Sale and rental subsidies in durable-good oligopolies*

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Abstract

We analyze the effects of per unit subsidies to sales and rentals of durable goods, considering subsidies in the present and subsidies in the future, in a situation where there is imperfect competition. We compare the effects on prices and quantities of changes in the subsidies and the welfare effects of those changes. We obtain some counterintuitive results. Intertemporal interactions among subsidies, the different implications of rentals and sales in the present, imperfect competition, the strategic behavior of each firm to steal sales from its rivals in the present and in the future and the inability of firms to commit in the present to future production are behind the results we obtain.

JEL classification codes: H32, L13, H21.

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1 Introduction

In some durable goods industries governments subsidize sales and rentals. This has been the case, for instance, with car sales in several countries and it also occurs with sales and rentals of houses. During the last years budget restrictions in many countries have implied reductions in subsidies to durable goods.

One reason to subsidize sales or rentals of durable goods is that the purchases of those goods shows greater variability over the business cycle than the purchases of non-durable goods and services, as it can be more readily postponed in times of economic weakness. As the services received from existing holdings of durable goods tend to be maintained even in the absence of any new purchases, its purchase can usually be postponed for long periods. As a result, consumer spending on durable goods is more volatile than spending on non-durable goods and services, and tends to be more closely related to the economic cycle. Durable consumer goods have in fact fallen as a percentage of GDP during recessions and risen during booms[1]. Given the weakness in demand for durable-goods during the slowdown, governments in several countries introduced temporary subsidies targeted at spending on consumer durables. These types of subsidies were particularly evident in economies that are large producers of durables. Motor vehicle subsidies for consumers were introduced in a number of countries, including China, Japan, the United States and some European nations, with higher car sales and production reported in many cases[2].

Moreover most western governments intervene heavily in the housing market. In this case equity reasons are also behind the decisions to subsidize purchases and rentals of houses. In the United States, the government has introduced temporary incentives to encourage housing activity through home-buyer tax credits. In the last years there have been some measures to enhance the rental market in Spain, in particular fiscal deductions and incentives, and purchases of houses by low and middle income households are also subsidized.

There have been some analyses of the effects of subsidies and taxes in the economic literature on durable goods. Chen, J., Esteban, S., Shum M., (2010) investigate the effectiveness of a sales tax reduction in stimulating sales and profits of durable goods manufactures. They show that the benefit of reducing

[1] See Black and Cusbert (2010) for an analysis of cycles in spending on durable goods in Australia over the past 50 years and during the actual economic slowdown, and for a comparison with the United States and other economies.

[2] Chen et al. (2010) consider a dynamic equilibrium model of durable goods and investigate the effectiveness of a sales tax reduction in stimulating sales and profits in the U.S. automobile industry

the sales tax, measured by its effect on firms profits and sales, greatly decreases with the product‘s durability. Goering and Boyce (1996) prove that, in a two period durable goods monopoly model, an excise tax on output may increase a durable goods monopolist´s commitment ability and market power. If the durable goods monopolist rents its output, the tax strictly decrease its profits, but if the monopolist sells its output its ability to increase first-period price and profits may be enhanced. To the best of our knowledge, the literature has analyzed the effects of subsidies in markets where firms can either rent or sell their production but not both.

Nevertheless, there are markets in which the good is sold but also rented, such as the housing market[3]. We analyze the effects of per unit subsidies to sales and rentals of durable goods, considering subsidies in the present and subsidies in the future, in a situation where there is imperfect competition. We compare the effects on prices and quantities of changes in each of the subsidies considered and we also study the welfare effects of those changes.

The decisions of the regulator on subsidies to sales and rentals of durable goods are taken without knowing well market conditions. Hence, the analysis of the effects of the subsidies to sales and rentals of durable goods seems more relevant than the study of the optimal levels of those subsidies. Our analysis is developed in a context where, as it occurs in reality, the regulator has not enough information to determine the optimal subsidies to sales and rentals of durable goods, but he sets some non-zero levels for the subsidies to those goods.

We consider a durable good industry where firms may both sell and rent their production (renting-selling firms). Bucovetsky and Chilton (1986) and Bulow (1986) show that a monopolist facing the threat of entry chooses to sell part of the units supplied, instead of renting them all. Carlton and Gertner (1989) show that, when there is not threat of entry, strategic interaction between rivals provides a reason for an oligopolist to choose to sell some of its output rather than rent it, in contrast to the behavior of a monopolist, which will choose to rent all its production. The reason for this behavior is that, when a firm sells a durable good in the present, it is depriving its rivals of current and future sales.

As Carlton and Gertner (1989) noted, when firms can both rent and sell their production, but they do not coordinate to rent them, then each firm, in equilibrium, will behave strategically and may sell part of its production,

[3] As indicated by Saggi and Vettas (2000), durable goods markets are primarily oligopolistic rather than monopolistic, and firms sell as well as lease goods. Examples include automobiles, house appliances, computers, copy machines, and machinery equipment.

although its profits would be greater if all of them only rented the good. While leasing helps solve the durable good problem, in the presence of competition leasing becomes less attractive. Sales allow a firm to capture part of the market, whereas leasing may result in loss of future market share to a rival since both firms can compete anew for consumers that leased in the past. By selling an extra unit today, a firm steals current and future sales from its rivals.

Moreover, in durable good industries firms face the time inconsistency problem first noted by Coase(1972). Each firm ignores the effect of its second period production on the value of the units bought by consumers in previous periods. If first period buyers have perfect foresight then they realize that, since the existing stock of units is held by buyers, firms have no incentive to take this capital loss into consideration in their future pricing behavior. Thus, if consumers are rational, this expected behavior of firms becomes an "expectation constraint" on each renting-selling firm.[4]

As we will discuss in section 5, when there are sales in a durable good market, future subsidies or increasing subsidies decrease firms' commitment ability to low production in the future. Analogously, present subsidies or decreasing subsidies increase firms' commitment ability with current buyers. Instead, in the case of non-durable goods subsidies benefit directly the firms without imposing them any costs, since there are not commitment problems with buyers.

