Optimal Pension Design in General Equilibrium

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Abstract

The present paper aims to quantify efficiency properties of real world social security systems of various institutional designs in order to identify an optimal pension design. Starting from a benchmark economy without social security, we introduce alternative pension systems and compare the costs arising from liquidity constraints as well as labor and savings distortions versus the benefits from insurance provision against income and lifespan uncertainty. Our findings highlight strong efficiency losses arising from both means-testing pension benefits against private assets and restricting the contribution base while indicating a positive impact of means-testing flat benefits against earnings-related benefits within pension systems resting on several tiers. Furthermore, our results suggest that the negative correlation between pension progressivity and pension generosity may be justified on efficiency grounds. In our model a single-tier universal earnings-related pension system yields the highest efficiency gains dominating flat benefits as well as two-tier systems of any form.

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1 Introduction

Pay-as-you-go financed public pension systems show a multitude of institutional designs reflecting the redistribution and insurance objectives of the respective governments and societies. Countries such as Australia, the United Kingdom, Ireland and New Zealand operate some form of flat-rate scheme which guarantees the same amount of benefits to retirees independent of their individual earnings and contribution history. These systems are specifically designed to prevent old-age poverty. In order to optimally target benefits to retirees in need while at the same time reducing the size of the program, benefits are typically means-tested. At the other end of the spectrum countries such as Germany, France or Italy operate retirement systems where the benefit level is linked to some measure of individual earnings during the working phase. Such earnings-related schemes are designed to provide retired individuals with means to keep an adequate standard of living compared to that during working life. In addition, the contribution base is often restricted to a certain multiple of the average wage. Finally, countries such as the Netherlands, Norway or Finland operate two-tier schemes aiming for a specific combination of both objectives. Interlinkages between the two tiers typically guarantee that benefits from different tiers are interdependent.

Given the vast diversity of pension systems it is important to understand the specific factors which determine the optimal institutional design. What is the optimal replacement rate that characterizes a pension system resting on one flat or earnings-related pillar? Should flat benefits be universal or should they be means-tested? What resources should be considered and what withdrawal rate should be applied in a means-test? What is the optimal structure of two-tier systems? What is the trade-off between optimal generosity and progressivity of benefits? In order to answer such questions the present paper attempts to quantify the main merits and costs of different institutional designs using a general equilibrium overlapping generations model where households decide about savings and labor supply under uncertainty. In our set-up the pension system increases welfare due to the insurance provision against labor income and longevity risk which is not provided by the market. At the same time contributions to the pension system distort labor supply and savings decisions and increase liquidity constraints. Consequently, the optimal pension design depends on the structure of individual preferences, the underlying uncertainty and the remaining governmental redistribution system. The simulation results highlight these linkages and allow to draw some conclusions for public policy.

Various previous studies have already analyzed specific issues of optimal pension design. Miles and Sefton (2003) as well as Sefton and van de Ven (2009) analyze the quantitative implications of various policy reforms for the UK's means-tested retirement benefit program using a partial equilibrium life-cycle model. Their results indicate a positive role for means-testing of benefits as long as the withdrawal rate is around 50 %. Kumru and Piggott (2009) extend this approach using a large scale general equilibrium stochastic overlapping generations model which is calibrated to UK data. They find that even a 100% taper rate for means-testing is optimal. Using a similar approach, Hugget and Ventura (1999) quantify the effects of replacing the current US system with a two tier system where the first tier comprises a compulsory defined contribution (DC) plan and the second tier guarantees a minimum income. They find that long run welfare decreases due to such a reform. Alternatively, Nishiyama and Smetters (2009) compute the parameter values for the optimal social security formula in the US which maximizes long run welfare. Kudrna and Woodland (2011) analyze the abolition of the means-test within the Australian pension system. In contrast to all previous studies their approach does not focus on long-run welfare consequences only. Instead they also consider transitional cohorts and compute compensating transfers which neutralize intergenerational income redistribution effects. However, they abstract from income uncertainty so that they do not take the insurance provision properties of the Australian pension scheme adequately into account. In contrast, Fehr and Habermann (2008) analyze the optimal progressivity of the German pension system in a model with uncertain income while considering both long-run and transitional cohorts. The resulting aggregate efficiency consequences are used to identify the optimal design. The numerical results indicate that the current German pension system should be more progressive.

The present study extends the previous work by Fehr and Habermann (2008) in various directions. First, the income process is modeled differently in order to distinguish between lifetime income classes that may differ in the income process and life expectancy. Second, our model is not calibrated to a specific economy but allows for a wide range of different institutional designs which may include the UK, the US or the German pension system as special cases. Third, simulations are presented with alternative specifications about bequest motives, labor supply, risk aversion and longevity to assess the sensitivity of the results. In order to quantify the macroeconomic, welfare and efficiency consequences of different pension formulas, we start from an initial long run equilibrium without social security. Next we introduce alternative pension systems and compute the transition path, the new long run equilibrium as well as the welfare consequences for different cohorts. Finally we incorporate lump-sum compensations in order to quantify the aggregate efficiency effect of a specific reform scenario. The latter is then used as the criterion to identify the optimal pension design.

The simulation results indicate an aggregate efficiency gain due to the introduction of pay-as-you-go financed pensions. The optimal replacement rate is around 40 percent for flat pensions and rises for earnings-related pensions. They also highlight the distortions arising from asset means testing and from restrictions of the contribution base. Finally, pension-testing in a two tier system might improve economic efficiency.

The remainder of the paper is organized as follows: the next section describes the general equilibrium model we use in our quantitative analysis. Sections 3 discusses the calibration of the initial equilibrium. Our simulation results are presented in Section 4, Section 5 concludes.

2 The model economy

2.1 Demographics and intracohort heterogeneity

Our model economy is populated by overlapping generations of individuals which may live up to a maximum possible lifespan of J periods. At each date t a new generation is born with its size normalized to unity, i.e. we assume zero population growth. At the beginning of life, individuals are assigned a skill level $s \in \mathcal{S}$ with an (exogenous) probability $N_{1,s}$. Since individuals face lifespan uncertainty, cohort sizes decrease over time, i.e. $N_{j,s} = \psi_{j,s} N_{j-1,s}$, with $\psi_{j,s} < 1$ denoting the time-invariant conditional survival probability of an individual of skill level s at the age of $j-1$ and $\psi_{J+1,s} = 0$.

Our model is solved recursively. At any given point in time t, agents are characterized by the state vector $z_j = (s, a_j, ep_j, \eta_j)$, with $j \in \mathcal{J} = \{1, \dots, J\}$ denoting the age of the individual, $a_j \in \mathcal{A} = [0, \infty)$ representing liquid assets held by the agent at the beginning of age j and $ep_j \in \mathcal{P} = [0, ep^{max}]$ marking the agent's accumulated earning points for earnings-related second pillar public pension claims. Furthermore, $\eta_j \in \mathcal{E}$ denotes an idiosyncratic shock to individual labor productivity.

At a given point in time t, the cohort of j-old agents is fragmented into subgroups $\xi_t(z_j)$ determined by the initial distribution at birth, the income process, mortality and the respective optimal decisions of its individuals over their life cycle. We define $X_t(z_j)$ as the corresponding cumulated measure of $\xi_t(z_j)$. As $\xi_t(z_j)$ only gives densities within cohorts and is not affected by cohort sizes,

$$\int_{\mathcal{A}\times\mathcal{P}\times\mathcal{E}} \mathrm{d}X_t(z_j) = N_{j,s} \qquad \text{and} \qquad \sum_{j\in\mathcal{J}}\sum_{s\in\mathcal{S}} N_{j,s} = \sum_{j\in\mathcal{J}} \int_{\mathcal{Z}} \mathrm{d}X_t(z_j)$$

with $\mathcal{Z} = \mathcal{S}\times\mathcal{A}\times\mathcal{P}\times\mathcal{E}$ holding $\forall\ t \in \{0,\ldots,\infty\}$. Furthermore, we define $Z_t = (\xi_t(z_j), B_{RAt}, \Psi_t)$ as the state of the economy at the beginning of period t, with B_{RAt} representing debt of the redistribution authority. Ψ_t marks the policy schedule at a point in time t. In the following, we will omit the time index t, the skill level s and the state indices z_j and Z_t whenever possible. Agents are then only distinguished according to their age j.

2.2 The household side

We assume an identical preference structure for all agents represented by a time-separable, nested CES utility function. By following the approach of Epstein and Zin (1991) we isolate the agent's relative risk aversion from intertemporal substitution. Abstracting from a specific bequest motive, a j-old individual decides about its optimal leisure ℓ_j, consumption c_j and asset holdings a_j, while the agent's time endowment is normalized to unity. The optimization problem of a representative j-old agent with the state z_j is formulated recursively as

$$V(z_j) = \max_{c_j\,,\,\ell_j} \left\{ u(c_j,\ell_j)^{1-\frac{1}{\gamma}} + \beta\left(\psi_{j+1}E\Big[V(z_{j+1})\Big]^{\frac{1-\frac{1}{\gamma}}{1-\mu}} + (1-\psi_{j+1})\mathcal{B}(a_{j+1})\right)\right\}^{\frac{1}{1-\frac{1}{\gamma}}}, \tag{1}$$

where the parameter β denotes the household's discount factor and γ marks the intertemporal elasticity of substitution between present and future consumption. The parameter μ defines the degree of the agent's relative risk aversion.[1] Since agents face an uncertain lifespan, expected utility $E[V(z_{j+1})]$ of the next period is weighted by the survival probability ψ_{j+1}. In the event of death they might benefit from a bequest motive $\mathcal{B}(\cdot)$.

