Firm growth and market structure: an agent based simulation approach

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ABSTRACT:

The stochastic and the deterministic models of firm growth have coexisted in the industrial organization theory despite their opposite predicted results. This paper compares both theories using an agent-based simulation approach. This methodology is a useful way to test a diversity of hypothesis in market dynamics and obtaining results quite accurate. Our results show that stochastic firm growth model generate a higher concentrated firm distribution, while the deterministic model generates a more homogeneous firm size distribution. Therefore, the economic implications of both models are very different and must be considered when adopting a theoretical framework.

Key words: firm growth, market structure, computational simulation

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1. INTRODUCTION

Traditionally, the industrial economics has applied the Game Theory as a main instrument to analyse the dynamics of the market. Although the results obtained with these techniques have been fruitful, they have usually forced to introduce excessively restrictive hypothesis that may affect the validity of the final conclusions.

In order to reduce the complexity of estimations, the majority of the models of firm dynamics have supposed that firms competing in the market are homogenous. However, this assumption differs substantially from the heterogeneity observed in the reality. Conversely, the scarcity of theoretical models introducing firms with different characteristics suffers from a drawback in that the number of agents is scarce limiting considerably the general applicability of their results. The main reason is that they do not consider the effects of the exponential increase of interactions when introducing more agents in the model.

Furthermore, theoretical models usually include a constant number of firms. This assumption supposes that those firms belong to markets where barriers to entry and exit are infinite. Additionally, the models using the Game Theory are deterministic and do not incorporate stochastic features which may be crucial in the natural selection process of active firms in the market.

Finally, the traditional models have problems to tackle medium term problems, because the applied methodology to solve either considers few periods of time or supposes an infinite temporal horizon. Both situations, few or infinite periods, move away from the interest of social agents, firms and public agents.

To face up to those disadvantages, the computer simulation offers a suitable alternative to the traditional methodologies because it is able to cope with satisfactorily the questions related to firm dynamics with a direct and straightforward approach[1].

This paper shows an implementation of the DRIADE[2]'s model of simulation of markets, developed by the Lab of Computational Finances from the University of Alcalá (Madrid).

[1] See Moss (2001) for an exhaustive analysis of the properties of the simulation based on agents in front of the Game Theory.

[2] Pablo (2000) shows the complete development of the model, including the mathematical code in MATLAB.

This model reveals the useful properties of simulation methodologies in order to cope with complex theoretical problems from the field of industrial economics. Concretely, we analyse the implications of firm growth on the product margins and the degree of market concentration.

The structure of this paper is the following: Next section reviews briefly the main theoretical approaches about firm growth. The third section shows the characteristics from the dynamic firm model used in the computational process. Following section implements the economic model with computational tools and shows its contrast. In the fifth section, the most outstanding results are presented. And finally, the main conclusions are summarised.

2. THE PROBLEM OF FIRM GROWTH IN INDUSTRIAL ECONOMICS

The firm growth constitutes a central issue in the core of the industrial economics since it affects the market structure and the survival likelihood of firms (Sutton, 1997).

The explanations to the firm growth pattern are basically two: on the one hand, the deterministic approach based on the neoclassic model and, on the other hand, the stochastic approach.

From the deterministic or technological approach, the firm growth is closely related to idea of optimum size. As a result, the firm growth is basically a more or less rapid process from which small firms endeavour to reach this optimum size (Williamson, 1975). In this context, the problem of growth is especially relevant to new entrants because their size is usually smaller than the active firms in the market (Segarra et al., 2002). In case those economies of scale play an important role on firm productivity, new firms will face up to serious problems to survive and to consolidate their presence in the market if they are not capable to grow quickly.

Solow (1971) tried to explain the reasons why firms increase their size at different rates. To cope with it, he assumes that firms choose their growth rate maximizing their net discounted current value. The main result from Solow's model is that firms try to reach the optimum size as soon as possible in order to minimize the costs of having a small size. Nevertheless, the existence of costs of adjustment causes that firms prefer to arrive at the equilibrium little by little instead of adjusting their size in one period.

This implies that a sector, where there appear economies of scale and firms have similar medium term curves of costs, will exist an inverse relationship between size and growth (Viner, 1932; Baumol, 1982; Schmalensee, 1989). The reason is that large firms have a lower necessity to increase their sizes than small firms because the costs of having an inefficient size are lower because large firms are nearer from their optimum size. Following this approach, the diversity of firm growth processes observed in the market is a transitory situation due to the fact that firms are in different stages of process of adjustment towards the optimum size. Moreover, the dispersion of firm size distribution in the market in the long run will be scarce since firms will have reached the equilibrium.

In order to analyse the stability of the market structure, Table 1 shows the average size of Spanish firms in manufacturing industries between 1994 and 2004. Given the empirical evidence, the results present a high stability in the average size of active firms in the manufactures. As a consequence, we should accept the existence of a stable structure of firms in the market.

Table 1. Average size of Spanish manufactures (1994-2004).

	1994	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**
Whole industries	15.45	15.63	14.73	15.33	15.70	16.18	16.10	16.64	16.84	16.92	16.51

Source: own elaboration from Survey of Manufactures (Encuesta Industrial), Spanish Statistics Institute.

Although the structure of the manufacturing industries maintains stable over time, the firm processes of entry, growth and exit is more complex and affects heterogeneously to firms. With this purpose, we aim to analyse the firm dynamics depending on the firm size grouping.

If we analyse the number of new entrants during the year 1994 and the percentage of firms having failed until the year 2000 (Table 2), we observe that 29,591 manufacturing firms were created from which 16,708 firms do not have any employee. From this pool of new entrants, a 54.06% of firms failed and leave the market during the observed period of six years. Nevertheless, the probability to fail affects differently depending on the firm size groups. In fact, we observe a negative relationship between the percentage of firms which failed and their initial firm size. Consequently, the larger the firm size when a firm is born, the higher its survival likelihood.

Table 2. Entrants in 1994 and percentage of firms which failed in 2000 depending on the firm size.

