An analysis of the trade balance for some OECD countries using periodic integration and cointegration

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Abstract

We analyze the recent imbalances in external accounts that have historically affected most of the developed countries from the point of view of trade balance. Following previous empirical studies (Husted (1992), Arize (2002) and Hamori(2009)) we analyzed the long-run relationship linking exports and imports, using quarterly data for Australia, Canada, Denmark, Sweden, United Kingdom, Norway, Switzerland, Japan, France, Italy, Netherlands, Finland and Spain. We use periodic integration and cointegration to deal with the seasonality and nonstationarity present in our time series and to analyze the long-run relationship between exports and imports. Finally in the case of France, Italy, Netherlands, Finland and Spain a clear change in the mean associated to the change of currencies it is observed in the data, hence we extend the test of periodic integration proposed by Boswijk and Franses (1996) allowing for a change in the mean.

VERY PRELIMINARY VERSION

1 Motivation

This paper analyzes the recent imbalances in external accounts that have historically affected most of the developed countries from the point of view of trade balance. The relevance of the disequilibria in the external balances during the last decade has renewed the academic interest for this issue. In particular, the case of the EMU countries deserves special attention as the union's net external position is close to equilibrium but many of their members present very large and persistent deficits or surpluses.

The approach of this paper follows the traditional theory that postulates the trade channel as the external adjustment mechanism. For this reason, the variables of interest are exports and imports of the countries analyzed.

Previous empirical studies, such as Husted (1992), Arize (2002) and, more recently, Hamori(2009) have analyzed the long-run relationship linking exports and imports using the cointegration methodology. Although the majority of the empirical evidence is based on annual or quarterly data and the latter can be affected by seasonal effects, to the best of our knowledge, the empirical literature has neglected the presence of seasonal non-stationary components giving rise to instabilities in the long-run relationships.

A simple way to deal with the instability of the relationships between elements of the current account, without assuming the existence of unobserved components, is the use of seasonal and periodical cointegration techniques. Seasonality is a phenomenon that has not received sufficient attention in the economic literature in general. The standard treatment is either to assume that the seasonality that appears in the time series is deterministic or, alternatively, to use a method to remove the seasonal component of the variables and estimate the models using seasonally-adjusted variables.

When it is assumed that the seasonality is deterministic, the normal practice is to use seasonal dummies, which implicitly assumes that seasonality is a deterministic phenomenon. In this case, the methods of seasonal adjustment commonly used are variants of the X-11 procedure (X-12 ARIMA and X-13 ARIMA SEATS) of the U.S. Census Bureau as well as procedures based on ARIMA-SEATS SPAN models developed at the Bank of Spain. However, these procedures usually corrupt the stochastic structure of the variables.

Ghysels (1990), Ghysels and Perron (1993) and del Barrio Castro et (2002) show that the removal of seasonality with X-11 and SEATS standard procedures introduces excessive persistence in the series, which reduces the power of unit root tests. Maravall (1993) shows how seasonal adjustment procedures induce non inverted moving average processes in the filtered series, invalidating the inference made in most of the unit root and cointegration tests. Olekalns (1994) extends this result to cases in which dummies or band-pass filters are used to remove seasonality. Abeysinghe (1994) shows that treatment of seasonal stochastic dummies leads to a spurious regression problem. In order to avoid these problems we intend to use a seasonal treatment that includes seasonal unit roots (Hylleberg, 1990, Hylleberg et al, 1995, Rodrigues and Taylor 2007) as well as periodical integration tests (Boswijk and Franses 1995, del Barrio Castro and Osborn, 2010) to determine the type of seasonality present in the non-stationary series analyzed. As shown in Ghysels and Osborn (2001), this point is crucial as it determines the type of cointegration between the set of variables analyzed. Specifically, if the series are seasonally integrated, long-term relationships can occur at each frequency, that is, "seasonal cointegration", (Lee, 1992 and Johansen and Schaumburg, 1999) or between the seasonal components of the series, namely "periodic cointegration" (Boswijk and Franses, 1995). However, if the series are periodically integrated, they can only be periodically cointegrated (del Barrio Castro and Osborn 2008). Moreover, if one does not take into account all the above mentioned and ignores the univariate properties of the series analyzed, it may originate problems of spurious correlations and unstable parameterization. Therefore, an important part of the instability observed in the estimates of traditional export-import relationships could be due to the omission of the above phenomena. Finally, another factor to consider is the modeling process where there is seasonal or periodical cointegration in the context of error correction models because the role that both types of cointegration can perform in improving the quality of the estimates and the stability of the parameters can be very relevant. The omission of common trends in seasonal frequencies or shared by the seasons of the series analyzed can lead to problems of omitted variables and instability in the estimated models.

Therefore, in this paper the econometric analysis consists of first determining the order of integration of the trade flows and then, if nonstationary, to test and estimate the existence of a long-run relationship between a country's exports and imports. Researchers confronted with nonstationary seasonal time series have two alternatives methods to deal with non-stationary seasonality Seasonal integration (SI) and or Periodic Integration (PI). Periodic Integration is more attractive than Seasonal Integration for the following reasons, first PI can arise naturally from the application of economic theory when the underlying economic driving forces, such as preferences or technologies, vary seasonally, as shown by Gersovitz and McKinnon (1978), Osborn (1988) and Hansen and Sargent (1993). Secondly, from an econometric perspective, PI is attractive because it implies that the seasons of the year are cointegrated with each other (Osborn (1991), Franses (1994)), and hence ensures that the patterns associated with the various seasons are linked in the long-run.

Finally we also extend the test of periodic integration proposed by Boswijk and Franses (1996) allowing for a changing mean in order to obtain the results for France, Italy, Netherlands, Finland and Spain, where a change in the mean in observed for exports and imports relative to the gross domestic product (see pictures 9 to 13)

The rest of the paper is organized as follows. Section 2 presents the theoretical background while section 3 states a review of the most relevant empirical literature. The econometric tests and the empirical results are reported in section 4 for the countries not affected by a change in the mean. Section 5 present the extension of the test for periodic integration allowing for a change in the mean, and also the empirical results for France, Italy, Netherlands, Finland and Spain. Finally, section 6 concludes.

2 Theoretical model

In this paper we follow Huster (1992) who presents a simple theoretical model of a small open economy with no government where there is a representative consumer. This economy produces and exports a composite good. The consumer can borrow and lend in the international markets using one-period instruments. His resources are output and profits from firms. that are used for consumption and savings. The consumer's budget constraint in the current period is:

$$C_0 = Y_0 + B_0 - I_0 - (1 + r_0)B_{t-1} \tag{1}$$

where C_0 is current consumption; Y_0 is output, I_0 is investment, r_0 is the one period world interest rate, B_0 is international borrowing that can be positive or negative, whereas $(1+r_0)B_{t-1}$ is the stock of debt by the agent (or the country's external debt). The budget constraint must hold for every period. Therefore, they can be combined to obtain the intertemporal budget constraint by iterating (1) forward:

$$B_0 = \sum_{t=1}^{\infty} \mu_t TA_t + \lim_{n\to\infty} \mu_n B_n \tag{2}$$

where $TA_t = X_t - M_t (= Y_t - C_t - I_t)$ represents the trade balance in period t (that is, income minus absorption), X_t are exports, M_t imports, $\lambda_0 = \frac{1}{(1+r_0)}$ and μ_t is the discount factor (the product of the first t values of λ. When the last term in equation (2) equals zero, the amount that a country borrows (lends) in international markets equals the present value of the future trade surpluses (deficits).

Assuming that the world interest rate is stationary, Husted(1992) expresses (1) as:

$$Z_t + (1 + r)B_{t-1} = X_t + B_t \tag{3}$$

where $Z_t = M_t + (r_t - r)B_{t-1}$. Solving forward as Hakkio and Rush (1991) do the next expression is obtained:

$$M_t + r_t B_{t-1} = X_t + \sum_{j=0}^{\infty} \lambda^{j-1}` [\Delta X_{t+j} - \Delta Z_{t+j}] + \lim_{j\to\infty} \lambda^{t+j} B_{t+j}, \tag{4}$$

where $\lambda = \frac{1}{(1+r)}$. The left-hand side consists of spending on imports and interest payments (receipts) on net foreign debt (assets). If we substract X_t from both sides and multiply by minus one, the left hand side becomes the economy's current account. Assuming that both Z_t and X_t are $I(1)$, (4) can be rewritten as:

$$X_t = \alpha + MM_t - \lim_{j\to\infty} \lambda^{t+j} B_{t+j} + \epsilon_t \tag{5}$$

where $MM_t = M_t + r_t B_{t-1}$.Assuming that the limit term equals zero, (5) we can obtain a testable equation:

$$X_t = a + b^* MM_t + e_t \tag{6}$$

where under the null hypothesis that the economy satisfies its intertemporal budget constraint, we expect $b = 1$ and e_t is stationary. Thus, if both variables are I(1), under the null, they are cointegrated, with a cointegrating vector $(1, -1)$.

We have also assumed earlier than the world interest rate is stationary. Therefore, the term $r_t B_{t-1}$ would also be stationary. In practice, we can test for cointegration between exports and imports when we believe that the adjustment works essentially through the trade channel. Alternative theories, such as Gourinchas and Rey (2007) consider that changes in assets valuations have been very important in the last twenty years. If this is the case, we should also account for valuation effects and our regression would suffer from an omitted variables bias.

3 Literature review

There are a few empirical studies that, in the last twenty years, have analyzed the trade channel adjustment of the external accounts. A summary is presented in the table below.

The evidence on cointegration is mixed. For a large group of countries there is cointegration between exports and imports, as in Hamori(2009) and Nayaran and Nayaran (2005), although the vector found is not frequently $(1, -1)$. The papers use either quarterly or annual data, in nominal and in real terms. Other papers analyze relative exports and imports over GDP. In none of the papers the authors consider the issue of seasonality, with the exception of Irandoust and Ericsson (2002), that use seasonally adjusted variables in their analysis.

Authors	Countries analyzed	Period	Variables
Azire(2002)	50, all continents	quart., 73-98	nom. X/GDP and M/GDP dom. cu
Fountas and Wu (1999)	US	quart., 67-94	X, M, real, nominal, relative
Hamori (2009)	G-7 countries	annu, 60-2005	X and M, mill. US $, trade bal
Herzer and Nowak-L. (2006)	Chile	annu, 75-2004	real X and M domest. currency
Husted (1992)	US	quart. 67-89	nom., real, differenced ratios X and M
Irandoust and Sjoo(2000)	Sweden	quart. 80-95	nom., real, X, M/GDP dom. currenc
Irandoust and Ericsson (2002)	Fr, G, I, Sw, UK, USA	quart. 71-97	real, log, seasonally adj.
Narayan and Narayan (2005)	22, least developed	annu. 60-2000	nominal X and M

4 Econometric techniques and results for time series without change in the mean

As in Azire (2002) we have decided to analyze the nominal ratio exp/gdp and imp/gdp in levels and in natural logs. In our case we have collected quarterly data (not seasonally adjusted) for the following countries: Australia, Canada, Denmark, Sweden, United Kingdom, Norway, Switzerland, Japan, France, Italy, Netherlands, Finland and Spain. The evolution of the ratios is depicted in figures 1 to 13. In all the cases the sample ends in 2009Q1 but it starts in 1960Q1 for Australia, 1961Q1 for the UK, 1975Q1 for Finland, 1977Q1 for Canada and Netherlands, 1978Q1 for Denmark and France and finally 1980Q1 for the remaining countries.

Note that in the case France, Italy, Netherlands, Finland and Spain we clearly observe a level shift (or change in the mean) that start in 1999Q1 associated with the change of the national currency for the Euro, hence we are going to analyze the evolution of these countries in a separate section as we need to deal with a structural break or a change in the mean

From pictures 1 to 8 we can observe that the ratios exp/gdp and imp/gdp show clear seasonal variation but without huge seasonal oscillations. Note also, that from the evolution of the time series presented in pictures 1 to 8, we do not observe a deterministic trending behavior in our data, hence we are going to consider only seasonal dummies in the deterministic part.

Researchers confronted with apparently nonstationary seasonal time series require methods of analysis that concurrently deal with the seasonal and nonstationary features of their data. Particularly within an economic context, the concept of Periodic Integration (PI) often provides a useful framework for such analysis for two reasons. Firstly, as shown by Gersovitz and McKinnon (1978), Osborn (1988) and Hansen and Sargent (1993), PI can arise naturally from the application of economic theory when the underlying economic driving forces, such as preferences or technologies, vary seasonally. Secondly, from an econometric perspective, PI can be attractive because it implies that the seasons of the year are cointegrated with each other (Osborn (1991), Franses (1994)), and hence ensures that the patterns associated with the various seasons are linked in the long-run. Indeed, the conventional class of integrated, or $I(1)$, time series form a special case of PI processes where the cointegrating vectors between adjacent (seasonal) observations have the form (1, -1).

