The present value model of U.S. stock prices revisited: long-run evidence with structural breaks, 1871-2010

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Abstract

According to several empirical studies, the Present Value model fails to explain the behaviour of stock prices in the long-run. In this paper we consider the possibility that a linear cointegrated regression model with multiple structural changes would provide a better empirical description of the Present Value model of U.S. stock prices. Our methodology is based on instability tests recently proposed in Kejriwal and Perron (2008, 2010) as well as the cointegration tests developed in Arai and Kurozumi (2007) and Kejriwal (2008). The results obtained are consistent with the existence of linear cointegration between the log stock prices and the log dividends. However, our empirical results also show that the cointegrating relationship has changed over time. In particular, the Kejriwal-Perron tests for testing multiple structural breaks in cointegrated regression models suggest a model of three or two regimes.

Keywords: Present value model; Stock prices; Dividends; Cointegration; Multiple Structural Breaks

JEL classification: C22, G12

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1   Introduction

One of the central propositions of modern finance theory is the efficient markets hypothesis (EMH), which in its simplest formulation states that the price of an asset at time t should fully reflect all the available information at time t.[1] This has often been tested by using the present value (PV) model of stock prices, since, if stock market return are not forecastable, as implied by the EMH, stock prices should equal the present value of expected future dividends. Over the last decades, the influence of the linear PV model to explain the behavior of aggregate US stock prices has been actively investigated. According to the linear PV model, stock prices are fundamentally determined by the discounted value of their future dividends, which derive their value from future expected earnings (e.g., see Campbell et al., 1997; Cochrane, 2001).

In a series of seminal papers, Leroy and Porter (1981) and Shiller (1981a, 1981b) provide empirical evidence against the linear PV model of stock prices and, consequently, against the EMH.[2] In conducting their empirical analysis, however, these authors rely on the hypothesis that the underlying data, such as stock prices and dividends, are characterized by stationarity around deterministic trends.

Making use of some recent advances in the econometrics of nonstationary processes, Campbell and Shiller (1987) present new evidence that, again, seems to be unfavorable to the linear PV model of stock prices. In particular, Campbell and Shiller show that real stock prices and dividends, on the hypothesis that are difference stationary, are not cointegrated. This outcome, which effectively rules out the presence of a long-run relationship between real stock prices and dividends, clearly bears negative implications for the PV model, in which dividends are supposed to be the major determinant of stock prices in the long run.

Since the work of Campbell and Shiller (1987), empirical studies of the validity of linear PV model of stock prices have been extensively conducted in the cointegration framework. The cointegration between stock prices and dividends has implications for return predictability, cash-flow predictability and the debate on rational bubbles. However, the empirical evidence is far from conclusive (e.g., see Campbell and Shiller, 1987, Diba and Grossman (1988), Froot and Obstfeld (1991), Craine (1993), Lamont (1998), and Balke and Wohar (2002)). In most studies, standard cointegration tests do not validate the cointegration hypothesis, which implicitly supports the "rational bubbles hypothesis". From a methodological point of view, if we take the long-run validity of the PV model, non-linearities, the low power of standard unit root tests, and structural breaks, are all three possible candidates for explaining persistent deviations from the equilibrium relationship between real stock prices and dividends.

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[1] See Fama (1970) for a definition of weak, semi-strong and strong efficiency, and Fama (1991) for alternative definitions in terms of return predictability.

[2] Many recent theoretical models, however, incorporate time-varying expected returns. This means that return predictability can coexist with EMH. See Joijen and Van Nieuwerburgh (2011) for a discussion of this alternative view.

With respect to non-linearities, recent research has found that the relationship between real stock prices and dividends may best be characterized by using a nonlinear PV model; see, e.g., Gallagher and Taylor (2001), Kanas (2003, 2005), Esteve and Prats (2008, 2010), MacMillan (2009), and MacMillan and Wohar (2010). On the other hand, some researchers have argued that the dividend-stock price relationship exhibits fractional cointegration, resulting from the high persistence of temporary deviations from the long run equilibrium between real stock prices and dividends (see, e.g., Caporale and Gil-Alana, 2004, Cuñado et al., 2005, and Koustas and Serletis, 2005). Finally, some empirical studies have used Markov switching models to detect regime shifts in the dividends process (when the cointegrating vector is subject to Markov regime shifts). These models have found the existence of different phases in stock markets, (see, e.g, Bonomo and Garcia , 1994, Schaller and Van Norden, 1997, Driffill and Sola, 1998, Psaradakis, Sola and Spagnolo, 2004, and Sarno and Valente, 2005).

As regard to this last group of studies, their authors have found that the long-run relationship between real stock prices and dividends and/or the dividend-stock price ratio can be potentially subject to regime changes when the following occurs: a) changes in expectations regarding dividends, following persistent temporary shocks to output or productivity (see, e.g., Psaradakis et al, 2004); b) changes in the dividend process itself, reflecting: i) changes in business cycle conditions that determine a more accurate valuation of equity premium as a result of changes in inflation and interest rates (see, e.g., Siegel, 1999, and Fama and French, 2002); ii) changes in corporate behavior, i.e., the switch towards share repurchasing and away from dividend payments in corporate payout policy (see, e.g., Carlson et al., 2002).

The lack of control for structural breaks in the series may be reflected in the parameters of the estimated models that, when used for inference or forecasting, can induce to misleading results. In general, structural breaks are a problem for the analysis of economic series, since they are usually affected by either exogenous shocks or changes in policy regimes. As a consequence, the assumption of stability in the long-run relationship between real stock prices and dividends would seem too restrictive, so that not allowing for structural breaks would be an important potential shortcoming of the past research using cointegration techniques. In our case the long-run relationship between real stock prices and dividends has probably changed due to alterations in monetary and fiscal policy, as well as because of reforms in the financial market and in the regulation of the stock market. Thus, the information content of the linear PV model of stock prices is subject to change over time and all the empirical modeling studies that did not take into account the possible changes and instabilities will fail to explain the variations in the relationship between real stock prices and dividends. Visual examination of these variables (see Figure 1 and Figure 2) might allow to think that the presence of some non-recurrent shocks with large magnitude might have affected the evolution of these variables, something that needs to be taken into account when assessing the stochastic properties of time series if meaningful conclusions are to be obtained (see, Perron, 2006).

Such structural changes in the long-run relationship between real stock prices

and dividends and structural breaks in the dividend-price relation have important implications for the return predictability, cash-flow predictability, and the descomposition of the variance of the dividend-price ratio (see, e.g., Lettau and Van Nieuwerburgh, 2008, and Koijen and Van Nieuwerburgh, 2011). In this context, using the approach suggested in Bai and Perron (1998), Lettau and Van Nieuwerburgh (2008) reported evidence for structural shifts in the mean of the dividend-price ratio in 1991 (one break model) and in 1954 and 1994 (two break model). Our results differ from the findings of the Lettau and Van Nieuwerburgh (2008). In particular, the Kejriwal and Perron (2008, 2010) tests for testing multiple structural breaks in cointegrated regression models suggest a model of three regimes, with the dates of the breaks estimated at 1944 and 1971, and a model of two regimes, with the date of the break estimated at 1944. Give that our paper employs longer time period and different econometric techniques that the work of Lettau and Van Nieuwerburgh (2008), our empirical analysis could add to understanding of the impact of structural breaks on stock prices movements.

