# Is Child Labor Efficient?

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**Abstract**

In a setting where parents determine the number of children to be born, we analyze child labor as an externality. Non-altruistic parents decide the amount of their children's time devoted to labor and, then, to education. Since child labor reduces the children's future income prospects –as her human capital is reduced–, a Millian inefficient allocation of resources might arise provided the return of human capital is lower than the return of physical capital. Empirical evidence is found on the fail of this condition and the existence of child labor. Finally, we propose a number of policies to improve efficiency, each affecting resource allocations and fertility decisions.
**Key words**: Child labor, Fertility, Millian efficiency, Human capital.
**JEL Codes**:

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# 1 Introduction

Child labor is a phenomenon frequently associated to the underdeveloped or developing countries but there also exists deveoped countries where children take part as labor force. According to ILO (International Labor Organization) 2006's estimates, about 218 million of children between ages of 5 to 17 worked in 2004 (see Fyfe 2007). The causes of child labor are strongly related with poverty and lack of education: child labor exists as the cause and as the result of poverty. The reason is it is an additional source of the household income, and acts as an income insurance in the case of parents unemployment or a large household size. This fact also explains why the population is so high in the economies with child labor: the costs of rearing children are low, which are mainly offset by the overall (discounted) benefits provided. Yet, a high percentage still exists throughout the world. Eradicating child labor, or at least its worst forms, is an issue that concerns families, governments –through active legal measures–, and international organizations –in helping the transition.[1] Thus, it is of interest if normative arguments can be provided to support them.

This paper examines under which conditions child labor is efficient when fertility decisions are endogenous. As an initial issue, it should make clear what child labor is. There not exists a global definition of *child labor*. In extreme forms, children carry out their tasks under hazardous conditions without healthy cares, forced to do illegal works, or treated like slaves. However, most of the working children work in legal employments with no risks for their health, development or education that the ILO consider as "safety works." The ILO states that the work done by children without the minimum age set up and under the worst forms of labor (slavery, prostitution, hazardous work) is defined as "child labor." So, except for those cases of brutal exploitation, if children contributes to the income support of their families, and also improves her personal development, what is wrong with child labor? There are two issues in play: an opportunity cost and an externality. The opportunity cost of the time a child spends working, and then earning income, is the present value of future income she will earn with the knowledge she acquires in this time. This involves individual, as well as society, benefits. The education is very important to the development of a economy. In countries where child labor is allowed and working children have to devote all their time to work, the economic progress is very slow. Thus, for the purposes of this paper, we will consider *child labor* as the work done by children that interferes with the children's accumulation of human capital.

The literature of child labor is very extensive. Many studies had approached the issue of child labor explaining the existence of this phenomenon, its causes, its consequences and the possible measures to face it (see Grootaert et al 1995 and Edmonds 2010 for two surveys). A first line of research aims to explain the existence of child labor and its effects on other

[1]Different institutions as ILO, UNICEF or the World Bank had written several reports to reveal the situations of the children around the word. There are also NGOs, such as *End Child Labor*, *Understanding Children's World* or *Free The Children*, which aim child labor to disappear. There is also international campaigns such as *Stop Child Labor* `http://www.stopchildlabour.eu/`, `http://www.stopchildlabour.net/`, *World Day Against Child Labour*, *International Initiative to End Child Labor* `http://endchildlabor.org/`; *GoodWeave's Campaign to End Child Labor* `http://www.goodweave.org/child_labor_campaign`; *Stop Child Labor: The child labor coalition*`http://clc.designannexe.com/`.

economic variables, such as education. Basu et al (1998) analyze the causes of child labor an the policy implications in a model that generates multiple equilibria in the labor market, where child labor is one of the possible outcomes. Fan (2004a) studies the existence of child labor from the trade-off of quantity and quality of children, in the tradition of Becker and Lewis (1989). Jafarey et al (2004) explain that the role that education plays in the economic growth and, therefore, in the economic development is too important. The factors that affect the persistence of child labor are those that parents have to face when deciding whether to send his children to work or not. Thus, the parent's poverty, the lack of credit, the low pecuniary returns to schooling, or the coordination failures in labor markets and intergenerational transmission mechanisms.

A second line of research aims to explain the existence of institutions that mitigate child labor. Doepke et al (2005) develop a positive theory of child labor regulations (CLR) where the education is the alternative to child labor. The result is a multiple politic-economic steady states, where child labor is supported or where child labor is banned. This explains the differences between countries about child labor regulations.

These two strands of the literature, in contrast with the present work, do not present a welfare analysis. This may shed doubts on the implementability of the policies proposed to reduce child labor.

A third line of research, focus on efficient issues. Baland et al (2000) study the conditions under which child labor is inefficient and its welfare implications. These authors find that under capital markets constraints (i.e., with non-negative savings), financial, fertility and bequests decisions child labor can be inefficient. Unlike their model we do not consider altruism motives, so no bequests decision will be taken. Two other works have extending this work by considering two-side altruism (i.e., parents are altruist towards their children, and children are altruist towards their parents). Bommier et al (2004) considers additionally that children derive disutility of working. They conclude that even with altruism of family members, capital markets constraints and children work disutility, child labor is inefficient. Rogers et al (2003) begin the same two-sided altruism model that Bommier et al, but they find a striking result: there is non-monotonicity between child labor and parental income; that is, an increase in parental income does not entail that schooling increases. Thus, households with higher incomes send also their children to work, a result that stems from the ascendant altruism assumed. Our paper differs from these two articles in that we do not consider descendant or ascendant altruism motives. This entails that children's education is not a concern for parents. In addition, none of them consider the decision of being born children, and how this fertility decision affects child labor.

An important departure of our model is the consideration of fertility issues. Only Baland et al (2000, Sec.IV) considers fertility issues and make some evaluation in welfare terms. They implicitly study a Benthamite social welfare function by considering the addition of the wellbeing of all children. In contrast we will consider a Millian social welfare function where average welfare is taking into account. Finally, we also differ from Baland et al (2000) on the policy proposals. These authors propose two policies for reducing child labor, but these policies are not founded on efficiency terms. This may shed some doubt on their political implementation. Unlike them, we evaluate our proposal on efficient grounds.

In this work we are concern with two decisions of parents (sending their offspring to work –instead to school–, and the number of children to be born), and the optimality of both simultaneous decisions. We study this issue by considering a simple two-period intertemporal model with endogenous fertility decisions and without any altruistic motives. A parent is the only agent that takes relevant economic decisions and decides his consumption and savings, the number of children and how to spend their children's time between labor and educational activities. The latter is an externality as this decision might affect the future economic prospects of children. The parent enhances welfare from consuming and having children, and each child's welfare is increased when consuming when adult, and is decreased when working when child. Children do not take any decision. We introduce the accumulation of human capital as a technology, with schooling time as an input, which increases the adult child's productivity and, then, her wages. In this setting, we compute the decentralized equilibrium in the case that complete and incomplete financial markets exist.

To study efficiency in our setting, where fertility decisions are endogenous, it requires the extension of the Pareto notion. The Pareto criterion allows one to one to rank feasible allocations using the preference orderings of all potential agents on those allocations in which they are alive. However, any two allocations with different population size cannot be ranked, since there is no way to know whether or not an agent who lives in one allocation $\boldsymbol{a}$ but not in other allocation $\boldsymbol{a}'$ is better off in the latter than he is in the former. To avoid this problem and preserve the partial order induced by the Pareto criterion, one needs to extend it to compare also allocations of different population size. According to Golosov et al (2007), there are at least two possible extensions of the Pareto criterion applicable to rank allocations with different fertility choices. To study efficiency, we first present three definitions of efficiency in our endogenous fertility setting: $\mathcal{A}-$efficiency –which compare only the allocation profiles of alive agents–, $\mathcal{P}-$efficiency –which compare the allocation profiles of all agents, after assuming a utility welfare for those non-born agents–, and $\mathcal{M}-$efficiency –which compare only symmetric allocation profiles of alive agents, i.e. considers the $\mathcal{A}-$efficiency notion and considers a symmetric treatment of identical agents. As will be shown, the most appropriate definition for our setting is the Millian dominance. Then, we identify Millian efficient allocations in our setting, to find that the decentralized allocations might not be efficient. This result could be expected because of the existence of an externality.

The main theoretical result in our work is that child labor might be efficient or not depending on the return of human capital with respect other productive assets. The accumulation of human capital from education improves the skills and productivity of the workforce, increase the social and economic returns and trigger the innovation and the technological progress. If the rate of return of human capital is higher than the rate of return of physical capital, the children could compensate their parent for refraining his present consumption with future income transfers. Then, child labor is inefficient. On the opposite case, the rate of return of human capital is lower than the rate of return of physical capital, most of the (Millian) efficient allocations involves child labor. Thus, child labor might be (Millian) efficient.

This work develops along the following sections. In Section 2 we present the model. In Section 3 we study the decentralized decisions of parents and children. In Section 4 we present the three definitions of efficiency in endogenous fertility settings and in Section 5 we identified the Millian efficient allocations in our model, to find that the decentralized allocations are Millian inefficient. To recover efficiency, we propose a number of policies in Section 7. Previously, in Section 6 we undertake an empirical analysis, to illustrate that those countries with child labor presents a rate of return of human capital lower than the rate of return of physical capital. Finally, we present a concluding section and extensions.

## 2 The model

We analyze child labor in a simple two-period intertemporal family decision model. We follow closely the Baland et al (2000)'s model that addresses child labor, but without any altruistic motives. The assumptions of our model are the following.

**Assumption 1.** ***Periods:*** *There are two periods in the economy, denoted to by* $t = 1, 2$.

**Assumption 2.** ***Agents.*** *There exist three types of agents: firms, parents and children. There are* $L^p$ *parents, each taken consumption and productive decisions in both periods, as well as financial and fertility decisions at period* $t = 1$. *In addition, at period* $t = 1$ *each parent decides his offsprings allocation of time between productive and educational activities. Children only take productive decisions at period* $t = 2$. *The other agents in the economy are firms. For simplicity, we consider a single representative firm who lives for both periods and has no children.*

**Assumption 3.** ***Commodities:*** *There exist five commodities. Two commodities exists at each period: a consumption good and labor; in addition, the number of children is also a commodity for parents.*

**Assumption 4.** ***Consumption Set:*** *The set of existing allocations is* $\mathcal{X} \in \Re^5_+$.

Observe that this assumption entails that the number of children is selected from a continuum.

**Assumption 5.** ***Preferences:*** *The individual preferences are represented by a continuous utility function that we assume strictly monotone, strictly quasiconcave and twicely differentiable. The parent's welfare is enhanced by consuming at periods* $t = 1$ *and* $2$, *and by rising children, so their utility function is*

$$U(c^y, c^o, n) = U(c^y) + U(c^o) + (n).$$

*Each child's welfare is enhanced by consuming in the second period, and –following Bommier et al (2004)– is decreased with the working hours at the first period. Her utility function is*

$$u(c^c, l^c) = u(c^c) + v(1 - l^c).$$

Two comments are in order. First, note that since each parent works in both periods and no disutility of labor exists, parents supply labor inelastically. The same applies for children at period $t = 2$. Second, note that, unlike Baland et al (2000), Bommier et al (2004) and Rogers et al (2003), we are assuming that no descendant altruism exists, so the parent's utility function does not include his children's welfare at period $t = 2$. This entails that the motives for taking care the children, and even bearing them, are softer. The introduction of altruism will only reinforce our results.