Taking into account the previous arguments and the evolution of the ratios exp/gdp and imp/dgp for each country (figures 1 to 8), we are going to focus on periodic integration as a possible source of non-stationarity in our data. In order to explicitly recognize the role of seasonality, it is often convenient to represent a univariate time series as $y_{s\tau}$, where the first subscript refers to the season (s) and the second subscript to the year (τ), as we have quarterly data s = 1, 2, 3, 4. For simplicity of exposition, we assume that data are available for precisely N years, so that the total sample size is $T = 4N$. Note that, throughout the paper, it is understood that $y_{s-k,\tau} = y_{4-s+k,\tau-1}$ for $s - k \leq 0$.

Applications of periodic processes within economics have focused on the autoregressive case, with the zero-mean p^{th} order periodic autoregressive, or $PAR(p)$ process, defined by

$$y_{s\tau} = \alpha_s + \phi_{1s}y_{s-1,\tau} + \phi_{2s}y_{s-2,\tau} + \cdots + \phi_{ps}y_{s-p,\tau} + e_{s\tau}, \quad s = 1,\ 2,\ 3,\ 4 \quad (7)$$

where $e_{s\tau}$ is white noise. In (7) we only consider seasonal intercepts α_s due to the ratio nature of the analyzed data. Note that all the coefficients in this process may vary over seasons $s = 1, ..., 4$. The conventional (nonperiodic) $AR(p)$ process is a special case with $\phi_{is} = \phi_i$ $(s = 1, 2, 3, 4)$ for all $i = 1, 2, \ldots, p$. However, in the presence of seasonality, it is important to consider the possibility that the process may be periodic, with at least some AR coefficients in (7) varying over the year.

Under the assumption that $y_{s\tau}$ is integrated of order 1, and using a similar notation to Boswijk and Franses (1996), (7) can also be written as

$$\begin{aligned}(y_{s\tau} - \varphi_s y_{s-1,\tau}) \quad &= \quad \alpha_s^* + +\psi_{1s}\left(y_{s-1,\tau} - \varphi_{s-1}y_{s-2,\tau}\right) + \cdots + \\ &\quad\quad +\psi_{p-1,s}\left(y_{s-p+1,\tau} - \varphi_{s-p+1}y_{s-p,\tau}\right) + e_{s\tau} \qquad (8)\end{aligned}$$

where $\prod_{s=1}^{4} \varphi_s = 1$. In the special case $\varphi_s = 1$ $(s = 1, 2, 3, 4)$, (8) may be a periodic $I(1)$ process, such that the first difference is a stationary $PAR(p-1)$ process. On the other hand, when $\prod_{s=1}^{S} \varphi_s = 1$ but not all $\varphi_s = 1$ $(s = 1, 2, 3, 4)$, (8) is a periodically integrated, or $PI(1)$, process with the quasi-difference $y_{s\tau} - \varphi_s y_{s-1,\tau}$ being stationary; see Ghysels and Osborn (2001, pp.153-155) for further discussion of these possibilities. In the latter case $y_{s\tau} - \varphi_s y_{s-1,\tau}$ may have constant coefficients over seasons, although for convenience we refer to it as a stationary PAR process.

Boswijk and Franses (1996) analyze the distribution of the Likelihood Ratio test statistic for the null of periodic integration $\prod_{s=1}^{S} \varphi_s = 1$ versus the alternative of $\prod_{s=1}^{S} \varphi_s < 1$ in (8), with this statistic defined by

$$LR = T \ln\left(\frac{RSS_0}{RSS_1}\right) \tag{9}$$

where RSS_0 and RSS_1 denote the residual sum of squares under the null hypothesis and from the unrestricted form (7), respectively. Under the null hypothesis of a $PI(1)$ or $I(1)$ process, they show that this statistic has the same asymptotic distribution as the squared Dickey-Fuller $t-$statistic for a conventional (nonperiodic) $I(1)$ process.

In order to implement the previous test (31) we need to determine the order p for the unrestricted and restricted models (7) and (8). To do that we follow Franses and Paap (2004) and use the AIC criteria to determine p using 5 as the maximin value. Franses and Boswijk (1997) also proposed a F-type statistic F_{per} to test the null of non periodic variation in the coefficients of (7) $H_0 : \phi_{js} = \phi_j$ for $j = 1, \cdots p$. The results of these tests are reported in tables 1.a to 8.a. Note that models (7) and (8) tend to have a lot of parameters, and also that in order to fit model (8) we will need non linear methods of estimation. Recently del Barrio Castro and Osborn (2011) have proposed two non-parametric tests (based on the Breitung (2002) and Stock (1999) unit roots tests) that allow us to circumvent the limitations of the Boswijk and Franses (1996) test. They propose to compute a variance ratio statistic for a given season s as

$$VRT_s = N^{-2}\frac{\sum_{\tau=1}^{N} \hat{U}_{s\tau}^2}{\sum_{\tau=1}^{N} \hat{u}_{s\tau}^2} \qquad s = 1, ..., 4 \tag{10}$$

where $\hat{U}_{s\tau}$ is the season-specific partial sum $\hat{u}_{s1} + \hat{u}_{s2} + \cdots + \hat{u}_{s\tau}$, with $\hat{u}_{s\tau}$ obtained as the OLS residuals $\hat{u}_{s\tau} = y_{s\tau} - \widehat{\boldsymbol{\beta}}_s' \mathbf{z}_\tau$ from a regression of observations for season s, $y_{s\tau}$ $(\tau = 1, ..., N)$, on $\mathbf{z}_\tau$ that collects the deterministic part, in our case $\mathbf{z}_\tau = 1$. In order to test the $PI(1)/I(1)$ null hypothesis, they use the average variance ratio statistic

$$VRT = 4^{-1}\sum_{s=1}^{4} VRT_s \tag{11}$$

where each VR_s is defined in (10).

Additionally, based on Perron and Ng (1996) and Stock (1999) they propose to apply for a single season s, the corresponding season-specific MSB test statistic:

$$MSB_s = \left(\frac{N^{-2}\sum_{\tau=1}^{N} \hat{u}_{s,\tau-1}^2}{\widehat{\gamma}_{sl}}\right)^{\frac{1}{2}} \qquad s = 1, ..., 4 \tag{12}$$

which requires an appropriate long-run variance estimator $\widehat{\gamma}_{sl}$ for the annual difference $\Delta u_{s\tau} = u_{s\tau} - u_{s,\tau-1}$ relating to season s. $\widehat{\gamma}_{sl}$ is obtained based on sample autocovariances using the Barlett and quadratic spectral kernels, following Newey and West (1994, equations (3.8) to (3.15) and Table 1) data-dependent bandwidth procedure.

As in the previous case they propose the use of the average MSB statistic

$$MSB = 4^{-1} \sum_{s=1}^{4} MSB_s. \tag{13}$$

del Barrio Castro and Osborn (2011) show that the distributions of the VRT (11) and MSB (13) is the same as those reported for the original tests proposed by Breitung (2002) and Stock (1999) respectively. The results obtained for these tests are also reported in tables 1.a to 8.a. Finally, $MSB_b$and MSB_q denote the statistic MSB with the Barlett and quadratic spectral kernels, respectively.

From the results of the F_{per} test we find clear evidence of periodicity in both exp/gdp and imp/gdp ratios as well as in their natural logs for the majority of the countries. Exceptions are the case of Norway for exp/gpd and ln(exp/gdp), Switzerland for imp/gdp and ln(imp/gdp), and Japan for ln(imp/gdp). For Canada, Sweden and Japan all the periodic integration tests (LR, MSB and VRT) do not reject the null of periodic integration. In the case of Australia we do no reject the null of periodic integration with the MSB and VRT tests. Concerning the LR test, we do not reject the null of periodic integration for imp/gdp and ln(imp/gdp) but we do reject the null with the LR for both exp/gdp and ln(exp/gdp). In the case of Denmark we only reject the null of periodic integration with the LR test for exp/gdp imp/gdp and ln(imp/gdp). In the UK the null is rejected only for imp/gdp with the LR and at 5% level of significance and with the MSB_q at 10% and for ln(imp/gdp) also at 10% for the LR and the MSB tests. For Norway we only reject the null with the MSB tests at 10% for exp/gdp and imp/gdp. Finally, in the case of Switzerland the null is rejected with the VRT test for exp/gdp and imp/gdp at 10% level of significance and for the natural logarithms of the variables at 10%. Overall we can conclude that we have found reasonable empirical evidence in favour that both ratios follow periodically integrated processes for all the counties.

As shown in Ghysels and Osborn (2001, pp.168-171) and del Barrio Castro and Osborn (2008), when the series follow PI processes, the only cointegration possibilities are periodic cointegration or nonperiodic cointegration, with cointegration for any one season implying cointegration for all seasons, that is, full cointegration. They also show that in order to have full nonperiodic cointegration the involved processes must share the same φ_s coefficients in (8). Note that full nonperiodic cointegration is equivalent to conventional cointegration. Hence if exp/gdp and imp/gdp or their natural logs are cointegrated with a $(1, -1)$ vector both processes must share the same φ_s coefficients in (8). In tables 1.b to 8.b we report these coefficients for the analyzed time series for all the countries. In tables 1.a to 8.a we also report the results obtained with the LR, MSB and VRT when applied to the difference between exp/gdp and imp/gdp , that is labelled dif, as well as to the difference between ln(exp/gdp) and ln(imp/gdp), that we label difln.

Finally, del Barrio Castro and Osborn (2008) propose a residual based LR

to test the null of not full periodic cointegration between periodically integrated processes and obtain their asymptotic distribution, in particular they show that the LR_{CR} statistics follow the squared distribution reported by Phillips and Ouliaris (1988) for the residual based ADF cointegration test. The results for the former test are also reported in tables 1.b to 8.b.

In the case of Canada and the UK we find clear evidence of cointegration with a $(1, -1)$ vector for the levels and the logs. Note that the coefficients in tables 2.b and 5.b are quite similar. Moreover, with the LR_{CR} test we also find evidence of full periodic cointegration as expected. In the case of Australia the results point to cointegration with vector $(1, -1)$ except for the variable dif with the VRT test. As in the previous cases, the coefficients in table 1.b are quite similar in levels and natural logs, and as expected we also find evidence of full periodic cointegration with the LR_{CR} test. In the case of Norway there is no evidence in favour of $(1, -1)$ cointegration but we detect full nonperiodic cointegration at a 10% level. Also note that in this case the coefficients φ_s are quite different. For Japan we find weak evidence of $(1, -1)$ cointegration, but strong full nonperiodic cointegration. Finally for Denmark, Sweden and Switzerland we do not find nonperiodic cointegration with vector $(1, -1)$ nor full periodic cointegration.

5 Testing for periodic integration in time series with a changing mean

In this section we extend the Periodic integration test proposed by Boswijk and Franses (1996) to the case where we allow for a change in the mean in the deterministic part of the periodic autorregressive process. In particular we consider the following four cases, that are the periodic counterpart of the case considered by Perron(1990) and Perron and Vogelsang (1992a) under the null hypothesis of periodic integration. Maekawa (1997) consider structural breaks in a periodically integrated processes but he only pay attention to the PAR(1) model and do not consider case the following model:

$$y_{s\tau} = \gamma_s D\left(NB\right)_{s\tau} + \varphi_s y_{s-1,\tau} + u_{s\tau}$$

where:

$$\begin{aligned}
s &= 1, 2, 3, 4 \quad \tau = 1, 2, 3, ..., N \\
D\left(NB\right)_{s\tau} &= 1 \text{ if } \tau = N_B + 1 \text{ otherwise } 0 \\
DU_{s\tau} &= 1 \text{ if } \tau > N_B \text{ otherwise } 0 \\
\prod_{s=1}^{S} \varphi_s &= 1 \\
\left(1 - \psi_{1s} L - \psi_{2s} L^2 - \cdots - \psi_{p-1,s} L^{p-1}\right) u_{s\tau} &= \varepsilon_{s\tau}
\end{aligned}$$

where N_B $(1 < N_B < N)$ is the date of break and we are going to assume that $N_B = \lambda N$, where λ is the fraction of break. As it is pointed out in Boswijk and Franses (1996) and In Ghysels and Osborn (2003) the key to explore the long run properties of PI processes is the vector of quarters representation and in particular the vector moving-average (VMA) representation:

$$\begin{aligned}
Y_\tau - Y_{\tau-1} &= (\Theta_0 + \Theta_1 B)\,\Psi(B)^{-1} E_\tau \\
\text{with} &: \\
Y_\tau &= \begin{bmatrix} y_{1\tau} & y_{2\tau} & y_{3\tau} & y_{4\tau} \end{bmatrix}' \quad E_\tau = \begin{bmatrix} \varepsilon_{1\tau} & \varepsilon_{2\tau} & \varepsilon_{3\tau} & \varepsilon_{4\tau} \end{bmatrix}' \\
\Theta_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ \varphi_2 & 1 & 0 & 0 \\ \varphi_2\varphi_3 & \varphi_3 & 1 & 0 \\ \varphi_2\varphi_3\varphi_4 & \varphi_3\varphi_4 & \varphi_4 & 1 \end{bmatrix} \Theta_1 = \begin{bmatrix} 0 & \varphi_3\varphi_4\varphi_1 & \varphi_4\varphi_1 & \varphi_1 \\ 0 & 0 & \varphi_4\varphi_1\varphi_2 & \varphi_1\varphi_2 \\ 0 & 0 & 0 & \varphi_1\varphi_2\varphi_3 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\end{aligned} \tag{14}$$

where B is the annual lag operator. Following the lines of Boswijk and Franses (1996) from (14) it is possible to write:

$$Y_\tau = Y_0 + ab'\Psi(1)^{-1} \sum_{j=1}^{\tau} E_j + C^*(1)\,E_\tau \tag{15}$$

with:

$$\mathbf{C}(1) = (\boldsymbol{\Theta}_0 + \boldsymbol{\Theta}_1) = \mathbf{ab}' \tag{16}$$

where

$$\begin{aligned}
\mathbf{a} &= \begin{bmatrix} 1 & \varphi_2 & \varphi_2\varphi_3 & \varphi_2\varphi_3\varphi_4 \end{bmatrix}', \\
\mathbf{b} &= \begin{bmatrix} 1 & \varphi_1\varphi_3\varphi_4 & \varphi_1\varphi_4 & \varphi_1 \end{bmatrix}'.
\end{aligned} \tag{17}$$

