

An Empirical Investigation of the Trade Benefit Function

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11/26/2011

Abstract

We introduce new definition, and estimation procedures of the trade benefit function, which allow researchers to generate estimable imports share functions. The method is an extension of the recent work of Chau and Färe. However, it is more general in permitting the estimation of import share systems, which are explicit in an unobservable variable but may lack a closed form representation in terms of observable variables. Applying this method with an appropriate estimator to the Japanese data, we find that the proposed method is operationally feasible. This opens up a further avenue for ultimately obtaining systems of import demand equations which are simultaneously more flexible and regular than those currently employed in applied demand analyses.

Keywords: Trade benefit functions; import share systems; numerical Inversion.

JEL Classification: D22, F11.

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1. Introduction

In an earlier paper, Chau and Färe (2011) advocated a more general use of trade benefit functions as an alternative representation of trade preferences. They showed that this function is dual to the standard trade expenditure function, allowing for a direct retrieval of the shadow price functions of imports from this function. Despite its obvious potential for policy applications, the trade benefit function has been virtually ignored in empirical work solely because the derived shadow price functions of imports are defined in terms of the level of unobservable utility.

This paper constitutes the first attempt to bridge the gap between the pure theory of trade benefit functions and its empirical implementation. In particular, it has two objectives: (a) to theoretically redefine the trade benefit function, which facilitates empirical analysis of trade preferences; (b) to introduce new estimation procedures of the Hicksian import share functions.

While the trade benefit function can directly yield Hicksian import share functions, they are usually explicit in the unobservable utility, but lack a closed-form representation in terms of the observable variables. The aforesaid problem, however, need not hinder estimation. A simple one-dimensional numerical inversion allows estimation of the parameters of any trade benefit function via the parameters of the implied Marshallian import share equations. The remainder of this paper introduces a new specification of the trade benefit function, and reports on an initial trial on the operational feasibility of the proposed method.

The remainder of the paper is organized as follows. Section 2 develops the theoretical foundations formally. Section 3 introduces a new specification for the trade benefit function. Descriptions of the data, estimation method and a summary of the empirical findings are provided in Section 4. Lastly, Section 5 recapitulates and

concludes.

2. Analytical Framework

Let \mathbf{x} be an N1-vector of consumption goods, \mathbf{y} an N1-vector of net outputs, \mathbf{v} an N2-vector of input endowments, and u the level of utility. Consider a small and open economy with perfect price flexibility and factor mobility across sectors. This economy is assumed to have a production possibility set Γ which is the set of net output vectors \mathbf{y} that are technically feasible given the endowment vector \mathbf{v} . The preferences of this economy are assumed to be represented by a social utility function $u = U(\mathbf{x})$ which is real, quasi-concave, and increasing in \mathbf{x} .

Denote by \mathbf{m} and \mathbf{e} the N1 dimensional vectors of imported and exported goods respectively, and \mathbf{g} the N1-vector of reference bundle of the import vector \mathbf{m} . The direct trade utility function is defined as:¹

$$U^T(\mathbf{m}, \mathbf{e}, \mathbf{v}) = \text{Max}_{\mathbf{x}, \mathbf{y}} \{U(\mathbf{x}) \text{ s.t. } \mathbf{m} = \mathbf{e} + \mathbf{x} - \mathbf{y}, (\mathbf{v}, \mathbf{y}) \in \Gamma\}, \quad (1)$$

which inherits the regularity conditions $\mathbf{R}U^T$; i.e., U^T is real, non-decreasing in (\mathbf{m}, \mathbf{v}) , non-increasing in \mathbf{e} , and quasi-concave in $(\mathbf{m}, \mathbf{e}, \mathbf{v})$. Following Luenberger (1992) and Chau and Färe (2011), define the trade benefit function as:

$$B(\mathbf{m}, \mathbf{e}, \mathbf{v}, u) = \text{Max}_b \left[b > 0 \text{ s.t. } U^T(\mathbf{m} - b\mathbf{g}, \mathbf{e}, \mathbf{v}) \geq u \right], \quad (2)$$

which measures the market access adjustments required to maintain a given level of

¹ See Meade (1952).

