Dynamic pricing, product innovation, and quality-based cost

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Preliminary version
Comments welcome

Abstract

Manufacturing and high-tech firms simultaneously conduct dynamic pricing and product innovation policies. In an optimal control framework, we model dynamic pricing and product innovation policies, when the production cost is based on product quality. We analyse the determinants of dynamic pricing and the impact of quality on price. We show that the dynamic pricing is linked to the class of the demand function. Under a general demand function, price dynamics is undetermined; price may decrease even if quality and cost increase. Under an additive separable demand function, price dynamics follows quality dynamics. Under a multiplicative separable demand function, price dynamics follows cost dynamics.

Keywords: Dynamic pricing, product innovation, quality-based cost
Code JEL: L11, O32

1 Introduction

In modern manufacturing industries such as automobile, electronic chips, or baby care, firms simultaneously set product pricing and product innovation policies. Product innovation enhances product quality. For example, a faster car, a more powerful computer, or a more absorbent diaper result from product innovation. If product quality raises the interest of the consumer, it also raises the cost paid by the firm. In effect, greater product quality implies first an investment in product innovation and second a higher production cost.

We study the determinants of pricing policy over time when product innovation enhances product quality and production cost is based on product quality.

We also investigate the impact of quality on price. We develop an optimal control model that copes with the following characteristics: the firm chooses the pricing and innovation (or product innovation if not otherwise stated) policies; the demand depends on price and quality (or product quality); the cost (or unit cost of production) is based on quality. The pricing policy is therefore connected to the dynamics of demand and supply. Demand and supply are linked to the preferences of the consumer and to the organisation of the firm (Saha 2007). The literatures on dynamic pricing and on innovation offer elements of response to our research.

The literature on dynamic pricing focuses on the properties of demand functions and gives pricing rules valid across different classes of demand functions (Kalish 1983, Dockner et al. 2000). It studies implicit but not explicit innovation (Chatterjee 2009). The modelling of demand supposes the launch of an innovative product with fixed characteristics. The modelling of supply assumes a learning effect based on cumulative production that reduces cost. An explicit innovation policy that enhances quality lacks from the analysis.

The literature on innovation investigates explicit innovation (Adner and Levinthal 2001, Vörös 2006, Lambertini and Mantovani 2009). Empirical and theoretical contributions show and explain that the main innovation is achieved at the beginning of the product life cycle (Bayus 1995, Saha 2007). Teng and Thompson (1996) or Lambertini and Mantovani (2009) note that most contributions use a static framework and specific innovation functions. The contributions also omit to deal with explicit pricing policy.

Teng and Thompson (1996), Mukhopadhyay and Kouvelis (1997), and Lin (2008) study both dynamic pricing and quality policies. Quality is chosen by the firm and doesn't result from innovation. The paper closest to ours is Chenavaz (2012), which considers the determinants of the dynamic pricing of a firm that invests both in product innovation (that augments product quality) and in process innovation (that reduces production cost). Chenavaz (2012) argues the importance of process innovation over product innovation as determinant of pricing. The main difference from its model is our quality-based cost. Quality-based cost rehabilitates product innovation as determinant of pricing and gives insights on the impact of quality on price.

A priori the effect of quality on price is three-fold. There are one effect on the supply-side and two effects on the demand-side. On the supply-side, the cost effect is obvious: with greater quality, cost and price increase. The cost effect is positive. On the demand-side, two effects - sales and markup effects - play in opposite directions. Greater quality increases sales and the sales increase even more with a lower price. The sales effect is negative. Greater quality increases markup and the markup increases even more with a higher price. The markup effect is positive. The final effect of quality on price is tied to cost, sales, and quality effects. The net result is therefore ambiguous.

To study the determinants of dynamic pricing and the effect of quality on price, we use general demand functions from the pricing literature and we generalise innovation functions from the innovation literature. Regarding research that studies pricing and innovation at the same time (Bayus 1995, Teng and

Thompson 1996, Vörös 2006, Chenavaz 2012), quality results explicitly from product innovation and cost is related to quality. Unlike research that uses numerical simulation (Bayus 1995, Adner and Levinthal 2001, Saha 2007), we derive analytical results.

