# When-Issued Markets and Treasury Auctions 

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## Introduction

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- 6\% of trading activity in the inter dealer Treasury market (Fabozzi and Fleming, 2005; Barclay et al, 2006).


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- Active when-issued (WI) market for Treasury securities:
- 6\% of trading activity in the inter dealer Treasury market (Fabozzi and Fleming, 2005; Barclay et al, 2006).
- Underpricing - Yields on Treasury securities are higher in the auction than in the when-issued market.
- In the order of $\frac{1}{2}$ to 1 basis point within minutes to the auction (Goldreich (2007), Bikhchandani et al (2000)).
- Higher underpricing if prices from previous days are considered (Lou, Yan and Zhang (2011)).


## Introduction

- If the price in the auction is lower, why dealers acquire securities in the when-issued market?
- Structure of the market:
- Single-price auctions do not allocate securities efficiently.
- Dealers can use WI market to improve allocation.
- Underpricing arises in equilibrium.


## Introduction

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- Structure of the market:
- Single-price auctions do not allocate securities efficiently.
- Dealers can use WI market to improve allocation.
- Underpricing arises in equilibrium.
- When-issued market increases efficiency (allocation).
- Prices in the when-issued market are a biased proxy for the true market value of securities.


## Related Literature

(1) When-issued markets and Treasury auctions

- Nyborg and Strabulaev (2004); Chatterjea and Jarrow (1998).
(2) Divisible good auction.
- Wilson (1979); Kyle (1989); Back and Zender (1993); Wang and Zender (2002); Ausubel et al (2011); Coutinho (2012)
(3) Forward Market and Market Power.
- Allaz and Vila (1993); Powell (1993), Green (1999).
(1) Empirical Treasury Auctions:
- Nyborg and Sundaransen (1996); Malvey and Archibald (1998); Bikhchandani et al (2000); Keloharju et al (2005); Goldreich (2007);


## Example: Two Investors, Two securities

## Example

- Two homogeneous Treasury securities.


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- Two dealers with constant valuation for the securities:

$$
\begin{aligned}
& A \\
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$$
\begin{aligned}
& A:(5,5) . \\
& B: \\
& B
\end{aligned}(3,3) .
$$

- Auction:
- Bids $\beta_{i}=\left(\beta_{i}^{1}, \beta_{i}^{2}\right) \in \mathbb{R}_{+}^{2} \quad i=A, B$.
- Units are allocated to the two highest bids.
- Single-price auction - dealers pay the same price $p^{50} \geq 0$ for acquired units.
- $p^{\text {so }} \equiv$ highest rejected bid.


## Example: Auction

- Claim: There is an equilibrium of the auction where dealers submit:

$$
\begin{aligned}
& \beta_{A}=(5,0) \\
& \beta_{B}=(3,0)
\end{aligned}
$$

- Why?


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$$

- Why?

| Dea. $A$ | Dea. $B$ | Aggregate |
| :---: | :---: | :---: |
| 5 | 3 | 5 |
| 0 | 0 | 3 |
|  |  | 0 |
|  |  | 0 |

- Payoff:

$$
\begin{aligned}
& U_{A}^{0}=(5-0) \times 1=5 \\
& U_{B}^{0}=(3-0) \times 1=3
\end{aligned}
$$

## Example: Auction

- Is there any profitable deviation for $A$ ?


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- What if $A$ acquires both securities?

| Dea. $A$ | Dea. $B$ | Aggregate |
| :---: | :---: | :---: |
| $\hat{\beta}_{A}^{1}$ | 3 | $\hat{\beta}_{A}^{1}$ |
| $\hat{\beta}_{A}^{2}$ | 0 | $\hat{\beta}_{A}^{2}$ |
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- Payoff with deviation:

$$
\begin{aligned}
\hat{U}_{A} & =5 \times 2-3 \times 2 \\
& =4
\end{aligned}
$$

## Example: When-Issued

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- $p^{w i} \in(3,5)$ arises in equilibrium of a double auction in the WI stage.
- Underpricing: $p^{w i}>p^{50}$.


## Main Model

## Economy

- Two divisible goods - Treasury securities, $q$, and Money, m.


