Pareto-improving redistribution in a competitive framework

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Abstract

Economic theory has proved that income redistribution in imperfect competitive markets can increase social welfare and lead to pareto-improving situations. This paper shows that under certain assumptions self-financing tax subsidy schemes can also have pareto-improving effects in perfect competitive markets, which stem from external economies of scale.

Keywords: Income redistribution; welfare; external economies of scale.

JEL classification: D31; D64; H2; L13.

1 Introduction

Previous studies have illustrated the fact that transferring resources between agents does not always hurt the donors. A government intervention through income transfers, for instance, could correct the market failures associated with imperfect competition and restore the Pareto efficiency. Dillén (1995) tackles this issue in a general equilibrium setting, while Thépot (2003) explores the case of a monopoly in which

* Helpful comments on earlier versions of this paper were received from Francisco Galera and Luis A. Gil-Alana. At the writing stage I benefited from the suggestions made by Zubin Mistry. Any errors are my own. Financial support from the Ministerio de Ciencia y Tecnologia (SEJ2004-04649, Spain) is gratefully acknowledged. Contact address: Universitat Internacional de Catalunya. Immaculada 22, 08017 Barcelona. Tel: (34) 932541800 - Fax: (34) 934187673. The author is also associate lecturer at Universidad Pompeu-Fabra and, at the time of writing the paper, was academic visitor at Imperial College London. *E-mail:*(pgarcia@cir.uic.es).

the implementation of a tax subsidy scheme brings about welfare improvements for all the agents involved.[1] Thépot uses the transfer mechanism as a device to increase the price elasticity of demand, which in the monopoly case must lead to a lower price. Under some assumptions, such a reduction in the price compensates the payment made by the contributing consumers, thereby leading to pareto-improving situations. However, note that even if his analysis goes beyond the monopoly case, it is always constrained to situations where some degree of market power exists.

In this paper a pareto-improving income redistribution is studied in the context of competitive markets. The analysis is made on the basis that economies of scale exist in an industry, implying that the aggregate supply function presents a negative slope in the long run. Using the distinction between internal and external economies of scale introduced by Marshall (1920, p.221), we recognize external economies when a fall in unit costs arises from an expansion of an industry (without a necessary increase in the size of individual firms). If the demand for a certain good becomes more elastic in this context, the total amount of trade in the industry can increase while the equilibrium price declines. Then, any policy that increases the elasticity of demand can potentially lead to pareto-improving situations.

The aim of this paper is to show how tax subsidy schemes can be implemented so that they lead to equilibria in which all the economic agents involved achieve a more advantageous situation.[2] Our approach is interesting because, in contrast with previous studies, it is applied within the context of competitive industries. Note that the results obtained by Dillén (1995) and Thépot (2003) were derived through cor-

[1] Dillén (1995) recommends that corrective tax-subsidy programs should be implemented by imposing taxes if one expects small effects on output while using subsidies wherever comparatively large output effects are foreseen. In Chipman (1970) the case of a general equilibrium model with perfect competition is examined in the presence of economies of scale.

[2] The possibility that income distribution improves along with welfare has been explored by applying different methodologies. As regards, for instance, Turkey's agricultural trade, Atici and Kennedy (2005) discuss alternative tax scheme policies involving the redistribution of the resulting revenue, which leads to pareto improving solutions. A different issue is whether government intervention can lead to improving efficiency allocations. Barros (2004) explores the case of the defense sector in Portugal, concluding that the impact of government subsidies is dubious. Similarly, using an stochastic frontier approach, Barros (2005) concludes that incentive regulation is not enforcing greater efficiency in the Portuguese defense industry.

recting market failures which arise in imperfect competition frameworks. However, pareto-improving policies are not surprising when some degree of market power exists, since regulations can bring forth greater efficiency by moving the equilibrium closer to a competitive situation. To ensure that the redistributive transfer does only affect the demand function in this industry (instead of transforming the elasticity of demand of other markets) we might consider that the tax applies upon consumption and the subsidy is granted in the form of a voucher.[3]

The paper is organized as follows. Having described in the introduction the motivation of the study, in Section 2 two alternative settings for the model are developed. Section 2.1 examines the case of a linear income transfer like the one described in Thépot (2003), but within a competitive framework. In Section 2.2 a more plausible implementation of the income redistribution is both described and analyzed. Section 3 comments on the implications of the main results, while the final section concludes.

2 Preliminaries of the model

The issue of pareto-improving income redistributions within a competitive framework can be studied using very generic specification of the underlying functions. The basic idea of the paper is illustrated in *Figure 1.* The initial demand of the industry $D_1(Q)$ is depicted as (sFE_1GH1) and the long-run aggregate supply as (dE_1E_2). The equilibrium is initially reached at (P_1, Q_1), where P denotes the price and Q accounts for the total quantity exchanged in the industry.

