Lagom regiO – a multi-agent model of several economic regions

Abstract

This paper presents Lagom regiO: a multi-agent model of several growing economic areas in interaction. The model is part of the Lagom model family, and its particular feature is that it locates agents in one of a user-chosen number of regions. It can thus be used to represent diverse economic areas by specifying characteristics of agents and their interaction network as depending on their regions. The description follows the Dahlem ABM documentation guidelines -

Sarah Wolf(1), Steffen Fürst (1), Antoine Mandel (2), Wiebke Lass (1), Daniel Lincke (1), Frank Meissner (1), Federico Pablo-Martí (3), Carlo Jaeger (1)

(1) Potsdam Institute for Climate Impact Research, Germany
(2) University Paris 1, France
(3) University of Alcalá, Spain

Corresponding author: federico.pablo@uah.es
Tlfno.: +34 918854231 Fax: +34 918854201
Facultad de CC. EE.y EE.
Plaza de la Victoria, s/n
28802 Alcalá de Henares, SPAIN

1   Introduction

Agent-based modelling is gaining importance in economics. In particular as the recent financial crisis has shed doubt on the usefulness of standard economic modelling approaches, which were not able to represent such an event, multi-agent models are used to study economic systems in which emergent system behaviour cannot be explained from the behaviour of some representative agents alone. The literature offers conceptual works on agent-based economic models, such as Tesfatsion and Judd [2006], as well as a variety of agent-based models on certain aspects of the economy, for example, leverage effects in financial markets [Farmer and Foley, 2009]. Multi-agent models of the economy as a whole are still rare, examples are the models by Gintis [2006, 2007], Dosi et al. [2008] and the EURACE model (see www.eurace.org).

The Lagom model family developed at the Potsdam Institute for Climate Impact Research (PIK) is based on Gintis' 2007 model. As discussed by Bilancini and Petri [2008], Gintis' model uses land rather than capital in the representation of production. As a step further, the Lagom models transfer the interaction between firms, households, a financial system, a government and import and export into a setting that involves heterogeneous capital goods with (possibly) diverse growth rates. The generic model, Lagom generiC, is presented in different versions in Mandel et al. [2009] and Mandel et al. [2010]. Lagom regiO adds the possibility of specifying different interacting economic areas to the generic model. For the interested reader, an executable file of the model is available under REFERENCE (ECF website?) and the source code can be requested there as well.

We distinguish the *abstract model* Lagom regiO from *applied models* that consist of this abstract model together with initial data. What is described here is the abstract model. The description follows the agent-based model (ABM) documentation guidelines developed at the 100th Dahlem Conference "New Approaches in Economics after the Financial Crisis" (Berlin Dahlem, August 28-31, 2010). Being a first application of the guidelines, which are still in a testing stage, the paper briefly discusses some experience made in using the guidelines.

As specified by the guidelines, the model description is divided into three parts. The first part provides an overview (Section 2), the second one explains general concepts underlying the model's design (Section 3), and the third part provides the functional specification of the model to some detail (Section 4). Questions that precede paragraphs are taken from the guidelines, we therefore suppress the repeated citation.

2 Overview

2.1 Rationale

What is the object under consideration? What is the intended usage of the model? Which issues can be investigated?

The rationale behind Lagom regiO is closely related to the purpose of Lagom generiC as described by Mandel et al. [2010]. Summarizing very briefly, Lagom generiC represents the evolution of economic systems over a time horizon of one to several decades. It is based on a multi-agent approach at the micro-economic level. One can use it to model macro-economic features of a system with heterogeneous capital goods accumulated through time at different rates, including endogenous sectoral technological change. Thus, Lagom generiC provides the opportunity to test micro-foundations specified outside the representative agent paradigm and may serve as a virtual laboratory with which effects of economic policy may be assessed qualitatively. As explained by Mandel et al. [2010], the approach is in line with the tradition of Arrow-Debreu general equilibrium theory [see pp. 3-4]. It can be seen as a generalisation to the tradition: in the same basic setting, instead of computing a unique optimal trajectory, a wider set of possible trajectories can be explored.

Lagom regiO generalizes Lagom generiC. The object under consideration is an economic area consisting of several connected regions. The model user chooses a number of regions $R \in \mathbb{N}$ and a number of sectors $S \in \mathbb{N}$ that represent the economic system under study with the desired degree of detail.[1] Lagom regiO thus enables its user to address various economic research questions by studying possible trajectories of an economy with heterogeneous regions in the multi-agent setting. Specific questions to be addressed are to be posed on a case to case basis for each applied model in which Lagom regiO is employed. These may range from theory generation to policy analysis. Example questions could range from reproducing, in the agent-based setting, stylized facts about economic growth in an economic area consisting of several countries to exploring consequences of green investment incentives on greenhouse gas emissions and on the economic development in the EU.

2.2 Agents

What kind of agents are considered in the model? Is there a refined taxonomy of agents? Are there agent groupings which are considered relevant?

The agents in the model are firms, households, a government, a financial system and import/export agents representing trade with the rest of the world. Firms belong to a sector and produce this sector's good. They are also located in a region which is fixed throughout a model run. Households are located in a region of origin at the beginning of a simulation, but may "migrate", that is, change their region membership during a run, depending on the amount of time worked in each region. Since a model household may represent many real world households, a simultaneous membership in several regions represents groups of people who have migrated to other regions. There is a government for each region, and an import/export agent for each sector. The financial system is one for the whole economic area.

2.3 Other entities

What are the other entities which are time-evolving but not decision-making?

Sectors and regions are time-evolving entities in the model that do not represent decision-making entities in the real world, but structure the model. Updating occurs for example by collecting statistics over all firms in a sector.

Regions provide the model's *spatial resolution*. Within a region, no further spatial differentiation between agents takes place. The model thus has a virtual spatial resolution depending on the

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[1]The notations R and S are fixed for these numbers throughout the whole paper.

regions chosen in an applied model. The model's *economic resolution*, as it were, is represented by the number of sectors which also determines the number of goods in the model, as is quite standard in economic modelling.

2.4 Boundaries

> *What are additional inputs to the model at runtime?*
> *Which outside influences on the model are hence represented?*

There are no model inputs at runtime.

2.5 Relations

> *What kind of relationships structure the agents' interactions?*
> *To which extent do these represent institutions?*

Agents' interactions are structured probabilistically, depending on the region(s) an agent belongs to. For firms, this means, partners for interaction are drawn uniformly from their region with a user-given probability (see the isolation parameter of the region in the initialization section below) and uniformly from the remaining regions all together with the remaining probability. For households, the region based probability distributions as used for firms are mixed, using as weights the membership in each region, which corresponds to the amount of working time spent there. Households have a total amount of working time equal to one, and the fraction of time a household is unemployed is counted as membership in its region of origin. Therefore the weights sum up to 1, and the mixture yields a probability distribution.

These interaction relationships represent labor and goods markets in the different regions, where agents interact more probably within the region(s) they belong to, but may also interact across region borders. Also, agents remember firms from which they have bought goods for one time-step, so that a temporary network for trade evolves between agents.

2.6 Activities

> *What kind of actions and interactions are the agents engaged into?*

Firms and import/export agents trade and households buy goods. Households make money by working for firms and by owning them. On the labor market, firms employ or lay off workers while households may search for jobs, accept job offers or quit jobs. Firms produce, households consume, and both firms and households as well as the import/export agents carry out accounting procedures, where firms may pay dividends and take loans from the financial system, and households may save money or take money out of their account. Firms may update their desired production and the prices at which they sell their goods. Both firms and households may change some of their characteristics by imitating others or by mutation.