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- $N$ dealers, indexed by $i \in \mathcal{I}=\{1, \ldots, N\}$, with valuation $V_{i}: \mathbb{R}_{+} \times \mathbb{R} \rightarrow \mathbb{R}$ :
- $V_{i}(q, m)=v_{i}(q)+m ;$
- $v_{i}^{\prime}(q)=v_{i}-\rho q$.


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- $V_{i}(q, m)=v_{i}(q)+m ;$
- $v_{i}^{\prime}(q)=v_{i}-\rho q$.
- Three periods:

$$
\begin{gathered}
\quad \mathrm{l} \\
t=-1 \\
\text { Dealers trade } \\
\text { When-issued securities }
\end{gathered}
$$

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## Auction

- Start by characterizing equilibrium in the auction stage.
- History $h=\left(p^{w i},\left\{\theta_{i}^{\text {wi }}\right\}_{\mathcal{I}}\right)$ from the WI market:
- $p^{\text {wi }}$ - price of WI securities.
- $\left\{\theta_{i}^{\text {wi }}\right\}_{\mathcal{I}}$ - WI positions.
- Stochastic quantity $Q \in[\underline{Q}, \bar{Q}] \subseteq \Re_{++}$of a Treasury security.
- Dealer $i$ submits a left continuous, weakly decreasing bid schedule $q_{i}^{A}(p, h): \Re_{+} \times \mathcal{H} \rightarrow \Re_{+}$.
- Stop-out price: $\tilde{p}^{\text {so }}$ such that $\sum_{\mathcal{I}} q_{j}^{A}\left(\tilde{p}^{\text {so }}, h\right)=\tilde{Q}$.
- Single-price auction: dealer $i$ gets $\tilde{\psi}_{i}^{A} \equiv q_{i}^{A}\left(\tilde{p}^{\text {so }}, h\right)$ units of the good and pays $\tilde{\psi}_{i}^{A} \times \tilde{p}^{\text {so }}$.


## Uniform Price Auction

Bid Schedules


## Uniform Price Auction



## Uniform Price Auction

Auction price, $\mathrm{Q}=6$


## Uniform Price Auction



## Uniform Price Auction

Bid Schedules


$$
q_{i}(p, h)
$$

—— Individual Bids -_ Aggregate

## Auction Stage

- Dealer $i$ faces the problem:

$$
\begin{gathered}
\max _{q_{i}(\cdot, h)} E\left[v_{i}\left(q_{i}\left(\tilde{p}^{s o}, h\right)+\theta_{i}^{w i}\right)-\tilde{p}^{s o} q_{i}\left(\tilde{p}^{s o}, h\right)\right] . \\
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\end{gathered}
$$

## Proposition

Let $\gamma \equiv \frac{N-2}{N-1}$ and $\theta_{i}^{\text {wi }} \leq \overline{\theta^{w i}}$, in the unique linear equilibrium, dealers submit:

$$
\begin{equation*}
q_{i}^{A}(P, h)=\frac{\gamma}{\rho}\left(v_{i}-P\right)-\gamma \theta_{i}^{w i} \tag{1}
\end{equation*}
$$

For a given realization of $\tilde{Q}$, the stop-out price satisfies

$$
\tilde{p}^{s o}=\tilde{p}^{W}-\rho \frac{1}{N-2} \frac{\tilde{Q}}{N}
$$

where $p^{W} \equiv$ Walrasian Price

## Equilibrium: Auction

- Dealers "shade" their bids:

$$
\operatorname{Bid}\left(q_{i}^{A}\right)=\underbrace{v_{i}^{\prime}\left(q_{i}^{A}+\theta_{i}^{w i}\right)}_{\text {Marginal Bennefit }}-\underbrace{\frac{d p^{s o}}{d q_{i}^{A}} \times q_{i}^{A}}_{\text {Shade }} .
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Bid Shading


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$$



## No When-Issued

## Corollary

No when-issued trade, i.e., $\theta_{i}^{\text {wi }}=0$ for $i \in \mathcal{I} \Longrightarrow$ the outcome of the auction is not efficient:

$$
v_{i}>v_{j} \Longleftrightarrow v_{i}^{\prime}\left(\tilde{\psi}_{i}^{F}\right)>v_{j}^{\prime}\left(\tilde{\psi}_{j}^{F}\right) .
$$

- Dealers with higher valuation end up with a higher marginal valuation for the security.


