# Endogenous timing in a mixed duopoly: Weighted welfare and price competition

Juan Carlos Bárcena-Ruiz [a] , Máximo Sedano [b]

[a] *Departamento de Fundamentos del Análisis Económico I, Facultad de Ciencias Económicas y Empresariales, Universidad del País Vasco, Avenida Lehendakari Aguirre 83, 48015 Bilbao, Spain. Tel.: +34 946013829; fax: +34 946013891. E-mail address: juancarlos.barcena@ehu.es*

[b] *Departamento de Economía Aplicada III, Facultad de Ciencias Económicas y Empresariales, Universidad del País Vasco, Avenida Lehendakari Aguirre 83, 48015 Bilbao, Spain. E-mail address: maximo.sedano@ehu.es*

**Abstract**

In this paper we analyze the endogenous order of moves in a mixed duopoly for differentiated goods. Firms choose whether to set prices sequentially or simultaneously. The private firm maximizes profits while the public firm maximizes the weighted sum of the consumer and producer surpluses (weighted welfare). It is shown that the result obtained in equilibrium depends crucially on the weight given to consumer surplus in weighted welfare. We obtain that if the weight is high enough firms decide prices simultaneously. If the weight takes an intermediate value, the public firm is the leader in prices and the private firm the follower. Finally, if the weight is low enough there are two sequential equilibria: the public firm is the leader in prices in one of them and the follower in the other.

*Keywords:* Mixed duopoly; Price competition; Endogenous timing; Weighted welfare.

*JEL Classification***:** L00; L30

## 1 Introduction

The literature on mixed markets has analyzed whether firms take decisions (on quantities or prices) sequentially or simultaneously, assuming that private firms maximize profits and public firms maximize social welfare.[1] In this regard, Pal (1998) shows that when firms produce a homogeneous good, they decide quantities sequentially in a mixed oligopoly. Matsumura (2003) and Lu (2006) extend this analysis to consider that the public firm competes with foreign private firms. They also find that decisions are taken sequentially. Bárcena-Ruiz and Garzón (2008) assume that a firm jointly owned by the public sector and private domestic shareholders (a semipublic firm) competes with the private firms, and show that, in equilibrium, firms take production decisions simultaneously. The papers cited above consider that firms compete in quantities. Under Bertrand competition, Bárcena-Ruiz (2007) shows that when firms produce a heterogeneous good they decide prices simultaneously in a mixed duopoly.

The papers cited above assume that public firms maximize social welfare. However, the objective function of the public firms can give a different weight to the consumer surplus than to the producer surplus, which can affect the result obtained in equilibrium.[2] Thus, in this paper we analyze the endogenous order of moves in a mixed duopoly under price competition,[3] assuming that the objective function of the public firm can give a different weight to consumer surplus than to producer surplus.

[1] This literature also analyzes whether other variables are decided sequentially or simultaneously. For example, Matsumura and Matsusima (2003) analyze the sequential choice of location in a mixed duopoly assuming a Hotelling-type spatial model.

[2] White (2002, p. 489) argues that "while the standard, equally-weighted welfare function may be desirable for normative reasons, based on utilitarianism or fairness doctrines (as in Harsanyi, 1995), it may be restrictive for purposes of predicting the behavior of actual public firms and the resulting market outcomes". White (2002) considers a weighted welfare function to analyze how the public firm responds to different objective functions, and how this impacts on the industry as a whole.

[3] There are few papers analyzing the case in which firms decide prices. Other papers analyzing a mixed market under price competition are Anderson et *al*. (1997), Tasnádi (2006) and Ogawa and Kato (2006), Bárcena-Ruiz (2008).

This paper addresses the issue of the endogenous order of moves by considering the observable delay game of Hamilton and Slutski (1990). We consider a mixed duopoly where firms decide the timing of their choosing of prices. It is shown that the result obtained in equilibrium depends crucially on the objective function considered by the public firm. If the weight given to consumer surplus in weighted welfare is high enough, firms decide prices simultaneously.[4] When this weight takes an intermediate value we obtain a sequential equilibrium in which the public firm is the leader in prices and the private firm the follower. Finally, when the weight given to consumer surplus is low enough there are two sequential equilibria: the public firm is the leader in prices in one of them and the follower in the other.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes whether firms set prices sequentially or simultaneously, and conclusions are drawn in Section 4.