Size	New entrants in 1994	(%) Failures until 2000
TOTAL	29591	54.06
0	16708	57.86
1-2	6284	52.43
3-5	3401	45.99
6-9	1586	44.58
10-19	1079	48.75
20-49	435	46.21
50 or +	98	36.73

Source: own elaboration from DIRCE (Directorio Central de Empresas), Spanish Statistics Institute.

This data is in concordance with a large number of empirical studies which suggest that the failure likelihood tends to decline with size and age during the first years of firm infancy while later they stabilize. Concretely, for the Spanish case Segarra et al. (2002) observe that the mortality declines significantly after the second period of having entered.

A further sep is the observation of the distribution of firms related to their initial size at 1994 and the distribution of surviving firms at the end of the period (Table 3). The empirical evidence shows that the percentage of firms created without employees represents the 56.46% of the total firms created during the year 1994. However, at the end of the 2000 the number of surviving firms which remain without employees were only a 31.93%. Consequently, the rest of size groups will gain weight in the final distribution (year 2000), especially the group with 1 and 5 employees.

Table 3. Distribution of manufacturing firms depending on the initial size.

Distribution of firms created in 1994 (%)

Initial size						
0	**1-2**	**3-5**	**6-9**	**10-19**	**20-49**	**50 o +**
56.46	21.24	11.49	5.36	3.65	1.47	0.33

Distribution of surviving firms in 2000 (%)

Initial size in 1994	Final size						
	0	**1-2**	**3-5**	**6-9**	**10-19**	**20-49**	**50 o +**
TOTAL	31.93	27.80	17.00	9.91	8.25	4.00	1.11
0	52.61	28.52	9.96	4.38	3.14	1.12	0.27
1-2	12.38	45.83	26.97	8.13	4.88	1.51	0.30
3-5	6.97	16.66	35.17	24.39	12.68	3.81	0.33
6-9	7.74	6.94	13.65	31.17	31.06	7.85	1.59
10-19	6.51	4.16	4.88	12.84	38.88	28.75	3.98
20-49	11.54	4.27	4.70	0.85	14.10	50.00	14.53
50 o +	12.90	1.61	0.00	1.61	0.00	8.06	75.81

Source: own elaboration from DIRCE (Directorio Central de Empresas), Spanish Statistics Institute.

In reference to the firm mobility depending on the initial size, there appears a double pattern. On the one hand, the smaller group (firms without employees) and the larger group (firms with more than 50 workers) offer a lower mobility: an 52.61% of firms which were born without employees, continue without any worker, and the 75.81% of firms created with more than 50 workers continue belonging to the same size group.

On the other hand, in the intermediate sizes there is a higher mobility rate. For example, only a 31.17% of firms employing between 6 and 9 workers in 1994 remain belonging to the same size group.

Furthermore, Table 3 shows how firms tend to grow in general. In other words, if we compare the percentage of firms changing their size towards a superior size group respect to their initial size, firms tend to redistribute in the larger groups (the right hand size). For instance, those firms which were initially created between 10 and 19 employees, the percentage of firms moving to a larger group (a 28.75%) is larger than the percentage of firms reducing their size (a 12.84%). In concordance with the literature of firm demography, new entrants are usually born with a lower size than the optimum (Geroski, 1995).

Conversely, the stochastic approach bases on the highly asymetric distribution of firm size in the market and which may be represented suitably with a diversity of theoretical distributions such as the log-normal, Yule or Pareto (Gibrat, 1931; Champernowne, 1937; Kalecki, 1945).

Those theories give a lower relevance to the technological and demand features considering that the evolution of firm sizes are influenced by a large number of explanatory variables which must be treated as random perturbations. Within this approach, the model of

"Gibrat's Law[3]" or the well-known "Law of the Proportional Effects" stands out. Gibrat's Law considers that firm growth is a pure random phenomenon. That means that firm growth and its variability are independent from firm size.

From an economic perspective, the main consequence is the independence between firm growth and size causes the inexistence of an optimum size. The main reason is that firm growth is not justified a process of adjustment (Sutton, 1997).

With the purpose of analysing the distribution of size firms in the during a medium period of time, Table 4 shows the coefficients of asymmetry and kurtosis of Fisher of the distribution of Spanish manufacturing firms between 1999 and 2005.

Table 4. Measures of the distribution of Spanish manufacturing firms (1999-2005).

	2005	2004	2003	2002	2001	2000	1999
Coefficient of asymmetry of Fisher	56,82	57,25	57,78	60,49	62,54	63,91	65,50
Coefficient of kurtosis of Fisher	3857,65	3921,78	4008,52	4437,47	4763,18	4960,51	5224,16

Source: own elaboration from DIRCE (Directorio Central de Empresas), Spanish Statistics Institute.

On the one hand, the coefficient of asymmetry has a value significantly larger than 0 what implies that the firm size distribution is asymmetric and positive. That means, that the number of small firms is larger than the number of firms with a large size. On the other hand, the positive value of the coefficient of kurtosis shows that the distribution is leptokurtic. In other words, firms are concentrated around the central values.

With all the above, we should confirm that the distribution of Spanish firms is the following. On the one hand, the asymmetric distribution remains constant over time in spite of the high mobility between different group sizes (Table 3) and in spite of the constant firm turnover.

In conclusion, the deterministic and stochastic theories of the firm growth are scarcely coincident and the majority of their predictions are opposite. Therefore, while the former has a larger microeconomic support, the latter approaches apparently to the reality, especially in reference to the size distribution observed in the markets.

The available evidence about firm growth is highly contradictory so it is not feasible to conclude which of both theories is the most appropriate. A wide range of empirical studies

[3] See Gibrat (1931).

have tried to test four different hypothesis with opposed implications about the evolution of the markets (McCloughan, 1995):

1. *Large firms grow more slowly than small firms* (Kumar, 1985; Evans 1987; Acs and Audretsch, 1990; Dunne and Hughes, 1994; Harhoff et al., 1998; Hart and Oulton, 1999; Fariñas and Moreno, 2000; Lotti et al., 2001). The acceptance of this hypothesis supports the deterministic theory.

2. *Large firms grow more rapidly than small firms* (Acs and Audretsch, 1990; Dunne and Hughes, 1994). Conversely, the validation of this hipótesis will refuse the deterministic theory while the stochastic approach would be accepted.