**Assumption 6.** ***Initial endowments:*** *Each child initial endowments are represented by the vector $\omega^c = (T_1^c, T_2^c, A_1^c)$, while each parent initial endowments are $\omega^p = (T_1^p, T_2^p, A_1^p, A_2^p, \theta)$. In both periods individuals –parents and children– are endowed with a single unit of time $T_t^i = 1$, with $i = p$ (parents), $c$ (children) and $t = 1, 2$. Parents only devote time to labor activities; children time can be devoted at period $t = 1$ to education $[e]$ or labor $[l^c]$, and only to labor activities at period $t = 2$. In addition, the individuals are endowed with $A_t^i$ efficiency units of labor, with $i = p$, $c$ and $t = 1, 2$. We will assume that children's units of efficient labor at period $t = 1$ is $A_1^c = 1$.*
*Finally, parents are endowed with the right to receive at period $t = 1$ a fraction $\theta \in [0, 1]$ of the productive returns of his children's labor revenue.*[2] *Along this paper, we will assume that there exists full expropriation, i.e. $\theta = 1$.*

Two comments are in order. First, at period $t = 1$ parents allocate their children's time endowments between education and labor activities; at period $t = 2$, adult children maintain separate households from their parents, with their income away from their parents control. Second, the children efficient units of labor at period $t = 2$, $A_2^c$, is not an endowment. We will assume that it depends on the human capital accumulated through education in period $t = 1$.

**Assumption 7.** ***Human capital technology.*** *In the period $t = 1$, each child accumulates human capital by devoting time to educational activities, $e$. Thus, the human capital stock at period $t = 2$ is the output of a technology that requires child's time. We will assume that this technology is a twice continuously differentiable, strictly increasing, and strictly concave function $h(e)$, with $h(0) = 1$.*

**Assumption 8.** ***Child's labor efficiency at period $t = 2$.*** *Each child's labor efficiency at period $t = 2$ depends on her accumulated human capital at period $t = 1$, $A^c(h(e))$. For simplicity, we will assume that this dependence is linear, i.e. $A^c(h(e)) = A_2^c h(e)$, where $A_2^c$ represent how useful are human capital for labor activities.*

**Assumption 9.** ***Children rearing cost function.*** *Rising children is costly. We will assume that this cost is a continuous, concave, differentiable function $b(n)$.*[3] *Along the paper we will assume it to be linear, i.e. $b(n) = bn$.*

**Assumption 10.** ***Production technology.*** *The representative firm produces the single consumption output good, which will be considered as the numeraire at each period (i.e., its price will be normalize to unity). The firm's technology is a constant return-to-scale technology that makes use efficient labor as input. As in Baland et al (2000), we will assume that this technology is linear, $f(Al) = Al$, where $A$ is efficient labor and $l$ the time devoted to labor by parents and children.*

[2] This rights may stem from cultural traditions, custom or are allowed by law. See Schoonbrodt et al (2010).

[3] Note that we are assuming that all rearing costs are included in this function, also those concerned with educational costs. In fact, education costs are considered to be a fixed cost per child. Alternatively, we could considered that rearing children involves a physical cost, $b(n)$, plus a variable cost of education depending on the number of hours the children study $E(e)$.

In exchange of labor, workers receive a compensation $w_t^i$ with $i = c$, $p$ and $t = 1$, 2 per efficient unit of labor.[4] We assume that the markets for young and old parental, child, and adult labor are all competitive with respective wage rates. Since firms have a linear technology, and produces efficiently, profits are zero and let all wages per unit of efficient labor be identical, i.e. $w_1^p = w_1^c = w_1$ and $w_2^p = w_2^c = w_2$. However, inside of the firm there exists difference between parents' and children's compensations. As a parent is more efficient than any children (e.g., parents undertake more activities), he receives more compensation. The output produced by any parent at period $t$ is $f(A_t^p T_t^p) = A_t^p T_t^p$. Observe that the parent's marginal productivity coincides with his efficient labor function. Thus, parents compensation is $w_t^p A_t^p$ for $t = 1$, 2. In the case the children work during the first period, the supply of child labor is $nw_1^c l^c$, where $l_c \in [0, 1]$ is the fraction of a child time that is allocated to work and $w_1^c$ is the children's labor compensation. In the second period, each child supplies $T_2^c = 1$ unit of labor which has a value in efficiency units that depends of the amount of education they received in the first period; that is, $A_2^c = A^c(h(1 - l_c))$.

Finally, following Baland et al (2000), we consider two financial market structure.

**Assumption 11.** ***Financial Markets.*** *The financial markets structure may be complete or incomplete. If complete, parents are able to transfer income across periods by saving in a productive security, $s$, that yields a real gross interest rate $R$.*

To conclude, we present some useful notation.

**Definition 1.** *An* **allocation** *is a vector $\boldsymbol{a} = (\mathbf{x}^p, \mathbf{x}^c, s) \in \mathcal{X}$, such that $\mathbf{x}^p = (c^y, c^o, n)$, $\mathbf{x}^c = (c^c, l^c)$.*

## 3 The decentralized decisions

### *3.1 The parent's problem*

In this model each parent is the only agent that takes relevant economic decisions. He takes his consumption, financial and fertility decisions and, in addition, their children's time allocation. Children do not take any decision, as parents are those who decide if the children work or receive education, and hence their human capital (and then their amount of consumption) at period $t = 2$. The parent maximizes his welfare subject to his budget constraints at each period, taking prices as given. At period $t = 1$ his source of income stems from his own labor $w_1^p A_1^p$ and for his child's labor $\theta w_1^c l^c n$, and he consumes, $c^y$, saves, $s$ and rise children $b(n)$. At period $t = 2$ his source of income is his labor income $w_2^p A_2^p$ and

[4] Baland et al (2000) consider the labor as units of efficient labor but take the wages as unit of human capital. Bommier et al (2004) however, take the wages as unit of efficient labor and explain that they are exogenous and are setting equal to one to simplicity.

the returns of his savings, $Rs$. The parent problem is the following,

$$\begin{aligned} & \max_{c^y, c^o, s, n, l^c} & & U\left(c^y, c^o, n\right) = U\left(c^y\right) + U\left(c^o\right) + U\left(n\right) & \\ \text{s.t.} \quad & c^y + b(n) + s & = \; & w_1^p A_1^p + \theta w_1^c l^c n & (1) \\ & c^o & = \; & w_2^p A_2^p + Rs & (2) \\ & l^c & \leq \; & 1 & (3) \\ & s & \geq \; & 0 & (4) \\ \text{given} \quad & R, w. & & & \end{aligned}$$

We will study the parent's decision under two scenarios depending on the financial structure of the markets. The case that financial markets are complete (Baland et al's *perfect capital markets* case); and, the case that financial markets are incomplete (Baland et al's *imperfect capital markets* case).

*3.1.1 Complete financial market case* When financial markets are complete there exists a (productive) security to transfer wealth to the future and savings are possible, i.e. $s > 0$. First order conditions are the following

$$\frac{U_y'\left(\mathbf{x}^p\right)}{U_o'\left(\mathbf{x}^p\right)} = R \tag{5}$$

$$\frac{U_n'\left(\mathbf{x}^p\right)}{U_y'\left(\mathbf{x}^p\right)} = \left[b'(n) - w_1^c\right] \tag{6}$$

jointly with restrictions (1)-(3) with equality, because of the monotonicity of preferences. The optimal condition at (3) with equality, entails that the parent will find $l^{c*} = 1$ as the optimum level of child labor; that is, children only work and do not study at all. This result is obtained from two reasons. Firstly, parents receive income from their children's labor revenue; thus, the higher quantity of child labor the higher parental income. Secondly, since each parent is not guided by altruistic motives, he does not care their offspring's present or future welfare; thus, children education has a high opportunity cost and, then, a parent will allocate all their children's time to work.

When savings are interior, we find from (5) that the real gross interest rate is equal to the Marginal Rate of Substitution (MRS) between present and future consumption. Notice that if the MRS would have been greater than the intertemporal gross interest rate the parent is saving too much, and he had found it worthy to save less and gave up a unit of his future consumption good to consume more in the present. He had kept doing so, until the marginal benefits of the consumption are equal to the marginal costs, that is, the returns of savings.

If financial markets are complete, condition (6) states that the marginal benefits of having another child $U_n'\left(\mathbf{x}^p\right)/U_y'\left(\mathbf{x}^p\right)$ equals the marginal (net) costs of having that child $(b'(n) - w_1^c)$. In fact, the marginal benefits are

$$\left(\frac{U_n'(n)}{U_y'\left(w_1^p A_1^p + w_1^c n - b(n) - s\right)} + w_1^c\right), \tag{7}$$

while the marginal costs are $b'(n)$. Observe that in the case that $(b'(n) - w_1^c) < 0$, the returns that a child yields to his parent are greater than the cost needed to support the child, and then parents will find optimal to bear an infinite number of children, to get an infinitely large consumption. Thus, we will assume that $b'(n) > w_1^c$. In this case, the optimal fertility decision is $n^*(s > 0)$.

Finally, observe that the optimal conditions (5)-(6) entail that the rate of return of the productive security and the children must be equalize

$$R = \frac{U_y'(\mathbf{x}^p)}{U_o'(\mathbf{x}^p)} = \frac{\frac{U_n'(\mathbf{x}^p)}{U_o'(\mathbf{x}^p)}}{[b'(n) - w_1^c]}.$$

*3.1.2 Incomplete financial market case* Consider now that there exists no (productive) security to transfer wealth to the future. Savings are not possible, i.e. $s = 0$, so we will find a corner solution. In this case, the first-order condition is

$$U_n'(\mathbf{x}^p) = U_y'(\mathbf{x}^p)\ [b'(n) - w_1^c]$$

jointly with restrictions (1)-(3) with equality. As in the previous case, since the parent has no altruistic motives and child labor increases his income, his decision will be that this child will work all their time, i.e. $l^{c*} = 1$. However, when financial markets are incomplete, other optimal decisions in this constrained setting are modified.

Now the parent's first period consumption increases because all income is consumed, as he cannot transfer wealth into the future (i.e., no financial decision can be taken). As a result, the marginal benefits of having a new child rise, as every new offspring will provide to her parent more income at period $t = 1$; that is, more income from additional child labor offsets in welfare terms the reduction in future consumption. Note that now the marginal benefits are now

$$\left(\frac{U_n'(n)}{U_y'(w_1^p A_1^p + w_1^c n - b(n))} + w_1^c\right), \tag{8}$$

while the marginal costs are $b'(n)$. Note that we need to keep the assumption $b'(n) > w_1^c$. If parents cannot transfer income across periods, they must increase their consumption at period $t = 1$, when young, as the marginal utility of present consumption decreases. Therefore the return of having another child increases, and then his optimal fertility decisions, $n^*(s = 0)$. We summarize this finding as follows.

