“Guns and Butter” Revisited: the Role of Deterrence

Giacomo De Luca* and Petros Sekeris*

Abstract

This paper explores the existence of deterrence equilibria in a general equilibrium model of guns and butter production. If fighting entails sufficiently low destruction, war is the unique equilibrium of the game. If, however, conflict generates sufficiently large damages, only mixed strategy equilibria survive, in which players randomize over their deterrence and war strategies. War, therefore, always occurs with positive probability for any positive investment in weapons.

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“Si vis pacem, para bellum”.
Vegetius

1 Introduction

War constitutes an inefficient way to settle disputes. Two sources of inefficiencies may be identified. On the one hand, resources are diverted from productive activities to build up armies. On the other hand, if a war occurs it entails direct costs (e.g. asset destruction, diseases, foregone trade). Nevertheless, human history is plagued by armed conflicts. A large body of literature has addressed this conundrum.

Ideally, one would desire peace without arms, which simultaneously solves both inefficiencies. Were parties able to credibly commit not to arm, such a scenario would be achieved. This kind of commitment, however, is highly demanding (e.g. third party enforcement). Recognizing the difficulty of refraining from arming, scholars have therefore identified conditions preventing the outbreak of conflicts. The ancient maxim by the Roman strategist Vegetius quoted above singles out deterrence as a powerful strategy to avoid war[1].

*Both authors belong to the CRED - University of Namur - Rempart de la Vierge, 8 - 5000 Namur, Belgium. Tel: +32 81 724838. E-mail: gdeluca@fundp.ac.be, psekeris@fundp.ac.be.

[1]Alternative solutions include credible commitment to transfer resources (Fearon, 1995; Powell, 2006), reducing information asymmetries (Powell, 1999; Fortna, 2008), and bargaining (Powell, 1996; Esteban and Sakovics, 2002; Slantchev, 2004; Leventoglu and Slantchev, 2007).

Following the Second World War, the Cold War environment boosted research around this topic. Two broad schools developed around the concept of deterrence in political science: structural deterrence theory, and decision-theoretic deterrence theory. The former school posits that power parity across potential opponents combined with high direct costs of war increases the likelihood of deterrence (Waltz, 1964; Intriligator and Brito, 1984). The latter school adopts a game-theoretic approach and emphasizes the role of the threats' credibility in deterring one's opponent. Thomas Schelling (1960) elaborates thoroughly on the concept of deterrence in his famous book "The Strategy of Conflict". According to him, a good strategist would prevent war through the "skillful nonuse of military forces". The core of his argument is that the success of deterrence rests on the credibility of the threat of retaliation. Although stressing the crucial role of the threat's credibility, Jervis (1976) warns about the risks associated to spiraling investments in arms. Indeed, arming to deter the opponent can lead to a vicious circle of increasing risk of a disastrous war (see also Glaser, 1997)[2]. A common feature of the political science literature on deterrence is the exogeneity of the parties' power. Indeed, even in game-theoretic works, agents often do not choose their optimal armament level.

Put aside Schelling, other economists have elaborated on the concept of deterrence in recent years (Garfinkel, 1990; Grossman and Kim, 1995; Neary, 1996; Chassang and Padro-i-Miquel, 2008; Jackson and Morelli, 2008). The added value of some of these models has been to endogenize militarization. Indeed, if deterrence is an equilibrium, it results from a deliberate rational *choice* by agents. Exogenously assuming deterrent levels of arms for both opposing parties does not *necessarily* make deterrence an equilibrium, as either party might have an incentive to modify its armament level. On the one hand, increasing one's arms level might make the expected payoff of war larger then the returns of peace, while, on the other hand there might be a a cost-saving reduction of weapons such that deterrence does not collapse.

In Garfinkel (1990) and Jackson and Morelli (2008), a deterrence equilibrium is obtained in a dynamic setting. An infinitely repeated game contributes to sustain cooperation by alleviating commitment problems. In particular, in Garfinkel (1990), if the discount factor is large enough, peace is sustained without any investment in arms. For lower values of the discount factors, positive amounts of weapons may be necessary to deter the outbreak of a war. Jackson and Morelli (2008) show that if the costs of war are intermediate, only mixed strategy equilibria survive, in which opposing parties mix over three strategies labeled "Hawk", "Dove" (accepting passively aggression), and "Deterrent". A similar result is obtained by Neary (1996) in a static framework. In this study, however, players mix over hawk and deterrence strategies only.

Grossman and Kim (1995) propose a static model of "guns and butter" in which each party sequentially chooses its defense and offense levels of armaments, before deciding whether or not to go to war. Deterrence results from

[2]The interested reader can refer to the first chapters of Zagare and Kilgour (2000) for a more complete review of the works on deterrence by political scientists.

producing sufficient defensive weapons in the first stage of the game such that investing in offensive weapons is not worthwhile for the opponent.