Figure 1

[3] The scope of this paper is presumably broader if considering the case of highly esteemed goods. Furthermore, it is more challenging if dealing with commodities which are supposed to be consumed at a rate of one-unit of good per period, like some types of medicines or the access to education. In such contexts, the ability of a policy to enlarge the total amount exchanged in the industry is actually its capability to enlarge the share of population that has access to this particular good. This is well suited to our purpose insofar as economies of scale can potentially arise in industries for these types of goods. A different issue is to state to what extent the possibility of implementing pareto-improving policies can actually be applied in existing industries.

Any change in the demand function increasing the equilibrium quantity while diminishing the price is potentially able to draw the approval of all the economic agents. Consider the case of an income redistributive policy which transforms the demand curve into D_2, (rFE_2H1) in *Figure 1.* This change provokes an alteration of the social welfare. In a competitive industry, the total welfare coincides with the consumer surplus, which for D_1 was given by the area $\widehat{sP_1E_1}$, whereas for D_2 is $\widehat{rP_2E_2}$. Accordingly, the increasing-welfare function is defined as:

$$W = \int_0^{Q_2} D_2(Q)dQ - P_2 \cdot Q_2 - \int_0^{Q_1} D_1(Q)dQ + P_1 \cdot Q_1 \tag{1}$$

This equation evaluates the increase in welfare derived from the implementation of a policy affecting the demand in the form shown in *Figure 1.* Any desirable social policy must yield increases in welfare, thereby enforcing expression (1) to be positive. Yet, for a policy to be pareto-improving, it must fulfill the more demanding requirement stated in the following inequality:

- Condition I: A pareto-improving transfer requires that all the consumers enjoy a welfare level at least as high as the one they had before transfer; i.e.,

$$P_1 - P_2 \geq t \tag{2}$$

Note that if inequality (2) is satisfied the increase in social welfare is ensured, but this relationship does not work in the opposite direction. We aim to illustrate the theoretical possibility of implementing policies leading to pareto-improving situations. For the sake of clarity, the two models presented thereafter are settled for linear specifications of the demand and supply functions. Further analytical extensions might be advisable in order to develop this result in a more general setting.

Consider the case of a market in which a continuum of consumers exists. The individuals' preferences are assumed to be identical, whereas the disposable income for this good differs among them. Firstly, the total quantity demanded in the industry at zero price is normalized to 1. Secondly, if the income of the population is uniformly distributed, the primary aggregate demand $D_1(Q)$ is of the form: $P = s - sQ$. Note that both linearity and normalization of the demand function are made for the sake of simplicity and do not entail loss of generality.[4]

We assume the following total cost function: $CT(x, Q) = F + ax^2 - bxQ$, where x is the production of the individual firm while F is the fixed set-up cost. The corresponding aggregate supply in the industry is a linear function with negative slope in the long run: $P = 2\sqrt{aF} - bQ$. In this expression the degree of external economies of scale is driven by b. Note that the form of the total cost function is such that the external economies do not alter the optimal level of production of the individual firms: $x = \sqrt{F/a}$. To ensure the existence of equilibrium in the industry (for positive values of price and quantity), the following assumption upon the parameters is made:

$$b < 2\sqrt{aF} < s \tag{3}$$

2.1 The case of a linear transfer redistribution

In this framework, a redistribution of income can be viewed as a mechanism aimed at increasing the price elasticity. Consider the linear redistributive transfer implemented through a self-financing tax subsidy scheme. This transfer redistribution works at splitting the whole population to three groups: (i) the donors (ii) the recipients, and (iii) the excluded consumers. Typically, the policy could be designed to maximize the increase in welfare or, alternatively, to minimize the number of excluded individuals.

Given the aggregate demand $D_1(Q)$, note that the total potential consumption

[4] Following the lines of Thépot (2003) we could consider a one-unit goods market. The analysis could then be applied to markets in which the good or service is considered something indispensable, like vaccinations, access to education, etc. Otherwise, the analysis is valid anyway if the tax is assigned upon consumption and the subsidy only applies to trade with this particular good. In this framework, the consumers' reservation prices for one unit of good will directly stem from their income and then s stands for the income of the richest consumer.

in the industry is $Q = 1$. A linear redistributive transfer would distort the effective demand for the Q_2 consumers affected, leading to the demand function D_2.