The governments set the tax rate and pay the unemployment assurance from the taxes collected. The financial system sets the interest rate and may give loans to firms.

3   Design Concepts

3.1   Time, activity patterns and activation schemes

What is the basic sequence of events in the model? Are activities by agents triggered by a central clock or by actions respectively messages sent by other agents? What is the interpretation of one time unit in the model?

Time is modeled discretely; a period, consisting of several steps, provides the model's *temporal resolution*. The real time interpretation of a period in the model is the unit of time taken for the computation of flows used as data (e.g., production). The sequence of steps, representing different markets (labor or goods) or policy interventions (e.g., interest rate updating), is fixed (for details see Table 1 in the Functional Specification Section). This means, actions are triggered by the "central clock" provided by the counting of periods. However, not all steps need to be carried out in each period; some steps have a periodicity chosen by the user. Further, some steps are optional, that is, they can be turned off by the model user.

3.2   Interaction protocols and information flows

What are the general properties of the protocols governing the interaction between agents? How is determined which agents can interact with each other? What kind of information is available to each agent? If agents interact within institutional frameworks like firms or markets, what are the main properties of these institutions?

Interactions on the labor and goods markets are bilateral. The matching occurs by agents observing samples of other agents (according to the regional structure and network connections evolving through time) and then choosing an agent from the sample to interact with, depending on the objectives for that interaction. For example when trading, the buyer observes sellers and then buys from the one with the cheapest price in the sample. Similarly, the genetic evolution is based on peers sampling: for imitation, the fittest agent from the sample is chosen. Agents engaged in an interaction observe and act on each other's characteristics and state variables. The details of negotiations are not explicitly modelled. (For details, see Sections 4.1.1, 4.1.4, 4.1.5, 4.1.15, 4.1.16, 4.1.19 and 4.1.20.)

3.3   Forecasting

Are agents in the model forward looking or purely backward looking? If agents are forward looking, what is the basic approach to modeling forecasting behavior?

Agents base their forecasts on the past experience. Firms determine their production and investment levels based on their previous and current success (see Section 4.1.13). Households forecast their expected income using exponential smoothing when allocating their wealth between consumption and savings (see Section 4.1.11). The financial system uses data from several previous periods to update the interest rate (Section 4.1.14).

3.4   Behavioural assumptions and decision making

Based on which general concepts is decision making behavior of the different types of agents modelled? If the decision making of certain agents is influenced by their beliefs, how are these beliefs formed?

Agents have bounded rationality and gain only partial information about their environment in local interactions. They base their operating decisions on rules of thumb (e.g., the Deaton thumb rule for household saving, see Section 4.1.11 or the Taylor rule for the interest rate, see Section 4.1.14). Further rules are parametrized by the agent's characteristics, such as determining demand from technology or consumption coefficients (see Section 4.1.1).

3.5 Learning

Are decision rules of agents changed over time? If yes, which types of algorithms are used to do this?

Learning is represented by the evolution of structural characteristics (e.g., technology or consumption coefficients, mark-up, wage reference) of firms and households. This occurs by imitation and mutation – (genetic) procedures which explicit the agents objectives (profit for the firm, utility for the household). Mutation introduces innovation. (Sections 4.1.15, 4.1.16, 4.1.19, 4.1.20.)

3.6 Population Demography

Can agents drop out of the population and new agents enter the population during a simulation run? If yes, how are exit and entry triggered?

Firms are created in profitable sectors and bankrupted in unprofitable ones (Sections 4.1.17, 4.1.18). The population of other entities is constant.

3.7 Levels of randomness

How do random events and random attributes affect the model?

There are two major sources of stochasticity. Randomness in the matching processes governs agents' interactions on the goods and labor markets. (Sections 4.1.1, 4.1.4, 4.1.5.) During the genetic evolution processes, random innovations, as well as random drawings of which agent imitates another one, and the random sample this agents then observes, lead to stochastic dynamics for prices, wages and technologies. (Sections 4.1.15, 4.1.16, 4.1.19, 4.1.20.)

3.8 Miscellaneous

Any important aspects of the used modeling approach that do not fit any of the items above should be explained here, for example, mathematical properties of the model.

From a mathematical perspective, the structure of the Lagom models can be interpreted as consisting of three interleaved kinds of processes: A deterministic dynamical system models the evolution of a set of economic agents embedded in a common environment. A superimposed stochastic process (i.e., the sampling) represents chance events that determine the (limited) information about other agents which an agent obtains. Based on this information, agents engage into transactions. A stochastic noise process of small intensity (i.e., imitation and mutation) perturbs the above processes. It represents the dynamics of social learning by which the strategies of agents are modified.

4 Functional specification

Lagom regiO is implemented in Scala [see Odersky and al., 2004] – an object-oriented, functional, statically typed programming language – using an extended version of MASON [see Luke et al., 2005], a discrete-event multi-agent simulation library in Java. Scala interoperates with Java (and therefore the MASON libary), but e.g. type inference, implicits and high-order methods allow to write code that is more concise and expressive than its Java equivalent.[2] The model provides a graphical user interface for setting the initial conditions of a simulation (see Section 4.2) and for observing model output. Many variables in the model can be dynamically represented at runtime in a set of line charts, bar diagrams, etc. Also, it is possible to write model variables into output files, allowing for further processing and analysis. This holds both for agents' individual state variables and for aggregate values, such as averages over all firms in a sector or region.

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[2]See blog entry "No, seriously, why Scala?" by David R. MacIver http://www.drmaciver.com/2007/12/no-seriously-why-scala/

4.1 Description of Agents and Other Entities, Action and Interaction

What, in detail, are the Agents and Other Entities in the model?

All types of agents and other entities were introduced in Section 2.2.

What Agent/Other Entity does what and in which order?

Actions with their actors, the order in which actions take place, whether they are optional and, if applicable, their periodicity, are presented in Table 1. In the column "who does what?" the actions that are further specified in Sections 4.1.1 through 4.1.20 below are set in italics.

Table 1: Activities.

step	periodicity	optional	who does what?
exchange		no	Firms and households *trade* in a random order.[a]
			For all sectors, the import/export agents *trade*.
			The financial system *updates the inflation rate*.
update benchmark wage	n	no	Sectors *update the benchmark wage*.
adjust employment	m	yes	Households, in a random order, *adjust employment*.
adjust workforce	m	yes	Firms, in a random order, *adjust workforce*s.
update labor productivity		yes	Sectors *update labor productivity*.
production		no	Firms *produce*; *emissions* are computed.
consumption		no	Households *consume*.
accounting		no	Firms *account*. The governments *set the tax rate* to pay unemployment assurance. Households *account*. Each sector's import/export agent *accounts*.
update price	q	yes	Firms *update prices*.
update desired production		yes[b]	Firms *update desired production*s.
setting interest rate	r	yes	The financial system *updates the interest rate*.
evolution of technologies	s	yes	For each sector and region, a number[c] of firms are drawn randomly. These *imitate technologies*. Firms *mutate technologies* with probability given by the mutation rate parameter. Analogously, households *imitate* and *mutate technologies*.
firms entry and exit	1, v	yes	Each period, sectors *bankrupt unproductive firms*. Each v-th period, *creating or bankrupting firms* yields a new firm generation per sector and region; sectors *replace unprofitable firms by the best firm*.
evolution of prices	w	yes	As described for technology evolution, firms *imitate prices* and/or *wages*, as well as *mutate prices* and/or *wages*.

