DURABLE GOODS, INCOMPLETE MARKETS AND ADJUSTMENT COST

Kamal Romero
Universidad Rey Juan Carlos *

Incomplete and Extremely Preliminary

Abstract

Empirical facts show that housing is one of the most important components of wealth for most families in the U.S.. Also durable goods have important characteristics that can potentially affects the pattern of wealth (asset) accumulation, such as credit constraints, durability among others. In order to analyze quantitatively the effects on asset accumulation and durables and wealth distributions, we include an illiquid asset and collateral credit in an otherwise standard heterogenous agents model. Specifically, an endogenous price version of Díaz-Giménez, Alvarez, Fitzgerald, and Prescott (1992) is presented, and the equilibrium properties evaluated. We depart from previous literature as we don't rely on an extremely persistent and volatile income process, we concentrate in how much inequality the inclusion of the indivisible durable good in presence of liquidity constraints generates.

Keywords: Wealth distribution, Illiquid assets, Adjustment cost.

JEL code: E21, G21, R21, E44

*Correspondence: Universidad Rey Juan Carlos, Campus de Vicálvaro, Edificio departamental, despacho 179. Paseo Artileros s/n, Madrid 28032. Teléfono: (34)914888023, E-mail: kamal.romero@urjc.es

1 Introduction

Beginning with Aiyagari (1994) and Huggett (1993), models where a continuum of ex-ante identical agents receives uninsurable idiosyncratic income shocks, constitute the main tool used by economists for understanding the determinants of wealth distribution[1]. These models in their standard version assume that agents have access to a single non-contingent liquid asset, but recently a group of papers highlights the potential role of introducing durable goods in order to explain better than the standard model issues such as precautionary saving and wealth distribution and to explore others like durable good distribution or the life cycle profile of durable consumption[2].

This role is given by the fact that durable goods have important characteristics that can potentially affects the pattern of wealth (asset) accumulation. Credit constraints (down-payment or collateral requirements) and the fact that durable goods can be used as collateral for credits affect both saving and the housing consumption, as agents has an additional motive to save to make the required down-payment and once they own the good they relax the borrowing constraint albeit at the cost of paying the mortgage and maintenance costs. Additionally, durables are illiquid and therefore changes in the size of the stock (and therefore consumption) are costly and infrequent.

Also, empirical facts show that housing is one of the most important components of wealth for most families. US data shows that Households holds 35% of their total assets in real estate and other durables an only 28% in equity (Fernandez-Villaverde and Krueger 2004), the ratio of housing to total wealth for the median household is around 0.7 (Gruber and Martin 2003), the poorest 80% of households hold, on average, 96.3 percent of their wealth as housing vs. 26.8% for the richest 20 percent (Díaz and Luengo-Prado 2006). If we distinguish between homeowners and renters, the fraction of wealth homeowners hold in stocks and bonds is 15.4% vs. 71.7% in housing (Platania and Schlagenhauf 2000).

In order to analyze quantitatively the effects on asset accumulation and durables and wealth distributions, we include an illiquid asset and collateral credit in an otherwise standard heterogenous agents model. Specifically, an endogenous price version of Díaz-Giménez, Alvarez, Fitzgerald, and Prescott (1992) is presented, and the equilibrium

[1] For a literature review on this topic see Castaneda, Diaz-Gimenez, and Rios-Rull (2003), Quadrini and Rios-Rull (1997) and Cagetti and Nardi (2006)

[2] See Díaz and Luengo-Prado (2006), Yang (2006), Gruber and Martin (2003), Platania and Schlagenhauf (2000) and Fernandez-Villaverde and Krueger (2004).

properties evaluated. Ours is a dynamic general equilibrium model where a continuum of ex ante identical agents are subject to uninsurable idiosyncratic shocks to their labor earnings. These households derive utility from consumption of a nondurable good and housing services provided by a stock of durable good which they can mortgage and which is costly to maintain and to sell. They can save in the form of a liquid non contingent asset. There is a intermediation technology that is costly and therefore creates a wedge between lending and deposit rates.

