Capital Utilization, Endogenous Growth and Persistence

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Abstract

This paper attempts to provide a simple model that generates predictions consistent with cross-country empirical evidence on persistence, long-term growth and capital utilization. On the one hand, as documented by Fatás (2000), in a cross section of countries there exists a strong positive correlation between the persistence of short-term fluctuations and long-term average growth rates. On the other hand, empirical evidence on capital utilization rates find large differences in capital utilization rates across countries as well as a positive correlation between capital utilization and per capita income. We show that by introducing capital utilization as an optimal choice into a standard AK model the degree of persistence is an increasing function of capital utilization rates by the way of long-term growth rates.

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1 Introduction

Empirical evidence on the persistence of output fluctuations documents large differences across countries. Campbell and Mankiw (1989) find using quarterly GNP data for the group of G-7 countries important differences in the estimates of persistence. Consistent with this evidence, Cogley (1990) reports significant differences in the variability of the permanent component of output in a similar sample of countries. Further, Fatás (2000) finds that there is a positive and significant correlation between the degree of persistence of short-term fluctuations and long-term average growth rates for a sample of countries that includes the G-7 countries and eight additional OECD countries. The standard RBC models with exogenous permanent productivity shocks predict no correlation between these variables. Fatás (2000) shows using a standard AK model, that the above mentioned positive correlation between persistence and growth can be obtained when the stochastic nature of the trend is endogenous.

The standard AK growth model considers the rate of depreciation as a constant and assumes that capital services are proportional to the underlying capital stock, as is usual in the growth literature. In such a setting, the marginal cost of capital utilization is equal to zero implying an optimal capital utilization rate equal to one. However, the empirical evidence on depreciation and capital utilization rates are not consistent with this theoretical assumption. Epstein and Denny (1980) and Kollintzas and Choi (1985) using aggregate US manufacturing data provide evidence against the standard assumption of a constant depreciation rate. Abadir and Talmain (2001) estimate time-varying depreciation rates for Canada, Germany, Japan and the UK. Moreover, Foss (1981), Orr (1989) and Beaulie and Mattey (1998) find upward trends for the capital utilization rate in the US.

Empirical evidence on capital utilization rates across countries is scarce, but it documents (i) a positive correlation between capital utilization and per capita income, and (ii) large differences in cross-country utilization rates. Kim and Watson (1974) find, using data for Pakistan, South Corea and the US, that the rate of capital utilization increases with per capita income. Mayshar and Halevy (1997) using data of 24,000 companies in ten European countries also report a positive correlation between the rate of capital utilization and income per worker. Anxo et al. (1995) report a large variation in utilization rates across Europe as well as much higher utilization rates in US manufacturing industries than in Europe. Finally, Gylfason and Zoega (2001) use data of 85 countries from the World Bank and find a positive correlation between depreciation and per capita income growth.

This paper studies the implications of capital utilization as an optimal decision on the dynamics of growth and persistence. Our theoretical explanation to the above mentioned cross-country positive correlation between persistence and growth follows the line of research suggested by Fatás (2000) and, in addition, is also consistent with empirical evidence on capital utilization rates across countries. In particular, we introduce the optimal choice of capital utilization in an AK-type endogenous growth model and, unlike the existing growth literature, we do not treat the depreciation rate of capital as a constant. Instead, the rate of depreciation is an increasing function of the capital utilization rate and hence it is endogenously determined. In such a setting, it is not optimal to fully utilize the capital and the model can generate cross-country persistence results that are consistent with both empirical observations.

The rest of the paper is structured as follows. In section 2 we briefly describe the endogenous growth model considered. Section 3 analyzes the persistence in output growth, and Section 4 concludes.

2 The Setup of the Model

The framework is a simplified, stochastic version of Chatterjee's (2005) model. In particular, it is an AK-type growth model augmented by endogenous capital utilization.

Consider a closed economy without a public sector. The economy is populated by a continuum of identical, infinitely lived agents which derive utility from the consumption of a final good and discount future utility at a rate $\beta \in (0,1)$. Preferences are given by $\sum_{t=0}^{\infty} \beta^t log(C_t)$, where C_t denotes consumption. We assume that the labor supply is inelastic and normalized to unity.