This paper proposes a static two-stage general equilibrium model of "guns and butter" in the spirit of Skaperdas (1992), Hirshleifer (1995), and Neary (1997), in which two agents choose their armament level in stage one, and whether to attack their opponent in the second stage. We show that if war entails enough destruction, then only mix strategy equilibria survive, as in Jackson and Morelli (2008) and Neary (1996). In our model, as in Neary (1996), agents mix over two strategies only: deterrence and aggression. As compared to both Jackson and Morelli (2008) and Neary (1996), we solve the model in a more general setting. The only requirement regarding the production function in our model is concavity. Moreover, while the cost of war is exogenous in the model of Jackson and Morelli (2008), we link it explicitly to the armament levels decided by the opposing agents in the first stage of the game. A critical implication of our analysis is that whenever weapons' production is positive deterrence is not perfect. In other words, if weapons are produced, there *always* exists a positive probability that a war breaks out.

The rest of the paper is organized as follows: in section 2 we present the model and derive the equilibria under the assumptions of non-destructive and destructive war, respectively; section 3 concludes by discussing the implications of our results for the conflict theory literature.

2 The Model

2.1 The Setting

Two risk-neutral agents[3] labeled 1 and 2 allocate their resource endowment, R^i ($i = \{1,2\}$), in guns, G^i, and butter, X^i. The agents' specific production technology $C^i(X^i)$ transforms butter into consumables. Guns create no wealth and their unique role is to improve the likelihood of appropriating the aggregate production in case of a dispute. More specifically, agent 1's probability of winning is denoted by $p(G^1, G^2)$. Agents 1 and 2 interact in a two-stage game. They allocate their resources between guns and butter in the first stage of the game. In the second stage, the players observe the output as well as the quantity of weapons produced and decide whether to wage a war or not. When no guns have been built, each player keeps his own production. If either player decides to attack the other the outcome is conflict and the ensuing payoffs are given by:

$$U^{1w}(G^1, G^2) = \delta p(G^1, G^2)\left(C^1(X^1) + C^2(X^2)\right) \tag{1}$$

$$U^{2w}(G^1, G^2) = \delta\left(1 - p(G^1, G^2)\right)\left(C^1(X^1) + C^2(X^2)\right) \tag{2}$$

where $\delta \in [0,1]$ is a parameter that captures the destructiveness of war, and the superscript w stands for war.

[3] In section 3 we discuss the implications of relaxing the risk-neutrality assumption.

If, however, both agents choose not to attack each other, the resulting utilities are given by:

$$U^{ip}(G^i) = C^i(X^i) \qquad\qquad i = \{1, 2\} \tag{3}$$

where the superscript p denotes peace.

We impose the following standard assumptions on the production technology:

Assumption 1. $C^i(0) = 0 \qquad \frac{\partial C^i}{\partial X^i} = C^i_1 > 0 \qquad \frac{\partial^2 C^i}{\partial X^i \partial X^i} = C^i_{11} < 0$

where the subscript k denotes the derivative with respect to the k^{th} argument.

Regarding the conflict technology, we restrict the analysis to the class of functions axiomatized by Skaperdas (1996)[4]:

$$p(G^1, G^2) = \frac{F(G^1)}{F(G^1) + H(G^2)} \tag{4}$$

Moreover, we impose the following assumption on the power functions $F(.)$ and $H(.)$:

Assumption 2. $\begin{cases} F_1(G^1) > 0 & \frac{F_{11}(G^1)}{F_1(G^1)} \leq 2\frac{F_1(G^1)}{F(G^1)} \\ H_1(G^2) > 0 & \frac{H_{11}(G^2)}{H_1(G^2)} \leq 2\frac{H_1(G^2)}{H(G^2)} \end{cases}$

Assumption 2 accommodates both convex and concave power functions. In the convex case, however, it restricts the degree of convexity for large values of $F(.)$ and $H(.)$. In other words, a powerful agent should not experience too high marginal increases in power. This assumption is used in Appendix A.1 to show quasi-concavity of the agents' war utilities as given by equations (1) and (2).

2.2 Agents' best responses

The agents optimally allocate their resources to either maximize their expected payoff of war, or to avoid the onset of a conflict.

The investment in guns and butter for agent 1, conditional on having war in the second stage of the game, results from the following maximization problem[5]:

$$max_{G^1} \;\; \delta p(G^1, G^2)\big(C^1(R^1 - G^1) + C^2(X^2)\big) \qquad s.t. \;\; 0 \leq G^1 \leq R^1 \tag{5}$$

Saving on notations, the first derivative with respect to G^1 equals:

$$U^{1w}_1 = p_1(C^1 + C^2) - pC^1_1 \tag{6}$$

In Lemma 4 we show that U^{1w} is quasi-concave in G^1. This allows us to derive agent 1's optimal war strategy.