## 2. The model

We consider an economy comprising two firms that produce a heterogeneous good. One firm is publicly owned and the other is privately owned; we denote these firms by 0 and 1, respectively. On the consumption side, there is a continuum of consumers of the same type. The representative consumer maximizes $U(q_0, q_1) - p_0q_0 - p_1q_1$, where $q_i$ is the amount of the good $i$ and $p_i$ is its price ($i$ = 0, 1). The function $U(q_0, q_1)$ is assumed to be quadratic, strictly concave and symmetric in $q_0$ and $q_1$:

$$U(q_0, q_1) = a(q_0 + q_1) - \frac{1}{2}((q_0)^2 + 2bq_0q_1 + (q_1)^2),\ 1>b>0,$$[5]

where parameter $b$ measures the degree to which goods are substitutes. Demand functions are thus given by:

---

[4] This result is also obtained by Bárcena-Ruiz (2007) by assuming that the consumer and producer surpluses have the same weight in social welfare.

[5] We assume that $b$<1 to ensure that the function $U(q_0, q_1)$ is strictly concave (see Vives, 1984).

$$q_i = \frac{a(1-b) - p_i + bp_j}{1-b^2},\ i \neq j;\ \ i, j = 0, 1. \tag{1}$$

The marginal cost of production of both firms is $c$. The profit of firm $i$ is given by:

$$\pi_i = (p_i - c)\, q_i,\ i \neq j;\ i, j = 0, 1, \tag{2}$$

where $q_i$ is given by (1). The private firm chooses the price, $p_1$, that maximizes its profit. The public firm chooses the price, $p_0$, that maximizes the weighted sum of the consumer surplus ($CS$) and the producer surplus ($PS$):

$$W = \alpha\, CS + PS,\ \alpha_L < \alpha < \alpha_H, \tag{3}$$

where $\alpha_L = \dfrac{b^3 + 4b^2 - 4}{b(1+b)}$ and $\alpha_H = \dfrac{4 + 2b - b^2}{2(1+b)}$.[6] As usual, the producer surplus is the sum of the profits of the firms $PS = \pi_0 + \pi_1$ and the consumer surplus is given by:

$$CS = U(q_0, q_1) - p_0 q_0 - p_1 q_1 = \frac{2a(1-b)(a - p_0 + p_1) + (p_0)^2 - 2bp_0 p_1 + (p_1)^2}{2(1-b^2)}. \tag{4}$$

We consider the observable delay game of Hamilton and Slutsky (1990) in the context of a price-setting mixed duopoly where the firms choose whether to set prices at $t$=1 or at $t$=2. We propose a two-stage game with the following timing. In the first stage, the firms decide whether to set prices at $t$=1 or at $t$=2. If one firm sets its price at $t$=1 and the other at $t$=2, the game is sequential. If both firms choose their prices in the same period, firms take decisions simultaneously. In the second stage, the firms decide their prices either

[6] The assumption $\alpha_L < \alpha < \alpha_H$ ensures that the two firms produce a positive output: $\alpha > \alpha_L$ ensures that the public firm produces a positive output level when it is the leader in prices; $\alpha < \alpha_H$ ensures that the private firm produces a positive output level when it is the follower in prices. The proof is relegated to the appendix.

sequentially or simultaneously. To obtain a subgame perfect equilibrium, the game is solved backwards.

## 3. Results

Given that firms may decide prices sequentially or simultaneously, we have three cases. First, the public firm and the private firm decide prices simultaneously. Second, the public firm decides its price before the private firm does. Finally, the private firm decides its price before the public firm does.

