathbf{J}\bar{\mathbf{R}}(z_t) + \mathbf{G}\right]\mathbf{P}(z_t) + \mathbf{K}\mathbf{R}(z_t) &= 0\\
\left[\mathbf{J}\bar{\mathbf{R}}(z_t) + \mathbf{G}\right]\mathbf{Q}(z_t) + \mathbf{J}\bar{\mathbf{S}}(z_t)\mathbf{N} + \mathbf{K}\mathbf{S}(z_t) + \mathbf{M} &= 0.
\end{aligned}$$

Then, solving for $\mathbf{R}(z_t)$ and $\mathbf{S}(z_t)$, we have

$$[\mathbf{K} + [\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(\mathbf{z}_t)][-\mathbf{A}(\mathbf{z}_t)]^{-1}\mathbf{C}(\mathbf{z}_t)]\mathbf{R}(\mathbf{z}_t) = -[\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(\mathbf{z}_t)][-\mathbf{A}(\mathbf{z}_t)]^{-1}\mathbf{B}(z_t^f) \tag{57}$$
$$[\mathbf{K} + [\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(\mathbf{z}_t)][-\mathbf{A}(\mathbf{z}_t)]^{-1}\mathbf{C}(\mathbf{z}_t)]\mathbf{S}(\mathbf{z}_t) = -\mathbf{J}\bar{\mathbf{S}}(\mathbf{z}_t)\mathbf{N} - \mathbf{M} - [\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(\mathbf{z}_t)][-\mathbf{A}(\mathbf{z}_t)]^{-1}\mathbf{D}. \tag{58}$$

Let

$$\begin{aligned}
\mathbf{T}(\mathbf{z}_t) &\equiv \mathbf{K} + [\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(\mathbf{z}_t)][-\mathbf{A}(\mathbf{z}_t)]^{-1}\mathbf{C}(\mathbf{z}_t)\\
\mathbf{U}(\mathbf{z}_t) &\equiv -[\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(\mathbf{z}_t)][-\mathbf{A}(\mathbf{z}_t)]^{-1}\mathbf{B}(z_t^f)\\
\mathbf{V}(\mathbf{z}_t) &\equiv -\mathbf{J}\bar{\mathbf{S}}(\mathbf{z}_t)\mathbf{N} - \mathbf{M} - [\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(\mathbf{z}_t)][-\mathbf{A}(\mathbf{z}_t)]^{-1}\mathbf{D}.
\end{aligned}$$

Then, using the relevant elements of $\mathbf{R}(z_t)$ and $\mathbf{S}(z_t)$, we can write

$$\begin{bmatrix}\mathbf{T}(\mathbf{z}_t) & 0\\ 0 & \mathbf{T}(\mathbf{z}_t)\end{bmatrix}\begin{bmatrix}\mathbf{R}(\mathbf{z}_t)\\ \mathbf{S}(\mathbf{z}_t)\end{bmatrix} = \begin{bmatrix}\mathbf{U}(\mathbf{z}_t)\\ \mathbf{V}(\mathbf{z}_t)\end{bmatrix}. \tag{59}$$

Given F_{0m}, F_{1m} and F_{1f}, $F(\mathbf{z}_t)$ takes the following expressions that include F_{2m}, F_{2f}, F_{3m} and F_{3f}:

- $\lim_{z_t^f \to \infty} F(0, z_t^f) = (1 + F_{1f})(F_{0m} + \frac{F_{1m}}{1 + \exp(F_{2m}F_{3m})})$
- $\lim_{z_t^m \to \infty} F(z_t^m, 0) = (F_{0m} + F_{1m})(1 + \frac{F_{1f}}{1 + \exp(F_{2f}F_{3f})})$
- $\frac{\partial}{\partial z_t^m} \lim_{z_t^f \to \infty} F(z_t^m, z_t^f)\Big|_{z_t^m = 0} = (1 + F_{1f})\frac{F_{1m}F_{2m}\exp(F_{2m}F_{3m})}{(1 + \exp(F_{2m}F_{3m}))^2}$
- $\frac{\partial}{\partial z_t^f} \lim_{z_t^m \to \infty} F(z_t^m, z_t^f)\Big|_{z_t^f = 0} = (F_{0m} + F_{1m})\frac{F_{1f}F_{2f}\exp(F_{2f}F_{3f})}{(1 + \exp(F_{2f}F_{3f}))^2}$.

Then, to find F_{2m}, F_{2f}, F_{3m} and F_{3f} we solve the following system of equations for the relevant elements of $\mathbf{R}(z_t)$ and $\mathbf{S}(z_t)$:

$$\begin{bmatrix} \lim\limits_{z_t^f \to l} \mathbf{T}(0, z_t^f) & 0 \\ 0 & \lim\limits_{z_t^f \to l} \mathbf{T}(0, z_t^f) \end{bmatrix} \begin{bmatrix} \lim\limits_{z_t^f \to l} \mathbf{R}(0, z_t^f) \\ \lim\limits_{z_t^f \to l} \mathbf{S}(0, z_t^f) \end{bmatrix} = \begin{bmatrix} \lim\limits_{z_t^f \to l} \mathbf{U}(0, z_t^f) \\ \lim\limits_{z_t^f \to l} \mathbf{V}(0, z_t^f) \end{bmatrix},$$

$$\begin{bmatrix} \lim\limits_{z_t^m \to l} \mathbf{T}(z_t^m, 0) & 0 \\ 0 & \lim\limits_{z_t^m \to l} \mathbf{T}(z_t^m, 0) \end{bmatrix} \begin{bmatrix} \lim\limits_{z_t^m \to l} \mathbf{R}(z_t^m, 0) \\ \lim\limits_{z_t^m \to l} \mathbf{S}(z_t^m, 0) \end{bmatrix} = \begin{bmatrix} \lim\limits_{z_t^m \to l} \mathbf{U}(z_t^m, 0) \\ \lim\limits_{z_t^m \to l} \mathbf{V}(z_t^m, 0) \end{bmatrix},$$

$$\begin{bmatrix} \left.\dfrac{\partial}{\partial z_t^m} \lim\limits_{z_t^f \to l} \mathbf{T}(z_t)\right|_{z_t^m=0} & 0 \\ 0 & \left.\dfrac{\partial}{\partial z_t^m} \lim\limits_{z_t^f \to l} \mathbf{T}(z_t)\right|_{z_t^m=0} \left.\dfrac{\partial}{\partial z_t^m} \lim\limits_{z_t^f \to l} \mathbf{C}(z_t)\right|_{z_t^m=0} \end{bmatrix} \begin{bmatrix} \lim\limits_{z_t^f \to l} \mathbf{R}(0, z_t^f) \\ \lim\limits_{z_t^f \to l} \mathbf{S}(0, z_t^f) \end{bmatrix} +$$

$$+ \begin{bmatrix} \lim\limits_{z_t^f \to l} \mathbf{T}(0, z_t^f) & 0 \\ 0 & \lim\limits_{z_t^f \to l} \mathbf{T}(0, z_t^f) \end{bmatrix} \begin{bmatrix} \left.\dfrac{\partial}{\partial z_t^m} \lim\limits_{z_t^f \to l} \mathbf{R}(z_t)\right|_{z_t^m=0} \\ \left.\dfrac{\partial}{\partial z_t^m} \lim\limits_{z_t^f \to l} \mathbf{S}(z_t)\right|_{z_t^m=0} \end{bmatrix} =$$

$$= \begin{bmatrix} \left.\dfrac{\partial}{\partial z_t^m} \lim\limits_{z_t^f \to l} \mathbf{U}(\mathbf{z}_t)\right|_{z_t^m=0} \\ \left.\dfrac{\partial}{\partial z_t^m} \lim\limits_{z_t^f \to l} \mathbf{V}(\mathbf{z}_t)\right|_{z_t^m=0} \end{bmatrix},$$

$$\begin{bmatrix} \left.\dfrac{\partial}{\partial z_t^f} \lim\limits_{z_t^m \to l} \mathbf{T}(z_t)\right|_{z_t^f=0} & 0 \\ 0 & \left.\dfrac{\partial}{\partial z_t^f} \lim\limits_{z_t^m \to l} \mathbf{T}(z_t)\right|_{z_t^f=0} \end{bmatrix} \begin{bmatrix} \lim\limits_{z_t^m \to l} \mathbf{R}(z_t^m, 0) \\ \lim\limits_{z_t^m \to l} \mathbf{S}(z_t^m, 0) \end{bmatrix} +$$

$$+ \begin{bmatrix} \lim\limits_{z_t^m \to l} \mathbf{T}(z_t^m, 0) & 0 \\ 0 & \lim\limits_{z_t^m \to l} \mathbf{T}(z_t^m, 0) \end{bmatrix} \begin{bmatrix} \left.\dfrac{\partial}{\partial z_t^f} \lim\limits_{z_t^m \to l} \mathbf{R}(z_t)\right|_{z_t^f=0} \\ \left.\dfrac{\partial}{\partial z_t^f} \lim\limits_{z_t^m \to l} \mathbf{S}(z_t)\right|_{z_t^f=0} \end{bmatrix} =$$