**Proposition 1.** *If child labor is allowed, then each parent will have more children when financial markets are imperfect with respect of the case that financial markets are perfect,* $n^*(s = 0) > n^*(s > 0)$*, to increase his present consumption* $c^{y*}(s = 0) > c^{y*}(s > 0)$*.*

The proof is simple. Consider that the number of children is the same at both financial market structures. When financial markets are complete the marginal benefits of having a child (7) are lower than when financial markets are incomplete (8), as the impossibility of transfer income across periods, $s = 0$, increases present consumption. Comparing both situations, complete and incomplete financial markets, the marginal costs of having a child are the same. However, if a parent cannot save, the marginal benefits of having children are higher; that is, the parent can give up a less quantity of present consumption to have another child and, then, improving his welfare. So, when child labor is allowed, the number of children under incomplete financial market are higher than if financial markets are complete.

*3.1.3 An example* For the two-period model with endogenous fertility and no altruistic behavior, consider that the parent is endowment is

$$\omega^p = (T_1^p, T_2^p, A_1^p, A_2^p, \theta) = (1, 1, A_1^p, A_2^p, 1),$$

and his preferences are represented by a Cobb-Douglas utility function

$$U(c^y, c^o, n) = \alpha Log(c^y) + \beta Log(c^o) + \gamma Log(n),$$

with $\alpha + \beta + \gamma = 1$. The parent's problem is maximize his utility function subject to the budgets constraints (1)-(4). From the Kuhn-Tucker Theorem, it is easy to find that the parent's optimal allocation of his child's labor time is

$$l^{c*} = 1.$$

As we explain before, this level is consequence of the lack of altruistic motives and fact that the parent's income rise with his child labor.

Our analysis has developed along the decisions being made by a parent under two scenarios: whether financial markets are complete or incomplete. If financial markets are complete, the savings are strictly positive, i.e. $s > 0$. In this case, from the first-order conditions (5)-(6) jointly with the budget constraints (1)-(2) with equality, we can find the parent's optimum values for the number of children, the savings amount, and the present and future consumption

$$\begin{aligned} n^*(s > 0) &= \frac{\gamma M(R)}{(b - w_1^c)} \\ (c^{y*}(s > 0), c^{o*}(s > 0)) &= (\alpha M(R), \beta M(R) R) \\ s^* &= \beta M(R) - \frac{w_2^p A_2^p}{R} > 0. \end{aligned}$$

where $M(R) = (w_1^p A_1^p R + w_2^p A_2^p)/R$ is the present value of the parent's labor income.

If financial markets are incomplete, it is impossible to transfer wealth across periods, and there is no savings, i.e. $s = 0$. In this case, from the first-order conditions (8) jointly with the budget constraints (1)-(2) with equality, we can find the parent's optimum values for the number of children, the savings amount, and the present and future consumption

$$\begin{aligned} n^*(s = 0) &= \frac{\gamma}{\alpha + \gamma} \frac{w_1^p A_1^p}{(b - w_1^c)} \\ (c^{y*}(s = 0), c^{o*}(s = 0)) &= \left( \frac{\alpha}{\alpha + \gamma} w_1^p A_1^p, w_2^p A_2^p \right) \\ s^* &= 0. \end{aligned}$$

Observe that in this economy with endogenous fertility and no altruistic motives when financial markets are imperfect and child labor is allowed the parents will have more children, $n^*(s = 0) > n^*(s > 0)$, as proved in Proposition 1. Also, as a result, the parent's present consumption increase, i.e. $c^{y*}(s = 0) > c^{y*}(s > 0)$.

### 3.2 The child's problem

Any child only takes a decision concerning her level of consumption at the period $t = 2$, as her level of education and labor supplied are chosen by her parent at the first period. When the child becomes an adult at period $t = 2$, she maintains separate households from her parent and no fraction of her income is appropriated by her parent.

The child maximizes her welfare subject to her budget constraint, taking prices as given.

$$
\begin{array}{rrcl}
\max_{c^c} & u_c(\mathbf{x}^c) & = & u\left(c^c\right) + v\left(1 - l^c\right) \\
\text{s.t.} & c^c & = & w_2^c A^c(h(e)) \\
 & l^c + e & = & 1 \\
\text{given} & l^c = l^{c*}. & &
\end{array}
$$

Observe that the child's source of income stems from the earnings from her efficient labor at period $t = 2$, which depend on the level of human capital accumulated from education activities at period $t = 1$. Time to education activities can only be devoted at the childhood and it was decided by her parent. The parent's optimal level of the child labor was found at Section 3.1 and set to $l^{c*} = 1$, so the child efficient labor at period $t = 2$ is $A_2^c = h(0) = 1$. Thus her consumption is $c^{c*} = w_2$.

## 4 Efficiency with endogenous fertility

The most commonly used optimality notion in standard normative economic analysis is that of Pareto efficiency. This notion of efficiency relies in turn on the well-known Pareto criterion to compare social alternatives, a criterion that allows one to construct a *partial ordering* on a set of alternatives from the *complete preference orderings* (defined on this set) of a *fixed* group of agents. An efficient allocation can be described as a maximal element of the partial order induced by the Pareto criterion on the set of feasible allocations.

With endogenous populations, one can still use the Pareto criterion to rank feasible allocations using the *partial orderings* of all potential agents, represented by the utility functions of the *living* agents. That is, an allocation can still be ranked as Pareto superior to another one if it is unanimously preferred by all potential agents according to their partial preference ordering. However, this implies that any two allocations with different population size cannot be ranked, since there is no way to know whether or not an agent who lives in one allocation $\boldsymbol{a}$ but not in other allocation $\boldsymbol{a}'$ is better off in the latter than he is in the former. To avoid this problem and preserve the partial order induced by the Pareto criterion, one needs to extend it to compare also allocations of different population size.

The literature has presented three extensions of the Pareto criterion to compare allocations with different population size.

### 4.1 $\mathcal{A}-$efficiency, $\mathcal{P}-$efficiency and $\mathcal{M}-$efficiency

Golosov et al (2007) have proposed two extensions of the Pareto criterion to compare allocations with different population size. The first of these extensions is referred to as the $\mathcal{A}-$dominance criterion and ranks any two allocations by applying the Pareto criterion using information of the preference profiles of those agents who are *alive* in the two allocations. The second extension, referred to as the $\mathcal{P}-$dominance criterion, is constructed from a previous assumption on the utility level obtained by *potential* non-born agents. This assumption, together with the utility functions that represent the preferences of the agents in those allocations in which they are alive, allows the authors to rank any two allocations of different population size by comparing (using the Pareto criterion and the utility level attributed to the unborn) the utility profiles of all potential agents in the two allocations. These two extensions of the notion of Pareto dominance give rise to two notions of efficiency, respectively referred to as $\mathcal{A}-$efficiency and $\mathcal{P}-$efficiency, to evaluate allocations in environments in which fertility decisions are endogenous. Observe that, since conditions establishing $\mathcal{P}-$dominance between any two allocations are stronger than those establishing $\mathcal{A}-$dominance, every $\mathcal{A}-$efficient allocation must be also $\mathcal{P}-$efficient. Thus, there might be a wide range of allocations that are $\mathcal{P}-$efficient independently of the utility attributed to the unborn.

Other authors, Schweizer (1997), Michel et al (2007) or Conde-Ruiz et al (2004, 2010) have proposed an alternative notion of efficiency, referred to as Millian efficiency (or $\mathcal{M}$-efficiency), to evaluate allocations with different population size. This notion results from combining the $\mathcal{A}-$dominance criterion to compare allocations with a restriction on the set of allocations that can be compared using that criterion. Somewhat surprisingly, Millian efficient allocations might are not, in general, $\mathcal{A}-$efficient (see Pérez-Nievas, et al 2011). In fact, if the agents are not altruistic towards their children, then any interior, Millian efficient allocation is $\mathcal{A}-$inefficient.

### *4.2 An example*

We illustrate with an example the three extensions of the Pareto criterion to endogenous fertility environments. Consider the same environment depicted in Section 2. A feasible allocation, $\boldsymbol{a} = (\mathbf{x}^p, \mathbf{x}^c, s)$, is an allocation that verifies the feasibility constraints.

Consider a feasible allocation $\boldsymbol{a} = (c^y, c^o, c^c, l^c, n, s)$, such that there not exists child labor, $l^c = 0$. Then, at period $t = 1$ the parent income stems from his own labor, $w_1^p A_1^p$, and he consumes, $c^y$, rise children, $bn$, and saves, $s$. At period $t = 2$ his source of income is his own labor, $w_2^p A_2^p$, and the returns of his savings, $Rs$. The child only takes decisions about her consumption at period $t = 2$. Her earnings depends of her human capital accumulation so with no child labor her efficiency units of labor are $A^c = h(1 - l^c) = h(1)$. That is,

$$\begin{aligned} c^y &= w_1^p A_1^p - bn - s \\ c^o &= w_2^p A_2^p + Rs \\ \overline{c}^c &= w_2^c A^c(h(1)). \end{aligned}$$

This allocation provides a welfare to the parent and children described by the utility function

$$\begin{aligned} & U^p\left(c^{y*}, c^{o*}, n^*\right) \\ u^c(\overline{c}^{c*}, l^{c*}) &= u^c(w_2^c A^c(h(1)), 0) \end{aligned}$$

*4.2.1 $\mathcal{A}-$efficiency* In this section we show that allocation $\boldsymbol{a}'$ is not $\mathcal{A}-$efficient, by finding another feasible allocation that $\mathcal{A}-$dominate this allocation. We find the new feasible allocation $\boldsymbol{a}''$ as follows.

Recall that allocation $\boldsymbol{a}'$ comprises a parent and $n'$ children, who do not work at all. Consider the case of a new allocation with an additional offspring. Recall that this new born is not taken into account for efficient comparisons, so this child can be treated in a different way to her siblings. Thus, we consider the new allocation that that all children are sent to schooling except the new born, who will be spend all her time at work. That is, the $n'$ already alive children will not work, $l_i^{c''} = 0$ for $i = 1...\ n'$, so each consumption do not change $c_i^{c''} = c^{c'}$ for $i = 1...\ n'$. On the contrary, the new born will, $l_{n'+1}^{c''} = 1$, and, since it is not taken into account on efficient grounds, she will not consume at period $t = 2$, i.e. $c_{n'+1}^{c''} = 0$. Then, since the $n'$ already alive children are equal in welfare terms, it is only left to show that the parent is better off under the feasible new allocation.

The parents consumption under the new allocation $\boldsymbol{a}''$ is

$$\begin{aligned} c^{y''} &= w_1^p A_1^p - bn' - b - s'' + w_1^c l_{n'+1}^{c''} \\ c^{o''} &= w_2^p A_2^p + Rs' - R\left[b - w_1^c l_{n'+1}^{c''}\right] + w_2^c A^c(h(0)). \end{aligned}$$

Two comments are in order. First, note that at period $t = 1$ the parent faces a new restriction where his income stems from his own labor, $w_1^p A_1^p$, and from his last child's income, $w_1^c \widehat{l^c}$ , and where his consumption is affected for the cost of the representative child. At period $t = 2$ his source of income stems from his own labor, $w_2^p A_2^p$, the returns of his savings, $Rs'$, and his representative child's labor, $w_2^c A^c(h(0))$. Second, assume that the consumption when young does not modify in the new allocation, i.e. $c^{y''} = c^{y'}$. Then, the savings and its returns are reduced as a new offspring brings with higher costs, $s' = s'' - b + w_1^c$. Then if $\left[-R\left[b - w_1^c \widehat{l^c}\right] + w_2^c A^c(h(0))\right] > 0$ the parent's future consumption will be increased by only be born an additional and appropriate all her present and future resources. If this is the case, the last child's returns are higher than the costs, a parent will find optimal to bear an infinite number of children. Note that this happens even under our assumption that children bearing children is costly, $b > w_1^c$.