² In Chau and Färe (2011), $B(\cdot)$ is represented in terms of the net import vector $(\mathbf{m}-\mathbf{e})$, implying that commonly used flexible functional forms such as the Translog could not be employed for empirical application. For instance, it is infeasible to specify the trade benefit function in terms of Translog since the logarithm of $(\mathbf{m}-\mathbf{e})$ may be undefined when $(\mathbf{m}-\mathbf{e})$ is non-positive. Accordingly, the trade benefit function has to be redefined as in (1), which facilitates the econometric analysis of this function.

utility in terms of import contraction or expansion in the direction \mathbf{g} .³ Clearly $u = U^T(\mathbf{m}, \mathbf{e}, \mathbf{v})$ if and only if:

$$B(\mathbf{m}, \mathbf{e}, \mathbf{v}, u) = 0. \quad (3)$$

Provided that U^T satisfies Conditions $\mathbf{R}U^T$, then the trade benefit function is real, increasing in (\mathbf{m}, \mathbf{v}) , decreasing in (\mathbf{e}, u) , concave in $(\mathbf{m}, \mathbf{e}, \mathbf{v})$, and satisfies a translation property:

$$B(\mathbf{m} + \alpha\mathbf{g}, \mathbf{e}, \mathbf{v}, u) = \alpha + B(\mathbf{m}, \mathbf{e}, \mathbf{v}, u). \quad (4)$$

Let \mathbf{p} be an N1 vector of the shadow prices of imports, and $\mathbf{r} = \frac{\mathbf{p}}{\mathbf{p}'\mathbf{m}}$ an N1 vector of the normalized prices of imports. The Hicksian normalized price functions of imports (R_i^H) are related to the trade benefit function via Hotelling-Wold Analogue:

$$R_i^H(\mathbf{m}, \mathbf{e}, \mathbf{v}, u) = \frac{p_i}{\sum_j p_j m_j} = \frac{\frac{\partial B(\mathbf{m}, \mathbf{e}, \mathbf{v}, u)}{\partial m_i}}{\sum_j \left[\frac{\partial B(\mathbf{m}, \mathbf{e}, \mathbf{v}, u) m_j}{\partial m_j} \right]}, \quad (5)$$

or in share form:⁴

$$W_i^H(\mathbf{m}, \mathbf{e}, \mathbf{v}, u) = \frac{p_i m_i}{\sum_j p_j m_j} = \frac{\frac{\partial B(\mathbf{m}, \mathbf{e}, \mathbf{v}, u)}{\partial m_i} m_i}{\sum_j \left[\frac{\partial B(\mathbf{m}, \mathbf{e}, \mathbf{v}, u) m_j}{\partial m_j} \right]}, \quad (6)$$

where W_i^H is the Hicksian import share function, and the superscript H is to indicate that (6) represents the Hicksian functions. Furthermore, the translation property (4) implies:

³ See Chau and Färe (2011).

⁴ See McLaren and Wong (2009, pp 1111-1113) for the derivation of Hotelling-Wold Analogue.

$$\sum_j \frac{\partial B}{\partial m_j} g_j = 1. \quad (7)$$

If one could invert (3) explicitly to give the implied trade utility function $U^T(\mathbf{m}, \mathbf{e}, \mathbf{v}) = u$, then the Hicksian system could be “Marshallianized” by replacing the u by $U^T(\mathbf{m}, \mathbf{e}, \mathbf{v})$; i.e.,

$$W_i^M(\mathbf{m}, \mathbf{e}, \mathbf{v}) = W_i^H[\mathbf{m}, \mathbf{e}, \mathbf{v}, U^T(\mathbf{m}, \mathbf{e}, \mathbf{v})], \quad (8)$$

where the superscript M is to indicate that (8) represents the Marshallian functions.