We contribute to the literature with new dynamic pricing rules based on analytical results. The pricing rules are only tied to the properties of demand functions and to the quality-based specification of cost. The pricing rules are independent of the innovation function. For general demand functions, price dynamics is ambiguous. Contrary to intuition, price may decline even if both quality and cost rise. For additively separable demand functions, price dynamics emulates quality dynamics; price rises with quality. For multiplicatively separable demand functions, price dynamics emulates cost dynamics; price rises with cost. In this case, price rises with quality if and only if the impact of quality on cost is strictly positive. The managerial implications of the additive and the multiplicative cases are straightforward and easy to apply.

2 General model formulation

2.1 Model development

This article studies a monopoly in an optimal control framework. A monopoly describes the situation of a firm that launches a new product or of a firm that protects its product by patent. The planning horizon is finite with length T. The time $t \in [0, T]$ is continuous.

2.1.1 Quality

The firm invests in product innovation $u(t) \in \mathbb{R}^+$ to improve product quality $q(t) \in \mathbb{R}^+$. Innovation $u(t)$ and quality $q(t)$ are control and state variables. Quality dynamics is

$$\dot{q}(t) = k(u(t)), \tag{1}$$

where $k : \mathbb{R}^+ \to \mathbb{R}$ is of class C^2. Hereafter, $\dot{z}$ denotes the time derivative of z, and z_x denotes the derivative of z with respect to x.

Where there is no confusion, we shall omit the function arguments. Quality q rises with innovation u and reaches diminishing returns: the marginal impact of innovation on quality is positive, but declines as innovation rises. Hence:

$$k_u > 0,\ k_{uu} < 0. \tag{2}$$

According to (1) and (2), there is no technological obsolescence and any quality improvement is cumulative as in Bayus (1995) or Saha (2007).

2.1.2 Cost

The cost of production function $c : \mathbb{R}^+ \to \mathbb{R}^+$ is of class C^2 and increases with quality q. As q is a state variable and not a control variable, function c doesn't require the second-order condition. The cost is $c = c(q(t))$ with

$$c_q \geqslant 0. \tag{3}$$

For example, a cost independent from quality $c_q = 0$ and a cost rising with quality $c_q > 0$ characterise the software and the hardware industries.

2.1.3 Demand

The price $p(t) \in \mathbb{R}^+$ is a control variable. The demand (or current demand) function $f : \mathbb{R}^{2+} \to \mathbb{R}^+$ is of class C^2. The demand f is a state variable that depends on the price p and the quality q. The cumulative sales are $x(t) \in \mathbb{R}^+$ and the current sales are $\dot{x}(t)$. There is no uncertainty and no stock issue; the demand equals the production. All the demand is satisfied; the demand equals the current sales:

$$\dot{x}(t) \equiv f(p(t), q(t)). \tag{4}$$

The demand falls with price and rises with quality. Moreover, the sensitivity of demand to price is smaller when the quality is greater:

$$f_p < 0,\ f_q > 0,\ f_{pq} \leqslant 0. \tag{5}$$

2.2 Model analysis

The current profit $\pi\,(t)$, with values in $\mathbb{R}$, is

$$\pi(t) = [p(t) - c(q(t))] f\,(p(t), q(t)) - u(t).$$

The firm simultaneously chooses the pricing and the innovation policies that maximise the intertemporal profit (or present value of the profit stream) over the planning horizon, according to the dynamics of demand and quality. The interest rate is $r \in \mathbb{R}$. The objective function of the firm is

$$\begin{aligned} & && \max_{p,u} \int_0^T e^{-rt}\pi(t)dt, \\ \text{subject to } \dot{x} &=&& f(p, q), \\ \dot{q} &=&& k(u). \end{aligned}$$

The shadow price (or current-value adjoint variable) λ represents the marginal value of quality. As there is neither diffusion nor learning effects, the shadow price of cumulative sales vanishes as in Mukhopadhyay and Kouvelis (1997) or Chenavaz (2012). We formulate the current-value Hamiltonian H with the shadow price $\lambda(t)$ for quality dynamics:

$$H(p, u, q, \lambda) = [p - c(q)]f(p, q) - u + \lambda k(u).$$

The Hamiltonian H measures the intertemporal profit. It sums the current profit $(p - c)f - u$ and the future profit λk.