$$= \begin{bmatrix} \left.\dfrac{\partial}{\partial z_t^f} \lim\limits_{z_t^m \to l} \mathbf{U}(z_t)\right|_{z_t^f=0} \\ \left.\dfrac{\partial}{\partial z_t^f} \lim\limits_{z_t^m \to l} \mathbf{V}(z_t)\right|_{z_t^f=0} \end{bmatrix},$$

for $l = -\infty$ or $l = +\infty$, where

$$\lim_{z_t^f \to -\infty} \mathbf{A}(0, z_t^f) = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1/vb & -1 & 0 & -\tau/b & -(1/v\Pi\delta + 1/\beta) & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -(1-\rho_R)\left(\alpha_0^y + \frac{\alpha_1^y}{1+\exp(\alpha_2^y\alpha_3^y)}\right) \\ 0 & 0 & 0 & -1 & 0 & 0 & -(1-\rho_\tau)\gamma_0^y \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\lim_{z_t^f \to +\infty} \mathbf{A}(0, z_t^f) = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1/vb & -1 & 0 & -\tau/b & -(1/v\Pi\delta + 1/\beta) & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -(1-\rho_R)\left(\alpha_0^y + \frac{\alpha_1^y}{1+\exp(\alpha_2^y\alpha_3^y)}\right) \\ 0 & 0 & 0 & -1 & 0 & 0 & -(1-\rho_\tau)(\gamma_0^y + \gamma_1^y) \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\lim_{z_t^m \to -\infty} \mathbf{A}(z_t^m, 0) = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1/vb & -1 & 0 & -\tau/b & -(1/v\Pi\delta + 1/\beta) & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -(1-\rho_R)\alpha_0^y \\ 0 & 0 & 0 & -1 & 0 & 0 & -(1-\rho_\tau)\left(\gamma_0^y + \frac{\gamma_1^y}{1+\exp(\gamma_2^y\gamma_3^y)}\right) \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\lim_{z_t^m \to -\infty} \mathbf{A}(z_t^m, 0) = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1/vb & -1 & 0 & -\tau/b & -(1/v\Pi\delta + 1/\beta) & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -(1-\rho_R)(\alpha_0^y + \alpha_1^y) \\ 0 & 0 & 0 & -1 & 0 & 0 & -(1-\rho_\tau)\left(\gamma_0^y + \frac{\gamma_1^y}{1+\exp(\gamma_2^y\gamma_3^y)}\right) \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\lim_{z_t^f \to -\infty} \mathbf{C}(0, z_t^f) = \begin{bmatrix} 1-\sigma & 0 \\ 0 & -(1/v\Pi\delta + 1/\beta) \\ (1-\rho_R)\left(\alpha_0^y + \frac{\alpha_1^y}{1+\exp(\alpha_2^y\alpha_3^y)}\right) & (1-\rho_R)\left(\alpha_0^\pi + \frac{\alpha_1^\pi}{1+\exp(\alpha_2^\pi\alpha_3^\pi)}\right) \\ (1-\rho_\tau)\gamma_0^y & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\lim_{z_t^f \to \infty} \mathbf{C}(0, z_t^f) = \begin{bmatrix} 1-\sigma & 0 \\ 0 & -(1/v\Pi\delta + 1/\beta) \\ (1-\rho_R)\left(\alpha_0^y + \frac{\alpha_1^y}{1+\exp(\alpha_2^y\alpha_3^y)}\right) & (1-\rho_R)\left(\alpha_0^\pi + \frac{\alpha_1^\pi}{1+\exp(\alpha_2^\pi\alpha_3^\pi)}\right) \\ (1-\rho_\tau)(\gamma_0^y + \gamma_1^y) & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\lim_{z_t^f \to -\infty} \mathbf{C}(z_t^m, 0) = \begin{bmatrix} 1-\sigma & 0 \\ 0 & -(1/v\Pi\delta + 1/\beta) \\ (1-\rho_R)\alpha_0^y & (1-\rho_R)\alpha_0^\pi \\ (1-\rho_\tau)\left(\gamma_0^y + \frac{\gamma_1^y}{1+\exp(\gamma_2^y\gamma_3^y)}\right) & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\lim_{z_t^f \to \infty} \mathbf{C}(z_t^m, 0) = \begin{bmatrix} 1-\sigma & 0 \\ 0 & -(1/v\Pi\delta + 1/\beta) \\ (1-\rho_R)\left(\alpha_0^y + \alpha_1^y\right) & (1-\rho_R)\left(\alpha_0^\pi + \alpha_1^\pi\right) \\ (1-\rho_\tau)\left(\gamma_0^y + \frac{\gamma_1^y}{1+\exp(\gamma_2^y\gamma_3^y)}\right) & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{B}(0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1/v\Pi\delta & 1/\beta & 1/\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_R & 0 & 0 & 0 & 0 \\ 0 & (1-\rho_\tau)\left(\gamma_0^b + \frac{\gamma_1^b}{1+\exp(\gamma_2^b\gamma_3^b)}\right) & 0 & \rho_\tau & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\lim_{z_t^f \to -\infty} \mathbf{B}(z_t^f) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1/v\Pi\delta & 1/\beta & 1/\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_R & 0 & 0 & 0 & 0 \\ 0 & (1-\rho_\tau)\gamma_0^b & 0 & \rho_\tau & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\lim_{z_t^f \to \infty} \mathbf{B}(z_t^f) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1/v\Pi\delta & 1/\beta & 1/\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_R & 0 & 0 & 0 & 0 \\ 0 & (1-\rho_\tau)\left(\gamma_0^b + \gamma_1^b\right) & 0 & \rho_\tau & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$\frac{\partial}{\partial z_t^m} \lim_{z_t^f \to l} \mathbf{A}(z_t)\bigg|_{z_t^m=0} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(1-\rho_R)\frac{\alpha_1^y\alpha_2^y\exp(\alpha_2^y\alpha_3^y)}{(1+\exp(\alpha_2^y\alpha_3^y))^2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$
\frac{\partial}{\partial z_t^m} \lim_{z_t^f \to l} \mathbf{C}(z_t)\Big|_{z_t^m=0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ (1-\rho_R)\frac{\alpha_1^y \alpha_2^y \exp(\alpha_2^y \alpha_3^y)}{(1+\exp(\alpha_2^y \alpha_3^y))^2} & (1-\rho_R)\frac{\alpha_1^\pi \alpha_2^\pi \exp(\alpha_2^\pi \alpha_3^\pi)}{(1+\exp(\alpha_2^\pi \alpha_3^\pi))^2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
$$

$$
\frac{\partial}{\partial z_t^f} \mathbf{B}(z_t^f)\Big|_{z_t^f=0} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (1-\rho_\tau)\frac{\gamma_1\gamma_2\exp(\gamma_2\gamma_3)}{(1+\exp(\gamma_2\gamma_3))^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.
$$

C.3 Finding F_4

Given $F_{0m}, F_{1m}, F_{1f}, F_{2m}, F_{2f}, F_{3m}, F_{3f}$, we have the following expression that includes F_4:

$$
F(0) = \frac{\left(F_{0m} + \frac{F_{1m}}{1+\exp(F_{2m}F_{3m})}\right)\left(1 + \frac{F_{1f}}{1+\exp\left(F_{2f}F_{3f}\right)}\right)}{1 - F4\frac{\exp\left(F_{2f}F_{3f}\right)}{1+\exp(F_{2m}F_{3m})}\frac{\exp(F_{2m}F_{3m})}{1+\exp\left(F_{2f}F_{3f}\right)}}
$$

Then, to find F_4 we solve the following system of equations:

$$
\begin{bmatrix} \mathbf{T}(0) & 0 \\ 0 & \mathbf{T}(0) \end{bmatrix} \begin{bmatrix} \mathbf{R}(0) \\ \mathbf{S}(0) \end{bmatrix} = \begin{bmatrix} \mathbf{U}(0) \\ \mathbf{V}(0) \end{bmatrix}.
$$

D Verifying Guessed Functional Form

Having obtained the coefficients of the logistic functions that characterize the solution, it is necessary to check that the guessed functional forms for $\mathbf{P}(z_t)$, $\mathbf{Q}(z_t)$, $\mathbf{R}(z_t)$, $\mathbf{S}(z_t)$ are indeed logistic. Recall the system of equations in (**??**)

$$
[\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(z_t)]\mathbf{P}(z_t) + \mathbf{K}\mathbf{R}(z_t) = 0 \tag{60}
$$

$$
[\mathbf{G} + \mathbf{J}\bar{\mathbf{R}}(z_t)]\mathbf{Q}(z_t) + \mathbf{J}\bar{\mathbf{S}}(z_t)\mathbf{N} + \mathbf{K}\mathbf{S}(z_t) + \mathbf{M} = 0 \tag{61}
$$

$$
\mathbf{A}(z_t)\mathbf{P}(z_t) + \mathbf{B}(z_t^f) + \mathbf{C}(z_t)\mathbf{R}(z_t) = 0 \tag{62}
$$

$$
\mathbf{A}(z_t)\mathbf{Q}(z_t) + \mathbf{C}(z_t)\mathbf{S}(z_t) + D = 0. \tag{63}
$$

Given that $\bar{\mathbf{R}}(z_t)$ and $\bar{\mathbf{S}}(z_t)$ can be approximated reasonably well by bivariate logistic functions with the same functional form as (34) (see Appendix E), (60) shows that if $\mathbf{R}(z_t)$ is a bivariate logistic function, then $\mathbf{P}(z_t)$ will also be a bivariate logistic function. From (62), if $\mathbf{P}(z_t)$ is a bivariate logistic function, then $\mathbf{R}(z_t)$ will be a bivariate logistic function. The same reasoning can be used to prove that both $\mathbf{Q}(z_t)$ and $\mathbf{S}(z_t)$ are bivariate logistic functions by using (61) and (63).