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[a]More precisely, there is a user input random trade order. If it is switched on, all firms and households are shuffled, else firms are shuffled and households are shuffled separately, and firms trade first.

[b]The two firm steps of adjusting prices and production are turned on or off by a single switch in the graphical user interface.

[c]The number is determined by multiplying a user-given firm replacement rate with the number of existing firms.

For each kind of Agents/Other Entities, what are the model state variables and parameters? List their type, that is, dimensions and admissible range, a short description of what they represent, units of measurement, how often they are updated (in models with different time scales) and how they are initialized.

Parameters[3] and state variables of agents and other entities are presented in Tables 2 to 7. These tables name each state variable and provide its type. Among the types of variables, some are self-defined: $\mathcal{S}, \mathcal{R}, \mathcal{F}, \mathcal{H}$ denote the types of our agents and other entities, sector, region, firm, and household. "ProductionFunction", "ConsumptionTechnology", and "Stats" are helper classes and I(type) denotes an interval of the given type. VM(n, type) denotes a ValueMemory, defined in order to remember values from the most recent n periods, where periodicity numbers refer to the notation of Table 1. Square brackets may denote either intervals or lists. No confusion should arise because in the former case, the brackets enclose two values, separated by a comma, which provide start and end point of the interval (e.g. $[0, 1]$) and in the latter case, they enclose another type, which is the type of the elements of the list (e.g. $[\mathcal{F}]$ for a list of firms).

For each variable, the tables further present a short description of what it represents in economic terms. Entries in the column "updating" specify how often a state variable is updated – while parameters are fixed for a simulation – the *periodicity* entry states the number of periods after which a variable may be updated, < 1 means the variable can change more than once per period.

The column "initialization" specifies how a parameter is set or a variable is initialized. This may be a fixed value such as 0 or *false*. An initial value may be *set* inside the model (such as the region of origin for the household, which is set to the region in which the household is created). It may be a value taken directly from the user's *input* into the model or a value *computed* from user input or other values. In both cases, no further specification means that the parameter/initial variable is equal for all agents/other entities of the given type. If a type is added, this means the input may differ according to this type: an $\mathcal{R}$, for example states, that each region has a parameter of its own, which can apply to the region itself or to all firms from that region. The notation (prob) further adds the information that the dependence is stochastic: random drawings with e.g. region-dependent probabilities constitute the computation or are included in it.

Units of measurement in the model are plain numbers for parameters/variables of type $\mathbb{N}$, such as the number of workers (if one wants to use thousands of workers for legibility, wages have to be adapted accordingly). Time units are user-chosen: a period represents whatever amount of time the user inputs. Economic parameters and variables are measured in money, the currency in the model is the same for all regions, so it remains implicit and depends on the interpretation coming with the data the user inputs.

> *What information and with whom does each kind of agent exchange for decision-making? When are state variables updated? How are state variables updated (specify equations, diagrams, or pseudo code for algorithms related to rules-of-thumb, learning, adaptation, forecasting, interaction, etc)?*

In the following, we provide details of the actions that were italicized in Table 1. These show which information agents' decisions are based on as well as in which steps and how state variables are updated. Note that the index t is used in various formulae to refer to time. However, the difference between $t - 1$ and t is not the same for all steps described. Rather, it is to be read locally for each step or sub-step: an index $t - 1$ denotes a state variable *before* the step under consideration has been carried out, an index t denotes the same variable *after* that step. This is not necessarily a complete period, since some variables may change more than once per period, for example, in different steps. Then, the variable with index $t - 1$ in a later step may have the value obtained with the index t in an earlier step. `State variables` and `parameters` from Tables 2 to 4 are highlighted using `typewriter` and sans serif fonts, respectively, however, spaces are used between words in the text instead of the original "camelCase" notation from the code that is used in the tables.

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[3]For further global parameters, see Section 4.2 on the initialization of the model below.

Table 2: Parameters and state variables of region and import/export

name	type	description		initialization
Region				
parameters				
name	String	Name for graphical output		input, $\mathcal{R}$
numHouseholds	$\mathbb{N}$	Number of households		input, $\mathcal{R}$
householdReplacementRate	$[0,1]$	imitation rate for households		input
numHouseholdsCopying	$\mathbb{N}$	# of households imitating others		computed
householdList	$[\mathcal{H}]$	Households from the region		computed
isolation	$[0,1]$	Interaction probability within region		input, $\mathcal{R}$
state variables			periodicity	
firmList	$[\mathcal{F}]$	Firms in the region	$1, v$	computed
Import/Export				
parameters				
productionSector	$\mathcal{S}$	Sector of the import/export agent		set
observedFirmsQuotient	$[0,1]$	rate of firms to observe to find suppliers		input
buyingProbabilities	$[0,1]^R$	Prob. of buying goods from regions		input
growthOfMaxImport	$\mathbb{R}_+$	growth of import limitation factor		input
state variables			periodicity	
maxImportInPeriod	$\mathbb{R}_+$	Bound for imports in period	1	computed
importLeft	$\mathbb{R}_+$	Amount left to import	< 1	computed
maxExportInPeriod	$\mathbb{R}_+$	Bound for exports in period	1	computed
exportLeft	$\mathbb{R}_+$	Amount left to export	< 1	computed
preferredFirms	$[\mathcal{F}]^S$	Firms from which goods were bought in the previous period	1	computed

4.1.1 Trade

All agents who trade have to find suppliers for the good from each sector. This works by observing a sample of firms from each sector. The number of firms included in the sample is computed by multiplying the sector's number of firms with the `observed firms quotient` for firms and for import/export, respectively the `observed suppliers quotient` for households. The firms with whom business was done in the previous period are stored in a list of `preferred firms`[4]. If these have a positive `inventory`, they are again included in the suppliers for the current period. To fill up the sample, further firms are drawn with region-based probabilities: for firms and households the formulae from Appendix A are used, import/export agents use the `buying probabilities` given by the model user. For firms and households, the import/export agent is added to the suppliers, while import/export agents do not buy from each other.

Given a new list of suppliers, the agent who trades determines his demand. **Firms** do this by checking the maximal available supply from the suppliers, adding the inventory of all firms from the suppliers list. For import/export agents, the inventory corresponds to the `import left`. Next, the maximal possible production from the available supply is computed, allocating the available supply for each sector between circulating and fixed capital in a way that yields the maximal production amount. If the `desired production` exceeds this value, only this much will be produced, otherwise, the firm wants to produce its `desired production`. It computes the circulating capital needed, and, if the conditions for investing are satisfied (i.e. if the variable `isInvest` is set to *true*), the fixed capital needed.

Households determine their demand by distributing their `money` holdings to the different sectors according to their `consumption technology coefficients`. Demand is then computed for each sector as the amount that can be bought with the money allocated to the sector, using

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[4]In the first period this list is empty.

the average price observed among suppliers. If this exceeds the total supply from all suppliers (summing up their `inventory`, respectively `import left`), demand is set to the maximal possible supply.

The demand of the import/export agent is the `export left` in the period. If the total available supply from the suppliers found is less than this amount, the demand is reduced to the supply found.

The thus determined demand is then bought, starting from the cheapest supplier, buying up to his whole supply, and continuing one by one with the next cheapest suppliers, until the demand is completely bought. The price in each transaction is the `price` at which the seller sells his good. For the buyer, `costs` are recorded, and the `stock`, `circulating capital` (and `fixed capital`, if applicable), or `export left` as well as the `money` holdings are updated. For the seller in the transaction, the `inventory` is decreased and the `money` holdings are increased accordingly. Finally, the `preferred firms` are updated to those firms from which goods have been bought in this step.