The $\mathcal{A}-$efficient allocation will be $(c^{y**}, c^{o**}, \overline{c^c}, l^c = 0, n+1, s')$.We are studying allocations with different size. Comparing both, the $n$-children family and the $n+1$-children family, we achieve the following results

$$\begin{aligned} U_p\left(c^{y*}, c^{o*}\right) &\leq U_p\left(c^{y*}, c^{o**}\right) \\ & \quad u_c(\overline{\mathbf{c}^{c*}}, l^{c*}) \\ u_{c_1}(\mathbf{c}^{c_1*}, \widehat{l^{c*}}) &= u_{c_1}(0, 1) \end{aligned}$$

In that case children are considered asymmetrically just because they born after and not because they have different preferences or abilities. The utility of the parent with more children (which are treated as slaves) is higher than the parent's utility with less children. A parent can consume more in the future if his child's income are higher than his child's costs, that is why a parent would have infinity number of children.

To summarize, we have found a feasible allocation $\boldsymbol{a}''$, where the newborn agents are asymmetrically treated, that $\mathcal{A}-$dominates the initial allocation $\boldsymbol{a}'$. The optimal allocation results to be an economy where (almost all) children are treated as slaves.

*4.2.2 $\mathcal{P}-$efficiency case* The second case we can show is the $\mathcal{P}-$efficient allocations. The non-born agents have attributed an utility function that represents their preferences. Through this utility function and the Pareto criterion is possible to compare allocations with different size. The main aim is to decide which utility function allocate to these individuals.

In our example, if we allocate an utility function to the non-born children, $u_c^n$, and this utility is smaller than the utility of the children with $\mathcal{A}-$efficiency (considerer as slaves)

$$u_c^n \leq u_{c_1}(\mathbf{c}^{c_1*}, \widehat{l^{c*}})$$

The utility $u_{c_1}(\mathbf{c}^{c_1*}, \widehat{l^{c*}})$ is the worst for the child. She spend all her time at work and cannot consume. If we allocate to the non-born children an utility identical or even more worst we can obtain a $\mathcal{P}-$efficient allocation.

*4.2.3 $\mathcal{M}-$efficiency case* Finally, we can obtain the $\mathcal{M}$-efficient allocation. In this case, the results combine $\mathcal{A}-$dominance criterion and symmetry of agents.

If the family grows-up and has another child, $n+1$ children, the parent will treat all his children identical. Assuming the initial conditions without child labor, $l^c = 0$, the agents constraints are,

$$\begin{aligned} c^y &= w_1^p A_1^p - bn - b - s'' \\ c^o &= w_2^p A_2^p + Rs - Rb \\ \overline{c^c} &= w_2^c A^c(h(1)) \end{aligned}$$

Two comments are in order. First, even when the parent treats all his children the same the savings and its returns will also change. With one more child and not child labor, the parent has to consider the cost of having that child so, $s' = s - b$. Second, note that at period $t = 2$ the parent income change....

The utilities of the agent

$$\begin{aligned} U_p\left(c^{y*}, c^{o*}\right) &\geqslant U_p\left(c^{y*}, c^{o***}\right) \\ u_c(\overline{\mathbf{c}^{c*}}, l^{c*}) &= u_c(w_2^c A^c(h(1)), 0) \end{aligned}$$

## 5 Inefficiency of the parent's optimal decisions

Baland et al (2000) explain the conditions under which child labor is efficient. This authors analyze in efficient terms the trade-off between child labor and education (human capital accumulation) under the existence of individuals altruistic decisions and capital markets constraints. They find that there exist an optimum level of child labor when the marginal returns of education equals the marginal returns of having a child. This condition also maximizes the family's income. However, they do not explain the definition of efficiency made used, or if the fertility decisions are taking under an exogenously environment.

In our case, following Baland et al, if fertility decisions are exogenous child labor is efficient when the marginal return of having a child is equal to the marginal cost of having that child, all in terms of income. However, when fertility decisions are endogenous things are different. In the previous section, we have presented three extensions of the Pareto efficient criterion to endogenous fertility settings; then, we argued that we will consider a symmetric treatment of every born children for welfare comparisons. In this section we will identify the Millian efficient allocations in our simple setting; then, we will compare them with the decentralized allocation found at the parent's and children's problem.

### *5.1 The social planner problem*

The social planner is an agent who takes the relevant economic decisions aiming to achieve a social goal. He maximizes his social welfare function, a weighted parent's and children's individual utility function, restricted to feasibility constraints.

For our purposes, we begin by choosing a particular social welfare function, the Millian social welfare function, consisting in the following average utility function of the alive agents,

$$U\left(c^{y}, c^{o}, n\right) + \frac{1}{n}\sum_{i=1}^{n} u(c_i^c, l_i^c).$$

Note that the Millian social welfare function treats identical agents (in endowments and preferences) exactly the same, so their average utility is what accounts for welfare considerations. This contrasts with the Benthamite social welfare function, based on the Jeremy Bentham's utilitarist dictum, "everybody to count for one, nobody for more than one," (see John Stuart Mill 1863, Chap.5), for which each agent weights the same for the Social Planner's welfare. It could be formalize in terms of our model as

$$U\left(c^{y}, c^{o}, n\right) + \sum_{i=1}^{n} u(c_i^c, l_i^c).$$

However, Bentham's welfare conception faces an important critique in our endogenous population setting: more agents, despite very poor, might result in a higher social welfare than few, but rich, agents.

To identify $\mathcal{M}-$efficient allocations we might solve the Social Planner problem that maximizes a Millian social welfare subject to the feasibility constraints, to find the optimal financial, fertility, consumption and labor allocations.

$$\begin{aligned}
&\max_{c^y,c^o,c^c,s,n,l^c,e} && U\left(c^{y}, c^{o}, n\right) + u(c^c, l^c) \\
\text{s.t.}\quad & c^y + b(n) + s &=\;& w_1^p A_1^p + \theta w_1^c l^c n && (9)\\
& c^o + nc^c &=\;& w_2^p A_2^p + n w_2^c A^c(h(e)) + Rs && (10)\\
& l^c + e &=\;& 1 && (11)\\
& s &\geq\;& 0 && (12)\\
&\text{given} && R,\, w.
\end{aligned}$$

Note that, as in most overlapping generations model with endogenous fertility decisions, the social planner problem is not a concave problem, so the interior solution needs not to be the efficient one (see the comments posed by Conde-Ruiz et al 2010, Sec.3). Unlike the overlapping generations setting, in our simple two-period setting there is no dynamic inefficiencies, so we can provide necessary and sufficient conditions to characterize $\mathcal{M}-$efficient allocations.

### *5.2 Necessary and sufficient conditions*

We will adapt Conde-Ruiz et al (2010, Sec.3) notation to our setting. Write $e_1$ for the amount of physical resources a parent receives from each of his children at period $t = 1$, and $e_2$ for the amount of physical resources a parent receives from each of his children at period $t = 2$.

With this notation, the condition for Millian efficiency can be stated as follows.

**Proposition 2.** *Every $\mathcal{M}-$efficient allocation $\widehat{\boldsymbol{a}} \in \mathcal{S}^*$ satisfies,*

$$U(\widehat{\mathbf{x}}^p) = \max_{(\mathbf{x}^p,s)\in\Re^4_+} \Big\{ U(\mathbf{x}^p) \; : \; c^y + b(n) + s \leq w_1 A_1^p + ne_1; \quad c^o = w_2 A_2^p + Rs + ne_2 \Big\} = W^p(\widehat{e}_1, \widehat{e}_2), \tag{13}$$

*and*

$$u(\widehat{\mathbf{x}}^c) = \max_{\mathbf{x}^c\in\Re^2_+} \Big\{ u(\mathbf{x}^c) : e_1 = w_1 l^c; c^c = w_2 A_2^c(h(e)) - e_2; l^c + e = 1 \Big\} = W^c(\widehat{e}_1, \widehat{e}_2) \tag{14}$$

Observe that $W^p(\widehat{e}_1, \widehat{e}_2)$ is the maximum utility that a parent can obtain if he is endowed with $\widehat{e}_1$ units of resources at period 1 and $\widehat{e}_2$ units of resources at period 2 from each of his children. Likewise, $W^c(\widehat{e}_1, \widehat{e}_2)$ is the maximum utility that a children can obtain if she is constrained to provide $\widehat{e}_1$ units of resources to her parent at period 1, and $\widehat{e}_2$ units of resources to her parent at period 2. Notice also that by strict monotonicity of preferences, the function $W^p$ is strictly increasing in both arguments, while the function $W^c$ is strictly decreasing in both arguments.

**Proof.** By contradiction. Suppose that $\widehat{\boldsymbol{a}}$ is an $\mathcal{M}-$efficient allocation, and suppose there exists a period $\tau$ for which the point $(\widehat{x}_\tau, \widehat{k}^o_{\tau+1})$ corresponding to the allocation $\widehat{\boldsymbol{a}}$ is not a solution to the optimization problem in (13). Select now a point $(\widetilde{x}_\tau, \widetilde{k}^o_{\tau+1}) \in \Re^4_+$ satisfying the two constraints in (13) in such a way that $u(\widetilde{x}_t) > u(\widehat{x}_t)$ is satisfied, and let $\widetilde{\boldsymbol{a}}$ be the allocation obtained from $\widehat{\boldsymbol{a}}$ by replacing the term $(\widehat{x}_\tau, \widehat{k}^o_{\tau+1})$ by such a point. Such an allocation is feasible because $(\widetilde{x}_\tau, \widetilde{k}^o_{\tau+1})$ must verify $\widetilde{c}^m_t + b_t(\widetilde{n}_\tau) + \widetilde{k}^o_{\tau+1} \leq \widehat{e}_\tau$ and $F_{t+1}(\widetilde{k}^o_{t+1}, \widetilde{n}_t) - \widetilde{c}^o_{t+1} \geq \widetilde{n}_t \widehat{e}$. Also, note that $\widetilde{\boldsymbol{a}}$ has been constructed in such a way that it satisfies $U_{t-1}(\widetilde{\boldsymbol{a}}) = U_{t-1}(\widehat{\boldsymbol{a}})$ for all $t \neq \tau$ and $U_{\tau-1}(\widetilde{\boldsymbol{a}}) = u(\widetilde{x}_\tau) > u(\widehat{x}_\tau) = U_{t-1}(\widehat{\boldsymbol{a}})$ for $t \neq \tau$, which implies that $\widehat{\boldsymbol{a}}$ is not Millian efficient, a contradiction that establishes Proposition 2.□

The indirect utility functions defined in (13) and (14) will allows us to identify the Millian efficient allocations at a $e_1$-$e_2-$diagram in the next section. First, we characterize the properties of the parent's indirect utility function, $W^p$; then, we study the properties of the child's indirect utility function, $W^c$.