E Computation of $\bar{\mathbf{R}}(z_t)$ and $\bar{\mathbf{S}}(z_t)$

Each of the elements of the matrices $\mathbf{P}(z_t), \mathbf{Q}(z_t), \mathbf{R}(z_t), \mathbf{S}(z_t)$ of the proposed solution takes the functional form

$$\begin{aligned} F(x, y; \eta, \alpha, \beta, \delta) &= \eta + \frac{\alpha_1}{1 + \alpha_3 \mathrm{e}^{-\alpha_2 x}} + \frac{\beta_1}{1 + \beta_3 \mathrm{e}^{-\beta_2 y}} + \\ &+ \frac{\delta_1}{1 + \delta_{3x} \mathrm{e}^{-\delta_{2x} x} + \delta_{3y} \mathrm{e}^{-\delta_{2y} y} + (1 - \delta_4) \delta_{3x} \delta_{3y} \mathrm{e}^{-\delta_{2x} x - \delta_{2y} y}}, \end{aligned}$$

where $\alpha = [\alpha_1, \alpha_2, \alpha_3]'$, $\beta = [\beta_1, \beta_2, \beta_3]'$ and $\delta = [\delta_1, \delta_{2x}, \delta_{3x}, \delta_{2y}, \delta_{3y}, \delta_4]$.

We need to compute

$$\mathbb{E}\left[F(x', y'; \eta, \alpha, \beta, \delta) | x, y\right] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(x', y'; \eta, \alpha, \beta, \delta) p(x', y' | x, y; \rho_x, \rho_y, \kappa) \, dx' dy',$$

where $x' = \rho_x x + \varepsilon_x$ and $y' = \rho_y y + \varepsilon_y$, $0 \le \rho_x \le 1$, $0 \le \rho_y \le 1$, and ε_x and ε_y are bivariate normal with zero mean, unit variance and correlation coefficient κ.

Following Maragakis et al. (2008), we can write

$$\begin{aligned} \mathbb{E}\left[\left.\frac{\alpha_1}{1 + \alpha_3 \mathrm{e}^{-\alpha_2 x'}}\right| x, y\right] &\approx \frac{\alpha_1}{1 + \alpha_3 \mathrm{e}^{-a_2 x}} \\ \mathbb{E}\left[\left.\frac{\beta_1}{1 + \beta_3 \mathrm{e}^{-\beta_2 y'}}\right| x, y\right] &\approx \frac{\beta_1}{1 + \beta_3 \mathrm{e}^{-b_2 y}}, \end{aligned}$$

where

$$a_2 = \frac{\rho_x}{\sqrt{\frac{1}{\alpha_2^2} + \frac{\pi}{8}}} \tag{64}$$

$$b_2 = \frac{\rho_y}{\sqrt{\frac{1}{\beta_2^2} + \frac{\pi}{8}}}. \tag{65}$$

Next, we need to approximate

$$G(x, y; \delta_{2x}, \delta_{3x}, \delta_{2y}, \delta_{3y}, \delta_4, \rho_x, \rho_y, \kappa) = \tag{66}$$

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(x', y') p(x', y' | x, y; \rho_x, \rho_y, \kappa) \, dx' dy', \tag{67}$$

where $h(x', y') = \left(\frac{1}{1 + \delta_{3x} \mathrm{e}^{-\delta_{2x} x'} + \delta_{3y} \mathrm{e}^{-\delta_{2y} y'} + (1 - \delta_4) \delta_{3x} \delta_{3y} \mathrm{e}^{-\delta_{2x} x' - \delta_{2y} y'}}\right)$, with

$$H(x, y; d_{2x}, d_{3x}, d_{2y}, d_{3y}, d_4) = \frac{1}{1 + d_{3x} \mathrm{e}^{-d_{2x} x} + d_{3y} \mathrm{e}^{-d_{2y} y} + (1 - d_4) d_{3x} d_{3y} \mathrm{e}^{-d_{2x} x - d_{2y} y}},$$

where

$$
\begin{aligned}
d_{2x} &= \frac{\rho_x}{\sqrt{\frac{1}{\delta_{2x}^2} + \frac{\pi}{8}}} \\
d_{2y} &= \frac{\rho_x}{\sqrt{\frac{1}{\delta_{2y}^2} + \frac{\pi}{8}}} \\
d_{3x} &= \delta_{3x} \\
d_{3y} &= \delta_{3y}
\end{aligned}
$$

are obtained by using the approach in Maragakis et al. (2008) to (67) in the limits $x = \infty$ and $y = \infty$.

To find d_4 we need the following results with respect to the bivariate logistic function given by

$$
\tilde{F}(x, y; \delta_{2x}, \delta_{2y}, \delta_4) = \frac{1}{1 + \mathrm{e}^{-\delta_{2x}x} + \mathrm{e}^{-\delta_{2y}y} + (1 - \delta_4)\mathrm{e}^{-\delta_{2x} - \delta_{2y}}} :
$$

$$
\begin{aligned}
\tilde{f}_{xy}(0, 0; \delta_{2x}, \delta_{2y}, \delta_4) &= \frac{\partial^2}{\partial x \partial y} \tilde{F}(x, y; \delta_{2x}, \delta_{2y}, \delta_4)\bigg|_{x=y=0} = \delta_{2x}\delta_{2y}\frac{4 - 3\delta_4 + \delta_4^2}{(4 - \delta_4)^3} \\
\tilde{f}_x(0; \delta_{2x}) &= \frac{\partial}{\partial x}\tilde{F}(x, \infty; \delta_{2x}, \delta_{2y}, \delta_4)\bigg|_{x=0} = \frac{\delta_{2x}}{4} \\
\tilde{f}_y(0; \delta_{2y}) &= \frac{\partial}{\partial y}\tilde{F}(\infty, y; \delta_{2x}, \delta_{2y}, \delta_4)\bigg|_{y=0} = \frac{\delta_{2y}}{4}.
\end{aligned}
$$

To conduct the approximation, it is necessary to approximate $\tilde{f}_{xy}(x, y; \delta_{2x}, \delta_{2y}, \delta_4)$ with a bivariate normal density function

$$
p(x, y; \sigma_x, \sigma_y, \kappa_{xy}) = \frac{1}{2\pi}\frac{1}{\sigma_x\sigma_y\sqrt{1 - \kappa_{xy}}} \exp\left(-\frac{1}{2\sqrt{1 - \kappa_{xy}^2}}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} - 2\frac{\kappa_{xy}xy}{\sigma_x\sigma_y}\right)\right)
$$

whose variances and correlation coefficient are chosen such that both functions coincide at the origin:

$$
\begin{aligned}
\tilde{f}_{xy}(0, 0; \delta_{2x}, \delta_{2y}, \delta_4) = \delta_{2x}\delta_{2y}\frac{4 - 3\delta_4 + \delta_4^2}{(4 - \delta_4)^3} &= \frac{1}{2\pi}\frac{1}{\sigma_x\sigma_y\sqrt{1 - \kappa_{xy}}} \\
\tilde{f}_x(0; \delta_{2x}) = \frac{\delta_{2x}}{4} &= \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_x} \\
\tilde{f}_y(0; \delta_{2y}) = \frac{\delta_{2y}}{4} &= \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_y}.
\end{aligned}
$$

These conditions yield

$$
\sigma_x = \frac{1}{\delta_{2x}}\sqrt{\frac{8}{\pi}}
$$

$$\sigma_y = \frac{1}{\delta_{2y}}\sqrt{\frac{8}{\pi}}$$
$$\kappa_{xy} = \sqrt{1 - \frac{1}{256}\frac{(4-\delta_4)^6}{(4-3\delta_4+\delta_4^2)^2}}.$$

A feature of $H(x, y; \delta_{2x}, d_{3x}, \delta_{2y}, d_{3y}, d_4)$ is

$$\frac{\partial^2}{\partial x \partial y} H(x, y; \delta_{2x}, d_{3x}, \delta_{2y}, d_{3y}, d_4)\big|_{x=\ln(\delta_{3x})/d_{2x}, y=\ln(\delta_{3y})/d_{2y}} = d_{2x} d_{2y} \frac{4 - 3d_4 + d_4^2}{(4-d_4)^3}.$$