The probability of a j-old agent to have a productivity shock η_{j+1} in the subsequent period conditional on the current productivity shock being η_j is represented by the distribution function $\pi_j(\eta_{j+1}|\ \eta_j)$. Expected utility is then given by

$$E\Big[V(z_{j+1})\Big] = \int_{\mathcal{E}} \pi_j(\eta_{j+1}|\ \eta_j)\cdot V(z_{j+1})^{1-\mu}\,\mathrm{d}\eta_{j+1} \qquad \text{and} \qquad \mathcal{B}(a_{j+1}) = \lambda_1\left(\lambda_2 + \frac{(1+r)a_{j+1}}{\lambda_3}\right)^{1-\frac{1}{\gamma}}$$

denotes the bequest function where the term λ_1 reflects the strength of the bequest motive and λ_2, λ_3 measure the extent to which bequests are conceived as a luxury good, see De Nardi (2004). The period utility function $u(c_j,\ell_j)$ in equation (1) is defined as

$$u(c_j,\ell_j) = \left[c_j^{1-\frac{1}{\rho}} + \alpha\ell_j^{1-\frac{1}{\rho}}\right]^{\frac{1}{1-\frac{1}{\rho}}},$$

[1] Note that for the special case $\mu = \frac{1}{\gamma}$ equation (1) simplifies to the traditional expected utility specification; see Epstein and Zin (1991), p.266.

with ρ denoting the intratemporal elasticity of substitution between consumption and leisure and α marking the age-independent leisure preference parameter.

The representative household maximizes (1) subject to the intertemporal budget constraint

$$a_{j+1} = a_j(1+r) + y_j(1-\tau_1) + p_j + m_j - \tau_2 cb_j + b_j - c_j + \nu_j. \tag{2}$$

We additionally assume that an individual does not hold any assets at birth and does not leave any intentional bequests, i.e. $a_1 = a_{J+1} = 0$. Furthermore, agents face credit market constraints, i.e. $a_j \geq 0 \quad \forall \quad j$. Households receive interest payments from liquid assets held in period j as well as gross labor income $y_j = w(1-\ell_j)e_j$, where e_j defines the agent's individual productivity and w as well as r denote the wage rate for effective labor and the gross interest rate, respectively. Individual productivity e_j is determined by the deterministic age-productivity profile and the productivity shock η_j which are both skill-dependent. Consumption expenditures are given by c_j.

The model features a detailed pension system with one or two tiers. The first tier comprises a basic pension m_j (either means-tested or not) which is financed by contributions levied on labor income with the rate τ_1. Pension benefits p_j from the second tier are earnings related and financed by the contribution rate τ_2 levied on the contribution base

$$cb_j = \min \left\{ \max[y_j - \theta_L \bar{y}\,;\, 0]\,;\, (\theta_H - \theta_L)\bar{y} \right\} \tag{3}$$

which may exempt incomes below a minimum threshold $\theta_L \bar{y}$ as well as incomes above a contribution ceiling $\theta_H \bar{y}$ where θ_L, θ_H denote specific fractions of the average income $\bar{y}$. Benefits from the public pension system are received after passing the mandatory retirement age of j_R.

As we abstract from modeling annuity markets, agents who do not survive leave their positive assets as accidental bequests. These bequests are aggregated for each income (or productivity) class and distributed as individual bequests b_j to employees in the last year before retirement within the respective income class.

Finally, agents may receive (or have to finance) specific compensation payments ν_j which are described in more detail below.

2.3 The production side

A large number of identical firms, the sum of which is normalized to unity, use the factors capital and labor to produce a single good with the Cobb-Douglas production technology

$$Y = \Phi K^{\epsilon} L^{1-\epsilon} \tag{4}$$

with Y, K and L denoting aggregate output, capital and labor, respectively. The parameter ϵ marks the share of capital in production while Φ represents a technology parameter which is adjusted in order to normalize the wage rate of effective labor to unity. Firms maximize their profits renting capital from aggregate private savings and hiring labor from households so that the marginal product of capital equals the market interest rate r plus the depreciation rate of capital δ and the marginal product of labor equals the wage rate for effective labor w, i.e.

$$r = \epsilon \Phi \left(\frac{L}{K} \right)^{1-\epsilon} - \delta \tag{5}$$

$$w = (1-\epsilon)\Phi\left(\frac{K}{L}\right)^{\epsilon} \quad (6)$$

holds.

2.4 The pension system

The public pension system of our model is resting on one or two tiers. The first represents a Beveridge-design system where claims may be means-tested against private assets and second tier pension-income. The second tier is given by a Bismarck-design system which provides individuals with an earnings-related pension, i.e. the level of individual benefits depends on the relative income position of agents during their working life until they reach the mandatory retirement age.

The first pillar of the pension system pays out a flat old-age pension m_j, the amount of which is determined by a replacement rate κ_1 of average labor earnings $\bar{y}$ in the economy and the taper rates φ_a and φ_p which define the means test against liquid assets and second pillar old-age pensions, i.e.

$$m_j = \max\left[\kappa_1\bar{y} - \varphi_a a_j - \varphi_p p_j\,;\,\underline{m}\right]. \quad (7)$$

Consequently, without a means test in place (i.e. $\varphi_p = \varphi_a = 0$), individual pension payments m_j are uniform for all retired agents. If first tier pensions are means-tested, pension claims are calculated as the difference between the flat general pension level $\kappa_1\bar{y}$ and the assessable liquid assets and second tier pensions. Of course, with a means-test in place the amount of m_j depends on individual characteristics of the retiree. The taper rates $\varphi_a, \varphi_p \in [0,1]$ determine the rate of the asset- and pension income-test, respectively: If φ_a (φ_p) takes a value of one, all liquid assets (second tier benefits) are taken into account and the initial first tier benefit $\kappa_1\bar{y}$ is reduced by one Euro for every additional Euro of private assets (second tier benefits) held or received by the agent. The reduction of flat benefits is restricted to $\underline{m}$ which could be zero or any positive figure.

Aggregating over all retirees yields aggregate expenditures on first tier pensions M_t in period t as

$$M_t = \sum_{j=j_R}^{J}\int_{\mathcal{Z}} m_j(z_j, Z_t)\mathrm{d}X_t(z_j). \quad (8)$$

Second tier pensions are earnings-related. In our model, the amount of the individual old age pension p_j depends on the accumulated normalized contributions (so-called "earning points") at retirement ep_{j_R} per working year and the replacement rate κ_2 of average labor earnings $\bar{w}$ in the economy, i.e.

$$p_j = \frac{ep_{j_R}}{j_R - 1} \times \kappa_2\bar{y}. \quad (9)$$

The accumulation of normalized contributions follows

$$ep_j = ep_{j-1} + \frac{cb_j}{(1-\theta_L)\bar{y}} \quad (10)$$

with the accumulation being determined by the individual contribution base relativ to the threshold level. If labor income y_j is below the minimum threshold $\theta_L\bar{y}$, no contributions are paid and no earning points are accumulated. If labor income is between the two threshold levels the accumulated earning points reflect the relative income position. Finally, for labor income above the contribution ceiling $\theta_H \bar{y}$ a maximum earning point is credited. Consequently, without a minimum threshold (i.e. $\theta_L = 0$) a representative agent earning the average income $y_j = \bar{y}$ over his working life accumulates exactly one (normalized) earning point, which in turn yields an individual pension of $p_j = \kappa_2 \bar{y}$. Aggregation gives total expenditures on earnings-related tier pensions P_t in a period t as

$$P_t = \sum_{j=j_R}^{J} \int_{\mathcal{Z}} p_j(z_j, Z_t) \mathrm{d}X_t(z_j). \tag{11}$$

In order to finance aggregate expenditures for flat pensions M_t as well as aggregate earnings-related pension expenditures P_t given by (8) and (11) in every period t, the pension system collects payroll contributions from labor income and the contribution base specified above. Since we assume time-invariant contribution rates to simplify the interpretation of the results, the budgets of the two tiers must be intertemporally balanced only. Consequently, we allow for debt $B_{M,t}, B_{P,t}$ within each tier which is adjusted in every transitional period such that

$$B_{M,t+1} = (1+r)B_{M,t} + M_t - \tau_1 w L_t \quad \text{and} \quad B_{P,t+1} = (1+r)B_{P,t} + P_t - \tau_2 \cdot CB_t \tag{12}$$

hold, where

$$CB_t = \sum_{j=1}^{j_{R-1}} \int_{\mathcal{Z}} cb_j(z_j) dX_t(z_j)$$

denotes the aggregate contribution base. The contribution rates τ_1, τ_2 are then computed from

$$\tau_1 = \frac{\sum_{t=1}^{\infty} M_t(1+r)^{1-t}}{\sum_{t=1}^{\infty} w L_t (1+r)^{1-t}} \qquad \text{and} \quad \tau_2 = \frac{\sum_{t=1}^{\infty} P_t(1+r)^{1-t}}{\sum_{t=1}^{\infty} CB_t (1+r)^{1-t}}.$$