Hence, with renting-selling firms, we have that renting and selling, without taxes or subsidies, is distorted away from the optimal for three reasons: the existence of imperfect competition in the production of the durable good, the commitment problem implied by the Coase conjecture and the strategic behavior of firms when choosing between renting and selling.

We consider in our analysis that parameters are such that there are both rentals and sales. In this way our model can be considered as an extension of the case considered by Carlton and Gertner, introducing per unit renting and selling subsidies.

We show that the effects on quantities and prices of a subsidy to rentals in the present and of a subsidy in the future are very similar, and that those effects

[4] Coase (1972) conjectured that if consumers have perfect information and are rational, then a monopoly seller of an infinitelly durable good without some commitment to limit future production would saturate the market with the competitive output "in the twinkling of an eye" (p.143). What Coase actually argued, called the Coase conjecture, is that a durable-goods monopolist selling a perfectly durable good in an infinite-period setting will not be able to exercise any market power. That is, because consumers anticipate that the price will quickly fall to marginal cost due to time inconsistency, they will be unwilling to pay more than marginal cost so the equilibrium is marginal-cost pricing in every period. The other main part of Coase's argument is that leasing or renting avoids the problem.

are almost opposite to the effects of a subsidy to purchases in the present. To favour total production and production in the future we should increase the subsidy to purchases in the present, or reduce the subsidy to rentals in the present or the subsidy in the future. An increase in the subsidy to rentals in the present favours short-run production. Most market prices change in the same direction as the subsidy to rentals in the present and the subsidy in the future, but they change in opposite direction to the subsidy to purchases in the present. Hence, there may be an overall pro-competitive effect of a reduction in the subsidy to rentals in the present or in the subsidy in the future.

We also show that total surplus (when the social cost of public funds is equal to 1) and consumer surplus change in the same direction as the subsidy to purchases in the present, but that consumer surplus changes in opposite direction to the subsidy to rentals in the present or the subsidy in the future and that total surplus may change in opposite direction to the subsidy to rentals in the present or to the subsidy in the future. As total surplus and consumer surplus diminish with a reduction in the subsidy to purchases in the present, that reduction would have to be justified on other criteria. Producers, instead, may favour reductions in the subsidy to purchases in the present or in the subsidy in the future, but they will oppose a reduction in the subsidy to rentals in the present.

We obtain some counterintuitive results. For instance, we prove that, for any of the subsidies considered, the cost of subsidies may change in opposite direction to the direction of change in the subsidy, or that the sale price in the present may change in opposite direction to the change in the subsidy to purchases in the present. Intertemporal interactions among subsidies, the different implications of rentals and sales in the present, imperfect competition, the strategic behavior of each firm to steal sales from its rivals in the present and in the future and the inability of firms to commit in the present to future production are behind the results we obtain.

As it occurs for some other results, the direction of the effect of subsidies on the cost of subsidies depends on the values of the parameters. Empirical work is thus required to estimate the values of the parameters that characterize each particular durable good industry.

Our results are obtained within a parametric (although standard) formulation. However, the simplicity of the model allows us to identify some key implications of strategic behavior in this context.

A message of this work is that to determine and understand the effects of subsidies to sales and rentals of durable goods we must consider the interactions among present and future, and sales and rentals, subsidies to those goods.

The paper is organized as follows: Section 2 introduces the model. The market decisions with sale and rental subsidies are investigated in section 3. The effects of subsidies on prices and quantities are studied in section 4. The welfare analysis of subsidies is presented in section 5. Finally, section 6 concludes. Some proofs and the restrictions in the parameters of the model required for the analysis are included in the Appendix.

2 Model

We consider an oligopolistic industry with $n \geq 2$ identical firms that produce a homogeneous durable good. Entry into the industry is assumed to be unprofitable or unfeasible. There are two discrete periods of time: present ($t = 1$) and future ($t = 2$). Firms may both sell and rent their production (renting-selling firms) in the first period. Given that the second period is the last one, renting is identical to selling in that period.

The inverse demand for services of the durable good is assumed to be constant over time. This inverse rental demand function for the services of the durable good in each period is $p(Q) = a - bQ$, where Q represents the quantity used by consumers in that period and $a, b > 0$.

All agents participating in the market have perfect and complete information and potential users of the good have perfect foresight. We consider that there exits a perfect second hand market for the durable good.

Our analysis is developed under three simplifying assumptions that do not affect the results. We consider, first, that the discount factor is 1. Moreover, we assume that the units of the durable good produced in the first period do not depreciate over time. Thus, every quantity used in the first period can be used in the second period without depreciation. Finally, the marginal cost of production of each firm is $c = 0$.

The analysis proceeds in two stages. In the first stage the regulator sets subsidies for the two periods. We consider that the regulator can commit to subsidies and announces those subsidies right at the beginning of the first period. In the second stage firms engage in quantity competition. Each firm chooses in every period its level of production and the division of production between renting and selling in the first period, considering as given the decisions on production, renting and selling of its competitors. Firms' choices are simultaneous. The objective of each firm is to maximize its discounted sum of profits. The competition game among firms is, therefore, non-cooperative.

The solution concept used is that of a subgame perfect Nash equilibrium in pure strategies. In the cases of selling firms and of renting-selling firms each firm maximizes in each period the present discounted value of profits starting from that period. Therefore, the solutions are derived by backward induction from the last period of the second stage.

The following notation will be used for quantities at the firm level (for the corresponding quantities at the industry level we will use a Q, instead of a q, and eliminate the i subscript):

q_{1i}^s: quantity sold by firm i in the first period,

q_{1i}^r: quantity rented by firm i in the first period,

$q_{1i}^s + q_{1i}^r$: quantity produced by firm i in the first period,

q_{2i}: quantity sold (or rented) by firm i in the second period,

$q_{2i} - q_{1i}^r$: quantity produced by firm i in the second period.