Let

$$\hat{F}(x, y; \delta_{2x}, \delta_{3x}, \delta_{2y}, \delta_{3y}, \delta_4) = \frac{1}{1 + \delta_{3x} \mathrm{e}^{-\delta_{2x} x} + \delta_{3y} \mathrm{e}^{-\delta_{2y} y} + (1-\delta_4)\delta_{3x}\delta_{3y} \mathrm{e}^{-\delta_{2x} x - \delta_{2y} y}}.$$

Then, d_4 is chosen to satisfy

$$\frac{\partial^2}{\partial x \partial y} H(x, y; \delta_{2x}, d_{3x}, \delta_{2y}, d_{3y}, d_4)\big|_{x=x_{0h}, y=y_{0h}} =$$
$$\frac{\partial^2}{\partial x \partial y} G(x, y; \delta_{2x}, \delta_{3x}, \delta_{2y}, \delta_{3y}, \delta_4, \rho_x, \rho_y, \kappa)\big|_{x=x_{0g}, y=y_{0g}},$$

where $x_{0h} = \ln(\delta_{3x})/d_{2x}$ and $y_{0h} = \ln(\delta_{3y})/d_{2y}$, and $x_{0g} = \ln(\delta_{3x})/\delta_{2x}$ and $y_{0g} = \ln(\delta_{3x})/\delta_{2x}$. That is,

$$\begin{aligned}
& d_{2x} d_{2y} \frac{4 - 3d_4 + d_4^2}{(4-d_4)^3} = \\
& = \frac{\partial^2}{\partial x \partial y}\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \hat{F}(x', y'; \delta_{2x}, \delta_{3x}, \delta_{2y}, \delta_{3y}, \delta_4) p(x', y' | x, y; \rho_x, \rho_y, \kappa)\, dx' dy'\right]_{x=x_{0g}, y=y_{0g}} \\
& = \frac{\partial^2}{\partial x \partial y}\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \hat{F}(\rho_x x + \varepsilon_x, \rho_y y + \varepsilon_y; \delta_{2x}, \delta_{3x}, \delta_{2y}, \delta_{3y}, \delta_4) p(\varepsilon_x, \varepsilon_y; \kappa)\, d\varepsilon_x d\varepsilon_y\right]_{x=x_{0g}, y=y_{0g}} \\
& = \rho_x \rho_y \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \tilde{f}_{xy}(\varepsilon_x, \varepsilon_y; \delta_{2x}, \delta_{2y}, \delta_4) p(\varepsilon_x, \varepsilon_y; \kappa)\, d\varepsilon_x d\varepsilon_y \\
& \approx \rho_x \rho_y \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p(\varepsilon_x, \varepsilon_y; \sigma_x, \sigma_y, \kappa_{xy}) p(\varepsilon_x, \varepsilon_y; \kappa)\, d\varepsilon_x d\varepsilon_y,
\end{aligned} \tag{68}$$

where

$$p(\varepsilon_x, \varepsilon_y; \sigma_x, \sigma_y, \kappa_{xy}) =$$
$$(2\pi)^{-1}\left(\sigma_x^2 \sigma_y^2 (1-\kappa_{xy}^2)\right)^{-1/2} \exp\left(-\frac{1}{2\sqrt{1-\kappa_{xy}^2}}\left(\frac{\varepsilon_x^2}{\sigma_x^2} + \frac{\varepsilon_y^2}{\sigma_y^2} - 2\frac{\kappa_{xy}\varepsilon_x\varepsilon_y}{\sigma_x\sigma_y}\right)\right)$$
$$p(\varepsilon_x, \varepsilon_y; \kappa) = (2\pi)^{-1}(1-\kappa^2)^{-1/2} \exp\left(-\frac{1}{2\sqrt{1-\kappa^2}}(\varepsilon_x^2 + \varepsilon_y^2 - 2\kappa\varepsilon_x\varepsilon_y)\right).$$

The RHS of (68) can be written as

$$\rho_x \rho_y (2\pi)^{-2} \left(\sigma_x^2 \sigma_y^2 (1-\kappa^2)(1-\kappa_{xy}^2)\right)^{-1/2} \int \exp\left(-\frac{1}{2}\varepsilon' A \varepsilon\right) d\varepsilon,$$

where $\varepsilon = (\varepsilon_x, \varepsilon_y)'$, and

$$A = \begin{bmatrix} \sigma_x^2 & \kappa_{xy}\sigma_x\sigma_y \\ \kappa_{xy}\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix}^{-1} + \begin{bmatrix} 1 & \kappa \\ \kappa & 1 \end{bmatrix}^{-1}.$$

The gaussian integral yields

$$\begin{aligned} \int \exp\left(-\frac{1}{2}\varepsilon' A \varepsilon\right) d\varepsilon &= 2\pi(\det(A))^{-1/2} \\ &= 2\pi \left[\frac{1-\kappa^2 - 2\kappa\kappa_{xy}\sigma_x\sigma_y + \sigma_x^2 + \sigma_y^2 + (1-\kappa_{xy}^2)\sigma_x^2\sigma_y^2}{(1-\kappa^2)(1-\kappa_{xy}^2)\sigma_x^2\sigma_y^2}\right]^{-1/2}. \end{aligned}$$

Hence, the RHS of (68) is

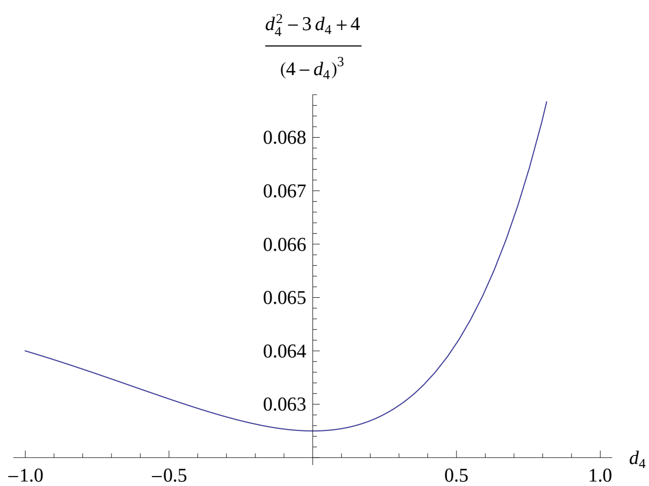
$$\rho_x \rho_y (2\pi)^{-1} \left[1-\kappa^2 - 2\kappa\kappa_{xy}\sigma_x\sigma_y + \sigma_x^2 + \sigma_y^2 + (1-\kappa_{xy}^2)\sigma_x^2\sigma_y^2\right]^{-1/2}.$$

Since $d_{2x}, d_{2y}, \sigma_x, \sigma_y$ and κ_{xy} are functions of δ_{2x}, δ_{2y} and δ_4, (68) allows finding d_4 using

$$\frac{4-3d_4+d_4^2}{(4-d_4)^3} \approx \frac{\rho_x\rho_y}{2\pi d_{2x} d_{2x}\sqrt{1-\kappa^2 - 2\kappa\kappa_{xy}\sigma_x\sigma_y + \sigma_x^2 + \sigma_y^2 + (1-\kappa_{xy}^2)\sigma_x^2\sigma_y^2}}. \tag{69}$$

Let $g(d_4)$ denote the function on the LHS of (69). Figure 7 shows $g(d_4)$.

Figure 7: $g(d_4)$



With all the parameters found, we can write

$$\mathbb{E}\left[F(x',y';\eta,\alpha,\beta,\delta)|x,y\right] \approx \eta+\frac{\alpha_1}{1+\alpha_3\mathrm{e}^{-a_2x}}+\frac{\beta_1}{1+\beta_3\mathrm{e}^{-b_2y}}+ \\ +\frac{\delta_1}{1+\delta_{3x}\mathrm{e}^{-d_{2x}x}+\delta_{3y}\mathrm{e}^{-d_{2y}y}+(1-d_4)\delta_{3x}\delta_{3y}\mathrm{e}^{-d_{2x}x-d_{2y}y}}.$$

F The Likelihood Function of the Simultaneous Equations Model with Time-Varying Coefficients and Stochastic Volatility