The maximum principle implies for λ

$$\dot{\lambda} = r\lambda - H_q = r\lambda + c_q f - (p-c) f_q;\ \lambda(T) = 0. \tag{6}$$

The first-order conditions for H maximisation are

$$H_p = 0 \Rightarrow p = c - \frac{f}{f_p}, \tag{7a}$$

$$H_u = 0 \Rightarrow k_u = \frac{1}{\lambda}. \tag{7b}$$

The pricing rule ($7a$) seems that of the static case. But as in Mukhopadhyay and Pangiotis (1997) or Chenavaz (2012), the price changes over time. Indeed the price is linked to cost and demand that depend on quality, and thereby, on innovation at any time t.

The innovation rule ($7b$) is that of Bayus (1995) and Chenavaz (2012). The higher is the shadow price of quality λ, the higher is the innovation u, and since the diminishing returns of innovation (2), the lower is the marginal impact of innovation on quality k_u.

The second-order conditions for H maximisation are

$$H_{pp} < 0 \Rightarrow 2 - \frac{f f_{pp}}{f_p^2} > 0, \tag{8a}$$

$$H_{uu} < 0 \Rightarrow \lambda k_{uu} < 0, \tag{8b}$$

$$H_{pp} H_{uu} - H_{pu} > 0. \tag{8c}$$

Equations (2) and ($8b$) and the transversality condition in (6) imply

$$\lambda(t) > 0,\ \forall t \in [0, T[, \tag{9}$$

which verifies that a better product quality always raises the intertemporal profit.

Integrate (6) with the transversality condition $\lambda(T) = 0$, the integrating factor e^{-rt}, and substitute ($7a$) gives

$$\lambda(t) = \int_t^T e^{-r(s-t)} \left(-c_q + \frac{\mu}{\tau} \frac{p}{q} \right) f\, ds, \tag{10}$$

with $\mu \equiv \frac{\partial f}{\partial q} \frac{q}{f}$ the quality elasticity of demand and $\tau \equiv -\frac{\partial f}{\partial p} \frac{p}{f}$ the price elasticity of demand.

According to (10), the shadow price of quality λ is the net result of the cost effect c_q and of the markup effect $\frac{\mu}{\tau} \frac{p}{q}$.

The cost effect c_q captures the marginal impact of product quality on the unitary production cost. The cost effect has negative impact on λ ($-c_q \leqslant 0$ since $c_q \geqslant 0$) because better quality fosters the cost, and therefore lowers the future profit. If cost is independent from quality ($c_q = 0$), the cost effect vanishes and only holds the demand effect as in Mukhopadhyay and Kouvelis (1997) or Chenavaz (2012). Alternatively, if the cost increases with quality ($c_q > 0$), the cost effect mitigates the shadow price λ. In this case, innovation rule (7b) predicts lower innovation.

The markup effect $\frac{\mu}{\tau}\frac{p}{q}$ captures the price increase that the consumer accepts to pay after a rise in product quality. The demand effect depends on the relative demand sensitivity to quality μ and demand sensitivity to price τ. The markup effect has a positive impact on λ ($\frac{\mu}{\tau}\frac{p}{q} > 0$ since $\mu > 0$ and $\tau > 0$) because quality promotes the willingness to pay, and so the future profit.

Equations (9) and (10) impose

$$c_q < \frac{\mu}{\tau}\frac{p}{q},\ \forall t \in [0, T[. \tag{11}$$

A positive shadow price of quality λ means that the markup effect $\frac{\mu}{\tau}\frac{p}{q}$ dominates the cost effect c_q. When quality rises, the firms gains more from higher markup than it looses from higher cost: the net result of better quality on profit is positive. The innovation gives a quality such that the cost of quality increase is below the price increase that consumers are willing to pay.

Result (11) differs from Teng and Thompson (1996) or Lin (2008) that find $c_q = \frac{\mu}{\tau}\frac{p}{q}$. The difference comes from the nature of quality q, a control variable in their case and a state variable in our case. In their case, the comparison of the first order conditions on price and quality imposes the equality $c_q = \frac{\mu}{\tau}\frac{p}{q}$. In our case, $\lambda > 0$ only implies the inequality $c_q < \frac{\mu}{\tau}\frac{p}{q}$. Our model is thus more flexible for determining the level of c_q, and thus of q. In particular it can analyse the situation $c_q = 0$. Quality q as a result of innovation $k(u)$ is then distinct from quality q as a choice variable.