Consider the case in which the linear transfer is implemented in the form of negative income tax scheme. The redistributive transfer may then be defined by:

$$T(Q) = \begin{cases} D_2(Q) - D_1(Q) & \text{for} \quad 0 < Q < Q_2 \\ 0 & \text{for} \quad Q_2 < Q < 1 \end{cases} \tag{4}$$

By impossing a self-financing transfer, characterized through an identical tax and subsidy rate t, half of the Q_2 consumers receive a subsidy (of t times their personal income), while the other half pay a tax. The remaining consumers $(1 - Q_2)$ experience neither a change in their income nor in their effective demand. The following expression states the condition for the transfer to be self-financing.

- Condition II: A self-financing income transfer redistribution requires that the total tax payment must be equal to the total subsidy; i.e.,

$$\int_0^{Q_i} D_1(Q) - D_2(Q)dQ = \int_{Q_i}^{Q_2} D_2(Q) - D_1(Q)dQ \tag{5}$$

This condition permits specifying the expression for the new effective demand. In the linear setting the intercept of D_2 is given by $s - t$. In order to obtain the slope, we solve $\int_0^{Q_2}(s - t - \beta Q) - (s - sQ)dQ = 0$, and express the result in terms of β. Then, once the self-financing transfer has been done, the demand function becomes:

$$D_2(Q) = \begin{cases} s - t - sQ - \frac{2tQ}{Q_2} & \text{for} \quad 0 < Q < Q_2 \\ s - sQ & \text{for} \quad Q_2 < Q < 1 \end{cases}$$

Next, we examine the conditions for a pareto-improving transfer. In our model, P_1 is given by $\frac{s(2\sqrt{aF}-b)}{s-b}$, while P_2 depends on t, as expressed by $\frac{s(2\sqrt{aF}-b)-bt}{s-b}$. In the context of the linear case, condition (2) can be expressed in a more specific manner.

$$2b \geq s \tag{6}$$

A pareto-improving transfer requires that a certain relationship between the degree of external economies of scale and the slope of the demand be fulfilled. Of course, inasmuch as the previous inequality is satisfied, the growth in total social welfare is only a natural corollary. This can be seen easily by looking at the increasing welfare function, which in this setting of the model takes the form:

$$W(t) = \frac{t}{2\,(s-b)^2}\left((2b-s)t + 2b\left(s - 2\sqrt{aF}\right)\right) \tag{7}$$

On the grounds that expressions (3) and (7) hold, $W(t)$ happens to be positive and, hence, establishing a positive t always results in improvements of social welfare.

Proposition 2.1. *In the linear case, the optimal linear transfer (to maximize the increase in welfare) is achieved at the level of the full-coverage tax rate.*

Proof: The value of $W(t)$ increases along with t, implying that its maximum value is achieved at the greater possible t. To show that, we compute the first derivative of this function with respect to t:

$$W'(t) = \frac{\partial W(t)}{\partial t} = \frac{1}{(s-b)^2}\left((2b-s)t + b\left(s - 2\sqrt{aF}\right)\right)$$

For values of the parameters in agreement with (3) and (7), and for any $t > 0$, this expression is positive, meaning that the improve in welfare grows bigger as t increases. Since the demand had been normalized, $Q \in (0,1)$, the policy that maximizes the total social welfare consists of implementing the t of full coverage, where $Q_2 = 1$. Therefore, the optimal solution, whenever $2\sqrt{aF} > b \geq \frac{s}{2} > \sqrt{aF}$, is given by:

$$t^* = t_f = 2\sqrt{aF} - b \tag{8}$$

where t^* denotes the tax-subsidy rate that maximizes the increase in welfare, and t_f denotes the rate of full coverage. This result is straightforward and yet quite surprising: the best policy is enforcing a perfect full coverage for the market.

This first example has been helpful to illustrate that pareto-improving policies can be implemented when positive economies of scale exist in an industry. Nevertheless, the result depends crucially on condition (7) and the linear transfer redistribution seems to be difficult to apply in reality. These are two good reasons for developing the following more plausible example.

2.2 A self-financing transfer in two steps

As in the previous example, for the analysis to be properly applied, it must deal with situations in which consumers are disposed to pay as much money as they can afford. Then, if economies of scale are present in the industry, we advocate that a self-financing income transfer can be implemented in two steps, yielding plausible pareto-improving situations.

In a market with these characteristics, the initial equilibrium indicates how many individuals can initially afford this good and what price they pay for it:

$$(Q_1, P_1) = \left(\frac{s - 2\sqrt{aF}}{s-b}, \frac{2\sqrt{aF} - b}{s-b} s \right)$$

We assume that the government is able to evaluate accurately the external economies of scale that operate in the industry. Note that our departure from *Section 2.1* is both due to the different nature of the tax subsidy scheme considered and to the timing of its implementation. On one hand, the tax consists now of a fixed identical amount paid by each of the Q_1 contributing consumers, whereas the subsidy is assumed to be proportional to the lack of disposable income of subsidized consumers. On the other hand, we presume that tax collecting takes place in an early stage (when the first group of consumers buy the good) while the provision of the subsidy occurs only afterwards. The process is described in *Figure 2* where, in accordance with our posterior analysis, the supply and demand curves are linear functions.