*5.2.1 Properties of the parent's indirect utility function, $W^p$* Note that the optimization problems in the definition of $W^p$ is a well-behaved concave program. Since utility functions are differentiable, an interior solution $(\mathbf{x}^p(\widehat{e}_1, \widehat{e}_2),\ s(\widehat{e}_t, \widehat{e}_{t+1})) >> 0$ to the optimization problem in (13) is characterized by the feasibility constraints

$$\begin{aligned} \widehat{c}^y + b(\widehat{n}) + \widehat{s} &= w_1 A_1^p + \widehat{n} e_1; \\ \widehat{c}^o &= w_2 A_2^p + R\widehat{s} + \widehat{n} e_2 \end{aligned}$$

together with the first order conditions

$$R = \frac{U_y'(\widehat{\mathbf{x}}^p)}{U_o'(\widehat{\mathbf{x}}^p)} = \frac{\frac{U_n'(\widehat{\mathbf{x}}^p)}{U_o'(\widehat{\mathbf{x}}^p)} - e_2}{b'(\widehat{n}) - e_1}. \tag{15}$$

We can find the slope of any indifference curve of the indirect utility function of the parent $W^p$ as follows. Since $W^p$ is strictly monotonic, the indifference curve given by all pairs $(e_1, e_2)$ for which $W^p(e_1, e_2) = W^p(\widehat{e}_1, \widehat{e}_2)$ is satisfied implicitly defines $e_2$ as a continuously differentiable, strictly increasing function of $e_1$. Define $m^p(\widehat{e}_1, \widehat{e}_2)$ as the slope of this indifference curve at the point $(\widehat{e}_1, \widehat{e}_2)$, which is always well defined. Differentiating at both sides of $W^p(e_1, e_2) = W^p(\widehat{e}_1, \widehat{e}_2)$, we obtain

$$m^p(\widehat{e}_1, \widehat{e}_2) = -\frac{\frac{\partial W^p(\widehat{e}_1, \widehat{e}_2)}{\partial e_1}}{\frac{\partial W^p(\widehat{e}_1, \widehat{e}_2)}{\partial e_2}}.$$

Also, the Envelope Theorem yields $\partial W^p(\widehat{e}_1, \widehat{e}_2)/\partial e_1 = \lambda_1(\widehat{e}_1, \widehat{e}_2)$ and $\partial W^p(\widehat{e}_1, \widehat{e}_2)/\partial e_2 = n_1(\widehat{e}_1, \widehat{e}_2)\lambda_2(\widehat{e}_1, \widehat{e}_2)$, where $\lambda_1(\widehat{e}_1, \widehat{e}_2)$ and $\lambda_2(\widehat{e}_1, \widehat{e}_2)$ are the Kuhn-Tucker multipliers for which the first order conditions of the optimization problem (13) are satisfied. Using these first order conditions, we obtain

$$\lambda_1(\widehat{e}_1, \widehat{e}_2) = n(\widehat{e}_1, \widehat{e}_2) U_y'(\mathbf{x}^p(\widehat{e}_1, \widehat{e}_2)), \text{ and } \lambda_2(\widehat{e}_1, \widehat{e}_2) = n(\widehat{e}_1, \widehat{e}_2) U_o'(\mathbf{x}^p(\widehat{e}_1, \widehat{e}_2));$$

since

$$R = \frac{U_y'(\mathbf{x}^p(\widehat{e}_1, \widehat{e}_2))}{U_o'(\mathbf{x}^p(\widehat{e}_1, \widehat{e}_2))}$$

yields

$$m^p(\widehat{e}_1, \widehat{e}_2) = -R. \tag{16}$$

This entails that the slope of the indifference curves are linear and constant.

*5.2.2 Properties of the child's indirect utility function, $W^c$* Next, we characterize the properties of the child's indirect utility function, $W^c$. Note that the optimization problems in the definition of $W^c$ is a well-behaved concave program. This indirect utility function for each child, (14), can be defined as

$$W^c(e_1, e_2) = u\left(w_2 A_2^c \left(h\left(1 - \frac{e_1}{w_1}\right)\right) - e_2; \frac{e_1}{w_1}\right).$$

Feasibility constraints restricts the range of transfers from each child to her parent. Thus, $l^c \in [0,1]$ implies that $e_1 \in [0, w_1]$; and $c^c \geq 0$ defines an upper bound frontier

$$e_2 = w_2 A_2^c \left( h \left( 1 - \frac{e_1}{w_1} \right) \right) \equiv \phi(e_1).$$

This frontier curve is decreasing and concave, $\phi'(e_1) < 0$ and $\phi'(e_1) < 0$, and its slope is defined as

$$\phi'(e_1) = -\frac{w_2}{w_1} A_2^c h' \left( 1 - \frac{e_1}{w_1} \right). \tag{17}$$

As before, we can find the slope of any indifference curve of the indirect utility function of the child $W^c$ as follows. Since $W^c$ is strictly monotonic, the indifference curve given by all pairs $(e_1, e_2)$ for which $W^c(e_1, e_2) = W^c(\widehat{e}_1, \widehat{e}_2)$ is satisfied implicitly defines $e_2$ as a continuously differentiable, strictly increasing function of $e_1$. Define $m^c(\widehat{e}_1, \widehat{e}_2)$ as the slope of this indifference curve at the point $(\widehat{e}_1, \widehat{e}_2)$, which is always well defined. Differentiating at both sides of $W^c(e_1, e_2) = W^c(\widehat{e}_1, \widehat{e}_2)$, we obtain

$$m^c(\widehat{e}_1, \widehat{e}_2) = -\frac{\frac{\partial W^c(\widehat{e}_1, \widehat{e}_2)}{\partial e_1}}{\frac{\partial W^c(\widehat{e}_1, \widehat{e}_2)}{\partial e_2}}.$$

Also, the Envelope Theorem yields $\partial W^c(\widehat{e}_1, \widehat{e}_2)/\partial e_1 = \lambda_1(\widehat{e}_1, \widehat{e}_2)$ and $\partial W^c(\widehat{e}_1, \widehat{e}_2)/\partial e_2 = n_1(\widehat{e}_1, \widehat{e}_2)\lambda_2(\widehat{e}_1, \widehat{e}_2)$, where $\lambda_1(\widehat{e}_1, \widehat{e}_2)$ and $\lambda_2(\widehat{e}_1, \widehat{e}_2)$ are the Kuhn-Tucker multipliers for which the first order conditions of the optimization problem (14) are satisfied. Using these first order conditions, we obtain

$$\lambda_1(\widehat{e}_1, \widehat{e}_2) = u_c'(\mathbf{x}^c(\widehat{e}_1, \widehat{e}_2)), \text{ and } \lambda_2(\widehat{e}_1, \widehat{e}_2) = u_l'(l^c(\widehat{e}_1, \widehat{e}_2)) - u_c'(\mathbf{x}^c(\widehat{e}_1, \widehat{e}_2)) w_2 A_2^c h' \left( 1 - \frac{\widehat{e}_1}{w_1} \right);$$

which yields

$$m^c(\widehat{e}_1, \widehat{e}_2) = -\frac{w_2}{w_1} A_2^c h' \left( 1 - \frac{\widehat{e}_1}{w_1} \right) + \frac{1}{w_1} \frac{u_l'(\mathbf{x}^c(\widehat{e}_1, \widehat{e}_2))}{u_c'(\mathbf{x}^c(\widehat{e}_1, \widehat{e}_2))}. \tag{18}$$

Since $u_l' < 0$, then at any transfer scheme the indifference curve is steeper than upper frontier function is higher than the $m^c(\widehat{e}_1, \widehat{e}_2) < \phi(\widehat{e}_1)$. This entails that the transfer scheme such that the child provides all the resources available in both period and works all hours, $(e_1, e_2) = (w_1, A_2^c(h(0)))$, provides the child the lowest welfare. Finally, since the child's welfare increasing towards the origin, the allocation such that no resources are provided to her parents, $(e_1, e_2) = (0, 0)$ results in her highest welfare allocation.

### *5.3 Characterizing Millian efficient allocations.*

The previous analysis allows us to characterize Millian efficient allocations which depend on the slopes of the parent's and children's indifferent curves $m^p$ and $m^c$ respectively. Whether child labor is $\mathcal{M}-$efficient or not, we will show that the key is the comparison of the returns of human capital, or the "usefulness" of the acquired knowledge to become

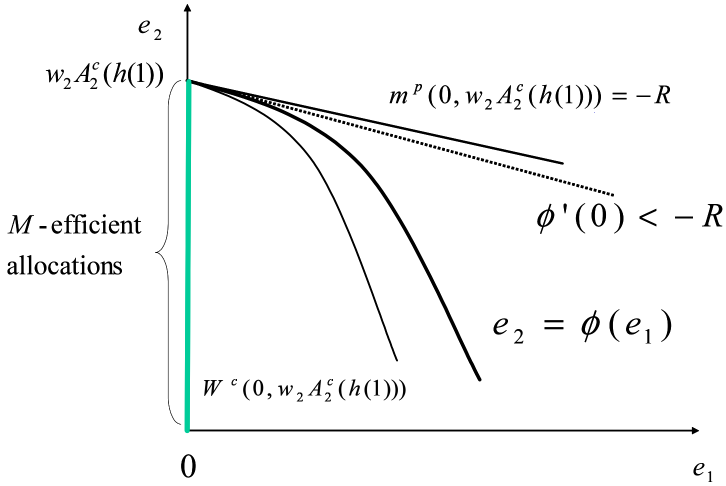


Figure 1: If the return of human capital is always higher than the return of physical capital, $Abs(\phi'(e_t)) > R$, then only allocations with no child-labor are Millian efficient.

efficient labor, and the return of exogenous securities. Provided the return in human capital is higher than the return in physical capital, child labor is not socially efficient.

Next, we present formally our argument. The key is the comparison of two slopes: (i) the slope of the parent's indifference curves $m^p$, (16); and, (ii) the slope of the upper bound frontier $\phi'$, (17). We can find three cases.

1. **The return of the human capital is *more productive* than the return of the productive security at any level of education:** $w_2 A_2^c h'(1) > R$. In particular, it is verified for the case that the child studies all the time $e = 1$ –i.e., $e_1 = 0$–, that is $m^p(0, w_2 A_2^c(h(1))) < \phi(0)$. Then only allocations with no child labor are Millian efficient. In any of these efficient allocations children are educated all their time, and their labor earnings at period $t = 2$ are redistributed among the child and her parent (any redistribution is efficient). See Figure 1. The $\mathcal{M}-$efficient allocations are defined as
$$(\widehat{e}_1, \widehat{e}_2) = (0, e_2) \text{ with } e_2 \in [0, w_2 A_2^c(h(1))].$$
Note that the allocation resulting for the no-transfer scheme $(\widehat{e}_1, \widehat{e}_2) = (0, 0)$ is always Millian efficient.