Previous attempts to include illiquid assets in similar settings in order to explain durables and wealth distribution were not able to generate sufficiently unequal distributions. Gruber and Martin (2003) succeed in increasing the precautionary saving but only find a marginal effect in the dispersion of wealth distribution, Díaz and Luengo-Prado (2006) obtains a distribution of houses less egalitarian than that of earnings for the total population but find that changes in the frictions that affect housing markets have limited impact on the wealth distribution.

From our viewpoint one of the possible reasons why the inclusion of durables does not have an important effect the wealth distribution, is the assumption that the illiquid asset is continuous subject to standard adjustment costs.

Following this idea we specify two model economies that differ in the specification of the durable good. First we assume an indivisible durable good and then we use a continuous durable with a specification of the adjustment cost such that consumers behave "as if" the good is indivisible (very infrequent changes in stocks). Both economies share the feature that they do not rely on a very persistent and volatile earnings process, used in Díaz and Luengo-Prado (2006) to match the wealth and durables distribution dispersions.

As we don't rely on an extremely persistent and volatile income process, our work may not be able to replicate the current wealth distribution, but analyzes how much inequality the inclusion of the indivisible durable good in presence of liquidity constraints generates

The paper is organized as follows. In the next section, we describe the model economy and define its equilibrium. In section 3, we describe the calibration, whereas in section 4 we proceed to make a numerical exercise for an economy like the one described in the third section. The last section concludes.

2 The Model Economy

The model economy analyzed in this paper is a version of Díaz-Giménez, Alvarez, Fitzgerald, and Prescott (1992) with endogenous prices and without Government[3]. Excluding Government we are left with two sectors: the household sector and the banking sector.

2.1 Environment

The economy is inhabited by a continuum of households of measure one who lives forever and are subject to an uninsured idiosyncratic shock that affects it's efficiency units of labor. These households derive utility from consumption and from housing services provided by an indivisible (or continuous subject to adjustment costs) stock of house which they can mortgage and is costly to maintain and sell. These consumers have access to an intermediation technology that is costly and therefore creates a wedge between lending and deposit rates.

We assume that the shock disturbances that affects consumers are identically and independently distributed across households and they follow a finite state Markov chain with probability transitions given by:

$$\pi(s' \mid s) = \Pr(s_{t+1} = s' \mid s_t = s)$$

where $s', s \in S = \{s_1, s_2\}$

We assume two productivity states, low and high.

We focus our analysis on steady states.

Intermediation technology

We assume households have access to a intermediation technology that is costly. This technology converts the composite good into deposits at a cost η_d per unit, and allows households to obtain collateralized loans at cost η_l per unit. These costs generate a wedge between lending and deposit rates.

We assume bank deposits belong to the finite set $\mathcal{D}$, and bank loans to the finite set $\mathcal{L}$.

[3]When prices are determined endogenously the Government loses the role of fixing prices such that these do not depend on the state variable distribution, therefore reducing the state space and simplifying the numerical problem (Rios-Rull 1995)

The set $\mathcal{L}$ is bounded above by ϕk_{t+1}, that is households at most can borrow up to a fraction of the resale value of their end of period housing stock. We fix a value $l \in \{0, \phi k_{t+1}\}$ that represents the maximum amount that households can borrow from banks.

Preferences and endowments

The household ordering of the perishable good (nondurable consumption) and housing services (durable consumption) is represented by a continuous differentiable, strictly concave and monotonically increasing utility function of the form:

$$E \sum_{t=0}^{\infty} \beta^t U(c_t, k_t'; s_t) \tag{2.1}$$

where c_t y k_t' represent nondurable consumption and housing services respectively. We assume $\beta \in (0, 1)$.

The amount of the composite good available to the household is given by the efficiency units of labor process $w(s_t)$ governed by the Markov process described above.