Which is the common trend representation of the PI process without considering deterministic terms, to obtain a equivalent representation to our case we only have to replace E_τ in (15) by $(\boldsymbol{\gamma} D(NB)_\tau + E_\tau)$, where $\boldsymbol{\gamma} = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \end{bmatrix}'$ and $D(NB)_\tau$ is the 4×1 vectors associated to $D(NB)_{s\tau}$. Hence after some rewriting we have:

$$\begin{aligned}
Y_\tau &= Y_0 + ab'\Psi(1)^{-1} \boldsymbol{\gamma} DU_\tau + C^*(1)\,\boldsymbol{\gamma} D(NB)_\tau + X_\tau \\
\textit{with} &: \\
X_\tau &= ab'\Psi(1)^{-1} \sum_{j=1}^{\tau} E_j + C^*(L)\,E_\tau
\end{aligned}$$

Note that in the previous expression the term $C^*(1)\,\boldsymbol{\gamma} D(NB)_\tau$ plays a role equivalent to the correction added by Perron and Vogelsang (1992a,1992b) to the initial analysis developed by Perron (1990) when testing for unit roots with a changing mean

Finally it is possible to summarize the main stochastic characteristics of a PI(1) process in the following Lemma due to Boswijk and Franses (1996).

5.1 The test.

In this section we present the test for PI that allows the presence of structural breaks in the deterministic part. Hence we are testing the null hypothesis $\varphi_1\varphi_2\varphi_3\varphi_4 = 1$ against the alternative hypothesis $\varphi_1\varphi_2\varphi_3\varphi_4 < 1$, we have the following PAR(p) unrestricted models:

$$\begin{aligned} \tilde{y}_{s\tau} &= y_{s\tau} - \hat{\mu}_s - \hat{\gamma}_s^* DU_{s\tau} \\ \tilde{y}_{s\tau} &= \sum_{j=0}^{p} \omega_{js} D\left(NB\right)_{s-j,\tau} + \sum_{j=1}^{p} \phi_{js} \tilde{y}_{s-j,\tau} + \varepsilon_{s\tau} \end{aligned} \tag{18}$$

Under the alternative the time series follows a stationary PAR(p) process. And the restricted models:

$$\tilde{y}_{s\tau} = \varphi_{s-1}\tilde{y}_{s-1,\tau} + \sum_{j=0}^{p-1} \omega_{js} D\left(NB\right)_{s-j,\tau} + \sum_{j=1}^{p-1} \psi_{js}\left(\tilde{y}_{s-j,\tau} - \varphi_{s-j}\tilde{y}_{s-j-1,\tau}\right) + \varepsilon_{s\tau} \tag{19}$$

with the restriction $\varphi_1\varphi_2\varphi_3\varphi_4 = 1$ imposed, but ψ_{js} unrestricted, with the estimation achieved using nonlinear least squares. We employ the test Likelihood Ratio test proposed by Boswijk and Franses (1996):

$$LR_{io}\left(\lambda\right) = N \ln\left(\hat{\sigma}_0^2/\hat{\sigma}^2\right)$$

In the following proposition we present the distribution of the test is the four different cases.

Proposition 1 *Under the null hypothesis of periodic integration the distribution of the likelihood ratio test statistic obtained from (18)/(19) is the following:*

$$\begin{aligned} LR_{io}\left(\lambda\right) &\Rightarrow \left[DE\left(\lambda\right)\right]^{-1}\left(\left[NU\left(\lambda\right)\right]\right)^2 \qquad (20) \\ where &: \\ \left[NU\left(\lambda\right)\right] &= \int_0^1 w\left(r\right)dw\left(r\right) - \lambda^{-1}w\left(\lambda\right)\int_0^{\lambda} w\left(r\right)dr + \\ &\quad + \left(1-\lambda\right)^{-1}\left[w\left(1\right) - w\left(\lambda\right)\right]\int_{\lambda}^1 w\left(r\right)dr \\ \left[DE\left(\lambda\right)\right] &= \int_0^1 \left[w\left(r\right)\right]^2 dr - \lambda^{-1}\left[\int_1^{\lambda} w\left(r\right)dr\right]^2 \\ &\quad - \left(1-\lambda\right)\left[\int_{\lambda}^1 w\left(r\right)dr\right]^2 \end{aligned}$$

Note results (20) is the square of the distribution reported in Perron and Vogelsang (1992a,1992b)

In table 9.a we report the empirical quantiles of the $LR_{io}(\lambda)$ test based on 20.000 replications and for a sample size of $\tau = 1000$ with $S = 4$. For $\lambda = 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7$ and 0.8. It is clear that the quantiles associated (20) is equivalent to the square of the quantiles reported in Perron (1988).

In order to check size and power performance of the $LR_{io}(\lambda)$ test we run a small monte-carlo experiment based in the following data generating process:

$$\begin{aligned} y_{s\tau} &= \varphi_s y_{s-1,\tau} + u_{s\tau} \quad s = 1,2,3,4 \\ with &: \\ a)\ \ \varphi_1 &= 0.9 \quad \varphi_2 = 1 \quad \varphi_3 = 1.25 \quad \varphi_4 = 1/(\varphi_1\varphi_2\varphi_3) \\ b)\ \ \varphi_1 &= 0.9 \quad \varphi_2 = 1 \quad \varphi_3 = 1.25 \quad \varphi_4 = 0.8/(\varphi_1\varphi_2\varphi_3) \\ c)\ \ \varphi_1 &= 0.9 \quad \varphi_2 = 1 \quad \varphi_3 = 1.25 \quad \varphi_4 = 0.5/(\varphi_1\varphi_2\varphi_3) \end{aligned} \tag{21}$$

with the combination of parameters $a)$ we are under the null of periodic integration, hence we will measure the empirical size of the test, and with the combinations $b)$ and $c)$ we are under the alternative and we will measure the empirical power of the test. We consider 3 alternative possibilities for $u_{s\tau}$:

$$\begin{aligned} i)\ \ u_{s\tau} &= \varepsilon_{s\tau} \quad \varepsilon_{s\tau}\tilde{\ }Niid(0,1) \\ ii)\ \ u_{s\tau} &= \varepsilon_{s\tau} - 0.5\varepsilon_{s-1,\tau} \quad \varepsilon_{s\tau}\tilde{\ }Niid(0,1) \\ iii)\ \ u_{s\tau} &= 0.5u_{s-1,\tau} + \varepsilon_{s\tau} \quad \varepsilon_{s\tau}\tilde{\ }Niid(0,1). \end{aligned}$$

The results are obtained for a sample size of $4N = 200$ and based on 5000 replications, we report the results obtained when the order of the fitted models (18)/(19) goes form 1 to 5. The results obtained for $i)$, $ii)$ and $iii)$ are collected in tables 9.b, 9.c and 9.d respectively. Clearly the best performance in terms of size and power it is obtained with and $PAR(1)$ for $i)$, with a $PAR(5)$ for $ii)$ and with a $PAR(2)$ for $iii)$. Note that we obtain a reasonable performance in terms of empirical size and power in small sample for the data generating process (21) without the presence of a structural break as in Perron and Vogelsang (1992) we also consider a monte carlo experiment with a change in the mean using the following data generating process:

$$\begin{aligned} x_{s\tau} &= -0.975 - 0.42DU_{s\tau} + y_{s\tau} \quad s = 1,2,3,4 \\ y_{s\tau} &= \varphi_s y_{s-1,\tau} + u_{s\tau} \\ u_{s\tau} &= 0.5u_{s-1,\tau} + \varepsilon_{s\tau} \quad \varepsilon_{s\tau}\tilde{\ }Niid(0,1) \\ with &: \\ a)\ \ \varphi_1 &= 0.9 \quad \varphi_2 = 1 \quad \varphi_3 = 1.25 \quad \varphi_4 = 1/(\varphi_1\varphi_2\varphi_3) \\ b)\ \ \varphi_1 &= 0.9 \quad \varphi_2 = 1 \quad \varphi_3 = 1.25 \quad \varphi_4 = 0.8/(\varphi_1\varphi_2\varphi_3). \end{aligned} \tag{22}$$

The results for this case are collected in table 9.e. Note that in this case we also obtain the best performance in terms of empirical size and power when the correct order of augmentation it is used, that is when a $PAR(2)$ order it is used to fit models (18)/(19). In this case we also obtain a reasonable performance in terms of empirical size and power.

Remark 2 *Following Carrion-i-Silvestre and Berenguer (2010), Zivot and Andrews (1992) ,Gregory and Hansen (1996) and Perron (1997) based on the results of Proposition 1 it is possible to establish for* $LR_{io}^{*} = \sup_{\lambda \in \Lambda} LR_{io}(\lambda)$:

$$LR_{io}^{*} \Rightarrow \sup_{\lambda \in \Lambda} \left\{ [DE(\lambda)]^{-1} ([NU(\lambda)])^{2} \right\} \tag{23}$$

where Λ *is closed subset of the interval* $(0, 1)$.

In this case using the supremum (sup) it is also possible to obtain a test that do not depend on λ and hence it is possible to use the critical values reported in table 1 of Perron and Vogelsang (1992b). Finally in the case of (20) the critical values are reported in Perron (1990) table 4.

5.2 Empirical results

In this subsection we collect the empirical results for France, Italy, Netherlands, Finland and Spain. It is clear for pictures 10 to 14 that for all the mentioned countries we observe a clear change in the mean associated with the change of the national currencies by the Euro. Hence in this section we use $LR_{io}(\lambda)$ as we know the break date. We determine the order on the unrestricted and restricted $PAR(p)$ models (18)/(19) with the AIC and BIC criteria starting for a maximum order of $p = 5$. In this case we also report the result obtained with the statistic F_{per} to test the null of non periodic variation in the coefficients in the model (18) $H_0 : \phi_{js} = \phi_j$ for $j = 1, \cdots p$. We also report the results obtained for the $LR_{io}(\lambda)$ test described in the previous subsection and finally assuming that there is a co-break we report the results obtained with the LR_{CR} to test the null of the absence of cointegration between the ratios exp/gdp and imp/gdp. These results could be found in tables 10 to 14. Note first that with the F_{per} test we obtain evidence against the null for Italy, Finland and in levels for Netherlands. We only clearly reject the null of periodic integration with the $LR_{io}(\lambda)$ test in the case of France. We find evidence of cointegration between the exp/gdp and imp/gdp in logs for the case of Netherlands and Finland, and reject the null of no cointegration for Spain but only at the 10%.

Finally it will be interesting to extend the approach of Gregory and Hansen (1996) to the case of periodic cointegration, in order to allow for breaks in the cointegration vector, but the this is part our future agenda...