2.5 Welfare and efficiency calculation

In order to assess the welfare effects of a policy reform on different cohorts we use the ex-ante expected utility of an agent before the productivity level is revealed. Consequently, expected utility of a newborn in period t is computed from

$$E\Big[V(z_1, Z_t)\Big] = \left[\int_{\mathcal{Z}} V(z_1, Z_t)^{1-\mu} dX_t(z_1)\right]^{\frac{1}{1-\mu}}$$

where assets and earning points are all zero. In order to compare welfare for a respective individual living in the reform year $t = 1$ before and after the introduction of the pension system, we compute the proportional increase (or decrease) in consumption and leisure ϕ which would make an agent in the baseline economy as well off as after the reform. Due to the homogeneity of the utility function the necessary increase (or decrease) in percent of resources is

$$\phi(z_j, Z_1) = \left[\frac{E\left[V(z_{j+1}, Z_1)\right]}{E\left[V(z_{j+1}, Z_0)\right]} - 1\right] \times 100 \tag{13}$$

Consequently, a value of $\phi(z_j, Z_1) = 1.0$ implies that this agent would need one percent more resources in the initial equilibrium to attain the expected utility level he receives after the introduction of the pension system. Aggregation of the percentage changes $\phi(z_j, Z_1)$ for each skill level s at age j across all asset levels, pension points and productivity shocks yields average welfare changes at age j for alternative skill levels s:

$$\bar{\phi}(s,j) = \frac{1}{N_{j,s}} \int_{\mathcal{A}\times\mathcal{P}\times\mathcal{E}} \phi(z_j, Z_1) dX_0(z_j), \tag{14}$$

For all newborn cohorts entering the labor market during the transition we only report the ex-ante welfare change for the respective cohort.

In order to asses aggregate efficiency consequences, we introduce a Lump-Sum Redistribution Authority (LSRA) in the spirit of Auerbach and Kotlikoff (1987), Fehr and Habermann (2008) or Kudrna and Woodland (2011) in a separate simulation. The LSRA treats those cohorts already existing in the initial equilibrium and newborn cohorts differently. To already existing cohorts it pays a lump-sum transfer (or levies a lump-sum tax) $\nu_j(z_j, Z_1), j > 1$ to bring their expected utility level after the reform back to the level of the initial equilibrium $E\left[V(z_j, Z_0)\right]$.

Since utility depends on age and state, these transfers (or taxes) have to be computed for every agent in the first year of the transition. Consequently, after compensation, their relative welfare change is $\phi^c(z_j, Z_1) = 0.0$. Furthermore, those who enter the labor market in period $t \geq 1$ of the transition receive a transfer $\nu_1(z_1, Z_t, V^*)$ which guarantees them an expected utility level of V^* through a (compensated) relative consumption change $\phi^c(z_1, Z_t)$ which is identical for all newborn future cohorts. Note that the transfers $\nu_1(z_1, Z_t, V^*)$ may differ among future cohorts but the expected utility level V^* is identical for all. This expected utility V^* is chosen so the present value of all LSRA transfers is zero[2]:

$$\sum_{j=2}^{J} \int_{\mathcal{Z}} \nu_j(z_j, Z_1) \mathrm{d}X_1(z_j) + \sum_{t=1}^{\infty} \nu_1(z_1, Z_t, V^*)(1+r)^{1-t} = 0. \tag{15}$$

In the first period of the transition the LSRA builds up debt (or assets) from

$$B_{RA,2} = \sum_{j=1}^{J} \int_{\mathcal{Z}} \nu_j(z_j, Z_1) \mathrm{d}X_1(z_j) \tag{16}$$

which has to be adjusted in each future period according to

$$B_{RA,t+1} = (1+r)B_{RA,t} - \nu_1(z_1, Z_t). \tag{17}$$

Of course, LSRA assets are also included in the asset market equilibrium condition.

If $\phi^c(z_1) > 0$ ($\phi^c(z_1) < 0$), all households in period one who lived in the previous period would be as well off as before the reform and all current and future newborn households would be strictly better (worse) off. Hence, the new policy is Pareto improving (inferior) after lump-sum redistributions.

2.6 Equilibrium conditions

An equilibrium path for a given policy schedule represents a set of value functions $\{V(z_j, Z_t)\}_{j=1}^{J}$, household decisions $\{c_j(z_j, Z_t), \ell_j(z_j, Z_t)\}_{j=1}^{J}$, distributions of unintended bequests $\{b_j(z_j, Z_t)\}_{j=1}^{J}$,

[2] Note that in order to avoid LSRA transfers causing major liquidity effects at any age, transfers are given as an annuity to agents before or after they retire.

measures of households $\{\xi_t(z_j)\}_{j=1}^J$ and relative prices of capital and labor $\{r, w\}$ that satisfy the following conditions $\forall t$:

1. The household decisions $\{c_j(z_j, Z_t), \ell_j(z_j, Z_t)\}_{j=1}^J$ solve the household decision problem (1) subject to the respective constraint (2).

2. Factor prices $\{r, w\}$ are competitive, i.e. (5) and (6) hold.

3. Aggregation holds so that

$$L_t = \sum_{j=1}^{J} \int_{\mathcal{Z}} [1 - \ell_j(z_j, Z_t)] e_j \mathrm{d}X_t(z_j) \tag{18}$$

$$C_t = \sum_{j=1}^{J} \int_{\mathcal{Z}} c_j(z_j, Z_t) \mathrm{d}X_t(z_j) \tag{19}$$

and K_t is derived from (5).

4. Defining $\mathbf{1}_{h=x}$ as an indicator function to return 1 if $h = x$ and 0 otherwise, the following law of motion for the measure of households $\{\xi_t(z_j)\}_{j=1}^J$ holds:

$$\xi_{t+1}(z_{j+1}) = \psi_{j,s} \int_{\mathcal{A}\times\mathcal{P}\times\mathcal{E}} \mathbf{1}_{a_{j+1}=a_{j+1}(z_j,Z_t)} \times \mathbf{1}_{ep_{j+1}=ep_{j+1}(z_j,Z_t)} \pi(\eta_{j+1}|\eta_j) \mathrm{d}X_t(z_j) \tag{20}$$

5. Unintended bequests satisfy

$$\int_{\mathcal{Z}} b_{j_R-1}(z_{j_R-1}, Z_{t+1}) \mathrm{d}X_{t+1}(z_{j_R-1}) = \sum_{j=1}^{J} \int_{\mathcal{Z}} (1 - \psi_{j+1,s})(1+r) a_{j+1}(z_j, Z_t) \mathrm{d}X_t(z_j) \tag{21}$$

6. The budgets of the pension system (12) and the lump-sum redistribution authority (17) are intertemporally balanced.

7. The goods market clears, i.e.

$$Y_t = C_t + \delta K_t + \mathcal{X}_t \tag{22}$$

holds for the small open economy, where $\mathcal{X}_t$ mark the net exports in period t.

3 Calibration of the initial equilibrium

In order to reduce computational time, each model period covers five years. We distinguish three skill classes ($S = 3$) where low-, medium- and high-skilled individuals initially represent 20, 55 and 25 percent of the population. Agents start their life at the age of 20 ($j = 1$), are forced to retire at age 60 ($j_R = 9$) and face a maximum possible life span of 100 years ($J = 16$). The conditional survival probabilities $\psi_{j,s}$ are computed from the year 2050 Life Tables reported in Bomsdorf (2003). Skill-specific differences in life expectancy are initially disregarded and considered in the sensitivity analysis. With respect to the preference parameters we set the intertemporal elasticity of substitution γ to 0.5, the intratemporal elasticity of substitution ρ to 0.6, the coefficient of relative risk aversion μ to 2.0 and the leisure preference parameter α to 1.6. This is within the range of commonly used values (see Auerbach and Kotlikoff, 1987, and Cecchetti et al., 2000) and yields a compensated (uncompensated) wage elasticity of labor supply of 0.55 (-0.02) in our benchmark. Finally, we abstract from a bequest motive and set the time discount factor β to 0.87 (which implies an annual discount rate of about 3 percent), in order to calibrate a realistic capital-output ratio.

With respect to technology parameters we choose the general factor productivity $\Phi = 1.38$ in order to normalize labor income and set the capital share in production ϵ at 0.35. The annual depreciation rate for capital δ is set at 6 percent. Table 1 summarizes the exogenous parameters.

Table 1: Parameter selection

Demographic parameters	Preference parameters	Productivity parameters	Technology parameters	Pension parameters
$N_{1,1} = 0.20$	$\gamma = 0.5$	$\varrho_1 = 0.3304$	$\Phi = 1.38$	$\kappa_1 = \kappa_2 = 0.0$
$N_{1,2} = 0.55$	$\rho = 0.6$	$\varrho_2 = 0.4712$	$\epsilon = 0.35$	$\varphi_a = \varphi_p = 0.0$
$N_{1,3} = 0.25$	$\mu = 2.0$	$\varrho_3 = 0.6284$	$\delta = 0.266$	$\theta_L = \theta_H = 0.0$
$J = 16$	$\alpha = 1.6$	$\varepsilon_1 = 0.1012$		
$j_R = 9$	$\beta = 0.87$	$\varepsilon_2 = 0.1120$		
$\psi_{j,s}$: Bomsdorf (2003)	$\lambda_1 = 0.0$	$\varepsilon_3 = 0.1467$		

Log-productivity for an individual of skill class s evolves over the life-cycle according to

$$\log e_j = \varsigma_0 + \varsigma_1 \cdot j + \varsigma_2 \cdot j^2/100 + \eta_j$$

where η_j follows an AR(1) process of the form

$$\eta_j = \varrho\eta_{j-1} + \varepsilon_j \quad \text{with} \quad \varepsilon_j \sim N(0, \sigma_\varepsilon^2).$$

The parameters for the skill-specific productivity profiles are derived from German household data, the estimation procedure is described in Fehr, Kallweit and Kindermann (2011). With rising skill level the AR(1) correlation coefficient ϱ increases from 0.3304 to 0.6284, while the transitory variance rises from 0.1012 to 0.1467.