We consider situations where there is renting and selling in $t = 1$ ($q_{1i}^s > 0$ and $q_{1i}^r > 0$) and where all units produced in $t = 1$ are used in $t = 2$ ($q_{2i} \geq q_{1i}^r$). These assumptions imply restrictions on the parameters of the model that we will obtain below. The quantity of the durable good used in $t = 2$ in the market will be $Q_2 + \delta Q_1^s$.

We consider that each buyer of a unit of the durable good in $t = 1$ receives a per unit subsidy equal to s_1^s, each renter of a unit of the durable good in $t = 1$ receives a per unit subsidy equal to s_1^r, and each consumer who purchases or rents a unit of the durable good in $t = 2$ receives a per unit subsidy equal to s_2. The regulator can commit to subsidies and announces these subsidies right at the beginning of the first period.

If there is not perfect rank correlation in the willingness to pay for the durable good by consumers, a consumer that has bought a unit in $t = 1$ may decide to sell that unit in $t = 2$. In this case we consider that the buyer of that second hand durable good in period 2 receives a subsidy equal to s_2 but the seller (who bought that durable good in period 1) pays s_2 in taxes to the regulator.

Let p_1^s, p_1^r and p_2 denote, respectively, the market price of a unit of the durable good sold in the first period, the market price of a unit of the durable good rented in the first period and the market price of a unit of the durable good sold or rented in the second period. It is:

$$p_1^s = a - bQ_1^s - bQ_1^r + (a - bQ_1^s - bQ_2) + s_1^s$$
$$p_1^r = a - bQ_1^s - bQ_1^r + s_1^r$$
$$p_2 = a - bQ_1^s - bQ_2 + s_2$$

The net prices paid by consumers are, respectively, $p_1^s - s_1^s$, $p_1^r - s_1^r$ and $p_2 - s_2$. We will consider that per unit subsidies and parameter values are such that $p_1^s - s_1^s \geq 0$, $p_1^r - s_1^r \geq 0$ and $p_2 - s_2 \geq 0$ (we will obtain below the restrictions on the parameters required to satisfy these conditions).

Note that the possibility of arbitrage by consumers implies:

$$p_1^s - s_1^s - (p_2 - s_2) = p_1^r - s_1^r$$

and we incorporate this condition into the analysis.

3 Market decisions with sale and rental per unit subsidies

In this section we study how subsidies affect the market levels of production, selling and renting. In period $t = 2$, each active firm i, with $i = 1, ..., n$, solves the following problem:

$$\max_{q_{2i}} (a - bQ_1^s - bQ_2 + s_2) q_{2i}.$$

The first order condition of this problem is:

$$a - bQ_1^s - bQ_2 - bq_{2i} + s_2 = 0, \tag{1}$$

that is, marginal revenue for firm i in the second period equal to marginal cost (which is 0). Adding up those n first order conditions over i we get:

$$q_{2i} = \frac{a - bQ_1^s + s_2}{b(n+1)}. \tag{2}$$

Note that firms face the time inconsistency problem first noted by Coase(1972). Each firm ignores the effect of its second period production on the value of the units bought by consumers in $t = 1$. However, first period buyers are rational and realize that each firm will choose its second period production to satisfy (1). Since the existing stock of units is held by buyers, firms have no incentive to take this capital loss into consideration in their future pricing behavior. Thus, if consumers are rational, (2) becomes an "expectation constraint" on a renting-selling firm.

In period $t = 1$, each firm chooses the levels of sales and rentals, q_{1i}^s and q_{1i}^r, that maximize the present value of its profits. Thus, each firm i, with $i = 1, ..., n$, solves the following problem:[5]

$$\max_{\{q_{1i}^r, q_{1i}^s\}} [(a - bQ_1^s - bQ_1^r + a - bQ_1^s - bQ_2 + s_1^s)q_{1i}^s + \\ + (a - bQ_1^s - bQ_1^r + s_1^r)\, q_{1i}^r + (a - bQ_1^s - bQ_2 + s_2)q_{2i}]$$

subject to (2). Assuming interior solutions, the first order conditions of this problem are:[6]

$$\begin{array}{l} a - bQ_1^s - bQ_1^r - bq_{1i}^s - bq_{1i}^r + s_1^r = 0 \\ a - bQ_1^s - bQ_1^r + \frac{a - bQ_1^s - ns_2}{n+1} + s_1^s - bq_{1i}^s - \frac{bq_{1i}^s}{n+1} - bq_{1i}^r - \frac{2(a - bQ_1^s + s_2)}{(n+1)^2} = 0 \end{array} \quad (3)$$

Observe from (1) and (3) that there is a symmetric market solution for firm decisions. The first equation in (3) equates marginal revenues from rentals in the first period to marginal cost. The second equation in (3) equates marginal revenues from sales in the first period to marginal cost.

Adding up over i each of the first order conditions in (3) and solving, we obtain:

$$\begin{array}{c} q_{1i}^s = \frac{a(n-1) + (n+1)^2(s_1^s - s_1^r) - \left(n^2 + n + 2\right)s_2}{(n^2+1)b} \\ q_{1i}^r = \frac{2a - s_1^s(n+1)^3 + s_1^r(n^3 + 4n^2 + 3n + 2) + s_2(n^3 + 2n^2 + 3n + 2)}{(n^2+1)(n+1)b} \\ q_{1i}^s + q_{1i}^r = \frac{a + s_1^r}{(n+1)b} \end{array} \quad (4)$$

By replacing q_{1i}^s in equation (1) we get:

$$q_{2i} = \frac{a - n\,(1+n)\,(s_1^s - s_1^r) + s_2(n^2 + n + 1)}{(n^2+1)b} \quad (5)$$

Therefore, it is:

$$\begin{array}{c} q_{2i} - q_{1i}^r = \frac{(n-1)a + (n+1)^2 s_1^s - 2s_1^r(n^2 + n + 1) - s_2(n+1)}{(n+1)(n^2+1)b} \\ q_{1i}^s + q_{2i} = \frac{na + (n+1)(s_1^s - s_1^r) - s_2}{(n^2+1)b} \end{array} \quad (6)$$

The restrictions on the parameters of the model implied by conditions $q_{1i}^s \geq 0$, $q_{1i}^r \geq 0$ and $q_{2i} \geq q_{1i}^r$ are presented in the Appendix. In particular, those restrictions are satisfied when there are no subsidies: $s_1^s = 0$, $s_1^r = 0$ and $s_2 = 0$.