Notice that the joint density function of $y_{i,t}$ and $\mathbf{v}_i,t$, conditional on $y_{i,t-1}$, for $i=1,2$, and the latent factors and stochastic volatility (these last two omitted from the density functions below) can be written as

$$\begin{aligned} p\left(y_{i,t},\mathbf{v}_{i,t}|y_{i,t-1},\Theta_{y_i}\right) &= p_y\left(y_{i,t}|y_{i,t-1},\mathbf{v}_{i,t},\Theta_{y_i},\Theta_{\mathbf{v}_i}\right)p_{\mathbf{v}}(\mathbf{v}_{i,t}|\Theta_{\mathbf{v}_i}) \\ &= p_y\left(y_{i,t}|y_{i,t-1},\mathbf{x}_{i,t},\mathbf{v}_{i,t},\Theta_{y_i},\Theta_{\mathbf{x}_i}\right)p_{\mathbf{x}}(\mathbf{x}_{i,t}|\mathbf{w}_{i,t},\Theta_{\mathbf{x}_i}), \end{aligned}$$

where, adding the conditionality on latent factors and stochastic volatility,

$$\begin{aligned} y_{1,t}|y_{1,t-1},\mathbf{x}_{1,t},\mathbf{v}_{1,t},z_t^m,\sigma_{R,t},\Theta_{y_1},\Theta_{\mathbf{x}_1} &\sim \mathbb{N}\left(\rho_R y_{1,t-1}+(1-\rho_R)\mathbf{x}_{1,t}\boldsymbol{\alpha}(z_t^m)+\sigma_{R,t}\left(\mathbf{v}_{1,t}\delta_1+e_t^R\right),1-\delta_1'\Psi_1\delta_1\right) \\ y_{2,t}|y_{2,t-1},\mathbf{x}_{2,t},\mathbf{v}_{2,t},z_t^f,\sigma_{\tau,t},\Theta_{y_2},\Theta_{\mathbf{x}_2} &\sim \mathbb{N}\left(\rho_\tau y_{2,t-1}+(1-\rho_\tau)\mathbf{x}_{2,t}\boldsymbol{\gamma}(z_t^f)+\sigma_{\tau,t}\left(\mathbf{v}_{2,t}\delta_2+e_t^\tau\right),1-\delta_2'\Psi_2\delta_2\right) \\ \mathbf{x}_{i,t}|\mathbf{w}_{i,t},\Theta_{\mathbf{x}_i} &\sim \mathbb{N}\left(\mathbf{w}_{i,t}\delta_i,\Psi_i\right),\ \text{for } i=1,2. \end{aligned}$$

Let $\mathbf{Y}_{i,t}=\{\mathbf{y}_{i,s}\}_{s=1}^t$, and be $\mathbf{X}_{i,t}$ and $\mathbf{V}_{i,t}$ be defined in a similar fashion. Let $\mathbf{Z}_{k,t}=\{z_s^k\}_{s=1}^t$ for $k=m,f$ and let $\mathbf{H}_{j,t}=\{\sigma_{j,s}\}_{s=1}^t$ for $j=R,\tau$. Then, the conditional log-likelihood functions of $\mathbf{Y}_{1,T}$ given $\mathbf{Y}_{1,T-1}$, $\mathbf{X}_{1,t}$, $\mathbf{V}_{1,t}$, $\mathbf{Z}_{m,t}$, $\mathbf{H}_{R,t}$, and of $\mathbf{Y}_{2,T}$ given $\mathbf{Y}_{2,T-1}$, $\mathbf{X}_{2,t}$, $\mathbf{V}_{2,t}$, $\mathbf{Z}_{f,t}$, $\mathbf{H}_{\tau,t}$ are, respectively,

$$\begin{aligned} \mathfrak{L}_T\left(\Theta_{y_1}(\hat{\Theta}_{\mathbf{x}_1})\right) &= \sum_{t=1}^T l_t\left(\Theta_{y_1}(\hat{\Theta}_{\mathbf{x}_1})\right), \\ \mathfrak{L}_T\left(\Theta_{y_2}(\hat{\Theta}_{\mathbf{x}_2})\right) &= \sum_{t=1}^T l_t\left(\Theta_{y_2}(\hat{\Theta}_{\mathbf{x}_2})\right), \end{aligned}$$

where

$$\hat{\Theta}_{\mathbf{x}_i}=\max_{\Theta_{\mathbf{x}_i}}\sum_{t=1}^T\log\left(p_{\mathbf{x}}(\mathbf{x}_{i,t}|\mathbf{w}_{i,t},\Theta_{\mathbf{x}_i})\right)$$

is the maximum likelihood estimator of $\Theta_{\mathbf{x}_i}$ for $i=1,2$, and

$$\begin{aligned} l_t\left(\Theta_{y_1}(\hat{\Theta}_{\mathbf{x}_1})\right) &= \log\left(p_y\left(y_{1,t}|y_{1,t-1},\mathbf{x}_{1,t},\mathbf{v}_{1,t},z_t^m,\sigma_{R,t},\Theta_{y_1},\hat{\Theta}_{\mathbf{x}_1}\right)\right), \\ l_t\left(\Theta_{y_2}(\hat{\Theta}_{\mathbf{x}_2})\right) &= \log\left(p_y\left(y_{2,t}|y_{2,t-1},\mathbf{x}_{2,t},\mathbf{v}_{2,t},z_t^f,\sigma_{\tau,t},\Theta_{y_2},\hat{\Theta}_{\mathbf{x}_2}\right)\right). \end{aligned}$$

This is the two-stage conditional maximum likelihood estimation procedure suggested by Vuong (1984). Let $\mathbf{X}_t = \cup_{i=1}^2 \mathbf{X}_{i,t}$, and let $\mathbf{V}_t$ be defined in a similar way. Let $\mathbf{Z}_t = \{\mathbf{z}_s\}_{s=1}^t$ and $\mathbf{H}_t = \{\mathbf{h}_s\}_{s=1}^t$. The log-likelihood function of $\mathbf{Y}_T$ given $\mathbf{Y}_{T-1}, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \mathbf{H}_T$ is

$$\mathfrak{L}_T\left(\Theta_y(\hat{\Theta}_{\mathbf{x}})\right) = \sum_{t=1}^T l_t\left(\Theta_y(\hat{\Theta}_{\mathbf{x}})\right),$$

where

$$l_t\left(\Theta_y(\hat{\Theta}_{\mathbf{x}})\right) = l_t\left(\Theta_{y_1}(\hat{\Theta}_{\mathbf{x}_1})\right) + l_t\left(\Theta_{y_2}(\hat{\Theta}_{\mathbf{x}_2})\right),$$

and $\hat{\Theta}_{\mathbf{x}} = \hat{\Theta}_{\mathbf{x}_1} \cup \hat{\Theta}_{\mathbf{x}_2}$.

G Bayesian Estimation

Let $p_{\mathbf{y}}(\mathbf{y}_t|\mathbf{X}_t, \mathbf{V}_t, \mathbf{Z}_t, \mathbf{H}_t, \Theta_y)$ denote the conditional density of $\mathbf{y}_t$ given $\mathbf{X}_t$, $\mathbf{V}_t$, $\mathbf{Z}_t$, $\mathbf{H}_t$ and Θ_y. Let $p_{\mathbf{z}}(\mathbf{z}_t|\mathbf{z}_{t-1}, \Theta_z)$ denote the conditional density of $\mathbf{z}_t$ given $\mathbf{z}_{t-1}$ and Θ_z. Let $p_{\mathbf{h}}(\mathbf{h}_t|\mathbf{h}_{t-1}, \Theta_h)$ denote the conditional density of $\mathbf{h}_t$ given $\mathbf{h}_{t-1}$ and Θ_h. Define $\mathbf{Z}_{t+1}^* = \{\mathbf{z}_s\}_{s=t+1}^T$ and $\mathbf{H}_{t+1}^* = \{\mathbf{h}_s\}_{s=t+1}^T$. Under this setup, the joint density of $\mathbf{Z}_T$, $\mathbf{H}_T$ and $\mathbf{Y}_T$ given $\mathbf{X}_T$, $\mathbf{V}_t$, Θ_z, Θ_h and Θ_y is given by

$$\begin{aligned} p(\mathbf{Z}_T, \mathbf{H}_T, \mathbf{Y}_T|\mathbf{X}_T, \mathbf{V}_T, \Theta_z, \Theta_h, \Theta_y) &= p_{\mathbf{z}}(\mathbf{Z}_T|\mathbf{X}_T, \mathbf{V}_T, \Theta_z)p_{\mathbf{h}}(\mathbf{H}_T|\mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \Theta_h)p_{\mathbf{y}}(\mathbf{Y}_T|\mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \mathbf{H}_T, \Theta_y) \\ &= p_{\mathbf{z}}(\mathbf{Z}_T|\Theta_z)p_{\mathbf{h}}(\mathbf{H}_T|\Theta_h)p_{\mathbf{y}}(\mathbf{Y}_T|\mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \mathbf{H}_T, \Theta_y), \end{aligned}$$

where the last equality follows from the Markov property of $\{\mathbf{z}_s\}_{s=0}^t$ and $\{\mathbf{h}_s\}_{s=0}^t$. Then, if $\mathbf{z}_0$ and $\mathbf{h}_0$ are assumed to be stochastic,

$$\begin{aligned} p_{\mathbf{z}}(\mathbf{Z}_T|\Theta_z) &= p_{\mathbf{z}}(\mathbf{z}_0|\Theta_z)\prod_{t=1}^T p_{\mathbf{z}}(\mathbf{z}_t|\mathbf{z}_{t-1}, \Theta_z), \\ p_{\mathbf{h}}(\mathbf{H}_T|\Theta_h) &= p_{\mathbf{h}}(\mathbf{h}_0|\Theta_h)\prod_{t=1}^T p_{\mathbf{h}}(\mathbf{h}_t|\mathbf{h}_{t-1}, \Theta_h), \\ p_{\mathbf{y}}(\mathbf{Y}_T|\mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \mathbf{H}_T, \Theta_y) &= \prod_{t=1}^T p_{\mathbf{y}}(\mathbf{y}_t|\mathbf{X}_t, \mathbf{V}_t, \mathbf{Z}_t, \mathbf{H}_t, \Theta_y), \end{aligned}$$