3. *The variability of firm growth rates diminishes with age and/or the firm size.* (Hart, 1962; Mansfield, 1962; Hymer and Pashigian, 1962; Singh and Whittington, 1975; Evans, 1987a and 1987b; Dunne and Hughes, 1994). The acceptance of this hypothesis supports partially the deterministic perspective.

4. *Existence of positive autocorrelation of first order in the firm growth* (Singh and Whitington, 1975; Chesher, 1979; Kumar, 1985; Wagner, 1992 and 1994; Almus and Nerlinger, 2000; Vennet, 2001). This hypothesis would accept the deterministic perspective.

In general the resultats are contradictory so we cannot conclude which model is the most appropriate. This lack of consensus has discouraged the interest for this issue during last years and, in the practice, the authors choose a model or another depending on their theoretical preferences or their simplicity to include in their main objective. This paper uses the simulation techniques based on agents to show how the election of the theoretical model, far from being neutral, has important effects on the results obtained. Therefore, the crucial process of firm growth should be investigated in the future.

3. THE DRIADE'S MODEL

The DRIADE's model of firm mobility has a modular structure providing him with a high degree of flexibility. This model facilitates the introduction of modifications and partial developments depending on the needs to analyse at a point of the time and without having to alter its main structure.

Module I: Initial structure of the market

The model starts from an initial market randomly generated which will evolve period after period by means of the firm entries and exits as well as the firm behaviour of firms competing in the market. This market is characterised with the introduction of the initial number of firms, the average capacity of firm production and its average degree of concentration with the Herfindahl rate.

The capacity or maximum level of production of firms at the initial period is distributed following a lognormal function. This hypothesis is based on extensive evidence which points out an asymmetric size distribution of entries (Ijiri and Simon, 1977).

Module II: Firm demography

The population of firms evolves over time due to the entrance and exit of firms.

The entrance of firms in the market has heterogeneous effects depending on their characteristics such as the size, competitiveness, costs, etc. (Acs et al., 1996; Segarra and Callejón, 2002). This characteristic provides more realism than those models of market dynamics with homogenous firms, which limit seriously the applicability of the model, especially if the entrants are considered equal to the active firms in the market. Furthermore, the heterogeneity of firms causes that the knowledge of their effects of the mobility on the individual firms is, at least, as interesting as their effects on the whole market distribution. Geroski (1995) points out that the effect of entrants over the market competitiveness is different depending to which kind of firms affect, for instance the smallest firms, or in case the new entrants influence uniformly over all the firms in the market.

Moreover, the survival likelihood of entrants depends considerably on the acceptance of their innovations introduced in the market. This aspect, essentially random, has a decisive impact on the creation of new firms in the markets (Audretsch, 1995).

In our model, the quantity of new firms deciding to enter every period of time depends on two different vectors of variables. On the one hand, variables which attract potential firms to enter in the market, basically the expected profitability; on the other hand, those variables which represent a barrier to the process of entering in the market.

The initial capacities of new firms follow also lognormal distributions similar to the active firms in the initial period but with different characteristics. The number of entrants every period is usually lower than the number of active firms so their distribution will tend to be under the distribution of the active firms. Moreover, given than the firm size of entrants is smaller, their mode will usually locate towards the left.

The probabilities that a firm leaves the market depend basically on their margin: the smaller their margin is, the lower their survival likelihood (in United Stated, Audretsch and Mahmood, 1995; in Japan Doi, 1999; in Holland Audretsch et al., 2000; in Spain, Segarra and Callejón, 2002;). In order to introduce the exit process, we define the survival likelihood of firms as a Bernoulli distribution with a parameter *p*, where *p* is a logistic function which depends on the margin

Consequently, firms presenting narrow margins, or even negative, will have low probabilities to survive in the medium term in the market especially if firms obtain negative economic results period after period. Conversely, successful firms will show high survival likelihood; nevertheless, positive results in one period do not guarantee the firm survival.

Since the margin from a firm depends on the market price, the number of firms exiting the market will be affected indirectly by all the variables which affect the price: quantity of production, importations and entrants.

Finally, the presence of firms with different sizes and different average costs causes that, given a level of market price, firms with high probabilities to survive will coexist in the market simultaneously with other unviable firms.

Module III: Functions of demand

The market price, at which active firms sell their products, is negatively affected by the level of production and the number of new firms.

In order to introduce a more realistic feature in the model, we consider two demand functions: firstly, one specific demand for products made by initially active firms and, secondly, another demand function for those firms which have been entering in the market over time. The introduction of both functions copes with the consumer's perception of an incomplete substitutability between the productions of active firms and new entrants and importations.

This way, the price at which the incumbents sell will be affected negatively by its production, as well as by the production carried out by the new companies. The demand equation for the established companies can be specified in the following way:

$$[1] \qquad P_{nt} = a_1 - a_2\ Q_{nt} - a_3\ Q_{et}$$

where P_{nt} is the price at which the companies settled down in the moment t sell its products, Qn is the production level, Qe the production of the incoming ones. The subindexes n and e refer to the populations of established and incoming firms respectively. The subindex t expresses the period to which the variable refers.

The function of demand of the incoming ones is similar:

$$[2] \qquad P_{et} = b_1 - b_2\ Q_{nt} - b_3\ Q_{et}$$

Module IV: Cost structure

Our model assumes that there is only on available technology equal for all firms composing each of the two groups of firms: active and new firms. Therefore, there will exist two different cost functions defining the level of average costs from each firm depending on their level of production[4]. The main reason why to distinguish between two different types of technologies for entrants and active firms is because new firms are usually more labour intensive and enjoy lower economies of scale than the active firms in the market. Obviously, those differences are due to the smaller firm size of new entrants (Audretsch and Mahmood, 1995).

Module V: Firm growth

The firms don't remain stable in the market, they change their size voluntarily in order to adapt to the market characteristics or because they are forced due to the circumstances.

[4] The cost functions we have applied belong to Cobb-Douglas technology with increasing returns to scale.