Figure 2

The issue allows for different approaches, among which we have chosen here the policy whose implementation is possibly easier to carry out than the others. For a policy to have greater credibility it must provide all the agents involved with incentives for participation. The way in which the transfer is implemented here implies that the producers eventually get extraordinary profits, although this feature is not essential to our results.[5]

Had the government not intervened in the industry, Q_1 would be the total quantity exchanged in the industry at price P_1. However, the provision of subsidies among individuals beyond Q_1 will permit a second wave of transactions in the market. The process may be described in two subsequent steps:

1. Some individuals pay the higher initial price P_1, which is considered the only way through which they can ensure enjoying the provision of this good. Otherwise, if waiting for the second round, they face a certain risk of not getting the commodity, as it may be exhausted. Then, consumers who are able to afford P_1 (those located between 0 and Q_1) are certainly willing to pay the initial price, on the grounds that they were already paying this price at the beginning.

2. At a second stage, consumers located between Q_1 and Q_2 receive from the government individual subsidies of the size that permits them to afford the subsequent prices. We consider that the price for each additional unit of this good declines along with the long-run aggregate supply function, starting at the level of P_1 and ending at the final price P_2.

[5] Various other arrangements could be conceived for the income transfer redistribution, which would lead to similar results to those we present here. The design of a transfer accounting for positive producers' surplus makes goes beyond the requirements of a pareto-improving transfer, since all the agents end up with improved welfare situations.

Firstly, note that the two-stage setting avoids any possibility of cheating on the part of the consumers. One might be suspicious that individuals who can actually afford the initial price would also try to benefit from the subsidy. Yet, if the government gives no information about the quantity that will be available at the subsidized price, such a threat of inefficiency in the allocation of the subsidy is avoided, given that the fear of being excluded prevents any attempts at cheating of the Q_1 initial consumers. Secondly, the manner in which the transfer is designed generates profits for the producers, thereby making them ready to meet the petitions made by the government. Consider, for instance, that the suppliers are encouraged to produce a larger amount of the commodity through the promise that the public sector will purchase the exceeding quantity (at the price indicated by the shape of long-run supply). Then, the government intervention enlarges the size of the market from Q_1 to Q_2, which permits producers to reduce the minimum average cost they incur from P_1 to P_2. Since the government agreed to pay prices greater than these, the policy conveys extraordinary profits for the producers while giving access to the market to additional consumers. The new demand function can then be considered as:

$$D(Q) = \begin{cases} s - t - sQ & \text{for} \quad 0 < Q < Q_1 \\ 2\sqrt{aF} - bQ & \text{for} \quad Q_1 < Q < Q_2 \\ s - sQ & \text{for} \quad Q_2 < Q < 1 \end{cases}$$

Note that, after the intervention, the competitive price in the industry is in fact P_2, indicating that the initial buyers are at the same time contributing consumers: they pay a higher price at the level of P_1. This account of the facts implies that the producers keep the amount $\widehat{kFIP_2}$ as profits. Alternatively, the arrangement could be done for the Q_1 initial consumers to pay the price $P_2 + t$, implying that the additional surplus would go to them, instead of the producers. Regardless of whether we assume the former or the latter, the government collects a total tax of $\widehat{sE_1Fr}$ with which to afford the subsidy payment.

To prevent cheating on the part of the second group of consumers, the subsidies could be granted through an auction process. Each participating consumer purchases one-unit good per period, but paying a different price. The first group of consumer paid P_1, but those located within Q_1 and Q_2, given that they do not know the quantity of good that will be available, bet for the good offering to pay as much as indicated by the aggregate demand. This is still less than the price the producers were promised to receive, and therefore, the government has to add part of the payment. The process finishes in Q_2, where the total tax equals the subsidy.