Table 2 reports some key numbers of the resulting benchmark equilibrium. Note that we abstract from skill-specific mortality rates initially. The model's income and wealth distribution is fairly equal, which simply reflects the fact that our model does not capture the extreme ends of the income distribution. Note that in all skill classes agents of the youngest cohort with negative productivity shocks would like to borrow because they expect a higher productivity (and therefore income) in the future. For older cohorts, the fraction of liquidity constraint agents decreases sharply and we hardly observe liquidity constrained households older than 40 years. Aggregate bequests of 9.7 percent of GDP are purely accidental since annuity markets are missing and we abstract from a bequest motive, i.e. $\lambda_1 = 0.0$ holds.

4 Simulation results

Based on the initial equilibrium without a pension system in period zero we introduce pension systems of various designs in period 1 and compute the resulting welfare changes without and with

Table 2: The initial equilibrium

Life expectancy	Interest rate	Gini index income	Liquidity constraints (20-29)
12.1[a]	2.4[c]	0.40	65[d]
Dependency ratio	Capital-output ratio	Gini index wealth	Liquidity constraints (30-39)
52.6[b]	4.5	0.56	23[d]

[a] i.e. 80.5 years. [b] ages 60+/20-59. [c] p.a. in %. [d] in %.

compensation payments. In the first two subsections we focus on pure Beveridge- and Bismarck-design pension schemes. First, we concentrate on flat pensions and highlight macroeconomic and welfare consequences of means-tested benefits. In the second subsection we compare the optimal replacement rates of either flat or earnings-related pensions and explore the effects of restricting the contribution base. The third subsection moves on towards two-tier systems which may be integrated in various forms. The last subsection presents some sensitivity analysis for our simulation results. In order to identify the optimal design, we compare the resulting aggregate efficiency effects from different institutional settings.

4.1 Optimal design of flat pensions

Before we discuss the results of our initial simulations it is useful to abstract from labor supply distortions and liquidity constraints in a thought experiment in order to highlight the insurance properties of the pension system. If employees were allowed to borrow against their future pension benefits, the optimal contribution rate of a flat pension system would be 100 percent. In this case everybody receives the same income during years of retirement and all income and longevity uncertainty is completely eliminated.

As soon as liquidity constraints and variable labor supply are taken into account, the optimal replacement rate (which determines the contribution rate) drops significantly as it now has to balance the benefits from income and longevity insurance against the losses from increased liquidity constraints and labor market distortions. The left part of Table 3 reports the resulting consequences of introducing a flat rate pension with a replacement rate of 40 percent of average income (i.e. $\kappa_1 = 0.4$) on labor supply, consumption and savings. In the left part of the Table we disregard any means-testing while in the right part a full means-test (i.e. $\varphi_a = 1.0$) is applied.

Table 3: Macroeconomic effects of flat benefits without and with asset means-test[a]

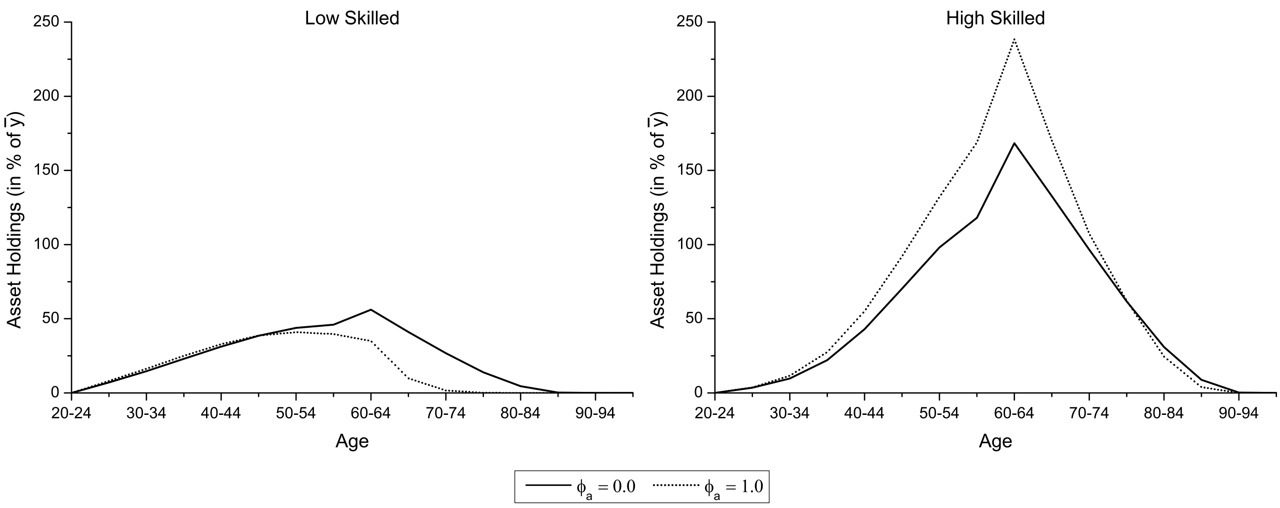
	Without means-test ($\varphi_a = 0.0$)				Full means-test ($\varphi_a = 1.0$)			
Period	Labor supply	Consumption	Private assets	Pension assets[b]	Labor supply	Consumption	Private assets	Pension assets[b]
1	-15.2	19.5	0.0	0.0	-10.4	13.8	0.0	0.0
3	-4.3	5.2	-36.3	0.0	-3.9	2.9	-35.4	11.8
5	-1.2	-4.3	-53.8	0.0	-2.2	-5.7	-54.0	20.3
∞	0.1	-11.9	-63.8	0.0	-2.5	-9.9	-62.5	24.7

[a]Changes are reported in percentage over initial equilibrium, $\kappa_1 = 0.4$. [b] In percent of GDP

If universal annual benefits amount to 40 percent of average income, the contribution rate τ_1 rises to 21.1 percent. As a consequence, labor supply and employment (as well as the capital stock and GDP) decreases sharply initially by more than 15 percent while consumption rises and private savings fall. In the transitional years younger individuals receive less bequests so that private consumption also falls by more than 10 percent in the long run compared to the initial level. The reduction of future income also reduces leisure demand and increases employment again. Changes in employment directly affect average income so that pension benefits adjust accordingly. Since the demographic structure remains unchanged, aggregate benefits in each period can be financed with a constant contribution rate so that the pension system does not need to build up resources for the future.

In the right part of table 3 further distortions are introduced by considering a system which tests pension payments against private asset holdings. Since now many medium- and high-skilled households receive zero or reduced pensions benefits, the contribution rate τ_1 only rises to 9.7 percent although the replacement rate remains unchanged at $\kappa_1 = 0.4$. Due to the lower contribution rate, labor supply and employment is reduced by a lesser amount initially, but now employment also falls significantly in the long run. This is surprising on first glance since both lower bequests and lower contribution rates should tend to increase labor supply. However, with means-tested pension benefits especially low- and middle-skilled households reduce labor supply sharply before retirement and consume their assets in order to increase their individual pension benefits. Figure 1 compares the savings profiles of low- and high-skilled households for universal and asset-tested pension schemes. While poor households decrease their (already low) savings due to the asset test, rich households save more since they do not receive pension benefits, see the right part of Figure 1.

Figure 1: Asset accumulation without and with means-testing



Consequently, our model generates the well documented result from the empirical literature that means-tested social insurance programs may be a part of the explanation why rich households save more than poor households, see Hubbard, Skinner and Zeldes (1995) or Dynan, Skinner and Zeldes (2004). In addition, our results are broadly in line with the results from Sefton, van de Ven and Weale (2008), who document that households react quite differently to changes in the asset test regime of retirement benefits depending on their position in the income distribution. Rising taper rates encourage poor households to save less while prompting richer agents to save a higher amount compared to a situation with universal benefits.

In the present parametrization the reaction of rich households seems to dominate so that aggregate savings decrease less than in the previous simulation without means-testing. The savings reduction of poor households also explains why the pension system now has to accumulate assets during the transition. Initially, private assets are given so that aggregate pension benefits are significantly lower. During the transition, low income households get rid of their assets before retirement so that aggregate pension benefits increase steadily. With constant contribution rates the latter has to be financed from the returns of pension assets.

Next we turn to welfare consequences for different cohorts in the reform year and the long run without and with compensation payments from the LSRA. As already explained above, we first compute the welfare changes of agents before their productivity is revealed and then derive an average welfare change for the different skill types in each cohort that already lives in the initial equilibrium. Therefore, Table 4 distinguishes in each cohort between “poor",“median", and “rich" households. “Poor" agents are the 20 percent of the cohort with the lowest skill level, “median" are those 55 percent who have a medium skill level and “rich" are those 25 percent of the cohort with the highest skills. For newborn cohorts along the transition path we are not able to disaggregate ex-ante welfare effects. Consequently, we report in the middle column the ex-ante welfare change of the whole cohort. The left part of Table 4 reports the results for the simulation without means-testing, while the right part reports the results of the simulation with means-testing.