Budget set

The fraction of the composite good that is not transformed into consumption takes the form of either housing services or bank deposits thats yields an interest r_d.

Like Díaz-Giménez and Puch (1998) we assume households can buy *discrete* amounts of housing stock that belongs to the set $\mathcal{K}$, $k \in \mathcal{K} = \{o, \kappa, 2\kappa, \ldots, n_k\kappa\}$. Also we study the case where the set $\mathcal{K}$ is continuous and the purchase of the housing stock is subject to an adjustment cost $\tau(k, k')$. We also assume that houses must be maintained, at a cost of $\mu > 0$ units of that period's composite good per unit of housing stock used during that period. Finally, in the discrete case, we assume that there is an irreversibility in the housing accumulation process. When a household decides to sell part of its housing stock it incurs a cost of ϕ units of the composite good.

Given the above information we can write the household budget constraint as:

$$c_t + x_t^d + \mu k_t' + r_l L_t \leq A_t + w(s_t) + x_t^s + L_t + r_d D_t \tag{2.2}$$

Where x_t^d and x_t^s represents sales and purchases of the housing stock respectively. L_t and D_t denotes household loans and deposits and A_t the beginning of period asset holdings.

We can now write the household problem as the maximization of 2.1 subject to the budget constraint 2.2 and:

$$L_t \leq l \leq \phi k_{t+1} \tag{2.3}$$

$$A_{t+1} = D_t - L_t \tag{2.4}$$

$$k_t = k_t' = k_t + x_t^d - \frac{x_t^s}{\phi} \tag{2.5}$$

$$A_{t+1} \in \mathcal{A} \quad k_{t+1}' \in \mathcal{K} \tag{2.6}$$

where 2.3 denotes the borrowing constraint that states that the maximum debt a household can incur is a fraction of the end of period value of the housing stock. That is, collateral lending.

2.4 and 2.5 represent the law of motion of financial assets and housing stock respectively.

Equilibrium

Our economy differs from those of Díaz-Giménez, Alvarez, Fitzgerald, and Prescott (1992) and Díaz-Giménez and Puch (1998) in the fact that the process that governs prices, in this case the interest rate is not given by the policy arrangement fixed by the Government. In our model the interest rate is endogenous and therefore a specific object of the equilibrium definition.

In order to reduce the dimensionality of the household problem we will limit our attention to stationary equilibria, where the interest rate and the distribution of agents across states are constant over time. This strategy permit us to solve agents decision rules for a interest rate that depends on a given distribution of agents and its correspondent law of motion (Rios-Rull 1995).

In each moment of time households are characterized by their position of assets and holdings of housing stock, as well as their productivity status $(a, k, s) \in \mathcal{S} = \mathcal{D} \times \mathcal{L} \times \mathcal{K} \times S$. The function $\Phi(a, k, s)$ $\Phi : \mathcal{S} \times \mathcal{B}_{\mathcal{S}} \rightarrow [0, 1]$ where $\mathcal{B}_{\mathcal{S}}$ is the Borel algebra of $\mathcal{S}$, represents the measure of agents of type (a, k, s), constant in the stationary equilibrium.

The consumer problem can now be formulated recursively as:

$$V(a,k,s) = \underset{c,k',x^d,x^s}{max} \{U(c,k',s) + \beta EV(a',k',s')\pi(s' \mid s)\} \tag{2.7}$$

$$\begin{aligned} & \text{subject to} \\ & c + x^d + d + \mu k' \leq a + w(s_t) + r_d d - r_t l + x^s + l \\ & L \leq l \leq \phi k \\ & a' \leq d - l \\ & k' = k + x^d - \frac{x^s}{\phi} \end{aligned}$$

where $a' \in \mathcal{A}$[4], $k' \in \mathcal{K}$.