The purpose of this paper is to advance the evidence on the empirical validity of the linear PV model of stock prices in two ways. In the first place, in order to avoid the econometric problems mentioned above, we make use of recent developments in cointegrated regression models with multiple structural changes. Specifically, we use the approach proposed by Kejriwal and Perron (2008, 2010) to test for multiple structural changes in cointegrated regression models. These authors develop a sequential procedure that not only enables detection of parameter instability in cointegration regression models but also allows for consistency in the number of breaks present. Furthermore, we test the cointegrating relationship when multiple regime shifts are identified endogenously. In particular, the nature of the long run relationship between real stock prices and dividends is analyzed using the residual based test of the null hypothesis of cointegration with multiple breaks proposed in Arai and Kurozumi (2007) and Kejriwal (2008).

In the second place, it is well known that misspecifications due to the non consideration of structural breaks can bias the analysis that is performed using the standard Dickey-Fuller (DF) test statistics for a unit root. Consequently, the analysis of the order of integration has to consider the presence of structural breaks. To this end, we have used the GLS-based unit root test statistics proposed in Kim and Perron (2009) and extended in Carrion-i-Silvestre et al. (2009) that allows a break at an unknown time under both the null and alternative hypothesis. The commonly used tests of unit root with a structural change in the case of an unknown break date assumes that if a break occurs it does so only under the alternative hypothesis of stationarity. The methodology developed by Kim and Perron (2009) and Carrion-i-Silvestre et al. (2009) solves many of the problems of the standard tests of unit root with a structural change in the case of an unknown break date. In our empirical analysis, we use annual data of US stock market for the period 1871-2010.

The rest of the paper is organized as follows. A brief description of the underlying theoretical framework is provided in section 2, the methodology and

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empirical results are presented in sections 3 and 4, respectively, and the main conclusions are summarized in section 5.

2 The standard present value model of stock prices

The basic theoretical framework for the analysis of the PV model of stock prices is analytically discussed in Campbell et al. (1997). Given the assumptions of rational expectations, risk-neutrality and market equilibrium the movement of shares prices over time is given by the PV of future cash flows or the arbitrage condition:

$$P_t = \frac{1}{1+R}(E_t P_{t+1} + E_t D_{t+1}) \tag{1}$$

where P_t is the real price of a share (or real stock price) at time t, D_t is the real dividend paid on the stock in time period t, $1/(1+R)$ is the discount factor, R is the constant expected stock return ($E_t[R_t + 1] = R$) and E_t is the expectations operator conditioned on information up to t.

A solution to equation (1) is provided by imposing the transversality condition (the no bubble condition) and substituting recursively for all future prices. After solving forward K periods and assuming that the expected discounted value of the stock price K periods from the present shrinks to zero as the horizon K increases, we can obtain an equation expressing the value of stock price as the expected value of future dividends out to the infinite future discounted at a constant rate and equal to the required rate of return (i.e., the so-called *dividend discount model* (DDM) of stock prices):

$$P_t = E_t \left[\sum_{k=1}^{\infty} \left(\frac{1}{1+R} \right)^k D_{t+k} \right] \tag{2}$$

The DDM can be used to illustrate the concept of cointegration. Following Campbell and Shiller (1987), if D_t follows a linear process with a unit root, so that ΔD_t is stationary, the stock price P_t will also follow a linear process with a unit root (ΔP_t is also stationary). In this case, the DDM reflected in equation (2) relates two unit-root processes for P_t and D_t. If we subtract a multiple of the dividend from both sides of (2), we obtain:

$$P_t - \frac{D_t}{R} = (\frac{1}{R})E_t \sum_{i=0}^{\infty} \left(\frac{1}{1+R} \right)^i \Delta D_{t+1+i} \tag{3}$$

The left hand side of (3) reflects the difference between the stock price and $(1/R)$ times the dividend, and the right hand side reflects the expected discounted value of the future changes in dividends. If changes in dividends are stationary, then the term of the left hand side (i.e., difference between the stock price and $(1/R)$ times the dividend) should also be stationary. In this case,

the DDM of stock prices should hold when stock prices and dividends are cointegrated (there is a linear combination of stock prices and dividends which is stationary), with a known cointegrating vector $(1, 1/R)'$.

Equation (3) is based on the assumption that expected stock returns are constant. However, this assumption contradicts empirical evidence since the latter suggests that stock returns are non predictable. If the expected stock return is time varying, then the exact PV model becomes nonlinear. Campbell and Shiller (1988a, 1988b) and Cochrane and Sbordone (1988) suggested an approximate loglinear PV model for use in this case:

$$p_t = \frac{k}{1-\rho} + E_t \sum_{j=0}^{\infty} \rho^j [(1-\rho)d_{t+1+j} - r_{t+1+j}] = \frac{k}{1-\rho} + p_{CF,t} - p_{DR,t} \quad (4)$$

where the lower case letters p, d, r denotes the logarithms of stock prices, dividends and the discount rate respectively, ρ and k are parameters of linearization, and $p_{CG,t}$ and $p_{DR,t}$ are the components of the stock price driven by cash flow (dividend) expectations and discount rate (return) expectations respectively.

We can re-write (4) in terms of the log dividend-price ratio, $d_t - p_t$, (or the log dividend yield, dy_t) as follows:[3]

$$d_t - p_t = -(k/1-\rho) + E_t \sum_{j=0}^{\infty} \rho^j [-\Delta d_{t+1+j} + r_{t+1+j}] \quad (5)$$

Equation (5) is used to test the DDM of stock prices when log dividends follow a unit root process, so that the log dividends and the log stock prices are nonstationary. In this case, changes in the log dividends are stationary, and from equation (5) the log dividend-price ratio is stationary provided that the expected stock return is stationary. This restriction implies that the log stock prices is a sum of a difference stationary random variable and a stationary random variable. Hence the log stock prices is also difference stationary. This restriction also implies that the log stock prices and the log dividends are cointegrated with a known cointegrating vector $(1, -1)'$.

Intuitively, equation (5) states that if future dividends are expected to grow, then current stock prices will be higher and the dividend yield will be low, while if the future discount rate (rate of return) is expected to be high, then current prices will be low and the dividend yield will be high.

Thus, we can test for the validity of the DDM of stock prices in two different ways. First, we can test for stationarity in the log dividend yield, dy_t. Second, we can test for cointegration between the log stock prices, p_t, and the log dividends, d_t.

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[3] The term dividend yield is used interchangeably with the price-dividend ratio, of which is the inverse, since in the literature of the PV model of stock prices the ratios D/P and P/D are both used and both are consistent within the PV model of stock prices context.

In the empirical section, we test the linear DDM of stock prices in the context of cointegration theory, using a log linear model such as:

$$p_t = \alpha + \gamma d_t + \varepsilon_t \tag{6}$$

3 Methodology

3.1 A linear cointegrated regression model with multiple structural changes

Issues related to structural change have received a considerable amount of attention in the statistics and econometrics literature. Bai and Perron (1998) and Perron (2006, 2008) provide a comprehensive treatment of the problem of testing for multiple structural changes in linear regression models. Accounting for parameter shifts is crucial in cointegration analysis since it normally involves long spans of data which are more likely to be affected by structural breaks. In particular, Kejriwal and Perron (2008, 2010) provide a comprehensive treatment of the problem of testing for multiple structural changes in cointegrated systems.