Note that, as import/export agents trade last, they can only buy the goods left in the firms' inventories after all other agents have traded. However, they never buy more than the `export left`.

4.1.2 Update inflation rate

The inflation rate I is computed using the Laspeyre price index:

$$I = \frac{\sum_{s=1}^{S} p_{s,t} \cdot q_{s,t-1}}{\sum_{s=1}^{S} p_{s,t-1} \cdot q_{s,t-1}} - 1$$

where $q_{s,t}$ is the quantity of purchased goods from sector s at time t

$p_{s,t}$ is the average trade price for goods from sector s at time t

The relevant quantities and prices are computed/stored in each sector's `statistics`. The financial system stores `inflation rate`s (see `memoryInflationRate`).

4.1.3 Update benchmark wage

For each sector and region, the `benchmark wage` is updated by multiplying its old value with the total growth factor of `labor productivity` over the previous n periods (computed from the `memoryLaborProductivityGrowthRate`).

4.1.4 Adjust employment

"Adjust employment" consists of the operations initiated by the households on the labor market. Households are shuffled into a random order and every household carries out the following steps: It checks its work contracts (see `firmWorkContractMap`). For every work contract, the specified wage is compared with the `fallback` of the household. If the wage is below the `fallback`, the job is quit by deleting the work contract and removing the worker from the firm's `worker list`. Afterwards the household checks whether it is fully employed (which is the case if its work amount is 1). If this is not the case, the household looks for new jobs going through a sample firms it observes (the size of the sample is determined by the `observed firms quotient`, and the probability distribution for drawing it depends on the household's `region membership` (see formula (3) in Appendix A). If the selected firm offers an acceptable work contract the household accepts the job offer by adding the work contract to its `firm work contract map`, and he is added to the firm's `worker list`. The amount of work specified in the contract is the minimum between the amount demanded by the firm and the household's available working time (which is the difference between 1 and the current work amount of the household). This procedure is repeated until the work amount of the household has reached 1 or the household has gone through all observable firms.

4.1.5 Adjust workforce

"Adjust workforce" consists of the operations initiated by the firms on the labor market. Every firm starts by updating its `target employment` which is done by computing the workforce needed to produce the `desired production`. It also computes the wage it will offer by multiplying its `wage reference` with the `benchmark wage` of its production sector.

If the current work amount (computed from the `worker list`) is smaller than the `target employment`, the firm tries to find new employees. It observes a sample of households (where the probability for observing a household is given by formula (1) in Appendix A) and offers a work contract with work amount 1.0 or the remaining work amount it needs, if that is smaller than 1.0. The household checks the work contract by comparing the `wage` offered with its `fallback`. If the contract is acceptable, the household sets the work amount in such a way that its total work amount is not above 1.0 (if necessary, the work amount of the contract is reduced). The household adds the contract to its `firm work contract list` and the worker is added to the firm's `worker list`. This procedure is repeated until all observable households have been offered a contract or the available work amount of the firm meets the `target employment`.

If, on the other hand, the firm's work amount, divided by $1 + b$ where b is the `buffer employment rate`, is above the `target employment`, the firm lays off workers, starting from those with the highest `fallback`. A worker who is laid off is removed from the `worker list` and his contract is deleted; the household's `firm work contract map` is adjusted. This is done repeatedly until the work amount adjusted with the buffer employment rate matches the `target employment`. The buffer smoothes the firm's employment behaviour.

For comparing firms according to recruitment efficiency later (see Section 4.1.19), the `labor capacity` of the firm is newly computed by dividing the total work amount (from all its workers) by its `target employment` (should the latter be zero, also the `labor capacity` is set to zero).

4.1.6 Update labor productivity

The model user chooses whether to update labor productivity endogenously, or to use an exogenous labor productivity growth rate (for these global parameters see Section 4.2 below). In the latter case, the `labor productivity` for all sectors in a region is increased by a factor $1 + g_r$ where g_r is the growth rate for that region given by the user.

Otherwise, labor productivity is updated endogenously, depending on investment. In each sector, the `labor productivity` for region r is basically increased by a factor $1 + i_{r,t}$ where $i_{r,t}$ is the regional investment rate. The idea is to compute this rate as the the temporal change rate in the value of the `regional fixed capital stock`, where the value is computed using a global average `price vector`.

$$i_{r,t} = \frac{\vec{p}_t \cdot \vec{c}_{r,t}}{\vec{p}_t \cdot \vec{c}_{r,t-1}} - 1$$

where $\vec{p}_t$ is the global average `price vector` over all sectors at time t

$\vec{c}_{r,t}$ is the vector of fixed capital in region r over all sectors at time t.

However, to be more realistic we do not want `labor productivity` to decrease. Therefore we set $i_{r,t}$ to 0 if it is negative and use the `regional maximal fixed capital stock` instead of the actual fixed capital from the previous period. The actual computation is:

$$i_{r,t} = \frac{\vec{p}_t \cdot \vec{c}_{r,t}}{\vec{p}_t \cdot \vec{c}_{r,t-1}^{max}} - 1$$

where $\vec{c}_{r,t}^{max}$ is the vector of maximal fixed capital in region r over all sectors recorded up to time t.

This ensures that after periods of decreasing capital stock labor productivity starts increasing only after the old capital stock level has been reached again.

4.1.7 Produce

Firms produce an amount corresponding to their `desired production`, if the possible production amount is at least that high. Otherwise the amount they produce corresponds to the possible production. The latter is determined by the resources of the firm (available amount of labor, fixed capital and circulating capital) and its current technology (specifying the amount of labor, fixed capital and circulating capital necessary to produce one unit). Consider a firm with `production sector` s. The production of, say, p units induces the following operations on the firm's stocks:

1. The inventory i is depreciated and the production is added to the inventory:

$$i_t = i_{t-1} \cdot (1 - \delta_s) + p$$

   where i_t denotes the `inventory` at time t,

   δ_s denotes the `inventory depreciation rate` of sector s.

2. For each input sector j, the amount of circulating capital used for the production of p units (computed using the inverse of the `production function`), is consumed by subtracting it from the `circulating capital` stock. Remaining circulating capital is depreciated using the corresponding `inventory depreciation rate` of the input sector.

3. For each input sector, the fixed capital necessary to produce p units (again, computed using the inverse of the `production function`) is depreciated. The value of the depreciated fixed capital, computed using the average trade price from the input sector's `statistics`, is added to the firm's `costs`.

$$f_{j,t} = f_{j,t-1} - u_j \cdot (1 - \tilde{\delta}_j)$$
$$c_t = u_j \cdot (1 - \tilde{\delta}_j) \cdot a_j$$

   where $f_{j,t}$ denotes the `fixed capital` stock the firm has from input sector j at time t

   u_j denotes the amount of fixed capital from input sector j used to produce p units,

   $\tilde{\delta}_j$ denotes the `fixed capital depreciation rate` of sector j

   a_j denotes the average trade price in input sector j.

   The `fixed capital depreciation rate` for each sector is computed from the initial economic data (see Section 4.2 on the initialization).