We also obtain:

$$\begin{array}{c} p_1^s - s_1^s = \frac{a\left(n + n^2 + 2\right) + 2n^2 s_1^r + n(n+1)s_2 - s_1^s n(n+1)^2}{(n^2+1)(n+1)} \\ p_1^r - s_1^r = \frac{a - ns_1^r}{n+1} \\ p_2 - s_2 = \frac{a - n(n+1)(s_1^s - s_1^r) + ns_2}{n^2+1} \end{array} \quad (7)$$

[5] If there is not perfect rank correlation in the willingness to pay for the durable good by consumers, the price that may be obtained in the market in $t = 2$ selling a unit of the durable good bought in $t = 1$ is $a - bQ_1^s - bQ_2$.

[6] Throughout the paper we assume that parameters are such that interior solutions exist.

$$\begin{array}{c}
p_1^s = \frac{a\left(n+n^2+2\right)+2n^2s_1^r+n(n+1)s_2-s_1^s(n-1)(n+1)}{(n^2+1)(n+1)} \\
p_1^r = \frac{a+s_1^r}{n+1} \\
p_2 = \frac{a-n(n+1)(s_1^s-s_1^r)+\left(n^2+n+1\right)s_2}{n^2+1} = \frac{a+s_2-\left(n^2+n\right)(s_1^s-s_1^r-s_2)}{n^2+1}
\end{array} \tag{8}$$

As

$$Q_2 \geq Q_1^r \Leftrightarrow p_1^s - s_1^s \geq p_1^r - s_1^r \geq p_2 - s_2,$$

for positive consumer prices we need also $a - bQ_1^s - bQ_2 \geq 0$. The restriction on the parameters implied by this condition is also obtained in the Appendix.

4 Effects of subsidies on prices and quantities

From the previous section it is immediate to obtain the effects on prices and quantities of changes in the level of any subsidy. In Tables 1 and 2 we summarize the directions of those effects (sign of the corresponding first partial derivative of the variable with respect to the subsidy). Signs $+$, $-$ and $+/-$ mean, respectively, a variation of the corresponding variable in the same direction as the subsidy, a variation in opposite direction to the subsidy or any direction of variation (same or opposite) is possible. We have written a 0 when the variable is independent of the level of the subsidy.

Table 1
Effects of subsidies: quantities

	q_{1i}^s	q_{1i}^r	q_{2i}	$q_{1i}^s + q_{1i}^r$	$q_{1i}^s + q_{2i}$	$q_{2i} - q_{1i}^r$
s_1^s	$+$	$-$	$-$	0	$+$	$+$
s_1^r	$-$	$+$	$+$	$+$	$-$	$-$
s_2	$-$	$+$	$+$	0	$-$	$-$

Table 2
Effects of subsidies: prices

	p_1^s	p_1^r	p_2	$p_1^s - s_1^s$	$p_1^r - s_1^r$	$p_2 - s_2$
s_1^s	$-$	0	$-$	$-$	0	$-$
s_1^r	$+$	$+$	$+$	$+$	$-$	$+$
s_2	$+$	0	$+$	$+$	0	$+$

From Tables 1 and 2 we can state:

Proposition 1 *Among the effects of subsidies on quantities and prices, we have:*

i) the directions of the effects on quantities and prices of s_1^r and s_2 are very similar, and those directions are almost opposite to the directions of the effects of s_1^s,

ii) total production and production in the second period change in the same direction as s_1^s and in opposite direction to s_1^r and s_2, *iii) most market and consumer prices change in opposite direction to s_1^s and in the same direction as s_1^r and s_2.*

From Tables 1 and 2 we have that there is an overall pro-competitive effect of an increase in s_1^s (or of a decrease in s_1^r or in s_2) and there is an overall anti-competitive effect of an increase in s_1^r or in s_2 (or of a decrease in s_1^s). That overall pro-competitive effect occurs through an induced increase in sales in period 1 which is not compensated by a reduction in rentals in the second period. A raise in s_1^s (or of a decrease in s_1^r or in s_2) increases the incentives of firms to steal market shares to its rivals in the present and in the future through an increase in sales in the present.

From i) in Proposition 1 we have that a simultaneous change in the same direction in s_1^s and in s_1^r (or in s_2) would be inefficient as it would imply a (partial) compensation of the effects on prices and quantities. The effects on prices and quantities would, instead, be reinforced if s_1^s and s_1^r (or s_2) change in opposite directions.

A reason for the similarity between the effects on prices and quantities of s_1^r and s_2 is that s_2 is paid in the future to all the units rented in the present, as they are also rented in the future. A simultaneous change in the same direction in s_1^r and in s_2 would reinforce most of the effects of a change in that direction in any of those subsidies.

While an increase in s_1^s favours purchasers of the good in the first period and an increase in s_1^r favours those consumers that rent the good in the present, we obtain that renters of the good in the second period pay a higher price when s_2 increases. Producers in the last period are able to increase the market price in an amount greater than the increase in s_2.