We first solve the case in which the prices are set simultaneously, denoted by superscript *S*. The private firm chooses the value of $p_1$ that maximizes (2), for $i$=1 and $j$=0. The public firm chooses the value of $p_0$ that maximizes (3). Solving these two problems simultaneously, we obtain the reaction functions in prices of the two firms:

$$p_1 = \frac{1}{2}(a(1-b)+c+bp_0)\,, \tag{5}$$

$$p_0 = bp_1 + \frac{(1-b)(c+a(1-\alpha))}{2-\alpha}. \tag{6}$$

Given that $b$>0, prices are strategic complements ($\frac{\partial p_i}{\partial p_j} > 0$, $i \neq j$; $i,j$=0,1). Thus, if one firm raises (lowers) its price the other firm reacts by raising (lowering) its price too.

From (5) and (6) the following is obtained:

$$p_0^S = \frac{a(1-b)(2(1-\alpha)+b(2-\alpha))+c(2-b\alpha)}{(2-b^2)(2-\alpha)}\,,$$

$$p_1^S = \frac{a(1-b)(2-\alpha+b(1-\alpha))+c(2-\alpha+b(1-b))}{(2-b^2)(2-\alpha)}\,,$$

$$\pi_0^S = \frac{(1-b)(a-c)^2(2(1-\alpha)+b(2-\alpha))}{(1+b)(2-b^2)(2-\alpha)^2}\,,\ \pi_1^S = \frac{(1-b)(a-c)^2(2-\alpha+b(1-\alpha))^2}{(1+b)(2-b^2)^2(2-\alpha)^2}\,,$$

$$CS^S = \frac{(a-c)^2(8-4\alpha+\alpha^2+b(4-6\alpha+\alpha^2)-3b^2-b^3(1-2\alpha))}{2(1+b)(2-b^2)^2(2-\alpha)^2},$$

$$PS^S = \frac{(a-c)^2(1-b)(8-8\alpha+\alpha^2+2b(2-\alpha)^2-b^2(1-\alpha^2)-b^3(2-\alpha))}{(1+b)(2-b^2)^2(2-\alpha)^2},$$

$$W^S = \frac{(a-c)^2(8-\alpha^2+b\alpha(2-\alpha)-b^2(9-2\alpha)-b^3+2b^4)}{2(1+b)(2-b^2)^2(2-\alpha)}.$$

Next we consider the case in which the public firm decides its price before the private firm does. We denote the leader firm by superscript $L$ and the follower firm by superscript $F$. The private firm chooses the value of $p_1$ that maximizes (2) for $i$=1 and $j$=0. Solving this problem, we obtain equation (5). The public firm chooses the value of $p_0$ that maximizes (3) taking into account equation (5). Solving this problem, we get:

$$p_0^L = \frac{a(1-b)(4(1-\alpha)+b(4-3\alpha))+c(4-b\alpha-2b^2)}{(4-3b^2)(2-\alpha)},$$

$$p_1^F = \frac{a(1-b)(2(2-\alpha)+2b(1-\alpha)-b^2)+c(4-2\alpha+2b-b^2(3-\alpha)-b^3)}{(4-3b^2)(2-\alpha)},$$

$$\pi_0^L = \frac{(1-b)(a-c)^2(4+b\alpha-b^2(4-\alpha)-b^3)(4(1-\alpha)+b(4-3\alpha))}{(1+b)(4-3b^2)^2(2-\alpha)^2},$$

$$\pi_1^F = \frac{(1-b)(a-c)^2(2(2-\alpha)+2b(1-\alpha)-b^2)^2}{(1+b)(4-3b^2)^2(2-\alpha)^2},$$

$$PS^L = \frac{(1-b)(a-c)^2(8-8\alpha+\alpha^2+b(8-8\alpha+\alpha^2)+b^2(1-\alpha))}{(1+b)(4-3b^2)(2-\alpha)^2},$$

$$CS^L = \frac{(a-c)^2(8-4\alpha+\alpha^2+b(\alpha-2)^2-3b^2-b^3)}{2(1+b)(4-3b^2)(2-\alpha)^2},$$

$$W^L = \frac{(a-c)^2(8-\alpha^2+b(2-\alpha)\alpha-b^2(7-2\alpha)-b^3)}{2(1+b)(4-3b^2)(2-\alpha)}.$$