4.1.8 Emissions

Based on the produced units, for each sector and region the emission of CO_2 is calculated. The emissions are assigned to the sectors that produce energy. For those sectors the numbers of produced units are multiplied with the `petajoule equivalent` factor and an exogenously given carbon intensity[5] for this sector and region. The carbon intensity depends on the fuel used for the energy production and the efficiency of the production process. The improvement (reduction) of the carbon intensity is modeled using a logistic function

$$c_t = c_{max} - \frac{c_{max} - c_{min}}{1 + e^{-s \cdot (t-o)}}$$

   where c_t denotes the carbon intensity at time t

   c_{max} denotes the `carbon intensity upper limit` parameter

   c_{min} denotes the `carbon intensity lower limit` parameter

   s denotes the `carbon intensity slope` parameter

   o denotes the `carbon intensity turning year` parameter.

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[5]There exist different definitions of carbon intensity, our's is: The amount of carbon (by weight) emitted per unit of energy consumed.

The parameters must be estimated by the model user, e.g. via a least-square optimization on data existing for the past years. The model user must also determine the `petajoule equivalent` so that in the first period the energy production of the model matches the real-world energy usage.

4.1.9 Consume

Consumption means that households use up the whole `stock` of goods, which was bought in the trading step. The consumption is recorded in the `statistics` and the `stock` is set to a vector of zeros.

4.1.10 Set the tax rate

For each region, the government updates the `average wage` earned in its region, and from this, the `unemployment wage` by multiplication with the user given `unemployment wage quotient`. The `unemployment rate` of a region is computed by summing up the unemployment time $(1-$ the work amount) of all households with this region of `origin` and dividing the sum by the number of households from that region (`numHouseholds`). These computations also provide the information how much `money` the government needs to pay unemployment assurance to the unemployed from the region. The `tax rate` is then set thus that the sum of taxes to be paid by households working in the region is equal to this amount of `money`.

4.1.11 Account

Wages, interests and dividends payment occur at this stage, profits, costs and monetary holdings are accordingly updated.

Firms add to their `costs` and subtract from their `money` the `wage`s which are payed to the employees. The accounting step of firms also includes interaction with the financial system. If the firm has a `debt` towards the financial system, `interest on debt` is computed according to the value of the `interest rate` currently posted by the financial system and subtracted from the firm's `money` holdings. If the `money` of a firm is now negative (which means that it has a deficit) it subscribes a new `debt` towards the financial system, which is accumulated with the existing debt and the `money` holdings are set to zero. If the `money` holdings are positive, the `debt` is reimbursed where applicable. This is done by paying the outstanding `debt` or, if that is above the monetary holdings, the complete `money` to the financial system.

If after the interaction with the financial system the `money` holdings are above zero, a share (given by the user input `dividend rate`) is paid as `dividend` to the firm's `owner`. Finally firms compute their `profit` by subtracting `costs` from revenues and their `profit rate` as the ratio between this period's `profit` and `last period's costs`.

The accounting step for **households** includes computing the income and taxes paid, as well as saving or un-saving money. The income consists of `employment income` and unemployment wage (that is, $(1-a) \cdot u_r$, where a is the amount of time worked for all employers and u_r the unemployment wage paid by the government from the household's region of `origin`), the interest on `savings` (according to the financial system's `interest rate`) and the `dividend`s from firms the household owns (see `ownedFirms`). The household's `net income` is computed by subtracting from all this the `taxes paid` to the governments of those regions in which a household is employed, owns firms, and his region of `origin`. The wage earned in a region is taxed by that region's government with its `tax rate`. Similarly, dividends are taxed in the region where they are earned (that is, the `region` of the firm owned), and the household's `savings` are taxed in the region of `origin`.

If the `net income` is above the `expected income`, the household saves an amount corresponding to the `savings rate` multiplied with the difference between `net` and `expected income`. If below, the difference is taken out of the `savings` (if these are high enough). Finally, the expected income of

the household is updated using the following formula:

$$i_t = (1 + g_t) \cdot ((1 - \alpha) \cdot i_{t-1} + \alpha \cdot n_t$$

where i_t denotes the expected income at time t

g denotes an `expected income growth rate`, explained below

α denotes an exponential smoothing factor, set to 0.1 in the code

n_t denotes the just computed `net income`

The expected income growth rate is set to the household's income growth rate from the previous period to the current period, if its `last period income` was positive. Otherwise, the growth rate is the current `inflation rate` of the financial system. The household's `money` holdings are updated accordingly.

The **import and export agents** for all sectors account by recomputing the maximal possible import and export (see `maxImportInPeriod` and `maxExportInPeriod`) for next period's trading step. For imports, this is

$$i_t = \max\{g \cdot i_{t-1}, l \cdot p_t, 0\}$$

where i_t denotes the maximal possible import at time t

g denotes the `growth of import limitation` parameter

l denotes the `initial import limitation factor`

p_t denotes the total production of the sector at time t.

The maximal export amount is computed as the product of the sector's total production multiplied with the `initial export limitation factor`.

4.1.12 Update prices

Each firm computes the `price` for a unit of its good by multiplying the unit costs with $1 + m$, where m is the `mark-up`. Unit costs are computed as `costs` divided by the `produced quantity`. However, there is an upper limit for the price: it cannot be higher than twice the current average trade price in the firm's `production sector` (taken from the sector's `statistics`).

4.1.13 Update desired production

Each firm updates its desired production by multiplying the previous `desired production` with a factor $1 + r$ where r denotes the `rate production up` that is computed depending on the firm's `inventory` in comparison with its `target inventory` and on its `profit rate` in comparison with the financial system's `interest rate`. First, the `target inventory` is updated, by multiplying the `sold quantity` with the `inventory/sales target`. Now, four possible cases lead to two different computations for r:

1. $r = 0$, that is, `desired production` stays the same,
   if `profit rate` < `interest rate`, and `target inventory` ≥ `inventory` (the firm is doing badly but wants a larger inventory)
   or if `profit rate` ≥ `interest rate`, and `target inventory` < `inventory` (the firm is doing well, but the inventory is still full);

2. if the firm is doing badly, and it still has an inventory larger than aspired, it wants to decrease its desired production, while, if it is doing well and wants to increase the inventory, it wants to increase the desired production. In both cases, it computes $r = t/i - 1$ where t is the `target inventory` and i the `inventory`.

   - For the case `profit rate` < `interest rate` and `target inventory` < `inventory`, $t/i < 1$, so that $r < 0$.

- For the case `profit rate` $\geq$ `interest rate` and `target inventory` ge `inventory`, $t/i \geq 1$, so that $r \geq 0$.

In both these cases, however, the change in desired production is bounded by the parameter `max rate production up` ρ by taking a maximum with $-\rho$, respectively a minimum with ρ.

Also, the conditions for investment in fixed capital (see `isInvest`) are set for each input sector by checking whether the `fixed capital` of the sector is sufficient to produce the `desired production` amount, scaled by a factor $(1 + \tilde{\delta}_j)(1 + g)$, where $\tilde{\delta}_j$ is the `fixed capital depreciation rate` of sector j and g is the initial growth rate of the firm's `production sector` in its `region`. This factor is a rough estimate for growth until the next period, in which the fixed capital would need to be bought: if the fixed capital is not sufficient, the variable `isInvest` is set to *true*, if fixed capital suffices, it remains *false*.

4.1.14 Update interest rate

The financial system recomputes the interest rate based on the Taylor rule. For this computation, the (global) unemployment rate is computed after each period by summing up the amounts of unemployment time of all households in the model (stored in the `unemployment rate memory`). With r the periodicity of this step (as in Table 1), the new interest rate is :

$$i_t = e + T + \alpha \cdot (\bar{i} - T) + \beta \cdot (U - \bar{u})$$

where i_t denotes the interest rate at time t

and $\bar{i}$ the average `inflation rate` of the periods $t - r, \ldots, t - 1$

e denotes the `equilibrium interest rate`

T denotes the `target inflation rate`

α denotes the `inflation rate adaptation coefficient`

β denotes the `unemployment rate adaptation coefficient`

U denotes the `natural unemployment rate` parameter

$\bar{u}$ denotes the average `unemployment rate` of the periods $t - r, \ldots, t - 1$

obtained from the `unemployment rate memory`.

