The Increase in College Premium and the Decline of Low Skill Wages, a Signaling Story.*

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Abstract

We introduce wealth heterogeneity and general conditions on wealth distribution (log-concavity and monotone likelihood) in Spence's (1973) education signaling model. With an improved access to college, this setup presents the reduction of non-college wages as a necessary condition for an increase in the college premium. This fits with the observed trends for the college premium, college enrollment and non-college wages in the US over the last thirty years.

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1 Introduction.

The increase in returns to higher education and the expansion in graduate supply in the US over the last decades of 20th century are a well documented

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fact[1]. Returns to higher education are measured according to the college wage premium, that is by differences in earnings between college and lower levels of education. As we can observe in the data (see *Figure 1*) the driving force of the expansion in the college premium is the reduction of low skill wages. Here we present a signaling explanation compatible with the three previous observations, the increase in college graduates' supply, the increase in returns to higher education and the decline in real earnings for less qualified workers.

Economic literature has made a notable and fruitful effort to understand the simultaneous increase in college premium and college enrollment. Most of these explanations focus their attention exclusively on the educational wage gap (see , Card and Di Nardo, 2002; Juhn et al., 1993 or Murphy and Welch, 1989 and 1992, among others). However, much less emphasis has been placed on disentangling why the expansion in the college premium is mainly driven by a reduction of low skill wages. Even, one of the most influential explanations, the canonical model, based in the demand and supply of imperfectly substitutable skills, has difficulties explaining the reduction of low skill wages. Acemoglu and Autor (2010) illustrates this lack in the following quotation, *"(...) despite its notable successes, the canonical model is largely silent on a number of central empirical developments of the last three decades, including, significant declines in real wages of low skill workers, particularly low skill males"*.

Here we revist the signaling interpretation of college education (also known as composition). We introduce wealth heterogeneity, similarly to Hendel et al. (2005), and very general conditions on wealth distribution, in Spence's (1973) original setup. Given an improve in access to college, this setup presents the reduction of low skill wages as a necessary condition for

[1]See Katz and Autor (1999) for a general survey of the literature and *Figure 1* to observe changes in returns to education and the increase of college graduates.

an increase in the college premium. This theoretical finding can be used as an observable testable implication for the feasability of the composition hypothesis. According to our results, with an improved access to college, composition hypothesis is a valid explanation for the expansion in the college premium only if it is accompanied by a reduction of non-college wages[2].

The intuition behind our idea is as follows. In a pure signaling model wages are equal to the average productivity of an education level. Then, college education can reveal (or partially reveal) the unobservable productivity or ability of individuals. Nevertheless, in a situation with imperfect credit markets and wealth heterogeneity, higher education is not only a signal of ability but also of indivuduals' (parents) wealth. The extent to which college education is more indicative of ability or wealth depends on which of the two elements is more determining for college enrollment. Increasing access to college alters the relationship between these two elements and hence the information provided by the signal. Imagine an extreme case in which college education is restricted to the richest individuals of high ability. Given the imperfection of credit markets, opening access to post-secondary education (i.e. reducing its cost) simultaneously attracts to college poor individuals with a high-ability and rich individuals with a low-ability. Consequently, only a minority of poor individuals with low-ability remain in the non-college education levels. This has ambiguous effects on the average productivity of college educated workers but will clearly reduce the non-college sector's productivity, moving the low-skill wage towards the low productivity. The informational role of higher education as a signal changes substantially in this process. College education does no longer reveal who are the high ability candidates, but lack of higher education discloses who are the less

[2] In the particular case of borrowing constrained agents the reduction of non-college wages is the unique driving force of an increase in the college premium.

able ones.

We use a stylized model with two types of ability, high and low, to show that some very general conditions on wealth distributions are sufficient to meet the previous intuition in more general cases, with differently abled individuals attending both educational levels. Log-concavity of the wealth distribution functions and the monotonicity of their likelihood guaranty that after an improvement in access to higher education, an increase in college premium is motivated by (or at least must be accompanied with) a reduction in non-college wages[3]. Given a context in which both wealth and ability are relevant to the education decision, these two conditions guarantee that the fraction of high ability types that abandon the non-college education level is relatively large according to their current presence in that sector, making the non-college wage decrease. Monotone likelihood is introduced in order to account for the potential positive correlation between wealth and ability, see Blau (1999) or Heckman (2011)[4].

2 Literature Review.

There is a large debate in the economic explanation of changes in college premium. These explanations can be divided into two blocks, corresponding

[3] We model the improvement in access as a reduction in tuition or as a reduction in education loans interest rate. Despite the increase in tuition observed during the period, there is evidence of an improvement in affordability of post-secondary education, see Archibald and Feldman, 2008 and 2010. In terms of our model, an improvement in affordability is equivalent to a reduction in tuition. Additionally, increases in tuition are mitigated if we pay attention to the net price of education, that is to the tuition minus all grants as is shown in Dynarski (2002), Hill et al. (2005) and Horn et al. (2002) or to the reduction of accommodation costs as a consequence of the implantation of new post-secondary institutions during the period.

[4]Just log-concavity is sufficient when considering independence between wealth and ability.

to the two main economic interpretations of education, human capital and signaling.

In the literature on human capital Groot and Oosterbeek (1994) or Layard and Psacharopoulos (1974) explain the increase in college premium as an improved productivity of education. Acemoglu (2002); Autor et al. (2005) and Katz and Murphy (1992); among others, construct supply and demand models with skill biased technology shocks, providing an appealing partial equilibrium theory for the increase in college premium. However, as we have already said all these models fail to explain the reduction in low skilled wages. There is the exception of Acemoglu and Autor (2010). They endogeneize the assigment of skills to different tasks and add an evolving technology that replaces labor in some tasks, to explain a reduction of non-college wages.

The second block of literature emphasizes the the signaling power of education, which associates the value of education with its positive correlation with individuals' unobserved skills. Section 5.1 in Riley (2001) provides a survey with the favorable and unfavorable evidence about the signaling hipothesis. When talking about changes in the college premium, we can divide signaling based explanations into two groups. On one hand, the first group associates the increase in college premium with an increase in the market price of the unobserved ability (Blackburn and Neumark, 1991[5] and 1993, Bound and Johnson, 1992 or Taber, 2001). Their explanation is based on the hypothesis that new technology complements better with unobserved ability.

[5] They show how if ability is a continuous variable and individuals sort in college education depending on their relative position in the ability rank, the composition hypothesis can only explain an increase in college premium if more than half of the population endorse post-secondary education (for single peak symmetric densities). However, our model and also Feldman (2004) present theoretical frameworks in which ability is not the only determinant for attending college but also individuals' wealth. This makes the Blackburn and Neumark's requirement unnecessary for an increase in college premium in a signaling model.

In this case, the increase in college premium is a consequence of the positive correlation between education and the improved productivity of unobserved skills.

Our argument belongs to a second group of signaling based explanations that deal with the composition effect. Lang (1986) is one of the first works, explaining the college premium by the composition effect. Bedard (2001) does not deal directly with changes in college premium, but suggests that composition plays an important role in the college enrollment decision and consequently, on returns to college. Hendel et al. (2005), Zheng (2010) and the present work use the increased access to college to explain changes in the college premium under a composition hypothesis. The main difference is that we use a static setting to focus separatedly on the wage of each educational level.

Some recent literature explicitly accounts for the non-exclusion between the previous explanations and study the contribution of each to the dynamics of college premium. Fang (2006) and Zheng (2010) distinguish between signaling and human capital. They asses that signaling can account for one third and near to one fifth of the increase in college premium, respectively.

Cunha et al. (2011), specifically separates between human capital and each of the two signaling explanations we have mentioned. They found that all of them play a role, altough the main effect is driven by the shifts in the demand and supply of human capital.

Our contribution to these debate is twofold. On one hand, as Carneiro and Lee (2005) point out, any estimation of the determinants of college primium that neglects composition will be biased. Then, whatever its relative contribution is, a good understanting of the composition effect is essential. On the other hand we provide an observable implication of composition. According to our results, composition can only explain an expansion in college premium if there is a reduction of non-college wages. This is the case in the United States, but is not generally true in all developed countries with an increase in college premium, see Ghose (2003).

3 The model.

To model wage formation we use a version of Spence's signaling model with wealth heterogeneity similar to Hendel et al. (2005). Individuals are endowed with an initial wealth and with a particular level of ability, perfectly correlated with their productivity and their academic performance. Then, with imperfect credit markets, their decision about acquiring college education does not only depend on their ability but also on their affordability of education.