Finally, we consider the case in which the private firm decides its price before the public firm does. The public firm chooses the value of $p_0$ that maximizes (3). Solving this problem, we obtain equation (6). The private firm chooses the value of $p_1$ that maximizes (2) for $i$=1 and $j$=0, taking into account equation (6). Solving this problem, we get:

$$p_0^F = \frac{a(2(1-\alpha)+b(2-\alpha)-b^2(1-\alpha))+c(2+b(2-\alpha)+b^2(1-\alpha))}{2(1+b)(2-\alpha)},$$

$$p_1^L = \frac{a(2-\alpha+b(1-\alpha))+c(2-\alpha+b(3-\alpha))}{2(1+b)(2-\alpha)},$$

$$\pi_1^L = \frac{(a-c)^2(2-\alpha+b(1-\alpha))^2}{4(1+b)^2(2-\alpha)^2},\ \pi_0^F = \frac{(a-c)^2(2(1-\alpha)+b(2-\alpha)-b^2(1-\alpha))}{2(1+b)^2(2-\alpha)^2},$$

$$CS^F = \frac{(a-c)^2(8-4\alpha+\alpha^2+2b(6-5\alpha+\alpha^2)+b^2(5-6\alpha+\alpha^2))}{8(1+b)^2(2-\alpha)^2},$$

$$PS^F = \frac{(a-c)^2(8-8\alpha+\alpha^2+2b(2-\alpha)^2-b^2(1-\alpha^2)}{4(1+b)^2(2-\alpha)^2},$$

$$W^F = \frac{(a-c)^2(8-\alpha^2+2b(4+\alpha-\alpha^2)-b^2(1-\alpha)^2)}{8(1+b)^2(2-\alpha)}.$$

Let $\alpha_1 = b$. From the results obtained in the three cases, the following result is obtained.

**Proposition 1.** *In equilibrium:*

*i) If* $\alpha \geq \alpha_1$*:* $p_0^S \geq p_0^L$, $p_0^F > p_0^S$, $p_1^L > p_1^S$, $p_1^S \geq p_1^F$, $PS^S \geq PS^L$, $PS^F > PS^S$, $CS^L \geq CS^S$, $CS^S > CS^F$;

*ii) If* $\alpha < \alpha_1$*:* $p_0^L > p_0^S$, $p_0^F > p_0^S$, $p_1^L > p_1^S$, $p_1^F > p_1^S$, $PS^L > PS^S$, $PS^F > PS^S$, $CS^S > CS^F$, $CS^S > CS^L$.

**Proof**. See Appendix.

We first analyze the case in which the weight given to the consumer surplus in the objective function of the public firm is great enough ($\alpha \geq \alpha_1$). In this case, when the private firm is the leader in prices it sets a higher price than in the simultaneous case since, as prices are strategic complements, this firm knows that the follower (the public firm) will also set a greater price. The private firm wants to increase prices to reduce market competition in order to increase profits.[7] When the public firm is the leader in prices it sets a price lower than or equal to that of the simultaneous case since, as prices are strategic complements, this firm knows that the follower (the private firm) will also set a lower price. The public firm wants to reduce prices in order to increase market competition and thus consumer surplus. As a result, when the private firm is the leader (follower) it sets a higher (lower or equal) price than in the simultaneous case: $p_1^L > p_1^S$ ($p_1^S \geq p_1^F$); when the public firm is the follower (leader) it sets a higher (lower or equal) price than in the simultaneous case: $p_0^F > p_0^S$ ($p_0^S \geq p_0^L$). Market competition is greater when the public firm is the leader than in the simultaneous case because both firms set lower or equal prices in the first case. Thus, the consumer surplus is greater or equal and the producer surplus is lower or equal in the first case: $CS^L \geq CS^S$ and $PS^S \geq PS^L$. By contrast, as market competition is lower when the private firm is the leader than in the simultaneous case, the consumer surplus is lower and the producer surplus is greater in the first case: $CS^S > CS^F$ and $PS^F > PS^S$.