6 Concluding remarks.

7 References.

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Table 1.a: Australia

	F_{per}	LR	MSB_b	MSB_q	VRT
exp/gdp	9,2262**	12,4398**	0,2741	0,2876	0,0576
imp/gdp	7,7790**	6,3479	0,3841	0,4076	0,0649
$ln(exp/gdp)$	6,9374**	12,6438**	0,2798	0,2825	0,0600
$ln(imp/gdp)$	11,2127**	7,1483	0,3329	0,3462	0,0618
dif	1,0388	18,3054**	0,1838**	0,1894**	0,0149
$difln$	1,8039	18,7643**	0,1817**	0,1944*	0,0098**

** and * statistically significant at a 5% and 10% respectively.

Table 1.b: Australia

	exp/gdp	imp/gdp	$ln(exp/gdp)$	$ln(imp/gdp)$
$\hat{\varphi}_1$	0,785	0,791	0,713	0,758
$\hat{\varphi}_2$	0,967	1,045	1,006	1,062
$\hat{\varphi}_3$	1,324	1,138	1,251	1,098
$\hat{\varphi}_4$	0,994	1,062	1,115	1,132
LR_{CR}	23,3558**		33,1368**	

Table 2.a: Canada

	F_{per}	LR	MSB_b	MSB_q	VRT
exp/gdp	9,2664**	3,5365	0,4179	0,4175	0,0765
imp/gdp	6,7301**	3,3886	0,4202	0,4195	0,0790
ln(exp/gdp)	7,0393**	3,6664	0,4465	0,4474	0,0797
ln(imp/gdp)	26,0935**	3,8353	0,4291	0,4283	0,0814
dif	3,4386**	9,5601**	0,1663**	0,1708**	0,0208
dif*ln*	2,6053**	13,0961**	0,1826**	0,1792**	0,0095**

Table 2.b: Canada

	exp/gdp	imp/gdp	ln(exp/gdp)	ln(imp/gdp)
$\hat{\varphi}_1$	1,1384	1,0853	1,1376	1,0690
$\hat{\varphi}_2$	0,9160	0,9110	0,8967	0,8756
$\hat{\varphi}_3$	0,9867	1,0424	1,0797	1,2053
$\hat{\varphi}_4$	0,972	0,970	0,908	0,886
LR_{CR}	29,9052**		29,5145**	

Table 3.a: Denmark

	F_{per}	LR	MSB_b	MSB_q	VRT
exp/gdp	4,7637**	9,4217**	0,3013	0,3047	0,0641
imp/gdp	3,9823**	14,4301**	0,1941	0,2103	0,0149
ln(exp/gdp)	3,4351**	6,9169	0,3046	0,3077	0,0624
ln(imp/gdp)	3,4995**	13,5095**	0,1981	0,2115	0,0149
dif	3,2743**	6,9876	0,4410	0,4329	0,0681
difln	3,1794**	8,1569*	0,4362	0,4385	0,0637

Table 3.b: Denmark

	exp/gdp	imp/gdp	ln(exp/gdp)	ln(imp/gdp)
$\hat{\varphi}_1$	0,9624	1,3365	0,9613	1,3120
$\hat{\varphi}_2$	1,0177	0,8306	0,9996	0,8743
$\hat{\varphi}_3$	1,2462	1,0054	1,3155	1,0011
$\hat{\varphi}_4$	0,8192	0,8960	0,7911	0,8708
LR_{CR}	7,5035		7,3776	

Table 4.a: Sweden

	F_{per}	LR	MSB_b	MSB_q	VRT
exp/gdp	3,5257**	3,8686	0,2777	0,2721	0,0368
imp/gdp	4,0235**	6,3992	0,2505	0,2491	0,0343
ln(exp/gdp)	3,9594**	3,5997	0,2732	0,2675	0,0348
ln(imp/gdp)	3,9580**	6,2928	0,2523	0,2501	0,0331
dif	4,3273**	5,4004	0,3202	0,3387	0,0250
difln	5,2382**	2,2068	0,3021	0,3313	0,0217

Table 4.b: Sweden

	exp/gdp	imp/gdp	ln(exp/gdp)	ln(imp/gdp)
$\hat{\varphi}_1$	1,1255	1,1227	1,0823	1,1039
$\hat{\varphi}_2$	0,9395	0,9824	0,9574	0,9999
$\hat{\varphi}_3$	1,1513	1,1266	1,2000	1,1273
$\hat{\varphi}_4$	0,8214	0,8048	0,8042	0,8036
LR_{CR}	7,1242		2,5414	

Table 5.a: United Kingdom

	F_{per}	LR	MSB_b	MSB_q	VRT
exp/gdp	0,2558	6,1229	0,2299	0,2483	0,0252
imp/gdp	3,5524**	9,6687**	0,2171	0,2301*	0,0308
ln(exp/gdp)	0,3915	5,8637	0,2752	0,2635	0,0274
ln(imp/gdp)	2,7003**	8,6335*	0,1964*	0,2038*	0,0351
dif	2,7003**	13,5919**	0,1785*	0,1690**	0,0131*
difln	3,7082**	15,3947**	0,1555*	0,1591**	0,0115*

Table 5.b: United Kingdom

	exp/gdp	imp/gdp	ln(exp/gdp)	ln(imp/gdp)
$\hat{\varphi}_1$	0,9866	0,9618	0,9578	0,9077
$\hat{\varphi}_2$	1,0675	1,1846	1,0768	1,1893
$\hat{\varphi}_3$	0,9547	0,9128	1,0082	0,9651
$\hat{\varphi}_4$	0,9946	0,9616	0,9617	0,9598
LR_{CR}	14,0012**		15,4674**	

Table 6.a: Norway

	F_{per}	LR	MSB_b	MSB_q	VRT
exp/gdp	1,4852	6,4036	0,1920*	0,1932*	0,0398
imp/gdp	2,1442**	2,7533	0,2944	0,2882	0,0650
ln(exp/gdp)	1,5214	6,4674	0,1928*	0,1944*	0,0382
ln(imp/gdp)	1,7184*	2,3716	0,2746	0,2721	0,0655
dif	2,9522**	3,9529	0,2589	0,2590*	0,0648
difln	2,4200**	3,3283	0,2762	0,2785	0,0670

Table 6.b: Norway

	exp/gdp	imp/gdp	ln(exp/gdp)	ln(imp/gdp)
$\hat{\varphi}_1$	1,0061	0,9471	1,0272	0,9604
$\hat{\varphi}_2$	0,9080	0,9299	0,8724	0,9245
$\hat{\varphi}_3$	1,2785	0,8408	1,3097	0,8800
$\hat{\varphi}_4$	0,8563	1,3506	0,8520	1,2798
LR_{CR}	9,8643*		9,5170*	

Table 7.a: Switzerland

	F_{per}	LR	MSB_b	MSB_q	VRT
exp/gdp	3,7263**	1,1613	0,2344	0,2351	0,0111*
imp/gdp	0,5875	4,3516	0,1997	0,1976*	0,0084**
ln(exp/gdp)	4,6170**	1,1163	0,2329	0,2342	0,0111*
ln(imp/gdp)	0,7261	4,0753	0,2037*	0,2031*	0,0085**
dif	5,0733**	2,1789	0,2294	0,2230	0,0121*
difln	6,2268**	1,9552	0,2381	0,2330	0,0124*

Table 7.b: Switzerland

	exp/gdp	imp/gdp	ln(exp/gdp)	ln(imp/gdp)
$\hat{\varphi}_1$	1,1748	1,0669	1,2062	1,1041
$\hat{\varphi}_2$	1,0906	1,0369	1,0615	1,0119
$\hat{\varphi}_3$	1,0924	1,0500	1,1387	1,0928
$\hat{\varphi}_4$	0,7145	0,8609	0,6859	0,8190
LR_{CR}	1,9734		1,7272	

Table 8.a: Japan

	F_{per}	LR	MSB_b	MSB_q	VRT
exp/gdp	4,3632**	3,2341	0,2863	0,2851	0,0246
imp/gdp	2,2274*	4,7804	0,3256	0,3267	0,0257
ln(exp/gdp)	2,8044**	2,9987	0,3039	0,3022	0,0248
ln(imp/gdp)	0,2432	3,2508	0,3208	0,3159	0,0255
dif	1,1161	15,8320**	0,2180	0,2190*	0,0112*
difln	1,0003	8,1958*	0,2420	0,2418	0,0173

Table 8.b: Japan

	exp/gdp	imp/gdp	ln(exp/gdp)	ln(imp/gdp)
$\hat{\varphi}_1$	0,9324	1,0434	0,9054	0,9786
$\hat{\varphi}_2$	1,1829	1,0514	1,1573	1,0267
$\hat{\varphi}_3$	1,0332	1,0310	0,9961	1,0125
$\hat{\varphi}_4$	0,8775	0,8841	0,9582	0,9831
LR_{CR}	14,1942**		12,1204**	

Table 9.a: Empirical quantiles of LR_{io}.

λ	0,9	0,95	0,975	0,99
0,2	8,5077	10,3599	12,1984	14,3974
0,3	9,0153	10,8519	12,5488	14,8566
0,4	9,3382	11,1417	12,9004	14,9936
0,5	9,3446	11,1893	12,9652	15,3002
0,6	9,3315	11,3013	13,0715	15,4670
0,7	8,9711	10,9550	12,7243	14,7483
0,8	8,4180	10,3099	12,1546	14,5669

Table 9.b: Empirical size and power of LR_{io} for (21) with i).

$\prod_{s=1}^{4}\varphi_s$	λ	$PAR(1)$	$PAR(2)$	$PAR(3)$	$PAR(4)$	$PAR(5)$
1	0,2	0,037	0,036	0,040	0,046	0,041
1	0,3	0,035	0,037	0,039	0,044	0,042
1	0,4	0,036	0,038	0,038	0,043	0,049
1	0,5	0,036	0,037	0,038	0,038	0,040
1	0,6	0,035	0,038	0,036	0,041	0,045
1	0,7	0,038	0,038	0,039	0,037	0,033
1	0,8	0,039	0,034	0,033	0,035	0,033
0,8	0,2	0,229	0,233	0,243	0,239	0,205
0,8	0,3	0,205	0,209	0,213	0,221	0,185
0,8	0,4	0,193	0,201	0,202	0,203	0,180
0,8	0,5	0,177	0,189	0,187	0,194	0,169
0,8	0,6	0,178	0,187	0,195	0,195	0,166
0,8	0,7	0,196	0,194	0,193	0,190	0,169
0,8	0,8	0,223	0,219	0,217	0,198	0,173
0,5	0,2	0,973	0,934	0,902	0,855	0,763
0,5	0,3	0,961	0,923	0,877	0,821	0,734
0,5	0,4	0,951	0,908	0,858	0,798	0,709
0,5	0,5	0,947	0,903	0,847	0,787	0,694
0,5	0,6	0,950	0,908	0,861	0,793	0,710
0,5	0,7	0,959	0,921	0,864	0,806	0,714
0,5	0,8	0,968	0,932	0,892	0,830	0,738

Table 9.c: Empirical size and power of LR_{io} for (21) with ii).

$\prod_{s=1}^{4} \varphi_s$	λ	$PAR(1)$	$PAR(2)$	$PAR(3)$	$PAR(4)$	$PAR(5)$
1	0,2	0,611	0,206	0,099	0,074	0,050
1	0,3	0,651	0,215	0,093	0,068	0,052
1	0,4	0,671	0,201	0,083	0,062	0,046
1	0,5	0,684	0,199	0,084	0,057	0,040
1	0,6	0,660	0,199	0,082	0,058	0,045
1	0,7	0,635	0,191	0,084	0,055	0,041
1	0,8	0,585	0,179	0,081	0,051	0,032
0,8	0,2	0,997	0,766	0,482	0,366	0,278
0,8	0,3	0,995	0,750	0,451	0,338	0,247
0,8	0,4	0,996	0,729	0,431	0,314	0,242
0,8	0,5	0,995	0,718	0,416	0,293	0,217
0,8	0,6	0,996	0,730	0,419	0,296	0,218
0,8	0,7	0,996	0,724	0,424	0,298	0,218
0,5	0,2	1,000	1,000	0,987	0,943	0,868
0,5	0,3	1,000	1,000	0,979	0,926	0,826
0,5	0,4	1,000	1,000	0,975	0,921	0,819
0,5	0,5	1,000	0,999	0,974	0,912	0,805
0,5	0,6	1,000	1,000	0,976	0,910	0,810
0,5	0,7	1,000	0,999	0,977	0,908	0,806
0,5	0,8	1,000	1,000	0,985	0,930	0,840

Table 9.d: Empirical size and power of LR_{io} for (21) with ii).