[4] See also Hirshleifer (1991).

[5] All results in the paper are derived solely for agent 1 for notational convenience.

Interior solution:

$$G^{1w}(G^2) = G^{1w}$$
$$\text{if } \; p_1\left(G^{1w}, G^2\right)\left[C^1\left(G^{1w}\right) + C^2\left(G^2\right)\right] - p\left(G^{1w}, G^2\right) C_1^1\left(G^{1w}\right) = 0 \qquad (7)$$

Corner solutions:

$$G^{1w}(G^2) = \begin{cases} 0 \;\text{ if }\; p_1\left(0, G^2\right)\left[C^1(0) + C^2\left(G^2\right)\right] - p\left(0, G^2\right) C_1^1(0) < 0 \\ R^1 \;\text{ if }\; p_1\left(R^1, G^2\right)\left[C^1\left(R^1\right) + C^2\left(G^2\right)\right] - p\left(R^1, G^2\right) C_1^1\left(R^1\right) > 0 \end{cases}$$

Alternatively, agent 1 can deter his potential opponent. His maximization problem is then the following:

$$max_{G^1} \quad C^1(R^1 - G^1)$$

$$s.t. \quad \begin{cases} C^2(G^2) \geq \delta\left(1 - p(G^1, G^2)\right)\left(C^1(G^1) + C^2(G^2)\right) \\ 0 \leq G^1 \leq R^1 \end{cases} \qquad (8)$$

Agent 1's optimal deterrence strategy (superscript d) is therefore given by:

Interior solution:

$$G^{1d}(G^2) = G^{1d} \;\text{ if }\; C^2(G^2) = \delta\left(1 - p(G^{1d}, G^2)\right)\left(C^1(G^{1d}) + C^2(G^2)\right) \qquad (9)$$

Corner solution[6]*:*

$$G^{1d}(G^2) = 0 \quad \text{ if } \quad C^2(G^2) > \delta\left(1 - p(0, G^2)\right)\left(C^1(0) + C^2(G^2)\right)$$

Agent 1 chooses the utility maximizing strategy between war and deterrence, for any guns levels of agent 2, G^2. Formally:

$$G^1(G^2) = \begin{cases} G^{1w}(G^2) & \text{if} \quad U^{1w}(G^{1w}, G^2) \geq U^{1p}(G^{1d}, G^2) \\ G^{1d}(G^2) & \text{otherwise} \end{cases} \qquad (10)$$

In the following two sections we derive the equilibrium results for non-destructive, and destructive conflicts, respectively.

2.3 Non-destructive war ($\delta = 1$)

In this section we consider war as a zero-sum activity. Under this assumption, the only cost of conflict to society is represented by the inefficient allocation of resources to non-productive (war) activities. Indeed, the socially optimal resource allocation requires that $G^1 = G^2 = 0$. In this setting, we show that the following proposition holds (see Appendix A.2 for a formal proof):

Proposition 1. *If conflict is not destructive, the war strategy always dominates the deterrence strategy.*

[6] Notice that $G^{1d} < R^1$ since $C^2(G^2) > \delta p(R^1, G^2)(0 + C^2(G^2)), \;\; \forall G^2$.

By the definition of a zero-sum game, if agent 1 prefers war, agent 2 necessarily prefers peace (and vice-versa). Proposition 1 states that both, nevertheless, prefer war to deterrence. Notice first that, if agent 1 prefers war, by the definition of the deterrence strategy, agent 1's optimal deterrence guns level is lower than his optimal war guns level, while the inverse is true for agent 2. The gains to agent 1 of reducing his guns investment to the deterrence level do not offset the foregone expected rewards of war. For agent 2, on the other hand, the burden of the deterring guns investment is too large as compared to the expected war payoffs.

Since only the war strategies are relevant in the absence of destruction, a well established result in conflict theory follows (Skaperdas, 1992; Skaperdas and Syropoulos, 1997):

Proposition 2. *If conflict is not destructive, there exists a unique stable pure strategy war equilibrium.*

For the proof see Appendix A.3.

As a consequence of Proposition 1, we obtain in Proposition 2 a standard result in the conflict literature: at equilibrium the agents invest inefficiently high amounts of resources in conflict activities and engage in war. Notice, however, that for particularly poor performing war technologies and/or for production functions exhibiting high marginal returns, the war equilibrium investment in guns may be nil for both players. Although such non-militarized war equilibria may indeed be observed in reality, they are not relevant for the purpose of our study. Indeed, in such configurations the very definition of deterrence is meaningless. In the following section we therefore restrict the analysis to situations where undeterred players would invest strictly positive resources in guns.