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- Can dealers improve the allocation trading when-issued securities?


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- Dealers with higher valuation end up with a higher marginal valuation for the security.
- Can dealers improve the allocation trading when-issued securities?
- Yes. Larger dealers acquiring WI securities from smaller dealers.
- WI positions affect how much dealers acquire in the auction and their final holdings of securities:

$$
\begin{aligned}
\frac{d \tilde{\psi}_{i}^{A}}{d \theta_{i}^{w i}} & =-\frac{N-2}{N-1} \\
\frac{d \tilde{\psi}_{i}^{F}}{d \theta_{i}^{w i}} & =\frac{1}{N-1}
\end{aligned}
$$

## No When-Issued



## When-Issued: Auction



When-Issued Market

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- Central inter-dealer broker runs a double auction:


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- Central inter-dealer broker runs a double auction:
- Dealer i submits a left continuous, weakly decreasing bid schedule $q_{i}^{\text {wi }}(p): \Re_{+} \rightarrow \Re_{+}$.
- When-issued price: $\tilde{p}^{w i}$ such that $\sum_{\mathcal{I}} q_{i}^{w i}\left(p^{w i}\right)=0$.
- Dealer $i$ gets $\theta_{i}^{w i} \equiv q_{i}^{w i}\left(p^{w i}\right)$ units of the good and pays $\theta_{i}^{w i} \times p^{w i}$.
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- $\theta_{i}^{w i} \leq \bar{\theta}^{w i}$.
- Dealers anticipate the equilibrium in the auction will be the linear one described previously.


## When-Issued

- Dealer i's problem:

$$
\begin{gathered}
\max _{q_{i}^{w i}(\cdot)} E\left[v_{i}\left(\psi_{i}^{F}\right)-p^{s o} \psi_{i}^{A}\right]-p^{w i} q^{w i}\left(p^{w i}\right) \\
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\text { subject to } \sum_{\mathcal{I}} q_{j}^{w i}\left(p^{w i}\right)=0
\end{gathered}
$$

## Proposition

There is a linear equilibrium where dealers submit the following bid schedules in the when-issued market:

$$
\begin{equation*}
q_{i}^{w i}(P)=\frac{\gamma}{(1-\gamma)^{2}} \frac{1}{\rho}\left(p^{w i}+(1-\gamma)^{2} \rho \theta_{i}^{c}-P\right) \tag{2}
\end{equation*}
$$

The equilibrium price and positions are given by:

$$
\begin{aligned}
p^{w i} & =(1-\gamma) E\left[p^{w}\right]+\gamma E\left[p^{s o}\right] \\
\theta_{i}^{w i} & =\frac{\gamma}{\rho}\left(v_{i}-\bar{v}\right)
\end{aligned}
$$

## When-Issued

## Corollary

## Underpricing: $p^{w i}>E\left[p^{5 o}\right]$

## When-Issued

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Underpricing: $p^{w i}>E\left[p^{50}\right]$

- The magnitude of underpricing is given by:

$$
p^{w i}-E\left[p^{s o}\right]=\frac{1}{(N-1)(N-2)} \rho \frac{E[Q]}{N}
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- Increasing in the size of the auction $\frac{E[Q]}{N}$ and slope of the demand $\rho$
- Decreasing in the number of dealers, $N$, even with constant $\frac{E[Q]}{N}$
- Dealers with higher (lower) than average $v_{i}$ will take long (short) positions in the WI market.


## When-Issued

- Limits to arbitrage:
- What is the payoff of a "sell high and buy low" strategy?
- Gain: $p^{w i}+\frac{\partial p^{w i}}{\partial \theta_{i}^{w i}} \theta_{i}^{w i}$
- Cost: $p^{\text {so }}+\frac{\partial p^{50}}{\partial \psi_{i}^{A}} \psi_{i}^{A}$
- In equilibrium, net gain $=-(1-\gamma) \rho E\left[\frac{Q}{I}\right]<0$


## When-Issued

Bid Schedules


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- Ranking Single vs. Multiple-price auction mechanisms using when-issued prices may be misleading
- e.g. Nyborg and Sundaransen (1996); Malvey and Archibald (1998); Goldreich (2007)
- The equilibrium value of $p^{w i}$ is affected by the auction mechanism.
- Future research!