There is a population of size one. A fraction π is high ability (H) with productivity $q_H \in \mathbb{R}_{++}$. The remaining $1-\pi$ mass is of low ability (L) with productivity $q_L \in \mathbb{R}_{++}$. As in Spence's original framework, the productivity of high ability types is greater than the one of low ability, that is $q_H > q_L$. We denote by $\bar{q}$ the population average productivity, i.e. $\bar{q} = \pi q_H + (1-\pi)q_L$.

Since ability affects productivity it is valued by firms, but unobservable. However, education can be used as a signal. To get education individuals have to pay the tuition, that we denote by T. But deciding to study does not immediatly lead to the credential. As in Zheng (2010), there is some risk of failing. Then, individuals have a probability P_i, $i = H, L$ of successfully finishing education. High ability individuals have a greater probability of obtaining the credential, in particular and without loss of generality we assume that $1 = P_H > P_L > 0$, which captures the Spence (1973) original idea that education is more challenging for low ability individuals. Once paid, tuition is never reimbursed, independently of failing.

The initial wealth of individuals is denoted by $b_i \in \mathbb{R}_+$. Endowments follow a continuous distribution, $F_i(b_i)$. In the case of obtaining a degree, individuals work in the qualified sector and a perceive a wage w^e, greater in equilibrium, than the one perceived in the case of working in the non-qualified sector (i.e. not enrrolling or failing), w^n. Workers are risk neural and only have incentives to study if their expected gain, i.e. the college premium, denoted $cp = w^e - w^n$, is sufficiently large relatively to their cost of education. At the same time, given the credit market imperfections, the education decision is also subject to individuals' initial wealth.

Firms can only observe workers' credentials and set wages according to their educational level, but cannot observe individuals' ability or wealth. Firms know the proportion of high-ability workers in the population as well as wealth distributions. Firms compete a la Bertrand, which implies that they fix wages according to the expected productivity conditional on having education or not. The expected productivity in each sector can be represented by g^s, $s = e, n$:

$$g^e = \frac{\pi(1-\rho_H)q_H + (1-\pi)(1-\rho_L)P_L q_L}{\pi(1-\rho_H) + (1-\pi)(1-\rho_L)P_L}$$
$$g^n = \frac{\pi\rho_H q_H + (1-\pi)(1-P_L(1-\rho_L))q_L}{\pi\rho_H + (1-\pi)(1-P_L(1-\rho_L))} \qquad (1)$$

Where ρ_i, with $i = H, L$, represents the proportion of each type in the non-college sector.

We consider first a situation with borrowing constrained agents that captures all the intuitions of the model. Afterwards, we extend to a situation with education loans.

4 Borrowing Constraints.

Without credit markets individuals are borrowing constrained and can only attend college if their wealth exceeds tuition. Then, an individual studies if and only if it is worth and it can be afforded. In the first place, an individual prefers to study if the expected gains are greater than in the case of not studying, i.e. $P_i w^e + (1 - P_i) w^n - T > w^n$ for $i = H, L$. We have assumed without loss of generality, that $P_H = 1$, then the incentives condition can be rewritten as, $cp > \frac{T}{P_i}$, with $i = H, L$. On the second place, individuals can afford college education if their wealth is greater than tuition fee, i.e. $b_i > T$ for $i = H, L$. We denote by b_i^* the lower wealth level of type i who studies in equilibrium. b_i^* is a function of college premium (endogenous part of the model) as well as of other exogenous variables (P_L, T). When $cp = \frac{T}{P_i}$, $i = H, L$, there is a mass of indifferent individuals of type i that can afford the education costs. For these cases we assume, without lost of generality, that more affluent individuals have priority to attend college. Then, given the continuity of wealth distributions and the monotonicty of education costs, $F(b_i^*)$ represents the proportion of type i in the non-college sector. That is, we can use $\rho_i = F(b_i^*)$ in the average productivities of expression (1). Therefore the average productivities are a function of each types' wealth threshold level, i.e. $g^s(b_H^*, b_L^*)$, for $s = e, n$. We write $\bar{b_i}$ and $\underline{b_i}$ for the upper and lower bound for type's i wealth, respectively.

We are interested in situations in which both heterogeneities are relevant, that is with individuals of different ability present in both educational levels. The following assumptions ensure that this is the case,

- *Assumption 0 (A0): There is at least one individual of each type that can afford the tuition and at least one who cannot. That is,* $\bar{b_i} > T$ *and*

$\underline{b_i} < T$, $i = H, L$.

- *Assumption 1 (A1): The tuition is lower than the difference between high productivity and population average productivity.* $T < q_H - \bar{q}$.

- *Assumption 2 (A2): For a college premium near to its upper bound, the expected earnings of low types exceed the cost of college. This is not true for a college premium equal to the difference between high productivity and population average productivity. That is,* $q_H - \bar{q} < \frac{T}{P_L} < q_H - q_L$.

A0 simply argues that at least the most affluent individual of each type can always afford the cost of education, while the poorest one cannot. $A1$ and $A2$ establish that differences in ability are relevant. Since firms pay according to the average productivity, the upper bound for college premium corresponds to the greatest difference in productivity, that is $q_H - q_L$. According to $A1$ and $A2$ both types have incentives to study when college premium have some value arbitrary close to its upper bound. But this is not true when college premium is equal to the difference between high productivity and population average productivity. In particular, according to $A1$, in this case high types have incentives to study. Conversely, $A2$ ensures that this is not true for low types, because of their strictly positive probability $1 - P_L > 0$ of failing.

For equilibrium we use Bayesian perfection. *A1* together with *A2* guarantees that firm's beliefs satisfying Cho and Kreps (1987) intuitive criteria must be that when nobody studies, only high types are interested in deviating by attending college, assigning the highest productivity, q_H, to the college wage. Therefore, it can not exist a pooling equilibrium with nobody getting education. On the other hand *A2* guarantees that pooling in education is not possible because in that case low ability types would strictly prefer not to study. The exclusion of pooling equilibria avoids the indeterminacy problem that might arise in the computation of average productivity when $F_i(b_i^*) = 0$ (or equals 1) for all $i = L, H$.

- *Assumption 3 (A3): The amount of high ability individuals that can afford the tuition is sufficient to attract some low types to college.*

$$F_H(T) < \frac{(1-\pi)(q_H - q_L - \frac{T}{P_L})}{\pi \frac{T}{P_L}}$$

This assumption guarantees that the proportion of high ability types that can afford the tuition cost is sufficiently large to sustain a college premium that incentives low types to study, i.e. $q_H - g^n(T, \bar{b}_L) > \frac{T}{P_L}$. Therefore at least the most affluent individuals of both types study in equilibrium.

We use $\tilde{w}^s$ for $s = e, n$ to denote equilibrium wages with $cp^* = \tilde{w}^e - \tilde{w}^n$ and $CP(b_H^*, b_L^*) = g^e(b_H^*(cp), b_L^*(cp)) - g^n(b_H^*(cp), b_L^*(cp))$ to denote the difference in productivities in equilibrium. As in Hendel et al. (2005) we refine the equilibrium with the notion of *tâtonnement stability.*[6].

Definition 1. *Equilibrium*
A signaling perfect Bayesian equilibrium is a set of choices of education based on ability and wealth level $s^*(q, b) \in \{e, n\}$, firms' beliefs about types for a given choice $\beta(H|s)$, and wages formation according to $g^e(b_H^*, b_L^*)$ and $g^n(b_H^*, b_L^*)$. Such that:

$$g^n(b_H^*(\tilde{w}^e - \tilde{w}^n), b_L^*(\tilde{w}^e - \tilde{w}^n)) = \tilde{w}^n$$
$$g^e(b_H^*(\tilde{w}^e - \tilde{w}^n), b_L^*(\tilde{w}^e - \tilde{w}^n)) = \tilde{w}^e$$

An equilibrium is stable if and only if:

$$\frac{\partial CP(b_H^*, b_L^*)}{\partial cp} < 1$$

[6]An equilibrium is tâtonnement stable when the college premium returns to the equilibrium level under slight perturbations of cp^*.

In the above defintion we have written productivities as $g^s(b_H^*(\tilde{w}^e - \tilde{w}^n), b_H^*(\tilde{w}^e - \tilde{w}^n))$ in order to emphasize the fix point nature of this problem, however we will generally simplify notation to $g^s(b_H^*, b_L^*)$.

The following lemma shows the existence and uniqueness of a stable equilibrium with both high ability and low ability types in both sectors.