In case the computation result is negative, the new interest rate is set to 0.

4.1.15 Imitate technologies

Imitation of technologies concerns production technologies for firms and consumption technologies for households. Each agent that has been chosen to imitate technologies (see Table 1) observes a random sample of its peers: other households or firms in the same sector. The sample is drawn with probabilities computed as described in Appendix A. The agent that imitates is included in the sample of peers, the sample is ordered according to a fitness function and the technology of the best is copied. This means, the agent never imitates a technology that is worse than its previous technology.

The best firm from a sample is determined by comparing the unit costs of production, that is, the cost of producing one unit of the good, using `average prices` for all goods, the firms' `wage` and their technology consisting of the `labor input coefficient`, the `fixed input coefficients` and the `circulating input coefficients`. The lower the unit production costs, the better the technology. Imitating the technology is done by copying all three types of coefficients.

In the case of households, the best household is the one with highest utility value, determined by the `consumption technology` chosen by the user. The utility can be a CES function or simply the sum of the amount of all goods consumed, it is computed for each household using its own `price vector` (that is, the average prices the household observed among its suppliers when trading). Imitating the

consumption technology is done by computing a convex combination of the consumption technology coefficients of the imitating and the best household:

$$c_{j,t} = \alpha \cdot b_j + (1 - \alpha) \cdot c_{j,t-1}$$

where $c_{j,t}$ denotes the imitating household's consumption coefficient for sector j at time t

b_j denotes the best household's consumption coefficient for sector j

α denotes the imitation factor.

4.1.16 Mutate technologies

If technology mutation occurs for a firm (which happens with a probability given by the firms' mutation rate parameter, see Table 1), its technology is randomly changed as follows. The labor input coefficient, the fixed input coefficients and the circulating input coefficients are randomly mutated, where the maximal possible change is bounded by the respective parameters mutation of labor input coefficient, mutation of circulating input coefficients and mutation of fixed input coefficients. A final normalisation operation ensures that the firm's technology remains, in standard economic theory terms, on the same isoquant of production, only the relative combination of inputs needed changes. The normalization operation depends on the production technology chosen by the user out of

- linear production function, a pure linear or a constant linear one, or a linear nested in Leontieff production function

- a Cobb-Douglas or Cobb-Douglas linear, non convex Cobb-Douglas based or an alternative non convex Cobb-Douglas based one,

- a constant elasticity, pure CES, and a CES nested in CES production function.

Similarly, if a household's technology is mutated, the consumption coefficients are mutated, bounded by the consumption coefficients mutation lower bound and consumption coefficients mutation upper bound, in such a way that the household's technology remains on the same utility curve (for the utility computation see Section 4.1.15 above).

4.1.17 Bankrupt unproductive firms

A firm is unproductive if its produced quantity is 0. The unproductive firm is bankrupted by laying off all workers and being deleted form the sector's and region's firm list. If there is money available, the debt is refunded as far as possible, and if there still is some money left after that, it is transferred to the owner who then deletes the firm from the list of his properties. It should be noted that there is a *minimal number of firms per sector* parameter (see Section 4.2 on the initialization) which insures that the number of firms does not get too low. If this would be the case, unproductive firms are not bankrupted.

4.1.18 Create or bankrupt firms

For each sector and region, a new firm generation is produced as follows. If the average profit rate is below the interest rate, given the mark-up behavior one can assume that there is overproduction. The less profitable firms are bankrupted (as described above) until the recomputed total production in this sector and region is below the total amount sold in that sector and region in the last trade step, or the number of bankrupted firms reaches a bankrupting-maximum which is computed from the sector's bankrupt factor and the number of firms given before this step. If the average profit rate, on the other hand, is above the interest rate, firms are created proportionally to the difference between profit and interest rate, if this difference does not exceed the firm creation rate. The number of firms to be created is then computed by multiplying the resulting rate with the number of firms

existing before the step, and the computed number of firms is created, initialized and added to the relevant lists (see initialization of firms in Section 4.2).

4.1.19   Imitate prices and imitate wages

Imitation of prices and wages are very similar to the imitation of technologies described in Section 4.1.15. Observing a sample of peers and ordering it (including oneself) occurs here just as described there. The fitness functions used for the order and the characteristics which are then copied from the "fittest" are the only things that remain to be described.

For prices, the fitness value $f_{A/B}$ of a firm A in relation to a firm B is determined as follows:

$$f_{A/B} = \min\left\{1.0, \frac{p(A)}{p(B)}\right\} + \min\left\{1.0, \frac{g(A)}{g(B)}\right\}$$

where $p(A)$ denotes the `profit rate` of firm A and

$g(A)$ the growth of the `profit rate` of firm A, computed from its `profit memory`

If $f_{A/B} > f_{B/A}$, firm A is fitter than firm B. The imitating firm copies the `mark-up` from the fittest firm.

For wages, greater fitness is simply greater `labor capacity` (in case these are equal for two firms, the one with the lower `wage reference` is fitter). The characteristic imitated in this step is the `wage reference`. The `wage` is immediately recomputed with this new `wage reference`.

4.1.20   Mutate prices and wages

Mutation of prices and mutation of wages occur independently for all firms with probability given by the `mutation rate` parameter. This means, both the mutation of different firms and the price and wage mutation of a given firm are independent (pseudo-) random events. If a firm mutates prices, its `mark-up` is changed randomly as follows:

$$m_t = m_{t-1} + (\sigma \cdot p \cdot m_{t-1})$$

where m_t denotes the `mark-up` of the firm at time t

σ is $+$ or $-$ chosen by a computational coin toss

p denotes the `price mutation factor`

If a firm mutates wages, the `wage reference` is changed randomly, using an analogous formula with the `wage mutation factor` instead of the one for prices.

4.2   Initialization

How is the model initialized? Which kind of input is needed?

The model's initialization requires a set of user-input parameters. These can be entered through a graphical user interface or by loading a csv-file. In the case the user does not supply either, the models runs using default parameters, which however are not supposed to represent any given real-world economy. Parameters can be categorized into global parameters determining the set-up of a simulation, agents' parameters (that were already included in the Tables 2 to 7 above, and will thus not be repeated here) and economic parameters, used to represent a given economic system.

Global set-up parameters are for example the switches for those steps that can be turned off, and the periodicities the user chooses for those steps where they exist. Details about these parameters are found in Table 1 so we do not repeat them in the following. Other parameters that also concern the setup of the simulation are given in Table 8.

Further, the parameters include some choices: whether

- sectors have the same size,

Table 8: Global parameters

name	type	description
numRegions	$\mathbb{N}$	number of regions (R)
numSectors	$\mathbb{N}$	number of sector (S)
startingYear	$\mathbb{N}$	Starting year for the simulation
numPeriodsPerYear	$\mathbb{N}$	number of periods per year[a]

[a]The number of periods per year needs to be chosen in a way that is compatible with the size of flow data compared to stock data in the initial economic data input.

- firms have the same size,

- firms and households trade in a random order or first firms trade and then households trade, each group in a random order (see Footnote a, Table 1)

- labor productivity is updated endogenously or supplied exogenously. In the latter case, an exogenous labor productivity growth rate in $[0, 1]$ needs to be supplied.