More specifically, Kejriwal and Perron (2008, 2010) consider a linear model with m multiple structural changes (i.e., $m + 1$ regimes) such as:

$$y_t = c_j + z'_{ft}\delta_f + z'_{bt}\delta_{bj} + x'_{ft}\beta_f + x'_{bt}\beta_{bj} + u_t \qquad (t = T_{j-1} + 1, ..., T_j) \tag{7}$$

for $j = 1, ..., m + 1$, where $T_0 = 0$, $T_{m+1} = T$ and T is the sample size. In this model, y_t is a scalar dependent I(1) variable, $x_{ft}(p_f \times 1)$ and $x_{bt}(p_b \times 1)$ are vectors of I(0) variables while $z_{ft}(q_f \times 1)$ and $z_{bt}(q_b \times 1)$ are vectors of I(1) variables.[4] The break points $(T_1, ..., T_m)$ are treated as unknowns.

The general model (7) is a partial structural change model in which the coefficients of only a subset of the regressors are subject to change. In our case, we suppose that $p_f = p_b = q_f = 0$, and the estimated model is a pure structural change model with all coefficients of the I(1) regressors and constant (slope and the intercept in (6)) allowed to change across regimes:

$$y_t = c_j + z'_{bt}\delta_{bj} + u_t \qquad (t = T_{j-1} + 1, ..., T_j) \tag{8}$$

Generally, the assumption of strict exogeneity is too restrictive and therefore the test statistics for testing multiple breaks are not robust to the problem of endogenous regressors. To deal with the possibility of endogenous I(1) regressors, Kejriwal and Perron (2008, 2010) propose to use the so-called dynamic OLS regression (DOLS) where leads and lags of the first-differences of the I(1) variables are added as regressors, as suggested by Saikkonen (1991) and Stock and Watson (1993):

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[4]The subscript b stands for "break" and the subscript f stands for "fixed" (across regimes).

$$y_t = c_i + z'_{bt}\delta_{bj} + \sum_{j=-l_T}^{l_T} \Delta z'_{bt-j}\Pi_{bj} + u_t^*, \qquad \text{if } T_{i-1} < t \leq T_i \qquad (9)$$

for $i = 1, ..., k+1$, where k is the number of breaks, $T_0 = 0$ and $T_{k+1} = T$.

3.2 Structural Break Tests

In this paper we test the parameter instability in cointegration regression using the tests proposed in Kejriwal and Perron (2008, 2010). They present issues related to structural changes in cointegrated models which allow both I(1) and I(0) regressors as well as multiple breaks. They also propose a sequential procedure which permits consistent estimation of the number of breaks, as in Bai and Perron (1998).

Kejriwal and Perron (2010) consider three types of test statistics for testing multiple breaks. First, they propose a $\sup Wald$ test of the null hypothesis of no structural break ($m = 0$) versus the alternative hypothesis that there are a fixed (arbitrary) number of breaks ($m = k$):

$$\sup F_T^*(k) = \sup_{\lambda \in \Lambda_\varepsilon} \frac{SSR_0 - SSR_k}{\hat{\sigma}^2} \qquad (10)$$

where SSR_0 denotes the sum of squared residuals under the null hypothesis of no breaks, SSR_k denotes the sum of squared residuals under the alternative hypothesis of k breaks, $\lambda = \{\lambda_1, ..., \lambda_m\}$ is the vector of breaks fractions defined by $\lambda_i = T_i/T$ for $i = 1, ..., m, T_i$, and T_i are the break date, and where $\hat{\sigma}^2$ is:

$$\hat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} \tilde{u}_t^2 + 2T^{-1} \sum_{j=1}^{T-1} \varpi(j/\hat{h}) \sum_{t=j+1}^{T} \tilde{u}_t \tilde{u}_{t-j} \qquad (11)$$

and $\tilde{u}_t(t = 1, ..., T)$ are the residuals from the model estimated under the null hypothesis of no structural change. Also, for some arbitrary small positive number ϵ, $\Lambda_\epsilon = \{\lambda :| \lambda_{i+1} - \lambda_i |\geq \epsilon, \lambda_1 \geq \epsilon, \lambda_k \leq 1 - \epsilon\}$.

Second, they consider a test of the null hypothesis of no structural break ($m = 0$) versus the alternative hypothesis that there is an unknown number of breaks, given some upper bound $M(1 \leq m \leq M)$:

$$UD \max F_T^*(M) = \max_{1 \leq k \leq m} F_T^*(k) \qquad (12)$$

In addition to the tests above, Kejriwal and Perron (2010) consider a sequential test of the null hypothesis of k breaks versus the alternative hypothesis of $k + 1$ breaks:

$$SEQ_T(k + 1|k) = \max_{1 \leq j \leq k+1} \sup_{\tau \in \Lambda_{j,\varepsilon}} T \left\{ SSR_T(\hat{T}_1, ..., \hat{T}_k) \right\} \qquad (13)$$

$$- \left\{ SSR_T(\hat{T}_1, ...\hat{T}_{j-1}, \tau, \hat{T}_j, ..., \hat{T}_k) \right\} / SSR_{k+1} \qquad (14)$$

where $\Lambda_{j,\varepsilon} = \left\{ \tau : \hat{T}_{j-1} + (\hat{T}_j - \hat{T}_{j-1})\varepsilon \le \tau \le \hat{T}_j - (\hat{T}_j - \hat{T}_{j-1})\varepsilon \right\}$. The model with k breaks is obtained by a global minimization of the sum of squared residuals, as in Bai and Perron (1998).

3.3 Cointegration tests with structural changes

Kejriwal and Perron (2008, 2010) show that their test can reject the null of no break in a purely spurious regression. If anything, their tests have power against spurious regression. In this sense, tests for breaks in the long run relationship are used in conjuction with tests for the presence or absence of cointegration allowing for structural changes in the coefficients.

In this paper, we use the residual-based test of the null of cointegration with an unknown single break against the alternative of no cointegration proposed in Arai and Kurozumi (2007). These authors developed a LM test based on partial sums of residuals where the break point is obtained by minimizing the sum of squared residuals. They considered three models: i) Model 1, a level shift; ii) Model 2, a level shift with a trend; and iii) Model 3, a regime shift.

The LM test statistic (for one break), $\tilde{V}_1(\hat{\lambda})$, is given by:

$$\tilde{V}_1(\hat{\lambda}) = (T^{-2} \sum_{t=1}^{T} S_t(\hat{\lambda})^2)/\hat{\Omega}_{11} \tag{15}$$

where $\hat{\Omega}_{11}$ is a consistent estimate of the long run variance of u_t^* in (9), the date of break $\hat{\lambda} = (\hat{T}_1/T, ..., \hat{T}_k/T)$ and $(\hat{T}_1, ...\hat{T}_k)$ are obtained using the dynamic algorithm proposed in Bai and Perron (2003).

The Arai and Kurozumi (2007) test may be quite restrictive since only a single structural break is considered under the null hypothesis. Hence, the test may tend to reject the null of cointegration when the true data generating process exhibits cointegration with multiple breaks. To avoid this problem, Kejriwal (2008) has extended the Arai and Kurozumi (2007) test by incorporating multiple breaks under the null hypothesis of cointegration. The Kejriwal (2008) test of the null of cointegration with multiple structural changes is denoted -with k breaks- as $\tilde{V}_k(\hat{\lambda})$.

4 Empirical results

In this section we re-examine the issue of the standard PV model of stock prices using instability tests to account for potential breaks in the long-run relationship between the log real stock prices and the log real dividends as well as the cointegration tests with multiple breaks. First, we use unit root tests to verify that the log real stock prices and the log real dividends are individually integrated of order one, and the log dividend yield ratio is stationary or integrated of order zero. Second, we test the stability of the log real stock prices and the log real dividends relationship (and select the number of breaks) using the test proposed in Kejriwal and Perron (2008, 2010). Third, we verify that the

variables are cointegrated with tests for the presence/absence of cointegration allowing for a single or multiple structural changes in the coefficients as proposed by Arai and Kurozumi (2007) and Kejriwal (2008), respectively. Finally, we estimate the model incorporating the breaks in order to study if the log real stock prices and the log real dividends relationship (the slope parameter γ) have altered over time.