$\prod_{s=1}^{4} \varphi_s$	λ	$PAR(1)$	$PAR(2)$	$PAR(3)$	$PAR(4)$	$PAR(5)$
1	0,2	0,016	0,043	0,053	0,070	0,155
1	0,3	0,019	0,044	0,059	0,079	0,153
1	0,4	0,019	0,040	0,054	0,067	0,144
1	0,5	0,023	0,043	0,052	0,065	0,139
1	0,6	0,018	0,043	0,052	0,064	0,141
1	0,7	0,017	0,037	0,044	0,058	0,136
1	0,8	0,017	0,036	0,044	0,061	0,133
0,8	0,2	0,003	0,229	0,229	0,217	0,203
0,8	0,3	0,002	0,208	0,214	0,202	0,187
0,8	0,4	0,002	0,199	0,201	0,190	0,172
0,8	0,5	0,003	0,196	0,194	0,189	0,170
0,8	0,6	0,002	0,185	0,187	0,174	0,165
0,8	0,7	0,002	0,191	0,189	0,176	0,160
0,8	0,8	0,001	0,207	0,198	0,180	0,167
0,5	0,2	0,067	0,874	0,831	0,776	0,699
0,5	0,3	0,048	0,835	0,789	0,726	0,650
0,5	0,4	0,040	0,829	0,781	0,717	0,633
0,5	0,5	0,038	0,822	0,770	0,707	0,621
0,5	0,6	0,041	0,837	0,780	0,703	0,632
0,5	0,7	0,048	0,829	0,776	0,707	0,633
0,5	0,8	0,065	0,863	0,814	0,743	0,660

Table 9.e: Empirical size and power of LR_{io} for (22).

$\prod_{s=1}^{4} \varphi_s$	λ	$PAR(1)$	$PAR(2)$	$PAR(3)$	$PAR(4)$	$PAR(5)$
1	0,2	0,016	0,041	0,053	0,071	0,154
1	0,3	0,017	0,037	0,046	0,061	0,143
1	0,4	0,019	0,043	0,057	0,072	0,145
1	0,5	0,019	0,041	0,053	0,063	0,145
1	0,6	0,019	0,038	0,048	0,066	0,134
1	0,7	0,017	0,034	0,041	0,050	0,125
1	0,8	0,016	0,036	0,041	0,056	0,134
0,8	0,2	0,005	0,235	0,232	0,217	0,202
0,8	0,3	0,003	0,221	0,221	0,210	0,185
0,8	0,4	0,002	0,203	0,212	0,199	0,174
0,8	0,5	0,002	0,191	0,180	0,173	0,165
0,8	0,6	0,002	0,197	0,198	0,183	0,170
0,8	0,7	0,001	0,205	0,198	0,188	0,170
0,8	0,8	0,002	0,221	0,211	0,192	0,169

Table 10: France

	F_{per}	$LR_{io}(\lambda)$	LR_{CR}
exp/dgp	0,2956	13,7320**	2,7537
imp/dgp	1,3455	15,6965**	
ln(exp/dgp)	0,1015	13,7380**	3,5439
ln(exp/dgp)	0,3885	13,9926**	

Table 11: Italy

	F_{per}	$LR_{io}(\lambda)$	LR_{CR}
exp/dgp	4,2104***	6,7669	4,2554
imp/dgp	6,7757***	8,7128	
ln(exp/dgp)	2,2115*	8,4031	4,7141
ln(exp/dgp)	2,8290**	8,9749	

Table 12: Netherlands

	F_{per}	$LR_{io}(\lambda)$	LR_{CR}
exp/dgp	6,1678***	12,7855**	4,9678
imp/dgp	6,8509***	8,8625	
ln(exp/dgp)	1,1100	9,6861*	15,3403**
ln(exp/dgp)	0,3862	9,9918*	

Table 13: Finland

	F_{per}	$LR_{io}(\lambda)$	LR_{CR}
exp/dgp	4,8178***	2,6434	3,9836
imp/dgp	3,1068**	13,8461**	
ln(exp/dgp)	3,2237***	4,1203	11,4335**
ln(exp/dgp)	2,0870*	6,9096	

Table 14: Spain

	F_{per}	$LR_{io}(\lambda)$	LR_{CR}
exp/dgp	0,9876	4,6657	10,7291*
imp/dgp	0,3175	10,4385*	
ln(exp/dgp)	0,6198	6,7820	10,0310*
ln(exp/dgp)	0,6668	11,3004*	

Proof. First note that from (14) it is possible to write:

$$\mathbf{y}_\tau - \mathbf{y}_{\tau-1} = (\mathbf{\Theta}_0 + \mathbf{\Theta}_1 L)\, \mathbf{\Psi}(L)^{-1}\mathbf{e}_\tau = \mathbf{C}(L)\,\mathbf{u}_\tau \tag{24}$$

with $\mathbf{u}_\tau = \mathbf{\Psi}(L)^{-1}\mathbf{e}_\tau$, them we have that:

$$\begin{aligned}\mathbf{y}_\tau &= \mathbf{y}_0 + \mathbf{C}(1)\sum_{j=1}^{\tau}\mathbf{u}_j + O_p(1)\\ &= \mathbf{y}_0 + \mathbf{ab}'\sum_{j=1}^{\tau}\mathbf{u}_j + O_p(1)\,.\end{aligned}$$

Replace E_τ by $(\boldsymbol{\gamma} D\,(NB)_\tau + u_\tau)$, hence we have:

$$\mathbf{y}_\tau = \mathbf{y}_0 + \mathbf{ab}'\sum_{j=1}^{\tau}\mathbf{u}_j + \mathbf{ab}'\boldsymbol{\gamma} DU_\tau + O_p(1)$$

As in Perron and Vogelsang (1992a) it is possible to write for $\tilde{y}_{s\tau} = y_{s\tau} - \hat{\mu}_s - \hat{\gamma}_s^* DU_{s\tau}$, where $\bar{y}_s^a = N_b^{-1}\sum_{\tau=1}^{N_b} y_{s\tau} = \lambda^{-1}N^{-1}\sum_{\tau=1}^{N_b} y_{s\tau}$ and $\bar{y}_s^b = (N-N_b)^{-1}\sum_{\tau=N_b+1}^{N} y_{s\tau} = (1-\lambda)^{-1}N^{-1}\sum_{\tau=N_b+1}^{N} y_{s\tau}$:

$$\begin{aligned}\tilde{y}_{s\tau} &= y_{s\tau} - \bar{y}_s^a = \mathbf{a}_s S_\tau - \mathbf{a}_s \bar{S}_a \quad if \quad \tau \le N_B\\ \tilde{y}_{s\tau} &= y_{s\tau} - \bar{y}_s^b = \mathbf{a}_s S_\tau - \mathbf{a}_s \bar{S}_b - \mathbf{a}_s\mathbf{b}'\boldsymbol{\gamma}\left(1-\lambda'\right)/\left(1-\lambda\right) \quad if \quad N_B \le \tau \le N_B' \qquad (25)\\ \tilde{y}_{s\tau} &= y_{s\tau} - \bar{y}_s^b = \mathbf{a}_s S_\tau - \mathbf{a}_s \bar{S}_b + \mathbf{a}_s\mathbf{b}'\boldsymbol{\gamma} - \mathbf{a}_s\mathbf{b}'\boldsymbol{\gamma}\left(1-\lambda'\right)/\left(1-\lambda\right) \quad if \quad N_B' \le \tau \le N\end{aligned}$$

with $S_\tau = \mathbf{b}'\sum_{j=1}^{\tau}\mathbf{u}_j$, $\bar{S}_a = N_b^{-1}\sum_{\tau=1}^{N_b} S_\tau = \lambda^{-1}N^{-1}\sum_{\tau=1}^{N_b} S_\tau$ and $\bar{S}_b = (N-N_b)^{-1}\sum_{\tau=N_b+1}^{N} S_\tau = (1-\lambda)^{-1}N^{-1}\sum_{\tau=N_b+1}^{N} S_\tau$. Additionally we define $\tilde{y}_{s\tau}^*$ as the residuals from a projection of $\tilde{y}_{s\tau}$ on $D\,(NB)_{s,\tau}$ in the case where we do not have serial correlation and as the the residuals from a projection of $\tilde{y}_{s\tau}$ on $D\,(NB)_{s,\tau}$ and its $p-1$ lags, assume for simplicity the absence of serial correlation, hence:

$$\begin{aligned}\tilde{y}_{s\tau}^* &= \mathbf{a}_s S_\tau - \mathbf{a}_s \bar{S}_a \quad if \quad \tau \le N_B\\ \tilde{y}_{s\tau}^* &= 0 \quad if \quad \tau = N_B + 1 \qquad (26)\\ \tilde{y}_{s\tau}^* &= \mathbf{a}_s S_\tau - \mathbf{a}_s \bar{S}_b - \mathbf{a}_s\mathbf{b}'\boldsymbol{\gamma}\left(1-\lambda'\right)/\left(1-\lambda\right) \quad if \quad N_B + 1 \le \tau \le N_B'\\ \tilde{y}_{s\tau}^* &= \mathbf{a}_s S_\tau - \mathbf{a}_s \bar{S}_b + \mathbf{a}_s\mathbf{b}'\boldsymbol{\gamma} - \mathbf{a}_s\mathbf{b}'\boldsymbol{\gamma}\left(1-\lambda'\right)/\left(1-\lambda\right) \quad if \quad N_B' \le \tau \le N\end{aligned}$$

Following the lines of the proof of Theorem 1 in Boswijk and Franses (1996) it is convenient to write (18)/(19) using conventional time subscripts and seasonal dummy variable notation (D_{st} taking the value unity when observation t falls in season s and zero otherwise). Employing this notation yields the representation (see Boswijk and Franses, 1996, p. 238):

$$\tilde{y}_t^* = \pi_1 D_{1t}\tilde{y}_{t-1}^* + \sum_{s=1}^{4}\varphi_s D_{st}\tilde{y}_{t-1}^* + \sum_{s=1}^{4}\sum_{j=1}^{p-1}\psi_{js}\left(D_{st}\tilde{y}_{t-j}^* - \varphi_{s-j}D_{st}\tilde{y}_{t-j-1}^*\right) + \varepsilon_t \tag{27}$$

where the restrictions $\varphi_1\varphi_2\varphi_3\varphi_4 = 1$ is imposed. Note that since the deterministic terms enter unrestrictedly then $\tilde{y}_t^*$ are the residuals as defined in (25)/(26). Let $\theta = \left[\theta_1, \theta_2', \theta_3'\right]'$ denote the full parameter vector with $\theta_1 = \pi_1$, $\theta_2' = [\varphi_2, \varphi_3, \varphi_4]$ and $\theta_3' = \left[\psi_{11}, \cdots, \psi_{1,p-1}, \cdots, \psi_{41}, \cdots, \psi_{4,p-1}\right]$. Under the null hypothesis $\pi_1 = 0$, hence this parameter is associated with the unit root while, φ_2, φ_3 and φ_4 are cointegration parameters (with φ_1 defined from the periodic unit root restriction as $\varphi_1 = (\varphi_2\varphi_3\varphi_4)^{-1}$), and θ_3 collects the parameters associated with the stationary regressors in (27). Let $z_t = \left[z_t^1, z_t^{2\prime}, z_t^{3\prime}\right]'$ be defined conformably with θ as $z_t = \partial\tilde{y}_t/\partial\theta$, and hence

$$\begin{aligned} z_t^1 &= D_{1t}\tilde{y}_{t-1}, \quad z_t^2 = H'u_t \quad u_t = [u_{1t}, u_{2t}, u_{3t}, u_{4t}]' \\ where &: \\ u_{st} &= D_{st}\tilde{y}_{t-1} - \sum_{i=1}^{p-1}\psi_{i,s+i}D_{s+i,t}\tilde{y}_{t-i-1} \quad s = 1,2,3,4 \\ H' &= \begin{bmatrix} -\frac{\varphi_1}{\varphi_2} & 1 & 0 & 0 \\ -\frac{\varphi_1}{\varphi_3} & 0 & 1 & 0 \\ -\frac{\varphi_1}{\varphi_4} & 0 & 0 & 1 \end{bmatrix}. \end{aligned} \tag{28}$$

Note that for z_t^1 we have that

$$\begin{aligned} \sigma^{-2}N^{-1}\sum_{t=1}^{T} z_t^1\varepsilon_t &= \sigma^{-2}N^{-1}\sum_{t=1}^{T} D_{1t}\tilde{y}_{t-1}\varepsilon_t = \sigma^{-2}N^{-1}\sum_{\tau=1}^{N}\tilde{y}_{4,\tau-1}\varepsilon_{1\tau} = \\ &= \sigma^{-2}N^{-1}\sum \mathbf{a}_4 S_{\tau-1}\varepsilon_{1\tau} - \sigma^{-2}N^{-1}\mathbf{a}_4\bar{S}_a\sum_{\tau=1}^{N_b}\varepsilon_{1\tau} - \\ &\quad -\sigma^{-2}N^{-1}\mathbf{a}_4\bar{S}_a\sum_{\tau=N_b+1}^{N}\varepsilon_{1\tau} + O_p(1) \end{aligned}$$

and

$$\begin{aligned}
\sigma^{-2}N^{-2}\sum_{t=1}^{T}\left(z_t^1\right)^2 &= \sigma^{-2}N^{-2}\sum_{t=1}^{T}\left(D_{1t}\tilde{y}_{t-1}\right)^2 = \sigma^{-2}N^{-2}\sum_{\tau=1}^{N}\left(\tilde{y}_{4,\tau-1}\right)^2 = \\
&= \sigma^{-2}N^{-2}\sum\left(\mathbf{a}_4 S_{\tau-1}\right)^2 + \sigma^{-2}N^{-1}\lambda\left(\mathbf{a}_4\bar{S}_a\right)^2 + \sigma^{-2}N^{-1}\left(1-\lambda\right)\left(\mathbf{a}_4\bar{S}_b\right)^2 + \\
&\quad -2\sigma^{-2}N^{-2}\mathbf{a}_4\bar{S}_a\sum_{\tau=1}^{N_b}\left(\mathbf{a}_4 S_{\tau-1}\right) - 2\sigma^{-2}N^{-2}\mathbf{a}_4\bar{S}_b\sum_{\tau=N_b+1}^{N}\left(\mathbf{a}_4 S_{\tau-1}\right) + o_p\left(1\right).
\end{aligned}$$