Lemma 1. *If A0 to A3 are satisfied then there is a unique equilibrium. This equilibrium is stable.*

As we show in the proof of this lemma (see the appendix), in equilibrium the college premium is always greater or equal to the education cost of low ability individuals ($\frac{T}{P_L} \leq cp^*$). This implies that all high types strictly prefer to attend college in equilibrium, but borrowing constraints prevent some of them from doing so. For low types it can be both a matter of incentives or borrowing constraints. We can distinguish two different types of equilibrium outcome according to this. One with a college premium strictly greater than low type's education cost (i.e. $cp^* > \frac{T}{P_L}$) and another in which both coincide (i.e. $cp^* = \frac{T}{P_L}$). We will refer to them as fully constrained and high-ability constrained equilibrium, respectively. In the first case both high and low individuals are borrowing constrained. In the second case low ability individuals are indifferent between studying or not, therefore only high ability types are borrowing constrained. Comparative statics' conclusions are different in these two cases.

Once we have stated our framework, the objective is to analyze the effects of opening access to education. We do comparative statics on tuition cost to analyze the effects of opening access to education on college premium and on wages[7]. We show that with borrowing constraints logarithmic concavity

[7]Tuition is the unique policy available in this basic model. In section 4 we introduce education loans and also look at the reduction of interest rate as an additional way of opening access to education.

and monotone likelihood are sufficient to guarantee that when opening access to education college premium increase because of a reduction in non-college wages. Moreover both wages go down in that case. Our results contribute to explain the already commented returns to education phenomena, presenting the decline in less educated jobs' wage as the driving force of the increase in returns to college.

Although not the only one, wealth is an important determinant in individuals' ability formation. The available evidence suggests a positive relationship between wealth and ability, see for instance Blau (1999). Recently, Cunha and Heckman (2010) showed that early years are determinant in the development of both cognitive and non-cognitive skills. Despite not being the direct reason for a better ability, affluent families can invest more resources on stimulating their child. The following assumption guarantees the positive correlation between wealth and ability,

- *Assumption 4 (A4): The likelihood ratio* $\frac{f_H(b)}{f_L(b)}$ *is monotone increasing.*

This assumption implies that lower types' wealth is stochastically dominated by high types, i.e. $F_H(b) \leq F_L(b)$, which implies that ability is positively correlated with wealth. We state our results for this case. However, the case with independence between wealth and ability is also considered and left as a remark.

Proposition 1.

- *I. When opening access to education in a fully constrained equilibrium:*

  i) Both wages, $\tilde{w}^e$ *and* $\tilde{w}^n$*, go down if wealth follows a log-concave distribution function and the likelihood ratio,* $\frac{f_H(b)}{f_L(b)}$ *is monotone increasing.*

  ii) The college premium increases if and only if:

$$|\frac{\partial CP(b_H^*, b_L^*)}{\partial b_H^*}| > |\frac{\partial CP(b_H^*, b_L^*)}{\partial b_L^*}|$$

- *II. When opening access to education in a high-ability constrained equilibrium:*

  i) The wage of the college educated sector, $\tilde{w}^e$ goes down.

  ii) The wage of the non-college sector, $\tilde{w}^n$, can either increase or decrease.

  iii) The college premium decreases.

If wealth distributions satisfy monotone likelihood and logartihmic concavity, improving access to higher education in a fully constrained equilibrium, i.e. $cp^* > \frac{T}{P_L}$, reduces both college and non-college wage[8]. Since the expected gains from education are greater than its cost for both types, everybody prefers to study. However, not all individuals can afford the education cost. Only those with a wealth greater than tuition, i.e. $b_i \geq T$, $i = H, L$, can enroll to college. Therefore, the wealth threshold level for both types is equal to tuition $b_i^* = T$ for $i = H, L$. A general reduction of education cost, relaxes borrowing constraints for both types by the same amount, i.e. $\frac{\partial b_i^*}{\partial T} = 1$, $i = H, L$. Then, we only need to look at the change in each wage involved by these identical changes in wealth threshold levels.

Obviously, since equilibrium wages are equal to the average productivity, high ability individuals getting education (i.e. a reduction in the wealth threshold level, b_H^*) make the college wage increase, $\frac{\partial g^e(b_H^*, b_L^*)}{\partial b_H^*} < 0$, and the non-college decrease, $\frac{\partial g^n(b_H^*, b_L^*)}{\partial b_H^*} > 0$. At the same time low ability individuals becoming educated have the opposite effect, i.e. $\frac{\partial g^e(b_H^*, b_L^*)}{\partial b_L^*} > 0$ and $\frac{\partial g^n(b_H^*, b_L^*)}{\partial b_L^*} < 0$. If the effect of high types predominates over the low types in college (non-college), then that sector's wage increases (decreases). The opposite is true if the effect of low types predominates.

[8] Notice the generality of log-concavity condition in An (1995) or Bagnoli and Bergstrom (2005)

Which type is more relevant on each wage depends on the relative amount of them that moves from one sector to the other as well as on their relative presence in each sector (i.e. on the relative value of density and cumulative functions). Given that wages are equal to the average productivity, the simultaneous reduction in both wages only takes place if the proportion of high types decreases simultaneously in both sectors. That is, if the proportion of high types that moves from non-college to college is greater than their current relative presence in non-collge, i.e. $\frac{F_H(b_H^*)}{1-P_L(1-F_L(b_L^*))} < \frac{f_H(b_H^*)}{P_L f_L(b_L^*)}$, but smaller than in the college sector, i.e. $\frac{f_H(b_H^*)}{P_L f_L(b_L^*)} < \frac{1-F_H(b_H^*)}{P_L(1-F_L(b_L^*))}$. This is guaranteed by log-concavity and the monotone likelihood ratio of distribution functions. It is equivalent to say that the effect of high types predominates over the low types in the non-college sector, while the opposite is true in the college educated one.

When tuition cost decreases in a fully constrained equilibrium there are ambiguous effects on college premium. Changes in the decision of each type affects the college premium in opposite directions. Obviously, increasing the presence of high ability indivduals in college increases the college premium, while increasing the presence of low ability reduces it. The final effect on college premium depends on whether it is more sensitive to the switching of high or low ability individuals (i.e. to changes in b_H^* or in b_L^*). This is determined by the relative size of changes and current presence of types in each sector, however log-concavity and monotone likelihood are not enough to guarantee a specific direction for this change.

Previous results change in the case of a high-ability constrained equilibrium, i.e. $cp^* = \frac{T}{P_L}$. In this case there is a mass of low ability types that are indifferent between studying or not. Then, we cannot find a closed form solution for b_L^*. We can determine the change in the wealth threshold level of high types ($b_H^* = T$, and $\frac{\partial b_H^*}{\partial T} = 1$), but not for low types. After a reduction in tuition some high ability types moves to the college sector (recall that high types strictly prefer to enroll college, since $cp^* = \frac{T}{P_L} > T$). But this also attracts some low ability types to college. The presence of an indifferent mass of low ability types implies that some of them decide to study until canceling out the increase in college premium generated by the entrance of high types and reducing it to the new value of tuition. The indifferent mass of low types gurantees that necessarily $\frac{\partial b_L^*}{\partial T} > \frac{\partial b_H^*}{\partial T}$. This reinforces the previous effects on the wage of the college educated sector, which is further reduced. Instead, this difference in the change of wealth threshold levels leads to an ambiguous result for non-college wages.

Corollary 1. *If wealth follows a log-concave distribution function with a monotone increasing likelihood ratio between high and low types, the driving force of an increase in the college premium is the reduction of non-college wage.*

We have seen that when the previous properties in wealth are satisfied returns to higher education can only increase in a fully constrained equilibrium. In this equilibrium both wages decrease. Then any increase in college premium must be motivated by a reduction of less educated workers' wage that exceeds the one of college wages.

5 Education Loans.

Previous results arise in the absence of a credit market for education. Two problems emerge from this simplification. First, education loans are a very extended solution to solve liquidity constraints blockage to education. Moreover, many programs to facilitate access to education consist in introducing advantageous conditions on education loans. Obviously these two issues remain uncovered in a model without credit markets. We modify Hendel et al. (2005) signaling model with wealth heterogeneity to deal with these issues. We can show that similar results take place under this more general framework. Moreover we can also provide results for the case of opening access to education through the reduction of education loans' interest rates.

The model is similar than before with the difference that now individuals can borrow to study. We denote by x the borrowing interest rate. We also assume that there is an interest rate for lending, that we normalize to 1 (in case of not spending their wealth on education, individuals keep exactly that level of wealth). However, credit markets are imperfect and the borrowing interest rate is higher than the lending one, i.e. $x > 1$. With education loans the individuals decision about enrolling to college only depends on the relative payoff perceived in each case, never on affordability as before. Anyway, borrowing is costly, so poorer individuals require a greater college premium to get education.