2.4 Destructive war ($\delta < 1$)

Assuming that war is destructive constitutes perhaps a more realistic hypothesis. This assumption implies that Proposition 1's result may not hold anymore. Indeed, the agents may find it optimal to adopt their deterrent strategy for some armament levels of the opponent, potentially changing the equilibrium outcome. The existence of an equilibrium in Proposition 2 has been proved for the setting in which the only relevant strategy is war. In the following proposition, we address the equilibrium existence in the present framework:

Proposition 3. *If conflict is destructive, an equilibrium always exists.*

A formal proof is provided in Appendix A.4.

Having proved the existence of an equilibrium, we investigate the conditions under which the various equilibria may arise. In particular, we want to isolate the conditions under which war and deterrence equilibria emerge. A first crucial qualification of the result in Proposition 3 is provided in the next proposition.

Proposition 4. *A pure strategy deterrence equilibrium never exists.*

The proofs can be found in Appendix A.5.

In proposition 1 we show that deterrence strategies are irrelevant when $\delta = 1$. War destruction creates scope for the deterrence strategy to be relevant for some armament levels. And yet, a pure strategy deterrence equilibrium never emerges. The intuition underpinning this result goes as follows. Take any initial combination of guns (G^{1o}, G^{2o}). For this distribution of power, at least one agent strictly prefers peace (U^{ip}) to war (U^{iw}) since war is a negative-sum game. Assume it is agent 1. The optimal level of guns to deter agent 1 are necessarily lower than G^{2o}. Indeed, agent 2 can cut his expenditure in guns and still deter agent 1. The reduction in G^2 increases agent 1's war payoff, until war becomes as attractive as peace to the latter. When agent 1 is made indifferent between war and peace, it must then be the case that agent 2 strictly prefers peace to war for this new distribution of power, since conflict is destructive. This process eventually reduces the guns to a level where at least one player finds it profitable to play according to his war strategy.

It is noteworthy that the war strategy remains optimal for some armament levels despite assuming that fighting is destructive. As a consequence, a war equilibrium may still arise. The next proposition identifies the necessary and sufficient conditions under which a pure strategy war equilibrium emerges.

Proposition 5. *A stable pure strategy war equilibrium exists if and only if* $\delta \geqslant \underline{\delta}$.

For a formal proof see Appendix A.6.

The result presented in Proposition 5 is fairly intuitive: a war equilibrium survives if the associated destruction is sufficiently low. The exact degree of destructiveness compatible with a war equilibrium depends on the combination of initial endowments, war technologies and production functions. When fighting is destructive enough, the war equilibrium collapses since at least one agent finds it optimal to deter his foe. By no means this implies that the deterring agent devotes no resources to guns. Indeed, he needs to wield enough power to dissuade any potential aggression. Interestingly, $\underline{\delta}$ needs not be small. In the perfectly symmetric case[7], for instance, even when fighting provokes minor damages, the war equilibrium collapses.

The findings of this section are summarized in the following theorem which presents the main result of this paper:

Theorem 1. *If aggregate spending in guns is strictly positive, war always occurs with positive probability.*

This result follows directly from Propositions 2, 3, and 4. Indeed, Proposition 2 ensures that when fighting is not destructive, war is the unique equilibrium of this game. Moreover, Propositions 3 and 4 imply that even highly damaging fighting cannot sustain peace as an equilibrium. For relatively low levels of destruction $(\delta \geq \underline{\delta})$, Proposition 5 shows that war is an equilibrium. Finally, the

[7]By perfect symmetry we assume that agents are symmetric in their endowment, war technology, and production technology.

only compatible equilibrium with relatively high levels of destruction ($\delta < \underline{\delta}$), is a mixed strategy equilibrium where agents mix over their war and deterrence strategies. War thus occurs with a positive probability for any destruction level.

A straightforward implication of Theorem 1 is that if war entails relatively large levels of destruction ($\delta < \underline{\delta}$), a scenario in which both agents choose their deterrence guns level in the first stage occurs with a positive probability. When this materializes, both agents own sufficient guns to deter one another, while none finds it optimal to initiate a war. An armed peace equilibrium results.

3 Concluding remarks

In this paper, we build a general equilibrium model of "guns and butter" in which we consider the often neglected deterrence strategy. We show that war is the only equilibrium outcome whenever fighting entails limited destruction. If, however, the losses associated to war are sufficiently large, only mixed strategy equilibria survive. Agents then follow the deterrent strategy with some probability and the war strategy with the complementary probability.