When-Issued: Sequence of Auctions

## When-Issued: Sequence of Auctions

- $T$ when-issued rounds before the auction.
- Let $\theta_{i}^{c} \equiv \frac{1}{\rho}\left(v_{i}-\bar{v}\right)$.


## Proposition

There is a unique linear sub game perfect equilibrium where dealers submit bid schedules at each period $t=-T, \ldots, 1$ :

$$
\begin{equation*}
q_{i}^{w i, t}(P)=\frac{1}{(1-\gamma)^{-2 t}} \frac{\gamma}{\rho}\left(A_{i}^{t}-P\right)-\gamma \theta_{i}^{t-1} \tag{3}
\end{equation*}
$$

where $A_{i}^{t} \equiv p^{w i, c}+(1-\gamma)^{-2 t} \rho \theta_{i}^{c}$.

## Corollary

The when-issued prices $\left\{p^{t}\right\}_{t=-T}^{1}$ are the same on all $t=-T, . ., 1$ and given by

$$
\begin{equation*}
p^{t}=p^{w i} \tag{57}
\end{equation*}
$$

## When-Issued: Sequence of Auctions

- Positions are build up over the entire when-issued period:

$$
\theta_{i}^{t}=\gamma \theta_{i}^{c}+(1-\gamma) \theta_{i}^{t-1}
$$

- Volume - Let $\Upsilon^{T} \equiv \frac{\sum_{i, T}\left(q_{i}^{m i t}\right)^{2}}{N}$, in equilibrium,

$$
\Upsilon^{T}=s_{v}^{2}\left(\frac{1-(1-\gamma)^{T}}{\rho}\right)^{2}
$$

where $s_{v}^{2} \equiv \frac{1}{N} \sum\left(v_{i}-\bar{v}\right)^{2}$.

## Corollary

Efficient outcome in the limit. When $T \rightarrow \infty$,

$$
\tilde{\psi}_{i}^{F} \rightarrow \tilde{\psi}_{i}^{W} .
$$

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## Conclusion

- First to model the strategic behavior of bidders when they participate in a when-issued market prior to a multiple unit auction.
- Market power + Single-Price auction $\Rightarrow$ When-issued trading + Underpricing.
- Prices in the when-issued market are a biased proxy to the true market value of the underlying security.
- When-issued market mitigates/vanishes uniform-price inefficiencies.

Thank you.

## Equilibrium: Auction

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## Equilibrium: Auction

(1)

## Equilibrium: When-Issued

## When-Issued

- Similar to the auction stage.
- However, marginal valuations functions are different. Remember that:

$$
\begin{aligned}
\frac{d \tilde{\psi}_{i}^{A}}{d \theta_{i}^{1}} & =-\gamma \\
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$$

- Two effects when increasing $\theta_{i}^{\text {wi }}$ :
- Increases utility by $v_{i}^{\prime}\left(\psi_{i}^{F}\right) \times(1-\gamma)$


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\begin{aligned}
\frac{d \tilde{\psi}_{i}^{A}}{d \theta_{i}^{1}} & =-\gamma \\
\frac{d \tilde{\psi}_{i}^{F}}{d \theta_{i}^{1}} & =(1-\gamma)
\end{aligned}
$$

- Two effects when increasing $\theta_{i}^{\text {wi }}$ :
- Increases utility by $v_{i}^{\prime}\left(\psi_{i}^{F}\right) \times(1-\gamma)$
- Decreases the cost in the auction stage by $\gamma \times p^{\text {so }}$ (constant)


## When-Issued



## When-Issued



## When-Issued



When-Issued
(2)

## Sequence of Auctions: Bid Schedules

When-Issued: Sequence of Auctions


When-Issued: Sequence of Auctions


When-Issued: Sequence of Auctions


When-Issued: Sequence of Auctions


## Bid Sequential Auction

(3)