In case of not studying, the workers' payoff is equal to the non-college wage plus their initial wealth, $w^n + b_i$. Otherwise the payoff depends on whether individual's wealth is enough to cover tuition. If this is the case, the payoff for studying is the expected earnings of enrolling to college minus the difference between their initial wealth and tuition, $P_i w^e + (1-P_i)w^n - T + b_i$. If wealth is insufficient to pay education cost, then individuals have to borrow, and their payoff is the expected earnings of enrolling to college minus the tuition and its associated borrowing costs, that is $P_i w^e + (1 - P_i)w^n - (T - b_i)(1 + x)$. Workers decide to invest on education if their payoff exceeds that of not studying. Then a worker of type i with a level of wealth $b_i < T$ borrow to study if and only if:

$$cp \geq \frac{b_i + (T - b_i)(1 + x)}{P_i}$$

On the other hand, an individual with $b_i \geq T$ studies if and only if:

$$cp \geq \frac{T}{P_i}$$

We denote with b_i^* the wealth threshold level determined by each type's indifferent individual. Because of monotonicity of borrowing costs, individuals with a wealth lower than the one of their corresponding indifferent type will not study, and the opposite will be true for those with a higher wealth.

Hendel et al. assume that low ability individuals never have incentives to study. Consequently, in their case the average productivity in the college educated sector is always q_H. Instead, we allow for low ability individuals to get education, which implies that both college and non-college sector's average productivity can change. In particular we analyze the pooling equilibrium with the presence of both types in each sector. To guarantee this we need the following assumptions,

- *Assumption 1'(A1′):* $T < q_H - \bar{q} < \frac{T}{P_L} < q_H - q_L < \frac{T}{P_L}(1 + x)$
- *Assumption 2'(A2′):* $\frac{T}{P_L} < T(1 + x)$.

These assumptions relax the requirement of low types never studying imposed in Hendel et al. (2005). Instead, we assume that low type individuals with a very small wealth never study (third inequality in the A1'). However, low ability but affluent individuals may be interested in acquiring education, at least for a college premium near to the maximum one (second inequality in A1'). First inequality guarantees that rich individuals with high ability are always interested in studying. A2' implies that paying the entire cost of education with a loan, is more expensive than the cost perceived by a low type that can afford tuition with own resources.

Lemma 2. *If $F_i(b_i)$, $i = H, L$ are continuously differentiable for any b_i and $\frac{\partial F_i(b_i^*)}{\partial b_i} \neq 0$ for all $b_i^* \in R_+$ and A1' and A2' are satisfied then there is at least one stable equilibrium.*

We want to restirct the outcome of the model to that situations in which both wealth and ability are relevant for college enrollment. The following assumption guarantees this,

- *Asumption 3'*($A3'$):

$$F_H(T) < \frac{(1-\pi)(q_H - q_L - \frac{T}{P_L})}{\pi \frac{T}{P_L}}$$
$$1 - F_L(\frac{T}{P_L}) > \frac{\pi(q_H - q_L - T(1+x))}{(1-\pi)\, T\, (1+x) P_L}$$

The first part of the assumption is identical to $A3$ in the previous section and avoids that only high ability types could study. The second part is an analogous condition to avoid that only low ability individuals do not study. The first case that we are preventing with this assumption, i.e. $CP(b_H^*, b_L^*) < \frac{T}{P_L}$, is analogous to the static model analyzed in Hendel et al. (2005), while the latter, i.e. $T(1+x) \leq CP(b_H^*, b_L^*)$, is exactly the opposite case (and leads to opposite conclusions)[9]. Specifically we present results for the more general case with $\frac{T}{P_L} < cp^*$[10].

Proposition 2. *Given a change in tuition in an equilibrium with $cp^* > \frac{T}{P_L}$:*

$$\frac{\partial CP(b_H^*, b_L^*)}{\partial T} < 0 \Longleftrightarrow \left|\frac{\partial CP(b_H^*, b_L^*)}{\partial b_L^*}\right| < \left|\frac{\partial CP(b_H^*, b_L^*)}{\partial b_H^*}\right| \Longleftrightarrow \frac{\partial b_L^*}{\partial T} < \frac{\partial b_H^*}{\partial T}$$

[9] Moreover we can understand from Hendel et al. (2005), that in these two cases opening access to education will push the college premium to the equilibrium analyzed here.

[10] When $\frac{T}{P_L} = cp^*$ results are exactly as in the borrowing constrained case.

$$\frac{\partial b_L^*}{\partial T} < \frac{\partial b_H^*}{\partial T} \Longrightarrow \frac{\partial b_H^*}{\partial T} > 0, \frac{\partial b_L^*}{\partial T} > 0$$

This proposition provides some equivalent conditions for an increase in the college premium. Since differently abled individuals obtain different net returns from education they are willing to borrow different amounts of money. Then, the change in the wealth threshold level is also different for each type, i.e. $\frac{\partial b_L^*}{\partial T} \neq \frac{\partial b_H^*}{\partial T}$.

By construction, the college premium increases with the college enrollment of high types, i.e. $\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*} < 0$, and decreases with the entrance of low types, i.e. $\frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*} > 0$ (recall that increasing the college enrollment of some type means a reduction in b_i^*, $i = H, L$). The premium increases if and only if it is more sensitive to the positive effect of high types' enrolling to college than to the negative effect of low types, i.e. $|\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}| < |\frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}|$. The sufficient and necessary condition for this is that high ability individuals react more positively to a change in tuition than low types $\frac{\partial b_H^*}{\partial T} > \frac{\partial b_L^*}{\partial T}$. Finally, the requirement to observe a stronger reaction of high ability types is that a greater mass of individuals prefers to study when it is cheaper, i.e. $\frac{\partial b_H^*}{\partial T} > 0, \frac{\partial b_L^*}{\partial T} > 0$.

Proposition 3. *When opening access to education by reducing tuition in an equilibrium with* $cp^* > \frac{T}{P_L}$ *increases the college premium:*
i) Non-college wage decreases if wealth follows a log-concave distribution function and the likelihood ratio, $\frac{f_H(b)}{f_L(b)}$*, is monotone increasing.*
ii) College wage increases if and only if:

$$\frac{f_H(b_H^*)}{f_L(b_L^*)} < \frac{1 - F_H(b_H^*)}{1 - F_L(b_L^*)} \frac{1 + \frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*} \frac{1-P_L}{x}}{1 + \frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*} \frac{P_L-1}{x}}$$

Since we are interested in explaining an increase in college premium, we focus in this case to analyze the change in each sector's wage. The monotonicity of the likelihood ratio together with log-concavity guarantees that the non-college wage is more sensitive to a change in high ability wealth threshold level rather than to a low ability one $|\frac{\partial g^n(b_H^*,b_L^*)}{\partial b_H^*}| > |\frac{\partial g^n(b_H^*,b_L^*)}{\partial b_L^*}|$. Combining this with the necessary condition for an increase in college premium, $\frac{\partial b_H^*}{\partial T} > \frac{\partial b_L^*}{\partial T} > 0$, and the sign of each expression, automatically guarantees a reduction in the non-college wage, $\frac{\partial g^n(b_H^*,b_L^*)}{\partial T} = \frac{\partial g^n(b_H^*,b_L^*)}{\partial b_H^*}\frac{\partial b_H^*}{\partial T} + \frac{\partial g^n(b_H^*,b_L^*)}{\partial b_L^*}\frac{\partial b_L^*}{\partial T} > 0$. Instead, for the wage of the college educated sector both effects go in opposite directions. On one hand log-concavity and monotone likelihood can guarantee that $|\frac{\partial g^e(b_H^*,b_L^*)}{\partial b_H^*}| < |\frac{\partial g^e(b_H^*,b_L^*)}{\partial b_L^*}|$, but on the other hand, high types reacts strongly to a reduction in tuition, $\frac{\partial b_H^*}{\partial T} > \frac{\partial b_L^*}{\partial T}$. Then final effect on the wage of the college educated sector depends on which of these two effects predominate. If the more influential (because of a lower presence) but less numerous (smaller change in wealth threshold level) low types' effect prevails over the less influential but more numerous high types that decides to study then the wage of the college educated sector decrease. The opposite is true if the later prevails.

On the other hand, college wage can either increase or decrease since log-concavity of distribution functions together with the monotonicity of the likelihood function is not sufficient to guarantee its reduction.