When the weight given to the consumer surplus in the objective function of the public firm is low enough ($\alpha < \alpha_1$), the incentive of that firm to behave aggressively in the product market is lower than when $\alpha \geq \alpha_1$. As in the preceding case, when the private firm is the leader in prices it sets a higher price than in the simultaneous case since, as prices are strategic complements, the public firm also sets a higher price, thus reducing market competition: $p_1^L > p_1^S$ and $p_0^F > p_0^S$. When the public firm is the leader in prices it sets a higher price than in the simultaneous case since, as the weight of the consumer surplus is low enough, the profit of the firms (the producer surplus) has a greater effect on the objective function of the public firm than the consumer surplus. Thus, in this case the public firm seeks to reduce market competition in comparison with the simultaneous case which means that both firms set greater prices: $p_0^L > p_0^S$ and $p_1^F > p_1^S$. Under sequential decisions, and independently of which firm is the leader, market competition is lower than when decisions

[7] In a private duopoly, when prices are set sequentially both the leader and the follower set higher prices than when prices are set simultaneously (see, Gal-Or, 1985; Dowrick, 1986; Hamilton and Slutsky, 1990). This is due to the fact that reaction functions in prices are upward sloping and, thus, if one firm raises its price the other firm reacts by raising its price too.

are taken simultaneously since prices are lower in the first case. As a result, the consumer surplus is greater and the producer surplus is lower under sequential decisions than under simultaneous decisions: $CS^S>CS^L$, $CS^S>CS^F$, $PS^L>PS^S$, and $PS^F>PS^S$.

Next we compare the weighted welfare and the profit of the private firm obtained in the sequential and simultaneous cases. We denote as Zone *I* the values of parameter $\alpha$ such that $\alpha_H > \alpha \geq \alpha_1$, as Zone *II* the values of parameter $\alpha$ such that $\alpha_1 > \alpha \geq \alpha_2$ and, finally, as Zone *III* the values of parameter $\alpha$ such that $\alpha_2 > \alpha > \alpha_L$, where $\alpha_2 = \dfrac{b(4+2b-b^2)}{(1+b)(4-b^2)}$. Zones *I* to *III* are shown in Figure 1.

**Proposition 2.** *In equilibrium:*

*i)* $W^L \geq W^S$, $W^S > W^F$, $\pi_1^L > \pi_1^S$ *and* $\pi_1^S \geq \pi_1^F$ *if* $\alpha_H>\alpha\geq\alpha_1$;

*ii)* $W^L > W^S$, $W^S \geq W^F$, $\pi_1^L > \pi_1^S$ *and* $\pi_1^F > \pi_1^S$ *if* $\alpha_1>\alpha\geq\alpha_2$;

*iii)* $W^L > W^S$, $W^F > W^S$, $\pi_1^L > \pi_1^S$ *and* $\pi_1^F > \pi_1^S$ *if* $\alpha_2>\alpha>\alpha_L$.

**Proof**. See Appendix.

[INSERT FIGURE 1 AROUND HERE]

We consider first the case in which the weight given to the consumer surplus is great enough: $\alpha_H>\alpha\geq\alpha_1$ (Zone *I* in Figure 1). Proposition 1 shows that, in this zone, when the private firm is the leader in prices both firms set a higher price than in the simultaneous case. This reduces market competition and increases the profit of the private firm ($\pi_1^L > \pi_1^S$). By contrast, when the public firm is the leader in prices, in this zone both firms set a lower price than in the simultaneous case. This increases market competition and reduces the profit of the private firm ($\pi_1^S \geq \pi_1^F$). Proposition 1 also shows that consumer surplus when the public firm is the leader is greater or equal to that of the simultaneous case ($CS^L\geq CS^S$), and in this last case is greater than if the public firm is the follower ($CS^S>CS^F$). Moreover, the producer surplus is greater when the public firm is the follower than in the simultaneous case

($PS^F$>$PS^S$) and in this last case is greater than or equal to the level if the public firm is the leader ($PS^S$≥$PS^L$). Given that in this zone $\alpha$ is great enough ($\alpha \geq \alpha_1$), the consumer surplus has a great enough weight in the objective function of the public firm. As a result, when that firm is the leader the greater consumer surplus has a stronger effect on weighted welfare than the lower producer surplus, implying that $W^L$>$W^S$. Moreover, when the public firm is the follower the lower consumer surplus has a stronger effect on weighted welfare than the greater producer surplus, implying that $W^S$>$W^F$.