To introduce this dynamic feature, the capacity of production for each firm in period *t* is defined as a function of the existing capacity during the previous period. With the purpose that this function is as flexible as possible in order to incorporate the two main theories about firm growth: the stochastic and the deterministic approaches. The introduction of both estimations has been previously analysed by Wissen (2000) who introduce an structural and a random component to the growth process. This author points out that the random component is necessary to avoid the convergence to the mean size. Other authors such as Richiardi (2004) analyse the effect of the introduction of homogeneous and heterogeneous firms as well as the firm turnover. His conclusions show that in order to obtain a Reasonable Distribution of firms such as Graphic 2, the model must incorporate heterogeneous firm growth rates or firm turnover.

The final equation defining the firm growth will be the following:

[1] $$d_t = \alpha\left(d_{t-1}\left(1+\frac{N(0,1)}{10}\right)\right)+(1-\alpha)\left(d_{t-1}\left(1+\lambda\,\frac{d_{opt}-d_{t-1}}{d_{t-1}}\right)\right)$$

where α and λ are coefficients which are bounded between zero and one, *N(0,1)* is a random variable which follows a normal distribution with average equal to zero and standard deviation equal to one and d_{opt} is the optimum size[5]. Therefore, we have a weighted average of both types of factors, where the coefficient α establishes the relative importance of each factor. In case that α is equal to one the firm growth is purely stochastic while in case that α is equal to zero the firm growth will be completely deterministic.

In the deterministic part, the coefficient λ shows the speed at which firms adjust their capacity reducing their gap in relation to the optimum. The value equal to the unity shows that firms arrive at the optimum size in only one period; conversely, the value equal to zero shows that firms do not change their production in order to reach to the optimum.

Nevertheless, it does not seem to be realistic to consider that all firms approach to the optimum at the same speed. The existence of constraints, basically of financial nature, may

[5] In case economies or diseconomies of scale exist, the model assumes that the optimum size is equal to the firm size with the smallest average costs. In case there are not economies of scale, the optimum firm size is the size of the largest firm.

limit the speed of firms which need to grow rapidly (Evans and Jovanovic, 1989[6]; Cabral and Mata, 2003[7]). In order to include this characteristic, the speed of adjustment for each firm towards the optimum is composed by two factors. The former is related with the technological level, it is constant and equal for all firms competing in the same sector, $\lambda_{0;}$ and another variable, which depends on the capacity of self-financing, $\lambda_1 IL_{t-1}$.

[2] $$\lambda = \lambda_0 + \lambda_1 \ IL_{\ t-1}$$

Consequently, the probabilities that a firm grow are influenced by the margins of profitability obtained in the previous period (Clarke and Davies, 1982). This is due to the fact that the margins will affect the capacity that a firm has in order to self-financing.

4. COMPUTATIONAL IMPLEMENTATION OF THE ECONOMIC MODEL

Having developed the model of firm mobility, we realised that it was rather difficult to implement the traditional mathematical instruments. Therefore, we decided to use computational techniques of multiagent simulation. This methodology, relatively new in the field of the economics, is receiving a growing interest because of the rapid development of computer tools. The reason is that all those new instruments led to apply the ideas about complexity, evolution and chaos in the social sciences.

The models with multiagents, as it is known, are composed by agents interacting among them and with their environment independently. Those agents are made up of small autoconstrained programs that agents are able to control their own actions based on their perception, complete or partial, from the environment where they evolve (Huhns and Singht, 1998). In the majority of the situations, the agents search for the achievement of some kind of objective, the survival in the environment, the obtaining of profits or its own growth. Obviously, all those concepts are more frequently applied to human beings than to computational programs (Gilbert and Troitzsch, 1999).

[6] Evans and Jovanovic (1989) show that financial constraints affect the decisions to invest.
[7] Cabral and Mata (2003) find that financial constraints may explain the skewness in the size distribution of young cohorts of firms.

Although the characteristics that agents may have are wide, Wooldridge and Jennings (1995) point out some of those that are usually present in the computational programs:

- *Autonomy:* The agents are able to act independently. Moreover neither their actions or their internal state are controlled by the exterior.
- *Social skills:* The agents interact among them by means of some kind of language.
- *Reactivity*: The agents are able to perceive their environment and respond to the received incentives.
- *Proactivity:* The agents not only are able to react to their environment but they also are able to carry out actions by their own initiative in order to achieve their objective.

To a large extent, those characteristics coincide with the expected features of the firm behaviour in the market.

Table 1 shows the characteristics of the agents in our model. Concretely, firms enjoy autonomy in their decisions, since they are not controlled by their competitors; they present social skills, because they are able to interact in the market giving and receiving information; additionally, they are reactive since they observe the market where they compete and they are able to respond to the received incentives; and finally, they are proactive since they can implement strategies to achieve their objectives.

Table 5. Characteristics of the agents in the model

Characteristic	Description of its implementation in the model
Autonomy	Firms decide their production individually trying to maximise their profits and survive in the market.
Social skill	The communication among agents is made, the same as it happens in the real markets, basically by means of the market prices. The decisions of production as well as the entries and exits affect to the prices which represent a signal of relative abundance or scarcity for the firms.
Reactivity	Firms respond to the variations of price increasing or decreasing their production taking into account their own characteristics and the market conditions. Depending on the number of competitors and the intensity of economies of scale the agents decide between two options: firstly, to adapt to the increases of production from the competitors reducing its own production, in order to maintain high market prices; secondly, to respond increasing its production to compensate the fall of market prices with the increase of market share.

Proactivity	Firms do not wait for changes in the market in order to define their own production. Conversely, they decide the most suitable behaviour every period.

The model developed with MATLAB™ due to its power and facilities to use. Additionally, this computational program uses specific mathematical language, therefore, the similarities between the economic and the computational modelisation makes easier the implementation of our model. As we have shown in the previous section, the program is structured by modules representing partial features from the whole problem such as the resolution of the equilibrium market price, the entrance of new firms or the decision of firm strategies. Furthermore, those modules may be modified freely without affecting the other modules, what makes possible the implementation of the *ceteris paribus* assumption and the analysis of the effects of changing the values of exogenous variables or the initial hypothesis on the model.