Finally we analyze the improvement of access to college by reducing the cost of borrowing to pay the tuition.

Proposition 4. *When opening access to education by reducing education interest rate in an equilibrium with $cp^* > \frac{T}{P_L}$:*
i) The college premium increases if and only if:

$$\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}\left(\frac{1}{x^2}CP(b_H^*,b_L^*) - \frac{T}{x^2}\right) + \frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}\left(\frac{P_L}{x^2}CP(b_H^*,b_L^*) - \frac{T}{x^2}\right) < 0$$

ii) Non-college wage decreases if wealth follows a log-concave distribution function and the likelihood ratio, $\frac{f_H(b)}{f_L(b)}$, is monotone increasing.

iii) College wage increases if and only if:

$$\frac{f_H(b_H^*)}{f_L(b_L^*)} < \frac{1-F_H(b_H^*)}{1-F_L(b_L^*)} \frac{(CP(b_H^*,b_L^*)P_L - T)x - \frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}(1-P_L)}{(CP(b_H^*,b_L^*) - T)x + \frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}(1-P_L)}$$

We obtain similar results when we consider a change in the education loans interest rates instead of tuition. The main difference is that when reducing education loans interest rate, there is always a a higher relative change in the presence of high ability types, $0 < \frac{\partial b_L^*}{\partial x} < \frac{\partial b_H^*}{\partial x}$, hence the increase in college premium is more likely to occur. The greater net returns to college for high types make them more willing to borrow than low types. Then a reduction in the interest rate of education loans represents a major cost savings for them. This implies the indifferent high type's wealth threshold level is more sensitive to changes in interest rate.

At the same time log-concavity and monotone likelihood ratio guarantees that high types abandoning the non-college sector's effect prevails over the one of low ability, $\frac{\partial g^n(b_H^*,b_L^*)}{\partial b_H^*} > \frac{\partial g^n(b_H^*,b_L^*)}{\partial b_L^*}$, pushing down the wage of that sector. The larger proportion of high types that abandon the non-college sector, $\frac{\partial b_L^*}{\partial x} < \frac{\partial b_H^*}{\partial x}$, reinforces this effect, making the wage in the non-college sector decrease. On the other hand, college wage can either increase or decrease since, as in the previous case, $\frac{\partial g^e(b_H^*,b_L^*)}{\partial b_i^*}$ and $\frac{\partial b_i^*}{\partial x}$ play in opposite directions.

Finally, in the borrowing constrained version of the model, an increase in education returns when reducing tuition cost is always induced by a reduction in the wage of the non-college sector, never by an increase in the educated one, since both wages are decreasing. An analogous version, for the case with education loans is presented in the following corollary.

Corollary 2. *If wealth follows a log-concave distribution function with a monotone increasing likelihood ratio between high and low types, an increase* *in the college premium must be accompanied by a reduction of non-college wages.*

The college wage sector can either increase or decrease, in this case. However, the reduction in the non-college sector always takes place. Then a more relaxed version of our testable implication is still valid, with an improved access to college, log-concavity and monotone likelihood, the composition hypothesis is only valid to explain an increase in college premium if we observe a reduction of low skill wages.

Finally the following remark covers the specific case of indpendence between wealth and ability, i.e. $F_H(b) = F_L(b) = F(b) \; \forall \; b$.

Remark 1. *If wealth and ability are independent, logarithmic concavity is sufficient to guarantee the results in the Propositions 3 and 4 and Corollary 2.*

When the distribution of wealth is the same for both types we do not longer need an assumption involving the relationship between both types distribution. Then log-concavity (which just concerns a single distribution function) is sufficient to reproduce the previous results.

6 Conclusions.

In the present paper we provide a signaling explanation for the well known paradox (observed over the last decades of the 20th century in the US) of the simultaneous expansion of returns to education together with an increase in the number of college graduates. Differently from previous works, our explanation fits perfectly with the empirical observation of the reduction in non-college wages as the main driving force of the college premium expansion.

With imperfect credit markets education is not only a signal of ability but also of individuals' (parents) wealth. Taking into account log-concavity of wealth distributions and a monotone likelihood monotonicity between high and low ability wealth distributions (to denote the positive correlation between wealth and ability), a reduction in education cost attracts both low-wealth, high-ability individuals and high wealth-low ability to college, in such a way that average ability in non-college is reduced. This stands for a reduction of low-skill wages.

Our results have potential implications on workers' welfare. As Spence pointed out in 1973 it might happen that *"Everyone would prefer a situation in which there is no signaling. No one is acting irrationally as an individual. Coalitions might profitably form and upset the signaling equilibrium"*. We have seen that the education cost works as a coordination device. Opening access to education debilitates this device and may lead to a situation as the one reported by Spence. But our case is still more dramatic; the increase in the number of individuals incurring into the signaling cost might be coupled with a reduction in all wages. If the reduction in wages exceeds the reduction in the education cost, then Pareto losses arise. However, we need to have in mind that the main determinant to this is the assumption of a costly but non-productive signal.

Finally, this piece of work can contribute to the economic debate on the source of changes in the college premium. Some recent literature estimates that changes in the college wage premium arise from the combination of several of the explanations present in this debate, Fang (2006), Zheng (2010) or Cunha et al. (2011). This calls for a correct understanding of each of these explanations. We can see that composition hypothesis fits well not only with the expansion of college premium and college enrollment but also with changes in each wage. Additionally the reduction of non-college wages arises as an observable and necessary condition to explain the expansion in college premium by the composition hypothesis.

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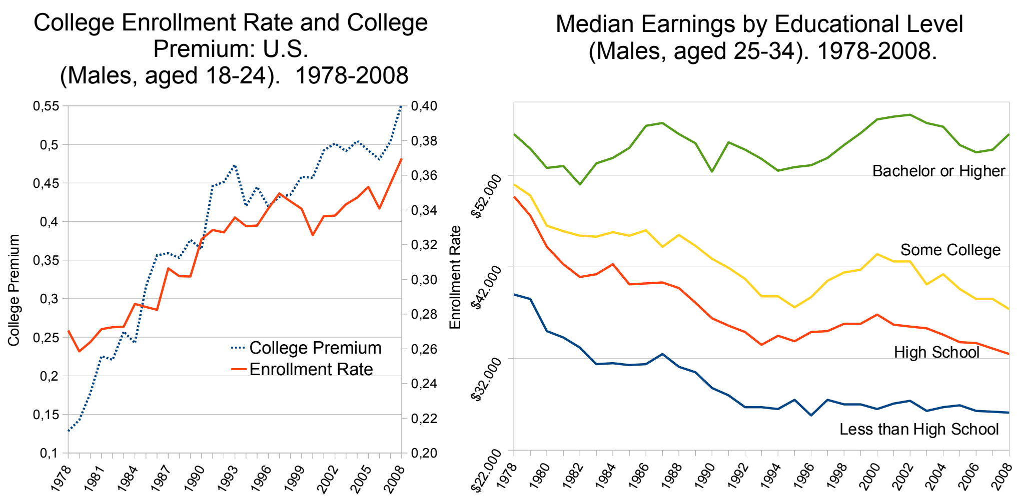
8 Appendix A

Figure 1: College Enrollment, College Premium and Wages.



Source:
Wages (constant 2008 dollars): National Center for Education and Statistics (NCES). The Condition of Education, 2011 (based on U.S. Census Bureau, Current Population Survey).

College Enrollment (as a percentage of all 18 to 24 year-old): National Center for Education and Statistics (NCES). Digest of Education Statistics, 2009. Table 204.

9 Appendix B

9.1 Proof of Lemma 1.

Call W^e and W^n the set of feasible wages for studying and not studying, respectively. First, we want to show that there is a fixed point.

In equilibrium necessarily $w^n < w^e$, then $F_H(b_H^*(w^e - w^n)) < F_L(b_L^*(w^e - w^n))$. Therefore, $g^e(b_H^*, b_L^*) \in [\bar{q}, q_H] = W^e$ and $g^n(b_H^*, b_L^*) \in [q_L, \bar{q}] = W^n$ which are compact and convex sets.

