Second-best emission taxation and environmental damage in durable-goods industries*

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Abstract

We analyze optimal second-best per unit emission taxes in a durable good industry under imperfect competition. The analysis is performed for three different types of emissions and for situations where the good is rented, sold or simultaneously sold and rented. We prove that over-internalization may occur in some situations, but that, in all contexts considered, the total expected tax paid per unit produced in the present is lower than the total expected environmental damage per unit produced in the present. We also compare optimal emission taxes in the different contexts considered and study the variation of optimal emission taxes with marginal environmental damage.

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1 Introduction

The production and consumption of durable goods can result in various types of pollution. For instance, environmental damage during the use of cars is the major contributor to air pollution in the form of smog and exhaust fumes, there are emissions such as smoke and water contamination when some durable goods are produced and solid waste at the end of a product's lifetime may also cause environmental damage. In order to make firms and consumers internalize this pollution damage, a regulator could consider imposing an emissions tax.

Over the past decade OECD countries have increased the number of environmentally related taxes imposed in order to reduce emissions. For example, a number of OECD member countries (as Denmark, Germany, Ireland, Luxembourg, Sweden, United Kingdom and Cyprus) are now applying some form of CO_2 related taxes on the use of motor vehicles. These taxes are paid annually by the owners of the vehicles in order to be allowed to use their vehicles.[1] There are also taxes on production of polluting durable goods. Moreover, the Government may assign to producers the responsibility, financial and/or physical, for the treatment or disposal of their products at the end of life. For instance, the European Directive 2000/53/CE requires car manufacturers to take back end-of-life vehicles free of charge and, in most EU countries, each manufacturer has decided to launch its own program by contracting with car dismantlers and shredders.

Economic literature has studied the relationship between environmental policy and market structure. Under perfect competition, external damage is fully internalized when the per unit emission tax equals the marginal external damage. Under a monopoly, however, as first noted by Buchanan (1969) complete internalization imposes an additional social cost by further restricting the already sub-optimal monopolist's output, so the optimal emission tax is less than the marginal external damage; see also Barnett (1980). As production under imperfect competition is below the efficient level, due to firms' market power, optimal emission taxes under imperfect competition are, in general, below marginal environmental damage. This implies underinternalization of environmental damage. However, the relevant literature has also demonstrated the possibility of overinternalization when there is imperfect competition. Katsoulacos and Xepapadeas (1995) show that under a fixed-number oligopoly

[1] See ACEA (2009) and OECD (2009) for an overview of the CO2-based taxation schemes implemented in some european countries.

the optimal emission tax falls short of the marginal external damage but that with free entry, so that the market structure is determined endogenously, the optimal tax may exceed the marginal environmental damage. Simpson (1995) also shows the possibility of overinternalization under a Cournot duopoly with asymmetric costs of production, in order to redistribute output from the less efficient producer to his more efficient rival.

In a durable goods setting, analysis of the internalization of environmental damage has centered on the relationship between the optimal emission tax in the present and marginal environmental damage in the present. Boyce and Goering (1997) derive that the optimal emission tax in the present may exceed marginal environmental damage under a monopoly that sells its product, when durability is exogenous, emissions occur during the production process and there are increasing returns to scale in production. Runkel (2002) shows that underinternalization results when there is an oligopoly of firms that rent their product, emissions occur during the production process and durability is exogenous. However, he finds that overinternalization may result with endogenous product durability. In a context with constant returns to scale in production and exogenous product durability, Runkel (2004) proves that the optimal waste taxes lie below the marginal environmental damage in the present and in the future, under a monopoly that sells its product, but that there may be overinternalization in the present when there is an oligopoly of firms that sell their product. He also extends the analysis of Goering and Boyce (1997) to show that overinternalization may occur under a monopoly that sells its product, with endogenous durability and constant returns to scale in production.

In the contexts considered in those publications there are several distortions from efficiency. First, environmental damage is not considered by producers under *laissez faire*. Moreover, imperfect competition implies a distortion from efficient provision. Finally, when the good is durable and firms sell at least part of their production in the present there is a possible intertemporal distortion due to the strategic behavior of each firm to steal sales from its rivals in the present and in the future, in a context where the intertemporal consistency problem first noted by Coase (1972) applies. If the regulator uses only one instrument (emission taxes) to correct for all distortions from efficiency, the emission taxes that maximize total surplus will, therefore, be second-best optimal emission taxes. We consider, like most of the relevant literature, that emission taxes are the only instrument available to the regulator.[2]

[2] First-best tax-subsidy schemes are investigated in Runkel (1999).

Some of the analyses of overinternalization with durable goods consider that emissions occur during the production process, as in Goering and Boyce (1997), while others center on situations where emissions occur at the end of the product's lifetime, as in Runkel (2004). One of the contributions of this paper is to study whether the results on overinternalization are affected by the type of emissions: to this end, emissions that occur during the production process, emissions proportional to the stock of the durable good in use and emissions that occur at the end of the product's life are considered.

This work investigates the optimal second-best emissions taxation under imperfect competition in durable goods industries when products are sold, rented or simultaneously sold and rented. To the best of our knowledge, no such analysis has been carried out previously for firms that rent and sell their good simultaneously. However, there are markets in which there is simultaneous renting and selling of the durable good[3]. Bucovetsky and Chilton (1986) and Bulow (1986) prove that a monopolist facing the threat of entry chooses to sell part of the units supplied, instead of renting them all. Carlton and Gertner (1989) show that, when there is no threat of entry, strategic interaction between rivals provides a reason for an oligopolist to choose to sell some of its output rather than rent it, in contrast to the behavior of a monopolist, which will choose to rent all its production.

The solution where firms rent the durable good is analogous to the solution where firms sell the good but they can precommit to current buyers that the value of their stock of durable good will be taken into account in future production (for instance, firms can precommit by offering best-price provisions). The solution where firms only sell the durable good corresponds to situations where firms do not have commitment ability and rentals are not feasible or not allowed by the regulator.[4] The solution with renting-selling firms refers to situations where firms may both rent and sell their production.

We prove that overinternalization may occur when emissions in each period are proportional to the stock of the product in use in that period and when they occur at the end of the product's lifetime, but there is no overinternalization when emissions occur during the production process. Overinternalization in the present requires firms to only sell their production.[5]

[3] As indicated by Saggi and Vettas (2000), durable goods markets are primarily oligopolistic rather than monopolistic, and firms sell as well as lease goods. Examples include automobiles, house appliances, computers, copy machines, and machinery equipment.

[4] Bullow (1982) offers examples of markets in which renting is not feasible.

[5] We show, however, that when we restrict the analysis to parameter values that guarantee, simultaneously, interior solutions for all market configurations (selling firms, renting firms and

Except when emissions occur during the production process, the environmental damage from a unit produced in the present is distributed throughout the lifetime of the product. Hence, a more adequate approach to the analysis of overinternalization in the present would be to compare the expected total emission tax paid per unit produced in the present and the expected total environmental damage caused by a unit produced in the present. We show that, in all cases considered in our analysis, the expected total emission tax paid per unit produced in the present is lower than the expected total environmental damage per unit produced in the present (those cases include the context with exogenous durability where Runkel (2004) obtains overinternalization). This result may provide an adequate perspective on the results on overinternalization in the previous durable goods literature.

We also compare the optimal emission taxes on renting firms, on selling firms and on renting-selling firms when parameter values guarantee interior solutions for the three market configurations. We find that, when emissions are proportional to the stock of the durable good or when emissions occur at the end of the product's lifetime, the optimal emission tax in the first period on renting firms is higher than the optimal emission tax on selling firms. Nevertheless, we show that the expected total emission tax in the present is higher for selling firms than for renting firms, under any type of emissions. Moreover, we study the variation in optimal emission taxes with marginal environmental damage.

The policy implications of these findings are substantial, as there are major polluting industries that produce durable goods and are highly concentrated (the car and aircraft industries, for instance). Knowing the characteristics of optimal emission taxes on durable goods industries is essential for public environmental policy.

The paper is organized as follows: Section 2 introduces the model. The social optimum is derived in Section 3. Section 4 investigates the market equilibrium with emission taxes. In Section 5 we obtain the second-best optimal emission taxes and study their characteristics for the situations where, in the present, firms only rent their product (renting firms, subsection 5.1), firms only sell their product (selling firms, subsection 5.2) and firms both sell and rent their production (renting-selling firms, subsection 5.3). Section 6 centers on the comparison of optimal emission taxes between the three situations considered in the previous section. Section 7 includes several extensions of the analysis. Finally, Section 8 concludes. All proofs are relegated to the Appendix.

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renting-selling firms), there is not overinternalization if firms only sell the durable good.

2 Theoretical framework

We consider an oligopolistic industry with $n \geq 2$ identical firms that produce a homogeneous durable good. Entry into the industry is assumed to be unprofitable or unfeasible. There are two discrete periods of time: present ($t = 1$) and future ($t = 2$). We study the cases where, in the first period, firms only sell their product, firms only rent their product and firms may both sell and rent their production. Given that the second period is the last one, renting is identical to selling in that period.