In order to create our model, the program generates several matrixes which compile data about all potential firms which may exist in the market. One of the features of the firm mobility, which presents a high degree of difficulty in the computational implementation, is the management of continuous agents (firms) entering and exiting without loosing valuable information about their stay in the market. Therefore, instead of representing the matrixes as the number of agents remaining active in the market every period, we decide to generate one matrix including all the relevant variables with an ample dimension in order to be able to compile all the potential firms which may enter in the market. Consequently, the dimension of the matrixes is the $t \times m$, where t is the number of periods that the simulation will run and m the maximum number of potential firms which may coexist simultaneously in the market. With the past of time, new firms enter in the market and we assign new columns in the right hand of the matrixes. As a result, the more towards the right the column is, the more recent the entry has produced.

It is worth to mention that the number of firms coexisting in the market in every period are significantly inferior to the potential firms and that, in the majority of situations, the matrix will reserve columns to potential firms that will never enter in the market. This way to implement the model, in spite of being slightly inefficient from the perspective of the use of the computational memory, has great advantages since it identifies rapidly the values of the variables every period for each firm.

In order to control the evolution of firms in the market, two auxiliary matrixes are created identifying respectively those firms which are active in the market (Table 6) and the periods that a firm has been active in the market (Table 7).

The structure of the matrix of active firms (Table 6) shows the structure that the rest of matrixes have. When new firms enter in the market, they appear in new columns where previously there were zeros. Nevertheless, the competitiveness expels the least efficient firms from the market and new columns with zeros appear signalling that a firm has left the market.

Table 6
Structure of matrixes
(Matrix of active firms)

Periods	Initial firms								New entrants										
1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0
4	1	1	0	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0
5	1	1	0	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0	0
6	1	1	0	1	1	0	1	0	1	1	1	0	1	1	0	0	0	0	0
7	1	1	0	1	1	0	1	0	1	1	1	0	1	1	1	0	0	0	0
8	1	1	0	1	0	0	1	0	0	1	1	0	0	1	0	1	0	0	0
9	1	1	0	1	0	0	1	0	0	1	1	0	0	1	0	1	1	0	0
10	1	1	0	1	0	0	1	0	0	1	0	0	0	1	0	1	0	0	0
...	.. .	.. .	.. .	.. .	.. .	.. .	.. .	.. .	.. .	.. .	.. .	.. .	.. .	.. .	.. .	.. .	.. .	.. .	.. .
t	1	0	0	1	0	0	0	0	0	1	0	0	0	1	0	1	0	0	0

The matrix of ages is similar to the active with the difference that instead of having ones, the values of the matrix coincides with the firm age remaining in the market every period. This matrix becomes highly useful to analyse how the age is, by itself, an element of survival since it lets agents to adapt their characteristics to the environment (Table 7).

Table 7
Matrix of firm ages

Periods				Initial firms										New entrants						
1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
2	2	2	2	2	2	0	2	2	1	0	0	0	0	0	0	0	0	0	0
3	3	3	3	3	3	0	3	3	2	1	1	0	0	0	0	0	0	0	0
4	4	4	0	4	4	0	4	4	3	2	2	1	0	0	0	0	0	0	0
5	5	5	0	5	5	0	5	5	4	3	3	2	1	1	0	0	0	0	0
6	6	6	0	6	6	0	6	0	5	4	4	0	2	2	0	0	0	0	0
7	7	7	0	7	7	0	7	0	6	5	5	0	3	3	1	0	0	0	0
8	8	8	0	8	0	0	8	0	0	6	6	0	0	4	0	1	0	0	0
9	9	9	0	9	0	0	9	0	0	7	7	0	0	5	0	2	1	0	0
10	10	10	0	10	0	0	10	0	0	8	0	0	0	6	0	3	0	0	0
...	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
t	t	0	0	t	0	0	0	0	0	t-2	0	0	0	t-4	0	t-7	0	0	0

After the introduction of exogenous variables, from the initial number of firms and from the coefficients defining the market characteristics (demand function, production function, kind of firm growth, function of exits and function of entries), the simulation starts.

With the help of different graphics, we may observe the evolution of main economic variables during the simulation: production, distribution of firm sizes, survival likelihood, employment, entry and exit rates, average of the price-cost margin, measure of Herfindahl, average marginal costs....

Having finished the simulation, all the results at firm and market level are stored in files in order to be analysed later by means of techniques such as the resampling of the firm population and Montecarlo techniques.

Verification and calibration

In order to verify how the model works avoiding the difficulties which are generated by existence of random variables, we fixed the pseudorandom number from simulations and we verify the correctness of each equation and subroutines, not only from a mathematic perspective but it also from the economic one. When we the model was verified using always the same initial values, we observed how the model worked with random seed values.

The calibration of the model may be done applying two alternative techniques, but not necessarily excluding, depending on the aim of each specific analysis. The first technique consists in the use of estimations for the coefficients and the exogenous variables obtained from empirical studies about real markets. The second technique consists in the introduction of alternative values, theoretically acceptable, with the purpose of evaluating the effect of changes in the parameters on the general behaviour of the model, using statistical methodologies of resampling.

Although it is possible, the contribution to the model with real data from specific markets is rather difficult due to the scarcity of data at individual firm level. Moreover, the election of a market among all the possible ones may confuse the analysis of the general problem, since it may not be useful to the obtaining of general results. Consequently, the calibration of the model has been made using theoretically acceptable values obtained by the literature related to this field, without having in mind any accurate image from any specific market.

This second technique has been used to analyse the effect of the firm growth on the markets applying three hundred simulations, one hundred for each one of the types of firm growth -deterministic, stochastic and mixed- in order to calculate lately the average and variance of the price-cost margin and the concentration[8].

The functions of costs and production are shaped as a standard Cobb-Douglas function; although the values taken for the exogenous variables, such as the total productivity of the factors or the costs of the productive factors, are arbitrary, they do not affect to the conclusions obtained in the model, since they may only be considered as a changes of scale.