Define the college premium as $cp = w^e - w^n$. Then the set $\mathcal{CP} = [0, q_H - q_L]$ containing all possible equilibrium values of $cp = w^e - w^n$, with $w^e \in W^e$ and $w^n \in W^n$, is also convex and closed. Consider a mapping from $\mathcal{CP}$ to itself, $G : \mathcal{CP} \to \mathcal{CP}$, with $G(cp) = g^e(b_H^*(cp), b_L^*(cp)) - g^n(b_H^*(cp), b_L^*(cp))$, notice that $G(cp)$ is the reduced form of the function $CP(b_H^*, b_L^*)$, in the text. According to assumptions $A0 - A3$, individuals' decisions and wage formation, $G(cp)$ is equal to,

$$G(cp) = \begin{cases} q_H - \bar{q} & \text{if } cp < T \\ q_H - \frac{\pi(F_H(b_H^*))q_H + (1-\pi)q_L}{\pi(1-F_H(b_H^*)) + (1-\pi)} & \text{if } cp = T \\ q_H - \frac{\pi(F_H(T))q_H + (1-\pi)q_L}{\pi(1-F_H(T)) + (1-\pi)} & \text{if } T < cp < \frac{T}{P_L} \\ \frac{(1-F_H(T) - P_L(1-F_L(b_L^*)))\pi(1-\pi)(q_H - q_L)}{(1-F_H(T)\pi + (1-F_L(b_L^*))(1-\pi)P_L)(F_H(T)\pi + (1-\pi)(1+P_L(1-F_L(b_L^*))))} & \text{if } \frac{T}{P_L} = cp \\ \frac{(1-F_H(T) - P_L(1-F_L(T)))\pi(1-\pi)(q_H - q_L)}{(1-F_H(T)\pi + (1-F_L(T))(1-\pi)P_L)(F_H(T)\pi + (1-\pi)(1+P_L(1-F_L(T))))} & \text{if } \frac{T}{P_L} < cp \end{cases}$$

Since $G(cp)$ has a closed graph and its image set is bounded, then it is upper-hemicontinuous with the property that the set $G(cp) \in \mathcal{CP}$ is nonempty and convex for every cp. Kakutani's theorem guarantees the existence of a fix point.

Call $g : W^e \times W^n \to W^e \times W^n$ with $g(w^e, w^n) = (g^e(b_H^*, b_L^*), g^n(b_H^*, b_L^*))$ where b_H^* and b_L^* are functions of w^e, w^n. Any fixed point of $g(w^e, w^n)$ is obiously a fixed point of $G(cp)$. In the opposite direction we can easily see that for all cp^* being a fix point of $G(cp^*)$, there is one fix point of $g(w^e, w^n)$. Consider a fix point, $G(cp^*) = cp^*$, then if we take, $\tilde{w}^e, \tilde{w}^n$ such that, $\tilde{w}^e = g^e(b_H^*(cp^*), b_L^*(cp^*))$, $\tilde{w}^n = g^n(b_H^*(cp^*), b_L^*(cp^*))$ this is a fixed point of $g(w^e, w^n)$.

To show uniqueness we demonstrate that the function $G(cp)$ crosses a single time with the 45^o line. Denoting by cp^* an equilibrium college premium, we can see it by steps,

1. $cp^* > T$.

Instead, assume an equilibrium college premium such that $cp^* \leq T$. Then by $A0 - A2$ and intuitive criteria, $g^e(b_H^*(cp^*), \bar{b_L}) = q_H$ and $g^n(b_H^*(cp^*), \bar{b_L}) \leq \bar{q}$ with $b_H^*(cp^*) = \bar{b_H}$ and non-college productivity equal to the population average productivity if $cp^* < T$. Then necessarily $G(cp^*) \geq q_H - \bar{q}$. But we know by $A1$, that $q_H - \bar{q} > T$. Then, $G(cp^*) > cp^*$ and consequently this could never be an equilibrium.

2. $cp^* \geq \frac{T}{P_L}$.

   Imagine on the contrary that there is an equilibrium college premium such that $cp^* < \frac{T}{P_L}$. We know by the previous step that $T < cp^*$. By $A0 - A2$, all high types want to attend college, that is $b_H^*(cp^*) = T$ but low types do not want to attend, $b_L^*(cp^*) = \bar{b}_L$. Then $g^e(T, \bar{b_L}) = q_H$ and $g^n(T, \bar{b_L}) = \frac{\pi F_H(T) q_H + (1-\pi) q_L}{\pi(1-F_H(T)) + (1-\pi)}$. Finally, $A3$ guarantees that $G(cp^*) = q_H - \frac{\pi F_H(T) q_H + (1-\pi) q_L}{\pi(1-F_H(T)) + (1-\pi)} \geq \frac{T}{P_L}$, leading to a contradiction.

3. Uniqueness.

   We can write $\frac{\partial G(\hat{cp})}{\partial cp} = \frac{\partial G(\hat{cp})}{\partial b_H^*} \frac{\partial b_H^*}{\partial cp} + \frac{\partial G(\hat{cp})}{\partial b_L^*} \frac{\partial b_L^*}{\partial cp}$. We know (by Kakutani's theorem) that there is at least one equilibrium and (by the previous steps) that in any equilibrium $cp^* \geq \frac{T}{P_L}$. Consider the top left equilibrium of the model, cp^{**}. For any value of the college premium at the right of cp^{**}, necessarily $cp > \frac{T}{P_L}$. Denote $\hat{cp} > cp^{**}$, then, wealth threshold levels must be, $b_i^*(\hat{cp}) = T$ for $i = H, L$, which means that $\frac{\partial b_i^*(\hat{cp})}{\partial cp} = 0$ for $i = H, L$. Then, $\frac{\partial G(\hat{cp})}{\partial cp} = 0$ for all $\hat{cp} > cp^{**}$. Hence, $G(cp)$ can not cross again the 45^o line, and cp^{**} is the unique equilibrium.

Finally the reasoning in step 3 also guarantees that the equilibrium is stable. In particular, since $cp^* \geq \frac{T}{P_L}$, necessarily $\frac{\partial b_H^*}{\partial cp} = 0$ and $\frac{\partial b_L^*}{\partial cp} \leq 0$ (negative only when $cp^* = \frac{T}{P_L}$. By construction we know that $\frac{\partial G(cp^*)}{\partial b_H^*} < 0$ and $\frac{\partial G(cp^*)}{\partial b_L^*} > 0$. Using these elements we can see that $\frac{\partial G(cp^*)}{\partial cp} \leq 0 < 1$ which guarantees stability.

9.2 Proof of Proposition 1.

i) Fully constrained equilibrium, $cp^* > \frac{T}{P_L}$.

We want to show that,

$$\frac{\partial g^s(cp^*)}{\partial T} = \frac{\partial g^s(cp^*)}{\partial b_H^*}\frac{\partial b_H^*}{\partial T} + \frac{\partial g^s(cp^*)}{\partial b_L^*}\frac{\partial b_L^*}{\partial T} > 0 \text{ for } s = e, n.$$

Given that we are considering a fully constrained equilibrium with $cp^* > \frac{T}{P_L}$, everybody prefers to study, rather than not doing so, but only those who can pay education cost study, that is $b_i^* = T$, so $\frac{\partial b_i^*}{\partial T} = 1$, for $i = L, H$.

Then, it is enough to look at the direct effect of each types' wealth threshold levels on each wage ($\frac{\partial g^s(cp^*)}{\partial b_H^*}$ and $\frac{\partial g^s(cp^*)}{\partial b_L^*}$) to determine the final effect. If $\frac{\partial g^s(cp^*)}{\partial b_H^*} + \frac{\partial g^s(cp^*)}{\partial b_L^*} > 0$ then sector's $s = H, L$ wage goes down after reducing T.

Computing the derivative of each wage with respect to b_H^* and b_L^* we obtain,

$$\frac{\partial g^e(b_H^*,b_L^*)}{\partial b_H^*} = -\frac{(1-F_L(b_L^*))f_H(b_H^*)P_L(q_H-q_L)\pi(1-\pi)}{(1-F_L(b_L^*))(1-\pi)+(1-F_H(b_H^*))\pi}$$
$$\frac{\partial g^e(b_H^*,b_L^*)}{\partial b_L^*} = \frac{(1-F_H(b_H^*))f_L(b_L^*)P_L(q_H-q_L)\pi(1-\pi)}{(1-F_L(b_L^*))(1-\pi)+(1-F_H(b_H^*))\pi}$$
$$\frac{\partial g^n(b_H^*,b_L^*)}{\partial b_H^*} = \frac{F_L(b_L^*)f_H(b_H^*)P_L(q_H-q_L)\pi(1-\pi)}{F_L(b_L^*)(1-\pi)+F_H(b_H^*)\pi}$$
$$\frac{\partial g^n(b_H^*,b_L^*)}{\partial b_L^*} = -\frac{F_H(b_H^*)f_L(b_L^*)P_L(q_H-q_L)\pi(1-\pi)}{F_L(b_L^*)(1-\pi)+F_H(b_H^*)\pi}.$$

Using these expressions we can see that both wages decrease with a reduction in T if the following holds,

$$P_L\frac{F_H(b_H^*)}{1-P_L(1-F_L(b_L^*))} < \frac{f_H(b_H^*)}{f_L(b_L^*)} < \frac{1-F_H(b_H^*)}{1-F_L(b_L^*)}$$

We can show that logarithmic-concavity of distribution functions and monotone likelihood ratio of $\frac{f_H(b_H^*)}{f_L(b_L^*)}$ are sufficient conditions.