At this stage we are in a position to analyze the effects of this income transfer program. The initial assumptions on the slope and intercepts of the functions stated in (3) still apply. Besides, in order for the transfer redistribution to be a self-financing one, the corresponding condition ought to be fulfilled, which yields:

$$tQ_1 = \int_{Q_1}^{Q_2} 2\sqrt{aF} - bQ - (s - sQ)dQ \tag{9}$$

Condition (10), in the present framework, leads to establishing a relationship between the value of the tax and the total number of consumers who eventually buy the good, Q_2. Specifically, for the transfer programme to be self-financing, we need:

$$Q_2(t) = \frac{s - 2\sqrt{aF} + \sqrt{2t}\sqrt{s - 2\sqrt{aF}}}{s - b} \tag{10}$$

It is also possible to express the change in prices as a function of t. Then, on the basis that the relationship between t and Q_2 shown in (10) holds, the condition for a pareto-improving redistribution now takes the following form:

$$t \leq \frac{s - 2\sqrt{aF}}{(s - b)^2} 2b^2 \tag{11}$$

As *Figure 2* illustrates, the increase in social welfare is the sum of areas $\widehat{kFIP_2}$

and $\widehat{E_1 I E_2}$. We can then define the specific form of the increasing-welfare function:

$$W(t) = \frac{s - 2\sqrt{aF}}{(s-b)^2}\left((2b-s)t + b\sqrt{2t}\sqrt{s - 2\sqrt{aF}}\right) \tag{12}$$

This expression is going to be useful in determining the optimal tax t^*, as well as in checking whether or not other possible values of t convey improvements in welfare. Now we are ready to obtain and examine the values of t for the three most significant situations: (I) the tax level t_f to reach the full coverage; (II) the level in which the tax t_p equals the drop in prices; and (III) the optimal tax t^* yielding the maximum increase in social welfare. These three values are going to be calculated in terms of the parameters. Thereafter, we discuss the conditions in which they have to be actually implemented.

(I) **Tax t_f of full-coverage.** In order to guarantee that the whole population gains access to the good, the government might apply the tax associated with full coverage. This solution corresponds to the point where the demand is perfectly satisfied, so that t_f is obtained by substitution of $Q_2 = 1$ in expression (10).

$$t_f = \frac{1}{2}\frac{(2\sqrt{aF} - b)^2}{s - 2\sqrt{aF}} \tag{13}$$

(II) **Tax t_p equalizing the drop in the price.** Another conceivable policy is establishing the level of t which provokes a market price drop of exactly the same amount as the tax. The tax level t_p is then calculated as the value for which condition (11) holds as a strict equality. The application of this level of income redistribution bears increases of welfare for the producers and for the second group of consumers.

$$t_p = \frac{s - 2\sqrt{aF}}{(s-b)^2}2b^2 \tag{14}$$

(III) **Tax t^* maximizing the increase in welfare.** The optimal level of taxation is such that it entails the greater growth of social welfare. Note that, if $s \leq 2b$, the function $W(t)$ does always increase along with t. If this is the case, the maximum level of welfare corresponds to a corner solution defined by the greatest feasible value of t. On the contrary, whenever $s > 2b$, the optimal level of taxation is calculated as the critical value of expression (12). The *Appendix* shows that the critical value defines a maximum for $s > 2b$. This critical value is congruent only if satisfying a number of additional conditions that we discuss later and is given by:

$$t^* = \frac{1}{2}\frac{s - 2\sqrt{aF}}{(s-2b)^2}b^2 \tag{15}$$

Some constraints must hold that each of the previous levels of t may be effectively implemented. These conditions depend on the value of the different parameters, which eventually determine how the magnitude of the three tax levels relate to each other. The discussion of this point is crucial in determining the outcome of the transfer programmes and the situations in which each of them ought to be implemented. Note that our approach limit us to considering those policies which are affordable, feasible and pareto-improving at the same time. The first issue is already enforced in our framework, which always considers self-financing income transfers, while the two other characteristics need further examination.

For the policy to be relevant, t must lay inside the feasible region. Given that in our model $Q \in (0, 1)$, the quantity associated with the prevailing tax level ought to be smaller than 1. (The redistribution scheme can never involve a number of consumers beyond the total number of them in the market). Alternatively, we can state that for the policy to be potentially applicable, the size of the tax cannot be greater than the value in expression (13), which can be interpreted as a feasible constraint.[6]

In addition to that, the tax level may require additional conditions that depend

[6] The three values of t previously reported are relevant depending on whether or not various conditions hold. Since these restrictions take the form of inequalities, the issue can be addressed applying the Kunh-Tucker technique. We have preferred adopting instead a more discursive analysis that is equally valid, thereby leading to the same conclusions.

on the goal aimed by the government. Then, in what follows, our analysis is arranged to suit two chief cases, which illustrate the most judicious policies that can possibly be implemented: to maximize the total increase in welfare; and to maximize the degree of coverage in the industry. The desirability of the former is beyond argument, whereas latter may often be a more prudent choice if considering the benefit of the poorest consumers. (Those who cannot initially afford the good).