Next we consider the case in which the weight given to the consumer surplus is low enough: $\alpha_2 > \alpha > \alpha_L$ (Zone *III* in Figure 1). Proposition 1 shows that, independently of which firm is the leader in prices, in this zone both firms set higher prices than in the simultaneous case. This reduces market competition and increases the profit of the private firm: $\pi_1^F > \pi_1^S$ and $\pi_1^L > \pi_1^S$. Proposition 1 also shows that the consumer surplus is greater and the producer surplus is lower in the simultaneous case than in the sequential cases: $CS^S$>$CS^L$, $CS^S$>$CS^F$, $PS^L$>$PS^S$, and $PS^F$>$PS^S$. Given that in this zone $\alpha$ is low enough ($\alpha < \alpha_2$), the consumer surplus has a low enough weight in weighted welfare. This means that, independently of which firm is the leader in prices, the lower consumer surplus has a lower effect than the greater producer surplus, implying that $W^L$>$W^S$ and $W^F$>$W^S$.

Finally, we consider the case in which the weight given to the consumer surplus takes an intermediate value: $\alpha_1 > \alpha \geq \alpha_2$ (Zone *II* in Figure 1). Proposition 1 shows that in the sequential cases, independently of which firm is the leader, both firms set higher prices than in the simultaneous case. This reduces market competition and increases the profit of the private firm: $\pi_1^F > \pi_1^S$ and $\pi_1^L > \pi_1^S$. Proposition 1 also shows that, in this zone, the consumer surplus is greater and the producer surplus is lower in the simultaneous case than in the sequential cases: $CS^S$>$CS^L$, $CS^S$>$CS^F$, $PS^L$>$PS^S$, and $PS^F$>$PS^S$. In this zone $\alpha$ takes an intermediate value ($\alpha_1 > \alpha \geq \alpha_2$) and the consumer (producer) surplus has a greater effect on weighted welfare when the public firm is the follower (leader). As a result, when the public firm is the leader in prices, the fact that the greater producer surplus has a greater effect on weighted welfare than the lower consumer surplus means that $W^L$>$W^S$. When the public firm is the follower in prices, the fact that the lower consumer surplus has a greater effect on weighted welfare than the greater producer surplus means that $W^S \geq W^F$.

Taking into account proposition 2, the following result is obtained.

***Proposition 3.*** *In equilibrium, firms decide prices simultaneously at t=1 if* $\alpha_H > \alpha \geq \alpha_1$. *For the remaining values of parameter* $\alpha$ *firms decide prices sequentially. In this case, the public firm set prices at t=1 and the private firm at t=2 if* $\alpha_1 > \alpha \geq \alpha_2$. *However, if* $\alpha_2 > \alpha > \alpha_L$ *there are two sequential equilibria: the public firm is the leader in prices in one of them and the follower in the other.*

From Proposition 2 it is obtained that if $\alpha_H > \alpha \geq \alpha_1$ both firms want to be the leader in prices and neither firm wants to be the follower ($\pi_1^L > \pi_1^S$, $\pi_1^S > \pi_1^F$, $W^L > W^F$, and $W^S > W^L$). Thus, in equilibrium they set prices simultaneously at t=1. In this zone, it is a dominant strategy for both firms to set prices at *t*=1.

If $\alpha_1 > \alpha \geq \alpha_2$, the public firm wants to be the leader in prices but it does not want to be the follower ($W^F > W^S$ and $W^S > W^F$). Moreover, the private firm obtains greater profit when prices are set sequentially rather than simultaneously ($\pi_1^F > \pi_1^S$ and $\pi_1^L > \pi_1^S$). As a result, in this zone the public firm sets prices at *t*=1 and the private firm at *t*=2. In this way, the public firm becomes the leader and prices are set sequentially.

Finally, if $\alpha_2 > \alpha > \alpha_L$, the two firms prefer to set prices sequentially rather than simultaneously ($\pi_1^F > \pi_1^S$, $\pi_1^L > \pi_1^S$, $W^L > W^S$ and $W^F > W^S$). This implies that there are two sequential equilibria: the public firm is the leader in prices in one of them and the follower in the other.