From lemma 1 in Boswijk and Franses (1996) it is possible to establish:

$$\begin{aligned}
\sigma^{-2}N^{-1}\sum\mathbf{a}_s S_{\tau-1}\varepsilon_{1\tau} &\Rightarrow \sigma^{-2}\omega a_s\int_0^1 w\left(r\right)dE_1\left(r\right) \\
\sigma^{-1}N^{-3/2}\sum\mathbf{a}_s S_{\tau-1} &\Rightarrow \sigma^{-1}\omega a_s\int_0^1 w\left(r\right)dr \\
\sigma^{-1}N^{-1/2}\sum\varepsilon_{1\tau} &\Rightarrow \sigma^{-1}E_1\left(1\right) \\
\sigma^{-1}N^{-3/2}\sum_{N_B}\mathbf{a}_s S_{\tau-1} &\Rightarrow \sigma^{-1}\omega a_4\int_{\lambda}^1 w\left(r\right)dr \\
\sigma^{-1}N^{-1/2}\sum_{N_B}\varepsilon_{1\tau} &\Rightarrow \sigma^{-1}\left(E_1\left(1\right)-E_1\left(\lambda\right)\right) \\
\sigma^{-2}N^{-2}\sum\left(\mathbf{a}_s S_{\tau-1}\right)^2 &\Rightarrow \sigma^{-2}\omega^2 a_s^2\int_0^1\left[w\left(r\right)\right]^2 dr \\
\sigma^{-1}N^{-1/2}\mathbf{a}_s\bar{S}_a &= \sigma^{-1}\lambda^{-1}N^{-3/2}\sum_{\tau=1}^{N_b}\mathbf{a}_s S_\tau \Rightarrow \sigma^{-1}\omega a_s\lambda^{-1}\int_0^{\lambda} w\left(r\right)dr \\
\sigma^{-1}N^{-1/2}\mathbf{a}_s\bar{S}_b &= \sigma^{-1}\left(1-\lambda\right)^{-1}N^{-3/2}\sum_{\tau=N_b+1}^{N}\mathbf{a}_s S_\tau \Rightarrow \sigma^{-1}\omega a_s\left(1-\lambda\right)^{-1}\int_{\lambda}^1 w\left(r\right)dr
\end{aligned}$$

Hence we have that:

$$\begin{aligned}
\sigma^{-2}N^{-1}\sum_{t=1}^{T}z_t^1\varepsilon_t &\Rightarrow \sigma^{-2}\omega a_4\left[\int_0^1 w\left(r\right)dE_1\left(r\right) - \lambda^{-1}\left[\int_0^{\lambda}w\left(r\right)dr\right]E_1\left(\lambda\right) + \right. \\
&\quad \left. -\left(1-\lambda\right)^{-1}\left[\int_{\lambda}^1 w\left(r\right)dr\right]\left(E_1\left(1\right)-E_1\left(\lambda\right)\right)\right] = \\
&= \sigma^{-2}\omega a_4\left[NU\left(E_1\right)\right] \qquad (29) \\
where &: \\
\left[NU\left(E_1\right),\left(\lambda\right)\right] &= \int_0^1 w\left(r\right)dE_1\left(r\right) - \lambda^{-1}\left[\int_0^{\lambda}w\left(r\right)dr\right]E_1\left(\lambda\right) + \\
&\quad -\left(1-\lambda\right)^{-1}\left[\int_{\lambda}^1 w\left(r\right)dr\right]\left(E_1\left(1\right)-E_1\left(\lambda\right)\right)
\end{aligned}$$

and:

$$\begin{aligned}
\sigma^{-2}N^{-2}\sum_{t=1}^{T}\left(z_t^1\right)^2 &\Rightarrow \sigma^{-2}\omega^2 a_4^2\int_0^1 [w(r)]^2\,dr - \sigma^{-2}\omega^2 a_4^2\lambda^{-1}\left(\int_0^\lambda w(r)\,dr\right)^2 - \\
&\quad -\sigma^{-2}\omega^2 a_4^2(1-\lambda)^{-1}\left(\int_\lambda^1 w(r)\,dr\right)^2 \\
&= \sigma^{-2}\omega^2 a_4^2\,[DE] \\
where &: \\
[DE(\lambda)] &= \int_0^1 [w(r)]^2\,dr - \lambda^{-1}\left(\int_0^\lambda w(r)\,dr\right)^2 \\
&\quad -(1-\lambda)^{-1}\left(\int_\lambda^1 w(r)\,dr\right)^2.
\end{aligned}$$

Note also that using Lemma 1 and (28) it is possible to establish:

$$\begin{aligned}
\sigma^{-2}N^{-1}\sum z_t^2\varepsilon_\tau &\Rightarrow \sigma^{-2}\omega H' A\Psi(1)'[NU(E)] \\
\sigma^{-2}N^{-2}\sum z_t^2 z_t^1 &\Rightarrow \sigma^{-2}\omega^2 H' A\Psi(1)' A_1[DE] \\
\sigma^{-2}N^{-2}\sum z_t^2 z_t^{2\prime} &\Rightarrow \sigma^{-2}\omega^2 H' A\Psi(1)'\Psi(1)AH[DE] \qquad (30) \\
where &: \\
[NU(E),(\lambda)] &= \int_0^1 w(r)\,dE(r) - \\
&\quad -\lambda^{-1}\left[\int_0^\lambda w(r)\,dr\right]E(\lambda) + \\
&\quad -(1-\lambda)^{-1}\left[\int_\lambda^1 w(r)\,dr\right](E(1)-E(\lambda)) \\
A &= diag[a_4,a_1,a_2,a_3] = diag[\varphi_2\varphi_3\varphi_4,1,\varphi_2,\varphi_2\varphi_3] \\
A_1 &= diag[a_4,0,0,0] = diag[\varphi_2\varphi_3\varphi_4,0,0,0].
\end{aligned}$$

Under the periodic unit root null hypothesis the PAR(p-1) regressors $D_{st}y_{t-j} - \varphi_{s-j}D_{st}y_{t-j-1}$ collected in the vector z_t^3 are stationary with

$$\begin{aligned}
\sigma^{-2}N^{-1}\sum z_t^3\varepsilon_\tau &\Rightarrow N(0,V_3) \\
\sigma^{-2}N^{-2}\sum z_t^3 z_t^{3\prime} &\rightarrow V_3.
\end{aligned}$$

Finally , reflecting the different rates of convergence for the parameter estimates corresponding to the nonstationary PI regressors and those for the stationary PAR(p-1) component in the augmented regression (7) or (**??**), we have that:

$$\begin{aligned}
N^{-2}\sum z_t^3 z_t^{2\prime} &= O_p(1) \\
N^{-2}\sum z_t^3 z_t^1 &= O_p(1).
\end{aligned}$$

The distribution of the LR test is established by Boswijk and Franses (1996) using

$$LR = \frac{\left(N\hat{\theta}_1\right)^2}{\left(Y_N^{-1}Q_\theta Y_N^{-1}\right)^{11}} + o_p\left(1\right). \tag{31}$$

where $Y_N = diag\left[N \times I_4, N^{1/2} \times I_{4(p-1)}\right]$, $\left(Y_N^{-1}Q_\theta Y_N^{-1}\right)^{11}$ is the first element of the principal diagonal of the inverse matrix $\left(Y_N^{-1}Q_\theta Y_N^{-1}\right)^{-1}$, $N\hat{\theta}_1$ is the first element of $\left(Y_N^{-1}Q_\theta Y_N^{-1}\right)^{-1} Y_N^{-1}q_\theta$, and q_θ and Q_θ are the score and negative of the Hessian matrix, respectively, formulated in terms of θ. Note that, as in Boswijk and Franses (1996),

$$\left(Y_N^{-1}Q_\theta Y_N^{-1}\right)^{-1} Y_N^{-1}q_\theta = \left(\sigma^{-2}Y_N^{-1}\sum z_t z_t' Y_N^{-1}\right)^{-1}\sigma^{-2}Y_N^{-1}\sum z_t\varepsilon_t.$$

From (29), (**??**) and (30) it is easy to see that

$$\begin{aligned} Y_N^{-1}Q_\theta Y_N^{-1} &\Rightarrow \begin{bmatrix} K'K\left[DE\left(\lambda\right)\right] & 0 \\ 0 & V_3 \end{bmatrix} \\ Y_N^{-1}q_\theta &\Rightarrow \begin{bmatrix} \sigma^{-1}K'\left[NU\left(E\right),\left(\lambda\right)\right] \\ N\left(0,V_3\right) \end{bmatrix} \\ where &: \\ K &= \frac{\omega}{\sigma}\left[A_1 \vdots \Psi\left(1\right)AH\right]. \end{aligned} \tag{32}$$

Therefore,

$$\left(Y_N^{-1}Q_\theta Y_N^{-1}\right)^{-1} Y_N^{-1}q_\theta \Rightarrow \begin{bmatrix} \left[DE\left(\lambda\right)\right]^{-1}\sigma^{-1}\left(K'K\right)^{-1}K'\left[NU\left(E\right),\left(\lambda\right)\right] \\ N\left(0,V_3^{-1}\right) \end{bmatrix}. \tag{33}$$

Note that $[DE,(\lambda)]$ is a scalar and also that for $\sigma^{-1}\left(K'K\right)^{-1}K'\left[NU\left(E\right),\left(\lambda\right)\right]$ it is possible to write:

$$\begin{aligned} \sigma^{-1}\left(K'K\right)^{-1}K'\left[NU\left(E\right)\right] &= \int_0^1 w\left(r\right)dS\left(r\right) - \\ &\quad -\lambda^{-1}\left[\int_0^\lambda w\left(r\right)dr\right]S\left(\lambda\right) \\ &\quad +\left(1-\lambda\right)^{-1}\left[\int_\lambda^1 w\left(r\right)dr\right]\left(S\left(1\right)-S\left(\lambda\right)\right) \\ where &: \\ S\left(r\right) &= \sigma^{-1}\left(K'K\right)^{-1}K'E\left(r\right). \end{aligned} \tag{34}$$

Now, partitioning $K = \left[K_1 \vdots K_2\right]$ to focus on the first element of $\left(Y_N^{-1}Q_\theta Y_N^{-1}\right)^{-1} Y_N^{-1}q_\theta$, namely $N\hat{\theta}_1$, (33) and (34) implies

$$\begin{aligned}
N\hat{\theta}_1 &\Rightarrow [DE(\lambda)]^{-1}\left\{\int_0^1 w(r)\,dS_1(r) - \right. \\
&\quad -\lambda^{-1}\left[\int_0^\lambda w(r)\,dr\right] S_1(\lambda) - \\
&\quad \left. -(1-\lambda)^{-1}\left[\int_\lambda^1 w(r)\,dr\right](S_1(1) - S_1(\lambda))\right\} \\
\textit{where} &: \\
S_1(r) &= \sigma^{-1}\left(K_1' M_2 K_1\right)^{-1} K_1' M_2 E(r) \\
M_2 &= I - K_2\left(K_2' K_2\right)^{-1} K_2'.
\end{aligned}$$

In Boswijk and Franses (1996) it is shown that $S_1(r) = \left(K_1' M_2 K_1\right)^{-1/2} w(r)$ hence we have:

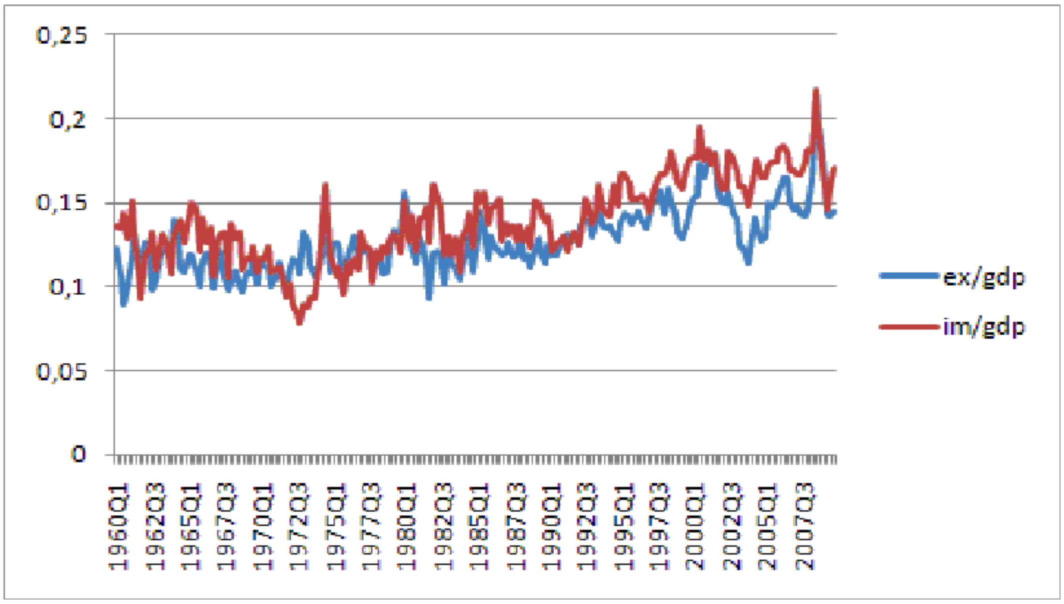
$$\begin{aligned}
N\hat{\theta}_1 &\Rightarrow \left(K_1' M_2 K_1\right)^{-1/2}[DE]^{-1}\left\{\int_0^1 w(r)\,dw(r) - \right. \\
&\quad -\lambda^{-1}\left[\int_0^\lambda w(r)\,dr\right] w(\lambda) - \\
&\quad \left. -(1-\lambda)^{-1}\left[\int_\lambda^1 w(r)\,dr\right](w(1) - w(\lambda))\right\} = \\
&= \left(K_1' M_2 K_1\right)^{-1/2}[DE(\lambda)]^{-1}[NU(\lambda)] \qquad (35) \\
\textit{with} &: \\
[NU(\lambda)] &= \int_0^1 w(r)\,dw(r) - \lambda^{-1}\left[\int_0^\lambda w(r)\,dr\right] w(\lambda) - \\
&\quad -(1-\lambda)^{-1}\left[\int_\lambda^1 w(r)\,dr\right](w(1) - w(\lambda))
\end{aligned}$$

note also that:

$$\left(Y_N^{-1} Q_\theta Y_N^{-1}\right)^{11} \Rightarrow \left(K'K\right)^{11}[DE]^{-1} = \left(K_1' M_2 K_1\right)^{-1}[DE(\lambda)]^{-1}. \qquad (36)$$

Then finally substituting (35) and (36) into (31) the required result is easily obtained.