The assumption supporting the mixed strategy equilibrium, namely that war is destructive, is almost a pleonasm. In addition to the loss of assets, war also entails other direct costs (e.g. casualties, diseases, decline in productivity, uncertainty). The literature on conflicts is well aware of these costs (Garfinkel and Skaperdas, 2007; Blattman and Miguel, 2008). And yet, some relevant contributions to conflict theory, although assuming war is destructive, fail to consider the deterrence strategy. In other words, in these models when agents decide to arm, they are *implicitly* assumed to go to war in the second stage of the game (e.g. Garfinkel and Skaperdas, 2000).

Alternative consequences of war may create the sufficient gap between the aggregate production under peace and war for deterrence to occur ($\delta < 1$). If, for instance, the trade volume between the two potential foes is negatively affected from war, as argued in the literature (Polachek, 1997; Dorussen, 1999), then both agents might refrain from attacking their commercial partner while maintaining the optimal armament level to deter one another. Complementarity in production leads to the same conclusions.

Lastly, assuming risk averse agents creates the necessary disparity in returns to obtain deterrence, as they prefer the certain outcome under peace to the risky bet of war. Therefore, explicitly considering the deterrence strategy in models like the ones of Skaperdas (1991) and Skaperdas and Gan (1995) would probably broaden the scope of their analysis.

A Appendix

A.1 Quasi-Concavity of U^{1w}

Regarding quasi-concavity, we closely follow the steps of Skaperdas (1992), and Skaperdas and Syropoulos (1997); we use Lemmas 1- 3 to establish Lemma 4 that guarantees quasi-concavity of U^{1w}.

Lemma 1. *Under Assumption 1,* $p_{11}p \leq 2(p_1)^2$

Proof. We shall make extensive use of p as given by (4), and to economize notation we do not write the F and H functions' arguments. Deriving (4) yields the following results:

$$p_1 = \frac{F'H}{(F+H)^2} = \frac{F'}{F}p(1-p) \tag{A-1}$$

$$\begin{aligned} p_{11} &= \frac{(F''p(1-p) + p_1F'(1-2p))F - (F')^2p(1-p)}{F^2} \\ &= \frac{FF''p(1-p)}{F^2} - \frac{2p^2(1-p)(F')^2}{F^2} \end{aligned}$$

Thus, $p_{11}p \leq 2(p_1)^2$ can be written as:

$$FF'' \leq 2(F')^2$$

And this last inequality is necessarily true by Assumption 1. □

Lemma 2. *Under Assumption 2,* $-p_{22}(1-p) \leq 2(p_2)^2$

Lemma 2 can be proved along the lines of Lemma 1's proof.

Lemma 3. $U_{11}^{1w} < 0 \qquad if \qquad U_1^{1w} \leq 0$

Proof. If $U_1^{1w} < 0$, the first order condition (6) can be written as:

$$\frac{p_1(C^{1w} + C^2)}{p} < C_1^{1w}$$

Replacing this term in U_{11}^{1w} which is obtained by using (6):

$$U_{11}^{1w} = p_{11}(C^{1w} + C^2) - 2p_1C_1^{1w} + pC_{11}^{1w} \tag{A-2}$$

This eventually gives us:

$$U_{11}^{1w} < \frac{C^{1w} + C^2}{p}\left[p_{11}p - 2(p_1)^2\right] + pC_{11}^{1w}$$

Because of the production function's concavity, a sufficient condition for establishing the result is therefore that $p_{11}p \leq 2(p_1)^2$, which is shown in Lemma 1. □

Lemma 4. $U_{11}^{1w} < 0 \quad, \forall G^2$

Proof. Denote by $\bar{G^1}$ the smallest value of G^1 such that $U_1^{1w}(G^1) < 0$. By Lemma 3 we can directly deduce that $U_1^{1w}(G^1) < 0, \quad \forall G^1 \geq \bar{G^1}$. To establish the present Lemma's result, we thus need that $U_1^{1w}(G^1) \geq 0 \quad, \forall G^1 < \bar{G^1}$, which is necessarily true since if there exists some $G^{1'} < \bar{G^1}$ such that $U_1^{1w}(G^{1'}) < 0$, then $\bar{G^1}$ is not the smallest value of G^1 such that $U_1^{1w}(G^1) < 0$. □

A.2 Proof of Proposition 1

Proof. Assume 1 wants to deter 2. Then:

$$\delta p(G^{1w}, G^2)(C^{1w} + C^2) < C^{1p} \tag{A-3}$$

The deterrent levels of guns for 1 must satisfy the following expression:

$$\delta \left(1 - p(G^{1p}, G^2)\right)(C^{1p} + C^2) = C^2 \tag{A-4}$$

which can be re-written as:

$$p(G^{1p}, G^2)(C^{1p} + C^2) = C^{1p} + C^2 - \frac{C^2}{\delta} \tag{A-5}$$

And since G^{1w} is the argmax of U^{1w}, it is necessary that the next expression be true:

$$p(G^{1w}, G^2)(C^{1w} + C^2) \geq p(G^{1p}, G^2)(C^{1p} + C^2) \tag{A-6}$$

Combining these results yields:

$$p(G^{1w}, G^2)(C^{1w} + C^2) > \delta p(G^{1w}, G^2)(C^{1w} + C^2) + (1 - 1/\delta)C^2 \tag{A-7}$$

which cannot be true if $\delta = 1$. □

A.3 Proof of Proposition 2

Existence:

Lemma 5. *Under Assumptions 1 and 2, a pure strategy equilibrium always exists.*

Proof. For the existence part of Proposition 2, we need that the strategy set of each player is compact and convex, and that each player's utility function is continuous in all the players' strategies, and quasi-concave in his own strategy.

¿From player 1's viewpoint (an analogous reasoning may be carried for player 2), the compactness and convexity of the strategy set is a direct consequence of $G^1 \in [0, R^1]$, while the continuity of U^{1w} follows from the continuity of p and C^1.

The above results together with Lemma 4 complete the proof of Lemma 5. □

Uniqueness:

Lemma 6. *Provided an equilibrium exists, it is unique and stable.*

Proof. We first prove that if an interior equilibrium exists, it is the unique stable equilibrium. We then show that if no interior equilibrium exists, the unique stable equilibrium (G^{1*}, G^{2*}) is such that *either* $G^{1*} = 0$ *or* $G^{1*} = R^1$.

To show the uniqueness of an interior equilibrium it is sufficient to prove that the composite function $\Gamma^1(G^1) = G^1(G^2) \circ G^2(G^1)$ is continuous, singled-valued, and that whenever it admits a fixed point, $\partial G^{1w}/\partial G^2 \cdot \partial G^{2w}/\partial G^1 < 1$. Continuity and single valuedness of $G^i(G^j)$ follow from the continuity of U^{1w} in G^1 and G^2, and from the quasi-concavity of U^{1w} in G^1.

Lemma 7. $U_1^{1w} = 0 \quad \Rightarrow \quad \partial G^{1w}/\partial G^2;\;\; U_2^{2w} = 0 \quad \Rightarrow \quad \partial G^{2w}/\partial G^1 = 0$

Proof. By using the implicit functions theorem on $G^1(G^2)$ and on $G^2(G^1)$, we obtain the two following equations:

$$\frac{\partial G^{1w}}{\partial G^2} = -\frac{U_{12}^{1w}}{U_{11}^{1w}} \qquad \frac{\partial G^{2w}}{\partial G^1} = -\frac{U_{12}^{2w}}{U_{22}^{2w}}$$

We construct the reasoning for U_{12}^{1w} alone since the proof follows the same lines for U_{12}^{2w}. By using U_1^{1w} as given by Equation (6), we can obtain the following term:

$$U_{12}^{1w} = p_{12}(C^{1w} + C^2) - p_1 C_2^2 - p_2 C_1^{1w} \tag{A-8}$$

Given, however, that we assume we are at the equilibrium, and superscripting the equilibrium variables by a star, we must have by Equation (6) and by an analogous reasoning for agent 2 that the two next equalities hold:

$$C_1^{1w*} = \frac{p_1^{w*}(C^{1w*} + C^{2w*})}{p^{w*}} \qquad C_2^{2w*} = -\frac{p_2^{w*}(C^{1w*} + C^{2w*})}{1 - p^{w*}}$$

Replacing these two optimality conditions in (A-8) yields:

$$U_{12}^{1w} = p_{12}^{w*}(C^{1w*} + C^{2w*}) + \frac{p_1^{w*}p_2^{w*}(C^{1w*} + C^{2w*})}{1 - p^{w*}} - \frac{p_1^{w*}p_2^{w*}(C^{1w*} + C^{2w*})}{p^{w*}}$$

Which, rearranged, gives:

$$U_{12}^{1w} = \frac{C^{1w*} + C^{2w*}}{p^{w*}(1 - p^{w*})}\Big[p^{w*}(1 - p^{w*})p_{12}^{w*} + p_1^{w*}p_2^{w*}(2p^{w*} - 1)\Big]$$

The square bracketed term, however, can be shown to be nil. Indeed, setting the bracketed term equal to zero and re-arranging it, we obtain:

$$-\frac{p^{w*}p_{12}^{w}}{p_1^{w*}p_2^{w*}} = \frac{2p^{w*} - 1}{1 - p^{w*}} \tag{A-9}$$

It can be shown that:

$$\frac{p_{12}}{p_2} = \frac{(1 - 2p)F_1}{F}$$

Replacing p_{12}/p_2 and p_1 as given by (A-1) in the LHS of (A-9) establishes the result. □