According to (7b) and (10), the demand-side with quality and the supply-side with cost both determine the product innovation policy u over time. The modelling takes into account the two major views in technological change. For the market pull view, innovation comes from the market. For the technology push view, innovation comes from the firm. The two views jointly explain most characteristics of technological change (Adner and Levinthal 2001).

Equations (6) ($\lambda(T) = 0$) and (9) ($\lambda(t) > 0$, $\forall t \in [0, T[$) imply $\exists\, t_1 \in [0, T[\, /\, \dot{\lambda}(t) < 0, \forall\, t \in [t_1, T[$. There is a time t_1 after which λ lowers. Moreover, according to (7b), $\frac{d}{dt}(k_u) = \frac{d}{dt}\left(\frac{1}{\lambda}\right) = -\frac{\dot{\lambda}}{\lambda^2}$. So, sign $\dot{k}_u = -$ sign $\dot{\lambda}$ and $\forall t \in [t_1, T[$, $\dot{k}_u > 0$. Recalling that $\dot{k}_u = k_{uu}\dot{u}$ and $k_{uu} < 0$ according to (2), we have sign $\dot{u} =$ sign $\dot{\lambda}$ and then

$$\exists\, t_1 \in [0, T[\, /\, \dot{u}(t) < 0,\ \forall t \in [t_1, T[. \tag{12}$$

Product innovation may increase ($\dot{u} > 0$) at the beginning of the planning horizon, from $t = 0$ to t_1. But product innovation falls ($\dot{u} < 0$) for the remaining planning horizon, from t_1 to T, even if the firm always conducts some innovation according to innovation rule (7b). If $t_1 = 0$, innovation declines over the whole planning horizon.

At the product launching, the consumer has interest in the product characteristics and in their improvement. At that moment, the willingness to pay is higher for better product quality. Over time, the consumer has less interest in quality and the willingness to pay falls.

In coherence with innovation theory and observation in chemical, automobile, or high-tech industries (Teng and Thompson 1996, Adner and Levinthal 2001), the firm supports product innovation especially at the beginning. Result (12) is tied to the interest of the consumer in price and quality and to the capability of the firm in quality and cost. At the beginning, innovation raises quality and satisfies the consumer. At that time, innovation develops new functionalities, product stability, or originates complementarity with other products. With product maturity, innovation becomes secondary and declines.

Variations of $p(t)$.

Based on Kalish (1983) and following Chenavaz (2011, 2012), the differentiation of ($7a$) with respect to t and the substitution of ($7a$) in the result give

$$\dot{p}\left(2-\frac{ff_{pp}}{f_p^2}\right)=\dot{c}+\dot{q}\left(\frac{ff_{pq}-f_pf_q}{f_p^2}\right). \tag{13}$$

Result (13), already in Chenavaz (2011, 2012), is only linked to the first-order condition on p ($7a$) and to its derivative with respect to t. As such it is robust from the dynamics of cost and quality.

A more accurate result appears if cost depends on quality. Consider (3) that gives $\dot{c}=\frac{dc}{dq}\frac{dq}{dt}=c_q\dot{q}$ and notice $-\frac{f_pf_q}{f_p^2}=\frac{\mu}{\tau}\frac{p}{q}$, equation (13) becomes

$$\dot{p}\left(2-\frac{ff_{pp}}{f_p^2}\right)=\dot{q}\left(c_q+\frac{ff_{pq}}{f_p^2}+\frac{\mu}{\tau}\frac{p}{q}\right). \tag{14}$$

Proposition 1. *For a general demand function, the dynamics of price is the result of three effects that play in opposite directions: the cost effect (positive), the sales effect (negative), and the markup effect (positive). The dynamics of price is undetermined.*

Proof. Immediate with (14). □

Results (13) and (14) stand for the general demand function (4). Result (13) holds for any specification of the dynamics of cost, while result (14) is tied to the quality dependence of cost (3). Because of the second-order condition ($8a$), the second factor on the left-hand side of (14) is positive. On the right-hand side, the impact of quality on price is the result of the cost effect on the supply-side and of the sales and markup effects on the demand-side. We already spoke briefly of the cost and the markup effects. We analyse now more deeply the three effects.