$$p(\mathbf{z}_t|\mathbf{Z}_{t-1}, \mathbf{Z}_{t+1}^*, \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T, \Theta_y, \Theta_z, \Theta_h) \propto \tag{70}$$
$$\begin{cases} p_{\mathbf{y}}(\mathbf{y}_t|\mathbf{X}_t, \mathbf{V}_t, \mathbf{Z}_t, \mathbf{H}_t, \Theta_y)p_{\mathbf{z}}(\mathbf{z}_t|\mathbf{z}_{t-1}, \Theta_z)p_{\mathbf{z}}(\mathbf{z}_{t+1}|\mathbf{z}_t, \Theta_z), & \text{if } t \le T-1 \\ p_{\mathbf{y}}(\mathbf{y}_t|\mathbf{X}_t, \mathbf{V}_t, \mathbf{Z}_t, \mathbf{H}_t, \Theta_y)p_{\mathbf{z}}(\mathbf{z}_t|\mathbf{z}_{t-1}, \Theta_z), & \text{if } t = T, \end{cases}$$

$$p(\mathbf{h}_t|\mathbf{H}_{t-1}, \mathbf{H}_{t+1}^*, \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \Theta_y, \Theta_z, \Theta_h) \propto \tag{71}$$
$$\begin{cases} p_{\mathbf{y}}(\mathbf{y}_t|\mathbf{X}_t, \mathbf{V}_t, \mathbf{Z}_t, \mathbf{H}_t, \Theta_y)p_{\mathbf{h}}(\mathbf{h}_t|\mathbf{h}_{t-1}, \Theta_h)p_{\mathbf{h}}(\mathbf{h}_{t+1}|\mathbf{h}_t, \Theta_h), & \text{if } t \le T-1 \\ p_{\mathbf{y}}(\mathbf{y}_t|\mathbf{X}_t, \mathbf{V}_t, \mathbf{Z}_t, \mathbf{H}_t, \Theta_y)p_{\mathbf{h}}(\mathbf{h}_t|\mathbf{h}_{t-1}, \Theta_h), & \text{if } t = T, \end{cases}$$

$$p(\Theta_y|\mathbf{Y}_T,\mathbf{X}_T,\mathbf{V}_T,\mathbf{Z}_T,\mathbf{H}_T,\Theta_z,\Theta_h) \propto p_{\mathbf{y}}(\mathbf{Y}_T|\mathbf{X}_T,\mathbf{Z}_T,\mathbf{H}_T,\Theta_y)p_{\Theta_y}(\Theta_y), \tag{72}$$

$$p(\Theta_z|\mathbf{Y}_T,\mathbf{X}_T,\mathbf{V}_T,\mathbf{Z}_T,\Theta_y,\Theta_h) \propto p_{\mathbf{z}}(\mathbf{Z}_T|\Theta_z)p_{\Theta_z}(\Theta_z), \tag{73}$$

$$p(\Theta_h|\mathbf{Y}_T,\mathbf{X}_T,\mathbf{V}_T,\mathbf{H}_T,\Theta_y,\Theta_z) \propto p_{\mathbf{h}}(\mathbf{H}_T|\Theta_h)p_{\Theta_h}(\Theta_h), \tag{74}$$

where $p_{\Theta_y}(\Theta_y)$, $p_{\Theta_z}(\Theta_z)$ and $p_{\Theta_h}(\Theta_h)$ are the prior densities of Θ_y, Θ_z and Θ_h, respectively.

From the posterior densities (70)-(74), the smoothing random draws are generated as follows:

Step 0. Take appropriate initial values for Θ_y, Θ_z, $\{\mathbf{z}_t\}_{t=0}^T$, Θ_h and $\{\mathbf{h}_t\}_{t=0}^T$.[10]

Step 1. Generate a random draw of $\mathbf{z}_t$ from $p(\mathbf{z}_t|\mathbf{Z}_{t-1},\mathbf{Z}^*_{t+1},\mathbf{Y}_T,\mathbf{X}_T,\mathbf{V}_T,\mathbf{H}_T,\Theta_y,\Theta_z,\Theta_h)$ for $t = 1, 2, \ldots, T$.

Draw $\mathbf{z}_t^{(i)}$ using the normal proposal density

$$\mathbf{z}_t^{(i)} \sim \mathbb{N}(\mathbf{z}_t^{(i-1)}, cK_t^z)$$

where K_t^z is the filtered variance-covariance matrix of the random coefficients of a random-coefficient specification of the policy rules, and c is a proper scale coefficient. The algorithm accepts $\mathbf{z}_t^{(i)}$ with probability r:

$$r = \min\left\{ \frac{p\left(\mathbf{z}_t^{(i)}|\mathbf{Z}_{t-1}^{(i-1)},\mathbf{Z}_{t+1}^{*(i-1)},\mathbf{Y}_T,\mathbf{X}_T,\mathbf{V}_T,\mathbf{H}_T^{(i-1)},\Theta_y^{(i-1)},\Theta_z^{(i-1)},\Theta_h^{(i-1)}\right)}{p\left(\mathbf{z}_t^{(i-1)}|\mathbf{Z}_{t-1}^{(i-1)},\mathbf{Z}_{t+1}^{*(i-1)},\mathbf{Y}_T,\mathbf{X}_T,\mathbf{V}_T,\mathbf{H}_T^{(i-1)},\Theta_y^{(i-1)},\Theta_z^{(i-1)},\Theta_h^{(i-1)}\right)}, 1 \right\}.$$

Step 2. Generate a random draw of Θ_y from $p(\Theta_y|\mathbf{Y}_T,\mathbf{X}_T,\mathbf{V}_T,\mathbf{Z}_T,\mathbf{H}_T,\Theta_z,\Theta_h)$.

(a) Partition $\Theta_{y_1} = \Theta_{y_{1.1}} \cup \Theta_{y_{1.2}}$, where $\Theta_{y_{1.1}} = \rho_R$ and $\Theta_{y_{1.2}} = \{\alpha_0^\pi, \alpha_1^\pi, \alpha_2^\pi, \alpha_3^\pi, \alpha_0^y, \alpha_1^y, \alpha_2^y, \alpha_3^y\}$. Partition $\Theta_{y_2} = \Theta_{y_{2.1}} \cup \Theta_{y_{2.2}}$, where $\Theta_{y_{2.1}} = \rho_\tau$ and $\Theta_{y_{2.2}} = \{\gamma_0^b, \gamma_1^b, \gamma_2^b, \gamma_3^b, \gamma_0^y, \gamma_1^y, \gamma_2^y, \gamma_3^y\}$.

(b) Generate a random draw of $\Theta_{y_1}^{(i)}$ sequentially, as follows:

i. Generate a random draw of $\Theta_{y_{1.1}}^{(i)}$ using a Beta proposal density (expressed in terms of mean and standard deviation)

$$\rho_R^{(i)} \sim \mathrm{Beta}(\hat{\rho}_R, \hat{\sigma}_{\hat{\rho}_R})$$

where $\hat{\rho}_R$ is the generalized least squares estimate of ρ_R in a constant-coefficient version of (41) given $\mathbf{H}_T^{(i-1)}$, and $\hat{\sigma}_{\hat{\rho}_R}$ is its generalized least squares standard

[10] The initial latent factors, $\{\mathbf{z}_t\}_{t=0}^T$, are smoothed estimates of a random coefficients model of the policy rules with constant volatility. The initial volatility processes, $\{\mathbf{h}_t\}_{t=0}^T$, are smoothed estimates of a stochastic volatility model with constant coefficients. The initial values of the parameters of the policy rules, Θ_y, are obtained from the maximization of the likelihood function given the initial processes for the latent factors and stochastic volatilities. The initial values for Θ_z come from a least-squares regression of current against lagged initial latent factors. The initial values for Θ_h come from a least-squares regression of current against lagged initial log stochastic volatilities.

error. The algorithm accepts $\Theta_{y1.1}^{(i)}$ with probability r:

$$r = \min\left\{\frac{p\left(\Theta_{y1.1}^{(i)} \cup \Theta_{y1.2}^{(i-1)}|\mathbf{Y}_{1,T}, \mathbf{X}_{1,T}, \mathbf{V}_{1,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}{p\left(\Theta_{y1.1}^{(i-1)} \cup \Theta_{y1.2}^{(i-1)}|\mathbf{Y}_{1,T}, \mathbf{X}_{1,T}, \mathbf{V}_{1,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}\frac{g(\rho_R^{(i-1)})}{g(\rho_R^{(i)})}, 1\right\},$$

where $g(\cdot)$ is the proposal density.