In our empirical analysis, we use annual data of US stock market for the period 1871-2010. The series on real stock prices and dividends are taken from Robert Shiller's website http://www.econ.yale.edu/~shiller/data.htm.[5] The evolution of the log stock prices, p_t, and the log dividends, d_t, appears in Figure 1 showing a close comovement between the two series. However, the plots also suggest that the association between p_t and d_t may have altered over time. The evolution of the log dividend yield, dy_t, is shown in Figure 2. It seems clear that these series are characterized a priori by at least one shift in the slope/or intercept of the trend function.

4.1   Stationarity of time series

The first step in our analysis is to examine the time series properties of the series by testing for a unit root over the full sample. We start the analysis of the order of integration of the time series involved in our study investigating the presence of structural breaks. This is an important feature provided that unit root tests can lead to misleading conclusions if the presence of structural breaks is not accounted for when testing the order of integration.[6] Therefore, the first stage of our analysis has focused on a pre-testing step that aims to assess whether the time series are affected by the presence of structural breaks regardless of their order of integration. This pre-testing stage of the analysis is a desirable feature, as it provides an indication of whether we should then apply unit root tests with or without structural breaks depending on the outcome of the pre-test. We have used the Perron and Yabu (2009) test for structural changes in the deterministic components of a univariate time series when it is a priori unknown whether the series is trend-stationary or contains an autoregressive unit root. The Perron and Yabu test statistic, called $Exp - W_{FS}$, is based on a quasi-Feasible Generalized Least Squares (FGLS) approach using an autoregression for the noise component, with a truncation to 1 when the sum of the autoregressive coefficients is in some neighborhood of 1, along with a bias correction. For given break dates, Perron and Yabu (2009) propose an F-test for the null hypothesis of no structural change in the deterministic components using the Exp function developed in Andrews and Ploberger (1994). Perron and Yabu (2009) specify three different models depending on whether the structural break only affects the level (Model I), the slope of the trend (Model II) or the level and the slope of the time trend (Model III).

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[5] The series are expressed in natural logaritms. The lowercase letters denote the logs of the variables.

[6] See Perron (1996).

The results of the $Exp - W_{FS}$ test are presented in Table 1. The results reported in Table 1 show that we find marginal evidence against the null hypothesis of no structural break. Thus, the null hypothesis of no structural break is only rejected at the 5% level of significance for d_t variable with Model III.

For the analysis of the order of integration without structural changes, we have used the M unit root test proposed in Ng and Perron (2001). In general, the majority of the conventional unit root tests (DF and PP types) suffer from three problems. First, many tests have low power when the root of the autoregressive polynomial is close to, but less than, the unit (Dejong et al., 1992). Second, the majority of the tests suffer from severe size distortions when the moving-average polynomial of the first differences series has a large negative autoregressive root (Schwert, 1989; Perron and Ng, 1996). Third, the implementation of unit root tests often necessitates the selection of an autoregressive truncation lag, k. However, as discussed in Ng and Perron (1995) there is a strong association between k and the severity of size distortions and/or the extend of power loss. More recently, Ng and Perron (2001) proposed a methodology that solves these three problems. Their method consists of a class of modified tests, called M^{GLS}, originally developed in Stock (1999) as M tests, with GLS detrending of the data as proposed in Elliot *et al.* (1996), and using the Modified Akaike Information Criteria ($MAIC$). Also, Ng and Perron (2001) have proposed a similar procedure [7] to correct for the problems of the standard Augmented Dickey-Fuller (ADF) test, ADF^{GLS}. Table 2 shows the results of M unit root tests of Ng and Perron (2001). First, the null hypothesis of non-stationarity cannot be rejected for p_t and dy_t at the 1% level of significance - the exception is for the ADF^{GLS} test for dy_t.[8] Second, the results reject the null hypothesis of non-stationarity for d_t at the 5% significance level.

For the analysis of the order of integration when structural changes are present, we have used the GLS-based unit root test statistics proposed in Kim and Perron (2009) and extended in Carrion-i-Silvestre et al. (2009) that allows multiples breaks at an unknown time under both the null and alternative hypothesis. The commonly used tests of unit root with a structural change in the case of an unknown break date (Zivot and Andrews (1992), Perron (1997), Vogelsang and Perron (1998), Perron and Vogelsang (1992a, 1992b)), assumed that if a break occurs it does so only under the alternative hypothesis of stationarity. The methodology developed by Kim and Perron (2009) and Carrion-i-Silvestre et al. (2009) solves many of the problems of the standard tests of unit root with a structural change in the case of an unknown break date.[9]

The results of applying for Model III the Carrion-i-Silvestre-Kim-Perron tests are shown in Table 3, allowing for up to one or two breaks, respectively. As can be seen in Table 2, the null hypothesis of a unit root with one or two structural breaks that affects the level and the slope of the times series cannot

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[7] See Ng and Perron (2001) and Perron and Ng (1996) for a detailed description of these tests.

[8] We base our analysis on the M^{GLS} unit root tests as they show better performance in finite sample than the ADF^{GLS} test statistic.

[9] See Carrion-i-Silvestre et al. (2009) for more details.

be rejected at the 5% level of significance, by any of the M^{GLS} and ADF^{GLS} tests.[10] The break points are estimated: i) at 1939 (one break model) and at 1932 and 1959 (two break model) for p_t; ii) at 1940 (one break model) and at 1886 and 1951 (two break model) for d_t; iii) at 1941 (one break model) and at 1945 and 1971 (two break model) for dy_t. Consequently, we can conclude that the three variables are $I(1)$ with structural breaks.

The unit root test results for dy_t, indicate that there is some evidence against stationarity behaviour. The fact that dy_t can be $I(1)$ with one or two structural breaks in the trend function points to the existence of a structural change in the equilibrium relationship between the log stock prices and the log dividends.

4.2 Long-run relationship

Once the order of integration of the series has been analyzed, we will estimate the long-run or cointegration relationship between p_t, and d_t. Given the relatively small sample size, we will estimate and test the coefficients of the cointegration equation by means of the Dynamic Ordinary Least Squares (DOLS) method from Saikkonen (1991) and Stock and Watson (1993), and following the methodology proposed by Shin (1994). This estimation method provides a robust correction to the possible presence of endogeneity in the explanatory variables, as well as serial correlation in the error terms of the OLS estimation. Also, in order to overcome the problem of the low power of the classical cointegration tests in the presence of persistent roots in the residuals of the cointegration regression, Shin (1994) suggested a new test where the null hypothesis is that of cointegration. therefore, in the first place, we estimate a long-run dynamic equation that includes the leads and lags of all the explanatory variables, i.e., the so-called DOLS regression:

$$p_t = c + \Phi t + \gamma d_t + \sum_{j=-q}^{q} \gamma_j \Delta d_{t-j} + v_t \tag{16}$$

Secondly, we use the Shin test, based on the calculation of two LM statistics from the DOLS residuals, C_μ and C_τ, in order to test for stochastic and deterministic cointegration, respectively. If there is cointegration in the demeaned specification given in (16), that occurs when $\Phi = 0$, this corresponds to a deterministic cointegration, which implies that the same cointegrating vector eliminates both deterministic and stochastic trends. But if the linear stationary combinations of $I(1)$ variables have nonzero linear trends (that occurs when $\Phi \neq 0$), as given in (16), this corresponds to a stochastic cointegration.[11] In both cases, the parameter γ is the long-run cointegrating coefficient estimated between p_t, and d_t.