In practice, however, such an explicit inversion of (3) in u is not always feasible. This depends heavily on the particular parametric form of B, and not every B has an explicit analytical inversion property. This paper considers the class of B for which such explicit inversion is not available, and exploits the fact that the implied Marshallian import share system (W_i^M) derived from any trade benefit functions can be expressed implicitly by the following equation system:

$$W_i^H(\mathbf{m}, \mathbf{e}, \mathbf{v}, u) = \frac{\frac{\partial B(\mathbf{m}, \mathbf{e}, \mathbf{v}, u)}{\partial m_i} m_i}{\sum_j \left[\frac{\partial B(\mathbf{m}, \mathbf{e}, \mathbf{v}, u) m_j}{\partial m_j} \right]}, \quad (9)$$

$$B(\mathbf{m}, \mathbf{e}, \mathbf{v}, u) = 0. \quad (10)$$

Providing B is strictly decreasing in u , then it becomes feasible to numerically invert B in (10) to express u as a function of \mathbf{m} , \mathbf{e} , and \mathbf{v} . Therefore, given a specific functional form for B and a vector of parameters $\boldsymbol{\theta}$, the corresponding import share system can be written as:

$$W_i^M(\mathbf{m}, \mathbf{e}, \mathbf{v}; \boldsymbol{\theta}) = W_i^H[\mathbf{m}, \mathbf{e}, \mathbf{v}, U^T(\mathbf{m}, \mathbf{e}, \mathbf{v}; \boldsymbol{\theta}); \boldsymbol{\theta}], \quad (11)$$

where $U^T(\mathbf{m}, \mathbf{e}, \mathbf{v}; \boldsymbol{\theta})$ is the numerical solution of the identity function $B(\mathbf{m}, \mathbf{e}, \mathbf{v}, u;$

$\theta) = 0$ for u , solved at the given values of \mathbf{w} , \mathbf{m} , \mathbf{e} , \mathbf{v} and θ . At each iterative step of the maximization of the likelihood function, there is a given set of parameter values. For these parameter values, (10) could be numerically inverted to recover the value of utility u consistent with the given values of commodities \mathbf{w} , \mathbf{m} , \mathbf{e} , and \mathbf{v} . Then, this value of utility can be used to eliminate the unknown value of u from the Hicksian import share system.

3. The Model

Using some intuition stemming from Rimmer and Powell's An Implicit Directly Additive Demand System (AIDADS), and Preckel, Cranfield, and Hertel (2007)'s Modified AIDADS, we assume that trade preferences are represented by the following trade benefit function:

$$B(\mathbf{m}, \mathbf{e}, \mathbf{v}, u) = M_1 + M_2 \cdot \frac{(\kappa Z_1 - Z_2)}{u^\mu}, \quad (12)$$

where M_k ($k=1, 2$) are positive, non-decreasing and concave functions of \mathbf{m} , and Z_1 (or Z_2) is a positive, non-decreasing and concave (or convex) function of \mathbf{v} (or \mathbf{e}). For the empirical application, we assume that the M_k and Z_k functions take the forms, respectively:

$$M_1 = \sum_j A_j m_j, \quad M_2 = \prod_j m_j^{\gamma_j}, \quad Z_1 = \prod_l v_l^{\phi_l}, \quad \text{and} \quad Z_2 = \prod_j e_j^{\xi_j}, \quad (13)$$

where $A_i = \frac{\alpha_i + \delta_i e^u}{1 + e^u} \forall i$ are the utility varying coefficients. We have seen from (7)

that $\sum_j \frac{\partial B}{\partial m_j} g_j = 1$, requiring:

$$\sum_j A_j g_j = 1 \quad \text{and} \quad \sum_j \frac{\gamma_j}{m_j} g_j = 0. \quad (14)$$