We define the operator Q that maps $\mathcal{M} \to \mathcal{M}$ where $\mathcal{M}$ is the set of finite measures over the measurable space $(\mathcal{S}, \wp(\mathcal{S}))$, such that $\Phi' = Q(\Phi)$ describes the probability that a household that belongs in this period to $a, k, s \in \mathcal{S}$ transits the next period to a subset $B_a \times B_k \times B_s \in \mathcal{S}$. This function summarizes the transition of households over $\mathcal{S}$ triplets, $Q : \mathcal{S} \times \wp(\mathcal{S}) \Rightarrow [0,1]$ where $\wp(\mathcal{S}) = C(\mathcal{A}) \times B(\mathcal{K}) \times D(S)$ is the product space over subsets of $\mathcal{A}, \mathcal{K}$ and S. Defining x as a vector in the (a,k,s) space, we can write the function Q as $Q(x,B) = \Pr\{\mathcal{S}_{t+1} \in B \mid \mathcal{S}_t = x\}$ for any $x \in \mathcal{S}$ and any set B σ-measurable.

We are now ready to define a stationary equilibrium:

Definition 2.1. A **stationary equilibrium** is a value function, $V(a,k,s)$, policy functions $c(a,k,s)$, $d(a,k,s)$, $l(a,k,s)$, $x^d(a,k,s)$, $x^s(a,k,s)$, $a'(a,k,s)$ and $k'(a,k,s)$, an interest rate r, a measure $\Phi \in \mathcal{M}$ and a law of motion for the measure of household types $\Phi' = Q(\Phi)$, such that:

- Given r_d and r_l determined by r and the intermediation technology , V_c solves the functional equation 2.7 and $c(a,k,s)$, $d(a,k,s)$, $l(a,k,s)$, $x^d(a,k,s)$, $x^s(a,k,s)$, $a'(a,k,s)$ y $k'(a,k,s)$ are the associated policy functions

- Markets clear:

$$\begin{aligned} L(\Phi) &= \sum_{a,k,s} \Phi(a,k,s) l(a,k,s) \text{ (Loan market)} \\ D(\Phi) &= \sum_{a,k,s} \Phi(a,k,s) d(a,k,s) \text{ (Deposit market)} \end{aligned}$$

[4] $\mathcal{A} = \mathcal{D} \times \mathcal{L}$

$$\sum_{a,k,s} \Phi(a,k,s)c(a,k,s) + \sum_{a,k,s} \Phi(a,k,s)a'(a,k,s) + \sum_{a,k,s} \Phi(a,k,s)k'(a,k,s) = \sum_{a,k,s} \Phi(a,k,s)w(s)$$

(Goods market)

$$\sum_{a,k,s} \Phi(a,k,s)a'(a,k,s) - \sum_{a,k,s} \Phi(a,k=0,s)k'(a,k,s) = 0$$

(Asset market)

where $\sum_{a,k,s} \Phi(a,k=0,s)k'(a,k,s)$ represents the measure of agents that consumes rental services

- The operator Q is generated by the decision rules and the transition matrix of s:

$$\Phi'(a',s') = \sum_{s} \sum_{a,k \in \Psi(a',k')} \Phi(a,s)P(s',s) \tag{2.8}$$

where:

$$\Psi(a',k') = \{(a,k) \ : \ a' = d(a,k,s) - l(a,k,s), \ k' = k(a,k,s) + x^d(a,k,s) - x^s(a,k,s)/\theta\}$$

- The measure of agents $\Phi(a,k,s)$ is stationary: $\Phi = Q(\Phi)$

3 Computational Procedure

Following is an outline of the algorithm used to compute the equilibria of the economy described in the paper.

Step 1 Given an interest rate, we solve the household decision problem described in 2.7 and obtain the vector of decision rules for assets and durables.

Step 2 Given the transition matrix for the earnings shock and the above policy rules, we iterate on 2.8 until $\Phi'(a',s') = \Phi(a,s)$

Step 3 We check if the asset and goods markets equilibrium holds, in that case we are done, otherwise we update the interest rate and go to step 1.