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[10] The critical values were obtained by simulations using 1,000 steps to approximate the Wiener process and 10,000 replications.

[11] See Ogaki and Park (1997) and Campbell and Perron (1991) for an extensive study of deterministic and stochastic cointegration.

The results of Table 4 show that the null of the deterministic cointegration between p_t and d_t is not rejected at the 1% level of significance, and the estimated value for γ is 1.66. But this estimate would be significantly different from one at the 1% level, according to a Wald test on the null hypothesis $\hat{\gamma} = 1$, distributed as a χ_1^2 and denoted by W_{DOLS} in Table 4. The results obtained are consistent with the existence of linear cointegration between the log stock prices, p_t, and the log dividends, d_t, with a vector (1, -1.66). Thus, the cointegration vector is not (1, -1), as predicted by the theory.

Accounting for parameter shifts is crucial in cointegration analysis, since this type of analysis normally involves long spans of data, which for this reason are more likely to be affected by structural breaks. In particular, our data covers one hundred and forty years of the history of the U.S. stock market, and during that period of time the long-run relationship between real stock prices and dividends has probably changed due to alterations in monetary and fiscal policy, as well as because of reforms in the financial market and in the regulations of the stock market. Thus, the information content of the linear PV model of stock prices is subject to change over time and all the empirical modeling studies that did not take into account the possible changes and instabilities will fail to explain the variations in this relationship between real stock prices and dividends. Therefore, as we argued before, it is very relevant to allow for structural breaks in our cointegration relationship.

We now consider the tests for structural change that have been proposed in Kejriwal and Perron (2008, 2010). Since we have used a 20% trimming, the maximum numbers of breaks we may have under the alternative hypothesis is 3. Moreover, the intercept and the slope in equation (16) are permitted to change. Table 5 presents the results of the stability tests as well as the number of breaks selected by the sequential procedure (SP) and the information criteria BIC and LWZ proposed by Bai and Perron (2003). The test and the SP results do no suggest any instability, although the information criteria BIC and LWZ select two breaks and one break, respectively, and provide evidence against the stability of the long run relationship. Overall, the results of the Kejriwal-Perron tests suggest: i) a model with two breaks estimated at 1944 and 1971 and three regimes, 1871-1944, 1945-1971 and 1972-2010; ii) a model with one break estimated at 1944 and two regimes, 1871-1944 and 1945-2010.[12]

Since the above reported stability tests also reject the null coefficient of stability when the regression is a spurious one, we still need to confirm the presence of cointegration among the variables. With that end in mind, we use the residual based test of the null of cointegration against the alternative of cointegration with unknown multiple breaks proposed in Kejriwal (2008), $\tilde{V}_k(\hat{\lambda})$.

Arai and Kurozumi (2007) show that the limit distribution of the test statistic, $\tilde{V}_k(\hat{\lambda})$, depends only on the timing of the estimated break fraction $\hat{\lambda}$ and the number of I(1) regressors m.[13] Since we are interested in the stability of

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[12] Note that this result is very similar to the change selected for the dividend yield series, dy_t, when we apply the Carrion-i-Silvestre-Kim-Perron tests for a unit root with multiple structural breaks (Table 3).

[13] In our case, the critical values for the test are then simulated for the corresponding break

the stock prices-dividends coefficient, γ, we only consider model 3 that permits the slope shift as well as a level shift. Table 6 shows the results of the Arai-Kurozumi-Kejriwal cointegration tests allowing for both two breaks and one break. As before, the level of trimming used is 15%. As a result we find that both tests, $\tilde{V}_2(\hat{\lambda})$ and $\tilde{V}_1(\hat{\lambda})$, cannot reject the null of cointegration with two structural breaks and one break at the 1% level of significance, respectively.

Therefore, we conclude that p_t and d_t are cointegrated with two structural changes estimated at 1944 and 1971 (model with two breaks) and with one structural change at 1944 (model with one break). The first break coincides approximately with the end of the Second World War and the boom in stock prices of the 1950s, while the second break coincides with the end of the Bretton Woods System, the oil price shock of 1973 and the collapse of the stock market in the early 1970s. Driffill and Sola (1998) obtain similar results using a regime-switching model that interprets the boom (the slump) as a response of the present-value stock price to a change of regime in an era of rapidly growing (declining) dividends. When a stochastic regime-switching is introduced in place of the bubble, they find that the fluctuations of stock prices that would have been explained as a bubble (e.g, Froot and Obstfeld, 1991) are now explained as breaks in the fundamental price that results from a change of regime, as we do in the present paper.

To compare the coefficients obtained from break models with those reported from models without any structural break, we estimate the cointegration equation (16) both with a two breaks model, as suggested by BIC criterion, and with a one break model, as suggested by the LWZ criterion. The results with the sub-samples are presented in Table 4.

First, the results of the C_μ statistics in the model with two breaks show that the null of the deterministic cointegration between p_t and d_t is not rejected at the 1% level of significance in the three regimes. The coefficient estimated between p_t and d_t (i.e, the long-run elasticity, γ) in a two-break model shows a tendency to increase over time (1.05, 2.23 and 4.51). Therefore, the coefficient in the first regime (1871-1944) is much smaller than the value obtain with the full sample (1.62); furthermore, the restriction on the estimate of γ being equal to one is clearly accepted, and the cointegration vector is (1, -1), as predicted by the theory.

Secondly, in the case of the model with one break, the results in Table 3 show that the null of the deterministic cointegration between p_t and d_t is not rejected at the 1% level of significance in the two regimes. Again, the estimated coefficient values in the first and second regimes increase over time (1.42 and 2.57). In both cases, according to a Wald test on the null hypothesis $\hat{\gamma} = 1$, the coefficient estimated is significantly different from one.

Overall, the results suggest that ignoring structural changes in the long-run cointegration relationship may understate the extend of correlation between the log stock prices, p_t, and the log dividends, d_t, since the response of the present-

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fractions using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of $i.i.d.$ $N(0, 1)$ random variables.

value stock price to a change of dividends increases over time.

5   Conclusions

In this paper we consider the possibility that a linear cointegrated regression model with multiple structural changes would provide a better empirical description of the Present Value model of U.S. stock prices.

To avoid the econometric problems mentioned in empirical literature, we make use of recent developments in cointegrated regression models with multiple structural changes. Specifically, we use the approach developed by Kejriwal and Perron (2008, 2010) to test for multiple structural changes in cointegrated regression models. These authors propose a sequential procedure that not only permits the detection of parameter instability in cointegration regression models but also allows for a consistent estimation of the number of breaks present. Furthermore, we test the cointegrating relationship when multiple regime shifts are identified endogenously. In particular, the nature of the long run relationship between the log stock prices and the log dividends is analyzed using the residual based test of the null hypothesis of cointegration with a single and/or multiple breaks proposed in Arai and Kurozumi (2007) and Kejriwal (2008), respectively. In the empirical analysis, we use annual data of the US stock market for the period 1871-2010.

The results obtained in our study are consistent with the existence of linear cointegration between the log stock prices and the log dividends, with a vector (1, -1.66). Thus, the cointegration vector is not (1, -1), as predicted by the theory. Additionally, the unit root test results for the dividend yield (or log of the dividend-price ratio) indicate that there is some evidence against stationarity behaviour. The results for the full sample (1871-2010) only support a "weak" version of the PV model of US stock prices.