The technology of the consumption good is described by the aggregate production function $Y_t = AZ_t u_t K_t$, where A is a scale parameter, $u_t K_t$ is the flow of capital services derived from the available capital stock, Y_t denotes the corresponding flow of output, and Z_t is a temporary exogenous shock that captures the state of the technology. As suggested by Taubman and Wilkinson (1970) and Calvo (1975) and following Chatterjee (2005), we define the rate of capital utilization u_t as the intensity (measured in hours per week) with which the available capital stock is used. In this way, firms are provided by an extra margin to vary output, say the intensive margin. The productivity shock Z_t is assumed to follow the autoregressive process: $zz_{t+1} = \rho\ zz_t + \varepsilon_{t+1}, \quad 0 < \rho < 1$, where zz_t denotes the log-deviations of variable Z_t from its steady state value and ε_t is a white noise.

In a closed economy without public sector all output is devoted to consumption or gross investment. Hence, the resource constraint of the economy is $C_t + I_t = Y_t$. The capital stock evolves according to $K_{t+1} = K_t(1 - \delta_t) + I_t$. Following Burnside and Eichenbaum (1996), we also assume that the rate of depreciation of the capital stock is a convex, constant elasticity function of its rate of utilization: $\delta(u_t) = d{u_t}^{\phi}$, where $\phi > 1$, $d > 0$ and $0 \leq \delta(u_t) \leq 1$. Note that, in contrast to the usual assumption in the growth literature, the marginal depreciation cost of capital utilization $\delta'(u_t)$ is variable. The parameter ϕ measures the elasticity of depreciation with respect to the rate of capital utilization.[1] As is already known, due to the sensitivity of the depreciation rate of capital to the choice of capital utilization, it may not be optimal to fully utilize the capital. It should be noted that this model collapses to the AK model considered by Fatás (2000) when full capital utilization is assumed.

2.1 The solution

In the absence of distortions, the allocations arising from a decentralized competitive economy coincide with those resultant from a centralized economy with a social planner. The dynamic program problem faced by the central planner is:

$$V(K_t, Z_t) = \max_{C_t, u_t} \left\{ log(C_t) + \beta E_t V(AZ_t u_t K_t - C_t + (1 - d{u_t}^{\phi})K_t, Z_{t+1}) \right\},$$

[1] A plausible range for parameter ϕ seems to be $[1.4, 2]$. See Dalgaard (2003) for a survey on this evidence.

given K_t and Z_t and where E_t is the expectations operator conditional on the information available up to period t. The objective function is concave and the constraints are convex. Hence, the following set of FOC's characterize the interior optimum:

$$AZ_t = \phi d {u_t}^{\phi-1}, \tag{1}$$

$$\frac{1}{C_t} = \beta E_t \left\{ \frac{1}{C_{t+1}} \left[1 + d {u_{t+1}}^{\phi} (\phi - 1) \right] \right\}, \tag{2}$$

$$K_{t+1} = K_t(1-\delta_t) + Y_t - C_t, \tag{3}$$

$$\lim_{t\to\infty} E_t \left\{ \beta^t \frac{1}{C_t} K_{t+1} \right\} = 0,$$

The interpretation of these optimality conditions is standard. Equation (1) determines the optimal choice of the capital utilization rate. The left hand-side of equation (1) represents the marginal benefit of capital utilization whereas the right hand-side is the marginal depreciation cost of capital utilization. Hence, in this setting, it is optimal to utilize capital less than fully, i.e. $u_t \in (0,1)$.[2]

Since variables grow in the long-run equilibrium, there are no steady state levels for the original variables. Let $y_t = \frac{Y_t}{K_t}$ and $c_t = \frac{C_t}{K_t}$ be the stationary variables for which we obtain the following steady state equilibrium: $\tilde{u} = \left(\frac{A}{\phi d}\right)^{\frac{1}{\phi-1}}$, $\tilde{c} = (1-\beta)\left[1 + \frac{(\phi-1)}{\phi} A\tilde{u}\right]$, $\tilde{y} = A\tilde{u}$, $\tilde{G} = \beta\left[1 + (\phi-1)\, d\tilde{u}^{\phi}\right]$, where $\tilde{G}$ denotes the gross growth rate of output.