Assume a unique stable interior equilibrium exists. This implies that if $\left(0, G^2(0)\right)$ and/or $\left(R^1, G^2(R^1)\right)$ are equilibria, then they are not stable. Indeed, if $G^1(G^2(0))$ $= 0$, then $\frac{\partial G^1}{\partial G^2}_{|G^1=0} > 1$. Similarly, if $G^1(G^2(R^1)) = R^1$, then $\frac{\partial G^1}{\partial G^2}_{|G^1=R^1} > 1$.
If, instead, a stable interior equilibrium does not exist, we have three potential cases. If $G^1(G^2(0)) = 0$ and $G^1(G^2(R^1)) < R^1$, then $\frac{\partial G^1}{\partial G^2}_{|G^1=0} < 1$. If $G^1(G^2(0)) > 0$ and $G^1(G^2(R^1)) = R^1$, then $\frac{\partial G^1}{\partial G^2}_{|G^1=R^1} < 1$. Lastly, if $G^1(G^2(0)) = 0$ and $G^1(G^2(R^1)) = R^1$, then either $\frac{\partial G^1}{\partial G^2}_{|G^1=0} < 1$ and $\frac{\partial G^1}{\partial G^2}_{|G^1=R^1} > 1$ (implying $(R^1, G^2(R^1))$ is a non-stable equilibrium), or $\frac{\partial G^1}{\partial G^2}_{|G^1=0}$ > 1 and $\frac{\partial G^1}{\partial G^2}_{|G^1=R^1} < 1$ (implying $(0, G^2(0))$ is a non-stable equilibrium). □

A.4 Proof of Proposition 3

Proof. We construct this proof for agent 1 alone since it extends straightforwardly to agent 2.

To establish the equilibrium existence we apply Kakutani's fixed point theorem to the correspondence $\Gamma^1 = G^1(G^2) \circ G^2(G^1)$.

Notice first that the strategy set, $G^1 \in [0, R^1]$, is non-empty, compact, and convex.

To prove the continuity of Γ^1 it is sufficient to show that $G^1(G^2)$ is continuous on the interval $[0, R^2]$. From (10) we know that $G^1(G^2)$ is either equal to $G^{1w}(G^2)$, or to $G^{1d}(G^2)$.

Regarding $G^{1w}(G^2)$, it is non-empty $\forall G^2 \subset [0, R^2]$. Moreover, $G^{1w}(G^2)$ is continuous on $[0, R^2]$ if $U^{1w}(G^1, G^2)$ is continuous in (G^1, G^2) and quasi-concave in G^{1w}. The continuity and quasi-concavity proofs can be found in Appendices A.1 and A.3.

Regarding the deterrent strategy $G^{1d}(G^2)$ it is also non-empty $\forall G^2 \subset [0, R^2]$. To prove continuity of $G^{1d}(G^2)$ we apply the implicit functions' theorem on the implicit function $\varphi(G^{1d}, G^2)$. This theorem states that if:

1. $\varphi(G^{1d}, G^2) = 0$,
2. $\varphi(G^{1d}, G^2)$ is continuous,
3. $\partial\varphi(G^{1d}, G^2)/\partial G^{1d} \neq 0$ at $G^{1d} = \hat{G}^1$,
4. $\varphi(G^{1d}, G^2)$ has continuous first partial derivative with respect to G^{1d} in a neighborhood $\hat{G}^1$,

then $G^{1d}(G^2)$ is continuous in G^2.
Denote $\varphi(G^{1d}, G^2) = C^2(G^2) - \delta\left(1 - p(G^{1d}, G^2)\right)\left[C^1(G^{1p}) + C^2(G^2)\right] = 0$.
Notice that $\varphi(G^{1d}, G^2)$ is continuous as each of its components is continuous.
Moreover $\partial\varphi(G^{1d}, G^j)/\partial G^{1d}$ is given by the following expression:

$$\frac{\partial\varphi(G^{1d}, G^2)}{\partial G^{1d}} =$$

$$\delta \left(p_1(G^{1d}, G^2) \left[C^1(G^{1d}) + C^2(G^2)\right] - \left(1 - p(G^{1d}, G^2)\right) C_1^1(G^{1d})\right) > 0$$

since $p_1 > 0$ and $C_1^1 < 0$.
Continuity of $\frac{\partial\varphi(G^{1d}, G^2)}{\partial G^{1d}}$ follows from the continuity of all the elements.
$G^1(G^2)$ is continuous in G^2 because both $G^{1w}(G^2)$ and $G^{1d}(G^2)$ are continuous in $G^2 \in [0, R^2]$, and because $G^1(G^2) = \lambda G^{1d}(G^2) + (1-\lambda)G^{1w}(G^2)$, $\forall G^2$ s.t. $U^{1d}(G^{1d}(G^2), G^2) = U^{1w}(G^{1w}(G^2), G^2)$, $\forall \lambda \in [0, 1]$.