The situation where firms sell their production but they can precommit to current buyers that the value of their stock of durable goods will be taken into account in future production is analogous to the situation where firms rent their output. We consider that, when firms only sell their output, they do not have commitment ability.

The inverse demand for services of the durable good is assumed to be constant over time. This inverse rental demand function for the services of the durable good in each period is $p(Q)$, where Q represents the quantity used by consumers in that period. We assume that marginal revenue is decreasing. The results presented in this work will focus on the case where $p(Q) = a - bQ$.

All agents participating in the market have perfect and complete information and potential users of the good have perfect foresight. We consider that there exits a perfect second hand market for the durable good.

The durable good depreciates with time: only a proportion δ of the units produced during the first period can be used in the second period. We consider that durability is exogenous, so as to focus on the comparisons of results between renting, selling and renting-selling firms and on the consequences of a change in the type of emissions. However, we are well aware from Bulow (1986) of the relevance of the choice of durability by producers in durable good markets.

The discount factor is the same for all agents participating in the market and it is represented by $\rho\epsilon[0, 1]$. All firms face the same production cost function which is supposed to be linear in output. The first and second period constant marginal cost of production are represented, respectively, by c_1 and c_2 (constant returns to scale).

Production, use or termination of the durable good causes damages external to the industry through the emission of pollutants. We consider that environmental damage in each period per unit of emission in that period is γ, with $\gamma > 0$. Moreover, emissions are proportional to the output levels. In section 7 we extend the analysis to situations where environmental damage per unit of emissions differs from one period to another and also to a context where the environmental damage function is non-linear on emissions and, besides, the inverse rental demand function is non linear.

Let us use parameters α and β to distinguish between the three types of emissions. If $\alpha = 1$ and $\beta = 0$ we have a situation where emissions occur during the production process. If $\alpha = 1$ and $\beta = 1$ we have a situation where emissions are proportional to the stock of product in use in the market. Finally, emissions occur at the end of the life of the product if $\alpha = 0$ and $\beta = 1$. The expected emissions per unit produced in period 1 are $(1-\delta)+\alpha\delta$ in the first period and $\beta\delta$ in the second period.[6] A unit produced in the second period implies emissions equal to 1 in that period. Hence, the environmental damages of a unit produced in period 1 are $(1-\delta)\gamma + \alpha\delta\gamma$ in the first period and $\beta\delta\gamma$ in the second period. The environmental damage from a unit produced in the second period is γ in that period. Therefore, the total environmental damage per unit produced in period 1, in period 1 units, depends on the type of emissions.

The analysis proceeds in two stages. In the first stage the regulator sets emission taxes for the two periods. We consider that the regulator can commit to emission taxes and announces those taxes right at the beginning of the first period. In the second stage firms engage in quantity competition. Each firm chooses in every period its level of production and, in the case of renting-selling firms, the division of production between renting and selling in the first period, considering as given the decisions on production, renting and selling of its competitors. Firms' choices are simultaneous. The objective of each firm is to maximize its discounted sum of profits. The competition game among firms is, therefore, non-cooperative.

The solution concept used is that of a subgame perfect Nash equilibrium in pure strategies. In the cases of selling firms and of renting-selling firms each firm maximizes in each period the present discounted value of profits starting from that period. Therefore, the solutions are derived by backward induction from the last period of the second stage.

The following notation will be used for quantities at the firm level (for the corresponding quantities at the industry level we will use a Q, instead of a q, and eliminate the i subscript):

[6] When $\alpha = 1$ the units of the good produced in $t = 1$ that can also be used in $t = 2$ cause emissions in the first period, and the contrary occurs when $\alpha = 0$. When $\beta = 1$ the units of the good produced in $t = 1$ that can also be used in $t = 2$ cause emissions in the second period, and the contrary occurs when $\beta = 0$.

q_{1i}^s: quantity sold by firm i in the first period,

q_{1i}^r: quantity rented by firm i in the first period,

$q_{1i}^s + q_{1i}^r$: quantity produced by firm i in the first period,

q_{2i}: quantity sold (or rented) by firm i in the second period,

$q_{2i} - \delta q_{1i}^r$: quantity produced by firm i in the second period.

We consider situations where all units produced in $t = 1$ that do not depreciate are also used in $t = 2$ (this implies, for all i, that $q_{2i} \geq \delta q_{1i}^r$). The quantity of the durable good used in $t = 2$ in the market will be $Q_2 + \delta Q_1^s$.

Let us denote by p_1^s, p_1^r and p_2, respectively, the (total) price paid by the buyer of a unit of the durable good in the first period, the (total) price paid by the renter of a unit of the durable good in the first period and the (total) price paid by the buyer (or renter) of a unit of the durable good in the second period. We have:

$$p_1^s = p(Q_1^s + Q_1^r) + \rho\delta p(\delta Q_1^s + Q_2)$$

$$p_1^r = p(Q_1^s + Q_1^r)$$

$$p_2 = p(\delta Q_1^s + Q_2),$$

Obviously, when firms only rent their product it will be $Q_1^s = 0$ and p_1^s will not be defined and when firms only sell their product it will be $Q_1^r = 0$ and p_1^r will not be defined. The possibility of arbitrage by consumers that implies:

$$p_1^s - \rho\delta p_2 = p_1^r$$

If emission taxes are paid by producers, p_1^s, p_1^r and p_2 are also the market prices. Our presentation will follow this situation. However, for some types of emissions the emission taxes on sales are charged to consumers. In that case the total price paid by buyers of the durable good equals the sum of the corresponding emission tax and market price, or price received by producers. As we will show in section 4, the analysis and results in this work remain unchanged when emission taxes are paid by consumers, instead of being paid by producers.

We will obtain below the restrictions on the parameters required to get non-negative quantities and non-negative (total) prices paid by buyers and renters of the durable good.

3 Social optimum

In the context stated in the previous section we have that:

Total surplus (TS)=Consumer surplus+Profits of firms

+Taxes paid - Emissions damage.

Let us denote by Q_{1u} the quantity of the durable good used in the first period and by Q_{2u} the quantity used in the second period. The quantity of the good used in the first period is equal to the sum of the quantity sold and the quantity rented in that period, that is, Q_{1u} is also the quantity of the durable good produced in $t = 1$. The quantity used in the second period will be equal to the sum of the quantity sold (or rented) in the second period and the non-depreciated part of the quantity sold in the first period (or equal to the quantity produced in the second period plus the non-depreciated part of the quantity used in the first period). Hence, $Q_{1u} = Q_1^s + Q_1^r$ and $Q_{2u} = \delta Q_1^s + Q_2$. With this notation we have:

$$\begin{aligned} TS = \textstyle\int_0^{Q_{1u}} p(Q)dQ + \rho \int_0^{Q_{2u}} p(Q)dQ - c_1 Q_{1u} - \rho c_2 (Q_{2u} - \delta Q_{1u}) \\ -\gamma \left((1 - \delta + \alpha\delta + \rho\beta\delta)Q_{1u} + \rho(Q_{2u} - \delta Q_{1u})\right) \end{aligned} \tag{1}$$

To obtain the social optimum we solve, using (1):

$$\max_{Q_{1u}, Q_{2u}} TS$$

The first order conditions of this problem are:

$$p(Q_{1u}) = c_1 - \rho\delta c_2 + \gamma(1 - \delta(1 - \alpha + \rho(1 - \beta))) \tag{2}$$

and

$$p(Q_{2u}) = c_2 + \gamma \tag{3}$$

In equation (3), price in the second period equals marginal production cost in that period plus marginal environmental damage. In equation (2), (rental) price in the first period equals net marginal expected production cost in that period plus net marginal expected environmental damage from a unit produced in $t = 1$. Net marginal expected production cost refers to marginal cost in the first period net of expected marginal cost saved in the second period as, with probability δ, a unit produced in the first period will be in use during the second period and it will allow a reduction in new production in $t = 2$. The interpretation of net marginal expected environmental damage is analogous, in terms of marginal environmental damages. The (rental) price in the first period incorporates the fact that production in that period allows to save on production costs and environmental costs in the second period.

From (3) we have that the price in the second period at the social optimum will be positive. To guarantee a non-negative price for rentals in the first period at the social optimum we assume throughout the paper that $c_1 > \delta\rho c_2$. If

$c_1 > \delta\rho c_2$ then the sale price in the first period at the social optimum will also be positive.

Using (3), equation (2) may be written as:

$$p(Q_{1u}) + \rho\delta p(Q_{2u}) = c_1 + \gamma(1 - \delta(1 - \alpha - \rho\beta)) \tag{4}$$

In this equation, the sale price in the first period (or the rental price in the first period plus expected marginal benefits in the second period from a unit produced in the first period) equals marginal production cost in that period plus expected marginal environmental damage from a unit produced in $t = 1$.