2.2.1 The government seeks maximizing the market coverage.

The scope of providing access to the industry to as many individuals as possible means that the government faces a twofold choice between (13) and (14). The function of total quantity transacted in the industry is an increasing function of t, implying that to reach the maximum level of coverage we look for the greatest value of t obeying the constraints. The solution to this problem is straightforward, since only the smallest of these two values is actually relevant. Effectively, the latter is not feasible if t_f is smaller than t_p, while the former infringes the pareto-improving condition if the opposite relationship holds. As a consequence, t_p is adopted if the following requirement, expressed in accordance with the parameters of our model, is verified:

$$2\sqrt{aF} \geq \frac{b(3s-b)}{s+b} \tag{16}$$

Then, the level of coverage and the corresponding increase in welfare for level t_p are calculated respectively as:

$$Q_2(t_p) = \frac{(s-2\sqrt{aF})(s+b)}{(s-b)^2} \quad \text{and} \quad W(t_p) = \frac{(s-2\sqrt{aF})^2}{(s-b)^4}2b^3$$

On the contrary, if condition (16) does not hold, the tax level must be settled as indicated by expression (14). It is important to note that the government succeeds anyway in its purpose of maximizing coverage. Moreover, in this case, procuring the maximum coverage leads to the most desirable outcome: the perfect coverage of the market. Obviously, the total quantity achieved from the setting of the full-coverage rate t_f takes the value 1, while the increase in welfare is given by the value of $W(t_f)$:

$$Q_2(t_f) = 1 \quad \text{and} \quad W(t_f) = \frac{s - 2\sqrt{aF}}{(s-b)^2}\left(2b(s-b) - s(\sqrt{aF} - b)\right)$$

Regardless of the size of the transfer, Q_2 is clearly greater than Q_1. Similarly, no matter which of the two tax levels is effectively adopted, welfare increases once the transfer has taken place. This can be verified by looking at the expressions $W(t_f)$ and $W(t_p)$: both are positive for parameters abide by condition (3).

2.2.2 The government aims to maximize the increase in total welfare.

Consider now that the intervention is designed to deliver the greatest possible increase in welfare. The value t^* is then calculated as the critical value of equation (12), but it defines a maximum only when the second order condition is verified:

$$2b < s \tag{17}$$

Otherwise, if this inequality does not hold, $W(t)$ is always an increasing function of t and the maximum welfare is found in a corner solution given again by the smallest value between t_f and t_p. Yet, even if condition (17) is fulfilled, it does not necessary mean that t^* can be applied in order to maximize welfare. This is because t^* is certainly unattainable if just one of the other additional conditions fails.

Let us analyze, then, these two other required conditions, on the basis that (17) holds true. First, for the tax level to be feasible it must be smaller than (13). (It is imperative that $t^* < t_f$; which is, naturally, equivalent to imposing $Q_2(t^*) < 1$). In accordance with the parameters of our model, this condition means that:

$$2b < 2\sqrt{aF} \tag{18}$$

In addition, for the policy to be pareto-improving the tax level must be smaller than expression (14), or $t^* < t_p$. Again, provided that condition (17) holds, the pareto-improving requirement imposes a more demanding relationship:

$$3b < s \tag{19}$$

The last result implies that expression (17) can be disregarded, since it is always fulfilled under the tighter condition (19). As a consequence, it can be stated that t^*, calculated as indicated by (15), will be the prevailing tax level insofar as conditions (18) and (19) happen to be true. The level of coverage and the welfare associated with this taxation are obtained respectively by substitution of t^* in expressions (10) and (12), and are positive for values of s strictly greater than $2b$:

$$Q_2(t^*) = \frac{s - 2\sqrt{aF}}{s - 2b} \quad \text{and} \quad W(t^*) = \frac{1}{2}\frac{(s - 2\sqrt{aF})^2}{(s-b)^2(s-2b)}b^2$$

In any other case (whenever one of the necessary conditions, either (18) or (19), fails) the optimal level of taxation to maximize welfare is determined by the minimum value between expressions (13) and (14).

3 Implications and policy recommendations

At this stage, we can consider what policy recommendations stem from our results. To illustrate the discussion, *Figure 3* summarizes the stylized findings arrived at so far. To avoid a confusing 3-dimensional graph, the representation is done in R^2, where the various relevant constraints are plotted in the space (s, b).

Figure 3

On one hand, the function following the relationship given in expression (16) determines the cases in which t_f will be adopted and those in which, instead, t_p will be the prescribed tax level. We represent equation (16) as a strict equality. Note that this function changes its position depending on the different values taken by $2\sqrt{aF}$. On the other hand, the necessary conditions (18) and (19) delimit the area where value t^* should be implemented (shadowed in *Figure 3*) in order to maximize welfare rather than coverage.