Are the initial values chosen arbitrarily or based on data? In the latter case, what kind of data is needed?

These set-up parameters are completely up to the user's choice, and should be chosen depending on the questions the users wants to investigate using the model. Parameters chosen for the representation of certain agents may be data-based (such as the dividend rate), theory-based (such as the production function for firms), or rather arbitrary, depending also on the questions to be studied using the model. If a real-world economy is to be represented, of course, as many parameters as possible should be based on the data concerning that economy. If, however, the goal is to study the influence of the genetic evolution on economic systems in general, a toy economy with synthetic economic data might be sufficient, while the parameters concerning genetic evolution of technologies, wages and prices need to be varied.

Initial economic data are the most obviously data-based parameters, and are required to represent a given economic system under consideration. The required data is closely related to national accounting data, and is summarized in Table 9.

Table 9: Initial economic data

parameter	type	explanation
initial unemployment rate	$[0,1]^R$	unemployment rate per region
number of workers	$\mathbb{N}^R$	real-world number of workers, used to calibrate how many workers a model household represents
wages	$\mathbb{R}_+^{R \times S}$	total wages paid per region and sector
production	$\mathbb{R}_+^{R \times S}$	total production per region and sector
consumption	$\mathbb{R}_+^{R \times S}$	total consumption per region and sector
growth rate	$[0,1]^{R \times S}$	initial growth rate per region and sector, used to calibrate first period growth
inventory depreciation rate	$[0,1]^{R \times S}$	inventory depreciation rate per region and sector
capital stock table	$\mathbb{R}_+^{R \times S \times S}$	for each region, a capital stock table as used in national accounting
input output table fixed capital	$\mathbb{R}_+^{R \times S \times S}$	analogously for fixed ...
input output table circulating capital	$\mathbb{R}_+^{R \times S \times S}$	... and for circulating capital IOtables

How is the initial state obtained from the input?

At initialization, the model constructs and initializes the government, the financial system, the regions, and within regions the households, the sectors, and within sectors the firms as well as the import/export agents. The following text describes, for all agents and other entities, how the user input is transformed into the initial state of the model. Where parameters or state variables are simply set to the input value, this can be seen in Tables 2 to 7 above (entry "user input" in the column *initialization*), so it will not be repeated here.

Government   The initial `unemployment wage` is computed for each region by multiplying the `unemployment wage quotient` with the initial desired wage in the economy, which itself is obtained as the result of dividing a the total consumption in the region (sum over all sectors (all rows) in the region's column of the `consumption` array from the initial economic data) by the region's `number of households`.

Financial system   The initial `interest rate` is set to the `equilibrium interest rate` from the user input. The first value in the `unemployment rate memory` is computed over all regions together after the first period.

Regions   For each region, the number of households which imitate other households later (see numHouseholdsCopying) is computed by multiplying the *household replacement rate* with the `number of households`. The user given `number of households` are constructed, with the corresponding `region of origin`, initialized as described right below, and added to the region's `household list`. After the initialization of households, their consumption technology coefficients are mutated using the parameter for the `initial consumption coefficients variation` to determine the range of mutation for the parameters. Households thus start out with slightly different coefficients, which in sum approximate the region's `consumption` data input.

Households   The consumption technology coefficients are computed as follows from the `consumption` input: for each sector, the coefficient is determined by dividing the consumption of the sector in the region of origin by the total consumption (sum over all sectors) in that region. The thus computed coefficients are saved as `initial consumption technology coefficients`, and mutated to yield the consumption technology coefficients for each individual household.

Savings, `money` and `expected income` are set to the number obtained by dividing the `region of origin`'s total consumption by its `number of households`, thus distributing the region's consumption equally among the households from that region. The `fallback` is drawn uniformly from the interval $[0, 1)$. The `list of owned firms` and the `list of employers` are filled for the first time in the initialization of the firms, as described below.

Sectors   Sectors are numbered by their order of appearance within the model, their `names` are used only for the graphical output. Dividing the `number of workers` in a sector and region by the sum of workers in the region, that is, the sum over sectors, one obtains the ratio of workers in the sector. Then, the number of model workers in the sector is computed by multiplying this ratio with the `number of households` in the region and with the "employment rate", that is, $1 - U$ where U is the region's initial `unemployment rate`. Dividing the `wages` for the sector by this number of model workers in the sector, one obtains the initial wage per household. The sector's `benchmark wage` is set to this value.

The `fixed capital depreciation rate` ($d = (d_1, \ldots, d_S)$ in the following) for the input from all sectors in this sector is computed from the initial economic data as follows: considering input into this sector from all sectors j, the data to consider are found in this sector's columns in the `fixed capital input output matrix` and the `capital stock matrix`. (The regional specification is suppressed in the following description to avoid cluttered formulae, in the model, the computations are done for each region, and the result is a matrix instead of a vector.) Initial gross investment rates are computed

by dividing, sector-wise, the fixed input by the capital stock. The fixed capital depreciation rate then is given by

$$d_j = (i_j - g)/(1 + g)$$

where i is the initial gross investment rate of sector j

g denotes the initial growth rate of this sector from the input data

Thus norming with growth, one obtains the net depreciation for goods from all sectors in this sector. The thus computed depreciation rate is needed for the initialization of firms, as described in detail in Appendix B.

Within the sector, the number of firms to be created per region is computed depending on the user choice whether sectors have the same number of firms. If so, the number of firms per region is simply divided by the number of sectors. Otherwise, the number of firms is determined as the size of the sector's production relative to the overall production in the region. For the thus determined numbers of firms per region in the sector, the desired production is computed, again depending on a user choice: if firms have the same size, the production in the sector and region is divided by the number of firms, otherwise, a desired production is chosen randomly, but in such a way that the sum turns out to be the production. The thus determined numbers of firms are created, having the respective sector as their production sector, the region as their location, and the respective desired production values. The firms are initialized as described in the following section. After initialization, their input coefficients for labor, circulating and fixed capital are mutated, as described above for the households' consumption coefficients. Then, for the mutated coefficients, each firm's initial mark-up is computed as described also in the firm section.

The import/export agent for the sector is constructed and initialized as described in the respective section.

Firms When a firm is created, the current period is recorded (see periodCreated). The initialization of the firm depends partly on whether this period is at the initialization of the model, that is, period 0, or during a simulation. We describe the procedure for firms created in period 0 and mention differences for later periods in footnotes.

The rate at which desired production changes (rateProductionUp) is computed as the minimum of two user inputs: the growth rate from the initial economic data and the maximal production increase parameter for firms. The initial inventory is computed by multiplying the sales/inventory target with the desired production as computed in the sector initialization. Then, the desired production is multiplied by a factor $(1 + \text{rateProductionUp})$ so as to include growth for the first period.

As for the households, firms have initial input coefficients and input coefficients for labor, circulating and fixed capital. The following describes the computations for the initial input coefficients; the individual coefficients for each firm are then obtained from these by mutation.[6] For all initial coefficients, the given production in the sector is multiplied with $(1 + \text{rateProductionUp})$ to accommodate growth in the first period. The result of this computation, production including growth, is then used to norm the input coefficients, that is, the following elements are divided by production including growth: for labor, the wages input (of the firm's production sector and region), and for circulating capital, the corresponding user input circulating input coefficients (the IO table to be chosen is that of the firm's region and the column in the table to be used is the one of its production sector). The computation for the fixed input coefficients is more involved, and is therefore explicitly described only in Appendix B. The computations ensure consistency between the initial data and the investment behaviour of the firms, where the first period in which firms invest are distributed over some periods at the beginning of a simulation. The fixed capital is also initialized so as to be consistent with investment behaviour.