ii. Generate a random draw of a transformation of $\Theta_{y1.2}^{(i)}$, $\tilde{\Theta}_{y1.2}^{(i)}$, using the normal proposal density

$$\tilde{\Theta}_{y1.2}^{(i)} \sim \mathbb{N}(\tilde{\Theta}_{y1.2}^{(i-1)}, cS_{ML}),$$

where S_{ML} is the variance-covariance matrix of the maximum likelihood estimator of $\tilde{\Theta}_{y1.2}$ given the initial latent factors and stochastic volatilities, and c is a scale coefficient. The algorithm accepts $\Theta_{y1.2}^{i}$ with probability r:

$$r = \min\left\{\frac{p\left(\Theta_{y1.1}^{(i)} \cup \Theta_{y1.2}^{(i)}|\mathbf{Y}_{1,T}, \mathbf{X}_{1,T}, \mathbf{V}_{1,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}{p\left(\Theta_{y1.1}^{(i)} \cup \Theta_{y1.2}^{(i-1)}|\mathbf{Y}_{1,T}, \mathbf{X}_{1,T}, \mathbf{V}_{1,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}, 1\right\}.$$

(c) Generate a random draw of $\Theta_{y2}^{(i)}$ sequentially, as follows:

i. Generate a random draw $\Theta_{y2.1}^{(i)}$ using a Beta proposal density (expressed in terms of mean and standard deviation)

$$\rho_\tau^{(i)} \sim \text{Beta}(\hat{\rho}_\tau, \hat{\sigma}_{\hat{\rho}_\tau}),$$

where $\hat{\rho}_\tau$ is the generalized least squares estimate of ρ_τ in a constant-coefficient version of (43) given $\mathbf{H}_T^{(i-1)}$, and $\hat{\sigma}_{\hat{\rho}_\tau}$ is its generalized least squares standard error. The algorithm accepts $\Theta_{y2.1}^{(i)}$ with probability r:

$$r = \min\left\{\frac{p\left(\Theta_{y2.1}^{(i)} \cup \Theta_{y2.2}^{(i-1)}|\mathbf{Y}_{2,T}, \mathbf{X}_{2,T}, \mathbf{V}_{2,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}{p\left(\Theta_{y2.1}^{(i-1)} \cup \Theta_{y2.2}^{(i-1)}|\mathbf{Y}_{2,T}, \mathbf{X}_{2,T}, \mathbf{V}_{2,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}\frac{g(\rho_\tau^{(i-1)})}{g(\rho_\tau^{(i)})}, 1\right\},$$

where $g(\cdot)$ is the proposal density.

ii. Generate a random draw of a transformation of $\Theta_{y2.2}^{(i)}$, $\tilde{\Theta}_{y2.2}^{(i)}$, using the normal proposal density

$$\tilde{\Theta}_{y2.2}^{(i)} \sim \mathbb{N}(\tilde{\Theta}_{y2.2}^{(i-1)}, cS_{ML}),$$

where S_{ML} is the variance-covariance matrix of the maximum likelihood estimator of $\tilde{\Theta}_{y2.2}$ given the initial latent factors and stochastic volatilities, and c is a scale coefficient. The algorithm accepts $\Theta_{y2.2}^{i}$ with probability r:

$$r = \min\left\{\frac{p\left(\Theta_{y2.1}^{(i)} \cup \Theta_{y2.2}^{(i)}|\mathbf{Y}_{2,T}, \mathbf{X}_{2,T}, \mathbf{V}_{2,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}{p\left(\Theta_{y2.1}^{(i)} \cup \Theta_{y2.2}^{(i-1)}|\mathbf{Y}_{2,T}, \mathbf{X}_{2,T}, \mathbf{V}_{2,T}, \mathbf{Z}_T^{(i)}, \mathbf{H}_T^{(i-1)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}, 1\right\}.$$

Step 3. Generate a random draw of $\mathbf{h}_t$ from $p(\mathbf{h}_t|\mathbf{H}_{t-1}, \mathbf{H}_{t+1}^*, \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \Theta_y, \Theta_z, \Theta_h)$

for $t = 1, 2, \ldots, T$**.** Draw $\mathbf{h}_t^{(i)}$ using the normal proposal density

$$\mathbf{h}_t^{(i)} \sim \mathbb{N}(\mathbf{h}_t^{(i-1)}, cK_t^h)$$

where K_t^h is the filtered variance-covariance matrix of the volatility of a constant-coefficient specification of the policy rules, and c is a proper scale coefficient. The algorithm accepts $\mathbf{h}_t^{(i)}$ with probability r:

$$r = \min\left\{\frac{p\left(\mathbf{h}_t^{(i)}|\mathbf{H}_{t-1}^{(i-1)}, \mathbf{H}_{t+1}^{*(i-1)}, \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T^{(i)}, \Theta_y^{(i)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}{p\left(\mathbf{h}_t^{(i-1)}|\mathbf{H}_{t-1}^{(i-1)}, \mathbf{H}_{t+1}^{*(i-1)}, \mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T^{(i)}, \Theta_y^{(i)}, \Theta_z^{(i-1)}, \Theta_h^{(i-1)}\right)}, 1\right\}.$$

Step 4. Generate a random draw of Θ_z from $p(\Theta_z|\mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T, \Theta_y, \Theta_h)$ sequentially, as follows:

(a) Partition $\Theta_z = \Theta_{z1} \cup \Theta_{z2}$ where $\Theta_{z1} = \{\rho_{z^m}, \rho_{z^f}\}$ and $\Theta_{z2} = \{\kappa\}$.

(b) Generate a random draw $\{\rho_{z^m}^{(i)}, \rho_{z^f}^{(i)}\}$ using two independent Beta proposal densities (expressed in terms of means and standard deviations)

$$\rho_{z^m}^{(i)} \sim \text{Beta}(\hat{\rho}_{z^m}, \hat{\sigma}_{\hat{\rho}_{z^m}})$$
$$\rho_{z^f}^{(i)} \sim \text{Beta}(\hat{\rho}_{z^m}, \hat{\sigma}_{\hat{\rho}_{z^f}}),$$

where $\hat{\rho}_{z^m}$ is the ordinary least squares estimate of ρ_{z^m} in (37) using $\{z_t^{m(i)}\}_{t=1}^T$, and $\hat{\sigma}_{\hat{\rho}_{z^m}}$ is its standard error. The same applies for $\hat{\rho}_{z^f}$ and $\hat{\sigma}_{\hat{\rho}_{z^f}}$, which come from the estimation of (38) using $\{z_t^{f(i)}\}_{t=1}^T$. The algorithm accepts $\{\rho_{z^m}^{(i)}, \rho_{z^f}^{(i)}\}$ with probability r:

$$r = \min\left\{\frac{p\left(\Theta_{z1}^{(i)} \cup \Theta_{z2}^{(i-1)}|\mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T^{(i)}, \Theta_y, \Theta_h\right)}{p\left(\Theta_{z1}^{(i-1)} \cup \Theta_{z2}^{(i-1)}|\mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T^{(i)}, \Theta_y, \Theta_h\right)}\frac{g(\rho_{z^m}^{(i-1)}, \rho_{z^f}^{(i-1)})}{g(\rho_{z^m}^{(i)}, \rho_{z^f}^{(i)})}, 1\right\}.$$

(c) Generate a random draw $\kappa^{(i)}$ using a four-parameter Beta proposal density with range on [-1,1] (expressed in terms of mean and standard deviation)

$$\kappa^{(i)} \sim \text{TransformedBeta}(\hat{\kappa}, (1-\hat{\kappa}^2)/\sqrt{n-1}),$$

where $\hat{\kappa}$ is the correlation coefficient between the residuals of equations (37) and (38) using as estimated coefficients $\rho_{z^m}^{(i)}$ and $\rho_{z^f}^{(i)}$ where corresponds. The algorithm accepts $\kappa^{(i)}$ with probability r:

$$r = \min\left\{\frac{p\left(\Theta_{z1}^{(i)} \cup \Theta_{z2}^{(i)}|\mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T^{(i)}, \Theta_y, \Theta_h\right)}{p\left(\Theta_{z1}^{(i)} \cup \Theta_{z2}^{(i-1)}|\mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{Z}_T^{(i)}, \Theta_y, \Theta_h\right)}\frac{g(\kappa^{(i-1)})}{g(\kappa^{(i)})}, 1\right\}.$$

Step 5. Generate a random draw of Θ_h from $p(\Theta_h|\mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T, \Theta_y, \Theta_z)$.

(a) Partition $\Theta_h = \Theta_{h1} \cup \Theta_{h2}$ where $\Theta_{h1} = \{\ln \sigma_R, \rho_{\sigma_R}, \ln \sigma_\tau, \rho_{\sigma_\tau}\}$ and $\Theta_{h2} = \{\eta_R, \eta_\tau\}$.