On one hand log-concavity guarantees that $\frac{F_i(b_H)}{F_i(b_L)} < \frac{f_i(b_H)}{f_i(b_L)} < \frac{1-F_i(b_H)}{1-F_i(b_L)}$ for $i = H, L$.

On the other hand monotone-likelihood implies that $\frac{F_H(b)}{F_L(b)} < \frac{f_H(b)}{f_L(b)} < \frac{1-F_H(b)}{1-F_L(b)}$ for all b.

The combination of these two conditions is sufficient to guarantee our objective. Dividing the right hand side inequality of log-concavity condition (using i=H) over the right hand side of the monotone likelihood one (evaluated at $b = b_L^*$),

$$\frac{\frac{f_H(b_H^*)}{f_H(b_L^*)}}{\frac{1-F_H(b_L^*)}{1-F_L(b_L^*)}} < \frac{\frac{1-F_H(b_H^*)}{1-F_H(b_L^*)}}{\frac{f_H(b_L^*)}{f_L(b_L^*)}}$$

Cancelling out some terms leads to

$$\frac{f_H(b_H^*)}{f_L(b_L^*)} < \frac{1-F_H(b_H^*)}{1-F_L(b_L^*)}$$

This guarantees the reduction of college wages. Proceeding in the same way we can see that $\frac{F_H(b_H^*)}{F_L(b_L^*)} < \frac{f_H(b_H^*)}{f_L(b_L^*)}$ is also true, which is a sufficient condition to guarantee that $P_L \frac{F_H(b_H^*)}{1-P_L(1-F_L(b_L^*))} < \frac{f_H(b_H^*)}{f_L(b_L^*)}$, increasing the non-college wage. The condition for college premium arises immediately from using the previous derivatives of wages and $\frac{\partial b_H^*}{\partial T} = \frac{\partial b_L^*}{\partial T} = 1$, in the derivative of the college premium, i.e. $\left(\frac{\partial g^e(b_H^*,b_L^*)}{\partial b_H^*}\frac{\partial b_H^*}{\partial T} + \frac{\partial g^e(b_H^*,b_L^*)}{\partial b_L^*}\frac{\partial b_L^*}{\partial T} - \frac{\partial g^n(b_H^*,b_L^*)}{\partial b_H^*}\frac{\partial b_H^*}{\partial T} - \frac{\partial g^n(b_H^*,b_L^*)}{\partial b_L^*}\frac{\partial b_L^*}{\partial T}\right)$

ii) High ability constrained equilibrium, $cp^* = \frac{T}{P_L}$.

There is a mass of low types with a wealth $b_L > T$ that are indifferent between studying or not (otherwise $b_L^* = T$ and comparative statics are equivalent to the case with $cp^* > \frac{T}{P_L}$). The indifferent mass of low ability types guarantees that they react strongly to a reduction in tuition, i.e. $db_L^* < db_H^* = dT < 0$ (we use the differential since $\frac{\partial b_L^*}{\partial T}$ does not exist in this case). Then given a small change in tuition, $dT \to 0$ and $\lim_{dT\to 0^-} dCP(b_H^*, b_L^*) = \frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}db_H^* + \frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}db_L^* = \frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}db_L^* < 0$

To study changes in each sector wage, we need to take into account that $|db_L^*| > |db_H^*| = |dT|$. Then in the college educated sector, $dg^e(b_H^*, b_L^*) = \frac{\partial g^e(b_H^*,b_L^*)}{\partial b_H^*}db_H^* + \frac{\partial g^e(b_H^*,b_L^*)}{\partial b_L^*}db_L^*$. Which is always positive if $\frac{f_H(b_H^*)}{f_L(b_L^*)}|dT| < \frac{1-F_H(b_H^*)}{1-F_L(b_L^*)}|db_L^*|$, which is guaranteed by monotone likelihood, log-concavity and $|db_L^*| > |dT|$. On the other hand, in the non-college sector, $dg^n(b_H^*, b_L^*) = \frac{\partial g^n(b_H^*,b_L^*)}{\partial b_H^*}db_H^* + \frac{\partial g^n(b_H^*,b_L^*)}{\partial b_L^*}db_L^*$. This is positive if, $\frac{F_H(b_H^*)}{F_L(b_L^*)}|db_L^*| < \frac{f_H(b_H^*)}{f_L(b_L^*)}|dT|$, which cannot be guaranteed by previous conditions.

9.3 Proof of Lemma 2.

We can proceed exactly as in the proof of Lemma 1 to show that W^e and W^n are compact and convex sets and the function $G : cp \to cp$, with $G(cp) = g^e(b^*_H(cp), b^*_L(cp)) - g^n(b^*_H(cp), b^*_L(cp))$, now becomes,

$$G(cp) = \begin{cases} q_H - \bar{q} & \text{if } cp < T \\ q_H - \frac{\pi(F_H(b^*_H))q_H + (1-\pi)q_L}{\pi(1-F_H(b^*_H)) + (1-\pi)} & \text{if } T \leq cp < \frac{T}{P_L} \\ \frac{(F_L(b^*_L)+F_H(b^*_H))\pi(1-\pi)(q_H - q_L)}{F_L(b^*_L)(1-\pi)+F_H(b^*_H)\pi(1+F_L(b^*_L)(1-\pi)-F_H(b^*_H)\pi)} & \text{if } \frac{T}{P_L} \leq cp \end{cases}$$

$G(cp)$ satisfies the same properties as in the proof of Lemma 1, so we can also apply Kakutani's fix point theorem. This also implies the existence of a fixed point for the wage of each sector (i.e. $\tilde{w}^e, \tilde{w}^n$ such that, $\tilde{w}^e = g^e(b^*_H, b^*_L)$,$\tilde{w}^n = g^n(b^*_H, b^*_L)$, where b^*_H and b^*_L are a function of cp.

Here we cannot show uniqueness but can show stability. To show that at least one college premium is stable consider cp^* as the fixed point of $G(cp)$ closest to the maximum college premium $(q_H - q_L)$. Since, by assumption $A1'$ $q_H - q_L > \frac{T}{P_L}$, then some low ability individuals will study, $b^*_L(q_H - q_L) \geq \frac{T}{L}$, and $g^e(b^*_H(q_H - q_L), b^*_L(q_H - q_L)) < q^H$, while $g^n(b^*_H(q_H - q_L), b^*_L(q_H - q_L))$ can never be lower than q_L. So $G(q_H - q_L) < q_H - q_L$. Because of continuity, previous inequality holds for any $\tilde{cp}$ such that $cp^* < \tilde{cp} < q_H - q_L$. We also know that for any college premium $\hat{cp}$, such that $\hat{cp} < T$, nobody studies, i.e. $b^*_H(\hat{cp}) = \bar{b}_H$ and $b^*_L(\hat{cp}) = \bar{b}_L$. Then $g^e(\bar{b}_H, \bar{b}_L) = q_H$ and $g^n(\bar{b}_H, \bar{b}_L) = \bar{q}$. Therefore $G(\hat{cp}) = q_H - \bar{q} > T > \hat{cp}$ where the first inequality arises from $A1'$.

Given that $\hat{cp} < \tilde{cp}$ and that $G(\hat{cp}) > \hat{cp}$ and $G(\tilde{cp}) < \tilde{cp}$, the function $G(cp)$ must cross the 45^o line from above at some cp^*, such that $\hat{cp} < cp^* < \tilde{cp}$, which guarantees stability, that is $\frac{\partial g^e(b^*_H, b^*_L) - g^n(b^*_H, b^*_L)}{\partial cp} < 1$.

9.4 Proof of Proposition 2.

The derivative of the college premium with respect to T is $\frac{\partial CP(b_H^*,b_L^*)}{\partial T} = \frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}\frac{\partial b_H^*}{\partial T} + \frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}\frac{\partial b_L^*}{\partial T}$.