Table 4: Welfare effects of flat benefits without and with means-testing[a]

Age in	Without means-test ($\varphi_a = 0.0$)				Full means-test ($\varphi_a = 1.0$)			
reform	Consumers			compen-	Consumers			compen-
year	poor	median	rich	sated	poor	median	rich	sated
75-79	45.9	39.2	29.1	0.0	25.3	20.8	13.8	0.0
65-69	35.1	29.9	22.0	0.0	16.6	13.1	8.1	0.0
45-49	8.9	6.6	3.5	0.0	3.3	1.1	-1.0	0.0
25-29	-4.2	-4.8	-5.6	0.0	-1.9	-3.0	-3.7	0.0
20-24		-6.1		0.8		-3.2		-1.5
∞		-6.2		0.8		-3.3		-1.5

[a]Changes are reported in percentage of initial resources, $\kappa_1 = 0.4$.

Not surprisingly, existing retirees benefit dramatically in both simulations, while future cohorts lose significantly. In addition, due to the progressive system, the benefit for poor households is much stronger than the one for rich households. Since the generosity of the system is reduced, means-testing mainly reduces the implied intergenerational redistribution. Older cohorts benefit significantly less since they receive lower pensions due to the asset test. At the same time, younger and future cohorts lose less since they benefit from lower contributions.

In order to assess the welfare implications of alternative pension designs, studies such as Nishiyama and Smetters (2008) or Kumru and Piggott (2009) neglect transitional issues and focus on the long run welfare effects. As shown in our simulation exercise, future cohorts typically lose after the introduction of a pension system but they lose less when the system is means-tested. Consequently, Kumru and Piggott (2009) conclude that the pension system should be fully means-tested. However, since transitional cohorts are neglected, long-run welfare effects themselves are not an adequate means of assessing the impact of the pension system on resource allocation. In order to quantify the efficiency consequences, one has to compare the transitional welfare gains and long run welfare losses. Consequently, we simulate both reforms with lump-sum compensation payments of the LSRA.[3] The compensated welfare changes for all generations alive in the initial equilibrium are then zero and newborn generations experience identical relative consumption increases or decreases. As shown in the forth column, the introduction of the pension system without means-testing increases aggregate efficiency by 0.8 percent of remaining resources. The positive efficiency effect is due to the fact that the benefits from income and longevity insurance dominate the efficiency costs from higher liquidity constraints and labor supply distortions. However, as reported in the right column, aggregate efficiency decreases by 1.5 percent when benefits are fully tested against private assets. Even though the means test reduces the total size of the system considerably which in turn yields lower labor supply and liquidity distortions through a lower contribution rate, strong additional distortions are introduced due to the asset test, completely offsetting the positive effects. This results in a decline in aggregate efficiency by 2.3 percentage points reported above. This is in line with Kudrna and Woodland (2011) who find that the abolition of a means test in Australia yields an efficiency gain of 1.2 percent of aggregate resources.

Recently, the UK taper rate for means-tested retirement benefits was reduced from 100 to 40 percent. Sefton, et al. (2008) have analyzed the long-run effects of this policy reform using a partial equilibrium life cycle model. They find that labor supply and savings of the poor increase significantly around retirement while middle-income households react in the opposite way. On aggregate, employment increases while savings decrease slightly. In the next step we explore the introduction of a means-tested pension system with a taper rate of only 40 percent. Comparing the macroeconomic effects in the right part of Table 3 and the left part of Table 5 we find exactly the same aggregate pattern. Employment falls slightly less with the reduced taper rate while assets are reduced even further in the long-run. Since benefits are now more generous, the contribution rate rises from 9.7 to 10.8 percent and the system has to build up less pension wealth. As one would expect, higher pension benefits increase the intergenerational redistribution from the currently young and future cohorts towards the currently elderly. The left part of Table 6 shows that welfare of existing pensioners rises significantly while long-run welfare decreases further (compared to the right part of Table 4). Kumru and Piggott (2009), who also find a reduction in long-run welfare due to the lowering of the taper rate, conclude that a 100 percent taper rate is optimal. However, if we take the welfare consequences for transitional cohorts into account, then the reduction of the taper rate is clearly efficiency enhancing. As shown in Table 6, the aggregate efficiency loss decreases from 1.5 percent of aggregate resources (with taper rate 1.0) to 1.3 percent (with taper rate 0.4).

Next, we keep the reduced taper rate and introduce a basic pension which limits the maximum benefit reduction to 25 percent. Since pension benefits are now much higher, the contribution rate increases from 10.8 to 17.1 percent. The right part of Table 5 shows that despite this significant rise in the contribution rate labor supply is reduced much less in the long run. Of course, this is due to the fact that the incentives to reduce income and savings right before retirement are now much lower. Consequently, private assets fall less than before and the required accumulation of pension wealth is much smaller.

The right part of Table 6 shows that the basic pension increases intergenerational redistribution to-

[3] We do not report the macroeconomic effects of simulations with compensation payments, but they are available on request.

Table 5: Macroeconomic effects of means-tested benefits without and with basic pension[a]

Period	Without basic pension ($\underline{m} = 0$)				With basic pension ($\underline{m} = 0.75\kappa_1\bar{y}$)			
	Labor supply	Consump-tion	Privat assets	Pension assets[b]	Labor supply	Consump-tion	Private assets	Pension assets[b]
1	-11.5	16.1	0.0	0.0	-12.9	17.3	0.0	0.0
3	-4.6	2.7	-38.7	11.9	-4.0	4.9	-33.2	1.5
5	-2.7	-6.6	-57.9	19.4	-1.4	-3.6	-50.7	3.2
∞	-2.2	-10.4	-64.5	22.2	-0.4	-11.0	-61.6	5.2

[a]Changes are reported in percentage over initial equilibrium, $\kappa_1 = \varphi_a = 0.4$. [b] In percent of GDP

wards the existing elderly. Studies which focus on long-run effects would therefore advise against the introduction of a basic pension. However, as the last column in Table 6 shows, due to lower labor supply and savings distortions, aggregate efficiency increases significantly after the introduction of the basic pension. While we still find that means-testing decreases economic efficiency, the combination with a basic pension at least generates an improvement of the resource allocation compared to the situation without a pension system in place.

Table 6: Welfare effects of means-tested benefits without and with basic pension[a]

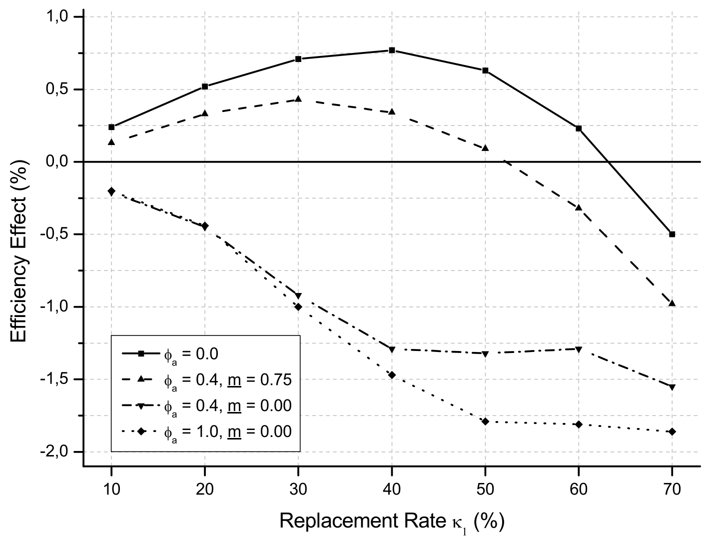
Age in	Without basic pension ($\underline{m} = 0$)				With basic pension ($\underline{m} = 0.75\kappa_1\bar{y}$)			
reform	Consumers			compen-	Consumers			compen-
year	poor	median	rich	sated	poor	median	rich	sated
75-79	30.0	24.3	16.1	0.0	40.2	34.2	25.1	0.0
65-69	18.7	14.8	9.4	0.0	29.9	25.3	18.4	0.0
45-49	3.9	1.6	-0.8	0.0	7.1	5.1	2.4	0.0
25-29	-2.0	-3.1	-3.9	0.0	-3.5	-4.1	-4.7	0.0
20-24		-3.5		-1.3		-5.0		0.3
∞		-3.5		-1.3		-5.1		0.3

[a]Changes are reported in percentage of initial resources, $\kappa_1 = \varphi_a = 0.4$.

Of course, the question is how efficiency effects change when we alter the size of the pension system. Figure 2 shows the aggregate efficiency effects for alternative replacement rates with and without means-testing and basic pensions.

Consider first the straight line which shows the benchmark case of a universal benefit system. With low replacement rates the benefits from insurance dominate the losses due to reduced liquidity and higher labor supply distortions so that higher replacement rates imply a rise in economic efficiency. The highest efficiency gains are realized in a system with a replacement rate of $\kappa_1 = 0.4$ which was discussed above. If the size of the system increases further efficiency gains decrease since now the cost from reduced liquidity and increased labor market distortions dominate the positive insurance effects. With a full means-test in place, aggregate efficiency effects turn negative for all replacement rates considered. Figure 2 also shows that efficiency losses increase sharply initially but then remain constant at roughly 2 percent of aggregate resources when the replacement rate is increased beyond 50 percent. This might reflect a balancing of the additional direct savings distortions by reduced

Figure 2: Universal vs. means-tested flat pensions



labor supply distortions due to reduced contribution rates. Figure 2 also shows that the reduction in the taper rate is efficiency enhancing as long as the replacement rate is beyond 40 percent. Finally, the introduction of the basic pension further increases economic efficiency. However, the highest efficiency gains are achieved without means-testing.