2. **The return of the human capital is *less productive* than the return of the productive security at any level of education:** $w_2 A_2^c h'(0) < R$. In particular, it is verified for the case that the child does not study at all $e = 0$ –i.e., $e_1 = w_1$–, that is $m^p(w_1, w_2 A_2^c(h(0))) > \phi(w_1)$. Then there exists Millian efficient allocations with child labor; for instance, the allocation $(\widehat{e}_1, \widehat{e}_2) = (w_2 A_2^c(h(0)), w_1)$ is Millian efficient. See Figure 2. The $\mathcal{M}-$efficient allocations are defined as follows. The interior Millian efficient allocations $(e_1, e_2)$ are those that verify $m^c(e_1, e_2) = R = m^p(e_1, e_2)$. These interior allocations exist provided there exists an $\widetilde{e}_2 > 0$ such that $m^c(w_1, \widetilde{e}_2) = R$. We could also find Millian efficient allocations with no child labor, provided there exists an $\breve{e}_2 \in (0, \widetilde{e}_2)$ such that $m^c(0, \breve{e}_2) = R$; that is, those allocations $(0, e_2)$ with

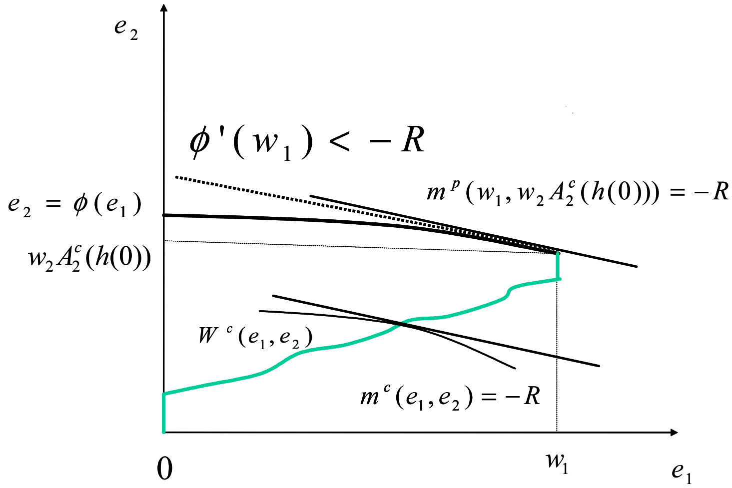


Figure 2: If the return of human capital is always lower than the return of physical capital, $Abs(\phi'(e_t)) < R$, there exists allocations that child-labor is Millian efficient.

$e_2 \in [0, \breve{e}_2]$. This is the case represented in Figure 2. If no $\widetilde{e}_2$ exists, then the Millian efficient allocations are defined onto the axes, and all Millian efficient allocations present child labor except the allocation $(0, 0)$.

3. **The return of the human capital is *as productive as* the return of the productive security for some level of education:** $w_2 A_2^c h'(e_1) = R$ for some $e_1 \in (0, w_1)$.

   In particular, it is verified that $m^p(w_1, w_2 A_2^c(h(e_1))) = \phi(e_1)$ for some $e_1 \in (0, w_1)$. Then there exists Millian efficient allocations with child labor See Figure 3. The $\mathcal{M}-$efficient allocations are defined as follows. The interior Millian efficient allocations $(\widehat{e}_1, \widehat{e}_2)$ are those that verify

   $$(\widehat{e}_1, \widehat{e}_2) \text{ with } m^p(\widehat{e}_1, w_2 A_2^c(h(\widehat{e}_1))) = R = m^p(\widehat{e}_1, w_2 A_2^c(h(\widehat{e}_1))).$$

   We could also find Millian efficient allocations with no child labor, provided there exists an $\breve{e}_2 \in (0, \widetilde{e}_2)$ such that $m^c(0, \breve{e}_2) = R$; that is, those allocations $(0, e_2)$ with $e_2 \in [0, \breve{e}_2]$. This is the case represented in Figure 3. If no $\breve{e}_2$ exists, then all Millian efficient allocations present child labor except the allocation $(0, 0)$.

Observe that with respect the conditions characterizing characterizing Pareto efficient allocations in an exogenous fertility setting

$$R = w_2 A_2^c h'(1 - \widehat{l}^c),$$

as in Baland et al (2000, p.669) the return of education needs not coincide with the return of the productive security unless the efficient solution is interior.

Child labor is troublesome because any kid is not allowed to accumulate enough human capital to be more productive in the future. This is true, as long as the usefulness of the human capital at the time of convert knowledge into productivity. This entails that when knowledge is not useful, or not enough useful, child labor might be efficient.

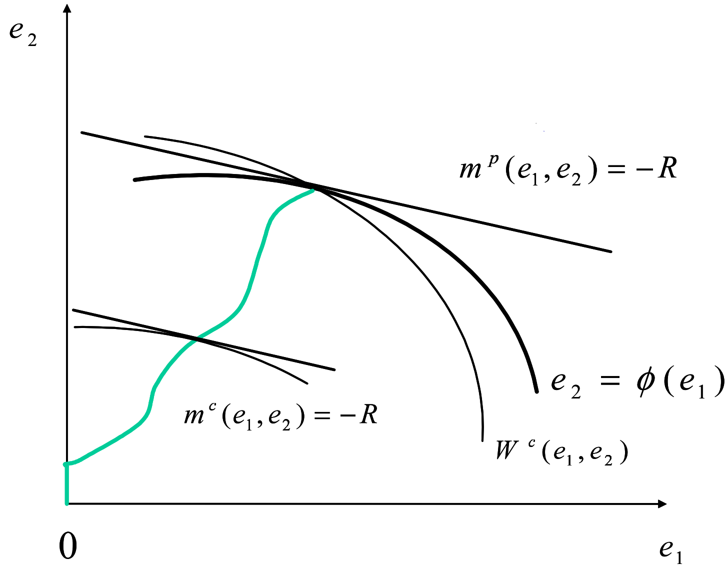


Figure 3: If the human capital is as productive the physical capital there exists allocations that child-labor is Millian efficient.

**Remark** Why child labor can be efficient? The key is the rate of returns of human capital and the interest rate of the economy. A parent has a number of children, who work. Children would like to bargain with their parent the alternative scheme: children do not work when child, and in return, they will compensate their parents when old. This scheme is sustainable as long as the reduction of welfare by reducing the present consumption is offset by the increase in welfare future by rising future consumption. This can be so if the rate of return of human capital is higher than the marginal rate of substitution. Otherwise, this alternative intertermporal scheme does not increase welfare, and then, an allocation with non-zero child labor rate is (Millian) efficient.

### *5.4 The inefficiency of decentralized equilibrium*

Observe that the decentralized equilibrium found in Section 3.1 is associated to the transfer scheme $(e_1^*, e_2^*) = (w_1, 0)$. It is easy to see at Figures 1 and 3 that the decentralized equilibrium is not Millian efficient is the rate of return on human capital is at least as productive as the rate of return on physical capital. This is also the case if the rate of return on human capital is lower than the rate of return on physical capital, unless there exist no interior $\mathcal{M}-$efficient allocation; i.e., there exist no $\widetilde{e}_2 > 0$ such that $m^c(w_1, \widetilde{e}_2) = R$ (see Figures 2).

This inefficiency stems from the parent's decision on his children education, i.e. an externality on his children future efficient labor. Any attempt to restore, or at least improve, the efficiency requires that the parent is compensated in period $t = 2$ for decreasing his income, and then his consumption, in period $t = 1$, as will study in Section 7.

### 5.5 *An example*

Consider the preferences of the parent is represented by the Cobb-Douglas utility function presented in Section 3.1.3. In this case, given any pair of transfers from each children $(e_1, e_2)$, the efficient allocations are

$$c^y(e_1, e_2) = \alpha M;\ c^o(e_1, e_2) = \beta RM;\ n(e_1, e_2) = \gamma \frac{RM}{bR - e_1 R - e_2}$$

That is the indirect utility function is

## 6 Empirical evidence

We conclude with a brief discussion of the empirical implications of our analysis. According to our theory, countries with not sufficiently high rate of return in human capital with respect the rate of return in physical capital will find child labor optimal. As shown in Section 5.3 this is the case for

$$\frac{w_2}{w_1} A_2^c h' \left(1 - \frac{e_1}{w_1}\right) \leq R.$$

To examine the empirical validity of this prediction, we compare the child labor rates with the difference between the rate of return between physical and human capital for a panel of 125 countries from 1960 to 2009, with observations at ten-year intervals. The data are detailed in the Appendix.

Figure 4 reports that countries below the bisectrix, the rate of return of physical capital are higher than the rate of return of human capital. For these countries child labor could be socially optimal. All countries are displayed in Table 1. Observe that, in general, those countries with a higher difference between the rate of return of physical capital and human capital are those with a positive child labor rate. Also observe that, for those underdeveloped countries with a lower difference between the rate of return of physical capital and human capital, we can suggest from the model that child labor will decrease by individual decisions. Altruism and children as an old-age insurance will reinforce this trend.

Finally, Table 2 reports similar results for the rate of physical and human capital computed from data obtained in a field study for five African countries.

## 7 Policy proposals: recovering efficiency

The previous welfare analysis demonstrate the misallocation resulting from the parent's individual decision on their offspring number and their time allocation. The reason of this inefficiency stems from the existence of an externality on the children's future income due to the present parent's child labor decision, which results in an underinvestment in his children's human capital. Thus, child labor is inefficient because of the existence of a negative external effect that parent does not internalize.

The parent is not concern with his children future prospects and does not bear in mind that each child burdens a cost when he decides to send her to work. To solve this market failure we can make use of the existing politic instruments. To apply a policy proposal we have to look for those which allow us to internalize the child's costs of work. It is necessary to find the way to make that the parents internalize that cost. In the literature we find out some options to solve this problem. Barro and Murphy (1989) propose a solution where as the family takes care of the children until they grow, the State makes something similar but at an aggregate level.

The main aim is to analyze if the first-order condition of our proposals are similar to the social planner and study if there is some Pareto improving policy.

### *7.1 Policy I*

Parent's Problem

$$
\begin{aligned}
&\max_{c^y,c^o,s,n,l^c} && U\left(c^y,c^o,n\right) && \\
\text{s.t.} \quad & c^y + b(n) + s &=\;& w_1^p A_1^p + \theta w_1^c l^c && (19)\\
& c^o &=\;& w_2^p A_2^p + Rs + \delta n && (20)\\
& l^c &\leq\;& 1 && (21)\\
& s &\geq\;& 0 && (22)\\
\text{given} \quad & R,\, w. &&&&
\end{aligned}
$$

Child's Problem

$$
\begin{aligned}
\max_{c^c} \quad U_c(c^c, l^c) \quad & && U\left(c^y,c^o,n\right) && \\
\text{s.t.} \quad & c^c &=\;& w_2^c A^c(h(e)) - \delta && (23)\\
& l^c + e &=\;& 1 && (24)\\
& s &\geq\;& 0 && (25)\\
\text{given} \quad & R,\, w. &&&&
\end{aligned}
$$

This policy sould to show that the parent allow to study to his children. $(\theta = 0, \delta)$ –> $c^y(0, \delta)$.