5 Welfare analysis

In this section we analyze how the sale and rental subsidies affect profits of firms, consumer surplus, cost of subsidies and total surplus. We have:

- Consumer surplus:

$$\begin{aligned}CS &= \int_0^{Q_1^s+Q_1^r}(a-bx)dx-(p_1^s-s_1^s)Q_1^s-(p_1^r-s_1^r)Q_1^r\\&+\int_0^{Q_1^s+Q_2}(a-bx)dx-(p_2-s_2)Q_2\\&=\frac{b}{2}\left((Q_1^s+Q_1^r)^2+(Q_1^s+Q_2)^2\right)\end{aligned}$$

- Producer surplus:

$$\Pi = p_1^sQ_1^s+p_1^rQ_1^r+p_2Q_2$$

- Cost of subsidies:

$$SUB = s_1^s Q_1^s + s_1^r Q_1^r + s_2 Q_2$$

We measure total surplus (TS) as the sum of consumer surplus and producer surplus net of the cost of subsidies. Let us consider that the social cost of public funds is equal to γ, with $\gamma \geq 1$. It will be:

$$TS = \pi + CS - \gamma SUB = a(Q_1^s + Q_1^r) + a(Q_1^s + Q_2)$$
$$-\frac{b}{2}\left((Q_1^s + Q_1^r)^2 + (Q_1^s + Q_2)^2\right) - (\gamma - 1)(s_1^s Q_1^s + s_1^r Q_1^r + s_2 Q_2)$$

Table 3 summarizes the directions of the effects of changes in the levels of the subsidies on welfare levels (the analysis of the first derivatives of the welfare variables with respect to each subsidy is relegated to the Appendix):

Table 3
Effects of subsidies: surplus and cost of subsidies

	CS	Π	SUB	TS $(\gamma = 1)$
s_1^s	$+$	$+/-$	$+/-$	$+$
s_1^r	$-$	$+$	$+/-$	$+/-$
s_2	$-$	$+/-$	$+/-$	$+/-$

From Table 3 we can state:

Proposition 2 *Among the effects of subsidies on welfare variables, we have:*

i) consumer surplus changes in the same direction as s_1^s and in opposite direction to s_1^r or s_2,

ii) producer surplus may change in opposite direction to s_1^s or s_2,

iii) if any of the subsidies is changed then the cost of subsidies may change in opposite direction to the direction of change in the subsidy, and

iv) when $\gamma = 1$, total surplus changes in the same direction as s_1^s and it may change in opposite direction to s_1^r or s_2.

Part i) in Proposition 2 is a consequence of the overall pro-competitive effect of an increase in s_1^s (or of a decrease in s_1^r or in s_2) and the overall anti-competitive effect of an increase in s_1^r or in s_2 (or of a decrease in s_1^s) that we have pointed out in the previous section. As an increase in s_1^r (or in s_2) would decrease consumer surplus, that increase would have to be rooted on equity criteria (renters of the durable good in the first period pay less when s_1^r is raised) or on other reasons.

A raise in a subsidy increases the willingness to pay of consumers for the type of consumption (sales or rentals, in the present or in the future) where that subsidy applies. Hence, it might seem that a raise in a subsidy, without changing the rest of subsidies, would increase producer surplus. However, there are other aspects to consider. A change in a subsidy affects the ability of producers to commit not to sell, or rent, large amounts of units of the durable good in the future and we know that an increase in this commitment ability may, by itself, increase producer surplus.[7] In this analysis a raise in s_2 reduces the ability of producers to commit not to sell large amounts of units of the durable good in the future, as sales in the future are favoured by the increase in the subsidy. Finally, although a raise in s_1^s or in s_1^r increases that commitment ability, as the subsidy in the future would become smaller with respect to the increased subsidy in the present, we have from Proposition 1 that a raise in s_1^s (or a decrease in s_1^r) increases the incentives of firms to steal market shares to its rivals in the present and in the future through an increase in sales in the present and this implies a pro-competitive effect that reduces producer surplus. The total effect of a change in a subsidy on producer surplus that we obtain in Proposition 2 is a consequence of the interaction among all these effects. In our analysis producers will favour increases in s_1^r and oppose reductions in that subsidy, but they may favor reductions in s_1^s or s_2 and oppose increases in these latter subsidies. The increase in producer surplus with a raise in s_1^r or in s_2 occurs as the effect on producer surplus of the increase in all market prices dominates the negative effect in total production. If producer surplus decreases with an increase in s_1^s, the effect on producer surplus of the decrease in all market prices dominates the positive effect in total production. When a subsidy is increased some quantities decrease and, if the increase is in s_1^s, market prices decrease or remain unchanged.

The result in part iii) in Proposition 2 occurs when the subsidy changed is not the only subsidy used by the regulator. For instance, the cost of subsidies may increase after a reduction in s_1^s as a consequence of the increase in rentals in the present (and of the corresponding cost of subsidies to rentals in the present). Analogously, the cost of subsidies may also increase after a reduction in s_1^r, or in s_2, as a consequence of the increase in sales in the present (and of the corresponding cost of subsidies to sales in the present).

Finally, from iv) in Proposition 2 we have that total surplus may decrease with an increase in s_1^r or in s_2, even if $\gamma = 1$. This is a consequence of the decrease in total production, after an increase in s_1^r or in s_2. The decrease in total surplus when s_1^s is reduced is a consequence of the induced decrease in total production without any change in production in the present. This decrease in

[7] See Goering and Boyce (1996).

total surplus implies that the effect on consumer surplus dominates when the profits of producers increase with that reduction in s_1^s as a consequence of the induced increase in the ability of producers to commit not to sell large amounts of units of the durable good in the future. As an increase in γ reduces total surplus, when $\gamma > 1$ it will be more likely to obtain a decrease in total surplus when any of the subsidies increases (and the contrary will occur when any of the subsidies is reduced.

When $s_1^s \neq s_1^r + s_2$ the regulator may decide to reduce the difference between s_1^s and $s_1^r + s_2$, that is, to approach the incentives to rent the good in the two periods with the incentives to purchase the good in the present. Consider that $s_1^s > s_1^r + s_2$ and the regulator prefers to approach s_1^s and $s_1^r + s_2$ modifying the subsidies paid in $t = 1$. In this case, the regulator could choose between a reduction in s_1^s and an increase in s_1^r in the same amount. We have already discussed the directions of the effects of a reduction in s_1^s and of an increase in s_1^r. However, if we compare also the amounts of those effects, it is easy to show from equations (4) to (8) that there is no difference in the amounts of the effects on sales in $t = 1$, rentals in the future, total production and future net and total prices. Nevertheless, in comparison with the reduction in s_1^s, the increase in s_1^r increases more rentals in $t = 1$ and p_1^s, reduces more production in the future, increases less $p_1^s - s_1^s$ and reduces $p_1^r - s_1^r$. If we compare the welfare effects of a reduction in s_1^s with those of an increase in s_1^r, considering changes in equal amounts in those subsidies, we may show, using the calculations in the proof of Proposition 2, that with an increase in s_1^r consumer surplus decreases less, producer surplus increases more (or increases, instead of decrease) and total surplus, when $\gamma = 1$, may increase (instead of decrease).