## 4. Conclusions

This paper studies a setting where firms decide about prices and the objective function of the public firm is the weighted sum of consumer and producer surpluses. A mixed duopoly is considered where the firms decide the timing of their choice of prices. It is shown that whether firms take decisions sequentially or simultaneously depends crucially on the objective function considered by the public firm. If the weight given to the consumer surplus in weighted welfare is high enough, firms decide prices simultaneously. However, when that weight is low enough firms take decisions sequentially.

*Appendix*

It can be shown that the output level of the firms in the different cases considered is:

$$q_0^S = \frac{a-c}{(1+b)(2-\alpha)},\ q_1^S = \frac{(a-c)(2-\alpha+b(1-\alpha))}{(1+b)(2-b^2)(2-\alpha)},\ q_0^L = \frac{(a-c)(4+b\alpha-b^2(4-\alpha)-b^3)}{(4-3b^2)(1+b)(2-\alpha)},$$

$$q_1^F = \frac{(a-c)(4-2\alpha+2b(1-\alpha)-b^2)}{(4-3b^2)(1+b)(2-\alpha)},\ q_0^F = \frac{a-c}{(1+b)(2-\alpha)},\ q_1^L = \frac{(a-c)(2-\alpha+b(1-\alpha))}{2(1+b)(2-\alpha)}.$$

It is easy to see that $q_0^L > 0$ if $\alpha > \alpha_L = \frac{b^3+4b^2-4}{b(1+b)}$. Similarly, $q_1^F > 0$ if $\alpha > \alpha_H = \frac{4+2b-b^2}{2(1+b)}$. Moreover, $q_0^S$, $q_1^S$, $q_0^F$ and $q_1^L$ are positive for $\alpha_L < \alpha < \alpha_H$.

**Proof of proposition 1.**

By comparing the prices obtained in the different cases we obtain:

$$p_0^S - p_0^L = \frac{2b(1-b^2)(a-c)(\alpha-b)}{(2-b^2)(4-3b^2)(2-\alpha)} \geq 0 \text{ if and only if } \alpha \geq \alpha_1,$$

$$p_0^F - p_0^S = \frac{b^3(a-c)(2-\alpha+b(1-\alpha))}{2(1+b)(2-b^2)(2-\alpha)} > 0 \text{ for all } (\alpha, b),$$

$$p_1^L - p_1^S = \frac{b^2(a-c)(2-\alpha+b(1-\alpha))}{2(1+b)(2-b^2)(2-\alpha)} > 0 \text{ for all } (\alpha, b),$$

$$p_1^S - p_1^F = \frac{b^2(a-c)(1-b^2)(\alpha-b)}{(2-b^2)(4-3b^2)(2-\alpha)} \geq 0 \text{ if and only if } \alpha \geq \alpha_1.$$

By comparing the consumer and producer surpluses obtained in the different cases we obtain:

$$PS^S\text{-}PS^L = \frac{(1-b)b(a-c)^2(\alpha-b)((4+b)\alpha+b^2(1-3\alpha)+b^3(1-\alpha))}{(2-b^2)^2(4-3b^2)(2-\alpha)^2} \geq 0 \text{ if and only if } \alpha\geq\alpha_1,$$

$$PS^F\text{-}PS^S = \frac{b^3(a-c)^2(2+b(1-\alpha)-\alpha)(4+b(2-\alpha)-b^2(1+\alpha))}{4(1+b)^2(2-b^2)^2(2-\alpha)^2} > 0 \text{ for all } (\alpha, b),$$

$$CS^L\text{-}CS^S = \frac{(1-b)b(a-c)^2(\alpha-b)(8+b(4-\alpha)-b^2(3+\alpha)-b^3)}{2(2-b^2)^2(4-3b^2)(2-\alpha)^2} \geq 0 \text{ if and only if } \alpha\geq\alpha_1,$$

$$CS^S\text{-}CS^F = \frac{b^2(a-c)^2(2+b(1-\alpha)-\alpha)(8-4\alpha+4b(3-\alpha)-b^2(2-\alpha)-b^3(5-\alpha))}{8(1+b)^2(2-b^2)^2(2-\alpha)^2} > 0 \text{ for all } (\alpha, b).$$