To complete the proof, the convex-valuedness of Γ^1 still needs to be established. Since both $G^1(G^2)$ and $G^2(G^1)$ have been shown to be convex-valued, Γ^1 is necessarily convex-valued. □

A.5 Proof of Proposition 4

Proof. Suppose a pure strategy deterrent equilibrium (G^{1d*}, G^{2d*}) exists. By the definition of such an equilibrium, it is necessary that:

$$C^1(G^{1d*}) > \delta p \left(G^{1w}(G^{2d*}), G^{2d*}\right) \left(C^1(G^{1w}(G^{2d*})) + C^2(G^{2d*}\right) \tag{A-10}$$

Moreover, by the definition of a deterrent equilibrium, G^{2d*} is such that the following equation must hold:

$$C^1(G^{1d*}) = \delta p \left(G^{1d*}, G^{2d*}\right) \left(C^1(G^{1d*}) + C^2(G^{2d*})\right) \tag{A-11}$$

Combining those two expressions we obtain:

$$\begin{aligned} p \left(G^{1d*}, G^{2d*}\right) \left(C^1(G^{1d*}) + C^2(G^{2d*})\right) > \\ p \left(G^{1w}(G^{2d*}), G^{2d*}\right) \left(C^1(G^{1w}(G^{2d*})) + C^2(G^{2d*}\right) \end{aligned} \tag{A-12}$$

which contradicts $G^{1w}(G^2)$ being the *argmax* of U^{1w}. □

A.6 Proof of Proposition 5

Proof. We first prove the $\Rightarrow$ part.

Take $\delta^1 : C^1 \left(G^{1d}(G^{2w*})\right) = \delta^1 p(G^{1w*}, G^{2w*}) \left(C^1(G^{1w*}) + C^2(G^{2w*})\right)$. Such a δ^1 exists and is unique. Indeed, when $\delta = 0$,

$$C^1 \left(G^{1d}(G^{2w*})\right) > \delta p(G^{1w*}, G^{2w*}) \left(C^1(G^{1w*}) + C^2(G^{2w*})\right)$$

Moreover, by Proposition 1, if $\delta = 1$, then $U^{1w*} \geq U^{1d}$. Since $C^1(.)$ is continuous, δ^1 exists.

To show uniqueness it is sufficient to prove the monotonicity of $\frac{\partial\left(U^{1d} - U^{1w*}\right)}{\partial\delta}$. Notice that G^{1d} is implicitly defined as follows:

$$C^2(G^2) = \delta \left(1 - p(G^{1d}, G^2)\right) \left(C^1(G^{1d}) + C^2(G^2)\right)$$

We thus obtain that $\partial G^{1d}/\partial\delta > 0$, which implies that $\partial C^1(G^{1d})/\partial\delta < 0$. Moreover, using equation (1) we deduce that $\partial U^{1w}/\partial\delta > 0$, thus implying that $\partial \left(U^{1d} - U^{1w*}\right)/\partial\delta < 0$, $\forall\delta$.

If $C^2\left(G^{2d}(G^{1w*})\right) \leq \delta^1\left(1-p(G^{1w*},G^{2w*})\right)\left(C^1(G^{1w*})+C^2(G^{2w*})\right)$, then $\underline{\delta}=\delta^1$.

Otherwise, following the above reasoning, there exists a unique $\delta^2=\underline{\delta}$, with $\delta^2>\delta^1$, and such that:

$$C^2\left(G^{2d}(G^{1w*})\right) = \delta^2\left(1-p(G^{1w*},G^{2w*})\right)\left(C^1(G^{1w*})+C^2(G^{2w*})\right)$$

We can therefore conclude that there exists a unique $\underline{\delta}=max\left\{\delta^1,\delta^2\right\}$.

The $\Leftarrow$ part is straightforward. War being an equilibrium implies:

$$C^1\left(G^{1d}(G^{2w*})\right) \leq \delta p(G^{1w*},G^{2w*})\left(C^1(G^{1w*})+C^2(G^{2w*})\right) \quad \text{(A-13)}$$

$$C^2\left(G^{2d}(G^{1w*})\right) \leq \delta\left(1-p(G^{1w*},G^{2w*})\right)\left(C^1(G^{1w*})+C^2(G^{2w*})\right) \quad \text{(A-14)}$$

Reduce the value of δ until either expression (A-13), or expression (A-14) (or both) hold with equality, and denote this δ as $\delta^{'}$. Any $\delta<\delta^{'}$ makes the pure strategy war equilibrium collapse. Setting $\underline{\delta}=\delta^{'}$ completes the proof.

□

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