6 Conclusion

We have analyzed the effects of per unit subsidies to sales and rentals of durable goods. We have considered subsidies to rentals and subsidies to sales in the present, and subsidies in the future. Intertemporal interactions among subsidies, the different implications of rentals and sales in the present, imperfect competition, the strategic behavior of each firm to steal sales from its rivals in the present and in the future and the inability of firms to commit in the present to future production are behind the results we obtain.

We have shown that the effects on quantities and prices of s_1^r and s_2 are very similar, and that those effects are almost opposite to the effects of s_1^s. To favour total production and production in the future we should increase s_1^s, or reduce s_1^r or s_2. An increase in s_1^r favours short-run production. Most market prices change in the same direction as s_1^r and s_2, but they change in opposite direction to s_1^s. Hence, there is an overall pro-competitive effect of an increase in s_1^s (or of a decrease in s_1^r or in s_2) and there is an overall anti-competitive effect of an increase in s_1^r or in s_2 (or of a decrease in s_1^s).

We have obtained that, for any of the three subsidies, the cost of subsidies may change in opposite direction to the direction of change in a subsidy, when that subsidy is not the only subsidy used by the regulator. As it occurs for some other results, the direction of the effect of subsidies on the cost of subsidies depends on the values of the parameters. Empirical work is thus required to estimate the values of the parameters that characterize each particular durable good industry.

We have also shown that total surplus (when $\gamma = 1$) and consumer surplus change in the same direction as s_1^s, but that consumer surplus changes in opposite direction to s_1^r and s_2 and total surplus may also change in opposite direction to s_1^r and s_2. As total surplus and consumer surplus diminish with a reduction in s_1^s, that reduction would have to be justified on other criteria. Producers, instead, may favour reductions in s_1^s or in s_2, but they will oppose a reduction in s_1^r.

Often governments consider reductions in subsidies for budgetary reasons. We could compare different alternatives for subsidies reduction. For instance, if we have to choose between a reduction in s_1^s and a reduction in s_2, we have that the directions of changes in quantities and prices under this latter option are better, except for the negative effect on rentals in the present of a reduction in s_2. Moreover, this latter reduction could also be better with respect to welfare considerations as it increases consumer surplus and it may not decrease total surplus.

Simultaneous reductions in subsidies could be also under consideration by governments. We know, for instance, from section 4 that a simultaneous reduction in s_1^s and in s_2 would be inefficient. If there is a simultaneous reduction, in the same amount, in both subsidies, it may be shown, however, from equations (4) to (8) that there would be a decrease in total production, purchases in the present and sale prices in the present and in the future. In a context where subsidies have to diminish, it would be better to opt for progressive reduction, with a greater reduction in s_2 than in s_1^s.

Per unit taxes are negative per unit subsidies. Our analysis can be applied to the analysis of per unit taxes on purchases or rentals of durable goods, provided that the restrictions on the parameters required to obtain renting and selling in period 1 hold. The effects of an increase in a tax would be the same as the effects of a reduction in the corresponding subsidy.

Sometimes, subisidies to durable goods are ad valorem, instead of per unit, with a maximum level for the subsidy. The analysis of this possibility is more involved analytically, but the message on the relevance of considering the interactions among present and future, and sales and rentals, subsidies and the rest of considerations discussed in this work remains.

Demand of durable goods may be non-stationary. To incorporate this possibility in our analysis consider that the inverse demand function for the services of the durable good in period 2 is: $p = a + k - bQ$, where k may be positive or negative, and that all agents in the market have correct expectations of that change in demand with time. In this case the effect of k is the same as the effect of a per unit subsidy equal to k in period 2. In the context of our model we have shown that production would respond to expectations of a change in the demand for the services of the durable good in the future with a lag. Production in the present would be unaffected, even if the change in future demand is correctly predicted. Production would only be modified in the future, when demand has changed. However, sales, rentals and the market and net price of sales in the present would all change with k.

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8 Appendix

8.1 Restrictions required on the parameters of the model

We have:

$$q_{1i}^s \geq 0 \Leftrightarrow s_1^s \geq \frac{(n+1)^2 s_1^r + \left(n^2+n+2\right) s_2 - a(n-1)}{(n+1)^2},$$

$$q_{1i}^r \geq 0 \Leftrightarrow s_1^s \leq \frac{2a + s_1^r(n^3+4n^2+3n+2) + s_2(n^3+2n^2+3n+2)}{(n+1)^3},$$

$$q_{2i} \geq q_{1i}^r \Leftrightarrow s_1^s \geq \frac{2s_1^r(n^2+n+1) + s_2(n+1) - a(n-1)}{(n+1)^2}$$

and

$$a - bQ_1^s - bQ_2 \geq 0 \Leftrightarrow s_1^s \leq \frac{a + n\,(n+1)\, s_1^r + ns_2}{n(n+1)}.$$