### *7.2 Policy II*

In that case we have a subsidy to education. The goverment offers a subsidy to the parent if he send to his children at school. Now we suppose that $\theta = 0$ so his source of incomes stem from his own labor $w_1^p A_1^p$ and the subsidy $\delta e$.

Parent's Problem

$$\begin{array}{lrcl} & \max_{c^y, c^o, s, n,} & & U\left(c^y, c^o, n\right) \\ \text{s.t.} & c^y + b(n) + s & = & w_1^p A_1^p + \theta w_1^c l^c + \delta e \quad (26) \\ & c^o & = & w_2^p A_2^p + Rs \quad (27) \\ & s & \geq & 0 \quad (28) \\ \text{given} & R,\ w. & & \end{array}$$

Child's Problem

$$\begin{array}{lrcl} & & \max_{c^c} & U_c(c^c, l^c) \\ \text{s.t.} & c^c & = & w_2^c A^c(h(e)) - \delta d e \quad (29) \\ & l^c + e & = & 1 \quad (30) \\ & s & \geq & 0 \quad (31) \\ \text{given} & R,\ w. & & \end{array}$$

$(\theta = 0, d, \delta)$

## 8 Conclusion

In this work, we have study the (Millian) efficiency of child labor when fertility decisions are endogenous. Child labor is troublesome because any kid is not allowed to accumulate enough human capital to be more productive in the future. This is true, as long as the usefulness of the human capital at the time of convert knowledge into productivity. This entails that when knowledge is not useful, or not enough useful, child labor might be efficient.

There are two extension of this work. First, a natural extension of this work is to study this issue in a dynamic setting, like Dessy (2000), Hazan et al (2002), and Doepke et al (2005). This will allow for normative analysis when study the welfare improving intertemporal policies to mitigate child labor when fertility decisions are endogenous (see Conde-Ruiz et al 2010, Sec.5).

As a second extension, note that a shortcoming of our model is that the rate of return on physical capital $R$ is exogenous. Note that this assumption is crucial for any policy to reduce child labor and increase the human capital in a country, and then escape from the poverty trap. The empirical work suggests that for a number of underdeveloped countries, an increase in the living conditions of population will easily increase the schooling of young citizens, and then decrease child labor. For other countries with higher real interest rate than the rate of return of human capital, an increase in human capital accumulation requires more investment on physical capital to reduce the marginal rate of capital. That is, the usefulness of learn knowledge contents at school requires an economic environment where such knowledge can be used and implemented.

The question of interrelationship between fertility choice and child labour was the focus of analysis in the classic works of Cain (1977) and Rosenzweig and Evenson (1977). In fact Cain (1977) attempted to explain why poor families in the village of Char Gopalpur in Bangladesh have large families by examining the economic contributions of the children to their families. Rosenzweig and Evenson (1977) in contrast examine the simultaneous determination of fertility decision, school enrolment decision and child labour decision by estimating a structural model for India. Both studies find evidence that child labour decisions are closely related to the decision on fertility. The issue of intergenerational persistence, though not explicit in their analysis, is also part of the broader story told by these authors: abject poverty has a lot to do with high fertility and low school enrolment, and the latter makes it impossible for the children to avoid poverty when they are adults.

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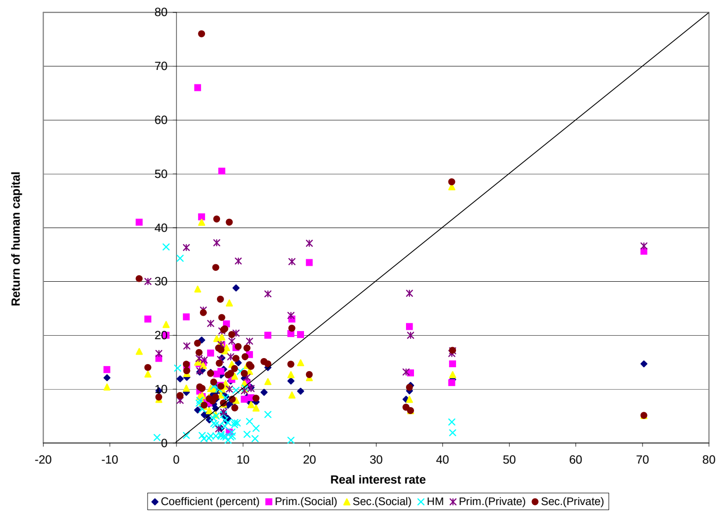


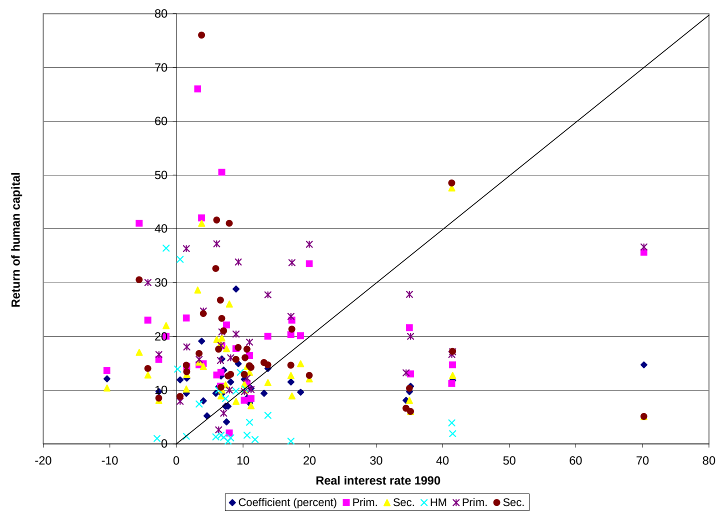


Figure 4: Real rate of return (horizontal axes) vs. rate of return of education -different measures- (horizontal axes). Above all countries, below countries with GDP per capital below 8 billion dollars.

| | *rPrimP* | | *r-HM* | | *r-Coef* | | *rSecP* | | *rPrimS* | | *rSecS* | | *cl90* | *gdp90* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Zimbabwe** | 62,72 | 1 | 75,42 | 1 | | | 30,82 | 2 | 68,12 | 1 | 31,72 | 2 | 31,86 | 366,73 |
| **Brazil** | 12,33 | 2 | | | 34,23 | 1 | 43,83 | 1 | 13,33 | 2 | 43,83 | 1 | 17,81 | 7180,6 |
| **Ecuador** | 5,71 | 3 | 20,91 | 2 | 11,01 | 5 | 5,61 | 5 | 8,11 | 4 | 10,11 | 5 | 6,54 | 4705,33 |
| **India** | 3,87 | 4 | -3,53 | 38 | -4,13 | 35 | -11,13 | 37 | | | | | 16,68 | 1407,72 |
| **Panama** | 2,40 | 5 | 6,90 | 7 | -5,60 | 43 | -12,90 | 42 | | | | | 4,53 | 5619,29 |
| **Chile** | *1,52* | *6* | | | *-0,78* | *20* | *-1,68* | *10* | *3,12* | *6* | *0,12* | *11* | *0,00* | *5636,4* |
| **Peru** | 1,25 | 7 | | | 6,35 | 6 | 7,85 | 4 | | | | | 3,16 | 4003,02 |
| **Argentina** | -2,35 | 8 | | | -2,55 | 27 | -6,45 | 26 | -0,65 | 8 | 0,65 | 8 | 6,65 | 6822,82 |
| **Yemen** | -3,61 | 9 | | | | | -34,61 | 57 | 4,39 | 5 | -19,61 | 49 | 21,57 | 1440 |
| **Tanzania** | -4,12 | 10 | -30,52 | 48 | -8,12 | 48 | -5,02 | 22 | | | | | 42,06 | 663,38 |
| **Uruguay** | -5,07 | 11 | | | 13,03 | 3 | 12,43 | 3 | 1,13 | 7 | 14,63 | 4 | 3,15 | 6049,54 |
| **Costa Rica** | -5,11 | 12 | 5,49 | 9 | -1,41 | 23 | -10,51 | 36 | -4,11 | 11 | -7,31 | 34 | 6,82 | 7464,17 |
| **Paraguay** | -6,12 | 13 | 17,08 | 3 | 6,08 | 7 | 2,98 | 6 | -2,72 | 9 | 4,88 | 6 | 9,91 | 3793,47 |
| **El Salvador** | -6,74 | 14 | 8,16 | 6 | 4,56 | 8 | -2,34 | 12 | -4,24 | 12 | -1,14 | 14 | 16,51 | 4020,4 |
| **Malawi** | -7,77 | 15 | 0,53 | 30 | | | -8,87 | 32 | -6,77 | 17 | -7,27 | 33 | 38,87 | 594,52 |
| **Pakistan** | -8,68 | 16 | -26,58 | 47 | -15,68 | 61 | -13,98 | 45 | | | | | 20,02 | 1931,24 |
| **Thailand** | -9,32 | 17 | 5,58 | 8 | -4,82 | 37 | -6,22 | 25 | | | | | 20,24 | 4454,07 |
| **Lesotho** | -10,77 | 18 | | | | | -21,97 | 52 | -5,97 | 14 | -13,87 | 45 | 23,48 | 906,83 |
| *Japan* | *-11,26* | *19* | *-5,76* | *43* | *-11,06* | *56* | *-8,26* | *30* | *-7,46* | *19* | *-6,46* | *31* | *0,00* | *28499,95* |
| **Honduras** | -11,84 | 20 | | | -0,34 | 17 | -14,34 | 47 | -9,24 | 23 | -10,74 | 39 | 9,89 | 3112,47 |
| *Cyprus* | *-12,77* | *21* | | | *-2,57* | *28* | *-4,37* | *21* | *-5,07* | *13* | *-4,17* | *21* | *0,00* | *15170,51* |
| **Bolivia** | -13,36 | 22 | | | -4,06 | 34 | 0,64 | 9 | -6,36 | 16 | 0,64 | 9 | 17,36 | 2801,02 |
| **Philippines** | -13,40 | 23 | 3,30 | 17 | -7,70 | 47 | -5,60 | 23 | -8,40 | 21 | -4,00 | 20 | 10,66 | 2067,54 |
| **Mexico** | -14,68 | 24 | 2,22 | 21 | -3,38 | 31 | -15,88 | 48 | -7,58 | 20 | -10,38 | 38 | 8,59 | 8788,96 |
| **Jamaica** | -15,10 | 25 | -4,50 | 41 | -23,50 | 62 | -10,40 | 35 | -12,40 | 27 | -2,60 | 18 | 0,21 | 7402,29 |
| **Nepal** | -15,12 | 26 | | | -8,22 | 49 | -7,02 | 28 | -14,22 | 29 | -6,62 | 32 | 48,31 | 867,4 |
| **China** | -15,96 | 27 | | | -10,16 | 54 | -11,36 | 38 | -12,36 | 26 | -10,86 | 40 | 15,24 | 1259,38 |
| **Colombia** | -16,92 | 28 | 5,48 | 10 | -3,22 | 30 | -3,92 | 18 | -9,22 | 22 | -0,62 | 13 | 7,21 | 4620,41 |
| *Singapore* | *-17,42* | *29* | *3,48* | *15* | *-8,32* | *51* | *-8,12* | *29* | *-11,92* | *25* | *-5,32* | *24* | *0,00* | *22980,8* |
| **Ethiopia** | -22,60 | 30 | | | -5,90 | 45 | -22,10 | 53 | -12,80 | 28 | -12,30 | 42 | 43,43 | 413,53 |
| **Gambia** | -23,00 | 31 | | | | | 1,40 | 7 | -19,40 | 33 | 2,00 | 7 | 40,15 | 846,21 |
| **Senegal** | -24,94 | 32 | | | | | -12,54 | 41 | -14,24 | 30 | -0,14 | 12 | 35,41 | 1125,15 |
| **Guatemala** | -27,83 | 33 | | | -8,93 | 52 | -11,93 | 40 | | | | | 18,28 | 4750,21 |
| **Papua New Guinea** | -30,76 | 34 | | | | | -35,16 | 58 | -6,36 | 15 | -12,96 | 44 | 21,74 | 1815,88 |
| **Nigeria** | -32,73 | 35 | | | | | -16,73 | 49 | -25,73 | 35 | -15,53 | 47 | 27,77 | 1180,27 |
| **Venezuela** | -38,87 | 36 | -3,97 | 40 | -11,97 | 58 | -17,17 | 50 | -25,97 | 36 | -12,77 | 43 | 1,90 | 7810,24 |
| **Ghana** | -42,07 | 37 | | | -24,67 | 63 | -34,57 | 56 | -35,57 | 37 | -30,57 | 51 | 14,55 | 843,86 |