The examination of the relationship between the parameters teaches us a number of things. For instance, the greater s is (intercept and slope of the aggregate demand) with respect to the other parameters, the greater the chances of reaching full-coverage in the market. We also learn that parameters a and F always operate together, meaning that what matters is the intercept of the aggregate supply (which includes the fixed set-up cost as well as the quadratic element of the variable cost). The greater $s\sqrt{aF}$ the more difficult reaching the full-coverage becomes.

Another implication of the model is that the preferred tax level is often the same regardless of whether the government wishes to maximize coverage or welfare. This is due to the fact that the best outcome is often the corner solution in any case. Only if b is sufficiently small, relative to the other parameters, it might be the case that the aim of maximizing welfare (associated in this case with an interior solution) deviates from the optimum level of pursuing the maximum coverage. Nevertheless, the main result of this paper is still to be stated and further discussed.

Proposition 3.1. *In competitive industries with economies of scale, it is always possible to implement self-financing tax subsidy programmes leading to pareto-improving situations in which additional consumers gain access to the market.*

Naturally, this proposition must be interpreted within the context of our model and on the grounds that some assumptions are respected. For instance, it needs to be the case that either the good is absolutely indispensable for consumers or the subsidy is not granted in cash, but as a voucher only exchangeable for this good.

Proof: In our model, either t_p is feasible or t_f is pareto-improving. Then, it is always the case that a positive t implies greater levels of coverage and welfare. The increase in coverage is ensured by the fact that $Q_2(t_f)$ and $Q_2(t_p)$ are both greater that Q_1.

$$Q_2(t_p) = \frac{s+b}{s-b}Q_1 \quad \text{and} \quad Q_2(t_f) = \frac{s-b}{s-\sqrt{aF}}Q_1$$

Note that factors $\frac{s+b}{s-b}$ and $\frac{s-b}{s-\sqrt{aF}}$ are greater than 1 for values of the parameters in accordance with condition (3). Similarly, there is no doubt about the increase in welfare once the transfer has been introduced. The value of $W(t_p)$ is always positive, whereas $W(t_f)$ is positive for the relevant case in which t_f is chosen, which occurs if expression (16) does not hold. But if this is the case, $2b(s-b)-s(2\sqrt{aF}-b) > b2\sqrt{aF}$ and, hence, $W(t_f)$ is greater than the following positive magnitude:

$$W(t_f) > \frac{s-2\sqrt{aF}}{(s-b)^2}b2\sqrt{aF}$$

Even if the result was derived within a linear framework, the analysis of *Figure 2* makes it manifest that it would remain the same for a broader (non-linear) setting.

4 Conclusion

This paper has advocated that implementing income transfer policies in competitive markets might be beneficial to society. The capability of the implemented policies to influence the elasticity of demand in the desired way, together with economies of scale in the industry, permits the realization of pareto-improving situations. The debate on the compatibility of economies of scale with perfect competition finds support in the literature, provided that the economies are external to the firms, which is precisely the topic analyzed here.[7] Besides that, a number of studies have documented examples of industries that experience external economies of scale. This feature is commonplace in various kinds of manufacturing industries, but it also

[7] For instance, Meade (1952, p. 33) states that perfect competition can prevail under conditions of increasing returns as long as these economies are external to individual firms. For a broader discussion on this issue, see Chipman (1965, p. 736-49). A description of models of trade with imperfect competition and scale economies is can be found in Salvatore (1993).

affects some of the most prominent markets, like those for cultural goods.[8] The presence of economies of scale in these types of industries is particularly well suited to the scope of this study.

The analysis of this issue, developed for linear specification of the functions, has focussed on two alternative settings of the redistributive transfer. Firstly, the case of a linear transfer redistribution has been examined. The eventual success of the policy to augment social welfare crucially depends on the relative magnitude of the external economies of scale with respect to the elasticity of the aggregate demand. In particular, the effective applicability of a pareto-improving policy is only warranted if the slope of the aggregate demand is smaller that twice the slope of the long-run supply in the industry. Furthermore, the best policy in this context is applying the linear transfer which leads to the full-coverage of the market.

Secondly, we have shown how a redistributive transfer can be implemented in two steps. The issue is encouraging, since these types of transfers are always implementable and bring about gains for all the economic agents. If the transfer is arranged in this way, the taxes collected in the first step of the process do not harm the incumbent consumers, while the subsidy improves the welfare of new consumers. To avoid the threat of cheating, the provision of the subsidies could be implemented through an auctioneer mechanism. This gives rise to a second wave of transactions in the industry, enhancing social welfare and gaining access to survival commodities for individuals who were initially excluded.