Differentiation of (12) after some manipulation gives the Hicksian import share

system:

$$\begin{aligned}
 W_i^H(\mathbf{m}, \mathbf{e}, \mathbf{v}, u) &= \frac{\frac{\partial B(\mathbf{m}, \mathbf{e}, \mathbf{v}, u)}{\partial m_i} m_i}{\sum_j \frac{\partial B(\mathbf{m}, \mathbf{e}, \mathbf{v}, u)}{\partial m_j} m_j} \\
 &= \frac{A_i m_i + \gamma_i \frac{M_2(\kappa Z_1 - Z_2)}{u^\mu}}{\sum_j \left[A_j m_j + \gamma_j \frac{M_2(\kappa Z_1 - Z_2)}{u^\mu} \right]}, \quad (15)
 \end{aligned}$$

which is referred to as the Rank Three Import Share System (RTISS). Several aspects of this system warrant discussion. First, the structure (12) maintains all of the regularity properties in $(\mathbf{m}, \mathbf{e}, \mathbf{v})$ of a trade benefit function over the regions $u > 0$ and $\kappa Z_1 \geq Z_2$ provided that the following conditions are satisfied:

$$\mu \geq 0, \kappa \geq 0, \text{ and } 0 \leq \alpha_i, \delta_i, \gamma_i, \xi_i, \phi_i \leq 1.$$

Second, the corresponding trade benefit function (12) is highly non-linear in its parameters and u , indicating that the value of u cannot be explicitly expressed in terms of parameters and other measurable variables. Thus, to convert (15) to an estimable Marshallian system, the u in (15) has to be replaced by the numerical inversion of (12) at $B = 0$. Lastly, in the spirit of Lewbel's (1991) definition, (15) is consistent with rank three preferences. This is potentially important since the model increases the flexibility of the price and expenditure effects as we move across the expenditure spectrum.

The impacts of import quantities and domestic input factors on the shadow prices of imports could be evaluated with the use Hicksian quantity elasticities $(E_{R_i m_j})$ and Hicksian elasticities of factor intensity $(E_{R_i v_j})$, which in the case of RTISS, is given by:

$$\begin{aligned}
 E_{R_i m_i} &= \frac{\partial \log(R_i)}{\partial \log(m_i)} = -\delta_{ii} + \frac{\delta_{ii} A_i m_i + \gamma_i \cdot \gamma_i \cdot \frac{M_2 (\kappa Z_1 - Z_2)}{u^\mu}}{A_i m_i + \gamma_i \frac{M_2 (\kappa Z_1 - Z_2)}{u^\mu}} \\
 &\quad - \left[\frac{A_i m_i + \gamma_i \cdot \left(\sum_j \gamma_j \right) \frac{M_2 (\kappa Z_1 - Z_2)}{u^\mu}}{M_1 + \left(\sum_j \gamma_j \right) \frac{M_2 (\kappa Z_1 - Z_2)}{u^\mu}} \right], \\
 E_{R_i v_i} &= \frac{\partial \log(R_i)}{\partial \log(v_i)} = \frac{\varphi_i \gamma_i \cdot \frac{M_2 (\kappa Z_1 - Z_2)}{u^\mu}}{A_i m_i + \gamma_i \frac{M_2 (\kappa Z_1 - Z_2)}{u^\mu}} - \frac{\varphi_i \cdot \kappa \left(\sum_j \gamma_j \right) \frac{M_2 \cdot Z_1}{u^\mu}}{M_1 + \left(\sum_j \gamma_j \right) \frac{M_2 (\kappa Z_1 - Z_2)}{u^\mu}}, \quad (16)
 \end{aligned}$$

where δ_{ii} is the Kronecker delta.

4. Data, Estimation Method and Results

The RTISS was estimated using annual Japanese data on five categories of imported/exported goods (Foodstuffs; Chemicals; Metallic Materials; Machineries and Equipments; and Miscellaneous Goods) and two categories of domestic inputs (labor and capital) covering the period 1969 to 2009. All raw data was obtained from the website of the Statistics Bureau of Japan,⁵ while the domestic input quantities, and import/export quantities are normalized to unity for 2000.