**Proof of proposition 2.**

$$W^L\text{-}W^S = \frac{b^2(1-b^2)(a-c)^2(\alpha-b)^2}{2(2-b^2)^2(4-3b^2)(2-\alpha)} > 0 \text{ for } \alpha > \alpha_1 \text{ and } 0 \text{ for } \alpha = \alpha_1.$$

$$W^S\text{-}W^F = \frac{b^2(a-c)^2(2+b(1-\alpha)-\alpha)(4\alpha-(4b-b^3)(1-\alpha)-b^2(2-\alpha))}{8(1+b)^2(2-b^2)^2(2-\alpha)} \geq 0 \text{ if and only if } \alpha\geq\alpha_2,$$

$$\pi_1^S - \pi_1^F = \frac{(1-b)b^2(a-c)^2(\alpha-b)(8(2-\alpha)+8b(1-\alpha)-b^2(12-5\alpha)-5b^3(1-\alpha)+b^4)}{(2-b^2)^2(4-3b^2)^2(2-\alpha)^2} \geq 0$$

if and only if $\alpha \geq \alpha_1$,

$$\pi_1^L - \pi_1^S = \frac{b^4(a-c)^2(2+b(1-\alpha)-\alpha)^2}{4(1+b)^2(2-b^2)^2(2-\alpha)^2} > 0 \text{ for all } (\alpha, b).$$

*Acknowledgments*

Financial support from Departamento de Educación, Universidades e Investigación del Gobierno Vasco (IT-223-07) and Ministerio de Ciencia y Tecnología and FEDER (SEJ2006-05596) is gratefully acknowledged.

*References*

Anderson, S., de Palma, A. and J-F. Thies (1997), "Privatization and efficiency in a differentiated industry", *European Economic Review* 41, pp. 1635-1654.

Bárcena-Ruiz, J.C. (2007), "Endogenous timing in a mixed duopoly: price competition", *Journal of Economics* 91, pp. 263-272.

Bárcena-Ruiz, J.C. (2008), "The decision to hire managers in mixed markets under Bertrand competition", *Japanese Economic Review*, forthcoming.

Bárcena-Ruiz, J.C. and M.B. Garzón (2008), "Endogenous timing in a mixed oligopoly with semipublic firms", Mimeo, University of the Basque Country.

Dowrick, S. (1986), "Von Stackelberg and Cournot duopoly: Choosing roles", RAND *Journal of Economics* 17, pp. 251-260.

Gal-Or, E. (1985), "First mover and second mover advantages", *International Economic Review* 26, pp. 279-292.

Hamilton, J.H. and S.M. Slutsky (1990), "Endogenous timing in duopoly games: Stackelberg or Cournot equilibria", *Games and Economic Behavior* 2, pp. 29-46.

Harsanyi, J.C. (1995), "Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility", Journal of Political Economy 63, pp. 309-321.

Lu, Y. (2006), "Endogenous timing in a mixed oligopoly with foreign competitors: the linear demand case", *Journal of Economics* 88, pp. 49-68.

Matsumura, T. (2003), "Stackelberg mixed duopoly with a foreign competitor", *Bulletin of Economic Research* 55, pp. 275-87.

Matsumura, T. and N. Matsushima (2003), "Mixed duopoly with product differentiation: Sequential choice of location", *Australian Economic Papers* 1, 18-34.

Ogawa, A. and K. Kato (2006), "Price competition in a mixed duopoly", *Economics Bulletin* 12, pp. 4-15.

Pal, D. (1998), “Endogenous timing in a mixed oligopoly”, *Economics Letters* 61, pp. 181-185.

Tasnádi, A. (2006), “Price vs. quantity in oligopoly games” I*nternational Journal of Industrial Organization* 24, pp. 541-554.

Vives, X. (1984), “Duopoly information equilibrium: Cournot and Bertrand”, *Journal of Economic Theory* 34, pp. 71-94.

White, M.D. (2002), “Political manipulation of a public firm's objective function”, *Journal of Economic Behavior and Organization* 49, pp. 487-499.

Figure 1

Illustration of Proposition 2