In the rest of this work we consider that $p(Q) = a - bQ$. Under this demand function for the services of the durable good in each period, we obtain, from the first order conditions of the problem of maximization of TS, that the social optimum is:[7]

$$\begin{aligned} Q_{1u}^* &= \tfrac{a-c_1+\delta\rho c_2}{b} - \gamma\tfrac{1-\delta(1-\alpha+\rho(1-\beta))}{b} \\ Q_{2u}^* &= \tfrac{a-c_2-\gamma}{b} \end{aligned} \tag{5}$$

and

$$Q_{2u}^* - \delta Q_{1u}^* = \frac{a(1-\delta) + \delta c_1 - c_2(1+\delta^2\rho)}{b} - \gamma\frac{1-\delta+\delta^2(1-\alpha+\rho(1-\beta))}{b}$$

We have that Q_{1u}^*, Q_{2u}^* and $Q_{2u}^* - \delta Q_{1u}^*$ diminish with γ under any of the three alternatives for the timing of emissions that we are considering. Moreover, note that Q_{2u}^* is independent of α and β and

$$Q_{1u}^*(\alpha = 1, \beta = 1) < Q_{1u}^*(\alpha = 1, \beta = 0) < Q_{1u}^*(\alpha = 0, \beta = 1).$$

We also have:

$$\begin{gathered} Q_{1u}^* > 0 \Leftrightarrow a - c_1 + \delta\rho c_2 > \gamma(1 - \delta(1 - \alpha + \rho(1 - \beta))) \text{ and} \\ Q_{2u}^* - \delta Q_{1u}^* > 0 \Leftrightarrow a - c_2 - \gamma \\ > \delta(a - c_1 + \delta\rho c_2 - \gamma(1 - \delta(1 - \alpha + \rho(1 - \beta)))). \end{gathered} \tag{6}$$

4 Market decisions with emission taxes

In this section we study how emission taxes affect the market levels of production, renting and selling. Let us denote by τ_1 and τ_2, respectively, the emission tax paid in the first period per unit of emission in that period and the emission tax paid in the second period per unit of emission in that period.

[7] Note that the second order conditions are satisfied. For the rest of maximization problems considered in this work it is not difficult to show that the corresponding second order conditions are also satisfied.

Hence, a unit produced in period 1 expects to pay $\beta\delta\tau_2$ in the second period and $(1-\delta+\alpha\delta)\tau_1$ in the first period. A unit produced in the second period pays τ_2 in that period.

With these taxes, in period $t=2$, each active firm i, with $i=1,...,n$, solves the following problem (we present the general case with renting and selling, but we know that when firms only rent their product it will be $Q_1^s=0$ and when firms only sell their product it will be $Q_1^r=0$):

$$\max_{q_{2i}} \left[(a-b(\delta Q_1^s+Q_2))q_{2i}-(\tau_2+c_2)(q_{2i}-\delta q_{1i}^r)-\beta\delta\tau_2\left(q_{1i}^s+q_{1i}^r\right)\right].$$

The first order condition of this problem is:

$$a-b(\delta Q_1^s+Q_2)-bq_{2i}=c_2+\tau_2 \tag{7}$$

In equation (7) we have that marginal revenue for oligopolist i in $t=2$ equals total marginal cost in that period. Adding up the n first order conditions (7) over i we get:

$$q_{2i}=\frac{a-b\delta Q_1^s-\tau_2-c_2}{b(n+1)}. \tag{8}$$

Note that when firms sell at least part of their output ($Q_1^s\neq 0$) they face the time inconsistency problem first noted by Coase (1972), which implies that the second period optimal production is implicitly determined by the first period production level. First period buyers realize that each selling (or renting-selling) firm will choose its second period production to satisfy (7). Since the existing stock of units is held by buyers, those firms have no incentive to take this capital loss into consideration in their future pricing behavior. Thus, if consumers are rational, (7) becomes an "expectation constraint" on a selling firm (or on a renting-selling firm). The higher is the discount factor, the more relevant is this "expectation constraint". When firms rent their output ($Q_1^s=0$), however, they are not constrained by consumer' expectations of future production behavior since they own the entire stock of the good.

In period $t=1$, each firm chooses the levels of sales and rentals, q_{1i}^s and q_{1i}^r, that maximize the present value of its profits. Thus, each firm i, with $i=1,...,n$, solves the following problem:

$$\max_{\left\{q_{1i}^r,q_{1i}^s\right\}}[(a-b(Q_1^s+Q_1^r)+\rho\delta(a-b(\delta Q_1^s+Q_2))$$

$$-(1-\delta+\alpha\delta)\tau_1-c_1)q_{1i}^s+(a-b(Q_1^s+Q_1^r)-(1-\delta+\alpha\delta)\tau_1-c_1)\,q_{1i}^r$$

$$+\rho((a-b(\delta Q_1^s+Q_2))q_{2i}-(\tau_2+c_2)(q_{2i}-\delta q_{1i}^r)-\beta\delta\tau_2\left(q_{1i}^s+q_{1i}^r\right))]$$

subject to (8). Assuming interior solutions, the first order conditions of this problem are (the first of these conditions is relevant when $Q_1^s\neq 0$ and the second is relevant when $Q_1^r \neq 0$; in the first condition it will be $Q_1^r = 0$ if firms only sell their output and in the second condition it will be $Q_1^s = 0$ if firms only rent their output):[8]

$$\begin{array}{c} a - b(Q_1^s + Q_1^r) + \rho\delta(a - b(\delta Q_1^s + Q_2) - (1 - \delta + \alpha\delta)\tau_1 - c_1) \\ -(b + \rho\delta^2 b + \rho\delta b \frac{dQ_2}{dq_{1i}^s})q_{1i}^s - bq_{1i}^r \\ -\rho b \frac{d(Q_2 - q_{2i})}{dq_{1i}^s} q_{2i} - \rho(b\delta q_{2i} + \beta\delta\tau_2) = 0 \\ \\ a - b(Q_1^s + Q_1^r) - (1 - \delta + \alpha\delta)\tau_1 - c_1 - b\left(q_{1i}^s + q_{1i}^r\right) \\ +\rho(\delta(\tau_2 + c_2) - \beta\delta\tau_2) = 0 \end{array} \quad (9)$$

with Q_2 given from (8).

Observe from (8) and (9) that there is a symmetric market solution for firms decisions. Moreover, note that conditions (8) and (9) hold also if emission taxes on sales are paid by buyers of the durable good. In this case buyers adjust their willingness to pay for the good to the emission taxes they will pay and the only change in the previous analysis is that the term $-\beta\delta\tau_2 q_{1i}^s$ will not be included in the maximization problem of $t = 2$. However, this change would not affect conditions (7) and (8) and the analysis and results in this work would remain unchanged.

From (8) and (9) we also have that the effects of the imposition of an emission tax in a period are the same as the effects of an increase, in the same amount, in the marginal production cost in that period when emissions occur in the production process, but not under the other two types of emissions.

Under *laissez faire*, firms do not take into account environmental damage in their decisions. However, imperfect competition implies a distortion from efficient provision. Moreover, when the good is durable and firms sell in the present at least part of their production, firms experience the commitment problems implied by the Coase conjecture and there is also a possible intertemporal distortion due to the strategic behavior of each firm to steal sales from its rivals in the present and in the future.

We consider that the regulator uses only one instrument (taxes on emissions from sales and rentals of the durable good) to correct for all those distortions from efficiency. In this context, the emission taxes that maximize total surplus are, therefore, second-best optimal emission taxes.

In the Appendix (subsection 10.1) we obtain from (8) and (9) the quantities sold and rented in each period, as a function of the emission taxes in both periods and of the rest of parameters, for renting firms, selling firms and renting-selling firms (equations A1 to A5). We show that often a quantity in one of the

[8] Throughout the paper we assume that parameters are such that interior solutions exist.

periods depends on the emission taxes in both periods, as a consequence of the durability of the good and of the distribution in both periods of emissions from units produced in the first period for some types of emissions. Moreover, we obtain that, without emission taxes, market production in period 1 may be greater than optimal production in that period.

In the following section we obtain the second-best optimal emission taxes for those situations where, in the first period, firms only rent their product, firms only sell their product and firms may both rent and sell their production. Notice that the amounts of the good used in each of these situations, when the corresponding second-best optimal emission taxes are imposed, will be Q_{1u}^* in $t = 1$ and Q_{2u}^* in $t = 2$, given by (5).

We also compare in the next section each optimal emission tax with the marginal environmental damage γ. Following the literature we consider that there is overinternalization of the environmental damage for an optimal emission tax if this tax is greater than γ.