An important remaining issue is the choice of reference bundle \mathbf{g} implied by (7). To simplify matters, we choose \mathbf{g} to be $(0, 0, \dots, 1)'$ implying that all valuations are made relative to the value of the imported miscellaneous goods. This choice of \mathbf{g} then implies the following parameter restrictions: $\alpha_5 = \delta_5 = 1$, and $\gamma_5 = 0$.

The major challenge in estimating the RTISS is that the unobservable utility level u is an argument in the import share system. That is, unlike the case where utility is an

⁵ Statistical Bureau of Japan: <<http://www.stat.go.jp>>

explicit function of observable variables, u does not drop of the system of import share functions. Accordingly, u remains explicit in the import share system and has to be estimated in addition to the parameters θ . This problem can be accomplished by using the GAUSS language which is ideally suited for handling the implicit representation of functional relationships. For this reason, the RTISS may be estimated by using the GAUSS 11.0 computer package with the modules NLSYS and CML.

For purposes of estimation, an error term e_{it} is appended additively in the import share system. The estimation method is non-linear full information maximum likelihood, and the last equation in each system, which is the budget share equation for miscellaneous goods (w_5), is deleted to ensure non-singularity of the error covariance matrix. As usual, the estimation should be independent of which equations are excluded. Preliminary analysis suggested the need to consider the serial correlation in estimated residuals. To rectify this problem, we introduce the first order autoregressive scheme based on a parameterization of the autocovariance matrix using the maximum likelihood algorithm of Moschini and Moro (1994).

Maximum likelihood estimation of the RTISS yielded the results reported in Table 1. Asymptotic t-ratios are also included although they must be interpreted with care given the non-linear nature of the model, and since one constraint ($0 \leq \gamma_3 \leq 1$) is binding. Overall, the proposed model yields a satisfactory fit as measured by the R^2 of each share equation. The serial correlation properties of the error terms as shown in the Durbin-Watson and Box-Pierce test statistics are not severely pathological, suggesting that serial correlation is satisfactorily handled by Moschini and Moro's (1994) method. More importantly, the RTISS satisfies the required regularity properties for all observations. Specifically, the estimated RTISS turns out to be

concave in $(\mathbf{m}, \mathbf{e}, \mathbf{v})$ and its fitted values are positive.

Parameter estimates of the RTISS could be used to compute Hicksian quantity and factor intensity elasticities of import share functions. These are evaluated at the sample means of the variables and reported in Table 2. The elasticities in the first part of the table are the Hicksian own/cross quantity ($E_{R_i m_j}$) elasticities. It appears that the own quantity elasticities are greater than minus one, suggesting that all imported goods are own quantity inelastic. It is also visible that all derived cross quantity elasticities (with the exception of $E_{R_2 m_4}$ and $E_{R_4 m_2}$) are negatively small, illustrating that most of the imported goods are slight substitutes whereas imported chemicals and machineries are weakly complementary.

Of more interest to trade economists are the factor intensity elasticities ($E_{R_i v_j}$), which measure the effects of changes in input factor endowments on the shadow prices of imports; the estimates are reported in the second part of Table 2. We see that increases in the endowments of labor and capital slightly raise the prices of imported food, chemical and machineries, implying that those imported goods and domestic inputs are weakly complementary. On the other hand, increasing the endowments of labor and capital marginally reduces the prices of imported metals and miscellaneous goods. It might be concluded that these imported items and domestic inputs are slight substitutes. One implication of these results is that removal of all import controls would have small but ambiguous effects on the demand for Japanese domestic input factors.

5. Concluding Remarks

This paper introduces new definition and estimation procedures of trade benefit functions intended to be applied in import demand study. Departing from a recent

paper by Chau and Färe (2011), we extend their work by proposing a new RTISS parameterization of the trade benefit function, which is estimable and can be easily constrained to satisfy the regularity conditions.