Let s_t be the proportion of income that is not being consumed: $c_t = (1-s_t)\phi d {u_t}^{\phi}$. In the steady state the saving rate is given by $\tilde{s} = \frac{\beta\left[1+(\phi-1)\tilde{\delta}\right]+\tilde{\delta}}{\phi\tilde{\delta}}$, where $\tilde{\delta} = \frac{1}{\phi}\tilde{y}$. Hence, the long-run solution to this model is characterized by a constant saving rate, a constant but not full capital utilization rate and a balanced growth path with output, consumption and capital growing at the same rate. These constant levels depend on the marginal product of capital services, A, and the elasticity of depreciation with respect to the rate of capital utilization, ϕ. Let us assume that A is a cross-country specific technological parameter. Notice that countries with a higher A will have both a higher $\tilde{u}$ and a higher $\tilde{G}$.

[2] This is in contrast to the existing growth literature which assumes a constant depreciation rate, implying a zero marginal cost of capital utilization and hence being optimal to fully utilize capital. As shown by Chatterjee (2005), there exists an optimal $u_t \in (0,1)$, under the mild condition $A < \phi d$.

We rewrite the equilibrium dynamics of the model in terms of the saving rate, s_t. Combining FOC's (1) and (2) and taking into account from the resource constraint (3) that $\frac{K_{t+1}}{K_t} = 1 - d\left(\frac{AZ_t}{\phi d}\right)^{\frac{\phi}{1-\phi}}(1-\phi s_t)$, the following expression is obtained:

$$\frac{1 - d\left(\frac{AZ_t}{\phi d}\right)^{\frac{\phi}{\phi-1}}(1-\phi s_t)}{(1-s_t)\phi d\left(\frac{AZ_t}{\phi d}\right)^{\frac{\phi}{\phi-1}}} = \beta E_t\left\{\frac{1 + d\left(\frac{AZ_{t+1}}{\phi d}\right)^{\frac{\phi}{\phi-1}}(\phi-1)}{(1-s_{t+1})\phi d\left(\frac{AZ_{t+1}}{\phi d}\right)^{\frac{\phi}{\phi-1}}}\right\}. \tag{4}$$

Since there is no closed form solution for the equilibrium, we approximate it by linearizing both equations around the steady state values. From (4) we obtain the following first order stochastic difference equation:

$$a_1 ss_t + a_2 zz_t = a_3 E_t\left(ss_{t+1}\right) + a_4 E_t\left(zz_{t+1}\right),$$

where xx_t denotes the deviation of variable X_t from its steady state value in logarithms. Since $zz_t = \rho zz_{t-1} + \varepsilon_t$, the solution is given by:

$$ss_t = \frac{\rho a_4 - a_2}{a_1 - \rho a_3} zz_t. \tag{5}$$

By linearizing around the steady state the resource constraint and by substituting the solution given by (5), we obtain the following expression for the deviations of capital growth from its steady state value:

$$\Delta kk_{t+1} = \theta zz_t, \quad \text{where} \quad \theta = \frac{\left(\frac{A^\phi}{\phi d}\right)^{\frac{1}{\phi-1}}}{1 + \left(1 - \frac{1}{\phi}\right)\left(\frac{A^\phi}{\phi d}\right)^{\frac{1}{\phi-1}}} = \frac{A\tilde{u}}{1 + (1-\frac{1}{\phi})A\tilde{u}}.$$

Therefore, shocks have an effect on capital accumulation. Plugging this expression into the production function and taking into account from (1) that $\Delta uu_t = \frac{1}{\phi-1}\Delta zz_t$, the deviations of output growth from its steady state value are given by the following moving average representation:

$$\Delta yy_t = A(L)\varepsilon_t = \frac{\left[\frac{\phi}{\phi-1} - \left(\frac{\phi}{\phi-1} - \theta\right)L\right]}{1-\rho L}\varepsilon_t,$$

where $A(L)$ is an infinite polynomial in the lag operator. Note that even though temporary shocks are considered, integrated time series are obtained.