- The cost effect c_q measures the marginal impact of quality on cost. The price raises with the marginal impact of quality on cost: the cost effect is positive. The cost effect is high for a manufacturing good such as automobile or computer chips, for which better quality is expensive. The cost effect is null for a digital good such as software or music, for which the marginal cost of production is null.

- The sales effect $\frac{ff_{pq}}{f_p^2}$ measures the increased sales following a price reduction, in the case of a quality rise. The sales decrease when the price increases: the sales effect is negative The sales effect is greater for product of mass consumption such as phone or television. The higher is the demand f, the higher is the sales effect. Also, the higher is the price sensibility of demand when quality is better f_{pq} (in absolute terms), the higher is the sales effect.

- The markup effect $\frac{\mu}{\tau}\frac{p}{q}$ measures the higher willingness to pay following a rise in quality. The markup increases when the price increases: the markup effect is positive. The markup effect increases with the quality elasticity of demand μ and decreases with the price elasticity of demand τ. The markup effect is high for an upmarket product with little competition. The markup effect is low for a downmarket product with much competition.

The sign of $\dot{p}$ depends on the cost, the sales, and the markup effects. The three effects play in opposite directions; the sign of $\dot{p}$ is unknown; the shape of the pricing policy is undetermined. The final effect of quality on price is also undetermined.

As a managerial implication, the firm should set the pricing policy according to the relative weight of the cost, sales, and markup effects. As a corollary to Proposition 1, if the sales effect is "large enough", the price falls even if cost and quality both raise.

Example 1. *Linear price-quality demand function.*

The specification of the demand (4) is

$$f = a_0 - a_1 p + a_2 q + a_3 \frac{q}{p}, \tag{15}$$

with $a_0,\ a_1,\ a_2$, and $a_3 > 0$.

The substitution of (15) into (14) yields

$$\dot{p}\left(2 - \frac{a_3 f q}{2p^3(a_1 + a_3\frac{q}{p^2})^2}\right) = \dot{q}\left(c_q - \frac{a_3 f}{p^2(a_1 + a_3\frac{q}{p^2})^2} + \frac{a_2 + \frac{a_3}{p}}{a_1 + a_3\frac{q}{p^2}}\right).$$

Proof. Substitute $\frac{\mu}{\tau}\frac{p}{q} = -\frac{f_q}{f_p}$, $f_p = -a_1 - a_3\frac{q}{p^2}$, $f_q = a_2 + \frac{a_3}{p}$, $f_{pp} = \frac{a_3}{2}\frac{q}{p^3}$, and $f_{pq} = -\frac{a_3}{p^2}$ in (14). □

On the left-hand side, the second factor is positive because of the second order condition (8a). On the right-hand side with regard to the last factor, the first term - the cost effect - is positive, the second term - the sales effect - is negative, and the third term - the markup effect - is positive. Therefore quality has an ambiguous impact on price.

If $a_1 = a_2 = 0$, the dynamics of price is

$$\dot{p}\left(\frac{3}{2} - \frac{a_0 p}{2a_3 q}\right) = \dot{q}\left(c_q - \frac{a_0 a_3}{p^2}\right).$$

On the right-hand side, $-\frac{a_0 p^2}{a_3 q^2}$ is the sum of the sales effect $(-\frac{a_0 p^2}{a_3 q^2} - \frac{p}{q})$ and the markup effect $(\frac{p}{q})$. The sales effect is strictly greater than the markup effect. If $\frac{dc}{dq} = 0$, price falls when quality raises. If $a_0 = 0$, the dynamics of price is

$$\frac{3}{2}\dot{p} = \dot{q} c_q.$$

As the sales effect $(-\frac{p}{q})$ equals the markup effect $(\frac{p}{q})$, both effects cancel out. If $\frac{dc}{dq} = 0$, price is constant over time.

3 Subclasses of the general formulation

The general demand function gives useful insights about the dynamic pricing policy. Result (14) is general but not easily applicable. The following specifications of the demand function give stronger results and clearer pricing rules. The gain in applicability outweighs the loss in generality.