In the case of durable goods we know, however, that a unit produced in period 1 might also be used in period 2. As the expected total tax per unit produced in the first period is $(1 - \delta + \alpha\delta)\tau_1 + \rho\beta\delta\tau_2$ and the expected total emission damage caused per unit produced in period 1 is $\gamma(1 - \delta + \alpha\delta + \rho\beta\delta)$, we define:

Definition 1. *There is overall overinternalization in the first period if:*

$$(1 - \delta + \alpha\delta)\tau_1 + \rho\beta\delta\tau_2 > \gamma(1 - \delta + \alpha\delta + \rho\beta\delta).$$

From this definition we have that there will not be overall overinternalization in the first period if there is not overinternalization in any of the two periods.

5 Optimal emission taxes and overinternalization

5.1 Renting firms

From (5) and (A1) we get that the optimal (second-best) emission taxes for renting firms may be written as:

$$\begin{aligned} \tau_1^{r*} &= \gamma - \tfrac{b}{n(1-\delta+\alpha\delta)}(Q_{1u}^* + \rho\delta(1-\beta)Q_{2u}^*) \\ \tau_2^{r*} &= \gamma - \tfrac{b}{n}Q_{2u}^* \end{aligned} \qquad (10)$$

From (10) we have:

Proposition 1. *If firms rent their output, then optimal emission taxes in the first and second periods do not imply overinternalization for the three types of emissions considered.*

If environmental damage were the only distortion in the market, the optimal emission tax in each period would be equal to γ. The optimal emission taxes would correct for overproduction in the market when environmental damage is not taken into account. Nevertheless, as imperfect competition, without taxes, induces underproduction, this additional distortion implies in our model an optimal emission tax in each period below γ. This result was already obtained in Runkel (2002) for the case of emissions that occur during the production process.

From (10) we also have that τ_1^{r*} and τ_2^{r*} are negative if γ is small and that they increase with γ, as we know from section 3 that Q_{1u}^* and Q_{2u}^* diminish with γ. Under perfect competition the optimal emission tax in each period is equal to γ (from (10) we have that $\tau_1^{r*} = \gamma$ and $\tau_2^{r*} = \gamma$ when $n \to \infty$)

5.2 Selling firms

From (5), (A2) and (A3) we get that the optimal (second-best) emission taxes for the durable good selling firms may be written as:

$$\begin{aligned} \tau_1^{s*} &= \gamma - \tfrac{b\left(\left(n+1+\rho\delta^2(\beta(n+1)-1)\right)Q_{1u}^* - \rho\delta(\beta(n+1)-2)Q_{2u}^*\right)}{n(n+1)(1-\delta(1-\alpha))} \\ \tau_2^{s*} &= \gamma - \tfrac{b}{n}(Q_{2u}^* - \delta Q_{1u}^*) \end{aligned} \tag{11}$$

Optimal emission taxes in periods 1 and 2 amend, simultaneously, for the distortion in production due to the oligopolistic market structure, for the distortion in production due to the strategic behavior of each firm to steal sales to its rivals in the present and in the future, and for environmental damage, taking into account the durability of the good, the type of emission and the intertemporal inconsistency problem first noted by Coase (1972). As in the case of renting firms we have that under perfect competition the optimal emission tax in each period is equal to γ (from (11) we have that $\tau_1^{s*} = \gamma$ and $\tau_2^{s*} = \gamma$ when $n \to \infty$).

From (11) we have:

Proposition 2. *If firms sell their output then:*

i) the optimal emission tax in the first period may imply overinternalization when $c_1 > c_2$ and emissions are proportional to the stock of the product or they occur at the end of the product's lifetime, but not when emissions occur in the production process;

ii) the optimal emission tax in the second period never implies overinternalization; and

iii) If firms sell their output, then there is not overall overinternalization in the first period for the three types of emissions considered.

Proof: See Appendix. ■

Proposition 2 indicates that the type of emission matters for overinternalization. We find that the optimal emission tax in the first period may be greater than γ only if the units produced in that period which are still in use in the second period cause environmental damage (and pay emission taxes) in this latter period.[9] As $c_1 > c_2$ is a necessary condition for overinternalization, some technological progress or learning by doing along time is also required.[10]

When there is overinternalization in the first period, however, we obtain in Proposition 2 that there is not overall overinternalization. The expected emission tax paid in the second period compensates for the excess over environmental damage of the expected emission tax paid in the first period. This possibility of compensation cannot occur when emissions occur during the production process and, in this case, there is not overinternalization in the first period.

The results on overinternalization cannot be explained considering only the incentives of firms in the first period. The incentives of firms in both periods and the interaction between emission taxes in the present and in the future, and their effects on firms decisions, jointly explain the results.

From (11) we have that τ_2^{s*} increases with γ, as $Q_{2u}^* - \delta Q_{1u}^*$ diminishes with γ, and it may be shown that, when $\alpha = 1$, τ_1^{s*} increases with γ. However, τ_1^{s*} may decrease with γ for some values of the parameters if $\alpha = 0$. For instance, when $\alpha = 0$, $\beta = 1$, $\delta = 0.95$, $\rho = 1$ and $n = 3$, we find that τ_1^{s*} decreases with γ. In this case, environmental damage occurs at the end of the product's life and, when γ increases, optimal emission taxes induce an increase in production in period 1 and a decrease in production in period 2 (this will imply a more equilibrated distribution of environmental damages between the two periods). Nevertheless, the expected total optimal emission tax per unit produced in the first period ($(1 - \delta + \alpha\delta)\tau_1^{s*} + \rho\beta\delta\tau_2^{s*}$) increases with γ, as we show in the following Proposition:

Proposition 3. *If firms sell their output, then the expected total optimal*

[9] This possibility of overinternalization in the first period in a durable goods industry with selling firms was already proved in Runkel (2004).

[10] From our analysis we cannot discard the possibility of overinternalization when $c_1 < c_2$ if the demand function for the services of the durable good is non linear.

emission tax per unit produced in the first period increases with γ for the three types of emissions considered.

Proof: See Appendix. ■

5.3 Renting-selling firms

As Carlton and Gertner (1989) noted, when firms can both rent and sell their production, but they do not coordinate to rent them, then each firm, in equilibrium, will behave strategically and may sell part of its production, although their profits would be greater if all of them only rented the good. The reason for this behavior is that, when a firm sells a durable good in $t = 1$, it is depriving its rivals of current and future sales. Hence, with renting-selling firms, we have that renting and selling, without taxes, is distorted away from the optimal for four reasons: the existence of imperfect competition in the production of the durable good, the commitment problem implied by the Coase conjecture, the strategic behavior of firms when choosing between renting and selling, and the no consideration of environmental damages by oligopolists.

From (5), (A4) and (A5), we find that the optimal emission taxes for renting-selling firms are:

$$\begin{aligned} \tau_1^{rs*} &= \gamma - b\tfrac{nQ_{1u}^* + \rho\delta(1-\beta)Q_{2u}^*}{n^2(1-\delta(1-\alpha))} \\ \tau_2^{rs*} &= \gamma - \tfrac{bQ_{2u}^*}{n^2} \end{aligned} \tag{12}$$

As in the case of selling firms, optimal emission taxes in periods 1 and 2 correct, simultaneously, for the distortion in production due to the oligopolistic market structure, for the distortion due to the strategic behavior of each firm selling part of its production to steal sales from its rivals, and for environmental damage, taking into account the durability of the good, the type of emission and the intertemporal inconsistency problem. Under perfect competition we have from (12) that, with renting-selling firms, the optimal emission tax in each period is equal to γ.

From (12) we have:

Proposition 4. *If firms rent and sell their output in the first period, then the optimal emission taxes in the first and second periods do not imply overinternalization for the three types of emissions considered.*

From (12) we also have that τ_2^{rs*} and τ_1^{rs*} increase with γ for the three types of emissions considered, as we know from section 3 that Q_{1u}^* and Q_{2u}^* diminish with γ. As a consequence, the expected total optimal emission tax per unit sold or rented in the first period $((1-\delta+\alpha\delta)\tau_1^{rs*}+\rho\delta\tau_2^{rs*})$ increases with γ.

Substituting (12) in (A4) we get:

$$q_{1i}^s(\tau_1^{rs*},\tau_2^{rs*})=\frac{(a-\gamma-c_2)(n-1)}{n^2\delta b}>0 \text{ and}$$
$$q_{1i}^r(\tau_1^{rs*},\tau_2^{rs*})=\frac{(n-1)(\gamma-a+c_2)+\delta n(a-\gamma-c_1+\delta\rho c_2+\delta\gamma(1-\alpha+\rho(1-\beta)))}{n^2\delta b}$$

The condition:

$$(n-1)(\gamma-a+c_2)+\delta n\left(a-\gamma-c_1+\delta\rho c_2+\delta\gamma\left(1-\alpha+\rho\left(1-\beta\right)\right)\right)>0 \quad (13)$$

is, therefore, required to obtain $q_{1i}^r(\tau_1^{rs*},\tau_2^{rs*})>0$. We can summarize this condition, together with conditions (6), as:

$$\delta\frac{n}{n-1}L>a-c_2-\gamma>\delta L>0.$$

where $L=a-c_1+\delta\rho c_2-\gamma(1-\delta(1-\alpha+\rho(1-\beta))$.