The applied functions of demand are linear and we suppose that they remain stable during the simulation. Although the introduction of functions of demand evolving with the degree of market maturity is easy, it was preferable to analyse the behaviour of the firms only because of reasons the firm mobility.

Validation and main results

Having made the simulations we have validated the model following the methodology pointed out by Kleijnen (1998) for situations where known values for the exogenous variables are not available. In other words, comparing the predictions from the model with the available evidence. This validation system has been also used by Wissen (2000).

[8] The average market margins were calculated as the arithmetic average of the measures of Lerner from firms; and the degree of concentration was calculated by means of the Lerner measure.

The developed model is absolutely compatible with the available evidence, which are rather scarce when analysing the evolution of the markets in the long term, as well as with the basic hypothesis in the industrial economics.

5. RESULTS

In the stochastic model, when α from equation 1 has a value equal to one, firms change their size randomly. Those modifications in the productive capacity, together with the existence of economies of scale, have a crucial impact on the survival likelihood of firms which changed their size in addition to their competitors. The reason is that those changes alter their own relative position in the market and, therefore, it alters their level of competitiveness.

The pure deterministic growth model, where α has a value equal to zero, allows the firms to adapt to the increase of the relative inefficiency caused by the entrance of a new market rival which is more efficient than the previous one. The entrance of this new efficient rival raises the level of the optimum firm size and, consequently, increases the firm growth rate.

In the mixed model (α is equal to 0.5), although firms change randomly their size, they are able to respond up to certain extent to the variations in the optimum size and the appearance of new rivals more efficient.

Graphics 1, 2 and 3 show the effect of the type of firm growth has on the price-cost margins. Those graphics show that the average profitability in the long term is higher in the stochastic model than in the deterministic or mixed model.

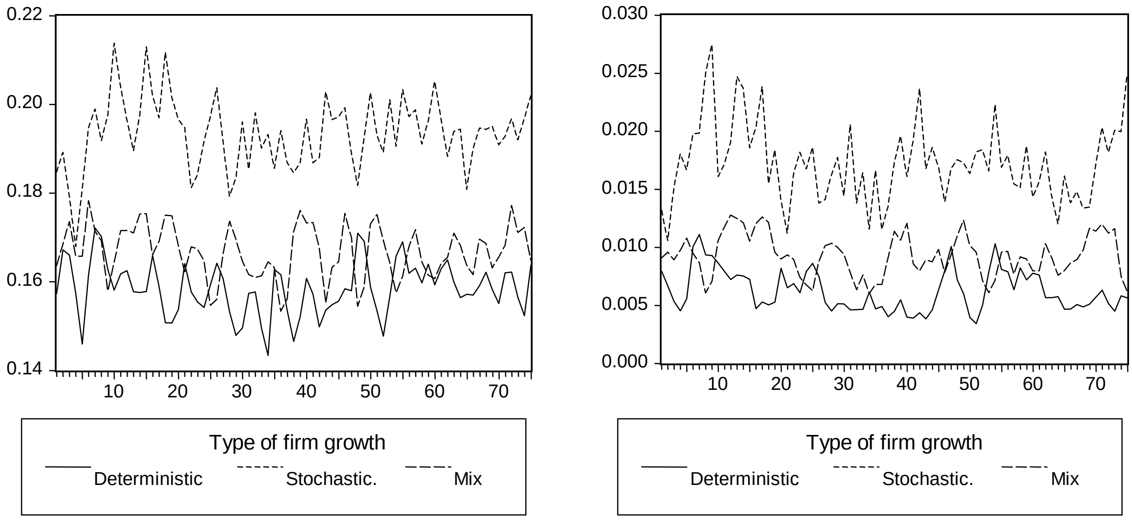
The main reason is that the variability of the firm sizes is lower than in the deterministic and mixed models. Therefore, the population of firms tend to be formed by firms with similar size and, consequently, with similar low profitability rates.

In case that the average profitability is high, the entries increase the number of firms in the market pressing down the profitability until the less efficient firms leave the market. In case that all firms have the same costs, there will not exist inefficient firms exiting the market, so there will be continuous entrants in the market until the profitability of all firms approaches to zero.

Conversely, in case of the stochastic model the coexistence of firms with different size causes that the average profitability is larger due to the coexistence of firms with profitability near to zero with others showing high margins. In this case, the dynamic process of entries ad exits will focus on firms which are more weak while the larger firms will not participate in the industrial dynamic.

Graphic 1
Evolution of margins om the market of active firms

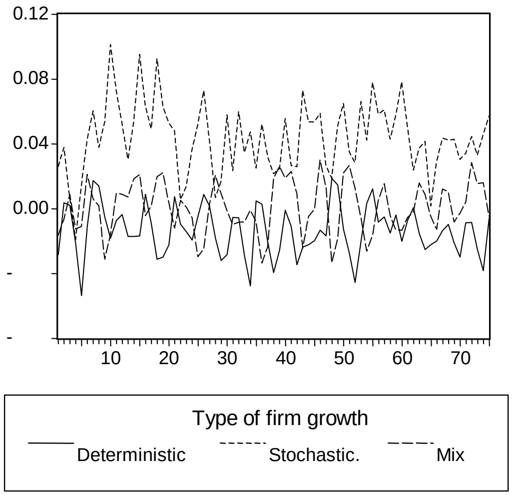
Averages Variances

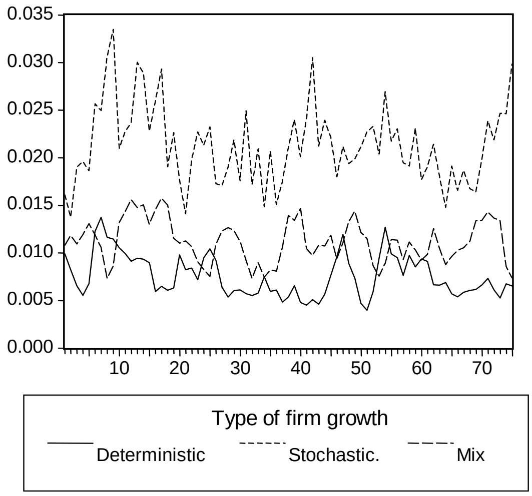


The differences between the markets of active firms and the entrants is due to the definition of the functions of demand. Since they are partially substitutive productions, the market of established firms is only affected partially by the entrants. Consequently, the competence is less intensive than in the market of new firms.