3.1 Additive separable demand function

A demand function additively separable in price and quality is the most simple and natural modelling. In the additively separable case, the demand (4) is

$$f = h(p) + l(q), \tag{16}$$

that implies $h_p < 0$, $l_q > 0$, and $f_{pq} = 0$ recalling (5).

The substitution of (16) into (14) yields

$$\dot{p}\left(2 - \frac{h h_{pp}}{h_p^2}\right) = \dot{q}\left(c_q + \frac{\mu}{\tau}\frac{p}{q}\right). \tag{17}$$

Proposition 2. *For an additive separable demand function, the dynamics of price is the result of two effects that play in the same direction: the cost effect (positive) and the markup effect (positive). The dynamics of price mimics the dynamics of quality.*

Proof. Immediate with (17). □

In the additively separable case, the sales effect (negative) vanishes. But the cost effect (positive) and the markup effect (positive) remain. The two remaining effects play in the same direction: the sign of $\dot{p}$ is the sign of $\dot{q}$; price results from a monotonically increasing transformation of quality. The shape of the pricing policy follows the shape of product quality. Product innovation determines dynamic pricing. As innovation rises, quality and price rise.

The managerial implication is simple: price rises with quality and the firm adopts a pricing policy that imitates the dynamics of quality.

Example 2. *Additive price-quality demand function.*

With $a_3 = 0$, demand function (15) is

$$f = a_0 - a_1 p + a_2 q. \tag{18}$$

The substitution of (18) into (17) gives

$$2\dot{p} = \dot{q}\left(c_q + \frac{a_2}{a_1}\right).$$

Proof. Substitute $\frac{\mu}{\tau}\frac{p}{q} = -\frac{l_q}{h_p}$, $h_p = -a_1$, $l_q = a_2$, and $h_{pp} = 0$ in (17). □

On the right-hand side with regard to the last factor, the first term - the cost effect - is positive and the second term - the markup effect - is positive. Therefore the product price rises with quality.

3.2 Multiplicative separable demand function

A demand function multiplicatively separable on price and quality holds relatively general and unconstrained. Such a simple and natural modelling is analytically tractable and explains well the data (Bayus 1995). In the multiplicatively separable case, the demand (4) is

$$f = h(p)l(q), \tag{19}$$

that implies $h_p < 0$, $l_q > 0$, and $f_{pq} = h_p l_q < 0$ recalling (5).

The substitution of (19) in (14) yields

$$\dot{p}\left(2 - \frac{hh_{pp}}{h_p^2}\right) = \dot{q}c_q = \dot{c}. \tag{20}$$

Proposition 3. *For a multiplicative separable demand function, the dynamics of price is the result of the sole cost effect (positive). The dynamics of price mimics the dynamics of cost.*

Proof. Immediate with (20). □

The markup effect (positive) and the sales effect (negative) are of same magnitude. They cancel each other out. The sole cost effect (positive) remains. If the marginal impact of quality on cost is independent from quality ($c_q = 0$), the price is constant over time ($\dot{p} = 0$). Alternatively, if the marginal impact of quality on cost strictly depends on quality ($c_q > 0$), the dynamics of price emulates the dynamics of quality (sign $\dot{p}$ = sign $\dot{q}$).

Considering that $\dot{c} = \dot{q}c_q$ gives an alternative analysis. The sign of $\dot{p}$ is the sign of $\dot{c}$ (independently of c_q); price results from a monotonically increasing transformation of cost. The shape of the pricing policy follows the shape of cost. The insight holds for any specification of the evolution of cost as discussed by Chenavaz (2012).

The managerial implication apply simply: price augments with cost and the firm adopts a pricing policy that imitates the dynamics of the cost of production. As the firm always conduct innovation, quality and cost rise.

Example 3. *Multiplicative price-quality demand function.*

Examples of multiplicatively separable functions are the exponential function $f = (a_0 + a_1q)e^{-a_2p}$, the isoelastic function $f = (a_0 + a_1q)(a_2p)^{-a_3}$, and the linear function $f = (a_0 + a_1q)(a_2 - a_3p)$. For exponential or isoelastic demand functions, equation (20) simplifies to

$$\dot{p} = \dot{q}c_q = \dot{c}.$$

Proof. For the exponential case, substitute $h_p = -a_2h$, $h_{pp} = a_2^2h$ in (20). The proof is similar for the isoelastic case. $\square$

As the cost effect remains, price dynamics mimics exactly cost dynamics. Considering the impact of quality, the cost effect is positive. Therefore, if the cost raises with quality ($c_q > 0$), the product price raises with product quality. Alternatively, if the cost is independent from quality $c_q = 0$, the product price is constant over time.