We should note that when condition (13) is fulfilled there is not overinternalization with selling firms for any of the three contexts of environmental damage considered: Condition (13) may be written as:

$$b(\delta nQ_{1u}^*-(n-1)Q_{2u}^*)>0$$

and this latter condition implies from (11) that, when $\beta=1$:

$$\begin{aligned}\tau_1^{s*}-\gamma &= -\frac{b(n+1+\rho\delta^2 nQ_{1u}^*-\rho\delta\left(n-1\right)Q_{2u}^*)}{n(n+1)\left(1-\delta(1-\alpha)\right)}\\ &= -\frac{b((n+1)Q_{1u}^*+\rho\delta(\delta nQ_{1u}^*-\left(n-1\right)Q_{2u}^*))}{n(n+1)\left(1-\delta(1-\alpha)\right)}<0\end{aligned}$$

If condition (13) is not fulfilled then the social optimum cannot be attained with emission taxes τ_1^{rs*} and τ_2^{rs*}. However, the regulator may induce the social optimum with emission taxes τ_1^{s*} and τ_2^{s*}, as it may be shown that $q_{1i}^r(\tau_1^{rs*},\tau_2^{rs*})<0$ implies $q_{1i}^r(\tau_1^{s*},\tau_2^{s*})<0$ and, therefore, firms will only sell the durable good and the solution with selling firms will result with emission taxes τ_1^{s*} and τ_2^{s*}. Hence, if $\beta=1$ there may also be overinternalization in the first period when we obtain that corner solution, with only sales, for the renting-selling competition game.

6 Comparison of optimal emission taxes

In this section we compare the optimal emission taxes with selling firms, with renting firms and with renting-selling firms, when parameter values guarantee, simultaneously, interior solutions for the three market configurations.

Boyce and Goering (1997), considering a durable goods monopolist and emissions during the production process, find that the optimal emission tax on a selling monopolist in any period is higher than the optimal emission tax on a renting monopolist in the same period. We show below that their result holds also for the case where the market structure is a Cournot oligopoly. However, we prove that the optimal emission tax on selling firms in $t = 1$ is always lower than the optimal emission tax on renting firms in that period, if emissions are proportional to the stock of the product in the market or if emissions occur at the end of the life of the product, when parameter values guarantee, simultaneously, interior solutions for the three market configurations. Nevertheless, when $\beta = 1$, a unit produced in the first period may result in emissions in $t = 2$ and, as a consequence, pay emission taxes in that period. Hence, when $\beta = 1$, the total expected optimal emission tax is the relevant emission tax on units produced in the first period to consider. In the next Proposition we also obtain that this expected optimal emission tax is greater for selling firms than for renting firms.

When condition (13) is fulfilled we can prove:

Proposition 5.

i) When emissions occur in the production process we have: $\tau_1^{s*} > \tau_1^{rs*} > \tau_1^{r*}$*;*

ii) when emissions are proportional to the stock of the product or when they occur at the end of the product's life we have: $\tau_1^{rs*} = \tau_1^{r*} > \tau_1^{s*}$*;*

iii) $\tau_2^{s*} > \tau_2^{rs*} > \tau_2^{r*}$ *for the three types of emissions considered; and*

iv) $(1-\delta+\alpha\delta)\tau_1^{s*} + \rho\beta\delta\tau_2^{s*} > (1-\delta+\alpha\delta)\tau_1^{rs*} + \rho\beta\delta\tau_2^{rs*} > (1-\delta+\alpha\delta)\tau_1^{r*} + \rho\beta\delta\tau_2^{r*}$ *for the three types of emissions considered.*

Proof: See Appendix. ■

There is not a direct relationship between the optimal emission taxes in the first period, with renting, selling, and renting-selling firms, and the amounts of the good that would be produced without taxes in that period. From (A1) and (A5) we have that, without taxes, the amount produced in the first period with renting-selling firms is equal to the amount produced in that period with renting firms. Moreover, from (A1) and (A2) we have that, without taxes, the difference between the amount produced in the first period with renting firms and the amount produced in that period with selling firms is positive, as that difference is equal to the amount produced in the first period with renting-selling firms, without taxes, multiplied by $\frac{(n^2+1)(n+1)\delta b}{(3n+3n^2+n^3+\delta^2\rho+n^2\delta^2\rho+1)(n+1)b}$ (this is the way that selling firms have to convince buyers in the first period that they will not flood the market in the second period). To attain the social optimum we have to consider the two periods simultaneously and note that the emission tax in the second period interacts with the emission tax in the first period.

Taking into account the condition for a positive level of renting with renting-selling firms, it is not difficult to show that, without emission taxes, the expected total environmental damage with selling firms is greater than expected total environmental damage with renting-selling firms and that this latter damage is greater than expected total environmental damage with renting firms. From the point of view of environmental damage the situation without taxes is worst with selling firms than with renting-selling firms and it is also worst in this latter case than with renting firms.

7 Extensions

Three extensions of our analysis are considered in this section: i) general inverse rental demand function for the services of the durable good and general environmental damage function, ii) environmental damage per unit of emission that differs from one period to another and iii) consideration that the types of emissions analyzed in this paper may occur simultaneously.

Consider a general inverse rental demand function for the services of the durable good, $p(Q)$, and a general environmental damage function where environmental damage in a period depends on total emissions in that period ($\gamma(E_i)$, where E_i are total emissions in period i), in a context where emissions occur at the end of the product's lifetime. We show in the Appendix (subsection 10.5) that in this context, that corresponds to the case with fixed durability in Runkel (2004), there is not overall overinternalization in the first period with selling firms (an extension of our result in Proposition 2-i) to that context).

Environmental damage in period 2 per unit of emission in that period could be different from the environmental damage in period 1 per unit of emission in the first period. In particular, consider that the environmental damage in period 1 per unit of emission in that period is γ and the environmental damage in period 2 per unit of emission in that period is $\kappa\gamma$, where κ may be different from 1. For instance, production technology may incorporate over time innovations that imply a reduction in environmental damage per unit of emission in $t = 2$ (in this case $\kappa < 1$) or cumulative pollution from production of all goods and services in the economy reduces the capacity of the environment to assimilate pollution in $t = 2$ (in this case, considering that the effect on cumulative pollution of production of the durable good that we are considering is negligible, $\kappa > 1$).[11] We show in the Appendix (subsection 10.6) that, when environmental damage per unit of emissions changes with time, all results on overinternalization and overall overinternalization presented for the case $\kappa = 1$ remain valid. We also show that, when emissions are proportional to the stock of the product or when they occur at the end of the product's life, and firms sell their output, the set of values of the parameters where there is overinternalization in the first period decreases with κ. Hence, compared to the case where $\kappa = 1$, overinternalization is more likely under those types of emissions if $\kappa < 1$ and it is less likely if $\kappa > 1$. Finally, we note that when emissions occur during the production process, Q^*_{1u} may increase with γ if κ is greater than 1 and big enough.

The types of emissions analyzed in this paper may occur simultaneously. If the three types of emissions occur simultaneously and the marginal environmental damage per unit of emission in any of these emission types is γ, then the environmental damage per unit produced in the second period would be 3γ and the environmental damage per unit produced in the first period would be $(3-\delta)\gamma$ in the first period and $2\delta\gamma$ in the second period. If, instead, the three types of emissions occur simultaneously and environmental damage per unit of emission during the production process is ν, environmental damage per unit of emission due to the use of the product is ζ and environmental damage per unit of emission due to the disposal of the product is η, the environmental damage per unit produced in the second period would be $\nu+\zeta+\eta$ and the environmental damage per unit produced in the first period would be $\nu + \zeta + (1 - \delta)\eta$ in the first period and $\delta(\zeta + \eta)$ in the second period. This possibility of simultaneous emission types may be incorporated in our analysis, as solving:

$$\begin{gathered}\nu + \zeta + \eta = \gamma \\ \nu + \zeta + (1 - \delta)\eta = (1 - \delta)\gamma + \alpha\delta\gamma \\ \delta(\zeta + \eta) = \beta\delta\gamma\end{gathered}$$

for α, β and γ, we obtain: $\alpha = \frac{\nu+\zeta}{\nu+\zeta+\eta}$, $\beta = \frac{\zeta+\eta}{\nu+\zeta+\eta}$ and $\gamma = \nu + \zeta + \eta$, with $0 < \alpha, \beta < 1$. The cases where only two of the three emission types occur may be analyzed in a similar way.