Summing up the results of this section we conclude that means-testing introduces dramatic savings and labor supply distortions and consequently should be abandoned. Previous studies such as Sefton and van de Ven (2009) or Kumru and Piggott (2009) which came to an opposite conclusion are biased since they neglect the welfare consequences of means-testing for transitional cohorts. We find an optimal replacement rate for flat pensions of roughly 40 percent of average labor earnings. It should be clear that the optimal replacement rate increases with relaxed liquidity constraints and with an increasing degree of risk aversion. On the other hand, with reduced risk aversion the optimal replacement rate will be quite lower. For example, with risk neutral preferences (i.e. $\mu = 0.0$) the optimal replacement rate without means-testing amounts to 30 percent and the respective aggregate efficiency gain is reduced to 0.3 percent of aggregate resources. This fall in aggregate efficiency is due to a complete disregard of the income insurance gains. Introducing the flat pension still yields positive effects from insurance against longevity.

4.2 Optimal earnings-related pensions and the structure of two-tier systems

In this section we focus on earnings-related benefits within single and multi-tier systems. The left part of Table 7 reports the macroeconomic consequences when we introduce a single-tier Bismarckian system with a replacement rate of 40 percent (i.e. $\kappa_2 = 0.4$). The resulting contribution rate is 20.9 percent. We can directly compare the macroeconomic consequences in the left parts of Tables 3 and 7. While assets and consumption react very similar, labor supply increases significantly in the long run with earnings-related pensions, since contributions to the latter imply substantially lower labor market distortions.

Table 7: Macroeconomic effects of earnings-related benefits without and with flat tier[a]

Period	Without flat tier ($\kappa_1 = 0.0, \kappa_2 = 0.4$) Labor supply	Consump-tion	Private assets	Pension assets[b]	With flat tier ($\kappa_1 = 0.2, \kappa_2 = 0.2$) Labor supply	Consump-tion	Private assets	Pension assets[b]
1	-7.3	24.5	0.0	0.0	-10.9	22.3	0.0	0.0
3	1.5	10.0	-34.5	-0.4	-1.3	7.7	-35.3	-0.5
5	4.2	0.6	-52.6	0.5	1.6	-1.8	-53.2	0.1
∞	5.6	-7.1	-63.6	0.8	2.9	-9.5	-63.9	0.3

[a]Changes are reported in percentage over initial equilibrium, $\theta_H = \infty$. [b] In percent of GDP

Comparing the left parts of Tables 4 and 8 indicates that flat and earnings-related benefits induce almost the same intergenerational redistribution. However, low-skilled households benefit more (are hurt less) by a system with flat benefits while high-skilled households benefit more (are hurt less) from earnings-related benefits. This is hardly surprising. Next, we compare the aggregate efficiency effects of the universal flat benefit (i.e. without means-testing) and the earnings-related benefit. Of course, the earnings-related system reduces labor supply distortions, but it also reduces the benefits from the insurance provision against income uncertainty. So on first glance it is not clear whether the earnings-related system outperforms the flat benefit system in terms of economic efficiency. As it turns out, given a replacement rate of 0.4 for both systems, earnings-related benefits slightly dominate flat benefits in terms of economic efficiency. The latter increases from 0.8 percent to 0.9 percent of economic resources.

Table 8: Welfare effects of earnings-related benefits without and with flat tier[a]

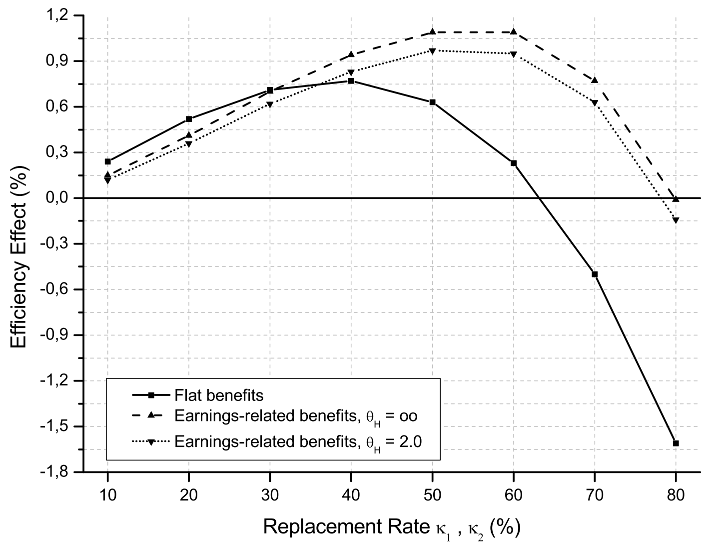
Age in reform year	Without flat tier ($\kappa_1 = 0.0, \kappa_2 = 0.4$) Consumers poor	median	rich	compen-sated	With flat tier ($\kappa_1 = 0.2, \kappa_2 = 0.2$) Consumers poor	median	rich	compen-sated
75-79	40.9	38.0	33.1	0.0	43.8	38.9	31.3	0.0
65-69	31.1	28.9	25.1	0.0	33.4	28.6	23.7	0.0
45-49	6.2	5.5	4.5	0.0	7.6	6.2	4.1	0.0
25-29	-4.9	-5.0	-5.0	0.0	-4.5	-4.9	-5.2	0.0
20-24		-6.2		0.9		-6.1		1.0
∞		-6.5		0.9		-6.3		1.0

[a]Changes are reported in percentage of initial resources, $\theta_H = \infty$.

Of course, the question is whether this last result holds for all replacement rates. Consequently, Figure 3 compares the resulting aggregate efficiency effects of the two systems for different replacement rates. With low replacement rates, benefits from insurance provision dominate labor supply distortions so that the flat-benefit system slightly outperforms the earnings-related benefit system. However, with an increasing size of the system the additional insurance gains become smaller while at the same time the additional costs from labor supply distortions rise. Consequently, with higher replacement rates the earnings-related system dominates in terms of economic efficiency. Of course, efficiency gains decrease finally in both systems but it should be clear from Figure 3 that the optimal replacement rate is clearly higher in the earnings-related system than in the flat benefit system.

Note that there exists a clear negative correlation between the progressivity of existing pension systems and their generosity. Previous explanations such as Conde-Ruiz and Profeta (2007) or Koethenbuerger et al. (2008) apply political economy arguments in order to explain this empirical observation. Our simulations indicate that pure efficiency arguments can be applied as well. Our result is also robust for alternative risk aversions as long as there exists some degree of risk aversion.[4] Of course, with a lower degree of risk aversion efficiency gains and (most likely) also the respective optimal replacement rates of the flat and earnings-related system fall.

Figure 3: Flat vs. earnings-related benefits



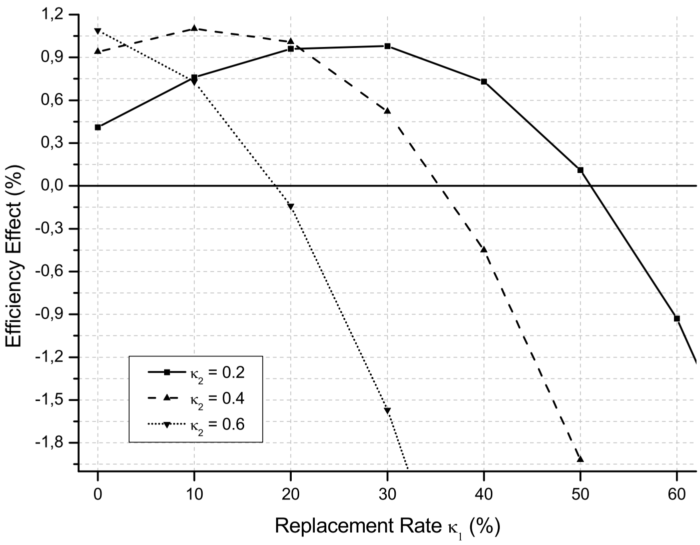
Finally, the dotted line in Figure 3 shows the situation where a contribution ceiling is applied amounting to the double of average labor income as in Germany ($\theta_H = 2.0$). Since the cutback of the contribution base also reduces the benefit level, the contribution rate remains constant at 20.9 percent. Rich households now contribute less and receive lower benefits. Therefore, they save more compared to the simulation without a contribution ceiling. At the same time elderly rich households benefit slightly less while younger rich households are hurt slightly less due to the introduction of the ceiling. Poor and medium income households are hardly affected. With a replacement rate of 40 percent the aggregate efficiency effect is slightly reduced from 0.9 to 0.8 percent. Figure 3 shows that this reduction in aggregate efficiency remains constant for all replacement rates considered.

The previous analysis has shown that the optimal replacement rate of a purely earnings-related tier is at 60 percent yielding an aggregate efficiency gain of 1.05 percent of aggregate resources. Of course, such an earnings-related system does hardly provide any insurance against income risk so that the question remains whether a combination of a flat and an earnings-related system could improve economic efficiency. In Figure 4 we try to answer that question.

The optimal replacement rate from the previous subsection with a purely earnings-related system (i.e. $\kappa_1 = 0.0, \kappa_2 = 0.6$) is shown as the top left point of the dotted line with an aggregate efficiency gain of 1.05 percent. The introduction of supplementary flat benefits obviously decreases economic

[4] Detailed simulation results with risk-neutral preferences are available upon request.