Our findings, even if attained from the analysis of linear functions, are valid beyond the linear framework. The study of non-linear specifications of the functions merits further research, although the basic conclusions presented here do not depend on linearity. Unlike other papers, our results were not derived from correcting market

[8] Broadberry and Marrison (2002) report strong evidence for the presence of external economies of scale in the cotton industry. Based on available data on trade, Marvasti (1994) stresses how highly populated countries dominate exportation of most cultural products such as books, motion pictures, recorded music, newspapers, etc. He ventures that the comparative advantage of these countries basically stems from economies of scale granted by population size.

failures associated with market power, but stem from the greater efficiency of taking into account external economies of scale.

5 Appendix

The increasing-welfare function, for the case in which a redistributing transfer is implemented in two steps, was reported in expression (12), which reads:

$$W(t) = \frac{s-2\sqrt{aF}}{(s-b)^2}\left((2b-s)t + b\sqrt{2t}\sqrt{s-2\sqrt{aF}}\right)$$

The first order condition to determine the critical value is then:

$$W'(t) = \frac{\partial W(t)}{\partial t} = \frac{s-2\sqrt{aF}}{(s-b)^2}\left((2b-s) + \frac{b\sqrt{2}}{2\sqrt{t}}\sqrt{s-2\sqrt{aF}}\right) = 0$$

Solving this equation leads to expression (15). Nonetheless, such a value corresponds to a maximum if and only if the second order condition holds. Therefore, we compute the second derivative of $W(t)$ with respect to t and, after substitution of t^*, obtain:

$$W''(t^*) = -\frac{1}{b^2}\frac{(s-2b)^3}{(s-b)^2}$$

Level t^* defines a maximum only if $W''(t^*)$ is negative, which obviously requires:

$$s > 2b$$

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Figure 1.

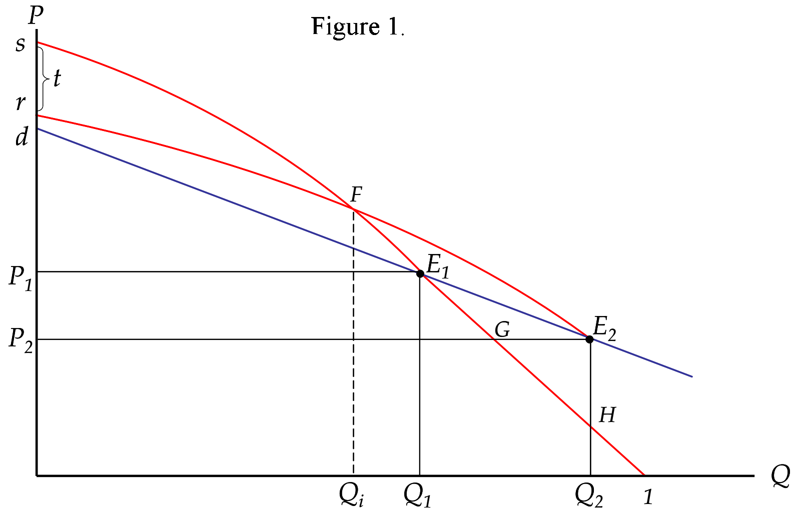

Figure 2.

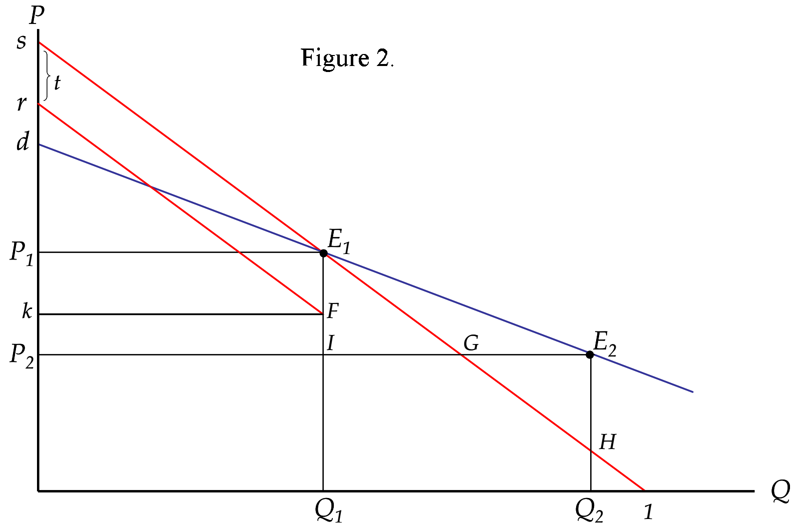

Figure 3.