8 Conclusion

We have analyzed optimal emission taxes in durable goods oligopolies. We have considered emissions that occur during the production process, emissions

[11] The case where production of the durable good results in non-negligible cumulative pollution is briefly considered in the next section.

proportional to the stock of the durable good and emissions that occur at the end of the product's lifetime. Moreover, we have studied the cases where firms only sell their product, firms only rent their product and firms may both sell and rent their production. We have compared optimal emission taxes with marginal environmental damage in each case. We have shown that, although the optimal emission tax in the first period may be, in some of those contexts, greater than the marginal environmental damage (overinternalization), the expected total tax paid per unit produced in the first period is, in all situations analyzed, smaller than the expected marginal environmental damage caused by a unit produced in that period. We think that this latter comparison is the correct one to make, and our results allow us to conclude that there will be overall underinternalization, putting in perspective some the results on overinternalization in the durable goods literature.

We have also compared the optimal emission taxes on renting firms, on selling firms and on renting-selling firms. We have obtained that, when emissions are proportional to the stock of the durable good or when emissions occur at the end of the product's lifetime, the optimal emission tax in the first period on renting firms is higher than the optimal emission tax on selling firms. Nevertheless, we have shown that the expected total emission tax in the first period is higher for selling firms than for renting firms, under any type of emissions.

Moreover, we have studied the variation in optimal emission taxes with marginal environmental damage. We have obtained that the optimal emission tax in the first period with selling firms may decrease with marginal environmental damage, but the expected total optimal emission tax per unit produced in the first period always increases with marginal environmental damage. In the rest of situations, all optimal emission taxes change in the same direction as the marginal environmental damage.

We show that, often, the quantity produced (sold or rented) in any period is affected by emission taxes in both periods, as a consequence of the durability of the good and of the distribution in both periods of emissions from units produced in the first period for some types of emissions. Hence, the optimal quantity in each period is attained through the selection of the adequate combination of emission taxes in the present and in the future.

It is important to determine the type of emission relevant in each situation to establish the corresponding optimal emission taxes. Moreover, as optimal emission taxes are second-best, we have pointed out that for some parameter values the optimal emission taxes may be negative.

Finally, we have discussed several extensions of our analysis. The model developed in this work could also be used to study the case where production of the durable good results in non-negligible cumulative pollution. Let us consider that environmental damage in the second period is equal to emissions from the production, use or termination of the stock of the good in that period plus pollution in the first period multiplied by υ, with $0 < \upsilon$ (durable pollution). This case of cumulative pollution requires only one modification in our model: the expected emissions per unit produced in the first period would be $(\beta\delta + \upsilon)\gamma$ in the second period. In this context, the case where emissions are proportional to the stock of the durable good in use, that we have analyzed in the previous sections, is analogous to a situation where emissions occur during the production process, there is cumulative production and $\upsilon = \delta$. The case where $\upsilon \neq \delta$, however, requires further research.

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10 Appendix

10.1 Emission taxes and quantities produced, rented and sold

When firms only rent the durable good let us denote the emission tax on units rented in the first period by τ_1^r and the emission tax on units rented in the second period by τ_2^r. From (8) and (9) we get:

$$q_{1i}^r = \frac{a-c_1+\delta\rho c_2-(1-\delta(1-\alpha))\tau_1^r+\delta\rho(1-\beta)\tau_2^r}{b(n+1)}$$

$$q_{2i} = \frac{a-c_2-\tau_2^r}{b(n+1)} \tag{A1}$$

$$q_{2i}-\delta q_{1i}^r = \frac{\left(a(1-\delta)+\delta c_1-c_2\left(1+\delta^2\rho\right)+\delta\tau_1^r(1-\delta(1-\alpha))-\tau_2^r\left(1+\rho\delta^2(1-\beta)\right)\right)}{b(n+1)}$$

Note that, without emission taxes, market production in period 1 may be greater than optimal production in that period. For instance, when $a=5$, $n=4$, $\gamma=1$, $\delta=0.98$, $\rho=0.95$, $c_1=2$, $c_2=1$, $b=1$, $\alpha=1$ and $\beta=1$, we have positive production levels and $Q^*_{1u} - nq^r_{1i} = -0.21$. However, when $\gamma = 0$ we have that, without taxes, $Q^*_{1u} - nq^r_{1i} > 0$.

When firms only sell the durable good let us denote by τ^s_1 and τ^s_2, respectively, the emission taxes on sales in $t = 1$ and in $t = 2$. From (8) and (9) we obtain:

$$\begin{array}{c} q^s_{1i} = \frac{1}{b\left((n+1)^3+\rho\delta^2(n^2+1)\right)}(a\left((n+1)^2 + \rho\delta\,(n-1)\right) \\ -c_1\,(n+1)^2 + c_2\delta\rho(n^2+n+2) \\ -\tau^s_1\,(1-\delta+\alpha\delta)\,(n+1)^2 + \tau^s_2\rho\delta\left(n^2+n+2-\beta(n+1)^2\right)), \end{array} \tag{A2}$$

$$\begin{array}{c} q_{2i} = \frac{1}{b\left((n+1)^3+\rho\delta^2(n^2+1)\right)}(a((n+1)^2 - \delta(n^2+n-\delta\rho)) \\ +n\delta c_1(n+1) - c_2((n+1)^2 + \delta^2\rho(1+n+n^2)) \\ +\tau^s_1 n\delta\,(1-\delta+\alpha\delta)\,(n+1) \\ -\tau^s_2((n+1)^2 + \delta^2\rho(1+n+n^2-\beta n(n+1)))) \end{array} \tag{A3}$$

and

$$\begin{array}{c} \delta q^s_{1i} + q_{2i} = \frac{1}{b\left((n+1)^3+\rho\delta^2(n^2+1)\right)}(a((n+1)^2 + \delta(1+n(1+\delta\rho))) \\ -\delta c_1(n+1) - c_2((n+1)^2 - \delta^2\rho) \\ -\tau^s_1\delta\,(n+1)\,(1-\delta+\alpha\delta) - \tau^s_2((1+n)^2 - \rho\delta^2(1-\beta(n+1)))) \end{array}$$

Without emission taxes, market production in period 1 may be greater than optimal production in that period. For instance, let $\rho = 0.5$, $\delta = 1$, $a = 10$, $n = 2$, $c_1 = 3$, $c_2 = 0.2$, $b = 1$, $\gamma = 1$, $\alpha = 1$ and $\beta = 0$. In this case production levels are positive and $Q^*_{1u} - Q^s_1(\tau_{1s} = \tau_{2s} = 0) = -1.36$. The difference between optimal production in the first period and market production in that period decreases with c_1 and increases with c_2. If $\gamma = 0$, however, we have that without taxes it is $Q^*_{1u} > Q^s_1$, taking into account the restrictions on the parameters that assure interior solutions for the case of renting-selling firms.

In the case of renting-selling firms each firm chooses simultaneously in every period its level of production and the division of production between renting and selling in the first period. Let us denote in that case by τ^{rs}_1 and τ^{rs}_2, respectively, the emission taxes in $t = 1$ and $t = 2$. From (8) and (9) we obtain:

$$\begin{array}{c} q^r_{1i} = \frac{1}{\delta b(n+1)(n^2+1)}(a\left(\delta - n^2 + n^2\delta + 1\right) \\ +c_2\left(n^2 + \delta^2\rho + n^2\delta^2\rho - 1\right) - \delta c_1\left(n^2+1\right) \\ -\tau^{rs}_1\delta\left(n^2+1\right)(\alpha\delta - \delta + 1) \\ -\tau^{rs}_2\left(\rho\delta^2\left(n^2+1\right)(\beta-1) + \left(1-n^2\right)\right)) \\ \\ q^s_{1i} = \frac{(n-1)(a-c_2-\tau^{rs}_2)}{\delta b(n^2+1)} \\ \\ q_{2i} = \frac{a-c_2-\tau^{rs}_2}{(n^2+1)b} \end{array} \tag{A4}$$

and

$$
\begin{gathered}
q_{1i}^s + q_{1i}^r = \tfrac{a - c_1 + \delta\rho c_2 - \tau_1^{rs}(\alpha\delta - \delta + 1) + \tau_2^{rs}\rho\delta(1-\beta)}{(n+1)b} \\
\delta q_{1i}^s + q_{2i} = \tfrac{n(a - c_2 - \tau_2^{rs})}{b(n^2+1)} \\
q_{2i} - \delta q_{1i}^r = \tfrac{1}{b(n+1)(n^2+1)}\left(a\left(n - \delta + n^2 - n^2\delta\right)\right. \\
-c_2\left(n + n^2 + \delta^2\rho + n^2\delta^2\rho\right) + c_1\delta\left(n^2+1\right)) \\
+\tau_1^{rs}\delta\left(n^2+1\right)(\alpha\delta - \delta + 1) \\
\left.+\tau_2^{rs}\left(\rho\delta^2\left(n^2+1\right)(\beta-1) + \left(1-n^2\right) - (n+1)\right)\right)
\end{gathered} \tag{A5}
$$

10.2 Proof of Proposition 2

i) and ii) If emissions occur in the production process ($\alpha = 1$ and $\beta = 0$) there is not overinternalization in the first period as:

$$\tau_1^{s*} - \gamma = -\frac{b}{n(n+1)}\left(\left(n+1-\rho\delta^2\right)Q_{1u}^* + 2\rho\delta Q_{2u}^*\right) < 0$$

There may be overinternalization in the first period if emissions are proportional to the stock of product in the market or if emissions occur at the end of the product's life. In the following numerical examples there is overinternalization in period 1: i) if $\rho = 0.5$, $\delta = 1$, $a = 3.5$, $n = 3$, $c_1 = 2.4$, $c_2 = 0.1$, $b = 1$, $\gamma = 1$, $\alpha = 1$ and $\beta = 1$, production levels are positive and $\tau_1^{s*} = 1.1313 > \gamma$, ii) if $\rho = 1$, $\delta = 0.5$, $a = n = 3$, $c_1 = 2.4$, $c_2 = 0.1$, $b = 1$, $\gamma = 1$, $\alpha = 0$ and $\beta = 1$, production levels are positive and $\tau_1^{s*} = 1.19792 > \gamma$.