An outline of the algorithm used to solve the households' decision problem is the following:

Step 1.1 Impose a grid on the household state space $\{\mathcal{A} \times \mathcal{K} \times S\}$ (finite state approximation)

Step 1.2 Initialize the value function $V_0(a, k, s)$ and the vector of decision rules

Step 1.3 We solve the household problem by value function iteration. Given the high dimensional state space we use a policy function accelerator described in Judd (1998) to speed up convergence.

These techniques are implemented in MATLAB.

4 Calibration

μ	0.00625
ϕ	0.2
β	0.96
ψ	4
α	0.333
α_k	0.108
γ	0.027
η_d	0.00821
η_l	0.00250

5 Preliminary Results

Experiment 1: Indivisible durable good

Interest Rates	0.0408
Homeowners Measure	1
Renters Measure	0
Indebt Agents	0.31116
Gini Coefficient	0.4 aprox.

The Results suggest that the indivisibility of the durable good generates a discontinuos policy function that results in a corner solution, where every agent purchase a house.

To solve this problem we plan to introduce a continuos durable good with the following specifications for the adjustment cost.

$$\tau(k, k') = \begin{cases} 0 & if \;\; H' = \delta_h H \\ (1 - \lambda_s)\delta_h H - (1 + \lambda_b)H' & otherwise, \end{cases}$$

and

$$\tau(k,k') = \begin{cases} 0 & if h' \in [(1-\mu_1)h, \ \ (1+\mu_2)h] \\ \rho_1 h + \rho_2 h' & otherwise, \end{cases}$$

References

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109(3), 659–684.

Cagetti, M., and M. D. Nardi (2006): "Wealth Inequality: Data and Models," NBER Working Papers 12550, National Bureau of Economic Research, Inc.

Castaneda, A., J. Diaz-Gimenez, and J.-V. Rios-Rull (2003): "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 111(4), 818–857.

Díaz, A., and M. J. Luengo-Prado (2006): "The Wealth Distribution With Durable Goods," Economics Working Papers we067027, Universidad Carlos III, Departamento de Economía.

Díaz-Giménez, J., F. Alvarez, T. Fitzgerald, and E. C. Prescott (1992): "Banking in computable general equilibrium economies," *Journal of Economic Dynamics and Control*, 16.

Díaz-Giménez, J., and L. Puch (1998): "Borrowing Constraints in Economies with Indivisible Household Capital and Banking: An Application to the Spanish Housing Market (1982-89)," *Investigaciones Económicas*, XXII(3), 469–499.

Fernandez-Villaverde, J., and D. Krueger (2004): "Consumption and Saving over the Life Cycle: How Important are Consumer Durables?," 2004 Meeting Papers 357b, Society for Economic Dynamics.

Gruber, J., and R. F. Martin (2003): "Precautionary Savings and the Wealth Distribution with Iliquid Durables," International Financial Discussion Papers 773, Board of Governors of the Federal Reserve System.

Huggett, M. (1993): "The Risk Free Rate in Heterogeneous Agent Incomplete Insurance Economies," *Journal of Economic Dynamics and Control*, (17), 953–969.

Judd, K. L. (1998): *Numerical Methods in Economics.* MIT Press, Cambridge, MA.

Platania, J. M., and D. E. Schlagenhauf (2000): "Housing and Asset Holding in a General Equilibrium Model," Mimeo.

QUADRINI, V., AND J. V. RIOS-RULL (1997): "Understanding the U.S. Distribution of Wealth," Quarterly Review 2, Federal Reserve Bank of Minneapolis.

RIOS-RULL, J. V. (1995): "Models with Heterogeneous Agents," in *Frontiers of Bussines Cycle Research*, ed. by T. F. Cooley, pp. 98–125. Princeton University Press, Princeton, N.J.

YANG, F. (2006): "Consumption along the life cycle: how different is housing?," Working Papers 635, Federal Reserve Bank of Minneapolis.