Figure 1: Australia



■

Figure 2: Canada

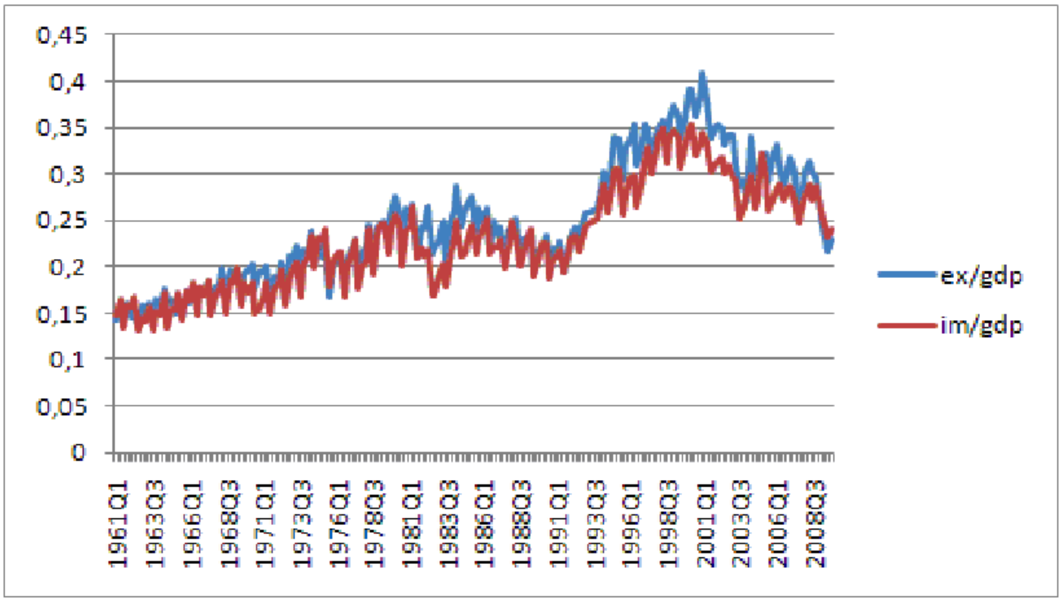


Figure 3: Denmark

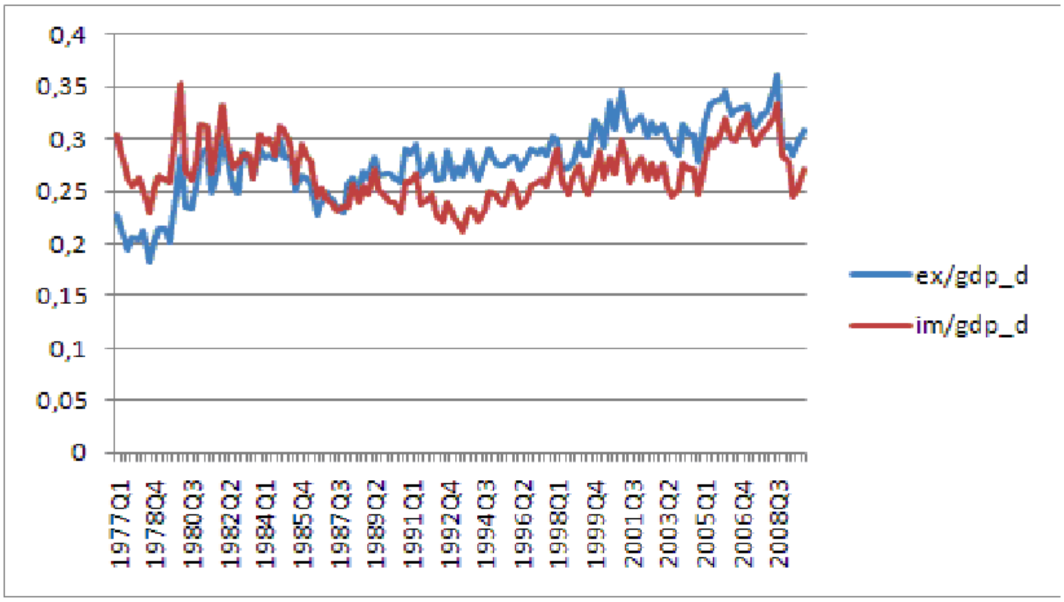


Figure 4: Sweeden

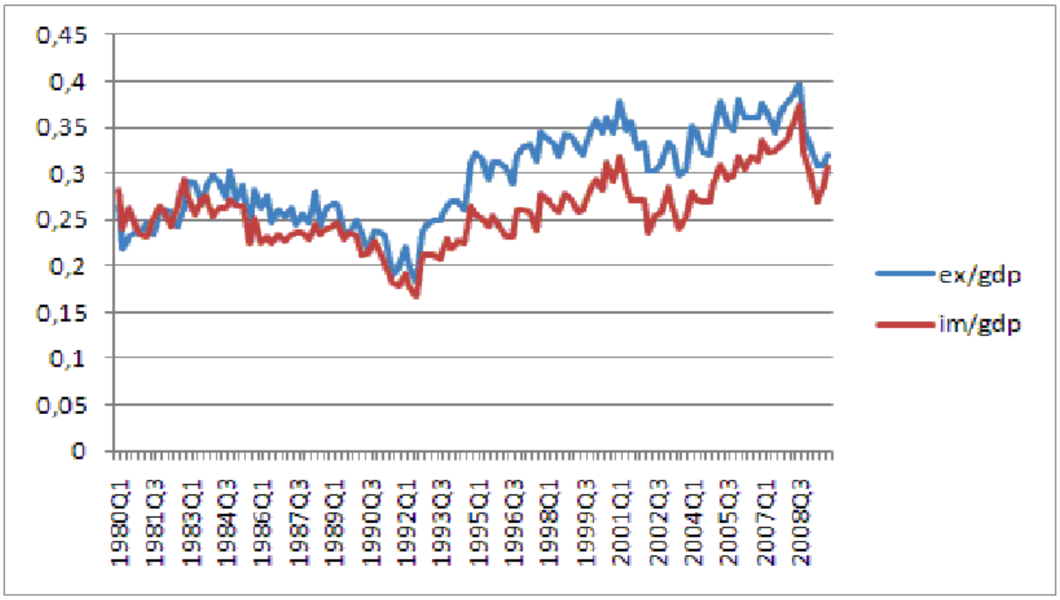


Figure 5: United Kingdom

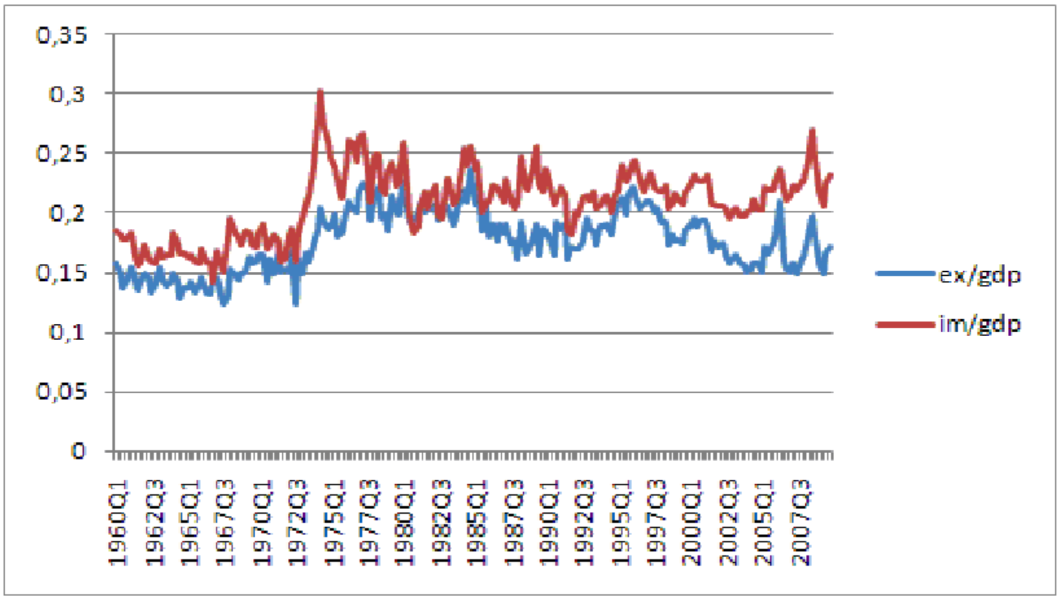


Figure 6: Norway

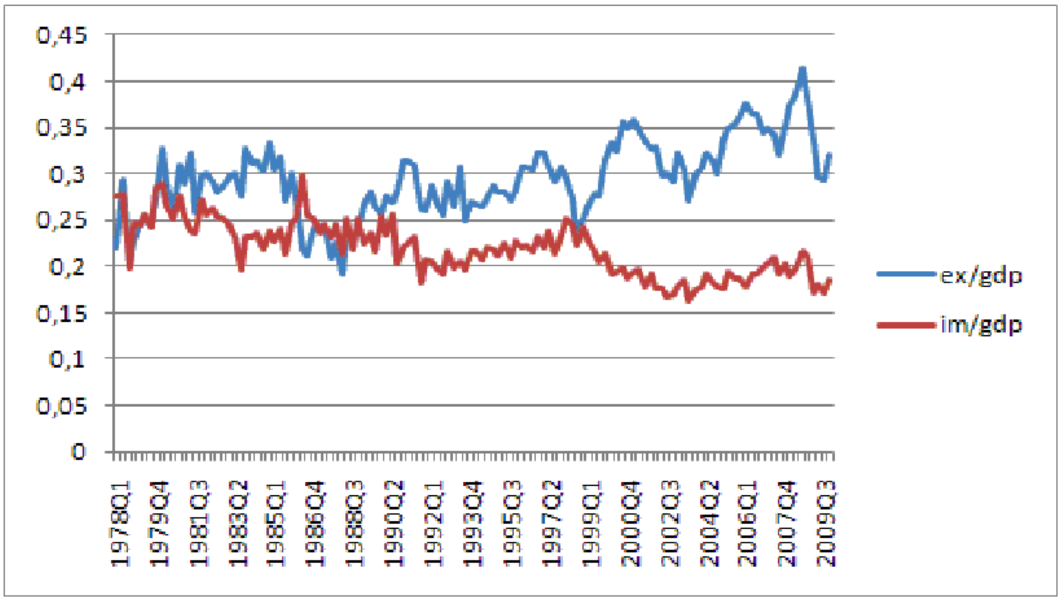


Figure 7: Switzerland

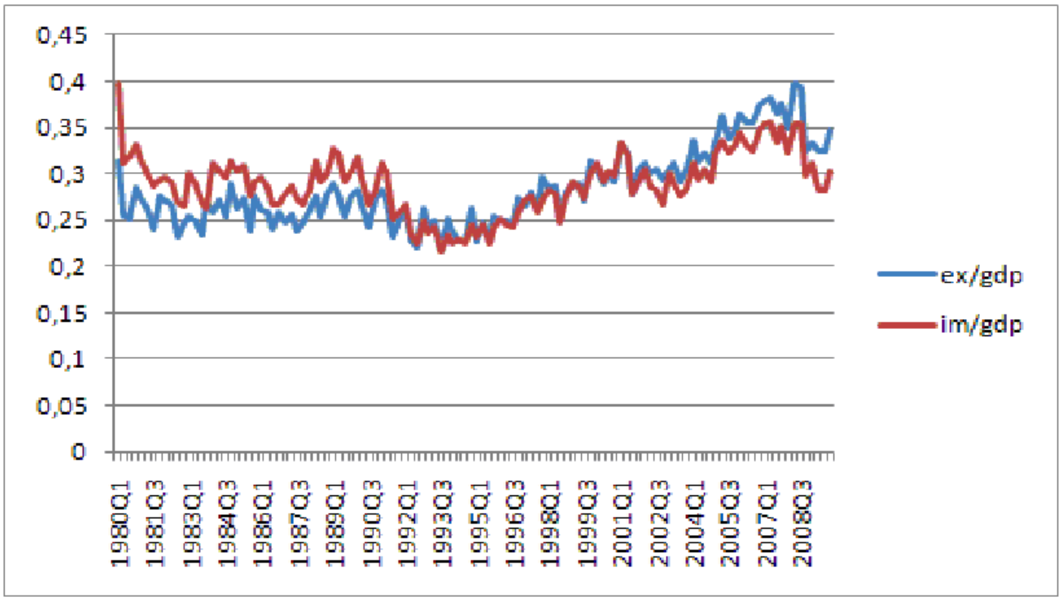


Figure 8: Japan

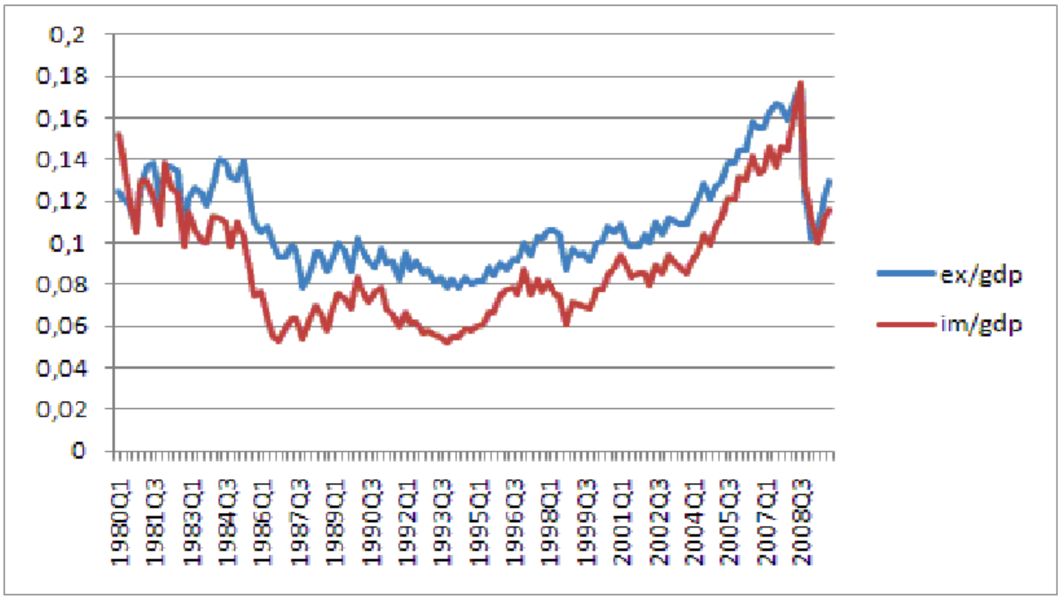


Figure 9: France

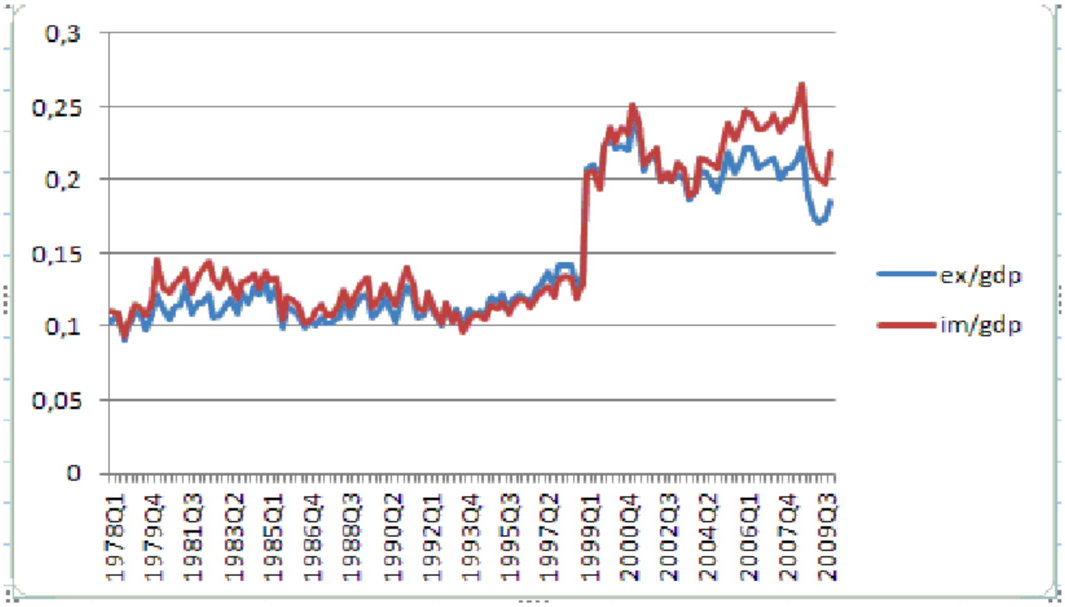


Figure 10: Italy

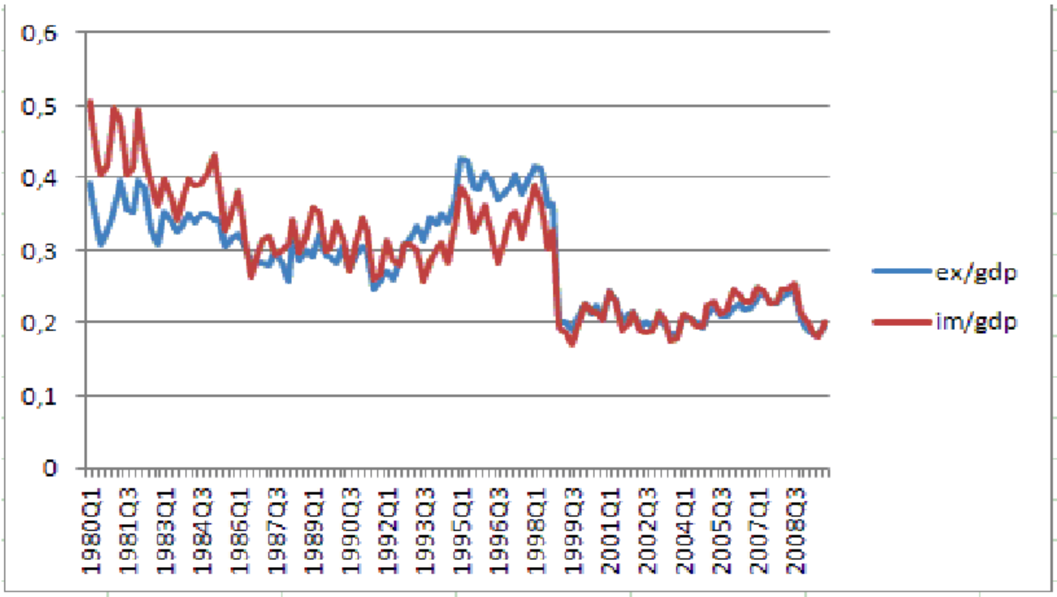


Figure 11: Netherlands

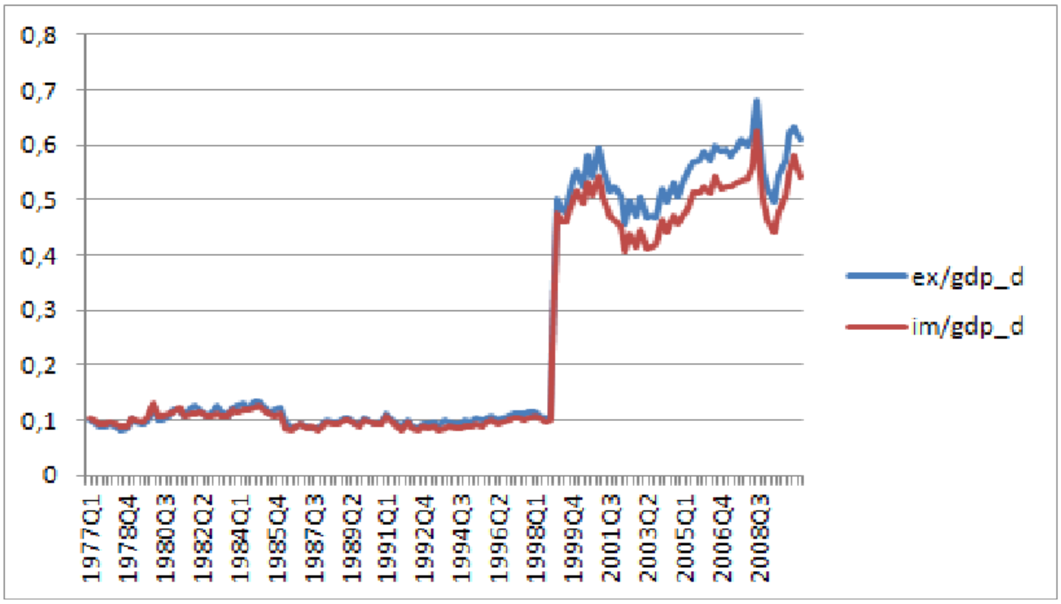


Figure 12: Finland

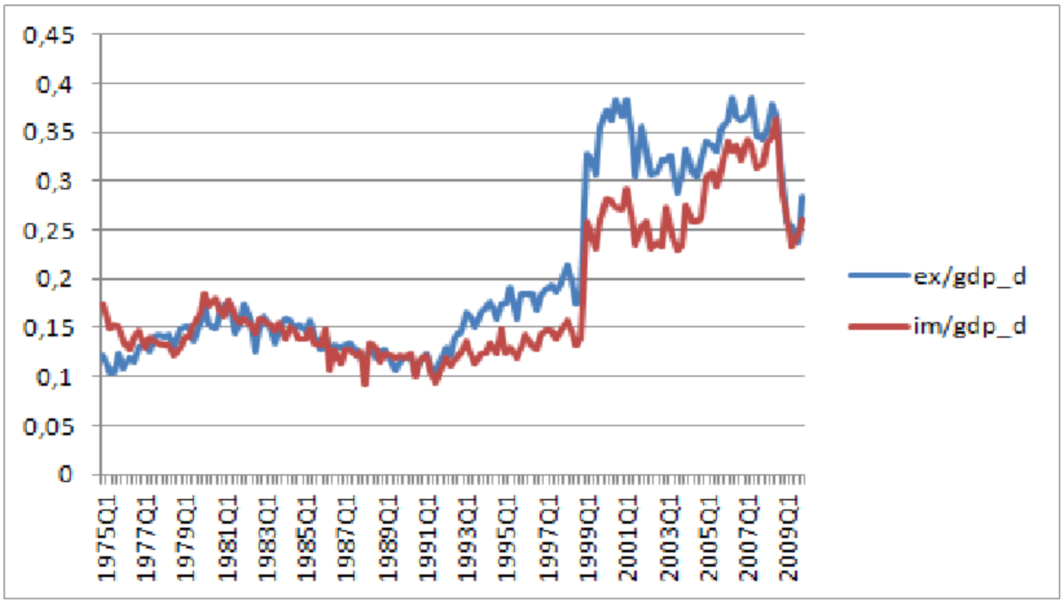


Figure 13: Spain