Graphic 2
Evolution of the margins in the market of new firms

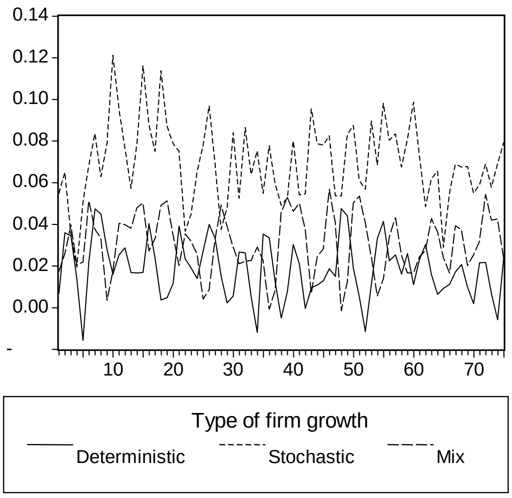
Averages Variances



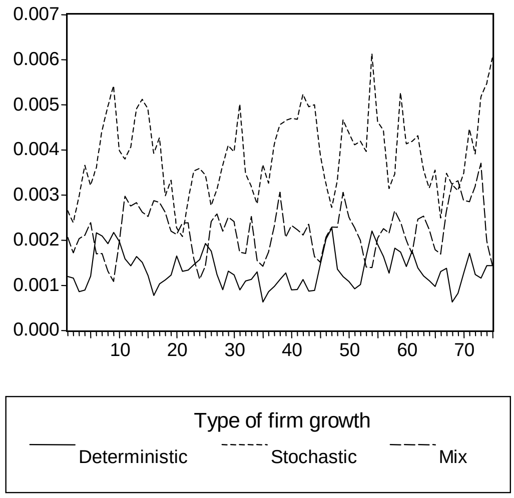


Graphic 3
Evolution of the margins in the whole market
(active and new firms)

Averages



Variances

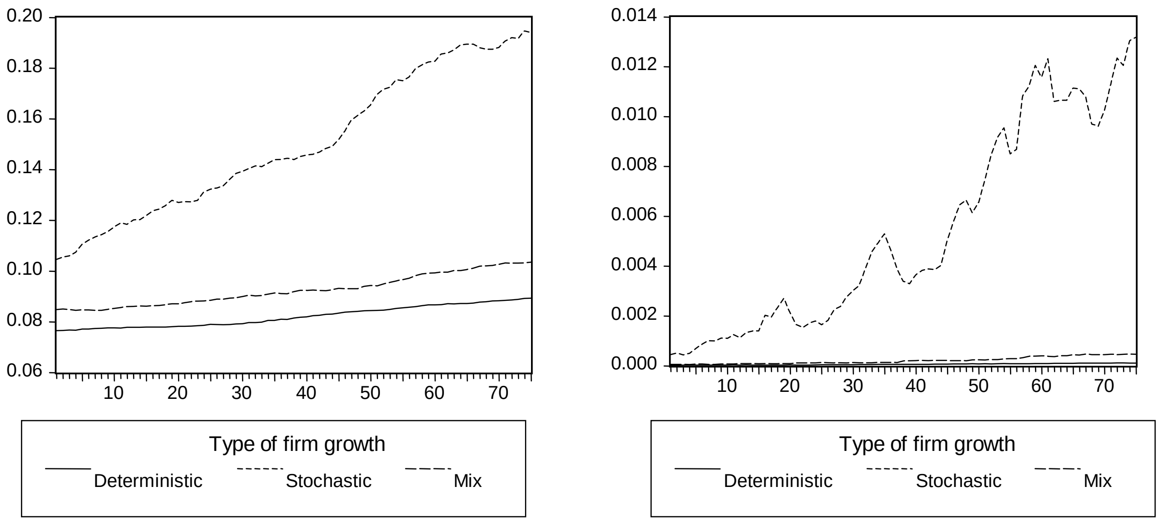


The influence of the firm growth in the evolution of the concentration may be appreciated in graphics 4 to 6.

From a theoretical perspective, the deterministic growth has an ambiguous effect on the concentration because it includes two effects with opposite consequences: on the one hand, the concentration diminishes because there is a decrease of the differences in the market share; on the other hand, there is an increase of the total production in the market which causes a fall in the market prices and, as a result, there is an expulsion of the less efficient firms. Consequently, the final results on the concentration are not obvious because there is a trend to exist a scarce number of firms in the market but with a low variability in the market shares. The results predicted by the stochastic growth theory are not more accurate than the previous ones. The reason is that, although the number of firms in the market will be likely higher than in the deterministic growth theory in the long term, the variability of the market shares will be highly different because there will be an increase of the inequality.

Graphic 4
Evolution of the concentration in the market of active firms

Averages Variances



Graphic 8 presents the evolution of the market concentration of initially established firms. The results show that the concentration increases over time among initially established firms and, more importantly, this is a common pattern for all the types of firm growth due to the fact that the initially established firms only may exit the market. Under these circumstances, in order to reduce the market concentration it will be necessary the rapid convergence in the level of production of firms with unequal productions. This rapid convergence will only happen in the deterministic and mixed models in case that the **lambda** parameter has high values. Those results are in concordance with Gallegati et al.'s (2003) simulation model who obtain that the heterogeneity among firm sizes is de to the differences of the financial accounts and their initial size.