The empirical results also show that the cointegrating relationship has changed over time, i.e., it is no stable. In particular, the Kejriwal-Perron tests for testing multiple structural breaks in cointegrated regression models suggest a model of three regimes, with the dates of the breaks estimated at 1944 and 1971, and a model of two regimes, with the date of the break estimated at 1944. The first break coincides approximately with the end of Second World War and the boom in the stock prices of the 1950s, while the second break coincides with the end of the Bretton Woods System, the oil price shock of 1973 and the collapse in the stock market in the early 1970s.

The estimate of long-run elasticity between the log stock prices and the log dividends in both break models shows a tendency to increase over time. Finally, only the results for the period 1871-1944 support a "strong" version of the PV model of stock prices, with the long-run coefficient equal to one.

Summing up, the results obtained in this study suggest that ignoring structural changes in the long-run cointegration relationship may understate the extend of correlation between the log stock prices and the log dividends, since the response of the present-value stock price to changes in dividends increases

over time.

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References

[1] Arai, Y. and Kurozumi, E. (2007): "Testing for the null hypothesis of cointegration with a structural break", *Econometric Reviews*, 26, 705-739.

[2] Bai, J. and Perron, P. (1998): "Estimating and Testing Linear Models with Multiple Structural Changes", *Econometrica,* 66, 47-78

[3] Bai, J. and Perron, P. (2003): "Computation and analysis of multiple structural change models", *Journal of Applied Econometrics*, 18, 1-22.

[4] Balke, N. S. and Wohar, M. E. (2002): "Low frequency movements in stock prices: A state-space decomposition", *Review of Economics and Statistics*, 84, 649-667.

[5] Bonomo, M. and Garcia, R. (1994): "Can a well-fitted equilibrium asset-pricing model produce mean reversion?", *Journal of Applied Econometrics*, 9, 19-29.

[6] Campbell, J.Y., Lo, A.W., and MacKinlay, A.C. (1997): *The Econometrics of Financial Markets*, Princeton, NJ: Princeton University Press.

[7] Campbell, J.Y. and Perron, P. (1991): "Pitfall and opportunities: what macroeconomists should know about unit roots", in O.J. Blanchard and S. Fisher, (Eds.), NBER macroeconomics Annual 1991. Cambridge MA, MIT Press.

[8] Campbell, J.Y. and Shiller, R. (1987): "Cointegration and tests of present value models", *Journal of Political Economy*, 95, 1062-1088.

[9] Campbell, J. Y. and Shiller, R. J. (1988a): "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors", *Review of Financial Studies*, 1, 195-227.

[10] Campbell, J. Y. and Shiller, R. J. (1988b): "Stock Prices, Earnings, and Expected Dividends", *Journal of Finance*, 43, 661-676.

[11] Campbell, J. Y. and Shiller, R. J. (1991): "Yield spreads and interest rate movements: a birds eye view", *Review of Economic Studies*, 58, 495-514.

[12] Caporale, G. M. and Gil-Alana, L. A. (2004): "Fractional cointegration and tests of present value models", *Review of Financial Economics*, 13, 245-258.

[13] Carlson, J.B., Pelz, E.A. and Wohar, M.E. (2002): "Will valuation ratios revert to historical means?", *Journal of Portfolio Management*, 28 (4), 23-35.

[14] Carrion-i-Silvestre, J.Ll., Kim, D. and Perron, P. (2009): "GLS-based unit root tests with multiple structural breaks under both the null and the alternative hypotheses", *Econometric Theory,* 25, 1754-1792.

[15] Cochrane, J. H. (2001): *Asset pricing*, Princeton University Press, Princeton NJ.

[16] Cochrane, J.H. and Sbordone, A.M. (1988): "Multivariate estimates of the permanent components of GNP and stock prices", *Journal of Economic Dynamics and Control,* 12, 255-296.

[17] Craine, R. (1993): "Rational bubbles. A test", *Journal of Economic Dynamics and Control*, 17, 829-846.

[18] Cuñado, J., Gil-Alana, L.A. and Pérez de Gracia, F. (2005): "A Test for Rational Bubbles in the NASDAQ Stock Index: A Fractionally Integrated Approach", *Journal of Banking and Finance*, 29, 2633-2654.

[19] DeJong, D.N.J., J.C. Nankervis, N.E. Savin and C.H. Whiteman (1992): "Integration versus trend stationary in time series", *Econometrica,* 60, 423-433.

[20] Diba, B. T. and Grossman, H. I. (1988): "Explosive rational bubbles in stock prices?", *American Economic Review*, 78, 520-530.

[21] Driffill, J. and Sola, M. (1998): "Intrinsic bubbles and regime-switching", *Journal of Monetary Economics*, 42, 357-373.

[22] Elliot, G., T.J. Rothenberg and J.H. Stock (1996): "Efficient test for an autoregressive unit root", *Econometrica,* 64, 813-836.

[23] Esteve, V. and Prats, M.A. (2008): "Are there threshold effects in the stock price-dividend relation? The case of the US stock market, 1871-2004", *Applied Financial Economics*, 18, 1533-1537.

[24] Esteve, V. and Prats, M.A. (2010): "Threshold cointegration and nonlinear adjustment between stock prices and dividends", *Applied Economics Letters*, 17, 405-410.

[25] Fama, E.F. (1970): "Efficient capital markets: a review of theory and empirical work", *Journal of Finance*, 25, 383-417.

[26] Fama, E.F. (1991): "Efficient capital markets II", *Journal of Finance*, 46, 1575-1671.

[27] Fama, E. F. and French, K. R. (2002): "The equity premium", *Journal of Finance*, 57, 637-659.

[28] Froot, K. and Obstfeld, M. (1991): "Intrinsic bubbles: The case of stock prices", *American Economic Review*, 81, 1189-1214.

[29] Gallagher, L. A and Taylor, M. P. (2001): "Risky Arbitrage, Limits of Arbitrage, and Nonlinear Adjustment in the Dividend-Price Ratio," *Economic Inquiry*, 39, 524-36.

[30] Kanas, A. (2003): "Non-linear cointegration between stock prices and dividends", *Applied Economics Letters*, 10, 401-405.

[31] Kanas, A. (2005): "Nonlinearity in the stock price-dividend relation", *Journal of International Money and Finance*, 24, 583-606.

[32] Kejriwal, M. (2008): "Cointegration with structural breaks: an application to the Feldstein-Horioka Puzzle", *Studies in Nonlinear Dynamics & Econometrics*, 12 (1), 1-37.

[33] Kejriwal, M. and Perron, P. (2008): "The limit distribution of the estimates in cointegrated regression models with multiple structural changes", *Journal of Econometrics*, 146, 59-73.

[34] Kejriwal, M. and Perron, P. (2010): "Testing for multiple structural changes in cointegrated regression models", *Journal of Business and Economic Statistics*, 28, 503-522.

[35] Kim, D. and Perron, P. (2009): "Unit root test allowing for a break in the trend function under both the null and alternative hypothesis, *Journal of Econometrics*, 148, 1-13.

[36] Koijen, R. and Van Nieuwerburgh, S. (2011): "Predictability of returns and cash flows", Annual Review of Financial Economics, 3, 467-491.

[37] Koustas, Z. and Serletis, A. (2005): "Rational Bubbles or Persistent Deviations from Market Fundamentals", *Journal of Banking and Finance*, 29, 2523-2539.

[38] Lamont, O. (1998): "Earnings and expected returns", *Journal of Finance*, 53, 1563-1587.