Hence, we require s_1^r, s_2 and s_1^s such that:

$$\max\left\{\frac{(n+1)^2 s_1^r + \left(n^2+n+2\right)s_2 - a(n-1)}{(n+1)^2}, \frac{2s_1^r(n^2+n+1)+s_2(n+1)-a(n-1)}{(n+1)^2}\right\} \leq s_1^s$$
$$\leq \min\left\{\frac{2a+s_1^r(n^3+4n^2+3n+2)+s_2(n^3+2n^2+3n+2)}{(n+1)^3}, \frac{a+n(n+1)s_1^r+ns_2}{n(n+1)}\right\}$$

i.e.,

$$\max\left\{s_1^r + \frac{\left(n^2+n+2\right)s_2}{(n+1)^2} - \frac{a(n-1)}{(n+1)^2}, s_1^r + \frac{s_2}{n+1} - \frac{a(n-1)-s_1^r(n^2+1)}{(n+1)^2}\right\} \leq s_1^s$$
$$\leq \min\left\{s_1^r + \frac{\left(n^2+n+2\right)s_2}{(n+1)^2} + \frac{2a+s_1^r(n^2+1)}{(n+1)^3},\ s_1^r + \frac{s_2}{n+1} + \frac{a}{n(n+1)}\right\}. \quad (9)$$

Condition (9) is satisfied if all subsidies are equal to 0. This condition is also satisfied if demand, as parametrized by a, is high enough. An example of situations where (9) is satisfied: $a = 100$, $n = 2$, $s_1^r = s_2 = 10$ and $7.7778 \leq s_1^s \leq 28.148$.

Note also that:

$$\frac{(n+1)^2 s_1^r + \left(n^2+n+2\right)s_2 - a(n-1)}{(n+1)^2} > \frac{2s_1^r(n^2+n+1)+s_2(n+1)-a(n-1)}{(n+1)^2}$$
$$\Leftrightarrow s_2 > s_1^r.$$

8.2 Proof of Proposition 2

i) We have:

$$CS = \frac{b}{2}\left(\left(\frac{(a+s_1^r)n}{b(n+1)}\right)^2 + \left(n\frac{na+s_1^s(n+1)-(n+1)s_1^r-s_2}{(n^2+1)b}\right)^2\right)$$

$$\frac{dCS}{ds_1^s} = \frac{(an-s_2-(n+1)s_1^r+(n+1)s_1^s)(n+1)n^2}{b(n^2+1)^2}$$

$$\frac{dCS}{ds_1^r} = b\left(\frac{an^2+n^2s_1^r}{b^2+2b^2n+b^2n^2} + \left(-n^2-n^3\right)\frac{an-s_1^r-ns_1^r-s_2+s_1^s(n+1)}{b^2+2b^2n^2+b^2n^4}\right)$$

$$\frac{dCS}{ds_2} = \frac{-(an-s_2-s_1^r+s_1^s-ns_1^r+ns_1^s)n^2}{b(n^2+1)^2}$$

From (9) we obtain:

$$s_1^r + \frac{s_2}{n+1} - \frac{a(n-1)-s_1^r(n^2+1)}{(n+1)^2} \leq s_1^s$$

$$\Leftrightarrow (s_1^s - s_1^r)(n+1) - s_2 + \frac{a(n-1)-s_1^r(n^2+1)}{n+1} \geq 0$$

$$\Rightarrow an - s_1^r - ns_1^r - s_2 + s_1^s\,(n+1) > 0,$$

as $an > \frac{a(n-1)-s_1^r(n^2+1)}{n+1}$. Hence, $\frac{dCS}{ds_1^s} > 0$ and $\frac{dCS}{ds_2} < 0$.

Note also that, as $\frac{a(n^2+n-1)-s_1^r}{n+1} > \frac{a(n-1)-s_1^r(n^2+1)}{n+1}$, it is from (9):

$$(s_1^s - s_1^r)(n+1) - s_2 + \frac{a(n^2+n-1)-s_1^r}{n+1} > 0$$

$$\Leftrightarrow a + s_1^r < (1+n)\left(an - s_1^r - ns_1^r - s_2 + s_1^s\,(n+1)\right)$$

$$\Leftrightarrow \frac{dCS}{ds_1^r} < 0.$$

ii) We have:

$$\frac{\partial \pi_i}{\partial s_1^s} = -2\left(\frac{na(n-1)+\left(n^2-n+1\right)(n+1)^2(s_1^r-s_1^s)+s_2\left(n^2+n^3+n^4+1\right)}{b(n^2+1)^2}\right)$$

$$\begin{aligned}\frac{\partial \pi_i}{\partial s_1^r} = \frac{2}{(n^2+1)^2(n+1)^2 b}&(a(n^2-n+n^3+2n^4+1)\\ &+s_1^r(3n+5n^2+2n^3+4n^4+3n^5+n^6+2)\\ &-s_1^s(n^2-n+1)(n+1)^4+s_2(n^2+n^3+n^4+1)(n+1)^2)\end{aligned}$$

$$\frac{\partial \pi_i}{\partial s_2} = \frac{2\left(an^2+s_2\left(2n^2+n^3+n^4+1\right)+\left(n^2+n^3+n^4+1\right)(s_1^r-s_1^s)\right)}{(n^2+1)^2 b}$$

From (9) it is:

$$s_1^s \leq s_1^r + \frac{\left(n^2+n+2\right)s_2}{(n+1)^2} + \frac{2a+s_1^r(n^2+1)}{(n+1)^3}.$$

Hence, we have:

$$\frac{\partial \pi_i}{\partial s_1^r} \geq \frac{2\,(n-1)\left(-n^2s_1^r+2an+s_2+2ns_2+n^2s_2+a\right)}{(n^2+1)\,(n+1)^2\,b}.$$

From (9) we also know that

$$s_1^r + \frac{s_2}{n+1} + \frac{a}{n(n+1)} > s_1^r + \frac{s_2}{n+1} - \frac{a(n-1) - s_1^r(n^2+1)}{(n+1)^2} \Rightarrow s_1^r < \frac{a}{n}.$$

As $\frac{a}{n} < \frac{2an+s_2+2ns_2+n^2s_2+a}{n^2}$, and

$$s_1^r < \frac{2an+s_2+2ns_2+n^2s_2+a}{n^2} \Leftrightarrow -n^2s_1^r + 2an + s_2 + 2ns_2 + n^2s_2 + a > 0,$$

we conclude that $\frac{\partial \pi_i}{\partial s_1^r} > 0$.