Figure 4: Two-tier systems without means-testing



efficiency immediately, since additional labor market distortions (due to rising contribution rates) dominate insurance gains.

Next, the generosity of the earnings-related system is reduced by considering replacement rates of 40 and 20 percent, respectively. Now economic efficiency rises initially with an increasing replacement rate of the flat tier.[5] As a consequence, the optimal replacement rate for the first tier rises to 10 and 30 percent given a second tier replacement rate of 40 and 20 percent, respectively. The combined optimal replacement rate of the two tiers therefore never increases beyond 60 percent of average labor income. Alternative combinations of the two tiers yield very similar aggregate efficiency gains of roughly one percent of aggregate resources in the respective optimum.

In Tables 7 and 8 we compare the macroeconomic and welfare effects of a pure earnings-related system (with replacement rate $\kappa_2 = 0.4$) and a mixed system (with replacement rates $\kappa_1 = \kappa_2 = 0.2$). Figure 4 shows that both yield very similar aggregate efficiency effects of roughly 1.0 percent of aggregate resources. While the aggregate contribution rate is quite similar in both systems (20.9 in the single-tier and 10.5 for each in the two-tier) employment is much lower with the two-tier system, since the flat tier distorts labor supply significantly. Table 8 also reveals that low-income households are slightly better off with the two-tier system which should not be surprising.

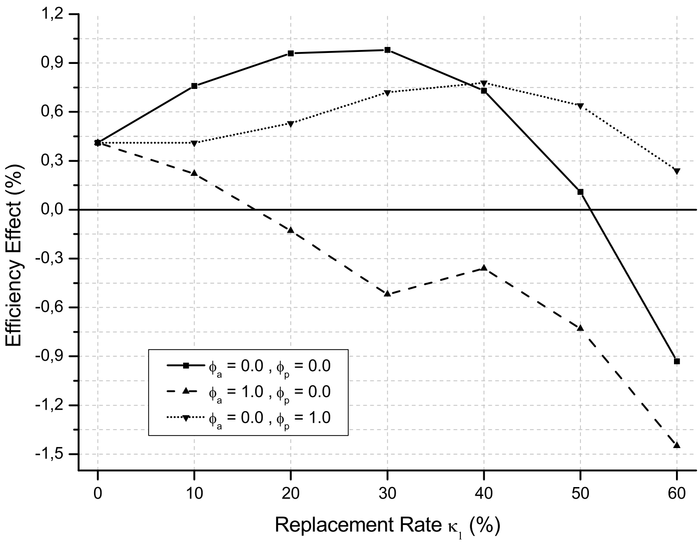
Up to now we have abstracted from means-testing. Of course, in a two-tier system it is possible to test flat benefits against private assets but also against benefits from the earnings-related second tier. Both tests allow to better target benefits to the poor and to reduce total outlays as well as contribution rates, so that labor market distortions decrease. However, while the asset test mainly distorts savings incentives of low income households (see the discussion above), the pension test introduces an additional source of labor supply distortions. With a pension test in place working more will earn additional benefits from the earnings-related tier but at the same time decrease benefits from

[5] Note again that the respective simulations without a flat tier from Figure 2 is also included in Figure 3 (i.e. when $\kappa_1 = 0.0$).

the flat tier. Consequently, agents especially in the low income end of the distribution have incentives to consume additional leisure, especially prior to retirement. In order to compare the efficiency consequences of asset- and pension-testing, Figure 5 fixes the replacement rate of the second tier at 20 percent and introduces a flat tier with alternative replacement rates, which is either means-tested against private assets (i.e. $\varphi_a = 1.0$, $\varphi_p = 0.0$) or against second tier benefits (i.e. $\varphi_a = 0.0$, $\varphi_p = 1.0$). For a better illustration the universal two-tier system from Figure 4 is also included.

As one would expect from the discussion above, aggregate efficiency declines significantly with asset-testing. Note, however, that with rising replacement rates the induced distortions from means-testing decrease, i.e. the solid and the dotted line reconvene.

Figure 5: Two-tier systems with means-testing ($\kappa_2 = 0.2$)



With a pension test of first tier benefits in place, economic efficiency also decreases for low replacement rates. However, if the replacement rate of the flat tier increases then aggregate efficiency also rises. If the replacement rates rises above 40 percent, pension-tested systems show higher aggregate efficiency gains than the universal counterfactuals. This is a surprising result on first sight, but it can be explained quite intuitively. Note that a means test against the second tier has hardly any direct impact on private savings. It mainly affects labor supply due to two reasons. First, the contribution rate of the first tier declines (which reduces labor supply distortions) and second, the implicit tax of the second tier increases (which increases labor supply distortions). For low replacement rates the second effect dominates the first so that aggregate efficiency decreases and for higher replacement rates it works in exactly the opposite direction. In order to better understand the difference between asset- and pension-testing, Table 9 compares the contribution rates of the two-tier schemes from Figure 5.

First, note that neither the rising replacement rate of the flat tier nor the introduction of means-testing does significantly affect the contribution rate of the second tier. If agents work less they receive lower earning points and pensions so that the relation between pension outlays and the contribution base remains fairly stable. The introduction of a universal flat tier dramatically increases labor supply distortions which explains the efficiency loss shown in Figures 4 and 5. Of course, means-testing reduces benefits of the first tier and, consequently, contribution rates decrease. Table 9 also reveals

Table 9: Contribution rates of alternative two-tier schemes [a]

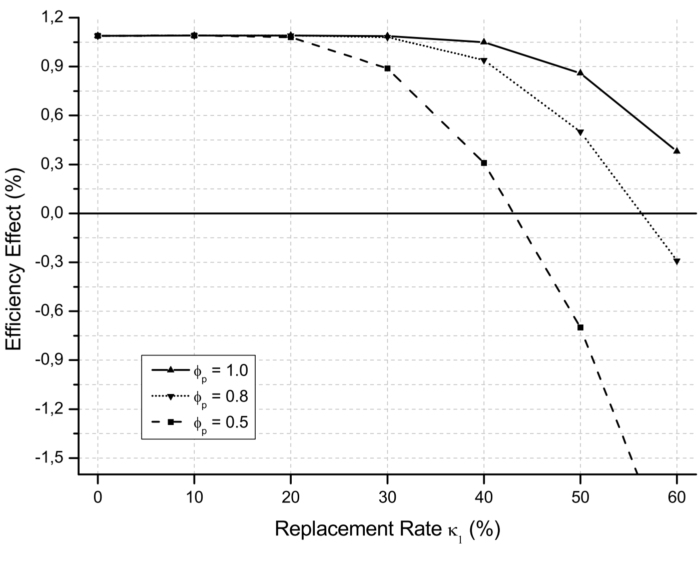
replacement rate (κ_1)	universal τ_1	universal τ_2	asset-tested τ_1	asset-tested τ_2	pension-tested τ_1	pension-tested τ_2
10	5.3	10.5	0.5	10.5	0.0	10.5
20	**10.5**	**10.5**	2.9	10.5	1.2	10.5
30	15.8	10.5	8.7	10.5	5.5	10.5
40	21.1	10.4	14.0	10.5	**10.6**	**10.5**
50	26.3	10.4	19.8	10.4	15.9	10.5
60	31.6	10.4	25.3	10.4	21.2	10.4

[a] In percent, $\kappa_2 = 0.2$.

that pension-testing reduces contribution rates further than asset-testing. Of course this is due to the fact that in the present model without bequest motives older cohorts completely run down their assets. Consequently, elderly cohorts will receive rising benefits from the first tier with asset-testing in place while flat benefits will be independent of age with pension-testing. Lower contribution rates and lower savings distortions explain why pension-testing outperforms asset-testing in terms of economic efficiency in Figure 5.

With Norway and Finnland two Scandinavian countries operate a two-tier system where flat benefits are pension-tested against the earnings-related second tier. In both countries second tier pensions replace at least 60 percent of previous earnings for an average earner while the flat pension amounts to roughly 30 percent of average income. The basic pension is means-tested against pension benefits from the earnings related scheme. While in Norway the taper rate amounts to 80 percent, Finnland only withdraws the basic pension by 50 cent for each additional Euro of earnings-related benefits, see OECD(2011). In order to understand the impact of variations in the taper rate, Figure 6 compares the aggregate efficiency effects of a fairly generous second tier (i.e. $\kappa_2 = 0.6$) which is combined with a pension-tested flat tier with alternative taper rates.

Figure 6: Pension-testing with alternative taper rates



The top left position shows again the situation – as in Figures 3 and 4 – where a single earnings-related tier with a replacement rate of 60 percent generates an aggregate efficiency gain of 1.1 percent. Without pension-testing the introduction of a flat tier immediately decreases economic efficiency, see Figure 4. Figure 6 reveals that pension-testing has a positive effect on economic efficiency. With a taper rate of 100 percent (i.e. $\varphi = 1.0$) economic efficiency remains initially constant and then decreases slightly when the replacement rate is higher than 40 percent. The reduction of the taper rate reduces economic efficiency so that the profile in Figure 6 approaches the respective profile in Figure 4 without a pension test. We therefore conclude that the higher taper rate in Norway seems to outperform the Finnish taper rate in terms of economic efficiency.

4.3 Sensitivity analysis

Of course, the numerical results reported above strongly depend on the specific parametrization and assumptions of our simulation model. With respect to the aggregate efficiency consequences mainly liquidity effects and distortions of labor supply and (with asset-testing) savings are negative while the provision of insurance against income and longevity risk has positive impacts. Consequently, aggregate efficiency curves are shifted upwards if we increase income uncertainty[6], relax borrowing constraints and dampen the elasticity of labor supply. At the same time, lower risk aversion, the existence of private annuity products or a progressive tax system would shift all efficiency curves downwards.