[39] Leroy, S. F. and Porter. R. D. (1981): "The present-value relation: Tests based on implied variance bounds", *Econometrica*, 49, 555-574.

[40] Lettau, M. and Van Nieuwerburgh, S. (2008): "Reconciling the return predictability evidence", The Review of Financial Studies, 21, 1607-1652.

[41] MacMillan, D.G. (2009): "Revisiting dividend yield dynamics and returns predictability: Evidence from a time-varying ESTR model", *The Quarterly Review of Economics and Finance*, 49, 870-883.

[42] MacMillan, D.G. and Wohar, M.E. (2010): "Stock return predictability and dividend-price ratio: A nonlinear approach", *International Journal of Finance and Economics*, 5 (4), 351-365.

[43] Newey, W. K. and West, K. D. (1987): "A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix", *Econometrica*, 55, 703-708.

[44] Ng, S. and P. Perron (1995), "Unit Root Tests in ARMA Models with Data Dependent Methods for the Selection of the Truncation Lag", *Journal of the American Statistical Association,* 90, 268-281.

[45] Ng, S. and Perron, P. (2001): "Lag length selection and the construction of unit root tests with good size and power", *Econometrica,* 69, 1529-1554.

[46] Ogaki, M. and Park, J.Y. (1997): "A cointegration approach to estimating preference parameters", *Journal of Econometrics*, 82, 107-134**.**

[47] Psaradakis, Z., Sola, M. and Spagnolo, F. (2004): "On Markov Error-Correction Models, with an Application to Stock Prices and Dividends", *Journal of Applied Econometrics*, 19, 69-88.

[48] Perron, P. (1997): "Further evidence on breaking trend functions in macro-economic variables", *Journal of Econometrics,* 80, 355-385.

[49] Perron, P. (2006): "Dealing with Structural Breaks", in Palgrave Handbook of Econometrics, Vol. 1: Econometric Theory, K. Patterson and T.C. Mills (eds.), Palgrave Macmillan, 278-352.

[50] Perron, P. (2008): "Structural Change", in The New Palgrave Dictionary of Economics, 2nd ed, S. Durlauf and L. Blume (eds.), Palgrave Macmillan.

[51] Perron, P. and Ng, S. (1996): "Useful modifications to some unit root tests with dependent errors and their local asymptotic properties", *Review of Economic Studies,* 63, 435-463.

[52] Perron, P. and Vogelsang, T.J. (1992a): "Nonstationarity and Level Shifts with an Application to Purchasing Power Parity", *Journal of Business and Economic Statistics*, 10, 301-320.

[53] Perron, P. and Vogelsang, T.J. (1992b): "Testing for a Unit Root in a Time Series with a Changing Mean: Corrections and Extensions", *Journal of Business and Economic Statistics*, 10, 467-470.

19

[54] Perron, P. and Yabu, T. (2009): "Testing for shifts in trend with an integrated or stationary noise component", *Journal of Business and Economic Statistics*, 27 (3), 369-396.

[55] Saikkonen, P. (1991): "Asymptotically efficient estimation of cointegration regressions", *Econometric Theory,* 7, 1-21.

[56] Sarno, L. and Valente, G. (2005): "Modelling and Forecasting Stock Returns: Exploiting the Futures Market, Regime Shifts and International Spillovers", *Journal of Applied Econometrics*, 20, 345-376.

[57] Schaller, H. and Van Norden, S. (1997): "Regime switching in stock markets returns", *Applied Financial Economics*, 7, 177-191.

[58] Schwert, G.W. (1989): "Tests for unit roots: A Monte Carlo investigation", *Journal of Business and Economic Statistics*, 7, 147-159.

[59] Shiller, R. J. (1981a): "The use of volatility measures in assessing market efficiency", *Journal of Finance*, 291-304.

[60] Shiller, R. J. (1981b): "Do stock prices move too much to be justified by subsequent changes in dividends?", *American Economic Review*, 421-436.

[61] Shin, Y. (1994): "A residual-based test of the null of cointegration against the alternative of no cointegration", *Econometric Theory*, 10, 91-115.

[62] Siegel, J. J. (1999): "The shrinking equity premium: Historical facts and future forecasts", *Journal of Portfolio Management*, Fall, 10-17.

[63] Stock, J.H. (1999): "A Class of Tests for Integration and Cointegration", in R.F. Engle and H. White (Eds.): *Cointegration, Causality and Forecasting. A Festschrift in Honour of Clive W.J. Granger*, Oxford University Press, pp. 37-167.

[64] Stock, J. H. and Watson, M. W. (1988): "Testing for common trends", *Journal of the American Statistical Association,* 83, 1097-1107.

[65] Stock, J. H. and Watson, M. W. (1993): "A simple estimator of cointegrating vectors in higher order integrated systems", *Econometrica,* 61, 783-820.

[66] Vogelsang, T.J., Perron, P. (1998): "Additional Tests for a Unit Root Allowing for a Break in the Trend Function at an Unknown Time", *International Economic Review*, 39, 1073-1100.

[67] Zivot, E. and Andrews, D.W.K. (1992): "Further evidence on the great crash, the oil-price shock, and the unit-root hypothesis", *Journal of Business and Economic Statistics,* 10, 251-270.

Table 1
Perron-Yabu test statistics to test the null hypothesis of no structural breaks
against the alternative hypothesis of one structural break ^{a,b}

Variable	Model	$Exp - W_{FS}$ test
p_t	II	-0.20
p_t	III	1.43
d_t	II	-0.28
d_t	III	4.38**
dy_t	II	-0.21
dy_t	III	2.07

Notes:

^{a} ** denotes rejection of the null hypothesis of unit root at the 5 % level of
significance.

^{b} The 5% critical values are taken from Perron and Yabu (2009), Table 2.b.
and 2.c.: Model II, 1.28% and Model III, 2.79.

Table 2

M unit root tests without structural breaks of Ng and Perron (2001) [a,b]

Variable	MZ_{α}^{GLS}	MZ_{t}^{GLS}	MSB^{GLS}	MP_{T}^{GLS}	ADF^{GLS}
p_t	-13.01	-2.53	0.195	7.07	-2.64
d_t	-22.29**	-3.33**	0.149**	4.10**	-3.66**
dy_t	-17.15	-2.89	0.169	5.50	-3.10**

Notes:

[a] ** denote significance at the 5% level of significance.

[b] The $MAIC$ information criterion is used to select the autoregressive truncation lag, k, as proposed in Ng and Perron (2011). The critical values are taken from Ng and Perron (2001), table 1.

Table 3

M unit root tests with multiple structural breaks of Carrion-i-Silvestre et al. (2009) [a,b,c]

Variable	Model	MZ_α^{GLS}	MZ_t^{GLS}	MSB^{GLS}	MP_T^{GLS}	Break date $\hat{T}_1$	$\hat{T}_2$
p_t	III	-5.00	-1.54	0.388	31.14	1939	—
p_t	III	-21.83	-3.29	0.150	10.31	1932	1959
d_t	III	-8.84	-1.96	0.222	17.97	1940	—
d_t	III	-19.05	-3.08	0.161	12.46	1886	1951
dy_t	III	-4.99	-1.57	0.316	32.49	1941	—
dy_t	III	-14.43	-2.62	0.181	13.72	1945	1971

Notes:

[a] A ** denote significance at the 5% level.

[b] The structural break affects the level and the slope of the time trend (Model III).

[c] The critical values were obtained by simulations using 1,000 steps to approximate the Wiener process and 10,000 replications.