| | *rPrimP* | | *r-HM* | | *r-Coef* | | *rSecP* | | *rPrimS* | | *rSecS* | | *cl90* | *gdp90* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Dominican Republic** | -72,87 | 38 | | | 2,83 | 10 | -2,87 | 14 | | | | | 19,10 | 4713,44 |
| **Liberia** | -93,63 | 39 | | | | | -25,13 | 54 | -35,63 | 38 | -11,63 | 41 | 21,84 | 519,16 |
| **Botswana** | -95,19 | 40 | | | -15,29 | 60 | -72,19 | 59 | -38,19 | 39 | -37,19 | 53 | 19,44 | 5800,82 |
| Italy | | | 12,43 | 4 | 11,03 | 4 | -3,57 | 16 | | | | | 0,43 | 24949,82 |
| **Mauritius** | | | 9,47 | 5 | | | | | | | | | 3,98 | 5011,89 |
| *Australia* | | | *4,18* | *11* | *-2,52* | *26* | *-2,62* | *13* | | | | | *0,00* | *25749,92* |
| *Germany* | | | *3,99* | *12* | *0,09* | *12* | *1,29* | *8* | | | | | *0,00* | *26163,48* |
| *Belgium* | | | *3,88* | *13* | | | *-14,22* | *46* | | | *-10,12* | *37* | *0,00* | *26230,67* |
| *Denmark* | | | *3,74* | *14* | *3,04* | *9* | | | | | | | *0,00* | *26299,42* |
| **Sri Lanka** | | | 3,45 | 16 | -3,05 | 29 | -8,65 | 31 | | | | | 2,85 | 1896,46 |
| **Malaysia** | | | 3,07 | 18 | -5,03 | 40 | -28,23 | 55 | | | | | 3,98 | 6062,04 |
| *Spain* | | | *2,85* | *19* | *-3,45* | *33* | | | *-3,65* | *10* | *-4,75* | *23* | *0,00* | *19965,42* |
| Portugal | | | 2,52 | 20 | -4,98 | 39 | | | | | | | 2,37 | 14893,4 |
| *Austria* | | | *2,01* | *22* | *-1,59* | *24* | *-5,69* | *24* | | | | | *0,00* | *27805,91* |
| *Netherlands* | | | *2,01* | *23* | *-0,99* | *22* | *-3,09* | *15* | | | *0,21* | *10* | *0,00* | *28121,6* |
| *Switzerland* | | | *2,00* | *24* | *-4,70* | *36* | | | | | | | *0,00* | *34952,63* |
| *Republic of Korea* | | | *1,91* | *25* | *-10,19* | *55* | *-6,79* | *27* | | | *-5,49* | *26* | *0,00* | *11437,17* |
| *Greece* | | | *1,51* | *26* | *-3,39* | *32* | *-4,09* | *20* | | | *-2,29* | *17* | *0,00* | *17825,63* |
| **Madagascar** | | | 1,22 | 27 | | | | | | | | | 37,59 | 916,97 |
| *New Zealand* | | | *1,22* | *28* | | | *-9,18* | *33* | | | *-7,78* | *35* | *0,00* | *19584,06* |
| *Sweden* | | | *0,68* | *29* | *-0,12* | *14* | | | | | *-5,62* | *27* | *0,00* | *26583,29* |
| *Iceland* | | | *0,38* | *31* | | | | | | | | | *0,00* | *27985,47* |
| *Norway* | | | *0,05* | *32* | *-0,15* | *15* | *-2,05* | *11* | | | *-1,85* | *16* | *0,00* | *32320,31* |
| *France* | | | *-0,30* | *33* | *-7,10* | *46* | *-11,90* | *39* | | | | | *0,00* | *25441,88* |
| *Canada* | | | *-0,96* | *34* | *-5,16* | *42* | *-4,06* | *19* | | | | | *0,00* | *27640,14* |
| *Finland* | | | *-1,67* | *35* | *-4,87* | *38* | | | | | | | *0,00* | *24422,27* |
| **Indonesia** | | | -1,72 | 36 | -0,32 | 16 | | | | | -4,32 | 22 | 11,30 | 2344,2 |
| **Zambia** | | | -2,58 | 37 | | | | | | | | | 16,92 | 1206,67 |
| **Kenya** | | | -3,58 | 39 | -9,38 | 53 | -9,38 | 34 | | | -3,38 | 19 | 43,36 | 1187,94 |
| *United Kingdom* | | | *-4,59* | *42* | *-5,09* | *41* | | | *-6,89* | *18* | *-5,79* | *29* | *0,00* | *24438,76* |
| *United States* | | | *-6,15* | *44* | *-5,65* | *44* | | | | | *-5,65* | *28* | *0,00* | *31636,85* |
| **Iran** | | | -7,56 | 45 | -12,06 | 59 | -21,66 | 51 | -15,66 | 31 | -18,06 | 48 | 6,84 | 6054,01 |
| **Algeria** | | | -13,53 | 46 | | | | | | | | | 3,26 | 4960,42 |
| **Sierra Leone** | | | -37,27 | 49 | | | | | -20,87 | 34 | -22,87 | 50 | 17,08 | 1046,21 |
| **Nicaragua** | | | | | 14,12 | 2 | | | 12,62 | 3 | 15,82 | 3 | 16,01 | 2178,54 |
| **Poland** | | | | | *0,54* | *11* | | | | | | | *0,00* | *7668,28* |
| Hungary | | | | | 0,07 | 13 | -3,83 | 17 | | | -1,63 | 15 | 0,35 | 11899,97 |

| | *rPrimP* | *r-HM* | *r-Coef* | | *rSecP* | | *rPrimS* | | *rSecS* | | *cl90* | *gdp90* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Hong(Kong?)* | | | *-0,65* | *18* | *-13,05* | *43* | | | *-9,55* | *36* | *0,00* | *22618,38* |
| **Burkina Faso** | | | -0,69 | 19 | | | -11,19 | 24 | -5,99 | 30 | 58,69 | 658,15 |
| **South Africa** | | | *-0,86* | *21* | | | *-18,86* | *32* | *-14,46* | *46* | *0,00* | *5389,66* |
| **Egypt** | | | -2,23 | 25 | | | | | | | 13,20 | 2647,79 |
| **Tunisia** | | | *-8,30* | *50* | *-13,30* | *44* | | | | | *0,00* | *3353,91* |
| **Morocco** | | | -11,21 | 57 | | | -45,91 | 40 | -5,41 | 25 | 10,56 | 2348,19 |
| **Uganda** | | | | | | | -69,52 | 41 | -32,12 | 52 | 46,84 | 553,1 |

Table 1: **Rate of returns on physical capital minus rate of return of human capital (different measures).** Country ranking for different measures. In italics countries with zero child labor rate at 1990, and in bold countries with GDP per capital below 11 billion dollars. *rPrimP* for real rate of return ($r$) minus Private rate of return of Primary Studies (Psacharopoulos 1993, Table A-1); *r-HM* for $r$ minus social rate of return (Mamuneasy et al 2006, Table 2); *r-Coef* for $r$ minus coefficient on Years of Schooling, Mincerian rate of return (Psacharopoulos 1993, Table A-2); *rSecP* for $r$ minus Private rate of return of Secundary Studies (Psacharopoulos 1993, Table A-1); *rPrimS* for $r$ minus Social rate of return of Primary Studies (Psacharopoulos 1993, Table A-1); *rSecS* for $r$ minus Social rate of return of Primary Studies (Psacharopoulos 1993, Table A-1); *cl90* Child labor rate (Doepke et al 2005); *gdp90* GDP per capital from Penn World Table (Heston et al 2011).

| | *rE+T* | | *r-E* | | *rPrimP* | | *r-HM* | | *r-Coef* | | *rSecP* | | *rPrimS* | | *rSecS* | | *cl90* | *gdp90* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Zimbabwe** | 34 | 1 | 26 | 2 | 62,72 | 1 | 75,42 | 1 | | | 30,82 | 2 | 68,12 | 1 | 31,72 | 2 | 31,86 | 366,73 |
| **Ghana** | 24 | 2 | 31 | 1 | -42,07 | 37 | | | -24,67 | 63 | -34,57 | 56 | -35,57 | 37 | -30,57 | 51 | 14,55 | 843,86 |
| **Kenya** | 19 | 3 | 16 | 3 | | | -3,58 | 39 | -9,38 | 53 | -9,38 | 34 | | | -3,38 | 19 | 43,36 | 1187,94 |
| **Cameroon** | 13 | 4 | 15 | 4 | | | | | | | | | | | | | 27,54 | 1896,82 |
| **Zambia** | 8 | 5 | 7 | 5 | | | -2,58 | 37 | | | | | | | | | 16,92 | 1206,67 |

Table 2: **Rate of returns on physical capital minus rate of return of human capital (different measures) in five African countries' Manufacturing Sector.** Country ranking for different measures. *rE+T* for $r$ minus return of human capital (Bigsten 1998, Table 7); *rE* for $r$ minus return of human capital (Bigsten 1998, Table 6). *rPrimP* for real rate of return ($r$) minus Private rate of return of Primary Studies (Psacharopoulos 1993, Table A-1); *r-HM* for $r$ minus social rate of return (Mamuneasy et al 2006, Table 2); *r-Coef* for $r$ minus coefficient on Years of Schooling, Mincerian rate of return (Psacharopoulos 1993, Table A-2); *rSecP* for $r$ minus Private rate of return of Secundary Studies (Psacharopoulos 1993, Table A-1); *rPrimS* for $r$ minus Social rate of return of Primary Studies (Psacharopoulos 1993, Table A-1); *rSecS* for $r$ minus Social rate of return of Primary Studies (Psacharopoulos 1993, Table A-1); *cl90* Child labor rate (Doepke et al 2005); *gdp90* GDP per capital from Penn World Table (Heston et al 2011).