Note that overinternalization requires $c_1 > c_2$. If $\beta = 1$ we have:

$$\tau_1^{s*} - \gamma = -\frac{b}{n(n+1)\left(1-\delta(1-\alpha)\right)}\left(\left(n+1+n\rho\delta^2\right)Q_{1u}^* - \rho\delta(n-1)Q_{2u}^*\right)$$

As $Q_{1u}^*(\alpha = 1, \beta = 1) < Q_{1u}^*(\alpha = 0, \beta = 1)$ and Q_{2u}^* does not depend on α, we have that $\tau_1^{s*}(\alpha = 1, \beta = 1) - \gamma > \tau_1^{s*}(\alpha = 0, \beta = 1) - \gamma$. Moreover, if $c_1 = c_2 = c$ it is:

$$
\begin{aligned}
\tau_1^{s*}(\alpha = 1, \beta = 1) - \gamma &= -\tfrac{b}{n(n+1)}\left(\left(n+1+n\rho\delta^2\right)Q_{1u}^* - \rho\delta(n-1)Q_{2u}^*\right) \\
&= -\tfrac{(a-c-\gamma)(1+n+\delta^2\rho n - \delta\rho(n-1)) + c(\delta\rho(n+1) + n\delta^3\rho^2)}{n(n+1)} < 0.
\end{aligned}
$$

Finally, we have $\frac{\partial(\tau_1^{s*}-\gamma)}{\partial c_1} > 0$ and $\frac{\partial(\tau_1^{s*}-\gamma)}{\partial c_2} < 0$. Hence, if we decrease c_1 or increase c_2, starting from any situation where $c_1 = c_2 = c$, that is, if $c_1 < c_2$, there will not be overinternalization.

In period 2 there is not overinternalization as:

$$\tau_2^{s*} - \gamma = -\frac{b}{n}(Q_{2u}^* - \delta Q_{1u}^*) < 0.$$

iii) We have:

$$\begin{aligned}&(1-\delta+\alpha\delta)\tau_1^{s*}+\rho\beta\delta\tau_2^{s*}-\gamma(1-\delta+\alpha\delta+\rho\beta\delta)\\ =\ &\frac{b}{n(n+1)}(-(n+1-\rho\delta^2)Q_{1u}^*-2\rho\delta\left(Q_{2u}^*\right))<0.\ \blacksquare\end{aligned}$$

10.3 Proof of Proposition 3

We have:

$\frac{d((1-\delta+\alpha\delta)\tau_1^{s*}+\rho\beta\delta\tau_2^{s*})}{d\gamma}=\frac{\left((n+1)^2(1-\delta(1-\alpha)+\delta\rho\beta)+\delta\rho(1-\delta-n+\delta^2(1-\alpha+\rho(1-\beta)))\right)}{(n+1)n}$.

If $\alpha=0$ and $\beta=1$, then:

$\frac{d((1-\delta+\alpha\delta)\tau_1^{s*}+\rho\beta\delta\tau_2^{s*})}{d\gamma}=\frac{\left(2n(1-\delta)+(1-\delta)+2\delta\rho+n\delta\rho+n^2(1-\delta)+\delta^3\rho+\left(n^2-\delta\right)\delta\rho\right)}{(n+1)n}>0.$

If $\alpha=1$ and $\beta=1$, then:

$\frac{d((1-\delta+\alpha\delta)\tau_1^{s*}+\rho\beta\delta\tau_2^{s*})}{d\gamma}=\frac{\left(2n+2\delta\rho+n\delta\rho+n^2-\delta^2\rho+n^2\delta\rho+1\right)}{(n+1)n}>0.$

If $\alpha=1$ and $\beta=0$, then:

$\frac{d((1-\delta+\alpha\delta)\tau_1^{s*}+\rho\beta\delta\tau_2^{s*})}{d\gamma}=\frac{\left(2n+\delta\rho-n\delta\rho+n^2-\delta^2\rho+\delta^3\rho^2+1\right)}{(n+1)n}>0.\ \blacksquare$

10.4 Proof of Proposition 5

We have:

$\tau_1^{rs*}-\tau_1^{s*}=\frac{((n-1)(\gamma-a+c_2)+\delta n(a-\gamma-c_1+\gamma\delta-\alpha\gamma\delta+\gamma\delta\rho+\delta\rho c_2-\beta\gamma\delta\rho))(\beta+n\beta-1)\delta\rho}{(\alpha\delta-\delta+1)(n+1)n^2}$,

$\tau_1^{rs*}-\tau_1^{r*}=\frac{(\gamma-a+c_2)(\beta-1)(n-1)\delta\rho}{(\alpha\delta-\delta+1)n^2}$ and

$q_{1i}^r\left(\tau_1^{rs*},\tau_2^{rs*}\right)=\frac{(n-1)(\gamma-a+c_2)+\delta n(a-\gamma-c_1+\gamma\delta-\alpha\gamma\delta+\gamma\delta\rho+\delta\rho c_2-\beta\gamma\delta\rho)}{n^2\delta b}$.

When $\alpha=1$ and $\beta=0$:

$\tau_1^{rs*}-\tau_1^{s*}=\frac{\delta\rho((n-1)(a-\gamma-c_2)-\delta n(a-\gamma-c_1+\gamma\delta\rho+\delta\rho c_2))}{(n+1)n^2}=\frac{-b\delta^2\rho q_{1i}^r(\tau_1^{rs},\tau_2^{rs*})}{(n+1)}<0$

and

$\tau_1^{rs*}-\tau_1^{r*}=\frac{(a-\gamma-c_2)(n-1)\rho\delta}{n^2}>0$ as $Q_{2u}^*>0$.

Hence, it is $\tau_1^{s*}>\tau_1^{rs*}>\tau_1^{r*}$ when $\alpha=1$ and $\beta=0$.

When $\beta=1$:

$\tau_1^{rs*}-\tau_1^{s*}=\frac{((n-1)(\gamma-a+c_2)+\delta n(a-\gamma-c_1+\gamma\delta-\alpha\gamma\delta+\delta\rho c_2))\rho\delta}{(\alpha\delta-\delta+1)(n+1)n}=\frac{b\delta^2\rho q_{1i}^r(\tau_1^{rs},\tau_2^{rs*})}{(n+1)(\alpha\delta-\delta+1)}>0$

and

$\tau_1^{rs*}-\tau_1^{r*}=\frac{(\gamma-a+c_2)(\beta-1)(n-1)\delta\rho}{(\alpha\delta-\delta+1)n^2}=0.$

Hence, it is $\tau_1^{rs*}=\tau_1^{r*}>\tau_1^{s*}$ when $\beta=1$.

We also have:

$\tau_2^{s*}-\tau_2^{rs*}=\frac{(n-1)(\gamma-a+c_2)+\delta n(a-\gamma-c_1+\gamma\delta-\alpha\gamma\delta+\gamma\delta\rho+\delta\rho c_2-\beta\gamma\delta\rho)}{n^2}=$

$=b\delta q_{1i}^r\left(\tau_1^{rs},\tau_2^{rs*}\right)>0$ and

$\tau_2^{rs*}-\tau_2^{r*}=\frac{(a-\gamma-c_2)(n-1)}{n^2}>0$ as $Q_{2u}^*>0$.

Hence, it is $\tau_2^{s*}>\tau_2^{rs*}>\tau_2^{r*}$.