It has been demonstrated that for a chosen trade benefit function, application of an analogue to the Hotelling-Wold Identity yields expressions for Hicksian import share functions. While these functions are explicit in the level of utility, in most cases they do not have a closed-form representation as corresponding Marshallian functions i.e. in terms of observable variables. This problem, however, need not hinder estimation, and can be solved by the numerical inversion estimation method, as discussed in Section 2. Overall, empirical findings indicate that the modeling and estimation procedures employed here are feasible and promising, and may prove beneficial for quantity and welfare analysis in the future when modeling systems of import demand functions.

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An Empirical Investigation of the Trade Benefit Function

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TABLE 1: Empirical Results: The Restricted Model

(Asymptotic t ratios in Parentheses)

Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
α_1	0.673 (4.328)	γ_1	0.299 (1.052)	ξ_1	1.066 (0.353)
α_2	0.072 (1.016)	γ_2	0.050 (0.420)	ξ_2	1.022 (1.099)
α_3	0.181 (2.447)	γ_3	0.000* —	ξ_3	1.045 (4.697)
α_4	0.074 (0.675)	γ_4	0.600 (4.456)	ξ_4	1.108 (0.215)
δ_1	0.694 (8.458)	ϕ_1	0.646 (0.734)	ξ_5	1.115 (0.186)
δ_2	0.059 (0.936)	ϕ_2	0.178 (0.745)	κ	2.012 (1.569)
δ_3	0.133 (5.163)			μ	0.201 (0.460)
δ_4	0.115 (2.424)				
R²		D-W Statistics		Box-Pierce χ^2 Statistics	($\chi^2_{5\%,6} = 12.592$)
Foodstuff	0.915	Foodstuff	1.656	Foodstuff	2.600
Chemicals	0.895	Chemicals	1.613	Chemicals	2.690
Metals	0.887	Metals	2.069	Metals	3.910
Machineries	0.985	Machineries	2.052	Machineries	2.990
Miscellaneous	0.973	Miscellaneous	1.774	Miscellaneous	2.600

* The constraint $0 \leq \gamma_3 \leq 1$ is binding, and hence no t-value is reported.

Table 2: Elasticity Estimates (Asymptotic Standard Errors in Parentheses)

Hicksian Quantity Elasticities^{a & b}					
Imported Category	$E_{R_1 m_1}$	$E_{R_1 m_2}$	$E_{R_1 m_3}$	$E_{R_1 m_4}$	$E_{R_1 m_5}$
Foodstuff	-0.423 (0.080)	-0.018 (0.007)	-0.056 (0.012)	-0.022 (0.023)	-0.481 (0.115)
Chemicals	-0.200 (0.033)	-0.339 (0.054)	-0.056 (0.012)	0.068 (0.050)	-0.481 (0.115)
Metals	-0.298 (0.037)	-0.027 (0.010)	-0.056 (0.012)	-0.129 (0.065)	-0.481 (0.115)
Machineries	-0.046 (0.041)	0.015 (0.012)	-0.129 (0.065)	-0.466 (0.055)	-0.481 (0.115)
Miscellaneous	-0.298 (0.037)	-0.027 (0.010)	-0.056 (0.012)	-0.129 (0.065)	-0.481 (0.115)
Hicksian Factor Intensity Elasticities					
Imported Category	$E_{R_1 v_1}$ (labor)		$E_{R_1 v_2}$ (capital)		
Foodstuff	0.001 (0.015)		0.000 (0.004)		
Chemicals	0.134 (0.039)		0.037 (0.011)		
Metals	-0.170 (0.096)		-0.047 (0.026)		
Machineries	0.602 (0.053)		0.166 (0.015)		
Miscellaneous	-0.170 (0.096)		-0.047 (0.026)		

^a The equality of some $E_{R_1 m_5}$ across goods is the consequence of the restriction $\gamma_5 = 0$ implied by the choice of \mathbf{g} , and the parameter estimate $\gamma_3 = 0$.

^b The index set for imports and exports: {1= Foodstuff, 2= Chemicals, 3= Metals, 4= Machineries, 5= Miscellaneous}.