Consider that $a = 100$, $b = 1$, $n = 10$, $s_1^s = 3$, $s_1^r = 2$ and $s_2 = 2$. In this case it is: $\frac{\partial \pi_i}{\partial s_1^s} = -3.9586$, and restrictions (9) are satisfied as:

$$\max\{-3.5868, -3.5868\} \leq s_1^s \leq \min\{4.1533, 3.0909\}.$$

Consider that $a = 13$, $b = 1$, $n = 2$, $s_1^s = 8.13$, $s_1^r = 6$ and $s_2 = 0.1$. In this case it is:$\frac{\partial \pi_i}{\partial s_2} = -0.5176$, and $\frac{\partial \pi_i}{\partial s_1^s} = 2.288$ and restrictions (9) are satisfied as:

$$\max\{4.644, 7.922\} \leq s_1^s \leq \min\{8.163, 8.2\}.$$

Instead, consider that $a = 100$, $b = 1$, $n = 10$, $s_1^s = 3$, $s_1^r = 2$ and $s_2 = 2$. In this case it is:$\frac{\partial \pi_i}{\partial s_2} = 4.1763$, and restrictions (9) are satisfied as:

$$\max\{-3.5868, -3.5868\} \leq s_1^s \leq \min\{4.1533, 3.0909\}.$$

iii) We have:

$$\begin{aligned} SUB &= s_1^s \left(n \frac{(a+s_2)(n-1) + (n+1)^2 (s_1^s - s_1^r - s_2)}{b(n^2+1)} \right) + \\ &\quad + s_1^r n \frac{2(a+s_2) - (n+1)^3 (s_1^s - s_1^r - s_2) + (n^2+1)(s_1^r - s_2)}{(n^2+1)(n+1)b} \\ &\quad + s_2 n \frac{a + s_2 - (n^2+n)(s_1^s - s_1^r - s_2)}{b(n^2+1)} \end{aligned}$$

$$\frac{dSUB}{ds_1^s} = \frac{\left(a(n-1) + 2s_1^s(n+1)^2 - 2s_1^r(n+1)^2 - 2s_2\left(n+n^2+1\right)\right)n}{(n^2+1)b}$$

$$\frac{dSUB}{ds_1^r} = \frac{2\left(a - s_1^s(n+1)^3 + s_2(n+1)\left(n+n^2+1\right) + s_1^r\left(3n+4n^2+n^3+2\right)\right)n}{(n^2+1)(n+1)b}$$

$$\frac{dSUB}{ds_2} = \frac{\left(a - 2\left(n+n^2+1\right)(s_1^s - s_1^r - s_2)\right)n}{(n^2+1)b}$$

Consider that $a = 40$, $b = 1$, $n = 10$, $s_1^s = 2.3$, $s_1^r = 2$ and $s_2 = 2$. In this case it is: $\frac{dSUB}{ds_1^s} = -1.129$, and restrictions (9) are satisfied as:

$$\max\{0.876, 0.876\} \leq s_1^s \leq \min\{4.063, 2.545\}.$$

Consider that $a = 75$, $b = 1$, $n = 10$, $s_1^s = 5.45$, $s_1^r = 4.9$ and $s_2 = 0.1$. In this case it is: $\frac{dSUB}{ds_1^r} = -0.72$ and $\frac{dSUB}{ds_2} = -2.465$ and restrictions (9) are satisfied as:

$$\max\{-0.585, 3.420\} \leq s_1^s \leq \min\{5.477, 5.590\}.$$

iv) When $\gamma = 1$ we have:

$$\begin{aligned} TS = 2an\tfrac{a(n-1)+(n+1)^2 s_1^s-(n+1)^2 s_1^r-\left(n^2+n+2\right)s_2}{b(n^2+1)} \\ +an\tfrac{2a-s_1^s(n+1)^3+s_1^r(n^3+4n^2+3n+2)+s_2(n^3+2n^2+3n+2)}{(n^2+1)(n+1)b} \\ +an\tfrac{a-n(1+n)s_1^s+n(n+1)s_1^r+s_2(n^2+n+1)}{b(n^2+1)} \\ -\tfrac{b}{2}\left(\left(\tfrac{(a+s_1^r)n}{b(n+1)}\right)^2+\left(n\tfrac{na+s_1^s(n+1)-(n+1)s_1^r-s_2}{(n^2+1)b}\right)^2\right) \end{aligned}$$

$$\frac{dTS}{ds_1^s} = \frac{(a+ns_2+n(s_1^r-s_1^s)(n+1))(n+1)n}{(n^2+1)^2 b}$$

$$\frac{dTS}{ds_1^r} = \frac{\left(s_1^s(n+1)^4-a(3+n+n^2-n^3)-(n+1)^3 s_2-2s_1^r((n+1)^4-2n(n^2+n+1)^2)\right)n^2}{(n^2+1)^2(n+1)^2 b}$$

$$\frac{dTS}{ds_2} = \frac{(a-ns_2+n(s_1^s-s_1^r)(n+1))n}{(n^2+1)^2 b}$$

From (9) it is: $s_1^s \leq s_1^r + \frac{s_2}{n+1} + \frac{a}{n(n+1)} \Leftrightarrow a + ns_2 + n\,(s_r - s_s)\,(n+1) > 0$. Hence, $\frac{dTS}{ds_1^s} > 0$.

Consider that $a = 21$, $b = 1$, $n = 10$, $s_1^s = 2.3$, $s_1^r = 2$ and $s_2 = 2$. In this case it is: $\frac{dTS}{ds_1^r} = -0.004$, and restrictions (9) are satisfied as:

$$\max\{2.289, 2.289\} \leq s_1^s \leq \min\{4.035, 2.373\}.$$

Consider that $a = 100$, $b = 1$, $n = 10$, $s_1^s = 1$, $s_1^r = 2$ and $s_2 = 2$. In this case it is: $\frac{dTS}{ds_2} = -0.029$, and restrictions (9) are satisfied as:

$$\max\{-3.587, -3.587\} \leq s_1^s \leq \min\{4.153, 3.091\}. \ \square$$