4 Discussion and conclusion

We studied an optimal control model in which product price and product innovation are the decision variables. Product innovation raises product quality. Production cost is based on quality. We analysed the relationships among price, product innovation, quality, and cost under different classes of demand functions. Derived from analytical results, the pricing rules only depend on the optimality condition of price.

The theoretical results confirm and expand prior results (Kalish 1983, Teng and Thompson 1996, Chenavaz 2012). Figure 1 shows the dynamics of innovation, quality, cost, and price over time, according to the class of demand function. According to (7*b*) and (10), innovation policy depends on the innovation and demand characteristics. Since (7*b*) and (12), the firm always conducts innovation but at a decreasing level over time. Product quality and production cost are therefore linked to the capabilities of the firm - innovation $k(u)$ and cost $c(q)$ - and to the preferences of the consumer - demand $f(p, q)$. As the firm always invests in innovation, quality and cost augment over time.

The impact of quality on price results from the cost (positive), the sales (negative), and the markup (positive) effects. The cost effect, on the supply-side, the sales and markup effects, on the demand-side, depend on the industry and on the product. The role of each effect, and the pricing rules are tied to the demand features.

- In the general case - $f(p, q)$ - the three effects play out. Proposition 1 states that the dynamics of price is undetermined. The impact of quality on price is also undetermined: even if quality and cost both rise, the price may fall if the sales effect is "large enough".

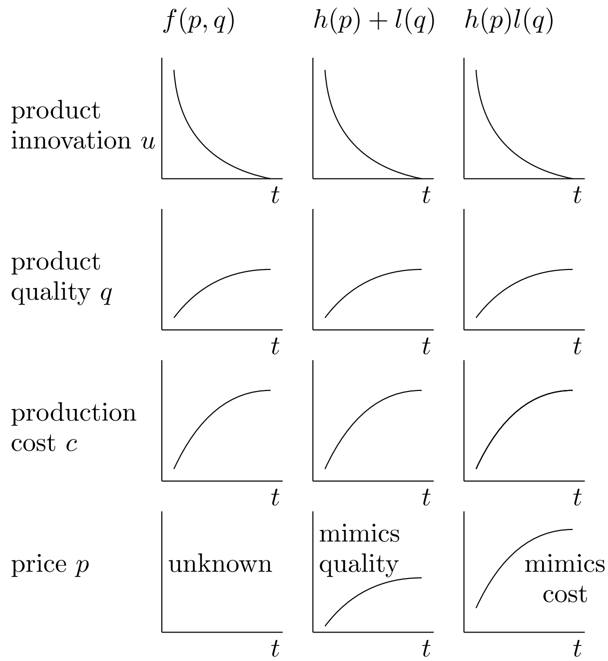

Figure 1: Product innovation, product quality, production cost, and pricing over time.

- In the additive case - $h(p)+l(q)$ - the sales effect vanishes. Both cost and markup effects play out. Proposition 2 shows that the dynamics of price mimics the dynamics of quality. Price increases with quality.
- In the multiplicative case - $h(p)l(q)$ - the sales and the markup effect cancel out each other. The sole cost effect plays out. Proposition 3 states that the dynamics of price mimics the dynamics of cost. Price increases with cost; price increases with quality if and only if cost strictly rises with quality.

Different market characteristics such as diffusion, learning, or competition may be analysed as temporal effects in a first analysis. A temporal effect $h(t)$ that affects the demand function in a multiplicatively separable way $f(p,q)h(t)$ is used by Kalish (1983), Bayus (1995), or Vörös (2006). Propositions 1, 2, and 3 are robust to a multiplicative separable temporal effect.

The pricing rules support easy managerial implications. A firm knows the marginal cost of quality c_q and can estimate the demand f. According to the nature of c_q and to the class of f, the firm applies the relevant pricing rule. The implications are simple and straightforward for the additive and the multiplicative cases. The pricing rules also predict the impact of quality on price.

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