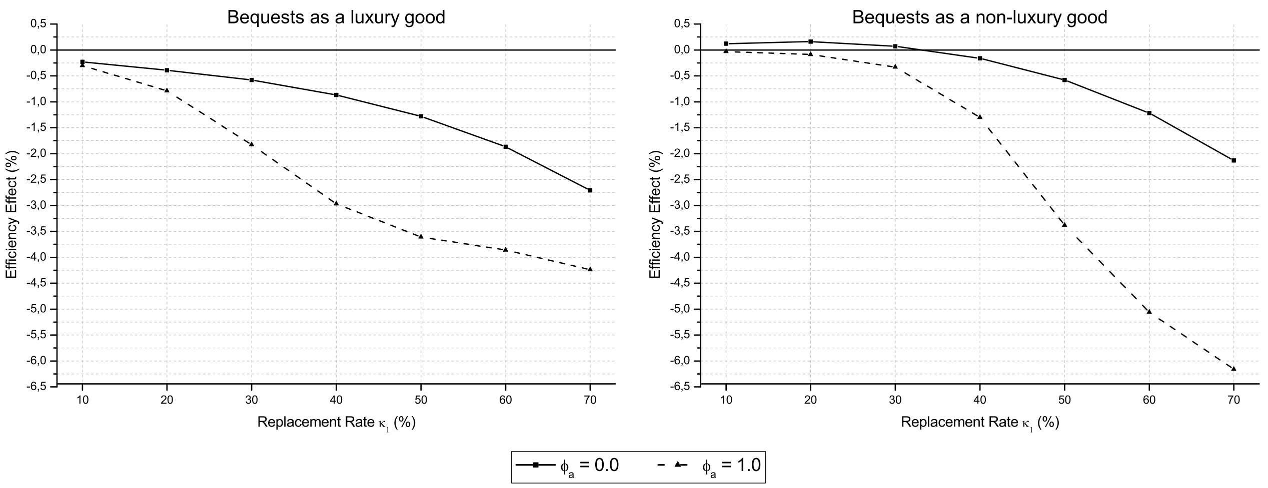
In the following sensitivity analysis we consider the impact of bequest motives and differential mortality. Both aspects seem to be relevant in practice and have not been covered so far by our model. If households have bequest motives of any kind, the positive effect of a given pension scheme regarding insurance provision against longevity is significantly reduced since households also benefit from positive asset balances which remain after death. The slower downsizing of assets at old age also changes the efficiency effects of asset-tested pension benefits.

In the following we explore in two separate simulation exercises the impacts of intentional bequests when bequests are either conceived as a luxury good and only rich agents have a bequest motive (i.e. $\lambda_1 = 13.2$, $\lambda_2 = 1$, $\lambda_3 = 10$) or all agents have a bequest motive (i.e. $\lambda_1 = 0.5$, $\lambda_2 = 0$, $\lambda_3 = 1$). The parameters determining the properties of the respective bequest motive are set to yield the same aggregate bequests in both situations. Of course, due to the bequest motive aggregate savings as well as bequests are higher in the initial equilibrium compared to the initial equilibrium considered before. Nevertheless, as we model a small open economy, higher savings are only affecting the trade balance so that the capital-output ratio in the economy remains the same as in Table 2.

The left part of Figure 7 compares the aggregate efficiency effects of introducing a universal vs. an asset-tested flat pension when intentional bequests modeled as a luxury good, i.e. only rich agents leave intentional bequests while poor agents behave in the same fashion as in the situation without a bequest motive. Obviously, due to the reduced (or even eliminated) longevity insurance provision, the universal pension system now yields much lower (and even negative) aggregate efficiency effects compared to the respective economy without a bequest motive considered in Figure 2. Without a bequest motive, asset-testing in Figure 2 mainly distorts the savings behavior of low- and medium-

[6] There is reason to believe that the present income process underestimates income risk since it does not include unemployment and disability risk during employment and long-term care risk during retirement.

Figure 7: Universal vs. means-tested flat pensions with bequest motives



income households while rich households only realize income effects due to lower contribution rates. Consequently, this distortion pattern is not affected by a bequest motive that only applies to rich households so that the loss in aggregate efficiency due to asset-testing in the left part of Figure 7 is quite similar as in Figure 2.

The right part of Figure 7 shows the same simulation exercise with all agents leaving small intentional bequests while aggregate bequests are the same as in the left part. The introduction of asset-testing has now very little efficiency impacts with low replacement rates since poor households have no incentive to change their saving behavior. With rising replacement rates more and more agents choose to run down their asset holdings faster in order to maximize their pension claims despite the present bequest motives. Consequently, the resulting aggregate efficiency loss due to asset-testing is even higher than before.

Therefore, the introduction of a bequest motive does not automatically affect the efficiency losses due to asset-testing. Only if poor households would like to leave bequest and if the replacement rate of the pension system is low then efficiency losses induced by asset-testing are dampened significantly.

In a second sensitivity test for our results we analyze the impact of skill-specific life expectancy. More specifically, we assume that high-skilled individuals can expect to live 2.5 years longer than the medium-skilled while life expectancy of low-skilled agents is 2.5 years lower than that of the medium-skilled. At first glance one would expect the high-skilled to benefit significantly more and the low-skilled to benefit less from the introduction of a pension system due to higher (lower) longevity insurance gains. However, one also has to take into account that saving rates now also rise significantly with skill level which dampens the impact of the pension system. Consequently, macroeconomic and welfare effects of pension systems do hardly change when we consider differential mortality.[7]

[7] Of course, simulation results are again available upon request.

5 Discussion

This paper aims at analyzing the optimal design of paygo-financed public pension systems. We apply an overlapping generations model that features realistic labor income and longevity risk, liquidity constraints and endogenous labor supply. Compared to previous studies that focus on optimal pension design, the present approach computes the whole transition path between steady states without and with social security. In addition, our study isolates the aggregate efficiency effects of the considered reforms in a separate simulation with compensating transfers. The latter are used to derive the following major quantitative and qualitative results.

First, our results highlight dramatic efficiency losses induced by asset means-testing of flat pensions due to substantial savings distortions. Previous studies which found a positive role for asset means-testing on long-run welfare grounds are misleading since they neglect transitional periods. Second, we find a trade-off between progressivity and generosity of the optimal design, i.e. the optimal replacement rate of an earnings-related pension system is higher than in a flat benefit system. Of course, this result only mirrors what is typically observed in real world pension systems, but previous studies only highlight a political economy rational. Third, our results indicate that the optimal replacement rate is about 40-50 percent of average income independent of the specific mix of the two tiers. Note that the optimal pension generosity is fairly independent of the specific model parametrization since most parameter changes do alter the level, but not the profile of aggregate efficiency. Fourth, while asset-testing clearly reduces economic efficiency, pension-testing could be efficiency enhancing. For countries such as Norway or Finland this implies that they should increase their taper rate for the pension test. Fifth, limitations of the contribution base at the lower or higher end of incomes decreases economic efficiency. This is mainly due to increased labor supply distortions arising from the jumps of the contribution rate.

Of course, although our model is very flexible with respect to alternative pension structures it still excludes various design issues by assumption. First, we abstract from any information problem governments face in reality when they design the pension system. We neglect disability risk and assume that the government can observe the expected individual life span. Consequently, all households quit working and receive their benefits at the same age in the model, there are no explicit choices for the labor market exit and/or the pension benefit take up. In practice the design of the transition rules into retirement are an important policy problem since the government neither observes the expected individual life span nor the physical condition of potential retirees. Simonovits (2003, 2006) has analyzed the optimal pension rule for flexible retirement which takes the information constraints of the government regarding individual lifespan explicitly into account. He finds that benefits should rise with the age of retirement more slowly than actuarially fair in order to dampen redistribution from the shorter-lived towards the longer-lived. Cremer, Lozachmeur and Pestieau (2008) analyze the optimality of an earnings test after retirement when the government cannot observe individual health and productivity. They show that an earnings test after retirement may be part of an optimal tax system in order to restrict the mimicking of agents with low productivity.

Second, our approach also completely abstracts from capital market uncertainty and the choice between funded and unfunded pension pillars. All optimal design issues considered are restricted within the unfunded pension pillar. In practice, stock market returns are uncertain as well as the implicit returns of the paygo-system. Taking the structure of aggregate risks explicitly into account, Matsen and Thogersen (2004) show that in most countries it is optimal to offer a mix of funded and unfunded social security. Although paygo-returns are lower they may be used to hedge against high-yielding capital market risk.

Finally, our approach only considers fully rational individuals and abstracts from myopic households who do not save enough for their retirement. The latter would give rise to paternalistic considerations that affect the optimal pension design. Cremer and Pestieau (2011) review recent studies that analyze the optimal pension design in such a context while Fehr, Habermann and Kindermann (2007) compare the efficiency consequences of pension reforms in economies with rational and myopic households in a very similar model. The consideration of myopic households will improve the aggregate efficiency of the considered pension system. However, it will hardly change the qualitative results of the present study discussed above.

Since the consideration of information constraints and aggregate risk is hardly feasible in the present model set up, we feel that despite its obvious limitations our approach is a very useful starting point for further discussions of the optimal pension design.

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