Given that $cp^* > \frac{T}{P_L}$ we know that the wealth threshold level for each type is $b_i^* = \frac{T(1+x)-P_iCP(b_H^*,b_L^*)}{x}$. Writing the derivative with respect to tuition, $\frac{\partial b_i^*}{\partial T} = \frac{1+x}{x} - \frac{P_i}{x}\frac{\partial CP(b_H^*,b_L^*)}{\partial T}$. Using this expression in the the derivative of the college premium we can see that,

$$\frac{\partial CP(b_H^*,b_L^*)}{\partial T} = \frac{\frac{1+x}{x}\left(\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*} + \frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}\right)}{1+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}\frac{1}{x} + \frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}\frac{P_L}{x}}$$

Stability (i.e. $\frac{\partial CP(b_H^*,b_L^*)}{\partial cp} < 1$) guarantees that the denominator of the previous expression is positive. Given that $\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*} < 0$ and $\frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*} > 0$, the following condition is necessary and sufficient for $\frac{\partial CP(b_H^*,b_L^*)}{\partial T} < 0$, $|\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}| > |\frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}|$.

Looking at the derivatives of the indifferent individuals' wealth ($\frac{\partial b_i^*}{\partial T} = \frac{1+x}{x} - \frac{P_i}{x}\frac{\partial CP(b_H^*,b_L^*)}{\partial T}$, $i = H, L$) we can see that $\frac{\partial b_H^*}{\partial T} > \frac{\partial b_L^*}{\partial T} \iff \frac{\partial CP(b_H^*,b_L^*)}{\partial T} < 0$.

Finally, looking at the derivatives of indifferent individuals' wealth ($\frac{\partial b_i^*}{\partial T} = \frac{1+x}{x} - \frac{P_i}{x}\frac{\partial CP(b_H^*,b_L^*)}{\partial T}$, $i = H, L$) it is straightforward to see that if the derivative of equilibrium college premium with respect to T is strictly negative (i.e.$\frac{\partial CP(b_H^*,b_L^*)}{\partial T} < 0$), then the derivative of both types wealth threshold level are strictly positive, while this implication is not necessarily true in the opposite direction.

9.5 Proof of Proposition 3.

We know by the Proposition 2 that when a reduction in tuition generates an increase in college premium (i.e. $\frac{\partial CP(b_H^*,b_L^*)}{\partial T} < 0$) it also generate a positive and strictly greater change in the wealth threshold level of high ability types (i.e. $\frac{\partial b_H^*}{\partial T} > \frac{\partial b_L^*}{\partial T} > 0$). Combining this with the previous inequalities in the proof of Proposition 1 (i.e. log-concavity together with monotone likelihood guarantees that $|\frac{\partial g^e(b_H^*,b_L^*)}{\partial b_H^*}| < |\frac{\partial g^e(b_H^*,b_L^*)}{\partial b_L^*}|$ and $|\frac{\partial g^n(b_H^*,b_L^*)}{\partial b_H^*}| > |\frac{\partial g^n(b_H^*,b_L^*)}{\partial b_L^*}|$) we can see that the non-college wage decreases while ambiguous results arise for the college wage,

$$\frac{\partial g^n(b_H^*,b_L^*)}{\partial T} = \frac{\partial g^n(b_H^*,b_L^*)}{\partial b_H}\frac{\partial b_H}{\partial T} + \frac{\partial g^n(b_H^*,b_L^*)}{\partial b_L}\frac{\partial b_L}{\partial T} > 0$$

Using that $\frac{\partial CP(b_H^*,b_L^*)}{\partial T} = \frac{\frac{1+x}{x}(\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*})}{1+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}\frac{1}{x}+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}\frac{P_L}{x}}$, we can write $\frac{\partial b_i^*}{\partial T} = \frac{(1+x)(1+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_{-i}^*}\frac{P_{-i}-P_i}{x})}{1+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}\frac{1}{x}+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}\frac{P_L}{x}}$. Using these expressions in the college wage, it increases if and only if,

$$\frac{\partial g^e(b_H^*,b_L^*)}{\partial T} = -\frac{(1-F_L(b_L^*))f_H(b_H^*)P_L(q_H-q_L)\pi(1-\pi)}{(1-F_L(b_L^*))(1-\pi)+(1-F_H(b_H^*))\pi}\frac{(1+x)(1+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}\frac{P_L-P_H}{x})}{1+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}\frac{1}{x}+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}\frac{P_L}{x}}$$

$$+\frac{(1-F_H(b_H^*))f_L(b_L^*)P_L(q_H-q_L)\pi(1-\pi)}{(1-F_L(b_L^*))(1-\pi)+(1-F_H(b_H^*))\pi}\frac{(1+x)(1+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}\frac{P_H-P_L}{x})}{1+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}\frac{1}{x}+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}\frac{P_L}{x}} > 0$$

Finally this can be arrenged to obtain that

$$\frac{\partial g^e(b_H^*,b_L^*)}{\partial T} > 0 \Longleftrightarrow \frac{f_H(b_H^*)}{f_L(b_L^*)} < \frac{1-F_H(b_H^*)}{1-F_L(b_L^*)}\frac{1+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}\frac{1-P_L}{x}}{1+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}\frac{P_L-1}{x}}$$

9.6 Proof of Proposition 4.

We already know from the previous proof that when college premium is greater than $\frac{T}{P_L}$, then $b_i^* = \frac{T(1+x)-P_iCP(b_H^*,b_L^*)}{x}$. Using the derivative of wealth threshold levels $\frac{\partial b_i^*}{\partial x} = \frac{T}{x^2} + P_i(\frac{CP(b_H^*,b_L^*)}{x^2} - \frac{1}{x}\frac{\partial CP(b_H^*,b_L^*)}{\partial x})$ in the derivative of the equilibrium college premium we obtain that,

$$\frac{\partial CP(b_H^*,b_L^*)}{\partial x} = \frac{\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}(\frac{1}{x^2}(\tilde{w}^e-\tilde{w}^n)-\frac{T}{x^2})+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}(\frac{P_L}{x^2}(\tilde{w}^e-\tilde{w}^n)-\frac{T}{x^2})}{1+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}\frac{1}{x}+\frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}\frac{P_L}{x}}$$

Since we know by stability that the denominator is always positive, by imposing negativity on the numerator we obtain part i) of the proposition.

Using the previous expressions for $\frac{\partial CP(b_H^*,b_L^*)}{\partial x}$ in the derivative of each indifferent type wealth we obtain that,

$$\frac{\partial b_H^*}{\partial x} = \frac{\frac{1}{x^2}(CP(b_H^*,b_L^*)-T) + \frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}(1-P_L)\frac{T}{x^3}}{1 + \frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}\frac{1}{x} + \frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}\frac{P_L}{x}} > 0$$

$$\frac{\partial b_L^*}{\partial x} = \frac{\frac{1}{x^2}(P_L CP(b_H^*,b_L^*)-T) - \frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}(1-P_L)\frac{T}{x^3}}{1 + \frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*}\frac{1}{x} + \frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*}\frac{P_L}{x}} > 0$$

The positive value of these expressions arise from the fact that $\frac{\partial CP(b_H^*,b_L^*)}{\partial b_H^*} < 0$ and $\frac{\partial CP(b_H^*,b_L^*)}{\partial b_L^*} > 0$, $P_L < 1$ and that we are in an equilibrium with $CP(b_H^*,b_L^*) > \frac{T}{P_L}$, which guarantee the positive value of the numerator. On the other hand stability guarantees that the denominator is also positive. Given that these derivatives are positive we can use $\frac{\partial b_i^*}{\partial x} = \frac{T}{x^2} + P_i(\frac{CP(b_H^*,b_L^*)}{x^2} - \frac{1}{x}\frac{\partial CP(b_H^*,b_L^*)}{\partial x})$ to conclude that $\frac{\partial b_H^*}{\partial x} > \frac{\partial b_L^*}{\partial x}$.

Given that $\frac{\partial b_H^*}{\partial x} > \frac{\partial b_L^*}{\partial x} > 0$ we can proceed as in the proof of the Proposition 3 to show that log-concavity and monotone likelihood are sufficient conditions to guarantee the reduction of non-college wages but not for the college ones.

9.7 Proof of Remark 1.

Proceeding exactly as in the proof of Proposition 1, it is easy to see that the sufficient and necessary condition to guarante that in equilibrium both wages go down after a reduction in T with ability independent wealth distribution is,

$$\frac{F(b_H^*)}{F(b_L^*)} < \frac{f(b_H^*)}{f(b_L^*)} < \frac{1-F(b_H^*)}{1-F(b_L^*)}$$

which is directly guranteed by log-concavity.