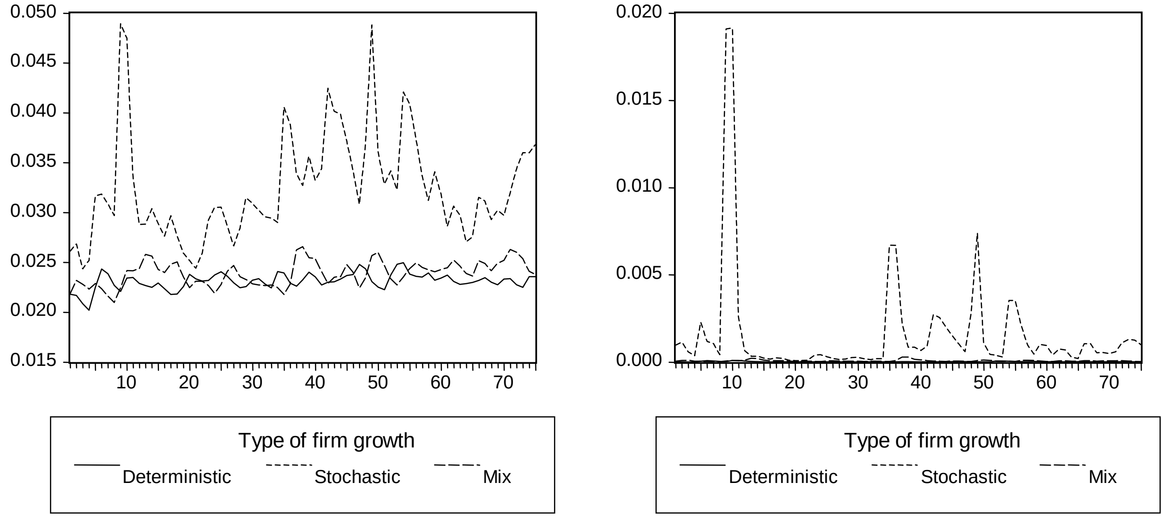
The increase of the differences of the market shares in the stochastic model, in addition to the scarce number of entrants, causes a rapid increase of the concentration. In the deterministic model the concentration is smaller, the model tends to reduce the differences among firm capacities and, therefore, there is a reduction of the concentration. Nevertheless, the increase in the average firm size, together with the existence of economies of scale, causes a fall of the market prices and the exit of firms which have been not able to acquire a size sufficiently large.

When observing the evolution of the market of new firms and for the whole firms (Graphics 5 and 6, respectively), there appears a similar behaviour. The predictions obtained on the evolution of the concentration in the long term are different depending on the model. The increase of the concentration is higher among the stochastic model because it does not have the tendency to the equivalence of market shares which reduces the market concentration.

Graphic 5

Evolution of the concentration in the market of new firms

Averages | Variances

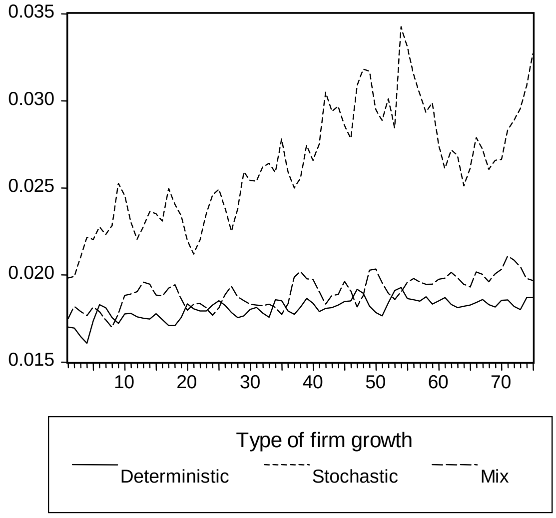


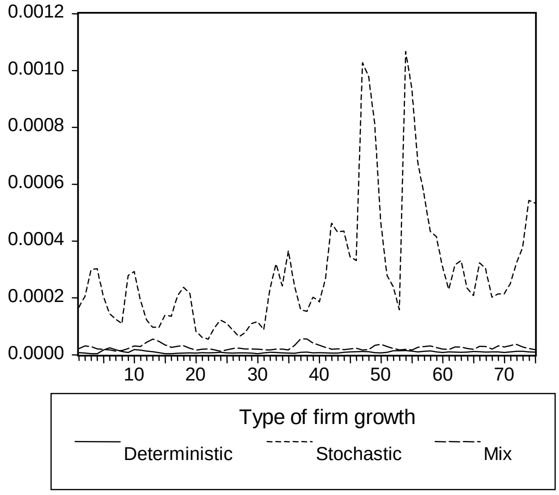
Furthermore, the variability of the estimations is rather different. While in the deterministic growth model and in the mixed model the variance remains low in the long term, the random model shows a growing variability in the estimation of the concentration. The reasons why there appears those different behaviours is that the stochastic model tends to generate an asymmetric distribution of firm size and, depending on the sectoral situation at each moment, it causes a different pattern on the concentration.

Graphic 6

Evolution of the concentration in the whole market (established and new firms)

Averages | Variances





The implications for the policy of defence of the competency are obvious. On the one hand, if we consider that the relevant growth of the firms competing in the market is the stochastic model, there will exist a higher trend to increase the concentration and, likely, a competition hazard. On the other hand, if the predictions show that in this case there is a larger variability, it will be necessary a more careful supervision than in the case of having a deterministic growth model whose predictions seem to be more reliable.

6. CONCLUSIONS

From our results, that the firm growth model is a key issue to explain the market structure, and that the implications of the deterministic and stochastic growth models are rather different. As we have shown, the introduction of a random component on the firm growth causes that the distribution of firm size concentrates on few large firms, while the absence of this component implies the homogeneous distribution of firm sizes.

Moreover, the analysis of firm growth and market structure through simulation estimations seems to be a powerful and adaptable tool to compare hypothesis in different scientific disciplines. The firm growth analysis has become an interesting field researched by different theories. In order to compare those different theories, the simulation creates artificial environments where the economist may analyse the reaction of the social agents. Therefore, the importance of simulation on firm growth is clear and industrial policy-makers should consider them in order to apply different socio-economic hypotheses. As we have seen previously, the implementation of simulation techniques let to cope easily with problems of industrial economics that with other traditional methodologies would have been impossible. Therefore, this study opens the door to researchers to investigate using novel techniques in the field of industrial economics.

Policies must improve the economic resources of policies among agents. Learning how to distribute the public resources seems to be a major challenge for economic growth. An important policy implication is that promotional efforts to job creation and increase of the competition should take into account the firm growth trends and the determinants of firm growth in order to apply efficient policies.

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