3 Persistence Results

This endogenous growth model has some important properties for growth and fluctuations. The model generates integrated time series, even when the underlying shocks are stationary. After the effects of these shocks vanish, output does not return to its trend level. That is, temporary shocks have permanent effects on output since they generate endogenous responses in the amount of resources allocated to growth. As a result, growth dynamics is an important component of the propagation mechanism in which the stochastic properties of the trend are endogenous. As argued by Fatás (2000), in this setting, output persistence is not simply equal to the persistence of disturbances, since shocks endogenously generate changes in the capital accumulation rate which result in persistent responses of output.

The permanent impact of a shock on the level of output equals the infinite sum of the moving average coefficients, which is $A(1)$. In the model under study this measure of persistence is given by:

$$A(1) = (\frac{\theta}{1-\rho}),$$

which is increasing in θ.

Cochrane (1988) suggested another measure of persistence as a weighted sum of autocorrelations $V = \lim_{J\to\infty}\left[1 + 2\sum_{j=1}^{J}(1-\frac{j}{J+1})\psi_j\right]$, where ψ_j is the jth autocorrelation of the growth rate of output. In the model under study, V is given by:

$$V = \frac{(1-\rho^2)\left[\theta^2 + 2\theta + \left(\frac{\phi}{\phi-1}\right)^2 - \frac{\phi}{\phi-1} - 2\frac{\phi}{\phi-1}\theta\right]}{(1-\rho)^2\left[\theta^2 + 2\rho\theta + \left(\frac{\phi}{\phi-1}\right)^2 + \frac{\phi}{\phi-1}(1-2\rho) - 2\frac{\phi}{\phi-1}\theta\right]},$$

which is also increasing in θ. [3]

Therefore, other things equal, countries with a higher marginal product of capital services (A) will have a higher steady-state capital utilization rate ($\tilde{u}$), a higher long-run growth rate ($\tilde{G}$), and a higher absolute response of capital growth to a given shock

[3]Note that, $\lim_{\phi\to\infty}\theta = \frac{A}{A+1-\delta}$ and hence, $A(1)$ converges to the corresponding value for the standard AK model considered by Fatás (2000). Note also that $\lim_{\phi\to\infty} V = \frac{\theta^2(1-\rho^2)}{(1-\rho)^2\left[\theta^2+2(1-\rho)(1-\theta)\right]}$ as in Fatás (2000).

(θ).[4] Regardless of the measure considered, persistence is increasing in θ, which yields a positive correlation between persistence and long-term growth rates as observed for the G-7 countries. Further, this result is also consistent with the observed differences in cross-country utilization rates as well as the positive correlation between depreciation and per capita income growth found by Gylfason and Zoega (2001).

To sum up, in this paper the degree of persistence is an increasing function of capital utilization rates by the way of long-term growth rates. The larger the capital utilization rate is, the larger both the growth rate and the permanent impact of a shock on the level of output. In this way, the model's predictions are consistent with two cross-country empirical observations: the positive correlation between persistence and growth and the positive correlation between capital utilization and per capita income.

4 Conclusions

Cross-country differences in output persistence have already been well documented by Campbell and Mankiw (1989) and Cogley (1990). Further, Fatás (2000) finds a strong positive correlation between the persistence of fluctuations and long-term average growth rates for a sample that includes the G-7 countries and eight additional OECD countries. The standard RBC models with exogenous productivity shocks cannot account for this evidence and Fatás (2000) shows that the standard AK endogenous growth model is able to generate this positive correlation.

However, although empirical evidence on capital utilization rates documents large differences across countries as well as a positive correlation between capital utilization and per capita income, the standard AK growth model assumes that capital services are proportional to the underlying capital stock and treats depreciation as a constant.

This paper attempts to reconcile the discrepancies between evidence and the standard predictions on persistence, long-term growth and capital utilization rates by allowing for depreciation rate to be sensitive to the rate of capital utilization in an otherwise standard AK model. In this setting, a full utilization rate of capital is not optimal and the implications of the model are consistent with observed cross-country results.

[4] The same result is obtained when the country-specific parameter is ϕ instead of A.

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