Finally, if we compare the total expected optimal emission tax per unit produced in the first period when $\beta=1$ we have that:

$$(1-\delta+\alpha\delta)\tau_1^{rs*}+\rho\delta\tau_2^{rs*}-((1-\delta+\alpha\delta)\tau_1^{r*}+\rho\delta\tau_2^{r*})=\rho\delta(\tau_2^{rs*}-\tau_2^{r*})>0$$

$$(1-\delta+\alpha\delta)\tau_1^{s*}+\rho\delta\tau_2^{s*}-((1-\delta+\alpha\delta)\tau_1^{rs*}+\rho\delta\tau_2^{rs*})=$$

$$=\frac{((n-1)(\gamma-a+c_2)+\delta n(a-\gamma-c_1+\gamma\delta-\alpha\gamma\delta+\gamma\delta\rho+\delta\rho c_2-\beta\gamma\delta\rho))\delta\rho}{(n+1)n^2}=\frac{b\delta^2\rho q_{1i}^r(\tau_1^{rs},\tau_2^{rs*})}{(n+1)}>$$

0. ■

10.5 Overall overinternalization with selling firms and general demand and damage functions

Consider a general inverse rental demand function for the services of the durable good, $p(Q)$, with $p'(Q)<0$, and a general environmental damage function where environmental damage in a period depends on total emissions in that period ($\gamma(E_i)$, with $\gamma'(E_i)>0$, where E_i are total emissions in period i), in a context where emissions occur at the end of the product's lifetime. We assume, as Runkel (2004), that, for any firm i:

$$p'(\delta Q_1+Q_2)+q_{2i}p''(\delta Q_1+Q_2)<0$$

From Runkel (2004) we have:

$$\tau_1^{s*}=\gamma'((1-\delta)Q_1^*)+\tfrac{z_1+z_2+z_3}{1-\delta} \text{ and}$$

$$\tau_2^{s*}=\gamma'(\delta Q_1^*+Q_2^*)+q_{2i}^*p'(\delta Q_1^*+Q_2^*),$$

where:

$$z_1=q_{1i}^*p'(Q_1^*)$$
$$z_2=\rho\delta^2q_{1i}^*p'(\delta Q_1^*+Q_2^*)\tfrac{\partial(\delta q_{1i}^*+q_{2i}^*)}{\partial(\delta q_{1i}^*)}$$
$$z_3=\rho\delta(\delta q_{1i}^*+q_{2i}^*)p'(\delta Q_1^*+Q_2^*)\tfrac{\partial(\delta Q_1^*+Q_2^*)}{\partial(\delta q_{1i}^*)}$$

We have:

$$(1-\delta)\tau_1^{s*}+\rho\delta\tau_2^{s*}=(1-\delta)\gamma'((1-\delta)Q_1^*)+\rho\delta\gamma'(\delta Q_1^*+Q_2^*)+H,$$

where:

$$H=z_1+z_2+z_3+\rho\delta q_{2i}^*p'(\delta Q_1^*+Q_2^*).$$

There will not be overall overinternalization if we prove that $H<0$. It is:

$$\begin{aligned}H &= z_1+\rho\delta^2q_{1i}^*p'(\delta Q_1^*+Q_2^*)(\frac{\partial(\delta q_{1i}^*+q_{2i}^*)}{\partial(\delta q_{1i}^*)}+\frac{\partial(\delta Q_1^*+Q_2^*)}{\partial(\delta q_{1i}^*)})\\ &\quad+\rho\delta q_{2i}^*p'(\delta Q_1^*+Q_2^*)(\frac{\partial(\delta Q_1^*+Q_2^*)}{\partial(\delta q_{1i}^*)}+1).\end{aligned}$$

Using the expressions in Runkel (2004) for the derivatives with respect to δq_{1i}^* we have:

$$\frac{\partial(\delta q_{1i}^*+q_{2i}^*)}{\partial(\delta q_{1i}^*)}+\frac{\partial(\delta Q_1^*+Q_2^*)}{\partial(\delta q_{1i}^*)}=\frac{p'(\delta Q_1^*+Q_2^*)}{(n+1)p'(\delta Q_1^*+Q_2^*)+Q_2^*p''(\delta Q_1^*+Q_2^*)}>0 \text{ and}$$

$$\frac{\partial(\delta Q_1^* + Q_2^*)}{\partial(\delta q_{1i}^*)} + 1 = -\frac{(n-1)p'(\delta Q_1^* + Q_2^*) + (Q_2^* - q_{2i}^*)p^{''}(\delta Q_1^* + Q_2^*)}{(n+1)p'(\delta Q_1^* + Q_2^*) + Q_2^* p^{''}(\delta Q_1^* + Q_2^*)} + 1 > 0.$$

As $z_1 < 0$ we conclude that $H < 0$. Hence, there is not overall overinternalization.

10.6 Overinternalization when environmental damage per unit of emission changes with time

To obtain the social optimum in this context we proceed as in section 3, noting that the environmental damage function to include in TS is now:

$$\gamma\left((1 - \delta + \alpha\delta + \kappa\rho\beta\delta)Q_{1u} + \rho\kappa(Q_{2u} - \delta Q_{1u})\right).$$

We obtain:

$$Q_{1u}^* = \tfrac{a - c_1 + \delta\rho c_2}{b} - \gamma\tfrac{1-\delta(1-\alpha+\rho\kappa(1-\beta))}{b},$$

$$Q_{2u}^* = \tfrac{a - c_2 - \kappa\gamma}{b}$$

and

$$Q_{2u}^* - \delta Q_{1u}^* = \frac{a(1-\delta) + \delta c_1 - c_2(1 + \delta^2\rho)}{b} - \gamma\frac{\kappa - \delta + \delta^2(1 - \alpha + \rho\kappa(1-\beta))}{b}.$$

Under any of the three alternatives for the timing of emissions that we have considered we obtain that Q_{2u}^* and $Q_{2u}^* - \delta Q_{1u}^*$ diminish with κ. However, Q_{1u}^* increases with κ if $\beta = 0$ and it is independent of κ if $\beta = 1$. As a consequence, when $\beta = 0$, Q_{1u}^* may increase with γ if κ is greater than 1 and big enough.

Note also that Q_{1u}^* is positive if and only if:

$$a - c_1 + \delta\rho c_2 > \gamma(1 - \delta + \alpha\delta - \kappa\delta\rho + \kappa\beta\delta\rho)$$

and

$$Q_{2u}^* - \delta Q_{1u}^* > 0 \Leftrightarrow a(1-\delta) + \delta c_1 - c_2(1+\delta^2\rho) > \gamma(\kappa - \delta + \delta^2 - \alpha\delta^2 + \kappa\delta^2\rho - \beta\kappa\delta^2\rho).$$

The optimal emission taxes with renting firms are:

$$\tau_1^{r*} = \gamma - \tfrac{b}{n(1-\delta+\alpha\delta)}(Q_{1u}^* + \rho\delta(1-\beta)Q_{2u}^*)$$

$$\tau_2^{r*} = \kappa\gamma - \tfrac{b}{n}Q_{2u}^*$$

and it is easy to show that τ_1^{r*} is independent of κ. Therefore, we have that, when emission damages differ between periods and firms rent their output, optimal emission taxes in the first and in the second periods do not imply overinternalization for any of the three types of emissions considered.

The optimal emission taxes with selling firms are:

$$\tau_1^{s*} = \gamma - \frac{b(\left(n+1+\rho\delta^2(\beta(n+1)-1)\right)Q_{1u}^* - \rho\delta(\beta(n+1)-2)Q_{2u}^*)}{n(n+1)(1-\delta(1-\alpha))}$$

$$\tau_2^{s*} = \kappa\gamma - \frac{b}{n}(Q_{2u}^* - \delta Q_{1u}^*)$$

If emissions occur in the production process ($\alpha = 1$ and $\beta = 0$) there is not overinternalization in the first period as:

$$\tau_1^{s*} - \gamma = -\frac{b}{n(n+1)}(\left(n+1-\rho\delta^2\right)Q_{1u}^* + 2\rho\delta Q_{2u}^*) < 0$$

When $\beta = 1$ we know from Proposition 2 that there may be overinternalization in the first period if $\kappa = 1$. As

$$\frac{\partial \tau_1^{s*}}{\partial \kappa} = \gamma\rho\delta\frac{1-n+\delta^2\rho(1-\beta(n+2)+\beta^2(n+1))}{n(n+1)(1-\delta(1-\alpha))} < 0,$$

we have that, when $\beta = 1$, the set of values of the parameters where there is overinternalization in the first period decreases with κ. Finally, there is not overall overinternalization in the first period when firms sell their output as:

$$\begin{aligned} & (1-\delta+\alpha\delta)\tau_{1s}^* + \rho\beta\delta\tau_{2s}^* - \gamma(1-\delta+\alpha\delta+\rho\kappa\beta\delta) \\ = \; & \frac{b}{n(n+1)}(-(n+1-\rho\delta^2)Q_{1u}^* - 2\rho\delta\left(\delta Q_{1u}^* + Q_{2u}^*\right)) < 0. \end{aligned}$$

The optimal emission taxes with renting-selling firms are:

$$\tau_1^{rs*} = \gamma - b\frac{nQ_{1u}^* + \rho\delta(1-\beta)Q_{2u}^*}{n^2(1-\delta(1-\alpha))}$$

$$\tau_2^{rs*} = \kappa\gamma - \frac{bQ_{2u}^*}{n^2}$$

Hence, we have that, when emission damages differ between periods and firms rent and sell their output, optimal emission taxes in the first and in the second periods do not imply overinternalization for any of the three types of emissions considered.