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[6]If the firm is created in a later period, it copies the coefficients from the best firm, instead of using mutated initial coefficients. It also copies wages and prices, instead of starting with the fixed user input values.

The firms' `target employment` is computed by dividing the labor needed for the `desired production`, that is `desired production` multiplied with the `labor input coefficient` and divided by the initial wage per household that was computed in the sector's initialization.

The firm's `owner` is drawn randomly according to a region-based probability distribution (Appendix A).[7]

To conclude the firm's initialization, the firm adjusts employment as described in Section 4.1.4.

Import Export The maximal number of goods that can be imported in a period is computed by multiplying the user input initial `import limitation factor` with the initial `production` in the sector, summed over all regions. The `import left` is set to the same number, and the analogous computation is carried out for export.

4.3 Run-time input

Does the model use input from external sources that drive the model? Are there data files or other models that represent these external processes? If so, what kind of data is required to feed the model at runtime? Include, if possible, references to relevant literature, or a description of the external models. If a model does not use external data, please state this here.

There is no external input to Lagom regiO while the model is running. All information needed for a simulation is present at the initialization. Even in the case of the sectors' state variable labor productivity, which can be determined "exogenously", at the initialization a `growth` rate is supplied, and this rate does not change during a model run.

5 Remarks on the guidelines

For giving a complete description of Lagom regiO, we had started out using the ODD protocol by Grimm et al. [2006, 2009]. While the protocol gave excellent basic guidelines for describing a multi-agent model, following it in detail proved very difficult. Meeting the aim of providing a description that enables the reader to re-implement the model was a pretty hopeless task for a model of the size of Lagom regiO. An audience for a description at that level of detail, which would easily have led to a text of 70 pages or more, seems hard to find. Further, the ODD protocol focuses on an applied model, that is, a model together with the data that it was used with to generate specific simulations. For a multi-purpose model as Lagom regiO, we preferred to give a more general description of the abstract model, that can then be referenced when describing various applied models. Questions like "how many entities of what type are there initially?" [Grimm et al., 2009, p. 9], refer to a simulation done with the model, not necessarily to the model itself. These become irrelevant when describing the abstract model alone, where the question should rather be "how is the initial number of entities of each type determined?". Therefore, some questions of the protocol would have made no sense for our purpose of describing the abstract model.

The ODD protocol would have been useful as guidelines for a description of Lagom regiO – when roughly followed.[8] However, the protocol authors explicitly state that

Using ODD means using exactly these identifiers in the order we specify here. We have seen manuscripts which claimed to "largely" follow the ODD protocol, but the order of elements was changed, elements were lumped, modified identifiers were used, or entire elements omitted. This inconsistent use was to be expected in a new standard and it pointed to some particularly unclear parts of ODDs original description. The purpose of a standard, however, is to be used just as defined.

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[7]If the firm is created at a later period, only the region is drawn, and the richest household from that region becomes the owner, transferring his savings to the firm to set the initial production level.

[8]In fact, a model from the Lagom family is described using ODD in Mandel et al. [2010]. However, it does not provide the full level of detail necessary for a re-implementation of the model, but makes reference to the documentation of the model code for several details.

Thus, roughly following the protocol is not what it was intended for. However, a description that would strictly follow the protocol would have been difficult for Lagom regiO, and probably useless because much too detailed.

The ABM guidelines used instead are more appropriate for the description of Lagom regiO. This is no surprise as they were developed explicitly with the purpose of describing economic models, and as they are intended as guidelines for describing abstract rather than applied models. The design concepts were thus more closely fitting for the context in which Lagom regiO is to be applied.

Concerning this particular model, which does not explicitly model message passing or similar procedures, the focus of the guidelines seemed to be somewhat strongly on interaction protocols. Similarly, institutions like markets are modelled rather indirectly in Lagom models (based on the observation of samples of agents and bilateral interactions with an in some way "best" agent from the sample), and thus some questions about institutions in the Design Concepts Section of the guidelines were superfluous for this model. While all points should be addressed in a description using the guidelines, we suggest to leave answers to the single questions under each point optional. Then, the model description can answer the questions which seem relevant for the model, without having to state repeatedly that other questions do not apply.

Further, the point "Description of Agents and Other Entities, action and interaction" seemed a bit repetitive of information that was already given in the Overview Section. Of course, it may be necessary to provide further detail on agents and other entities here, however, the question "What, in detail, are the Agents and Other Entities in the model?" should already have been answered among the points "Agents" and "Other entities" in the Overview Section. It might be useful to change the description of this bullet point so that the question for detail about agents and other entities immediately links to the description of their state variables and parameters. In the current version, the intermediate question "What Agent/Other Entity does what and in which order?" interrupts this link, and should probably be moved further down for a smoother flow of elements of the description compiled according to the guidelines.

Finally, the need to list all state variables of an agent might be relaxed. In the code, some variables may be defined only for computational or performance reasons. For a description of a model as a conceptual piece of work, however, it should suffice to list those variables that are relevant for an agent's behaviour and characteristics. That is, given that the aim of these guidelines is not to enable a re-implementation of a model, the Section Functional Specification in the current guidelines is probably more technically focused than necessary.

6   Conclusions and outlook

The present text describes the model Lagom regiO and should provide a thorough insight into it. Readers interested in further detail of the implementation can request the source code under LINK. This text was meant as a general description of the model that can serve as a reference for other works which present simulation results obtained with the model.

At the same time, the present text is a first application of the Dahlem ABM documentation guidelines. Following these guidelines was an interesting work. With some minor difficulties, it showed that the guidelines are a feasible construction for describing an abstract model of this "size".

Further work with the model can be classified into three broad tasks. First, on a rather practical side, the model is currently being calibrated for using it with a dataset for Europe and the Mediterranean. Our choice of the resolution of the model is 5×5: the five sectors are Agriculture, Energy, Construction, Manufacturing and Services, and the five regions are Germany, the CoreEU (Austria, Belgium, Denmark, Finland, France, Luxembourg, Netherlands, Sweden, UK), the PeripheryEU (Cyprus, Greece, Ireland, Italy, Malta, Portugal, Spain) the EastEU (Bulgaria, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Slovakia, Slovenia) and the Southern Shore Mediterranean countries. As a first task in this setting, we want to reproduce stylized facts about the economy in this region of the world for the decade from 1995 to 2005.

Second, further development of the model will become necessary as resulting from the first task. For example, population growth, which differs largely between the EU and the Southern Shore Mediterranean countries, needs to be represented in the model in order to relate results to real economic developments such as growth per capita etc. Also, to investigate economically driven migration, such phenomena as population growth need to be included. Another goal is to further specify the model so that renewable energies can be explicitly represented, in order to investigate emission reduction policies. The government agent, so far rather rudimentarily represented, should also be further developed, so that more complex questions of policy making can be studied using the model.

Finally, from a mathematical point of view, the model should be simplified in order to investigate its mathematical structure in more detail. The huge amount of state variables present at the moment makes it difficult to grasp what actually happens in a simulation from a mathematical perspective. In a sense, agents that are more "simple-minded", that is, use much less information to make their decisions, or have less different actions to perform, could facilitate the analysis of the underlying networks, of effects of imitation and mutation, or of the really essential economic features arising from the kind of sample-based bilateral interactions. Investigating these features from a more theoretical perspective seems worthwhile for example for the formation of prices, in comparison to the Walrasian Auctioneer paradigm in standard economics.

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