(b) Generate a random draw $\{\ln \sigma_R^{(i)}, \rho_{\sigma_R}^{(i)}, \ln \sigma_\tau^{(i)}, \rho_{\sigma_\tau}^{(i)}\}$ using the independent proposal densities (expressed in terms of means and standard deviations)

$$\begin{aligned}
\rho_{\sigma_R}^{(i)} &\sim \text{Beta}(\hat{\rho}_{\sigma_R}, \hat{\sigma}_{\hat{\rho}_{\sigma_R}}) \\
\rho_{\sigma_\tau}^{(i)} &\sim \text{Beta}(\hat{\rho}_{\sigma_\tau}, \hat{\sigma}_{\hat{\rho}_{\sigma_\tau}}) \\
c_R^{(i)} &\sim \mathbb{N}\left(\hat{c}_R, \hat{\sigma}_{c_R}\right) \\
c_\tau^{(i)} &\sim \mathbb{N}\left(\hat{c}_\tau, \hat{\sigma}_{c_\tau}\right),
\end{aligned}$$

with $\ln \sigma_R^{(i)} = c_R^{(i)}/(1-\rho_{\sigma_R}^{(i)})$ and $\ln \sigma_\tau^{(i)} = c_\tau^{(i)}/(1-\rho_{\sigma_\tau}^{(i)})$, where $\hat{c}_R, \hat{\rho}_{\sigma_R}$ are the least squares estimates of $(1-\rho_{\sigma_R})\ln \sigma_R$ and ρ_{σ_R} in (39), respectively, using $\{\ln \sigma_{R,t}^{(i)}\}$, and where $\hat{c}_\tau, \hat{\rho}_{\sigma_\tau}$ are obtained similarly from (40) using $\{\ln \sigma_{\tau,t}^{(i)}\}$. The algorithm accepts $\{\ln \sigma_R^{(i)}, \rho_{\sigma_R}^{(i)}, \ln \sigma_\tau^{(i)}, \rho_{\sigma_\tau}^{(i)}\}$ with probability r:

$$r = \min\left\{ \frac{p\left(\Theta_{h1}^{(i)} \cup \Theta_{h2}^{(i-1)}|\mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T^{(i)}, \Theta_y, \Theta_z\right)}{p\left(\Theta_{h1}^{(i-1)} \cup \Theta_{h2}^{(i-1)}|\mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T^{(i)}, \Theta_y, \Theta_z\right)} \frac{g(\ln \sigma_R^{(i-1)}, \rho_{\sigma_R}^{(i-1)}, \ln \sigma_\tau^{(i-1)}, \rho_{\sigma_\tau}^{(i-1)})}{g(\ln \sigma_R^{(i)}, \rho_{\sigma_R}^{(i)}, \ln \sigma_\tau^{(i)}, \rho_{\sigma_\tau}^{(i)})}, 1 \right\}.$$

(c) Generate a random draw $\{\eta_R^{(i)}, \eta_\tau^{(i)}\}$ using the inverted gamma proposal densities

$$\begin{aligned}
\eta_R^{(i)} &\sim \text{IG}(\tilde{\xi}'^{R(i)}\tilde{\xi}^{R(i)}, \text{df}) \\
\eta_\tau^{(i)} &\sim \text{IG}(\tilde{\xi}'^{\tau(i)}\tilde{\xi}^{\tau(i)}, \text{df}),
\end{aligned}$$

where $\tilde{\xi}_t^{R(i)} = \ln \sigma_{R,t}^{(i)} - (1-\rho_{\sigma_R}^{(i)})\ln \sigma_R^{(i)} - \rho_{\sigma_R}^{(i)} \ln \sigma_{R,t-1}^{(i)}$ and $\tilde{\xi}_t^{\tau(i)} = \ln \sigma_{\tau,t}^{(i)} - (1 - \rho_{\sigma_\tau}^{(i)})\ln \sigma_\tau^{(i)} - \rho_{\sigma_\tau}^{(i)} \ln \sigma_{\tau,t-1}^{(i)}$ are residuals, and $\tilde{\xi}^{R(i)}$ and $\tilde{\xi}^{\tau(i)}$ are vectors that stack the respective residuals. The algorithm accepts $\{\eta_R^{(i)}, \eta_\tau^{(i)}\}$ with probability r:

$$r = \min\left\{ \frac{p\left(\Theta_{h1}^{(i)} \cup \Theta_{h2}^{(i)}|\mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T^{(i)}, \Theta_y, \Theta_z\right)}{p\left(\Theta_{h1}^{(i)} \cup \Theta_{h2}^{(i-1)}|\mathbf{Y}_T, \mathbf{X}_T, \mathbf{V}_T, \mathbf{H}_T^{(i)}, \Theta_y, \Theta_z\right)} \frac{g(\eta_R^{(i-1)}, \eta_\tau^{(i-1)})}{g(\eta_R^{(i)}, \eta_\tau^{(i)})}, 1 \right\}.$$

Step 6. Repeat steps 1-4 N times to obtain N random draws of $\mathbf{Z}_T$, $\mathbf{H}_T$, Θ_y, Θ_z and Θ_h.

In steps (ii)-(vi) the random draws of $\mathbf{Z}$, $\mathbf{H}$, Θ_Y, Θ_z and Θ_h are updated. This sampling method is referred to as the Gibbs sampler. To generate the random draws of $\mathbf{z}_t$, $\mathbf{h}_t$ for $t = 1, 2, \ldots, T$, Θ_y, Θ_z and Θ_h, I use the Metropolis-Hastings (M-H) algorithm. That is, the Gibbs sampler and the M-H algorithm are combined to obtain the smoothing random draws from the state-space model. Appendix G shows the choice of prior densities that determine the posterior densities (72)-(73).

H Markov Chain Plots

Figure 8: Markov Chain Policy Rule Parameters

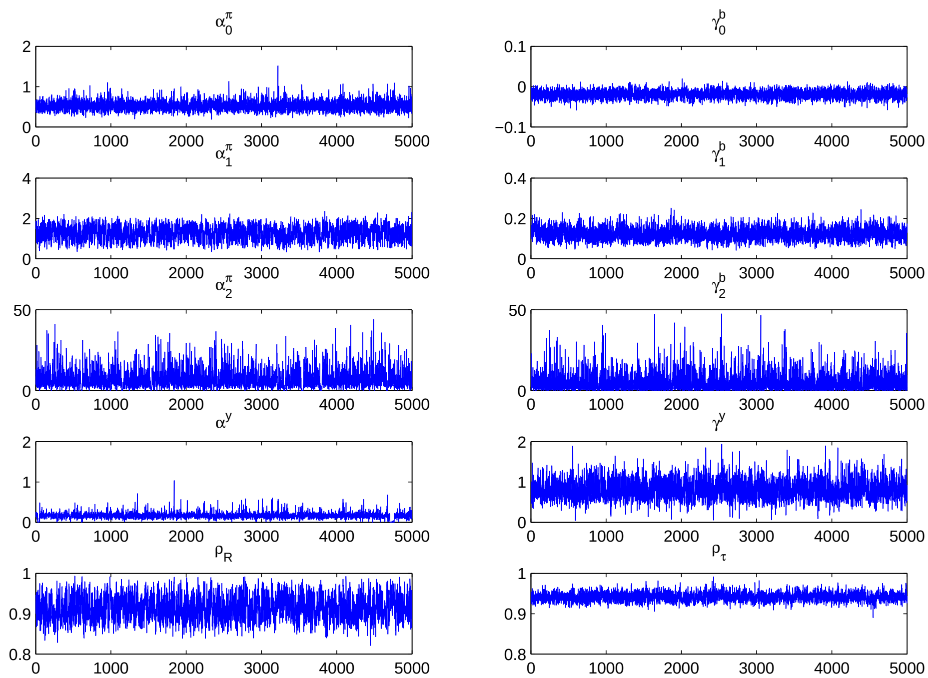
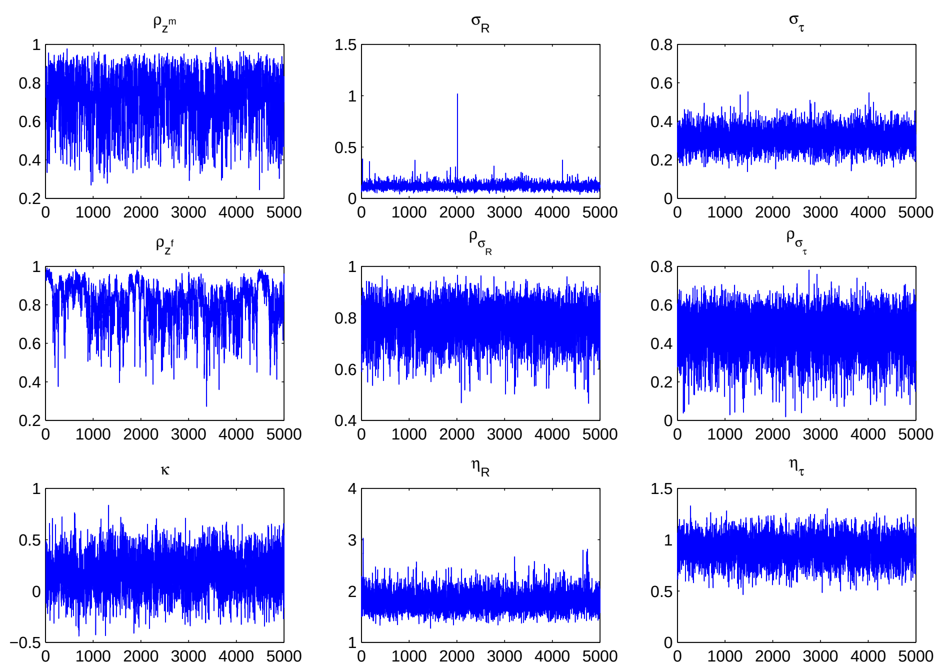


Figure 9: Markov Chain Parameters of Latent Factors and Stochastic Volatility



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