Table 4
Estimation of long-run relationships: Stock-Watson-Shin[a,b,c,d]
cointegration tests

Parameter estimates	Model without structural breaks	Two breaks model			One break model	
	Full sample 1871-2010	First regime 1871-1944	Second regime 1945-1971	Third regime 1972-2010	First regime 1871-1944	Second regime 1945-2010
c	1.62	2.87	0.11	-6.28	2.87	-0.88
	(3.27)	(12.13)	(0.25)	(-4.43)	(12.13)	(-0.85)
γ	1.66	1.05	2.23	4.51	1.05	2.57
	(7.88)	(8.89)	(13.07)	(8.86)	(8.89)	(6.94)
C_μ	0.187	0.142	0.149	0.048	0.142	0.130
W_{DOLS}	9.88***	0.04	52.04***	47.58***	0.04	18.00***

Notes:

[a] t-statistics are in brackets. Standard Errors are adjusted for long-run variance. The long-run variance of the cointegrating regression residual is estimated using the Barlett window which is approximately equal to $INT\left(T^{1/2}\right)$ as proposed in Newey and West (1987).

[b] We choose $q = INT\left(T^{1/3}\right)$ as proposed in Stock and Watson (1993).

[c] C_μ is LM statistics for cointegration using the DOLS residuals from deterministic cointegration, as proposed in Shin (1994). *, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively. The critical values are taken from Shin (1994), table 1, from $m = 1$.

[d] W_{DOLS} is a Wald test on the null hypothesis $\hat{\beta} = 1$, distributed as a χ_1^2. *, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively.

Table 5
Kejriwal-Perron tests for testing multiple structural breaks
in cointegrated regression models: equation (9) and (16)

	Specifications[a]		
$y_t = \{p_t\}$	$z_t = \{1, d_t\}$	$x_t = \{\emptyset\}$	$M = 3$
	$q = 2$	$p = 0$	$h = 27$
	Tests[b]		
$\sup F_T(1)$	$\sup F_T(2)$	$\sup F_T(3)$	$UD\max$
6.07	4.09	3.20	6.07
	Number of Breaks		
	Selected	Breaks	
		$\hat{T}_1$	$\hat{T}_2$
SP	0	—	—
LWZ	1	1944	—
BIC	2	1944	1971

Notes:

[a] y_t, z_t, q, p, h, and M denote the dependent variable, the regressors, the number of $I(1)$ variables (and the intercept) allowed to change across regimes, the number of $I(0)$ variables, the minimum number of observations in each segment, and the maximum number of breaks, respectively.

[b] The critical values are taken from Kejriwal and Perron (2010), Table 1.10 (critical values are available on Kejriwal-Perron website), trending case with $q_b = 1$.

Table 6
Arai-Kurozumi-Kejriwal cointegration tests with multiple structural breaks: equation (9) and (16)^{a,b}

|	Two breaks model				On break model		
Test $\tilde{V}_2(\hat{\lambda})$	$\hat{\lambda}_1$	$\hat{T}_1$	$\hat{\lambda}_2$	$\hat{T}_2$	Test $\tilde{V}_1(\hat{\lambda})$	$\hat{\lambda}_1$	$\hat{T}_1$
0.096***	0.52	1944	0.72	1971	0.076***	0.54	1944

Notes:

^{a} *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

^{b} Critical values are obtained by simulation using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of $i.i.d.$ $N(0,1)$ random variables.

Critical values:	10%	5%	1%
$\tilde{V}_2(\hat{\lambda})$	0.077	0.101	0.149
$\tilde{V}_1(\hat{\lambda})$	0.104	0.128	0.201

Figure 1: U.S. real stock prices and dividends
1871-2010
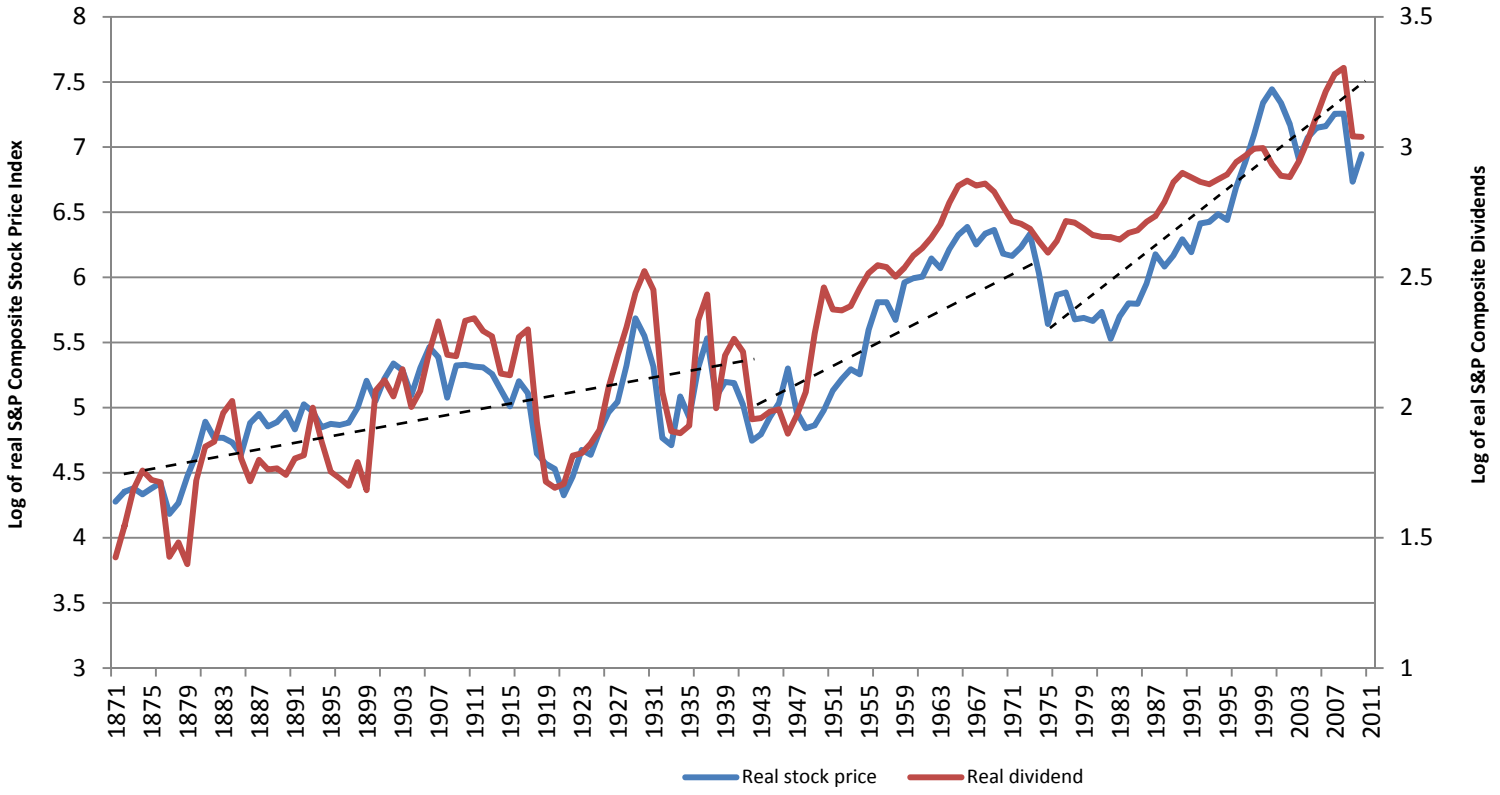
Figure 2: U.S. real dividend-stock price ratio
1871-2010